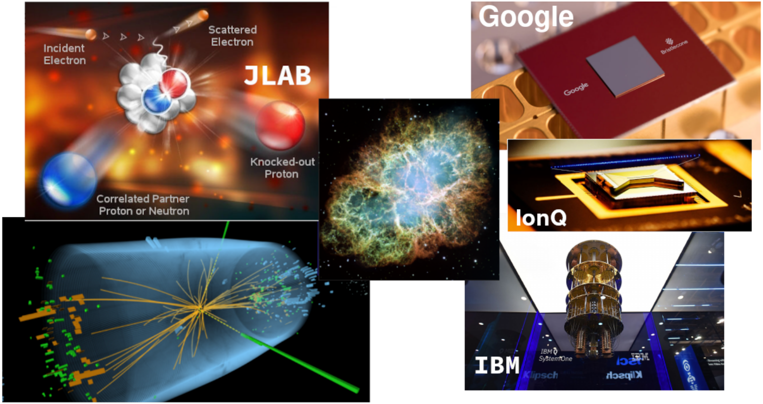
Quantum Simulation for Nuclear and Neutrino Physics

Alessandro Roggero



TIFPA
Trento Institute for
Fundamental Physics
and Applications

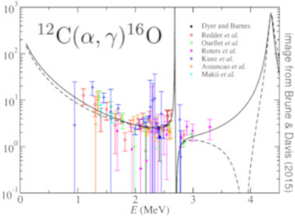
MPA Summer School

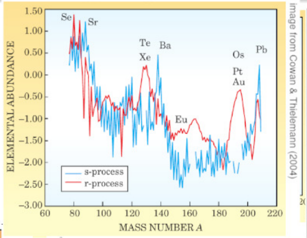
Chiemsee – 10-15 Sep, 2023

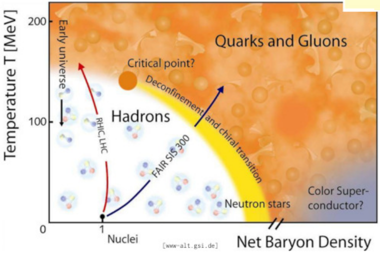
Plan for these lectures

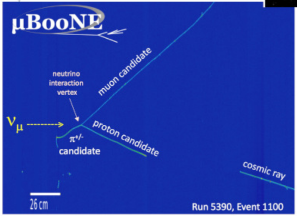
Quantum Simulation for Nuclear Physics

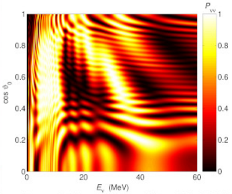
Quantum Simulation for Neutrino Physics









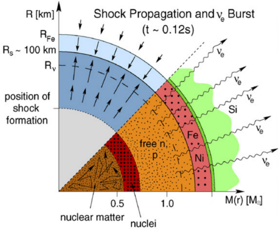


Duan, Fuller, Carlson, Qian PRL(2006)

Neutrino's roles in supernovae

figures from Janka et al. (2007)

- efficient energy transport away from the shock region (burst)



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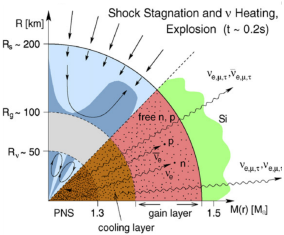
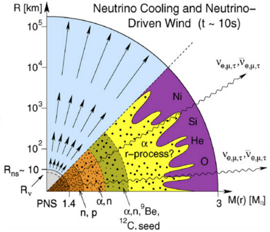


- regulation of electron fraction in ν-driven wind (nucleosynthesis)



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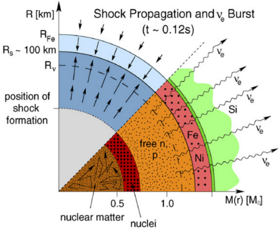

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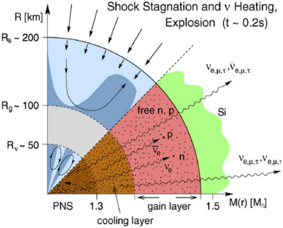

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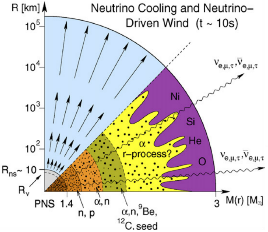
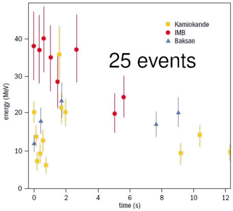
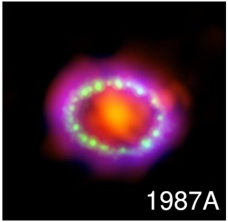


$\approx 10^{58}$ neutrinos emitted in few sec.





Neutrino oscillations in astrophysical environments

We know that neutrinos can display flavor oscillations in vacuum, does it matter in a core-collapse supernova?

