Factorization of Weak Annihilation Amplitudes in Nonleptonic *B*-Meson Decays

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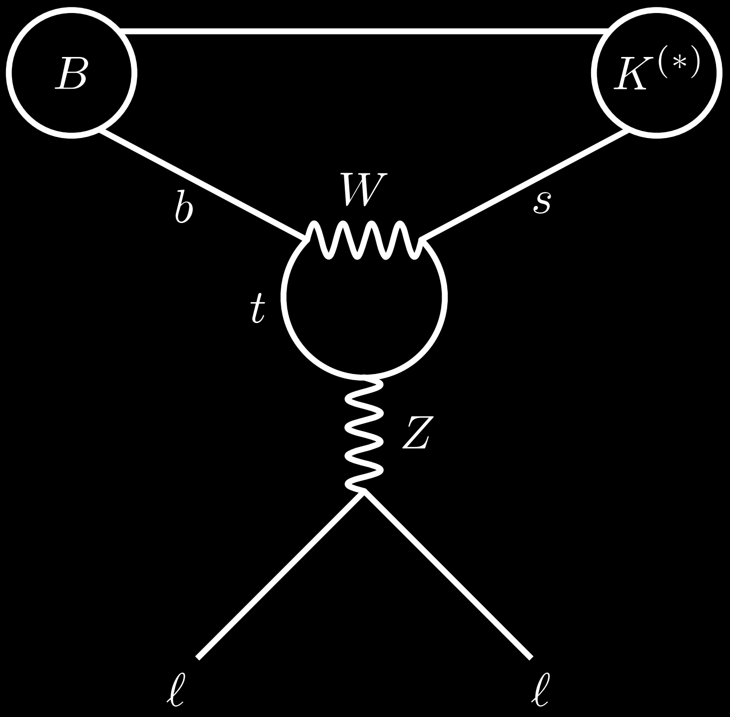
B-anomalies: $b \to s\,\ell^+\ell^-$

- lepton universality parameters

$K^{*0} \equiv K^*(892)^0$

$$R_{K,K^*} = \frac{\Gamma(B^{(\pm,0)} \to K^{(\pm,*0)}\mu^+\mu^-)}{\Gamma(B^{(\pm,0)} \to K^{(\pm,*0)}e^+e^-)}$$

- SM prediction: $R_K = R_{K^*} = 1 + \mathcal{O}(m_e/m_\mu)$



$$Q_{9V} = (\bar{s}\,b)_{V-A} \sum_\ell (\bar{\ell}\,\ell)_V$$

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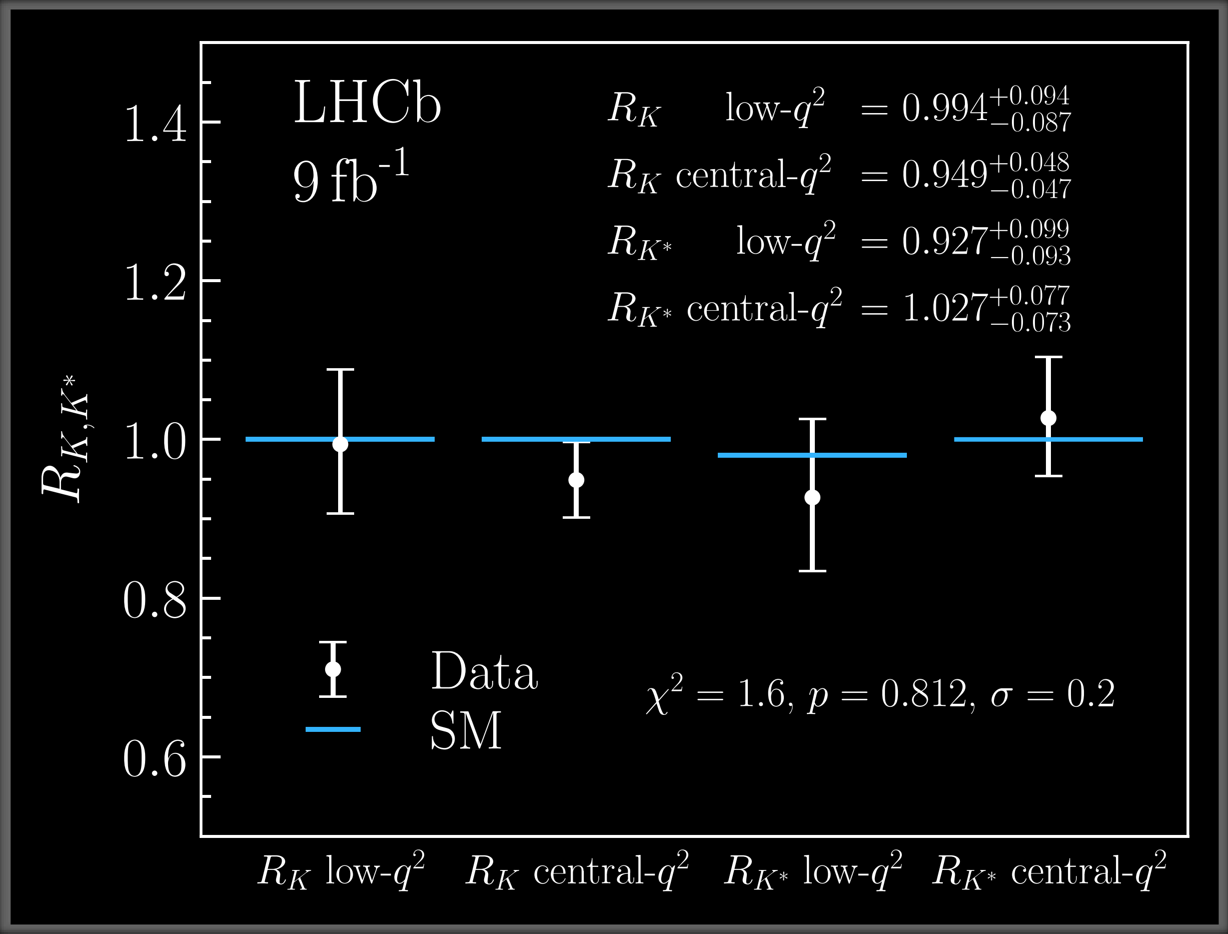
- SM prediction: $R_K = R_{K^*} = 1$

- latest LHCb measurement from 2022
  - **agrees** with SM prediction [LHCb 2022]



B-anomalies: $b \to s\,\ell^+\ell^-$

- angular distribution in $B^0 \to K^{*0}\mu^+\mu^-$
- branching fraction of $B_s^0 \to \phi\,\mu^+\mu^-$

