The study of nuclear structure far from stability

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Introduction

Stable nuclei are qualitatively described by “simple” models

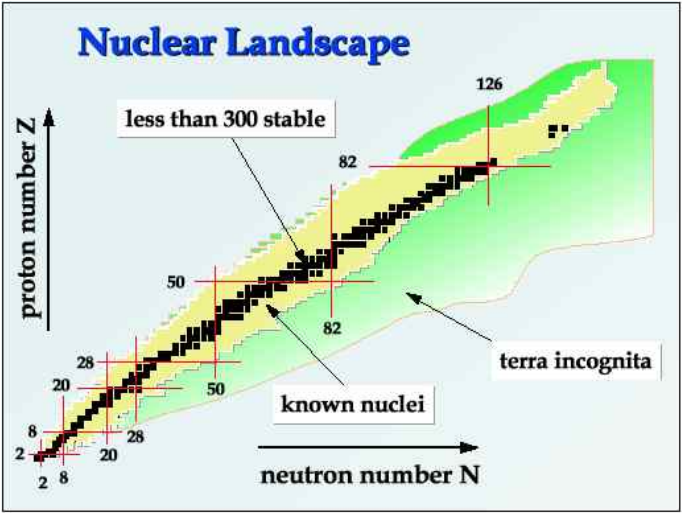
- (semi-empirical) liquid-drop model
- (basic) shell model

New techniques enable ab-initio methods (A-body models)

What happens far from stability ?
Experimentally, Radioactive-Ion Beams (RIB) available since 80s
⇒ study of structure far from stability
⇒ discovery of exotic structures

- halo nuclei
- shell inversions

Nuclear Landscape



1. Basic features in nuclear structure
    - Liquid-drop model
    - Shell model
2. Ab-initio nuclear models
3. Superheavy nuclei
4. Radioactive-Ion Beams
5. Oddities far from stability : halo nuclei
6. Experimental techniques
    - Active targets
    - Electron-ion collider
7. Summary

Electron scattering

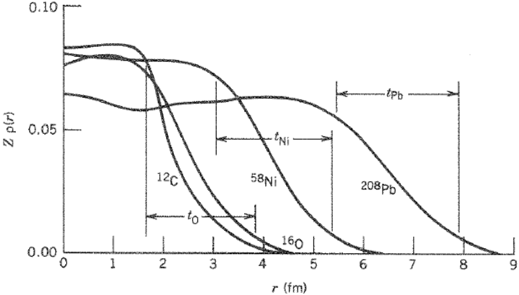
Nuclear charge distributions can be studied by electron scattering
At the Born approximation

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_R}{d\Omega} \, |F(\boldsymbol{q})|^2,$$

with the nuclear form factor

$$F(\boldsymbol{q}) \propto \int \left\langle \Psi \left| \sum_{j=1}^{Z} \delta(\boldsymbol{r} - \boldsymbol{r}_j) \right| \Psi \right\rangle e^{i\boldsymbol{q}.\boldsymbol{r}} \, d\boldsymbol{r}$$

Charge distributions in (stable) nuclei



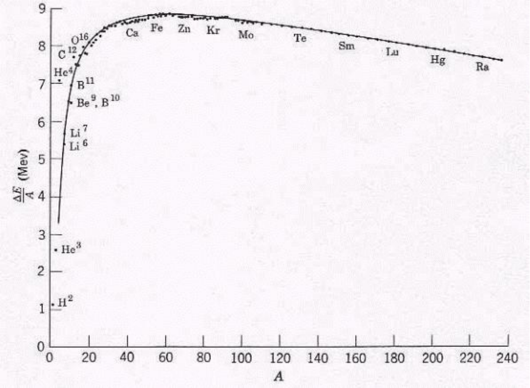
- constant density ρ_0 out to the surface (saturation)
- same skin thickness t

(Stable) nuclei look like liquid drops of radius $R \propto A^{1/3}$

Liquid-drop model

Bethe-Weizsäcker semi-empirical mass formula

$$B(Z,N) = a_v A -$$



Liquid-drop model

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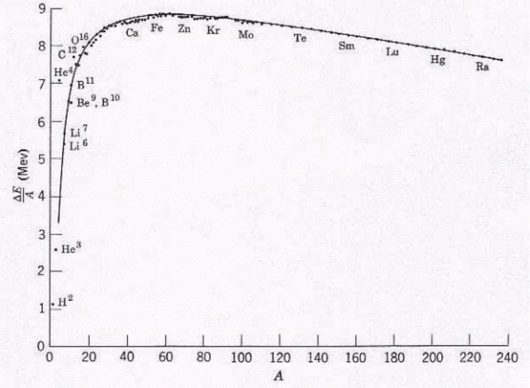
$$B(Z,N) = a_v A - a_s A^{2/3} -$$



