Structure of light hypernuclei in the framework of Fermionic Molecular Dynamics

Martin Schäfer, Jiří Mareš

Nuclear Physics Institute, Řež, Czech Republic

H. Feldmeier, T. Neff

GSI Helmholtzzentrum für Schwerionenforschung, Darmstadt, Germany

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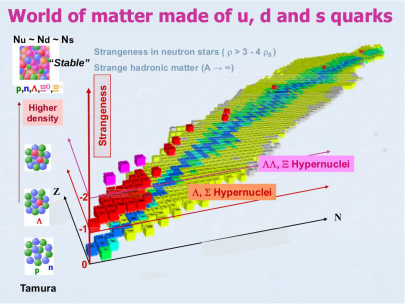
Hypernuclei

Hypernucleus

- nuclear system which contains besides nucleons and protons also one or more **hyperons**



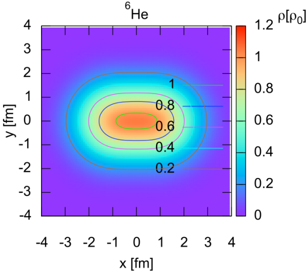
Why to study hypernuclei ?

- test models of BB interactions (meson exchange models, quark models, chiral models, ...)
- test nuclear models (RMF, RPA, NCSM, FMD, ...)
- test models of hadrons (SU(3) symmetry, quark models, ...)
- hypernuclear production (test reaction mechanisms)
- hypernuclear decays (study of weak interaction)
- **no Pauli blocking for hyperons** (probe the nuclear core)
- astrophysics (neutron stars, ...)

Introduction

Main goal

- study of light hypernuclei (shell vs. cluster structure)
- information about the ΛN interaction
- modification of the nuclear core due to Λ



$\xrightarrow{\Lambda}$?

Objectives :

- develop Fermion Molecular Dynamics for hypernuclei
- calculations of ground and excited states of s-shell (p-shell) hypernuclei (Λ separation energy B_Λ, ρ_N and ρ_Λ densities, rms radii)

Fermionic Molecular Dynamics

(H. Feldmeier, Nucl. Phys. **A 515** (1990) 147)
(T. Neff, H. Feldmeier, Nucl. Phys. **A 738** (2004) 367)

system of interacting fermions described by an antisymmetrized many-body function $|Q\rangle$

Antisymmetrization

- many-body wave function approximated by a **Slater determinant**

spatial part of a single particle state represented by a **Gaussian wave packet**

$$\langle \vec{x} | \, q_k \rangle = \exp\left(-\frac{(\vec{x} - \vec{b}_k)^2}{2a_k} \right) \otimes \left| \chi_k^{\uparrow}, \chi_k^{\downarrow} \right\rangle \otimes |t\rangle$$

- complex width a_k, complex $\vec{b}$, complex $\chi^{\uparrow}$ and $\chi^{\downarrow}$ spin parameters (12 real parameters for each particle)

Minimization

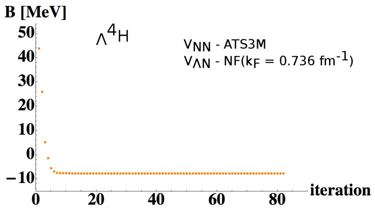
Time-independent variational calculation

$$E_{min} = \min_{q_1,\dots,q_n} \frac{\langle Q | \hat{T}_k + \hat{V}_{NN} + \hat{V}_{\Lambda N} - \hat{T}_{cm} | Q \rangle}{\langle Q | Q \rangle}$$

- gradient method (gradients evaluated analytically to ensure numerical stability)
- minimization with respect to single particle state parameters $q_k = \{a_k, \vec{b}_k, \chi_k^{\uparrow}, \chi_k^{\downarrow}\}$

Result

- minimization yields an **intrinsic state** which is not parity and total angular momentum eigenstate J^{π}
- **broken symmetries** have to be **restored**



Symmetries

(T. Neff, H. Feldmeier, Eur. Phys. **J 156** (2008) 69)

Parity projection

- parity projected state $|Q;\pi\rangle = \hat{P}^{\pi}\,|Q\rangle$

$$\hat{P}^{\pi} = \frac{1}{2}(\hat{1} + \pi\hat{\Pi})$$

Total angular momentum projection

- total angular momentum eigenstate is projected out of the minimized intrinsic state

$$|Q; J^{\pi} MK\rangle = \hat{P}^{J}_{MK}\hat{P}^{\pi}\,|Q\rangle$$

- total angular momentum projector $\hat{P}^{J}_{MK}$

$$\hat{P}^{J}_{MK} = \frac{2J+1}{8\pi^2}\int d\Omega D^{J\,\star}_{MK}(\Omega)\hat{R}(\Omega)$$

K-mixing

(T. Neff, H. Feldmeier, Eur. Phys. **J 156** (2008) 69)

Orthogonal eigenstates

$$|Q; J^{\pi} M \kappa\rangle = \sum_{K} |Q; J^{\pi} M K\rangle \, C_{K}^{J^{\pi} \kappa}$$