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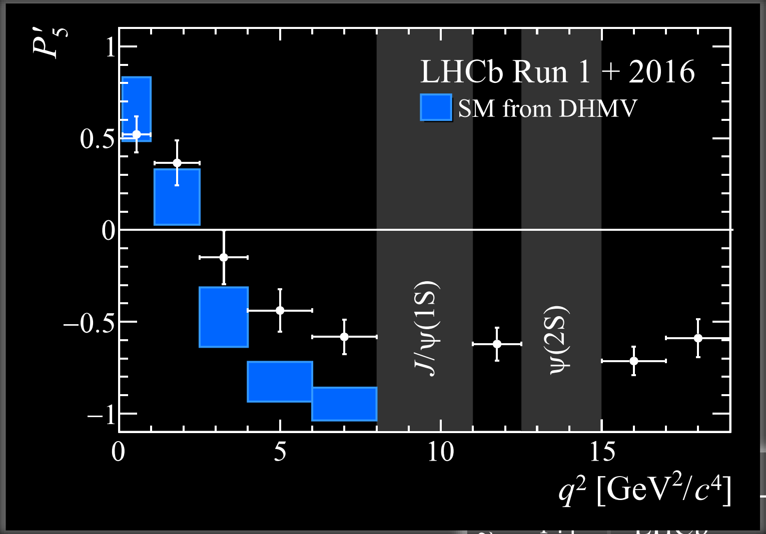
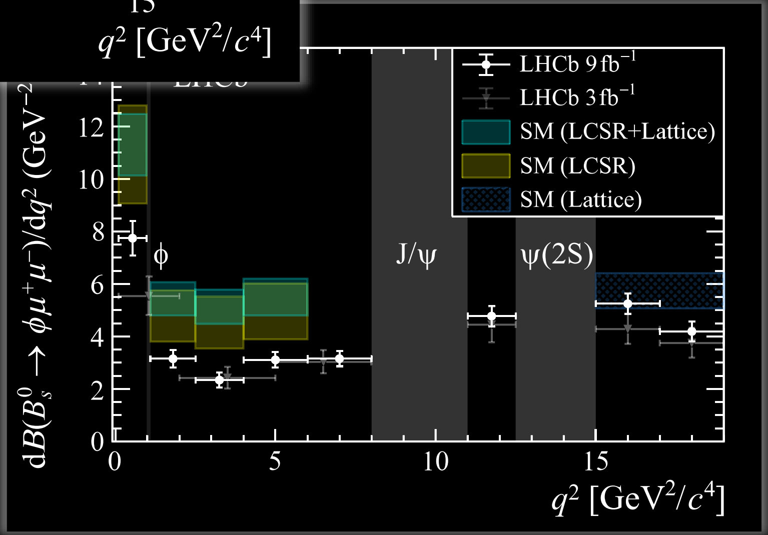
- LHCb measurements
  - $> \mathbf{2\sigma}$ deviation [LHCb 2020]
  - $> \mathbf{3\sigma}$ deviation [LHCb 2021]



$B^0 \to K^{*0}\mu^+\mu^-$



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- angular distribution in $B^0 \to K^{*0}\mu^+\mu^-$
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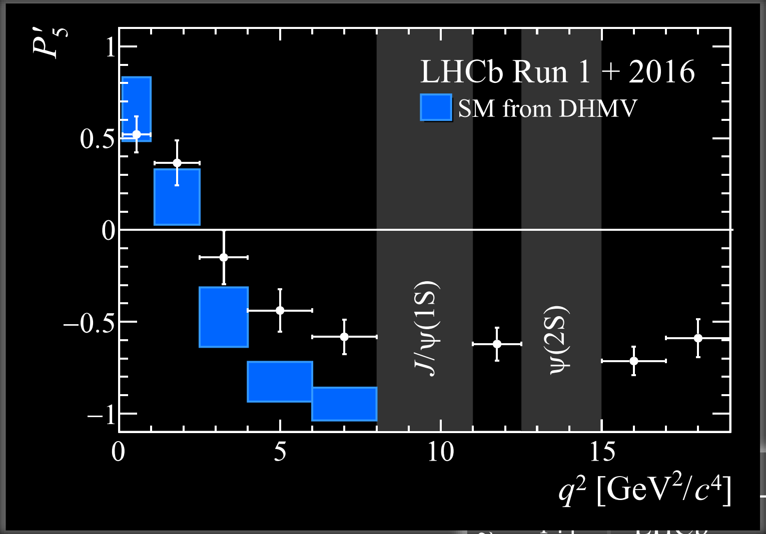
- LHCb measurements
  - $> \mathbf{2\sigma}$ deviation [LHCb 2020]
  - $> \mathbf{3\sigma}$ deviation [LHCb 2021]

- possible explanation: shift in Wilson coefficient C_{9V}



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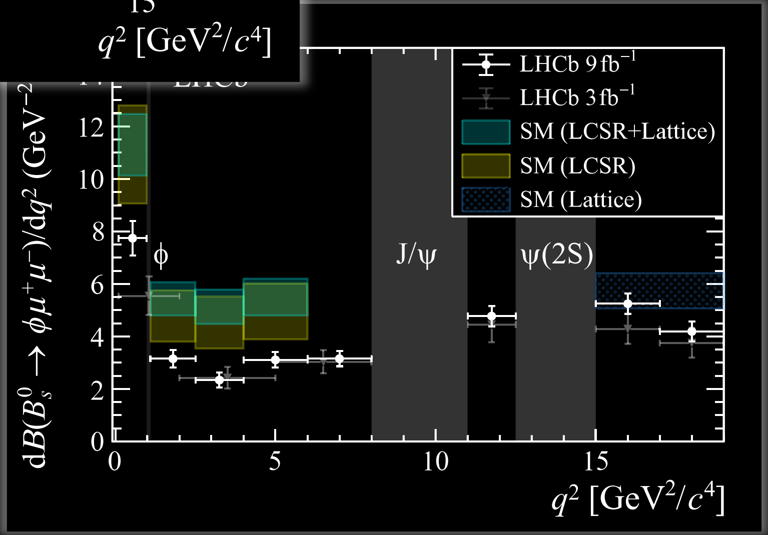
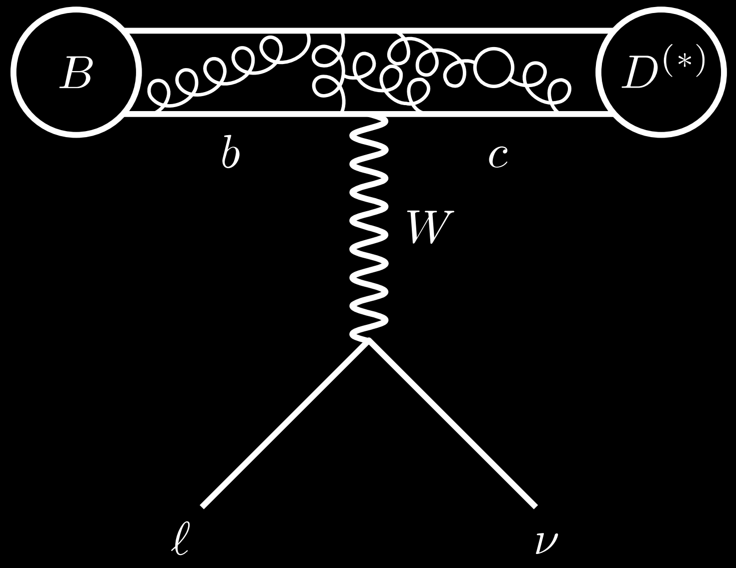


B-anomalies: $b \to c\,\ell\,\nu$

- branching fractions of semitauonic decays

$$R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)}\tau\,\nu_\tau)}{\Gamma(B \to D^{(*)}\mu\,\nu_\mu)}$$