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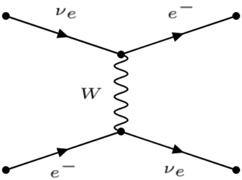
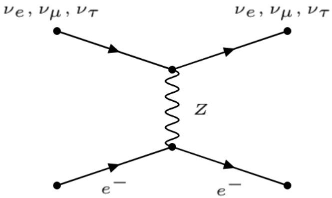
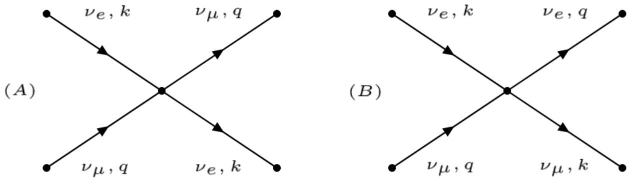
- energy deposition behind shock and in the wind proceeds through charge-current reactions (large differences in $\nu_e - \nu_{\mu/\tau}$)
- neutrino oscillation rates can get enhanced through elastic forward scattering with high density external matter (MSW effect)





Neutrino-neutrino forward scattering

Fuller, Qian, Pantaleone, Sigl, Raffelt, Sawyer, Carlson, Duan, ...



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- off-diagonal term (B) equivalent to flavor/momentum exchange between two neutrinos
    - total flavor is conserved

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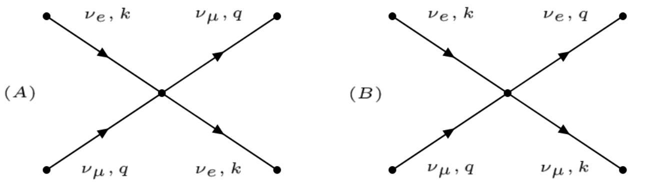

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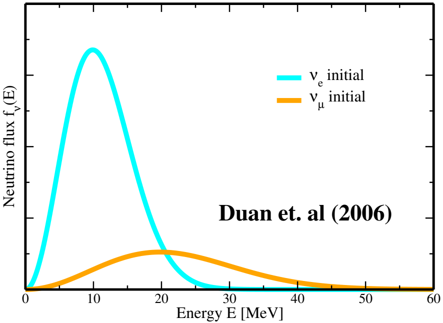

Important effect if initial distributions are strongly flavor dependent

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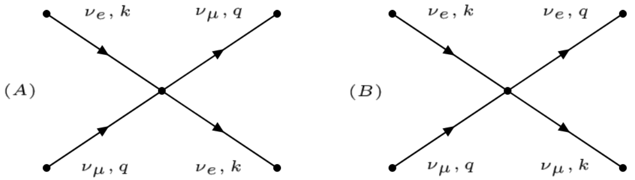

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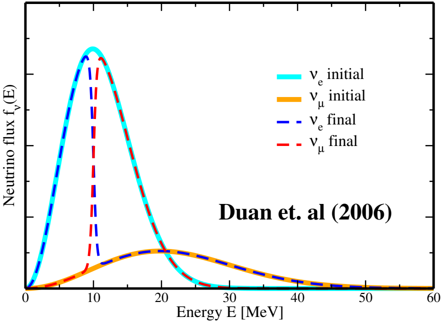


Two-flavor approximation and the iso-spin Hamiltonian

Consider two active flavors (ν_e,ν_x) and encode flavor amplitudes for a neutrino with momentum p_i into an $SU(2)$ iso-spin:

$$|\Phi_i\rangle = \cos(\eta_i)|\nu_e\rangle + \sin(\eta_i)|\nu_x\rangle \equiv \cos(\eta_i)|\uparrow\rangle + \sin(\eta_i)|\downarrow\rangle$$

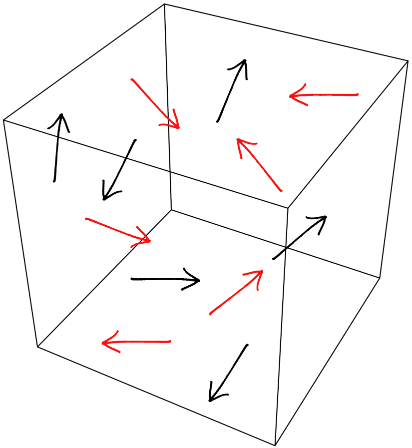
A system of N interacting neutrinos is then described by the Hamiltonian

$$H = \sum_i \frac{\Delta m^2}{4E_i}\vec{B}\cdot\vec{\sigma}_i + \lambda\sum_i \sigma_i^z + \frac{\mu}{2N}\sum_{i<j}(1-\cos(\phi_{ij}))\vec{\sigma}_i\cdot\vec{\sigma}_j$$

- vacuum oscillations: $\vec{B} = (\sin(2\theta_{mix}), 0, -\cos(2\theta_{mix}))$
- interaction with matter: $\lambda = \sqrt{2}G_F\rho_e$
- neutrino-neutrino interaction: $\mu = \sqrt{2}G_F\rho_\nu$
  - dependence on momentum direction: $\cos(\phi_{ij}) = \frac{\vec{p}_i}{\|\vec{p}_i\|}\cdot\frac{\vec{p}_j}{\|\vec{p}_j\|}$

for a full derivation, see e.g. Pehlivan et al. PRD(2011)