Generalized eigenvalue problem

$$(\hat{H} - \hat{T}_{cm}) \, |Q; J^{\pi} M \kappa\rangle = E^{J^{\pi} \kappa} \, |Q; J^{\pi} M \kappa\rangle$$

$$\sum_{K'} H^{J^{\pi}}_{K,K'} \, C_{K}^{J^{\pi} \kappa} = E^{J^{\pi} \kappa} \sum_{K'} N^{J^{\pi}}_{K,K'} \, C_{K}^{J^{\pi} \kappa}$$

$$H^{J^{\pi}}_{K,K'} = \langle Q| \, (\hat{H} - \hat{T}_{cm}) \hat{P}^{J}_{KK'} \hat{P}^{\pi} \, |Q\rangle$$
$$N^{J^{\pi}}_{K,K'} = \langle Q| \, \hat{P}^{J}_{KK'} \hat{P}^{\pi} \, |Q\rangle$$

V_{NN} and $V_{\Lambda N}$ potentials

NN two-body potentials

- V2-M0.0, V2-M0.6 (A. Volkov, Nucl. Phys. **74** (1965) 33)
- MTV (UCOM modified*)
  (R. Malfliet, J. Tjon, Nucl. Phys.**A127** (1969) 161)
- ATS3M (UCOM modified*)
  (I. Afnan, Y. Tang, Phys. Rev. **175** (1968) 1337)

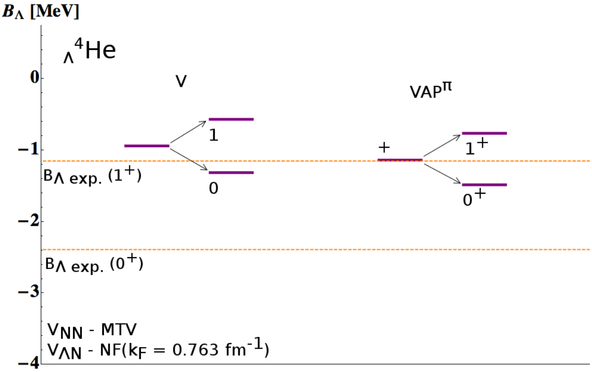
* UCOM (H. Feldmeier, T. Neff, R. Roth, J.Schnack, Nucl. Phys. **A632** (1998) 61)

ΛN two-body potential

- G-matrix transformed YNG (Jülich, Nijmegen)
- k_F dependence (Y.Yamamoto et. al, PTP Suppl. **117** (1994) 361)

$$V_{\Lambda N}(r) = \sum_i^3 (a_i + b_i k_F + c_i k_F^2) \exp\left\{-\frac{r^2}{\beta_i^2}\right\}$$

Parity projection of energy levels in $^{4}_{\Lambda}\mathrm{He}$

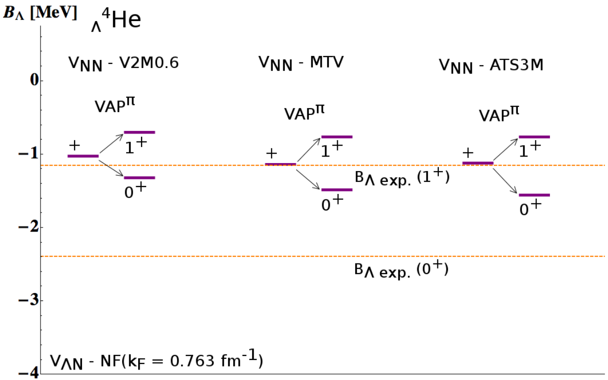


$\mathrm{V} \rightarrow$ variation without parity projection

$\mathrm{VAP}^{\pi} \rightarrow$ variation of the parity projected state

Parity projection $\hat{P}^{\pi}$ increases B_{Λ}

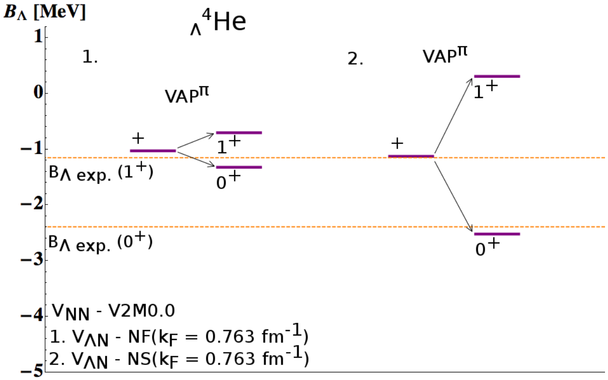
V_{NN} dependence of energy levels in $^{4}_{\Lambda}$He



