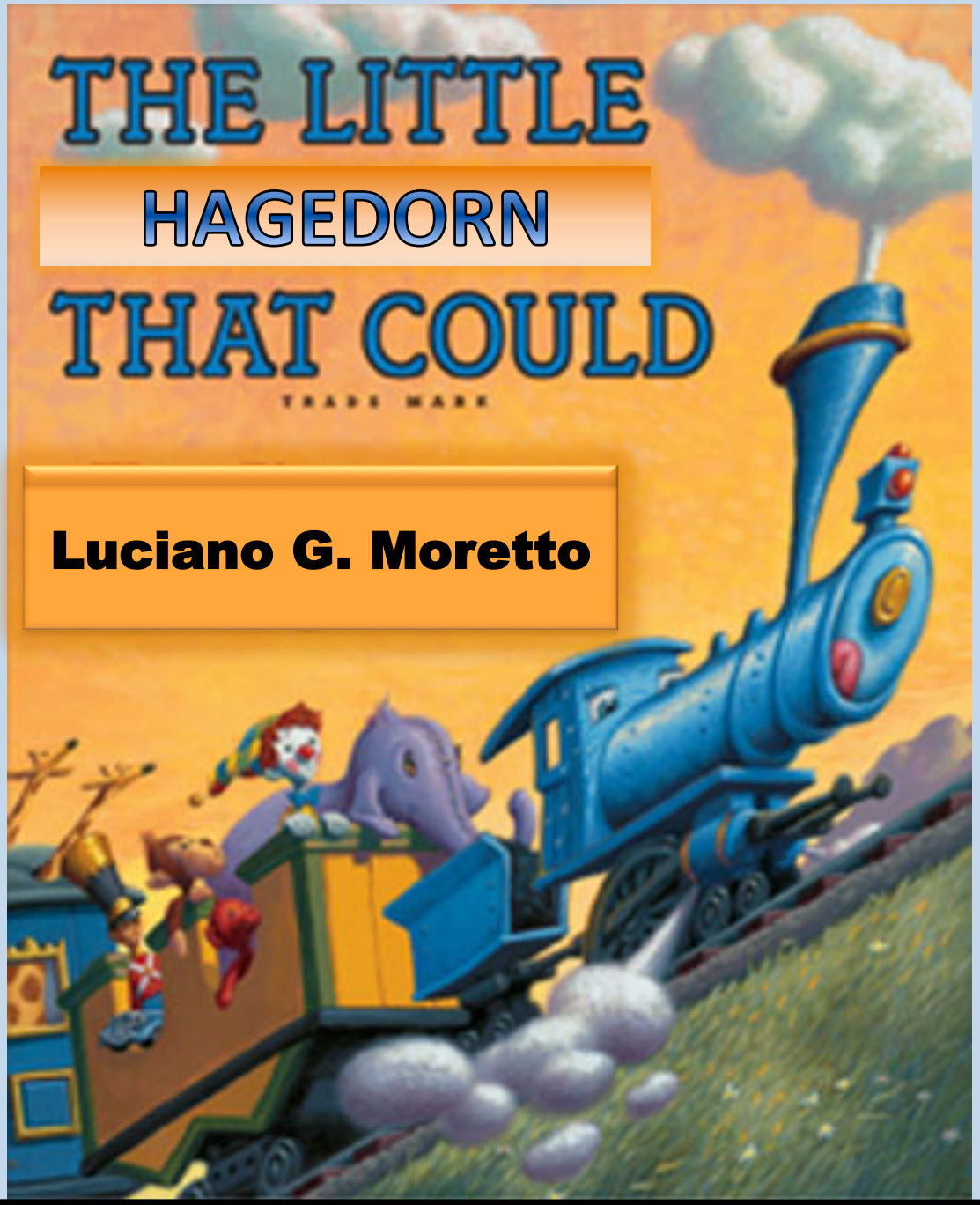
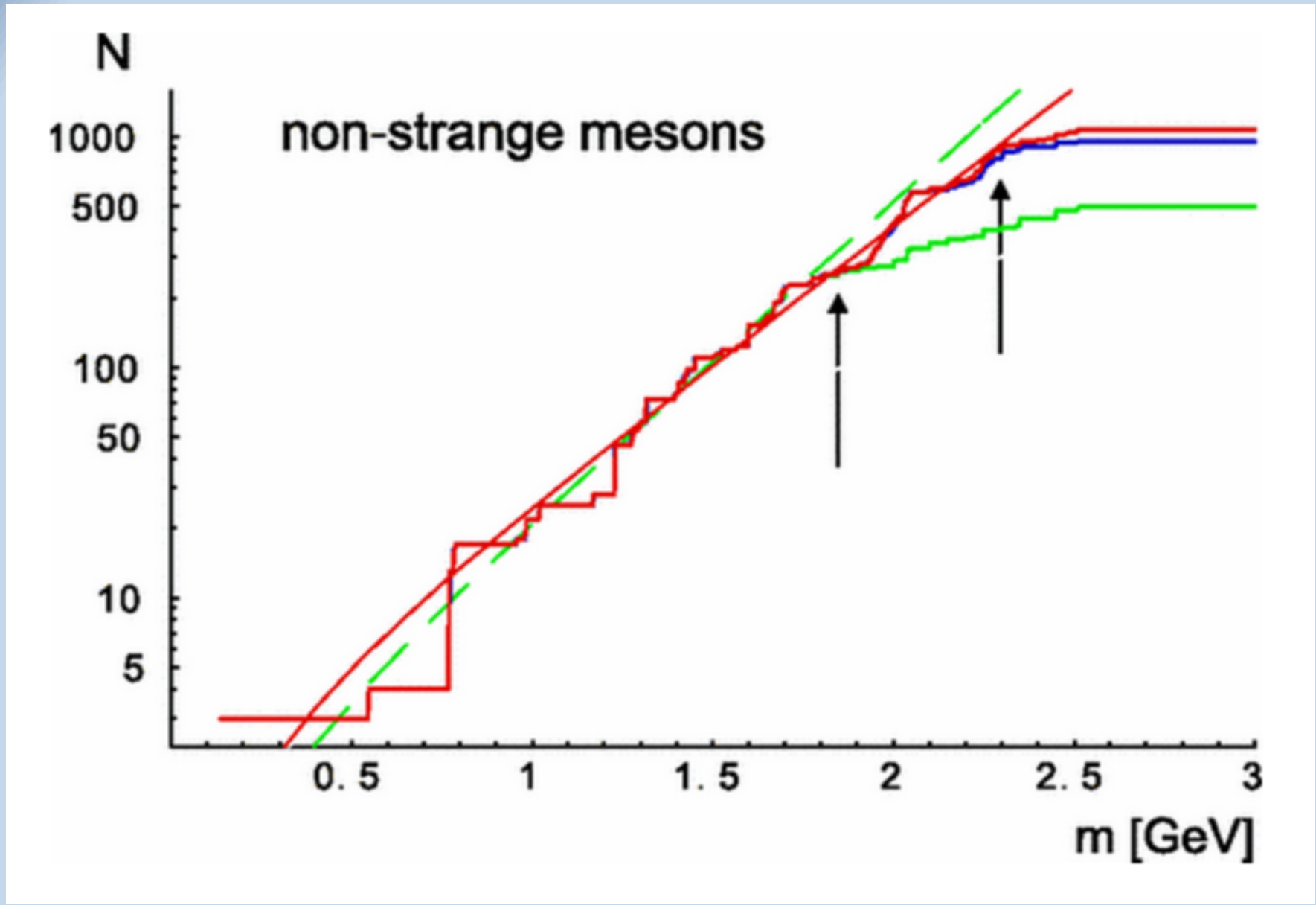
$\ln\rho(E)$