Finite size effects and thermodynamic limit



$$H = \sum_{i=1}^{N} \vec{B}_i \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j}^{N} v_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- the quantum system is defined in some finite volume V
- we have a finite number N of neutrinos within the box
- the neutrino density ρ_ν (and thus μ) is given by N/V

For astrophysically relevant predictions need to understand how the system behaves when $V \to \infty$ and $N \to \infty$ while keeping $\rho_\nu = N/V$ constant

The mean-field approximation

The equations of motion for the polarization vector $\vec{P}_i = \langle \vec{\sigma}_i \rangle$ are

$$\frac{d}{dt}\vec{P}_i = \left(\frac{\Delta m^2}{4E_i}\vec{B} + \lambda\hat{z} \right) \times \vec{P}_i + \frac{\mu}{2N}\sum_{j\neq i} \left(1 - cos(\phi_{ij})\right) \langle \vec{\sigma}_j \times \vec{\sigma}_i \rangle$$

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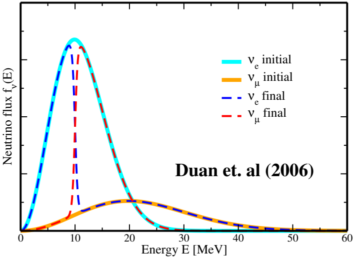
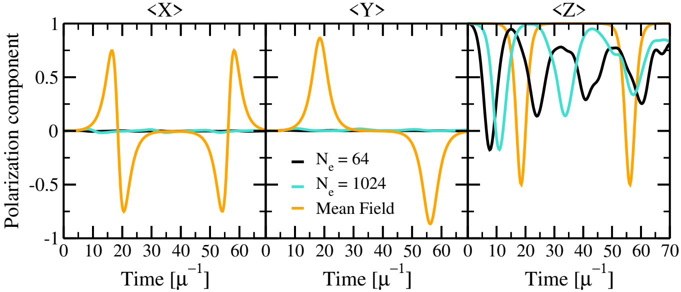
In this way we obtain a closed system of $3N$ coupled differential equations

- efficient solutions for systems containing $N \approx \mathcal{O}(10^{4-5})$ neutrino amplitudes [$\approx \mathcal{O}(100)$ energies and $\approx \mathcal{O}(100)$ angles]



Beyond mean field effects: a simple example

J. Martin, AR, H. Duan, J. Carlson, V. Cirigliano PRD(2022)



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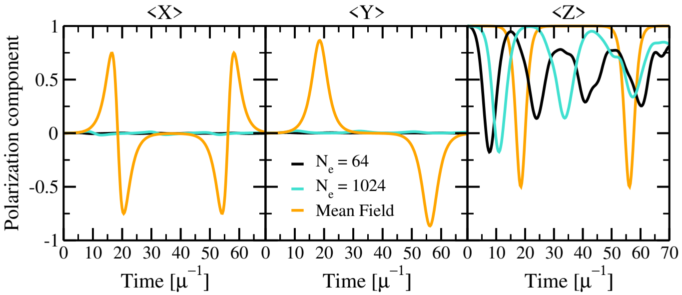

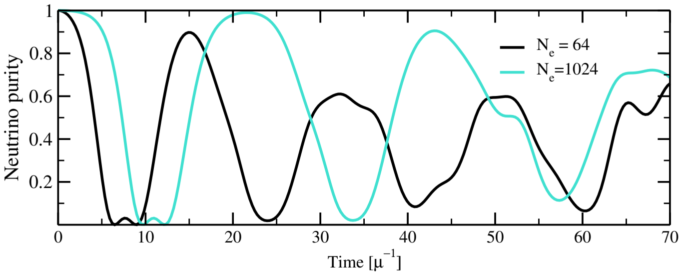

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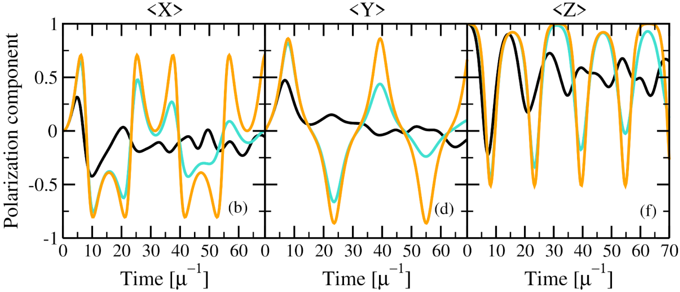
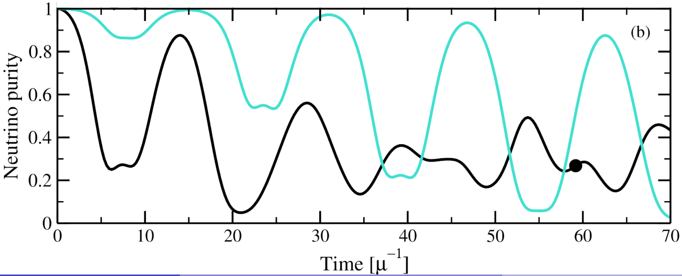


