Width difference and semileptonic asymmetry in B mesons

MPA Summer school 2023

Josua Scholze with Prof. Luca Silvestrini

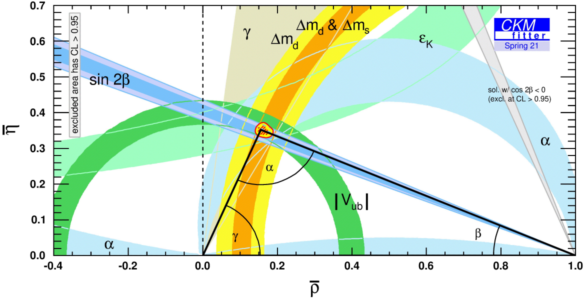
11.09.2023

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

SAPIENZA UNIVERSITÀ DI ROMA

Testing the Standard Model

- Flavor observables (e.g.: ΔM_d, ΔM_s) put strong constrains on the Standard Model
- Unitarity triangle by CKMfitter, UTfit:



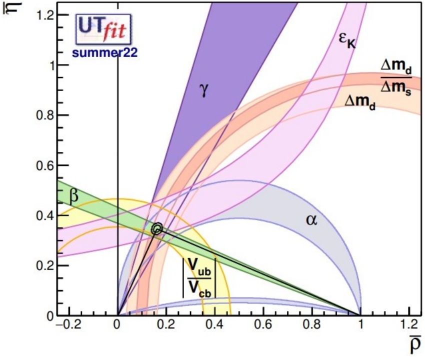
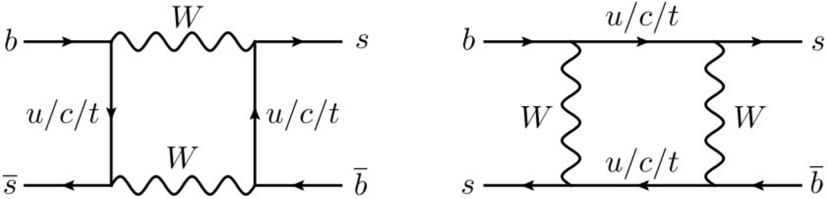


Table of Contents

Mixing of neutral B mesons

- flavor eigenstates of B mesons defined by quark content: $B_s = (s, \bar{b}),\ \bar{B}_s = (\bar{s}, b)$
- weak interaction allows mixing:



[Gerlach]

Mixing of neutral B mesons

- time evolution: $i\frac{d\psi(t)}{dt} = \hat{H}\psi(t), \quad \psi(t) = \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$ with

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{21} - i\frac{\Gamma_{21}}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix}$$

- without mixing $\hat{H}$ would be diagonal
- Hermiticity and CPT invariance requires:
  $M_{21} = M_{12}^*, \quad \Gamma_{21} = \Gamma_{12}^*, \quad M_{11} = M_{22}, \quad \Gamma_{11} = \Gamma_{22}$

Mixing of neutral B mesons

- time evolution: $i\frac{d\psi(t)}{dt} = \hat{H}\psi(t), \quad \psi(t) = \begin{pmatrix} |B(t)\rangle \\ |\bar{B}(t)\rangle \end{pmatrix}$ with

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M_{11} - i\frac{\Gamma_{11}}{2} \end{pmatrix}$$

- diagonalization $\hat{H}$ gives mass eigenstates: $|B_\mathrm{H}\rangle = p\,|B\rangle + q\,|\bar{B}\rangle\,, |B_\mathrm{L}\rangle = p\,|B\rangle - q\,|\bar{B}\rangle$
- M_{12}: off-shell contribution from: u, c, t, W
- Γ_{12}: on-shell contribution from: u, c

Physical observables

- Three independent observables: (in B system: $|\Gamma_{12}| \ll |M_{12}|$)
  - Mass difference: $\Delta M = M_{\mathrm{H}} - M_{\mathrm{L}} \approx 2|M_{12}|$
  - Width difference:

$$\Delta\Gamma = \Gamma_{\mathrm{L}} - \Gamma_{\mathrm{H}} = -\mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right)\Delta M$$