THE LITTLE
HAGEDORN
THAT COULD
TRADE MARK
Luciano G. Moretto

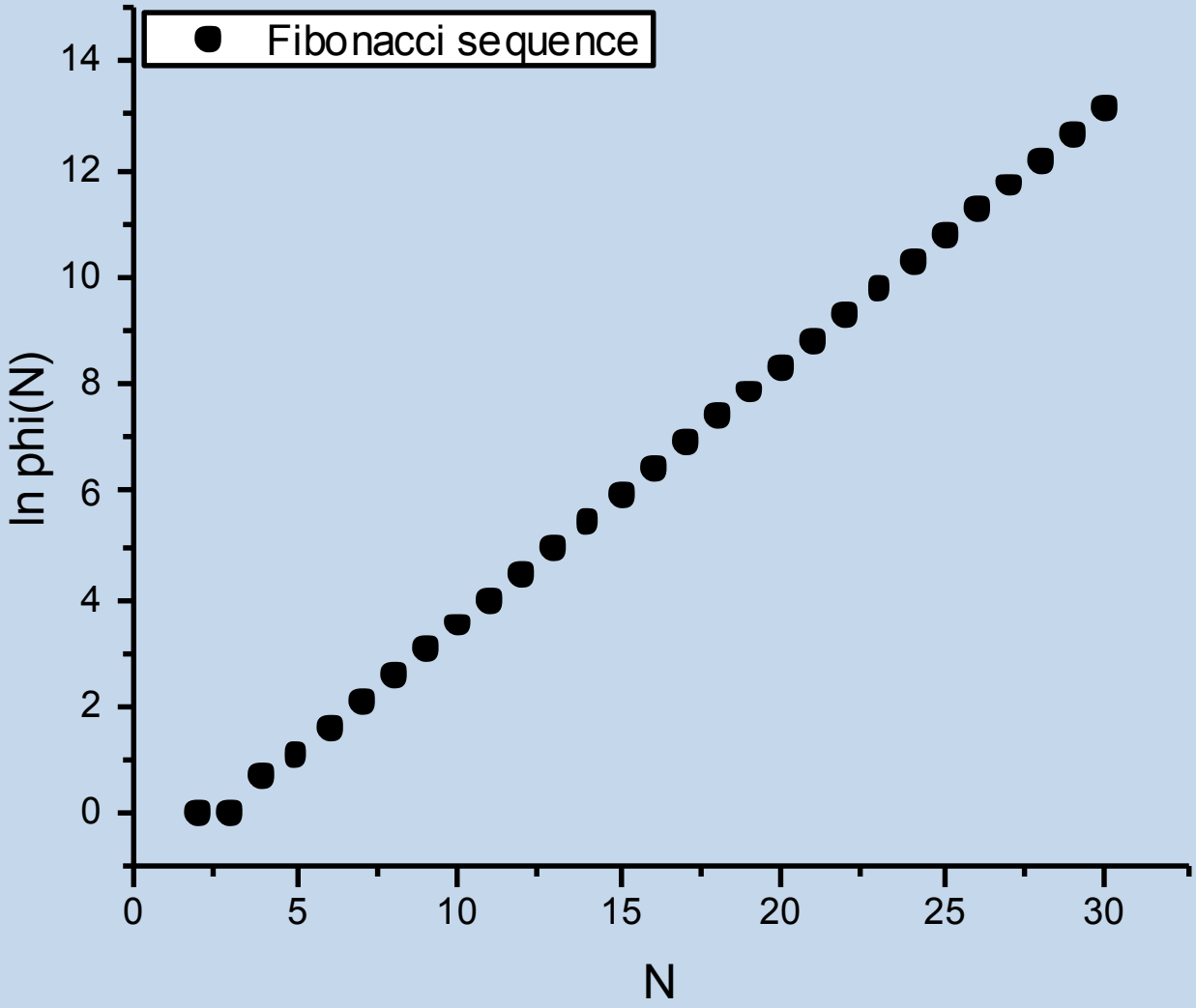
E

The big Hagedorn that could'nt



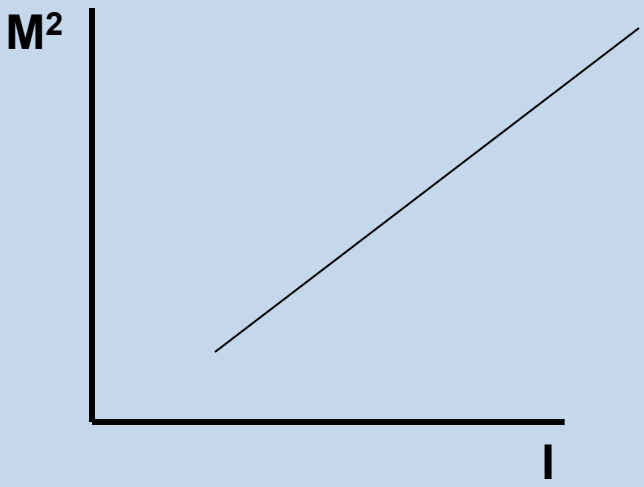
The (too) many ways of obtaining the Hagedorn spectrum (given the experimental evidence!!)

1. Bootstrap
2. Mit Bag
3. Regge Trajectories
4. Fractal shapes (if no surface energy)
5. ----------


Fibonacci sequence
ln phi(N)
N
14
12
10
8
6
4
2
0
0
5
10
15
20
25
30

Regge Trajectories

$M^2 = kl$



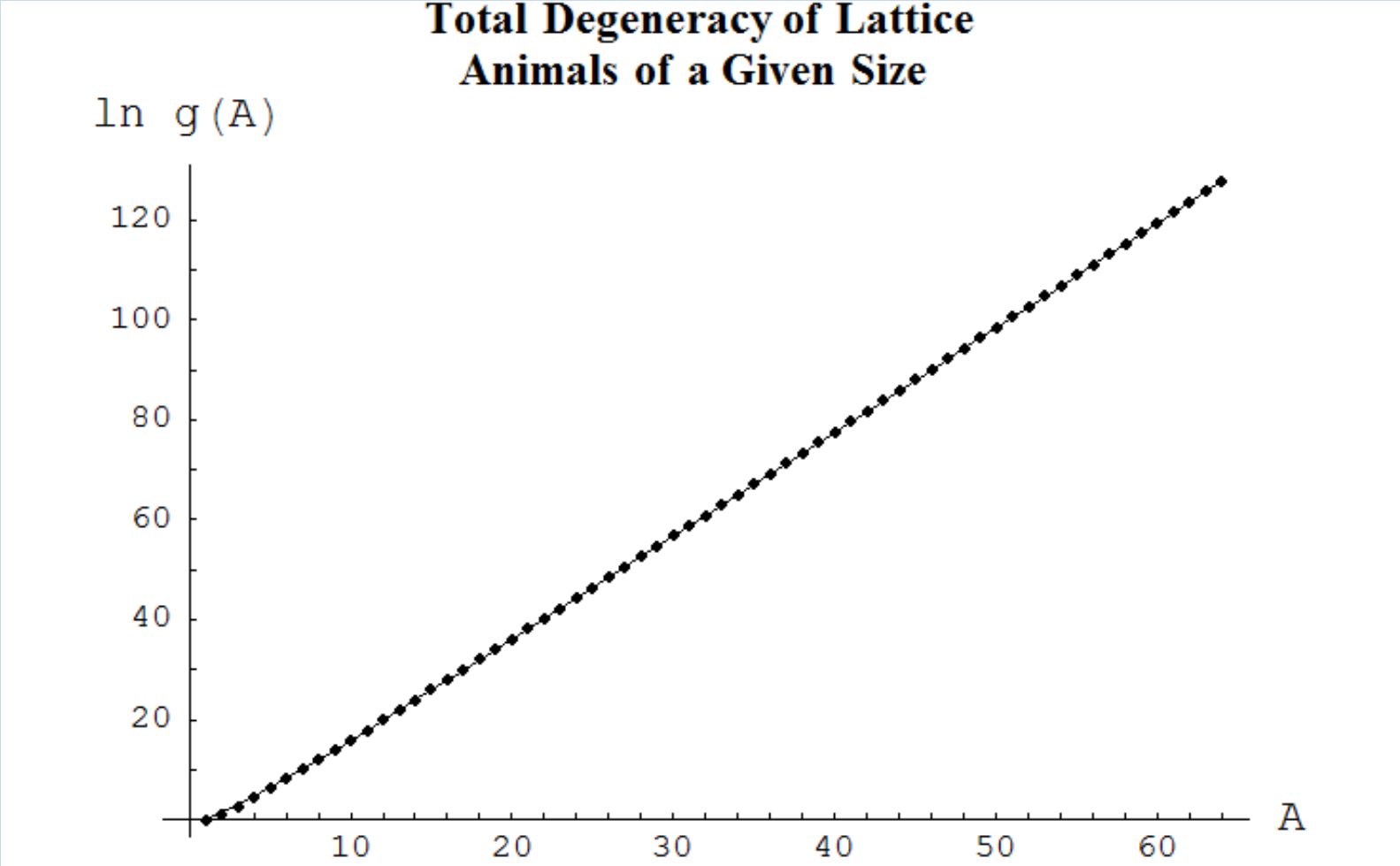
$M^2 = kn\hbar$

In how many ways can one write $n = \sum_{n'' < n} n'$

Euler's Partitio Numerorum

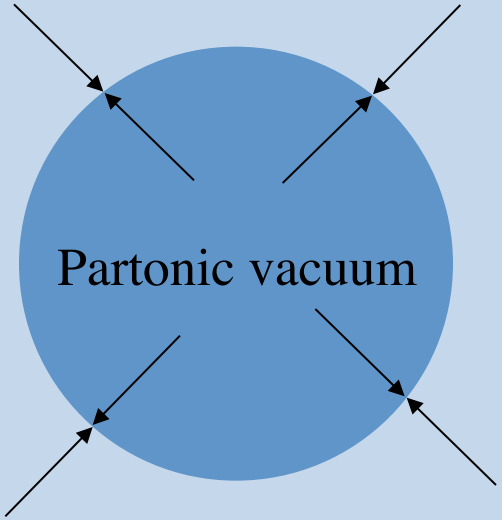
$$P(n) \approx \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{2n/3}} \approx e^{bM} = P(M)$$