Liquid-drop model

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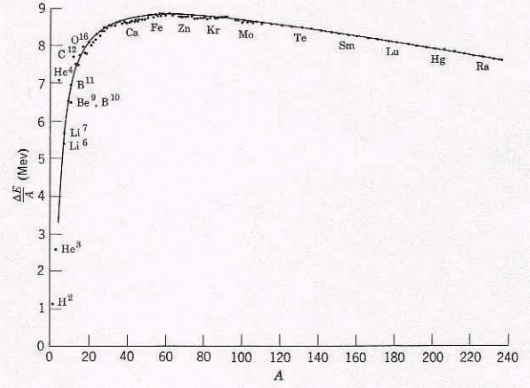
$$B(Z,N) = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} -$$



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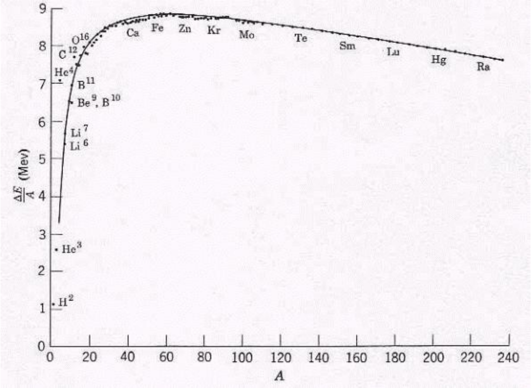
Bethe-Weizsäcker semi-empirical mass formula

$$B(Z,N) = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} +$$



Liquid-drop model

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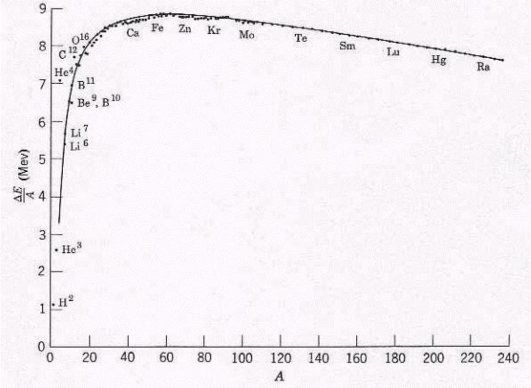
$$B(Z,N) = a_v A - a_s A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$



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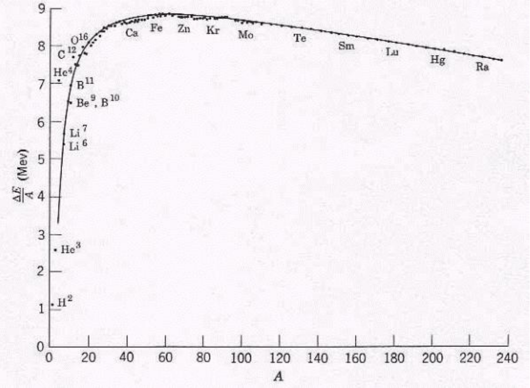
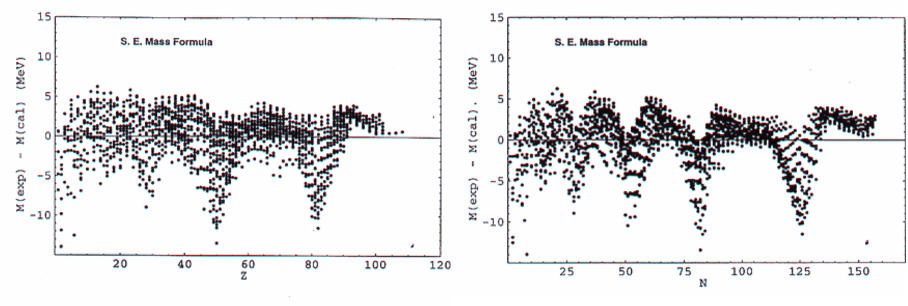
Exoenergetic reactions :

- fission of heavy nuclei (nuclear power plants, atomic bomb)
- fusion of light nuclei (stars, thermonuclear weapons)



Variation from the semi-empirical mass formula



More bound systems at Z or $N = 2, 8, 28, 50, 82, 126$

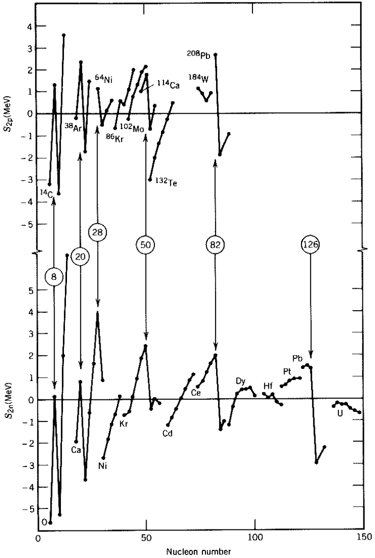
magic numbers

⇒ shell structure in nuclei as in atoms ?