  - semileptonic asymmetry:

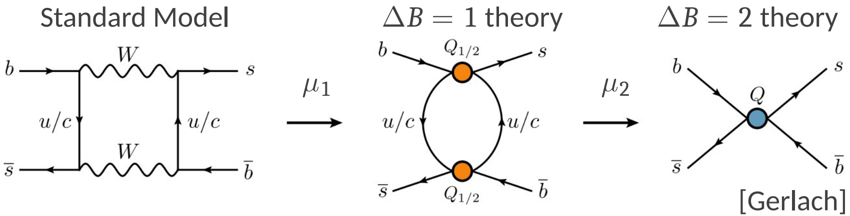
$$a_{\mathrm{sl}} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right), \quad \text{experimentally: } \frac{\Gamma\left(\bar{B}(t) \to \bar{l}\nu_l X\right) - \Gamma\left(B(t) \to l\bar{\nu}_l X\right)}{\Gamma\left(\bar{B}(t) \to \bar{l}\nu_l X\right) + \Gamma\left(B(t) \to l\bar{\nu}_l X\right)}$$

- Up to now measured: ΔM_s, ΔM_d, $\Delta\Gamma_s$

Table of Contents

Obtaining Γ_{12}

- general procedure:



- Heavy Quark Expansion in $\Lambda/m_b \approx 0.05$
- decomposition of Γ_{12}:

$$\Gamma_{12} = -\left[\lambda_c^2\,\Gamma_{12}^{cc} + 2\,\lambda_c\,\lambda_u\,\Gamma_{12}^{uc} + \lambda_u^2\,\Gamma_{12}^{uu}\right]$$
$$= -\lambda_t^2\left[\Gamma_{12}^{cc} + 2\,\frac{\lambda_u}{\lambda_t}\,(\Gamma_{12}^{cc} - \Gamma_{12}^{uc}) + \frac{\lambda_u^2}{\lambda_t^2}\,(\Gamma_{12}^{cc} - 2\Gamma_{12}^{uc} + \Gamma_{12}^{uu})\right]$$

Obtaining Γ_{12}

- decomposition of Γ_{12}:

$$\Gamma_{12} = -\lambda_t^2 \left[\Gamma_{12}^{cc} \; + \; 2\,\frac{\lambda_u}{\lambda_t} \; (\Gamma_{12}^{cc} - \Gamma_{12}^{uc}) \; + \; \frac{\lambda_u^2}{\lambda_t^2} \; (\Gamma_{12}^{cc} - 2\Gamma_{12}^{uc} + \Gamma_{12}^{uu}) \right]$$

- in terms of Wilson coefficients C_i and $\Delta B = 2$ matrix elements:

$$\Gamma_{12} \; \propto \; \sum_i C_i \, \langle B | H_i^{\Delta B = 2} | \overline{B} \rangle$$

- for $a_{\mathrm{sl}} = \mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)$: Γ_{12}^{cc} doesn't contribute $\Rightarrow$ depends on m_c

Why do we calculate the ratio Γ_{12}/M_{12}?

- decomposition of Γ_{12}:

$$\Gamma_{12} = -\lambda_t^2 \left[\Gamma_{12}^{cc} + 2\,\frac{\lambda_u}{\lambda_t}\,(\Gamma_{12}^{cc} - \Gamma_{12}^{uc}) + \frac{\lambda_u^2}{\lambda_t^2}\,(\Gamma_{12}^{cc} - 2\Gamma_{12}^{uc} + \Gamma_{12}^{uu}) \right]$$

- in terms of $\Delta B = 1$ Wilson coefficients C_i and $\Delta B = 2$ matrix elements:

$$\Gamma_{12} \propto \sum_i C_i \,\langle B|H_i^{\Delta B=2}|\overline{B}\rangle$$

- factor λ_t^2 appears also in M_{12}: cancels in the ratio Γ_{12}/M_{12}
- M_{12} contains just one factor $\langle B|H_i^{\Delta B=2}|\overline{B}\rangle \Rightarrow$ cancellation with Γ_{12} possible