$$g(A) = 0.120705\ A^{-1.38675}\ \exp(2.11982\ A)$$



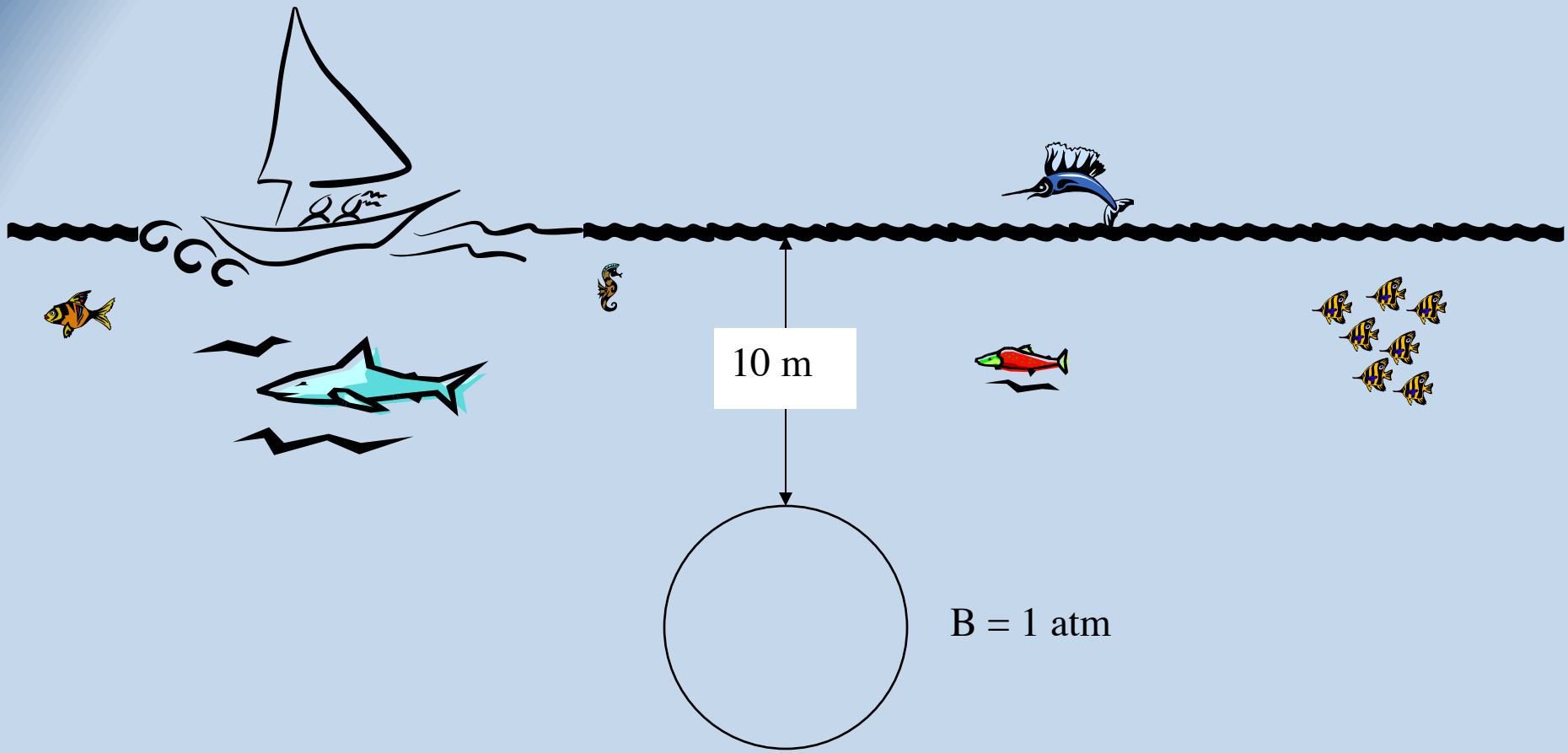
The partonic world (Q.G.P.)
(a world without surface?)

- The M.I.T. bag model says the pressure of a Q.G.P. bag is constant:
  - $p = \frac{g\pi^2}{90} T_{\mathrm{H}}^4 = B$; g: # degrees of freedom, constant $p = B$, constant $T_{\mathrm{H}} = \left(B \frac{90}{g\pi^2} \right)^{\frac{1}{4}}$.
- The enthalpy density is then
  - $\varepsilon = \frac{H}{V} = \frac{E}{V} + p = \frac{g\pi^2}{30} T_{\mathrm{H}}^4 + B$
- which leads to an entropy of
  - $S = \int \frac{\delta Q}{T} = \int_0^H \frac{dH}{T} = \frac{H}{T_{\mathrm{H}}} \equiv \frac{m}{T_{\mathrm{H}}}$
- and a bag mass/energy spectrum (level density) of
  - $\rho(m) = \exp(S) \propto \exp\left(\frac{m}{T_{\mathrm{H}}} \right)$.



Hadronic vacuum

Origin of the bag pressure



- To make room for a bubble of volume V an energy $E = BV$ is necessary.
- To stabilize the bubble, the internal vapor pressure $p(T)$ must be equal to the external pressure B.
- Notice that the surface energy coefficient in this example is not obviously related to the volume energy coefficient.

Primer on first order phase transitions in micro-canonical systems

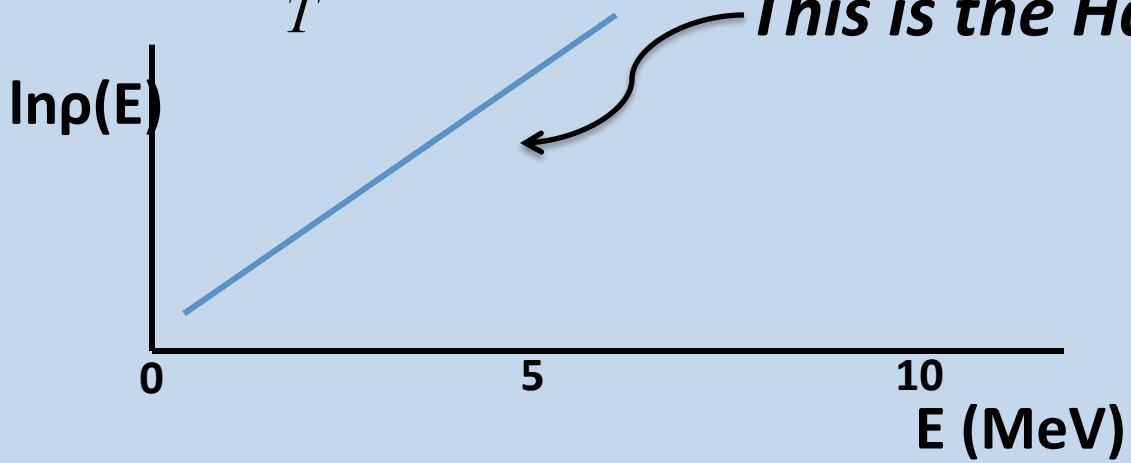
Linear Dependence of Entropy with Energy !

$$\rho(E) = \exp(S) = \exp(\frac{E}{T})$$

$$\frac{\partial S}{\partial E} = \frac{1}{T}$$

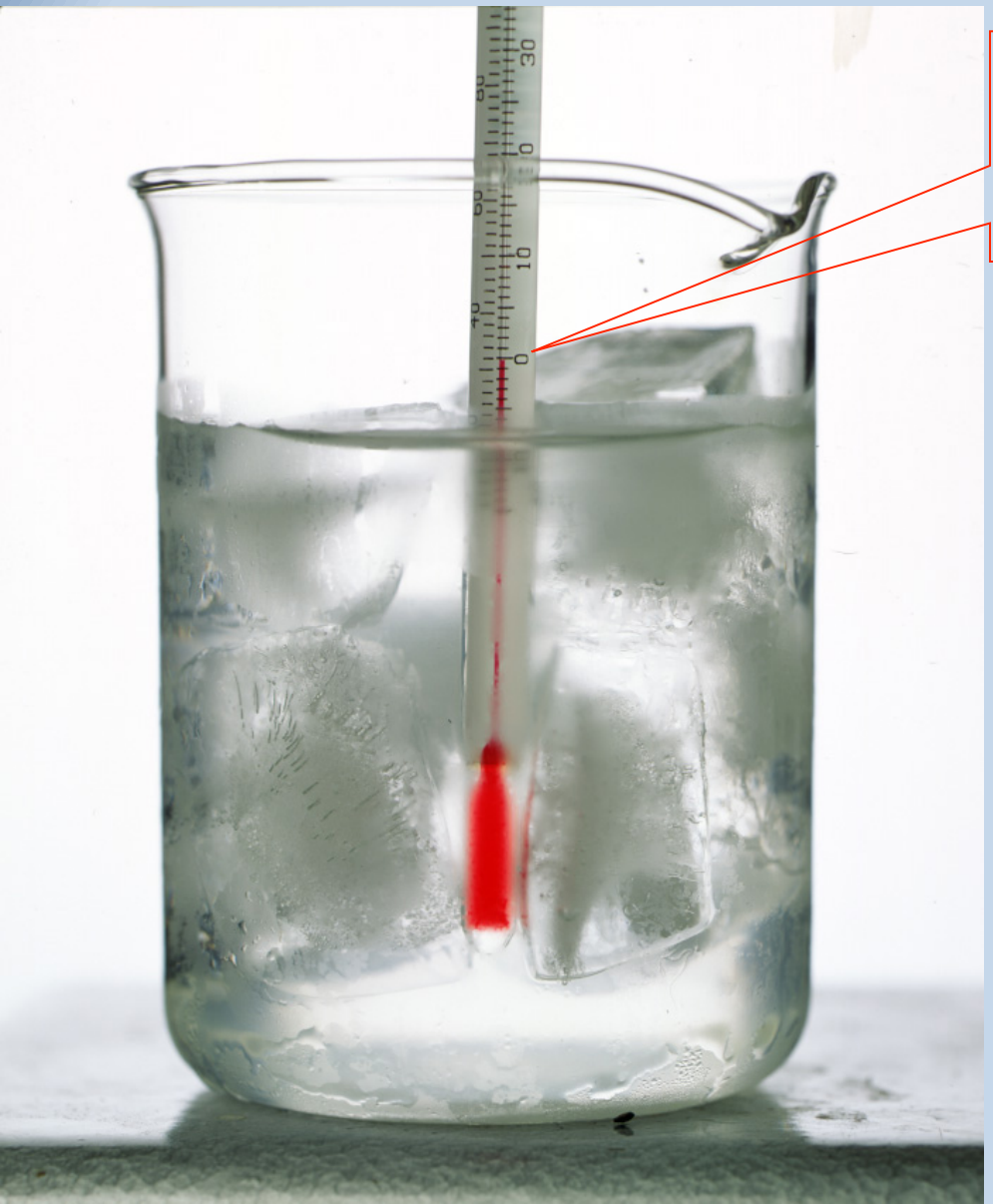
$$S(E) = KE = \frac{E}{T}$$

This is the Hagedorn!



Energy goes in, Temperature stays the same

Can a "thermostat" have a temperature other than its own?