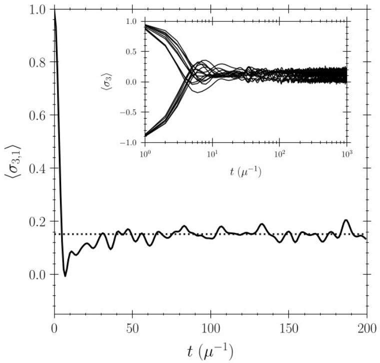


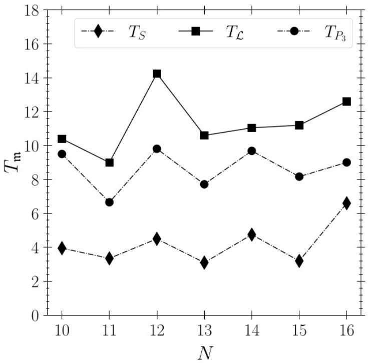
Beyond mean field effects: a (less) simple example

J. Martin, D. Neill, AR, H. Duan, J. Carlson, arXiv:2307.16793

$$H = \frac{\mu}{2N} \sum_{i<j} (1 - \vec{v}_i \cdot \vec{v}_j) \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- spherical sym.: $\vec{v}_i \cdot \vec{v}_j \longrightarrow const.$
- axial sym.: $\vec{v}_i \cdot \vec{v}_j \longrightarrow v_i^z v_j^z$

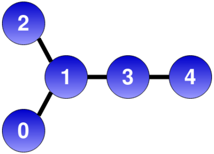




Interacting neutrino systems can thermalize!

Quantum simulation of collective neutrino oscillations

$$H = \sum_i \omega_i \vec{B} \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j$$



- with only 2 flavors direct map to spin 1/2 degrees of freedom (qubits)
- only one- and two-body interactions $\Rightarrow$ only $\mathcal{O}(N^2)$ terms
- all-to-all interactions are difficult with reduced connectivity

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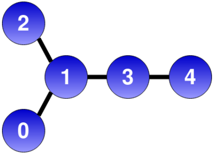

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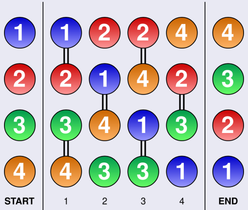
SWAP network



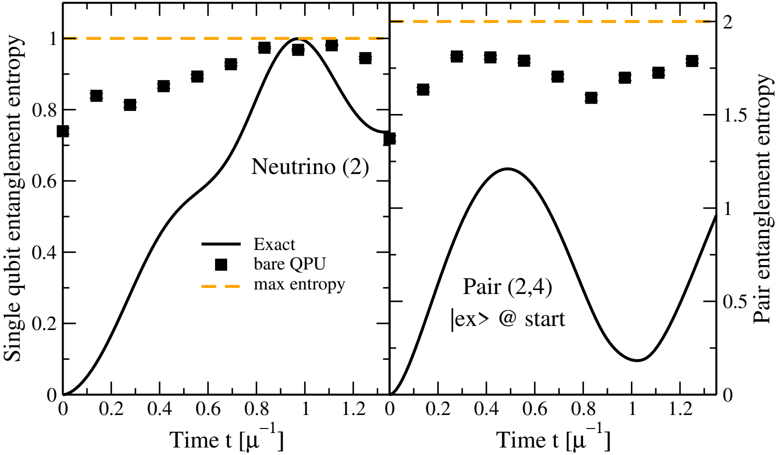
- SWAP qubits every time we apply time-evolution to neighboring terms
- in N steps we perform full evolution using only $\binom{N}{2}$ two qubit gates
  - NOTE: final order will be reversed

Kivlichan et al. PRL (2018)

B.Hall, AR, A.Baroni, J.Carlson PRD(2021)

Entanglement evolution and error mitigation with $N = 4$

B.Hall, AR, A.Baroni, J.Carlson PRD(2021)