Table of Contents

Comparison with measurement

Theory predictions by Gerlach et al. [2205.07907]

$$\Delta\Gamma_s = (0.076 \pm 0.017)\,\text{ps}^{-1}$$
$$a_{\text{sl}}^s = (2.19 \pm 0.14) \times 10^{-5}$$
$$\Delta\Gamma_d = (2.16 \pm 0.47) \times 10^{-3}\,\text{ps}^{-1}$$
$$a_{\text{sl}}^d = (-5.04 \pm 0.33) \times 10^{-4}$$

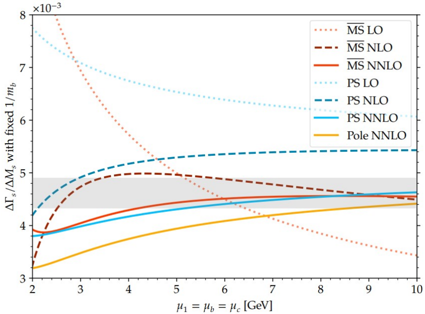
Experimental values by HFLAV [2206.07501]

$$\Delta\Gamma_s = (0.083 \pm 0.005)\,\text{ps}^{-1}$$
$$a_{\text{sl}}^s = (-60 \pm 280) \times 10^{-5}$$
$$\Delta\Gamma_d = (0.7 \pm 6.6) \times 10^{-3}\,\text{ps}^{-1}$$
$$a_{\text{sl}}^d = (-21 \pm 17) \times 10^{-4}$$

- I confirmed the theory uncertainties with MC simulations

Renormalization scale dependence

- μ_1 scale dependence shrinks by including higher orders
- Potential Subtracted (PS) and $\overline{\text{MS}}$ scheme behave better than the pole scheme



Gerlach et al. [2205.07907]

Summary

- Implemented $\Delta\Gamma$ and a_{sl} for different mass schemes in `HEPfit`
- Reevaluated the uncertainties for $\Delta\Gamma_s$ and a_{sl} with MC simulations
- Next step: get constraints for the Standard Model with a complete UT analysis
- Future: extension to New Physics models possible

Thank you.
Any questions?

General sources

- ”Effective Theories for Quark Flavour Physics” by Silvestrini
- ”Meson width differences and asymmetries”, thesis by Gerlach
- ”CP violation in the B_s^0 system” by Artuso et al.
- ”Gauge Theory of Weak Decays” by Buras
- ”HEPfit Manual” by de Blas et al.

$\Delta B = 1$ Effective Hamiltonian

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \Bigg\{ \Bigg[\Bigg(V_{cb}^* V_{ud} \left(C_1 Q_1 + C_2 Q_2 \right) + V_{cb}^* V_{cd} \left(C_1 Q_1^c + C_2 Q_2^c \right) + (c \leftrightarrow u) \Bigg)$$

$$- V_{tb}^* V_{td} \left(\sum_{i=3}^{6} C_i Q_i + C_{8G} Q_{8G} \right) \Bigg] + \Big[d \to s \Big] \Bigg\} + h.c.$$

- operator in traditional basis [hep-ph/9211304], [hep-ph/0308029]:

$$Q_1 = (\bar{b}_i c_j)_{V-A} (\bar{u}_j d_i)_{V-A} \,, \qquad Q_2 = (\bar{b}_i c_i)_{V-A} (\bar{u}_j d_j)_{V-A} \,,$$

$$Q_1^c = (\bar{b}_i c_j)_{V-A} (\bar{c}_j d_i)_{V-A} \,, \qquad Q_2^c = (\bar{b}_i c_i)_{V-A} (\bar{c}_j d_j)_{V-A} \,,$$

$$Q_3 = (\bar{b}_i d_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A} \,, \qquad Q_4 = (\bar{b}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A} \,,$$

$$Q_5 = (\bar{b}_i d_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A} \,, \qquad Q_6 = (\bar{b}_i d_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A} \,,$$

$$Q_{8G} = \frac{g_s}{8\pi^2} m_b \bar{b}_i \sigma^{\mu\nu} \left(1 - \gamma^5 \right) t_{ij}^a d_j G_{\mu\nu}^a$$