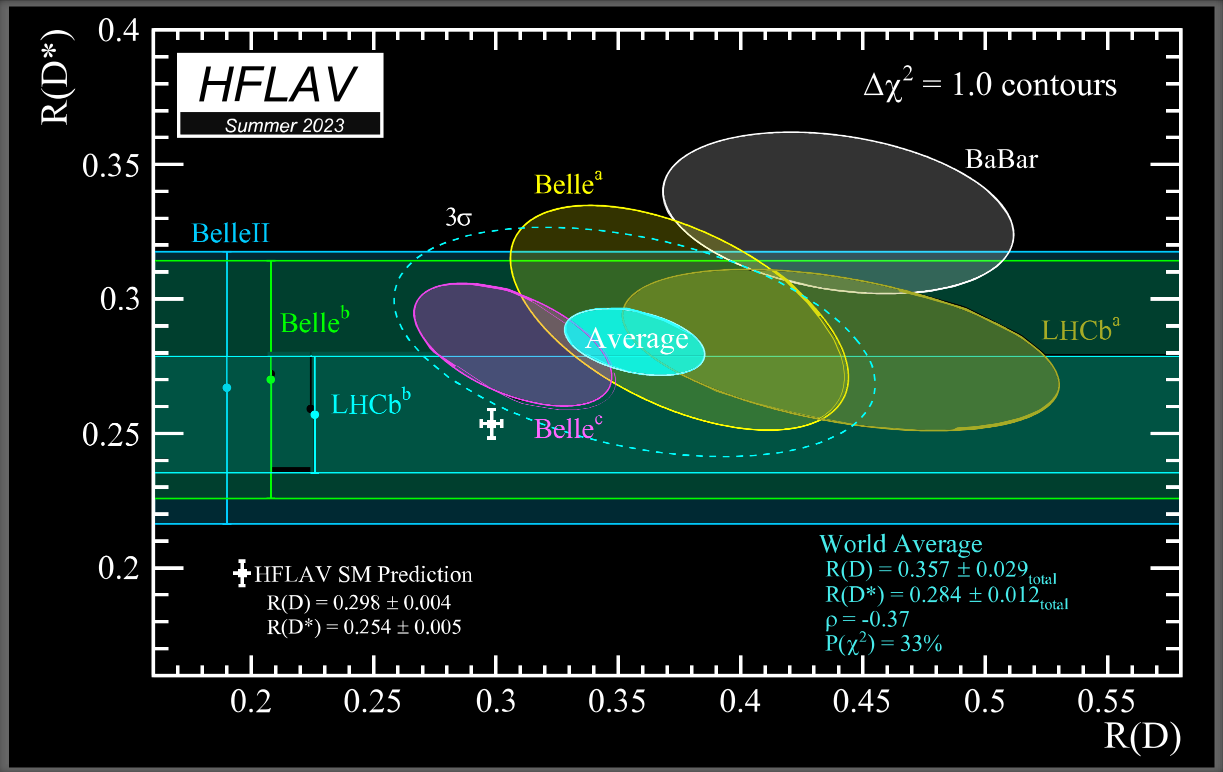
$$Q^q_{sl} = (\bar{q}\,b)_{V-A}\sum_\ell (\bar{\ell}\,\nu_\ell)_{V-A}$$

B-anomalies: $b \to c\, \ell\, \nu$

- branching fractions of semitauonic decays

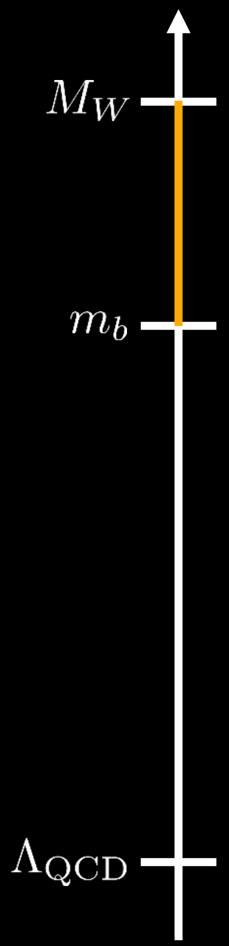
$$R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)} \tau\, \nu_\tau)}{\Gamma(B \to D^{(*)} \mu\, \nu_\mu)}$$

- HFLAV results from 2023
  - **$> 3\sigma$** deviation [HFLAV 2023]



Weak effective Hamiltonian [Buchalla *et al.* 1995]

- integrate out "heavy" physics at scale $\mu \sim M_W$
- obtain effective operators describing low-energy physics



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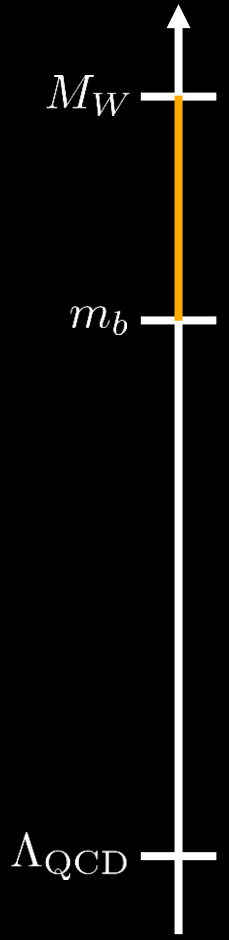
- Example: $\Delta B = 1$

$C_i(\mu)\, Q_i(\mu)$

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qd}^* \Big[C_1\, Q_1^q + C_2\, Q_2^q + \sum_{i=3}^{10} C_i\, Q_i + C_{7\gamma}\, Q_{7\gamma} + C_{8g}\, Q_{8g} + C_{9V}\, Q_{9V} + C_{10A}\, Q_{10A} \Big] + \text{h.c.}$$

$$+ \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}\, C_{sl}\, Q_{sl}^q + \text{h.c.}$$



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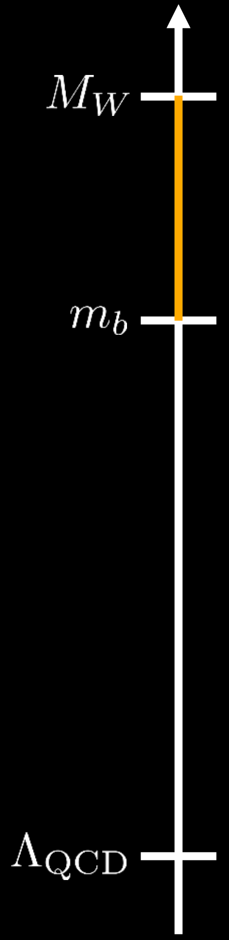
$$+ C_{8g}\, Q_{8g} + \cancel{C_{9V}\, Q_{9V}} + \cancel{C_{10A}\, Q_{10A}} \Big] + \text{h.c.}$$

$$\cancel{+ \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb}\, C_{sl}\, Q_{sl}^q} + \text{h.c.}$$

weak annihilation

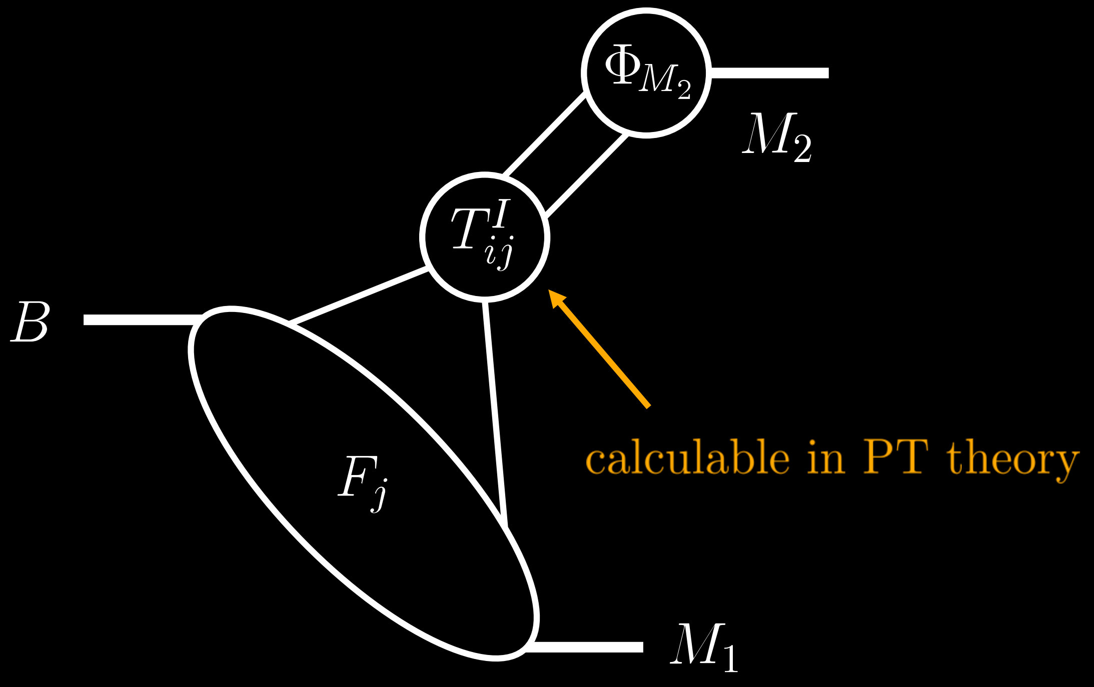


QCD Factorization [BBNS 2000]