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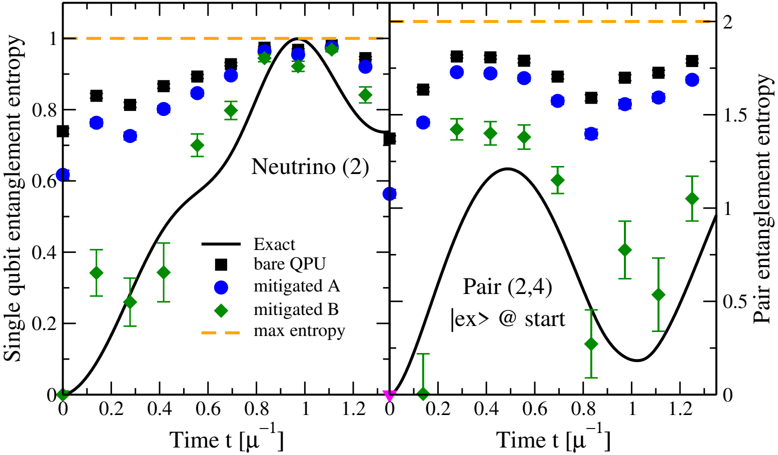
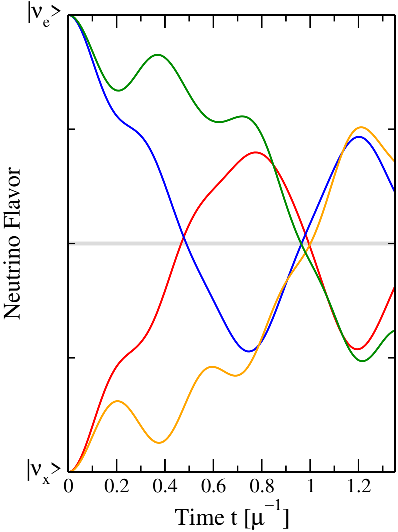


- Dechoerence with environment leads to increase in measured entropy
- Noise impact on observables can be modeled and effect mitigated

Accuracy in flavor evolution

Entanglement is useful to understand collective oscillation mechanism but priority is to predict flavor evolution. How's the current (2020) accuracy?



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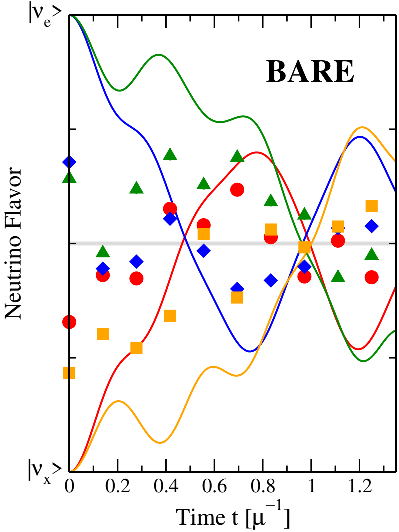

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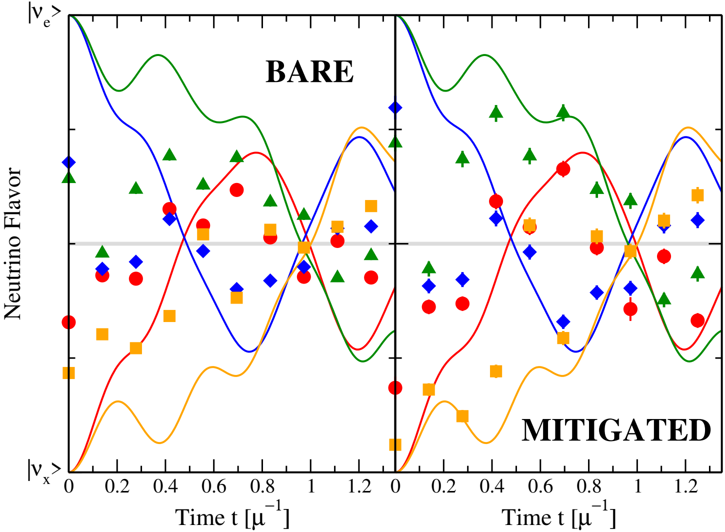

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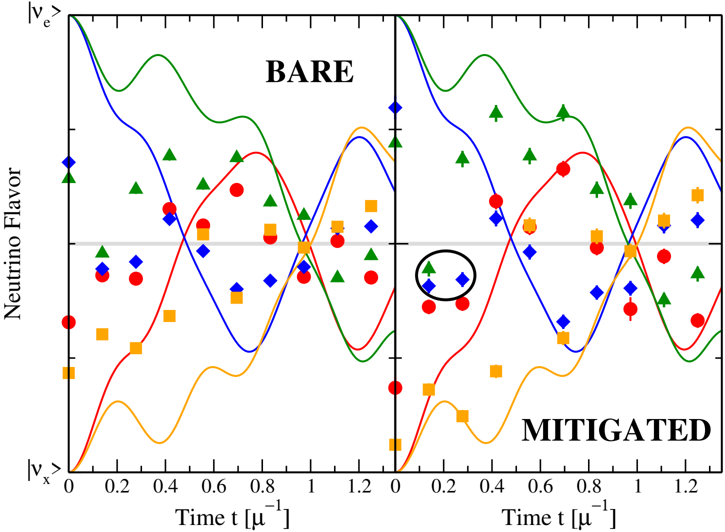

Fidelity of quantum hardware is improving fast

The device used for the previous results was Vigo with a QV of 16

$QV = 2^n \approx$ we can run n full layers on n qubits with fidelity $\geq 66\%$