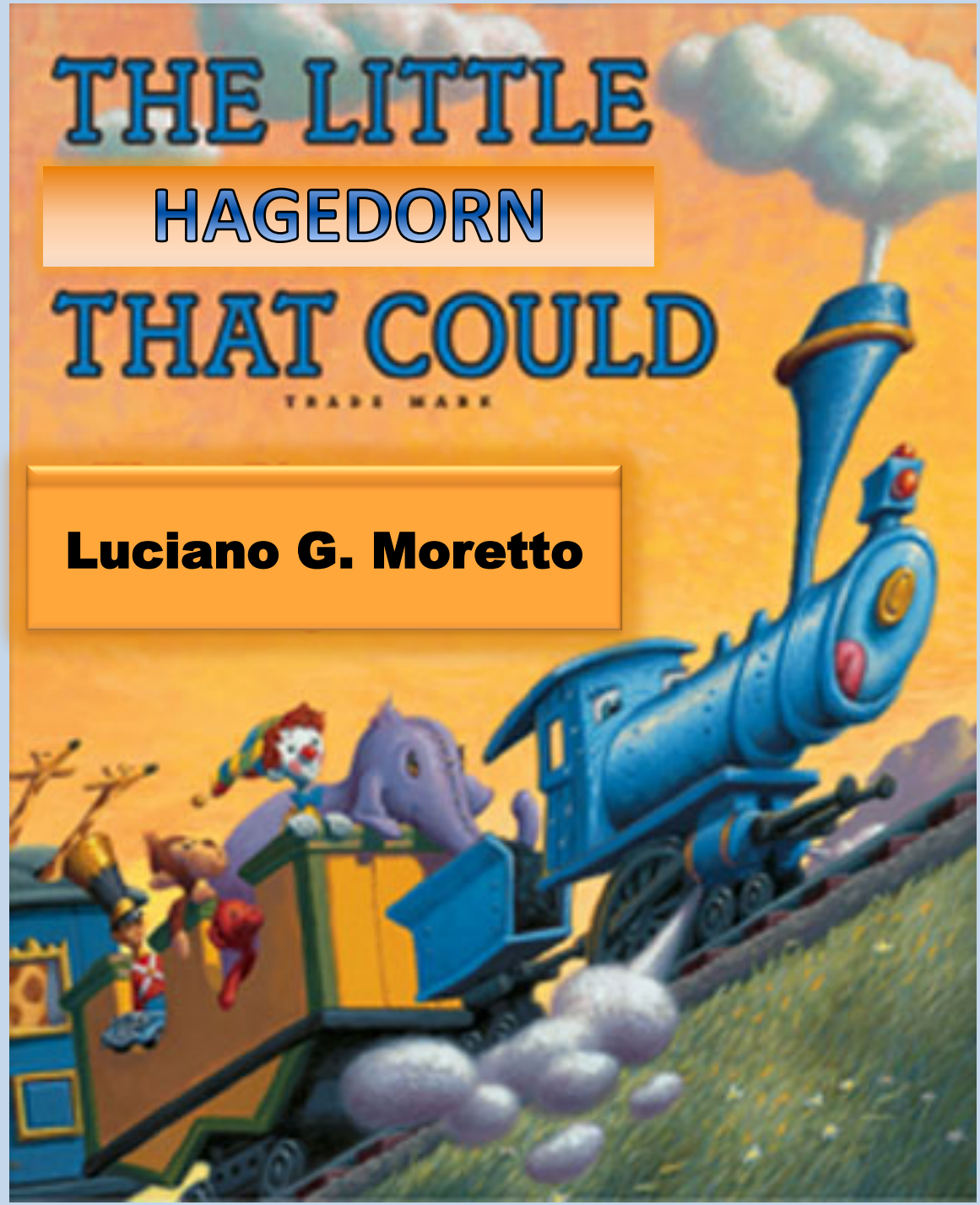
$T = T_c = 273\text{K}$

or

$0 \le T \le 273\text{K}$

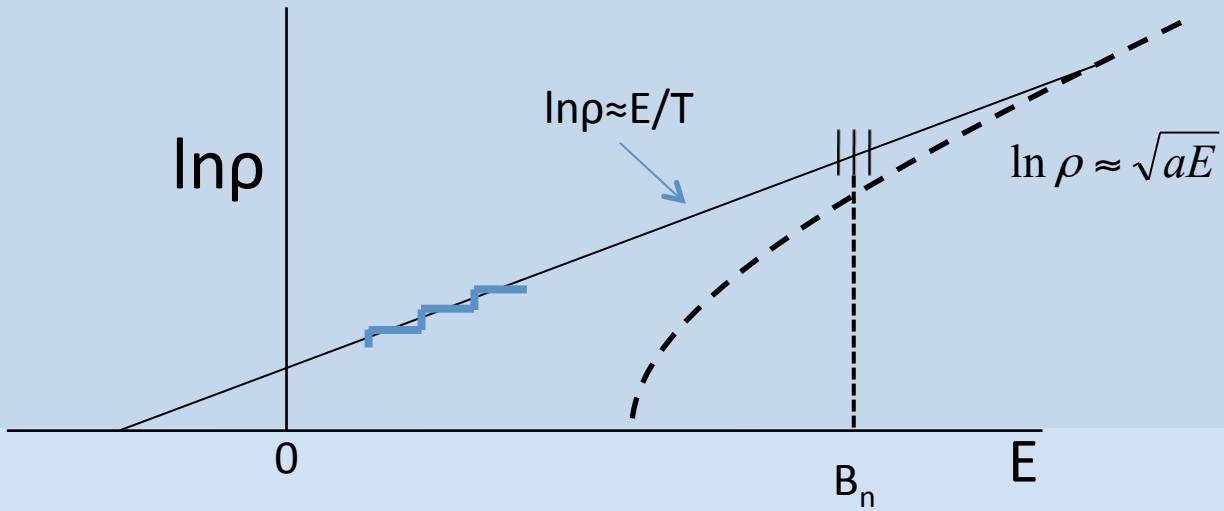
?

- $S = S_0 + \dfrac{\Delta Q}{T} = S_0 + \dfrac{E}{T_0}$
- $\rho(E) = e^S = e^{S_0 + \frac{E}{T_0}}$
- Is T_0 just a "parameter"?
- $Z(T) = \int dE \rho(E) e^{-E/T} = \dfrac{T_0 T}{T_0 - T} e^{S_0}$
- According to this, a thermostat, can have any temperature lower than its own!


THE LITTLE
HAGEDORN
THAT COULD
TRADE MARK
Luciano G. Moretto

Memories….

Gilbert and Cameron



Low energy level counting …..exponential?
Neutron resonances ……. 1 point
Higher energy……..Fermi gas

Global Solution : matching a Fermi Gas to an exponential dependence

Away from shells $T_{G.C.} \approx T_{Cr\ pairing} = 2\Delta/3.5$

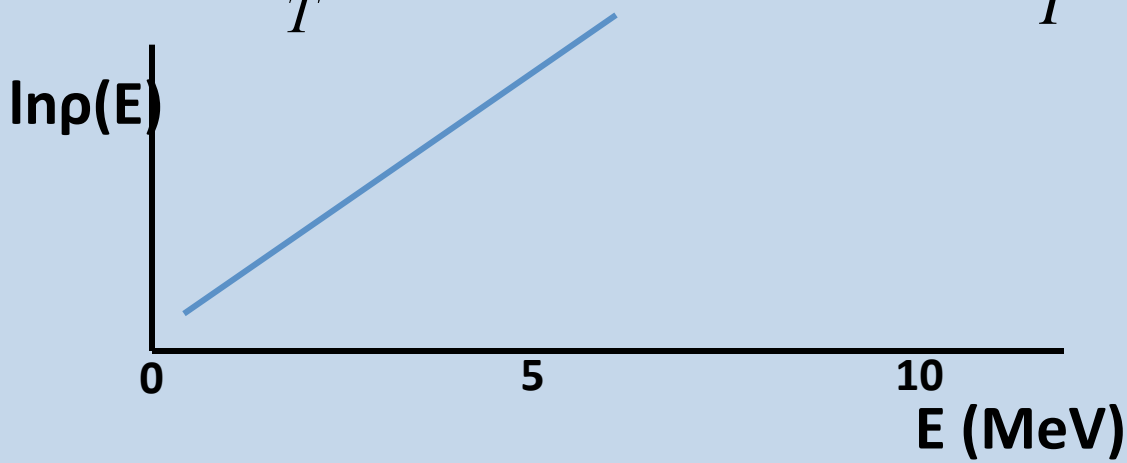
Universal 1st Order Low Energy Phase Transition in Nuclei and the Magic of Parallels

L. G. Moretto, A.C. Larsen, M. Guttormsen and S. Siem

Hallmark of 1st order phase transition in micro-canonical systems?

Linear Dependence of Entropy with Energy !