- amplitude factorization for $B \to M_1\, M_2$ with M_1, M_2 both light

$$\langle M_1 M_2 | Q_i | B \rangle = \sum_j F_j^{B\to M_1}(m_2^2) \int_0^1 dx\, T_{ij}^I(x)\, \Phi_{M_2}(x)$$

$$+ (M_1 \leftrightarrow M_2)$$



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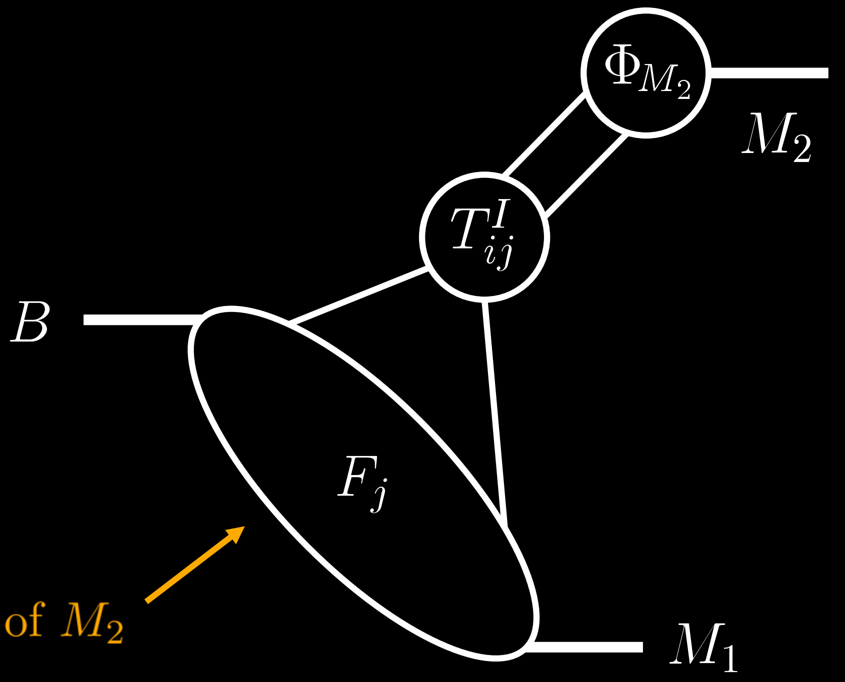
form factor



independent of M_2

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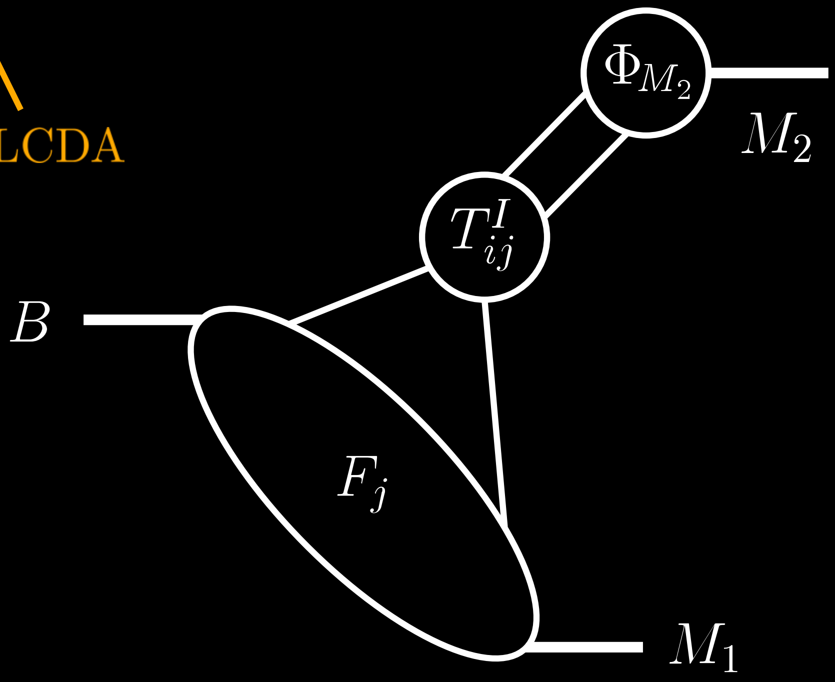
LCDA

process independent



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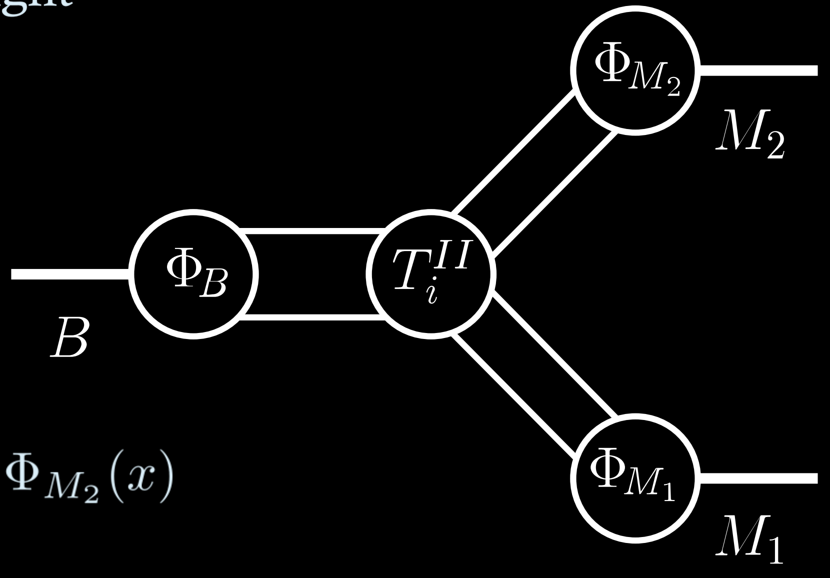
$$+ (M_1 \leftrightarrow M_2)$$

$$+ \int_0^1 d\xi\, dx\, dy\, T_i^{II}(\xi, x, y)\, \Phi_B(\xi)\, \Phi_{M_1}(y)\, \Phi_{M_2}(x)$$

LCDAs



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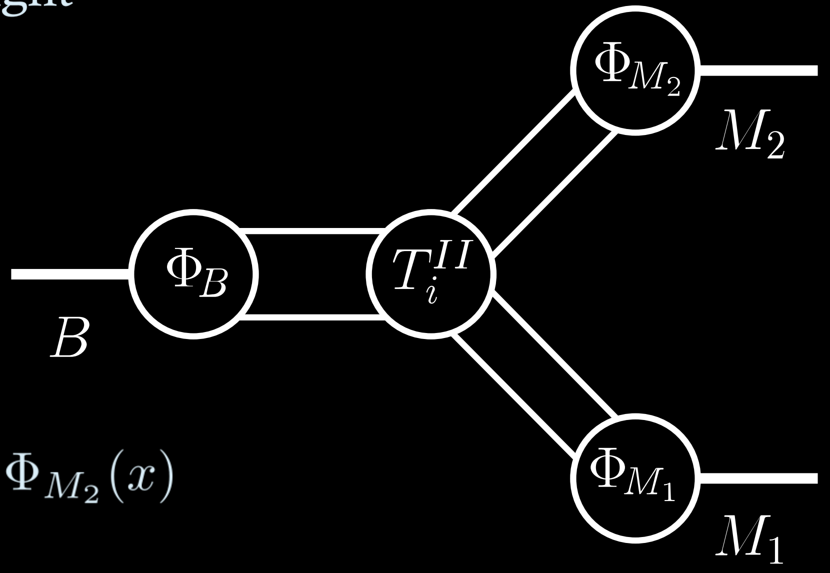