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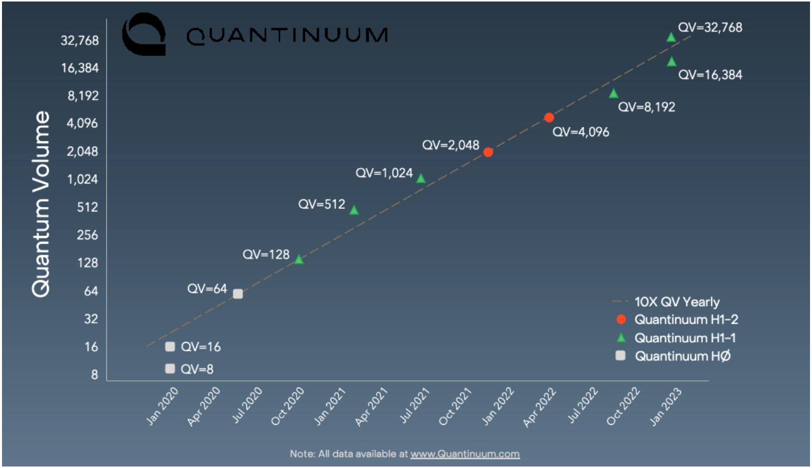

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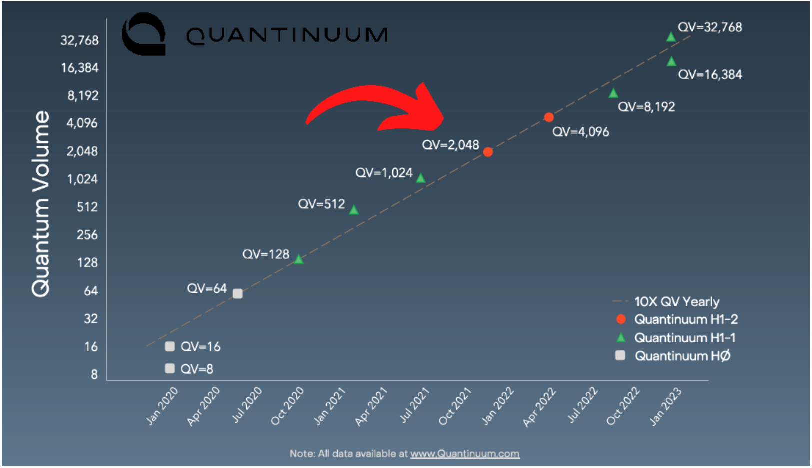
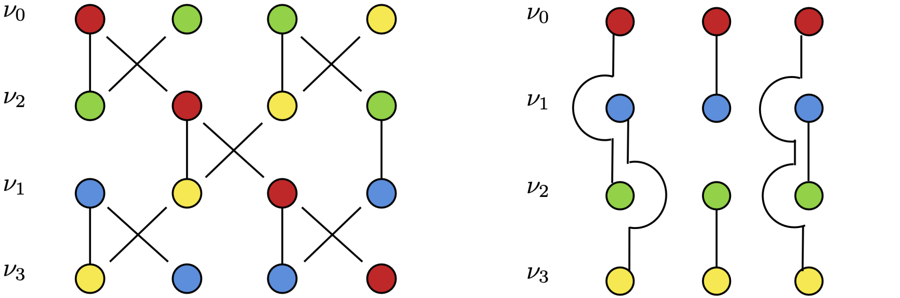


Connectivity advantage with trapped ions

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

- all-to-all connectivity allows a reduction in circuit depth and the possibility of exploring different orderings for the decomposition

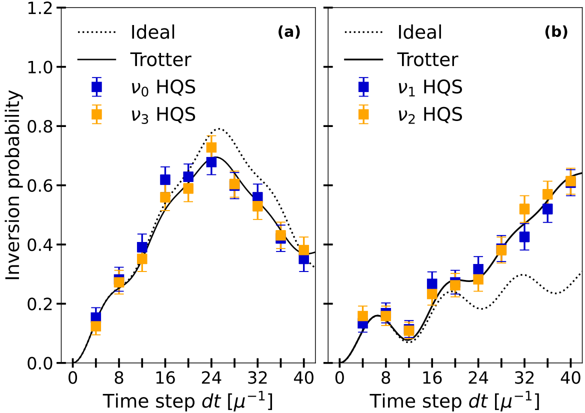


- removing SWAPs allows for a big **reduction in number of rotations**
- **very low infidelities**: $\approx 5 \times 10^{-5}$ one-qubit, $\approx 3 \times 10^{-3}$ two-qubit