$$S(E) = KE = \frac{E}{T}$$ **or** $$\rho(E) = \exp(\frac{E}{T})$$

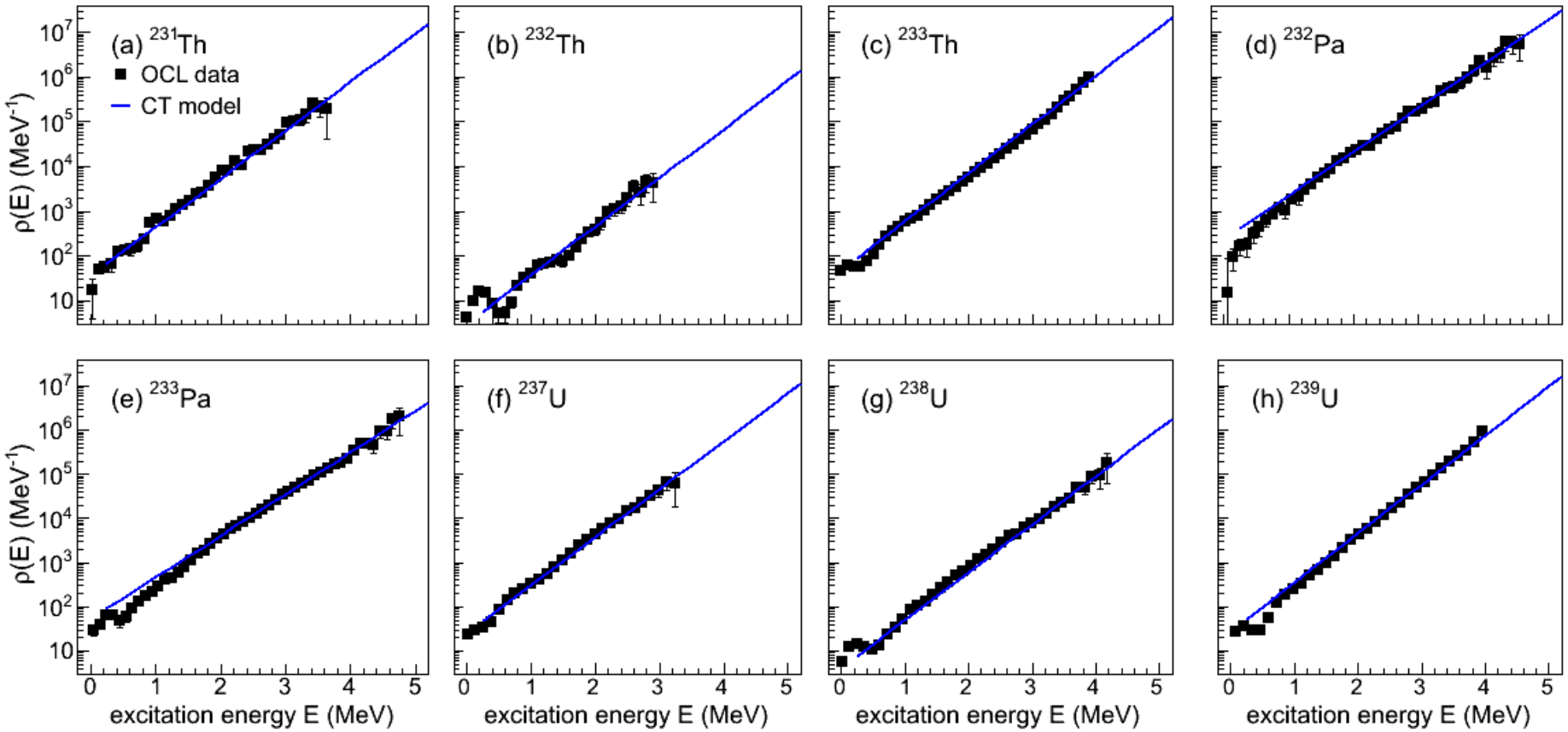


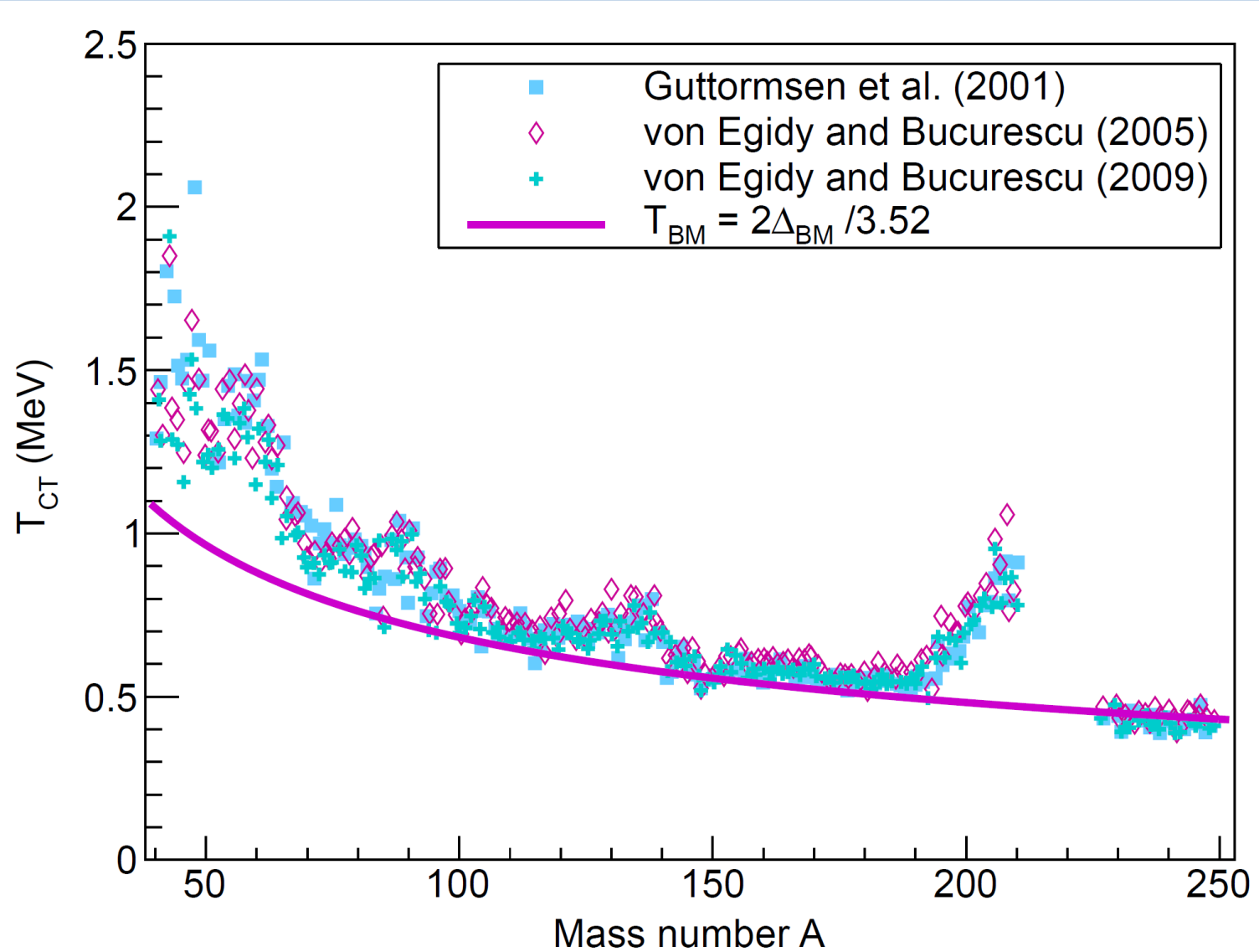
This is universally observed in low energy nuclear level densities

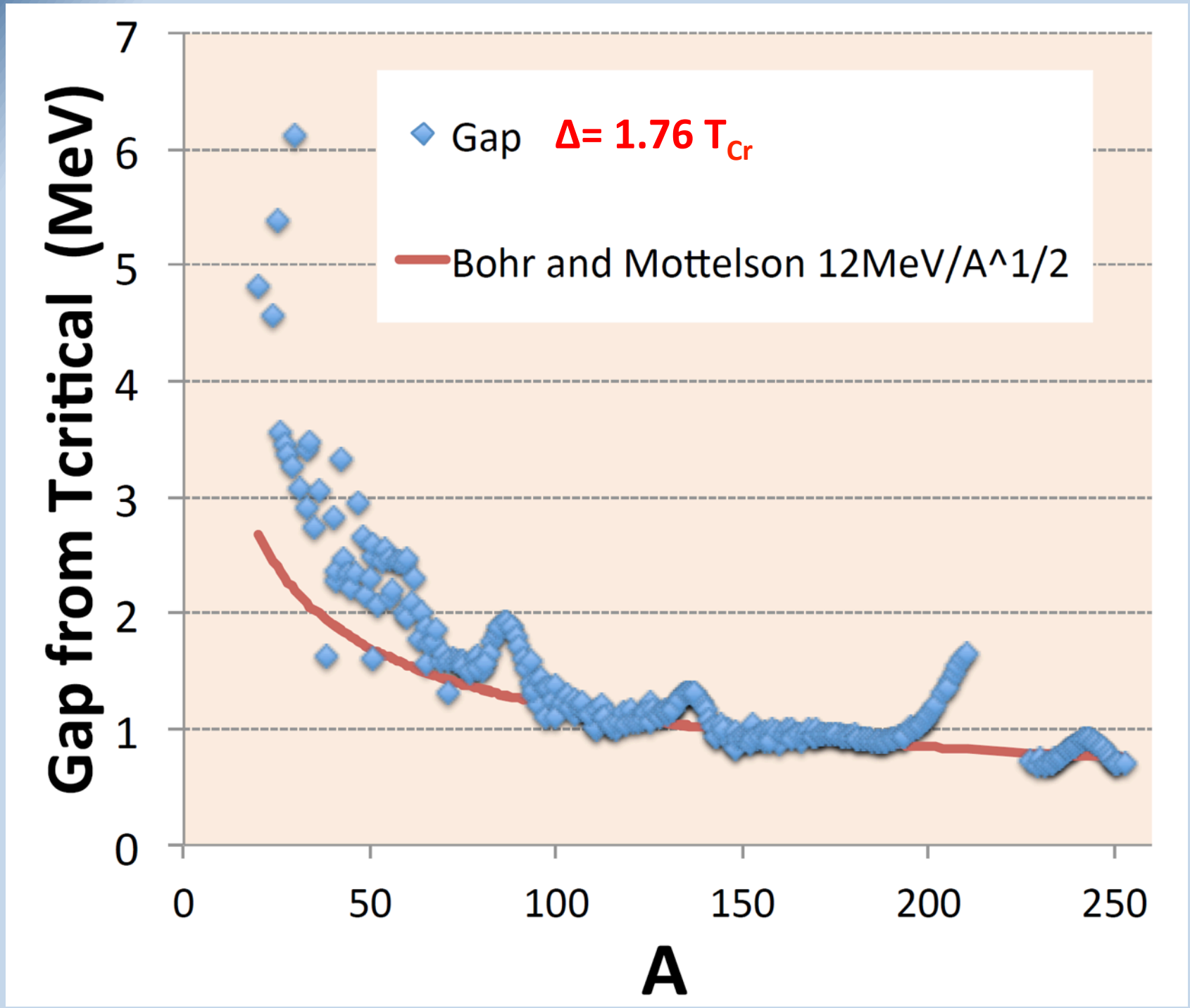
T is the micro-canonical temperature characterizing the phase transition

Energy goes in, Temperature stays the same

Level densities, actinides




Guttormsen et al. (2001)
von Egidy and Bucurescu (2005)
von Egidy and Bucurescu (2009)
$T_{BM} = 2\Delta_{BM}/3.52$
T_{CT} (MeV)
Mass number A
0
0.5
1
1.5
2
2.5
50
100
150
200
250