leading power in $\Lambda_{\rm QCD}/m_b$

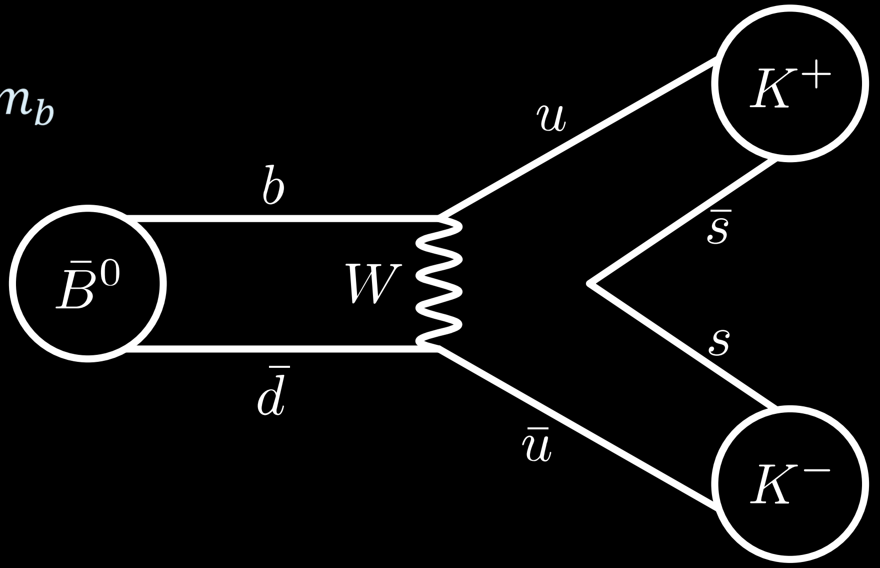
LCDAs

Weak Annihilation [BBNS 2001]

- effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qb} V_{qd}^* \left[C_1\, Q_1^q + C_2\, Q_2^q + \sum_{i=3}^{6} C_i\, Q_i + C_{8g}\, Q_{8g} \right] + \text{h.c.}$$

- contains contributions that are subleading in $\lambda \equiv \Lambda_{\text{QCD}}/m_b$



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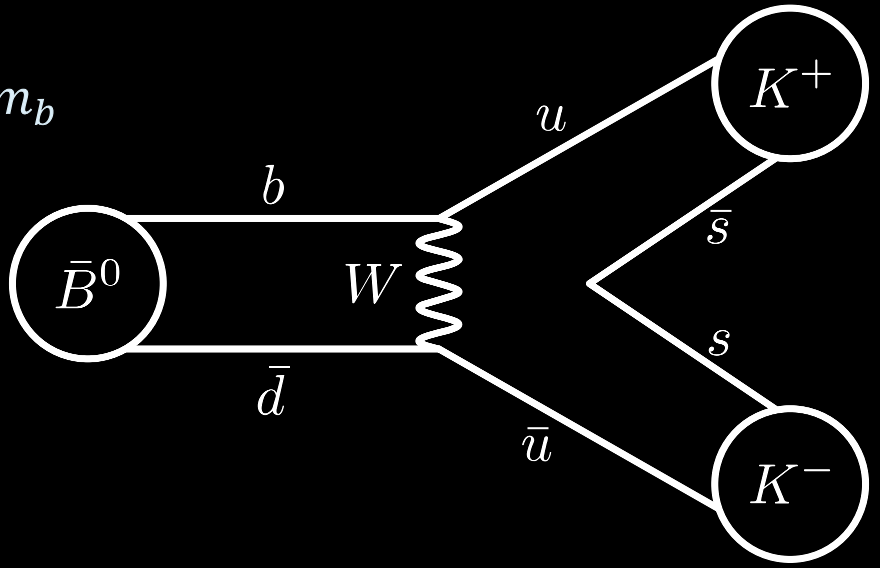
- **not** included in previous factorization formula

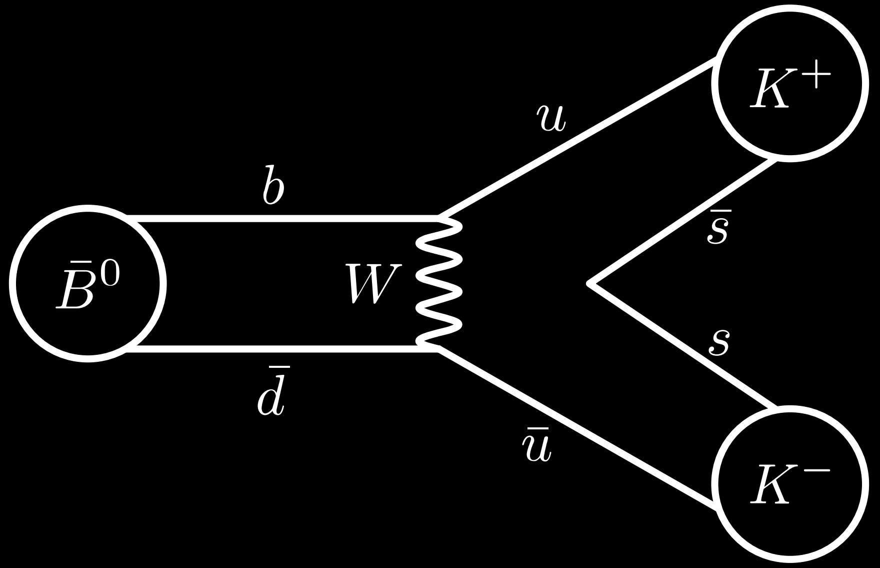


Weak Annihilation [BBNS 2001]

- leading order contribution

$$A_1^i = \pi\alpha_s \int_0^1 dx\, dy \left\{ \Phi_{M_1}(y)\, \Phi_{M_2}(x) \left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y} \right] + \frac{4\mu_{M_1}\mu_{M_2}}{m_b^2} \frac{2}{\bar{x}y} \right\}$$

$\bar{x} \equiv 1 - x$
$\bar{y} \equiv 1 - y$



Weak Annihilation [BBNS 2001]

- leading order contribution

$\Phi_{M_2}(x) \sim 6x\bar{x}$

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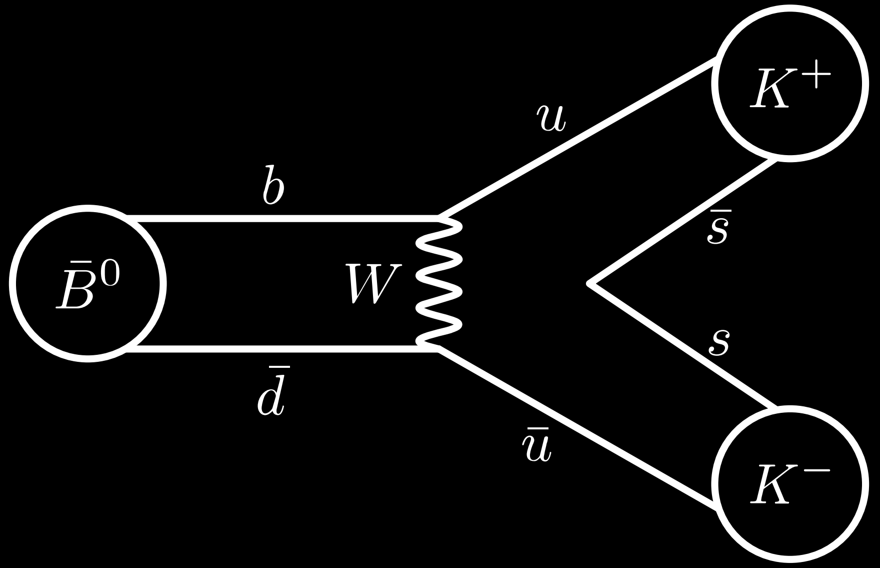
- endpoint divergent integral for $x \to 1$