Recent progress in porting the scheme to trapped ions

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

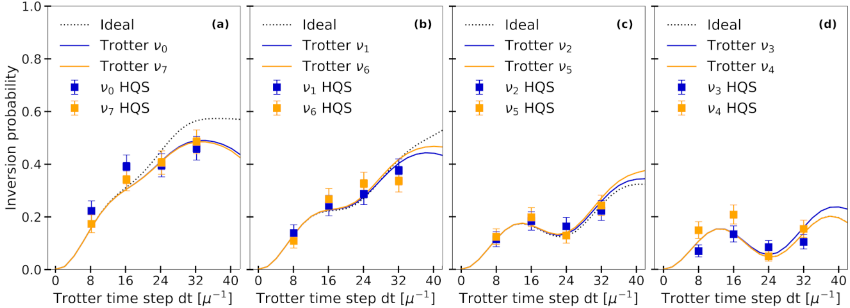
$N = 4$ neutrinos, one time step



Recent progress in porting the scheme to trapped ions II

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

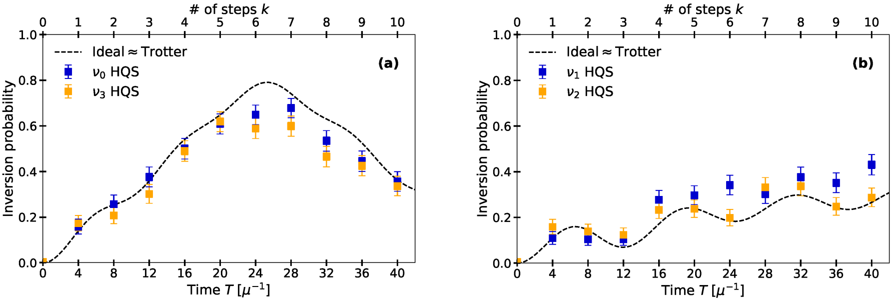
$N = 8$ neutrinos, one time step



Recent progress in porting the scheme to trapped ions III

V.Amitrano, AR, P.Luchi, F.Turro, L.Vespucci, F.Pederiva, arXiv:2207.03189 (2022)

$N = 4$ neutrinos, multiple time steps



Last two points required: ≈ 350 two-qubit gates over 8 qubits

Scaling to large system sizes

In most cases the entire cost of the simulation comes from time evolution since the initial state preparation is trivial if we start in a product state

$$H = \sum_i \omega_i \vec{B} \cdot \vec{\sigma}_i + \frac{\mu}{2N} \sum_{i<j} J_{ij} \vec{\sigma}_i \cdot \vec{\sigma}_j = H_\nu + H_{\nu\nu}$$

In simpler case where $\omega_i = \omega$ then $[H_\nu, H_{\nu\nu}] = 0$ (2-body dominates)

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- QSP [Low&Chuang(2016)]: $r = \mathcal{O}\left(T\lambda_H + \log\left(\frac{1}{\epsilon}\right)\right) = \mathcal{O}\left(T\mu N + \log\left(\frac{1}{\epsilon}\right)\right)$

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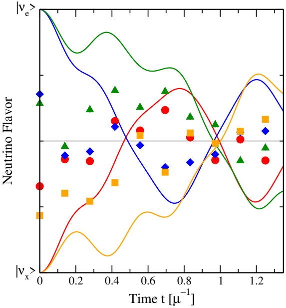
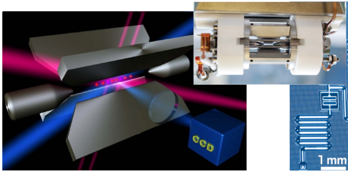
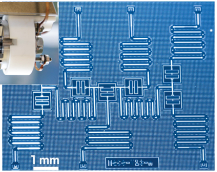
High order Trotter formulas achieve better gate cost for large N!

Current limitations of digital quantum simulations



current and near term digital quantum devices have limited fidelity and might not scale much beyond $N = \mathcal{O}(10)$ neutrinos in next years





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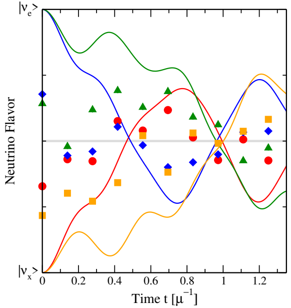

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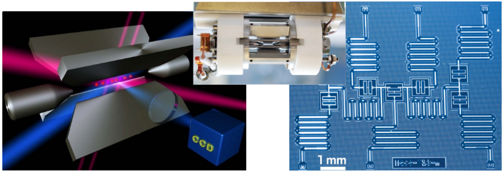


Possible paths to scalability in the meantime

- Analog Quantum Simulators

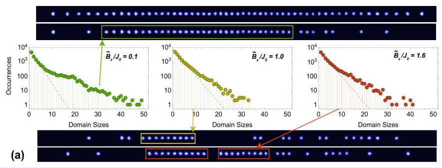


figure from Zhang et al Nature(2017)

- Describe low entanglement states with Tensor Networks

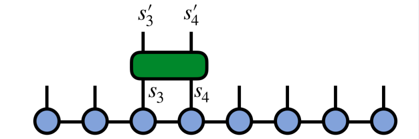
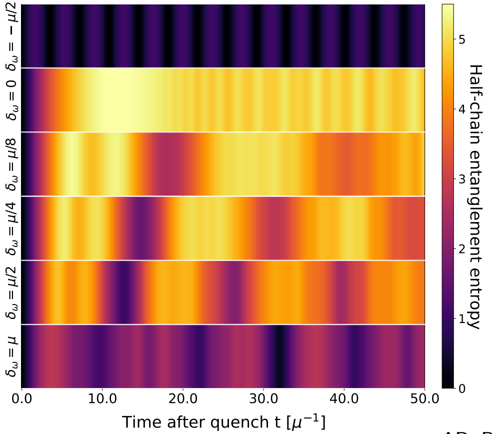


image from itensor.org

Collective oscillations with MPS

$$H = -\frac{\delta_\omega}{2}\left(\sum_{i\in\{1,\dots,N/2\}}\sigma_i^z - \sum_{i\in\{N/2+1,\dots,N\}}\sigma_i^z\right) + \frac{\mu}{2N}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j\ ,$$