$\Delta B = 1$ Effective Hamiltonian

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \Bigg\{ \Bigg[\Bigg(V_{cb}^* V_{ud} \left(C_1 Q_1 + C_2 Q_2 \right) + V_{cb}^* V_{cd} \left(C_1 Q_1^c + C_2 Q_2^c \right) + \left(c \leftrightarrow u \right) \Bigg) - V_{tb}^* V_{td} \left(\sum_{i=3}^{6} C_i Q_i + C_{8G} Q_{8G} \right) \Bigg] + \Big[d \to s \Big] \Bigg\} + h.c.$$

- to diminish problems with γ_5:

  alternative basis by Chetyrkin, Misiak and Münz [hep-ph/9711280]
  known up to NNLO and transformation to traditional basis up to NLO

Matching procedure

- matching of Standard Model (SM) to Weak Effective Theory (WET): get $\Delta B = 1$ Wilson coefficients $C_i(\mu_0 \approx m_W)$
- use Renormalization Group Equation (RGE): $\mu \frac{d}{d\mu} \vec{C}(\mu) = \vec{\gamma} \vec{C}(\mu)$
- matching to $\Delta B = 2$ Hamiltonian at: μ_1
- RGE to obtain scale μ_2 of the $\Delta B = 2$ operator matrix elements

Operator basis for $\Delta B = 2$

- Result: $\Gamma_{12} = \frac{G_F^2 m_b^2}{24\pi M_B}\left[H(z)\langle B|Q|\bar{B}\rangle + H_S(z)\langle B|Q_S|\bar{B}\rangle + \widetilde{H}_S(z)\langle B|\widetilde{Q}_S|\bar{B}\rangle\right] + \Gamma_{1/m_b}$
- with dimension 6 operators:

$$Q = \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_i\,\bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_j$$

$$Q_S = \bar{s}_i\left(1+\gamma^5\right)b_i\,\bar{s}_j\left(1+\gamma^5\right)b_j$$

$$\widetilde{Q}_S = \bar{s}_i\left(1+\gamma^5\right)b_j\,\bar{s}_j\left(1+\gamma^5\right)b_i$$

$$R_0 = \frac{1}{2}\alpha_1 Q + Q_S + \alpha_2\widetilde{Q}_S = \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$$

, at LO in α_s: $\alpha_1 = \alpha_2 = 1$

- coefficients: $H_i = H_i(C_j(\mu_0, \mu_1), \mu_1, \mu_2)$

Operator basis for $\Delta B = 2$

- Result: $\Gamma_{12} = \frac{G_F^2 m_b^2}{24\pi M_B}\left[H(z)\langle B|Q|\bar{B}\rangle + \cancel{H_S(z)\langle B|Q_S|\bar{B}\rangle} + \widetilde{H}_S(z)\langle B|\widetilde{Q}_S|\bar{B}\rangle\right] + \Gamma_{1/m_b}$
- with dimension 6 operators:

$$
\begin{aligned}
Q &= \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_i\,\bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_j\\
Q_S &= \bar{s}_i\left(1+\gamma^5\right)b_i\,\bar{s}_j\left(1+\gamma^5\right)b_j\\
\widetilde{Q}_S &= \bar{s}_i\left(1+\gamma^5\right)b_j\,\bar{s}_j\left(1+\gamma^5\right)b_i\\
R_0 &= \frac{1}{2}\alpha_1 Q + Q_S + \alpha_2\widetilde{Q}_S = \mathcal{O}\left(\frac{\Lambda}{m_b}\right), \text{ at LO in } \alpha_s\text{: } \alpha_1 = \alpha_2 = 1
\end{aligned}
$$

- old choice: use Q and Q_S [hep-ph/9808385], implemented from [hep-ph/0308029]
- better alternative: use Q and $\widetilde{Q}_S$ [hep-ph/0612167] to cancel $\langle B|Q|\bar{B}\rangle$ in $\Delta\Gamma/\Delta M$