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$\bar{x} \equiv 1 - x$
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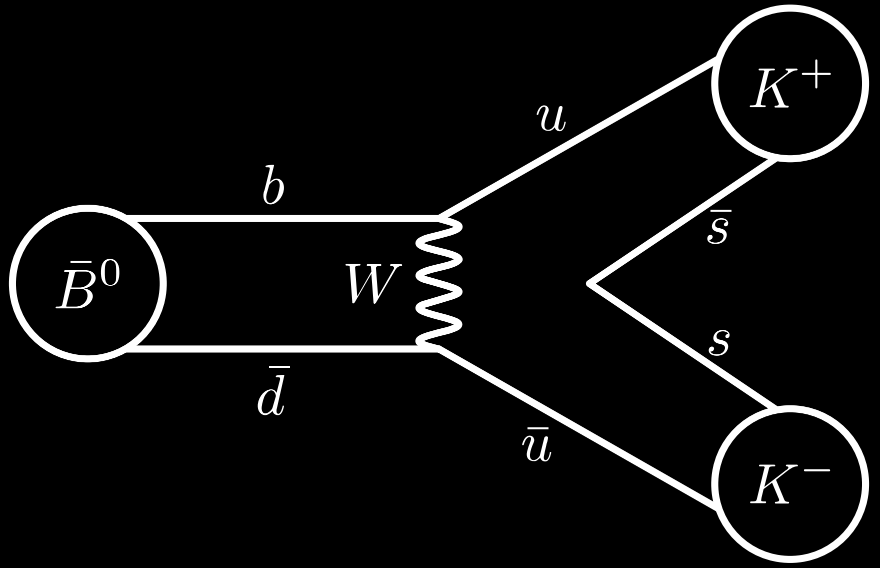
- endpoint divergent integral for $x \to 1$
- similar contribution divergent for $y \to 0$
- **How to deal with these endpoint divergences?**



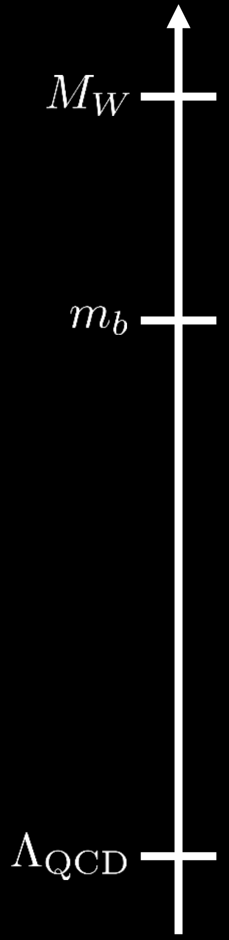
Soft-Collinear Effective Theory (SCET)

- momenta in B-Meson CMS

$$n^\mu = (1,0,0,1) \qquad n\cdot\bar{n} = 2 \qquad p_{K^-} = m_B\, n$$

$$\bar{n}^\mu = (1,0,0,-1) \qquad n^2 = \bar{n}^2 = 0 \qquad p_{K^+} = m_B\, \bar{n}$$

$$v^\mu = (1,0,0,0) \qquad n+\bar{n} = 2v \qquad p_B = m_B\, v$$



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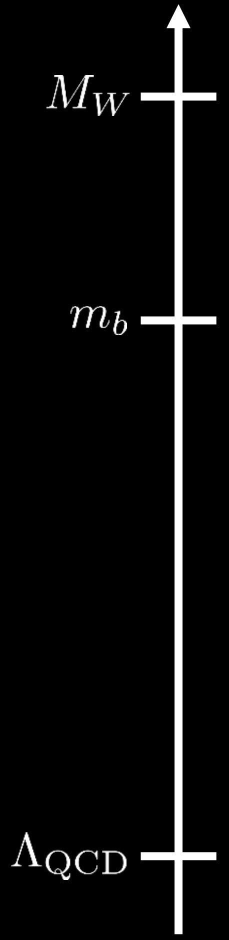
$$v^\mu = (1,0,0,0) \qquad n + \bar{n} = 2v \qquad p_B = m_B\, v$$

- decompose momenta

$$p^\mu = (n\cdot p)\,\frac{\bar{n}^\mu}{2} + (\bar{n}\cdot p)\,\frac{n^\mu}{2} + p_\perp^\mu \qquad p \sim (n\cdot p, \bar{n}\cdot p, p_\perp)$$



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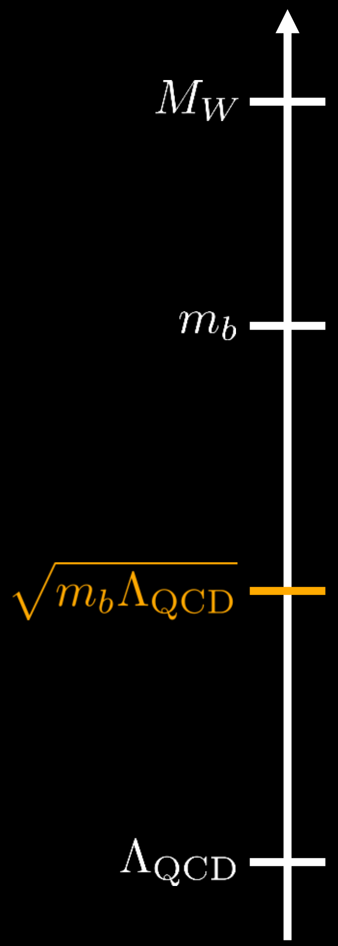
- relevant momentum regions

$$p_h \sim m_b\,(1,1,1) \qquad p_{hc} \sim m_b\,(\lambda,1,\lambda^{1/2}) \qquad p_c \sim m_b\,(\lambda^2,1,\lambda)$$

$$p_{\bar{hc}} \sim m_b\,(1,\lambda,\lambda^{1/2}) \qquad p_{\bar c} \sim m_b\,(1,\lambda^2,\lambda)$$

$$p_s \sim m_b\,(\lambda,\lambda,\lambda)$$

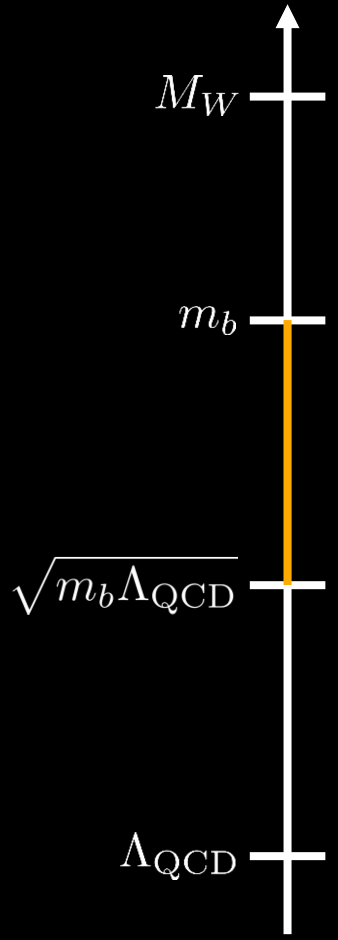


Matching onto SCET-1

- split fields according to momentum scaling: $\phi \to \phi_{hc} + \phi_{\overline{hc}} + \phi_s$

$$\mathcal{L}^{\text{SCET}-1} = \mathcal{L}_{hc} + \mathcal{L}_{\bar{h}c} + \mathcal{L}_s + \mathcal{L}_{hc+s} + \mathcal{L}_{\bar{h}c+s}$$