Gap from Tcritical (MeV)
7
6
5
4
3
2
1
0
0
50
100
150
200
250
A
Gap
Δ= 1.76 T_Cr
Bohr and Mottelson 12MeV/A^1/2

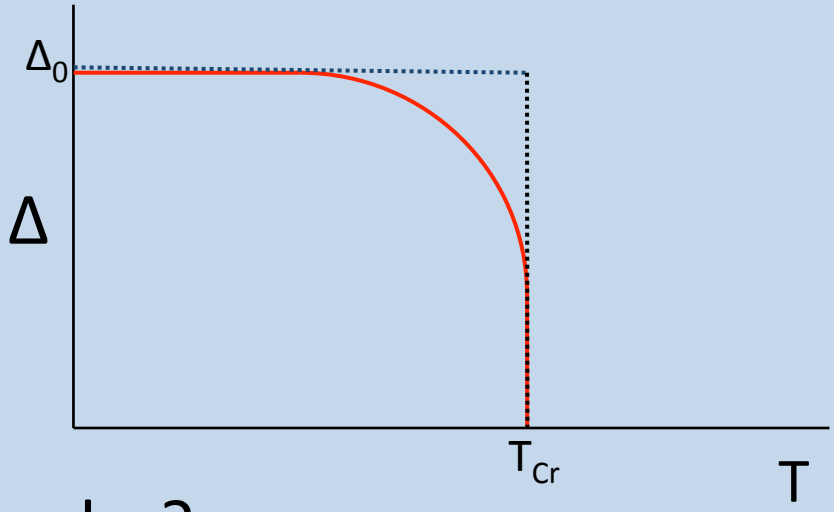
What causes the phase transition?

1. In non magic nuclei — Pairing

2. In magic nuclei — Shall gap

BCS Phase Transition

2nd order



Nearly 1st order?

$$Q_{Cr} = 4\ln 2 g T_{Cr}$$

 # quasi particle at T_{Cr}

$$E_{Cr} = \frac{1}{2} g\Delta_0^2 + \frac{\pi^2}{3} g T_{Cr}^2$$

Energy at criticality

$$\frac{E_{Cr}}{Q_{Cr}} = \frac{3.53\pi}{16\ln 2}\Delta_0 = \Delta_0$$

!

Fixed energy cost per quasi particle up to criticality : little blocking ?

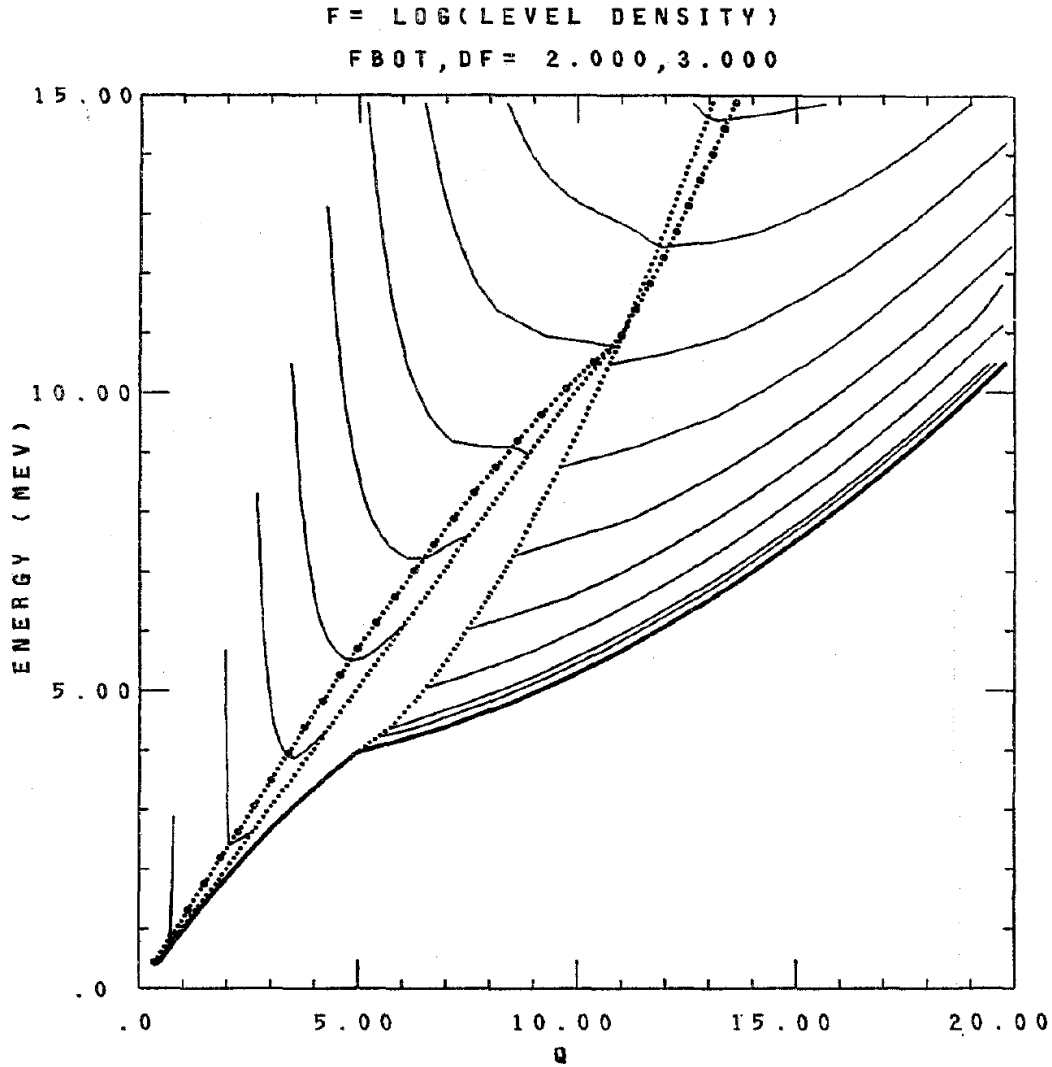
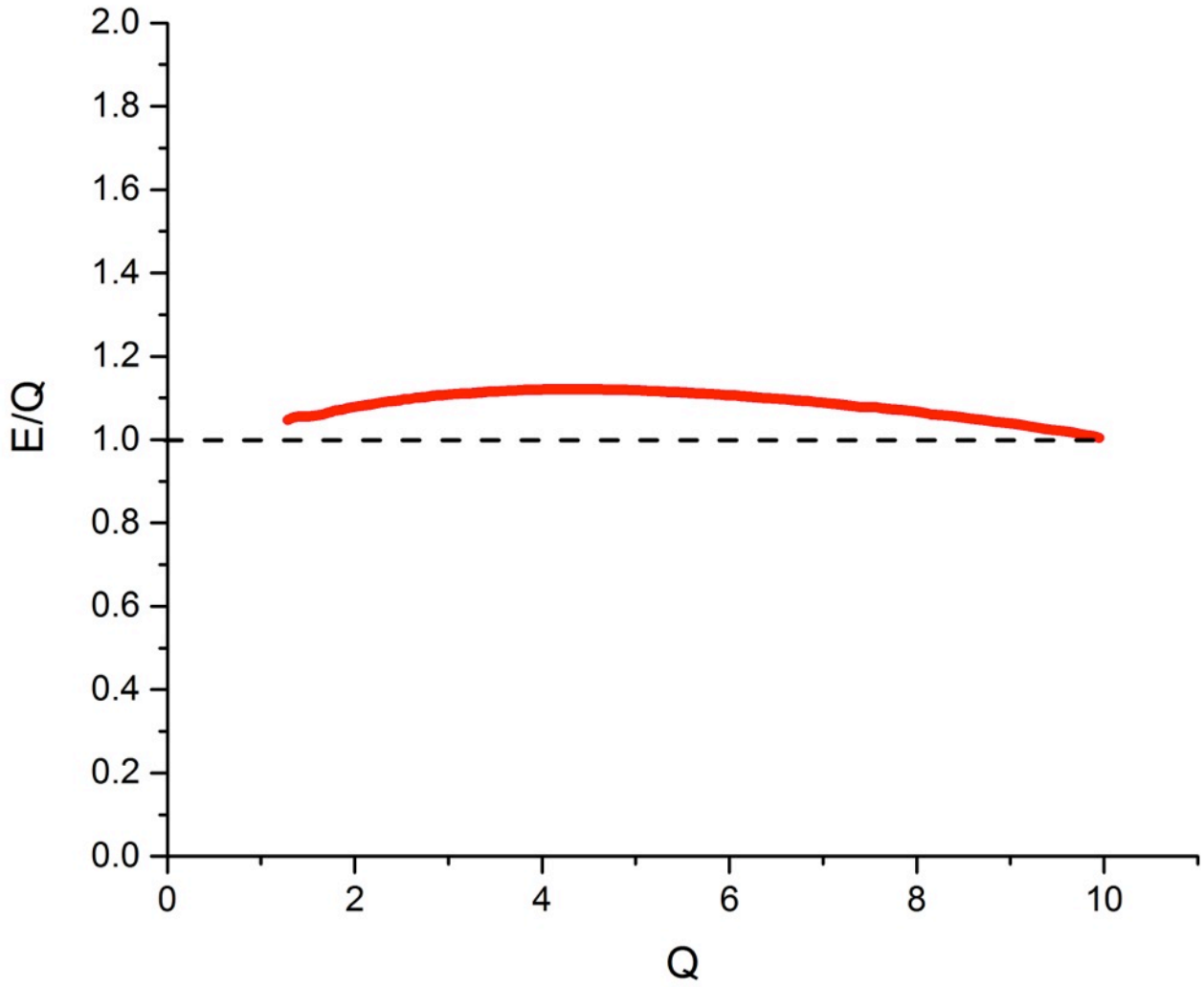


Fig. 14. Lines of constant level densities in the E, Q plane. The calculation refers specifically to a nucleus with $g = 7.0\ \mathrm{MeV}^{-1}$ and with $\Delta_0 = 1.0$ MeV. The lowest level density line has a value $\ln\rho = 2.0$. The higher lines are plotted in steps of $3.0 \ln\rho$.