Switch to the RI scheme for $\Delta B = 2$ operators

- renormalization prescription for the RI scheme [hep-ph/9501265]:

$$\langle F|Q_i|I\rangle_\lambda = \langle F|Q_i|I\rangle_{\text{tree}}$$

- ensures to all orders: $\langle B|R_0|\bar{B}\rangle = \mathcal{O}\left(\frac{\Lambda}{m_b}\right)$
- conversion only known to NLO [hep-lat/0110091]:

$$\begin{pmatrix} \langle Q(\mu)\rangle \\ \langle Q_S(\mu)\rangle \\ \langle \widetilde{Q}_S(\mu)\rangle \end{pmatrix}_{\overline{\text{MS}}} = \left[\mathbb{1} + r_{123}\frac{\alpha_s(\mu)}{4\pi}\right] \begin{pmatrix} \langle Q(\mu)\rangle \\ \langle Q_S(\mu)\rangle \\ \langle \widetilde{Q}_S(\mu)\rangle \end{pmatrix}_{\text{RI}}, r_{123} = \frac{1}{9}\begin{pmatrix} -42+72\log 2 & 0 & 0 \\ 0 & 61+44\log 2 & -7+28\log 2 \\ 0 & -25+28\log 2 & -29+44\log 2 \end{pmatrix}$$

$1/m_b$ corrections

- Beside R_0, the operators
  - $R_1 = \frac{m_s}{m_b}\, \bar{s}_\alpha (1+\gamma_5) b_\alpha\, s_\beta (1-\gamma_5) b_\beta$
  - $R_2 = \frac{1}{m_b^2}\, \bar{s}_\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma_5) D^\rho b_\alpha\, s_\beta \gamma_\mu (1-\gamma_5) b_\beta$
  - $R_3 = \frac{1}{m_b^2}\, s_\alpha \overleftarrow{D}_\rho (1+\gamma_5) D^\rho b_\alpha\, s_\beta (1+\gamma_5) b_\beta$

  and $\widetilde{R}_i$ (with interchanged colour indices α, β) occur
- known to LO in α_s and parameterized by [hep-ph/0612167]:

$$\widetilde{\Gamma}^{ab}_{12,1/m_b} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[g_0^{ab} \langle B_s | R_0 | B_s \rangle + \sum_{j=1}^{3} \left[g_j^{ab} \langle B_s | R_j | B_s \rangle + \widetilde{g}_j^{ab} \langle B_s | \widetilde{R}_j | \overline{B} \rangle \right] \right]$$

Resummation of logarithms

- dominant z-dependent contribution at order α_s^n from $\alpha_s^n z \ln^n z$
- change renormalisation scheme [hep-ph/0307344]:

$$z = \frac{\overline{m}_c^2(\overline{m}_c)}{\overline{m}_b^2(\overline{m}_b)} \to \bar{z} = \frac{\overline{m}_c^2(\overline{m}_b)}{\overline{m}_b^2(\overline{m}_b)} \approx \frac{z}{2}$$

- important for semileptonic asymmetry (of order z)

Different mass renormalization schemes

- $\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_B}\left[H^{ab}(z)\langle B|Q|\bar{B}\rangle + \widetilde{H}_S^{ab}(z)\langle B|\widetilde{Q}_S|\bar{B}\rangle\right] + \mathcal{O}(\Lambda_{\text{QCD}}/m_b)$
- $H^{ab} = H_0 + \alpha_s H_1 + \alpha_s^2 H_2$ were calculated in pole mass scheme
- for numerical evaluation: switch scheme of the prefactor m_b^2 to $\overline{\text{MS}}$ or PS
- general scheme transformation: $m_b = m_b' \left(1. + \alpha_s \Delta m_b^1 + \alpha_s^2 \Delta m_b^2\right)$
- adapt the expansion of H^{ab} to:

$$H = H_0 + \alpha_s \left[H_1 + 2\Delta m_b^1 H_0\right] + \alpha_s^2 \left[H_2 + 2\left(\Delta m_b^1\right) H_1 + \left(2\Delta m_b^2 + \left(\Delta m_b^1\right)^2\right) H_0\right]$$