$$\leftarrow \mathcal{L}^{\text{HQET}} + \mathcal{L}_s^{\text{QCD}}$$



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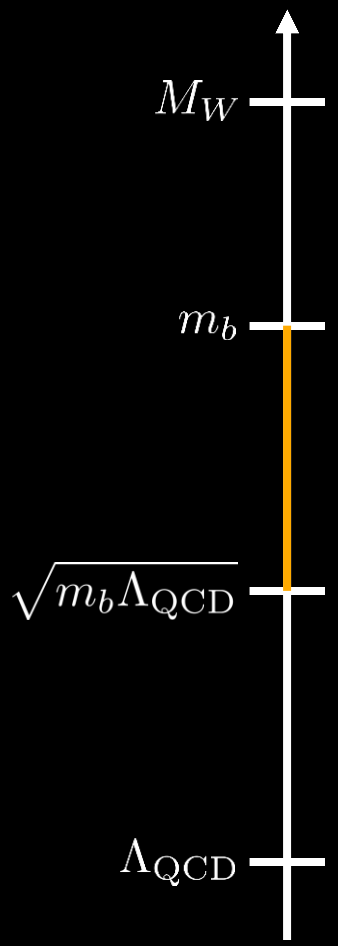
$\mathcal{L}^{\mathrm{HQET}} + \mathcal{L}_s^{\mathrm{QCD}}$

- match weak effective Hamiltonian onto SCET-1

$$\mathcal{M} = \sum_n H_n(\mu) \otimes \langle K^+ K^- | O_n(\mu) | \bar{B}^0 \rangle$$

integrate over momentum fractions



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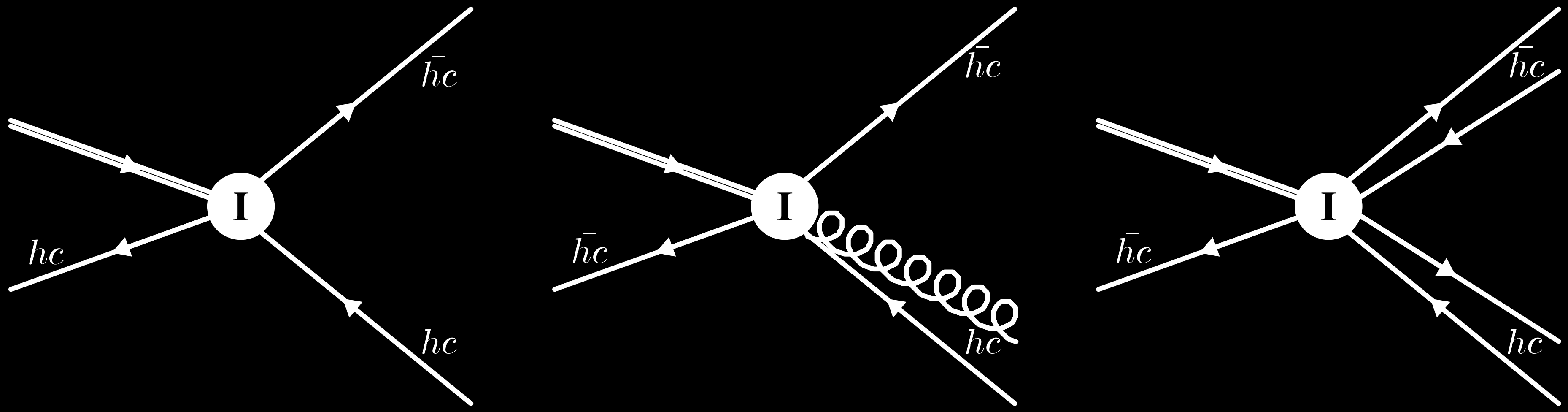
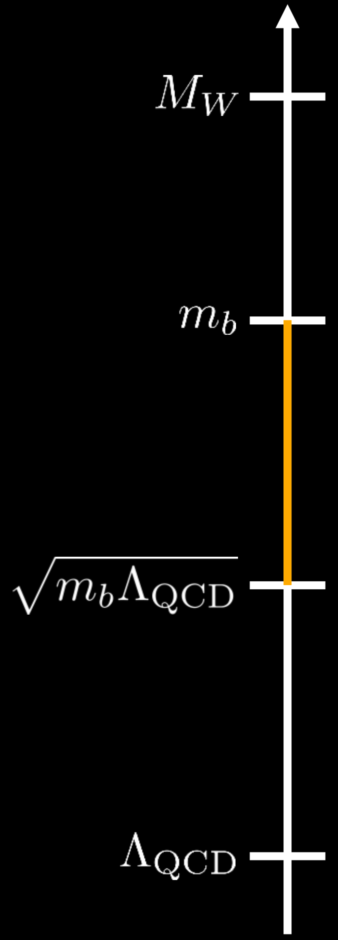


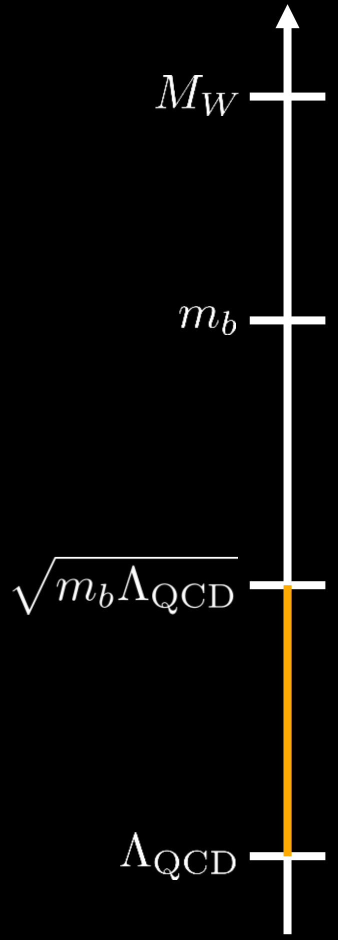


Matching onto SCET-2

- split fields further: $\phi_{hc} \to \phi_c + \phi_s$ and $\phi_{\overline{hc}} \to \phi_{\bar{c}} + \phi_s$

$$\mathcal{L}^{\text{SCET}-2} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

⟵ no $s - c$ interactions at leading power in λ



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← no $s - c$ interactions at leading power in λ

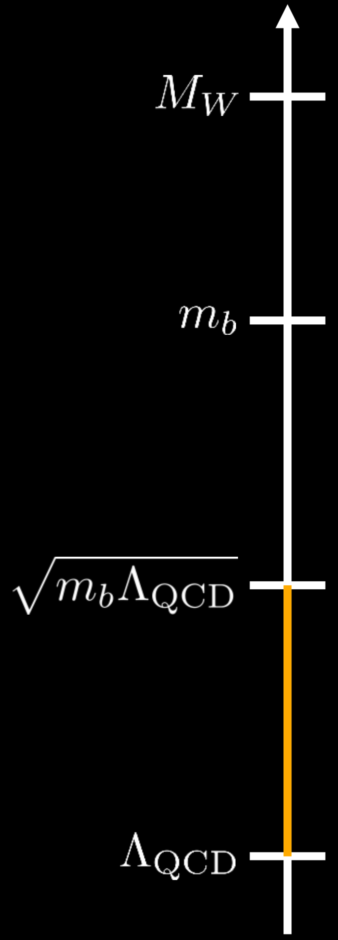
- match SCET-1 operators onto SCET-2

$$\mathcal{M} = \sum_n J_n(\mu) \otimes H_n(\mu) \otimes \bar{J}_n(\mu) \otimes \langle K^+ K^- | \mathcal{Q}_n(\mu) | \bar{B}^0 \rangle$$