2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
E/Q
0
2
4
6
8
10
Q

Pairing: Fixed Energy cost/ quasi particle up to T_{CR} !

Is this consistent with blocking?

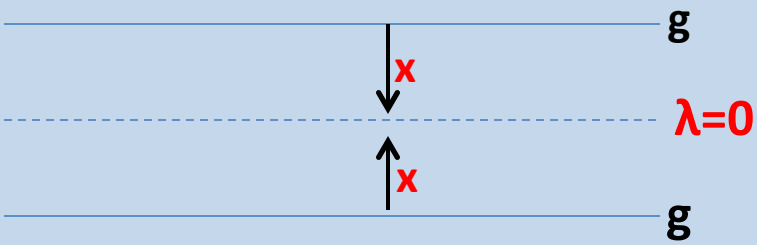
$$E_k = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$$

Δ goes down

$(\varepsilon_k - \lambda)$ goes up

Proof:

$$E_{Cr}/Q_{Cr} = 1/2$$



for x=0 $\quad E_{Cr}/Q_{Cr} = ½\ \Delta_0$

for x>0 $\quad E_{Cr}/Q_{Cr} \rightarrow \Delta_0$

1st order phase transition implies two phases

Superfluid phase ⟷ **gas of independent quasi particles**

What fixes the transition temperature?

constant entropy per quasi particle

Remember Sackur Tetrode

$$S = N\ln\left(\frac{V}{N}\frac{4}{3}\frac{\pi p^3}{h^3}\right) = N\ln(\#\,states\,/\,quasiparticle)$$

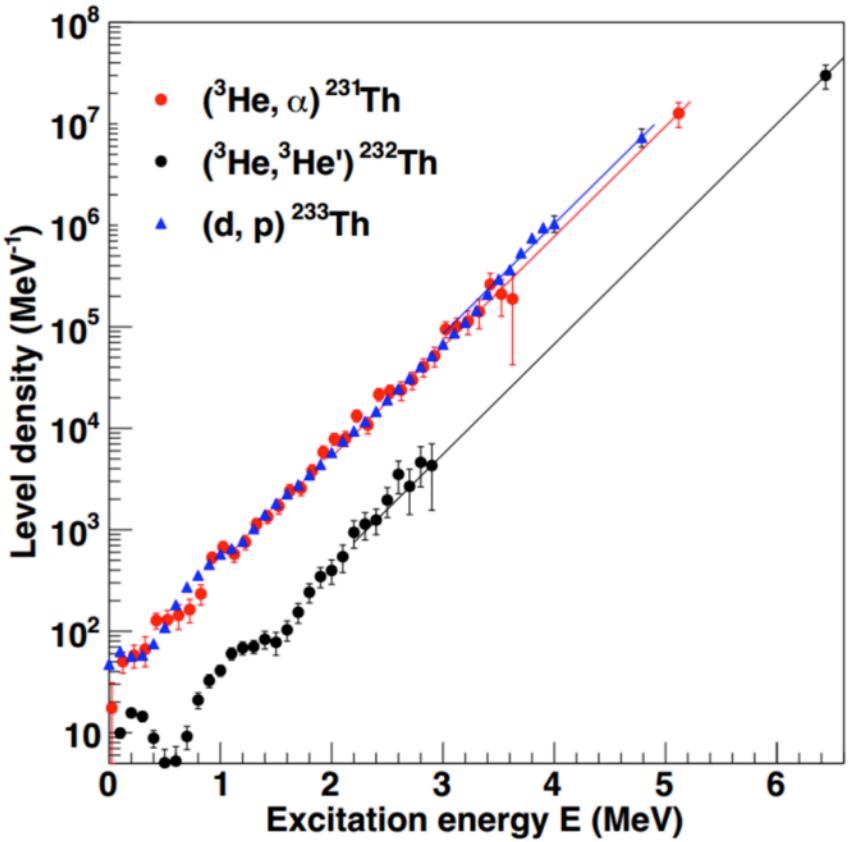
Entropy / Quasi Particle

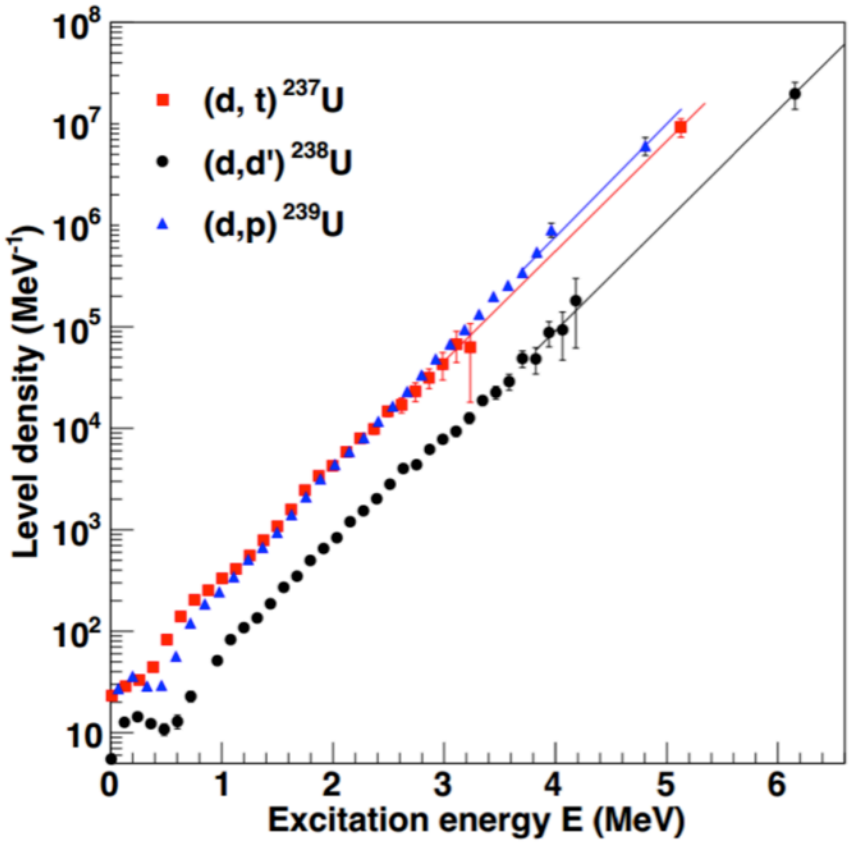
$$S = \frac{Q\Delta_0}{T_{Cr}} = \frac{Q\Delta_0 3.53}{2\Delta_0}$$

$$\frac{S}{Q} = \frac{3.53}{2} = 1.76$$

Constant-temperature level densities in the quasi-continuum of Th and U isotopes

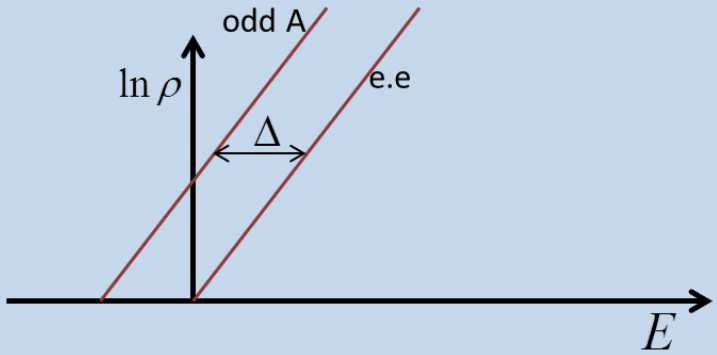
M. Guttormsen,[1,*] B. Jurado,[2] J.N. Wilson,[3] M. Aiche,[2] L.A. Bernstein,[4] Q. Ducasse,[2]
F. Giacoppo,[1] A. Görgen,[1] F. Gunsing,[5] T.W. Hagen,[1] A.C. Larsen,[1] M. Lebois,[3] B. Leniau,[3]
T. Renstrøm,[1] S.J. Rose,[1] S. Siem,[1] T. Tornyi,[1] G.M. Tveten,[1] and M. Wiedeking[6]





Testing the picture:

a) Even-Odd horizontal shift….



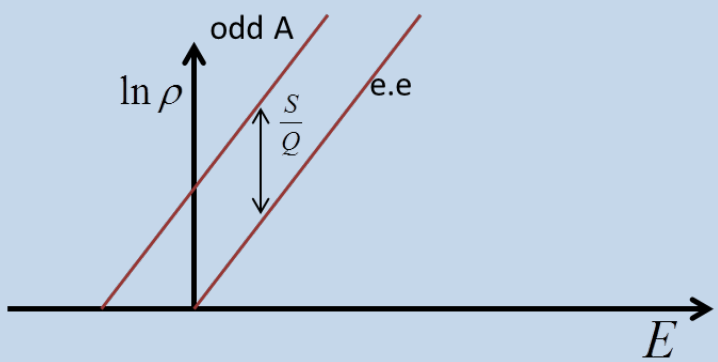
should be compared with even-odd mass differences