Λ separation energy B_Λ slightly changes with V_{NN}

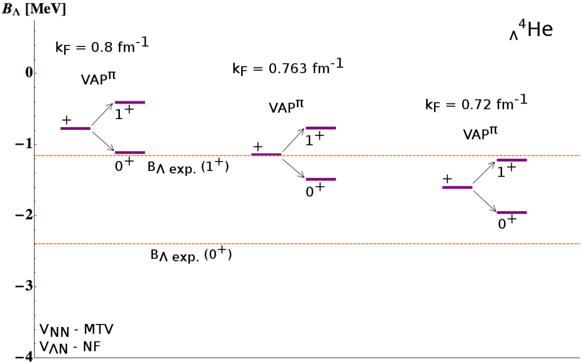
$B(^3\text{He}; \text{V2M0.6}) = -7.18$ MeV
$B(^3\text{He}; \text{MTV}) = -6.45$ MeV
$B(^3\text{He}; \text{ATS3M}) = -5.40$ MeV

$V_{\Lambda N}$ dependence of energy levels in $^4_\Lambda$He



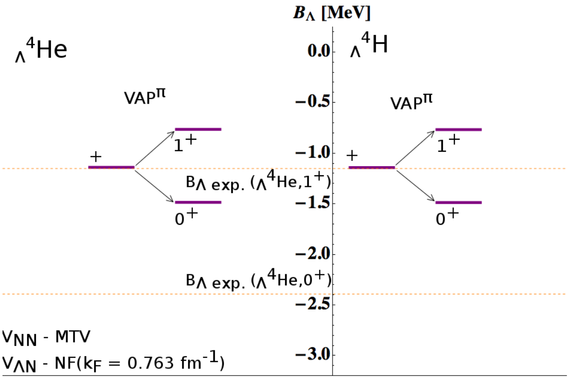
Substantial difference between Λ separation energies as well as $|B_\Lambda(0^+) - B_\Lambda(1^+)|$ for various $V_{\Lambda N}$

k_F dependence of energy levels in $^{4}_{\Lambda}$He



Strong Fermi momentum dependence in the $V_{\Lambda N}$ part
(k_F acts as a scaling factor)
$k_F = 0.8\ \mathrm{fm}^{-1}$(Y.Yamamoto et al, PTP Suppl. **117** (1994) 361)
$k_F = 0.753\ \mathrm{fm}^{-1}$ (^{3}He rms radius approximation)
$k_F = 0.72\ \mathrm{fm}^{-1}$(test value)

Mirror hypernuclei ${}^4_\Lambda$He and ${}^4_\Lambda$H



→ no difference in B_Λ between ${}^4_\Lambda$He and ${}^4_\Lambda$H using YNG $V_{\Lambda N}$

$B_{\Lambda\ \mathrm{exp.}}({}^4_\Lambda\mathrm{He};\ 0^+)$ = -2.39 MeV

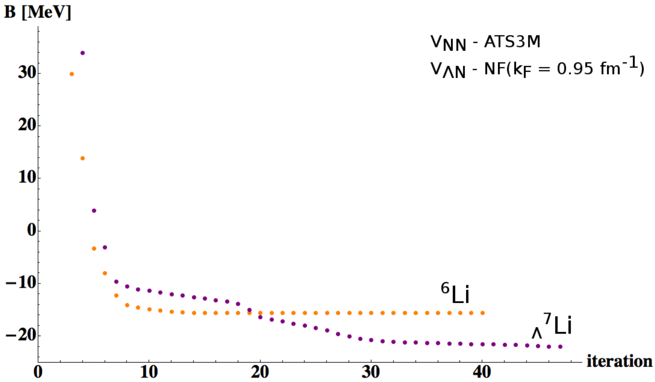
$B_{\Lambda\ \mathrm{exp.}}({}^4_\Lambda\mathrm{He};\ 1^+)$ = -1.15 MeV

$B_{\Lambda\ \mathrm{exp.}}({}^4_\Lambda\mathrm{H};\ 0^+)$ = -2.04 MeV

$B_{\Lambda\ \mathrm{exp.}}({}^4_\Lambda\mathrm{H};\ 1^+)$ = -1.04 MeV

p-shell hypernucleus $^7_\Lambda$Li

- **preliminary** results
- extensive computational complexity
- $k_F = 0.95\ \mathrm{fm}^{-1}$ (Y.Yamamoto et al, PTP Suppl. **117** (1994) 361)



Conclusions

- FMD for hypernuclei developed
  - calculations of s-shell hypernuclei ${}^{4}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{He}$
  - relevance of the symmetry restoration (B_Λ, projected 0^+ and 1^+ state)
  - weak $\mathrm{V_{NN}}$ dependence of B_Λ
  - strong $\mathrm{V_{\Lambda N}}$ dependence of B_Λ
  - strong $\mathrm{k_F}$ dependence of B_Λ
    ($\mathrm{k_F}$ acts as a scaling parameter of YNG $\mathrm{V_{\Lambda N}}$ potentials)
- preliminary results for ${}^{7}_{\Lambda}\mathrm{Li}$

Next steps :

- calculations of p-shell hypernuclei
- more sophisticated interactions (Argonne V18, $\mathrm{V_{\Lambda N}}$ potentials with $\Lambda - \Sigma$ mixing, chiral $\mathrm{V_{NN}}$ and $\mathrm{V_{\Lambda N}}$ potentials)
- ΛΛ hypernuclei