↓

$$\Phi_B^{(n)}(\mu)\, \Phi_{K^+}^{(n)}(\mu)\, \Phi_{K^-}^{(n)}(\mu)$$

so far unknown (LC)DAs ↗



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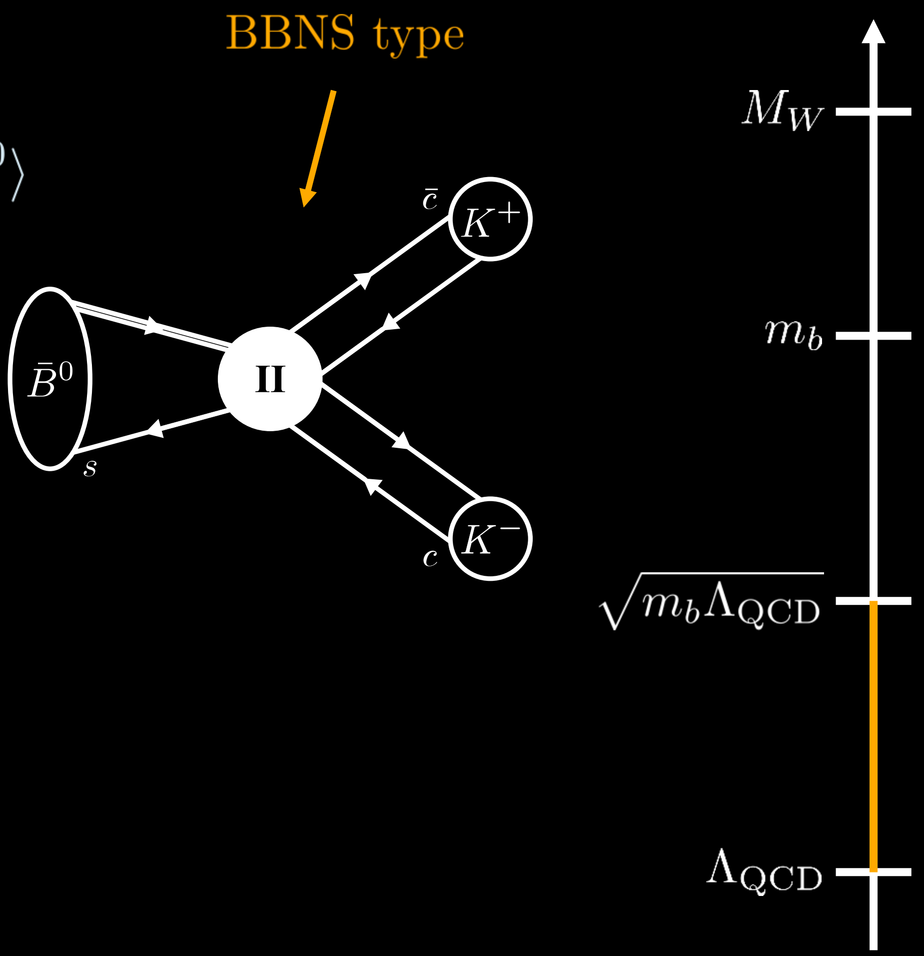

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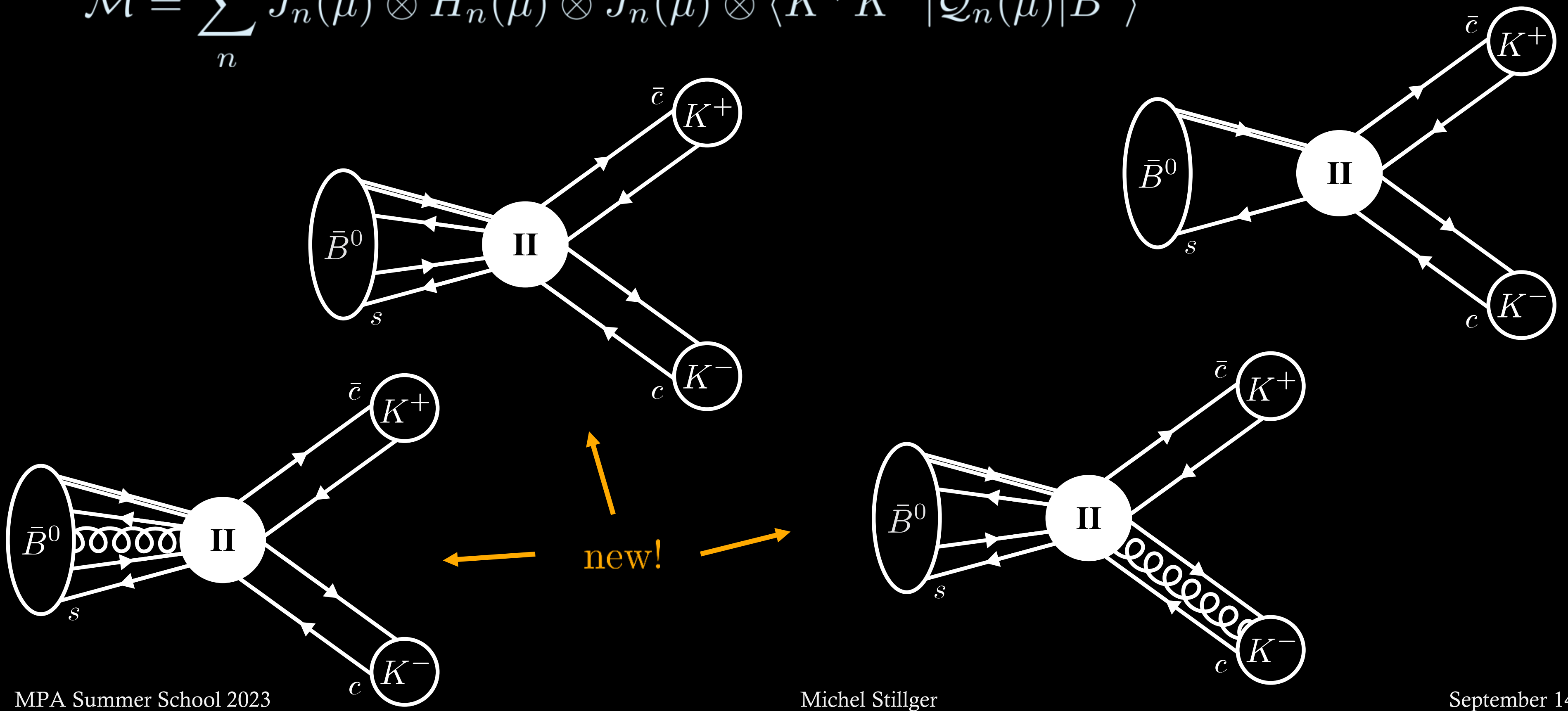

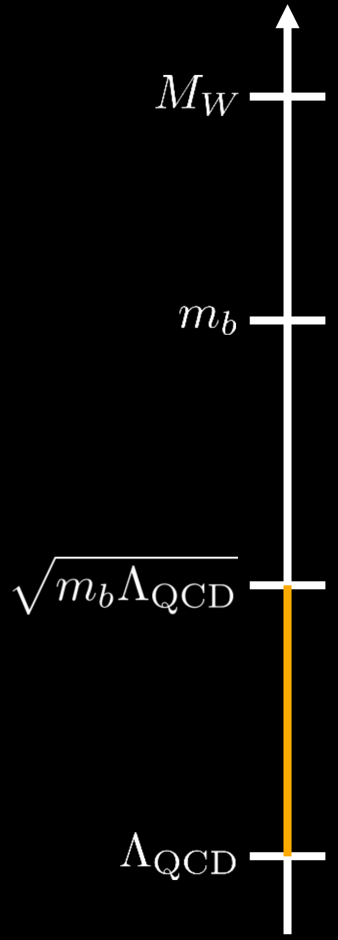

Conclusion

- B-anomalies → precise theoretical predictions needed
- leading power factorization understood since 20 years
- NLP factorization more complicated → endpoint divergences
- systematic study of weak annihilation amplitudes using SCET
  - new **universal** (LC)DAs
  - need to study endpoint behavior