$$A = \frac{12}{A^{\frac{1}{2}}} MeV$$

b) Relationship between the above shift and the slope 1/T

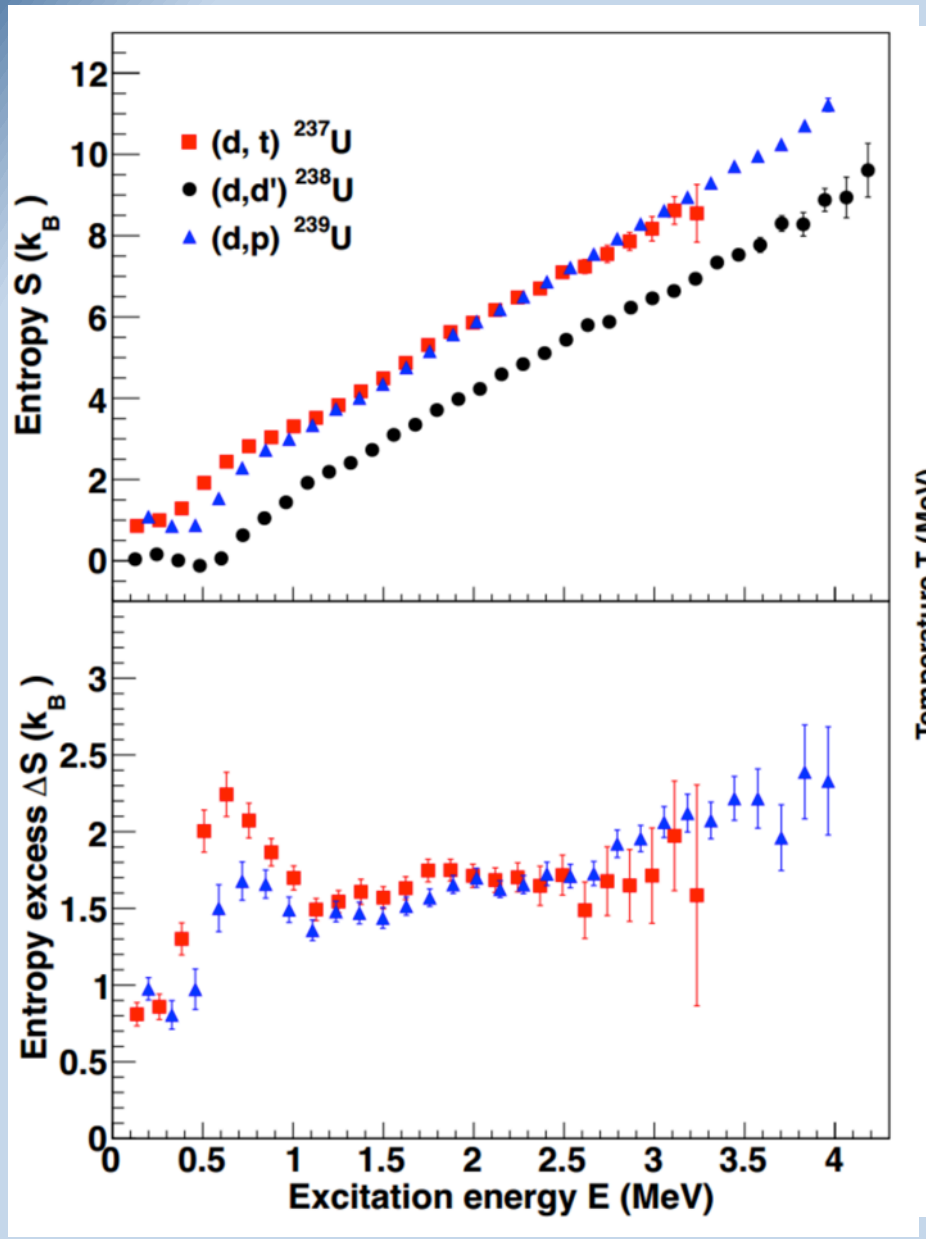
$$T = \frac{2\Delta}{3.53} MeV$$

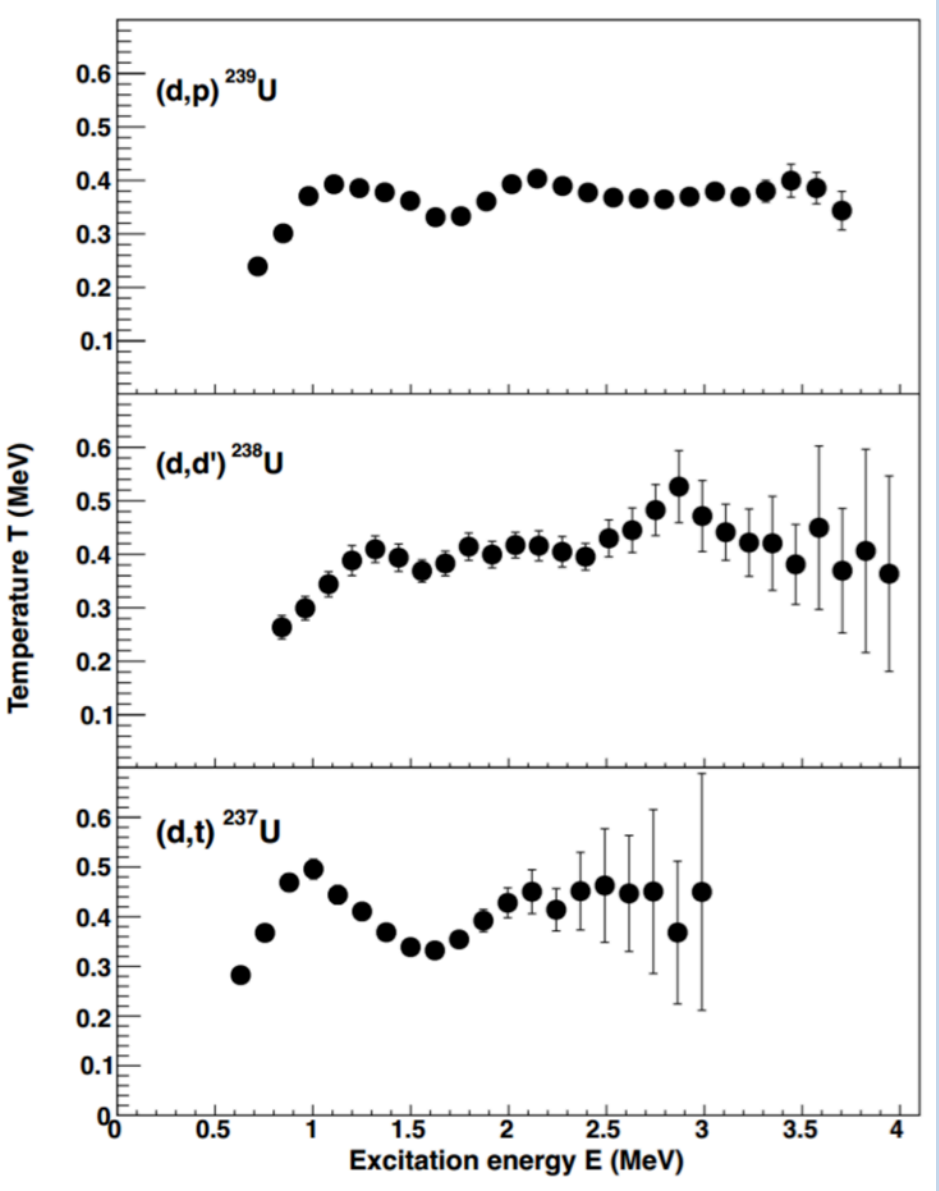
c) Vertical shift or "entropy excess"



$$\frac{S}{Q} = \frac{3.53}{2} = 1.76$$

Nuclide	T_{CT} (MeV)	Δ_{CT} (MeV)	T_{BM} (MeV)	Δ_{BM} (MeV)	T_{eo} (MeV)	Δ_{eo} (MeV)	ΔS (k_B)
^{148}Sm	0.51(1)	0.90(2)	0.56	0.99	–	–	–
^{149}Sm	0.46(1)	0.81(2)	0.56	0.98	0.51(6)	0.9(1)	2.0(2)
^{160}Dy	0.60(1)	1.05(1)	0.54	0.95	–	–	–
^{161}Dy	0.58(2)	1.01(3)	0.54	0.95	0.57(6)	1.0(1)	1.9(2)
^{162}Dy	0.60(1)	1.05(2)	0.54	0.94	–	–	–
^{163}Dy	0.57(2)	1.00(4)	0.53	0.94	0.51(6)	0.9(1)	1.9(2)
^{164}Dy	0.56(1)	0.99(1)	0.53	0.94	–	–	–
^{166}Er	0.52(1)	0.92(1)	0.53	0.93	–	–	–
^{167}Er	0.56(2)	0.99(4)	0.53	0.93	0.51(6)	0.9(1)	2.0(2)
^{170}Yb	0.57(1)	1.00(1)	0.52	0.92	–	–	–
^{171}Yb	0.55(1)	0.96(1)	0.52	0.92	0.51(6)	0.9(1)	1.8(2)
^{172}Yb	0.54(1)	0.95(1)	0.52	0.91	–	–	–
^{231}Th	0.41(1)	0.72(2)	0.45	0.79	0.51(11)	0.9(2)	2.4(4)
^{232}Th	0.34(1)	0.60(2)	0.45	0.79	–	–	–
^{233}Th	0.40(2)	0.70(2)	0.45	0.79	0.51(11)	0.9(2)	2.3(4)
^{232}Pa	0.44(1)	0.77(2)	0.45	0.79	0.40(6)	0.7(1)	1.7(2)
^{233}Pa	0.45(1)	0.79(2)	0.45	0.79	–	–	–
^{237}U	0.40(1)	0.70(2)	0.44	0.78	0.40(6)	0.7(1)	1.9(2)
^{238}U	0.42(1)	0.74(2)	0.44	0.78	–	–	–
^{239}U	0.37(1)	0.65(1)	0.44	0.78	0.37(3)	0.65(5)	2.5(5)


(d, t) 237U
(d,d') 238U
(d,p) 239U
Entropy S (kB)
Entropy excess ΔS (kB)
Excitation energy E (MeV)


(d,p) 239U
(d,d') 238U
(d,t) 237U
Temperature T (MeV)
Excitation energy E (MeV)

Low energy level densities for nuclei away from shells vademecum for beginners…….

1) Get T_{Cr} from $\Delta=12/A^{1/2}$

2) Write $\ln\rho(E)=S(E)=E/T$

3) Shift horizontally by Δ or 2Δ for odd or odd-odd nuclei

Conclusions

1) The "universal" linear dependence of $S=\ln\rho$ with E at low energies is a clear cut evidence of a first order phase transition

2) In non magic nuclei the transition is due to pairing. The coexisting phases are
   a) superfluid
   b) ideal gas of quasi particles

1) In magic nuclei the transition is due to the shell gap