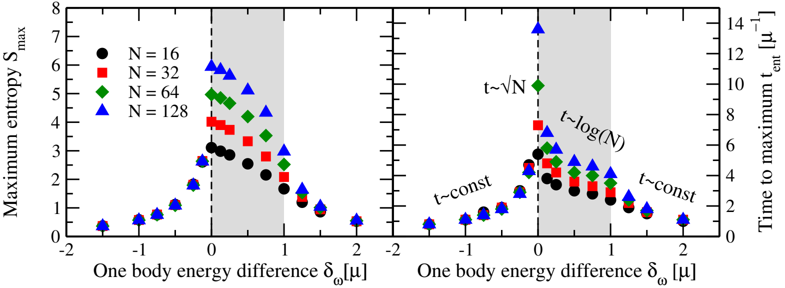
MF predicts no evolution, MPS has oscillations for $0 \le \delta_\omega/\mu \lesssim 1$



AR, PRD 104, 123023 (2021)

Collective oscillations and entanglement scaling

AR, PRD 104, 103016 (2021) & PRD 104, 123023 (2021)



Why is this interesting?

- entanglement scaling provides general criterion for appearance of collective modes in full many-body treatment
- entropy scaling as $\log(N)$ $\Rightarrow$ large ab-initio simulations possible
- MPS method fails when entanglement too large $\Rightarrow$ we can use this to detect interesting regimes to study on quantum simulators!

Summary and perspectives

- collective neutrino oscillations are an interesting **strongly coupled** many-body system driven by the **weak interaction** with possible important impact on flavor dynamics in extreme environments
- even the basic 2-flavor model for collective oscillations poses a challenging many-body problem well suited to quantum technologies
  - Hamiltonian is two-local but all-to-all $\rightarrow$ best suited for trapped-ions
- first calculations on small scale digital devices show promise in studying flavor evolution and achievable fidelity is advancing at a rapid pace (recent $N = 12$ simulation [Illa & Savage, PRL (2023)])
- analog trapped ion devices are an ideal platform to study mid-size systems as the interactions can be embedded in a natural way
- tensor network methods can help push the boundary of classical simulations and identify interesting regimes to study with simulators
- can the spin-model describe neutrinos in supernovae correctly?

Error mitigation with zero-noise extrapolation

Li & Benjamin PRX(2017), Temme, Bravy, Gambetta PRL(2017), Endo,Benjamin,Li PRX(2018)

Zero noise extrapolation

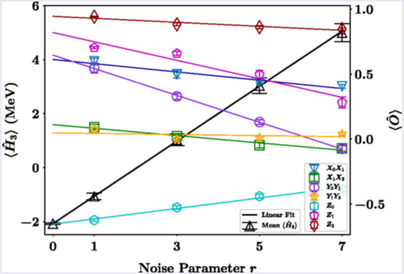
For small enough noise we can write

$$M(\epsilon) = M_0 + \epsilon M_1 + \frac{\epsilon^2}{2} M_2 + \dots$$

Using two points $\epsilon_2 = \eta\epsilon_1$ we have

$$M_0 \approx M(\epsilon_1) - \frac{M(\epsilon_1) - M(\epsilon_2)}{\eta - 1}$$

picture from Dumitrescu et al. PRL(2018)



- for moderate ϵ other parametrizations (like exp) might be more useful

$$M(\epsilon) = M_0 e^{-\alpha\epsilon} \Rightarrow M_0 \approx M(\epsilon_1)\left(\frac{M(\epsilon_2)}{M(\epsilon_1)}\right)^{\frac{\epsilon_1}{\epsilon_1 - \epsilon_2}}$$

In that case it is very beneficial to ensure $M(\epsilon \to \infty) \to 0$ (mitigated B)

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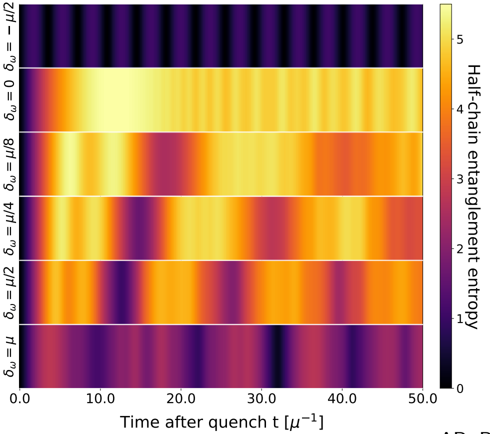

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