Two-nucleon separation energy

Same magic numbers in S_{2p} and S_{2n}

$\Rightarrow$ more bound at shell closure
cf. ionisation energies of atoms

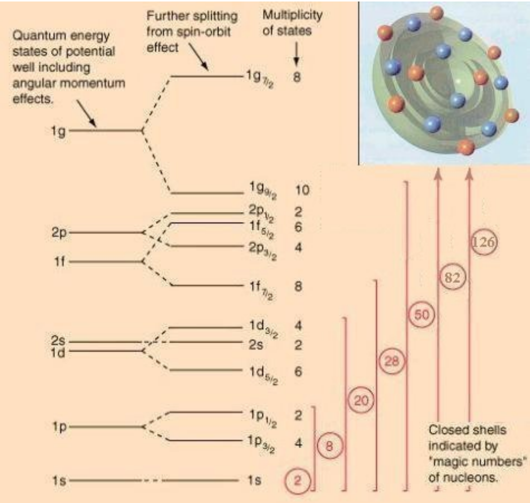


Shell model

Developed in 1949 by M. Goeppert Mayer, H. Jensen and E. Wigner (NP 1963)

As electrons in atoms,
nucleons in nuclei
feel a mean field
and arrange into shells

Spin-orbit coupling is crucial to get right ordering of shells



Nowadays

Can we go beyond these models ?

Can we build ab-initio models ?
i.e. based on first principles

- nucleons as building blocks
- realistic N-N interaction

What happens away from stability ?

- Is nuclear density similar for radioactive nuclei ?
- Are magic numbers conserved ?
- Is there an island of stability for heavy nuclei ?

A-body Hamiltonian

Nuclear-structure calculations : A nucleons (Z protons+N neutrons)

Relative motion described by the A-body Hamiltonian

$$H = \sum_{i=1}^{A} T_i + \sum_{j>i=1}^{A} V_{ij}$$

⇒ solve the A-body Schrödinger equation

$$H\,|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$\{E_n\}$ is the nucleus spectrum

Realistic N-N interactions

V_{ij} not (yet) deduced from QCD
⇒ phenomenological potentials
fitted on N-N observables :
d binding energy,
N-N phaseshifts

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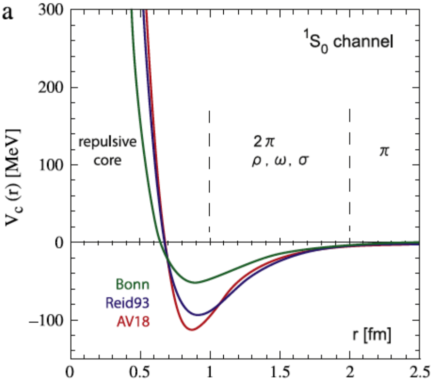
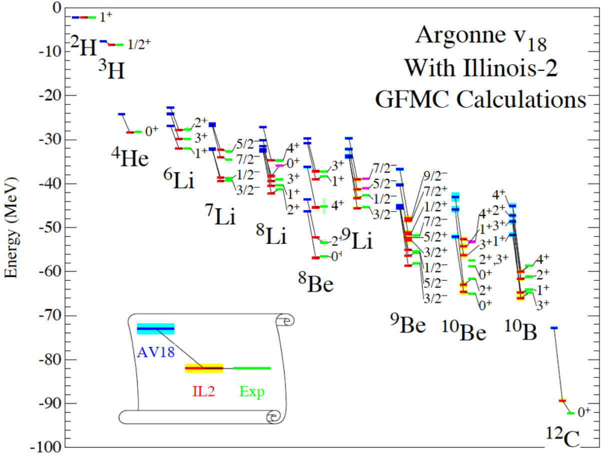
Ex. : Argonne V18, CD-Bonn,…



Light nuclei calculations



[R. Wiringa, Argonne]

Three-body force

Need three-body forces to get it right…

$$H = \sum_{i=1}^{A} T_i + \sum_{j>i=1}^{A} V_{ij} + \sum_{k>j>i=1}^{A} V_{ijk} + \cdots$$

But there is no such thing as three-body force…

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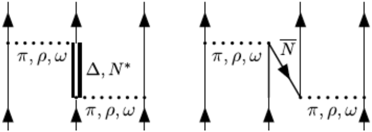
They simulate the non-elementary character of nucleons
⇒ include virtual Δ resonances, $\bar{\mathrm{N}}$…



Phenomenological 3-body interaction fitted on $A > 2$ levels : IL2
Alternatively, derived from EFT

Effective Field Theory

EFT is an effective quantum field theory based on QCD symmetries with resolution scale Λ that selects appropriate degrees of freedom : nuclear physics is not built on quarks and gluons,
but on nucleons and mesons

EFT provides the nuclear force with a systematic expansion in Q/Λ

- gives an estimate of theoretical uncertainty

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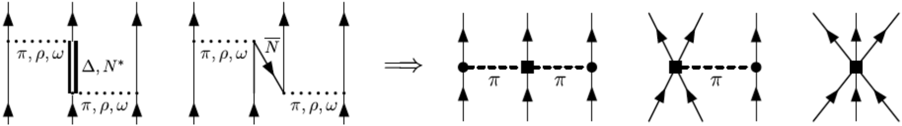
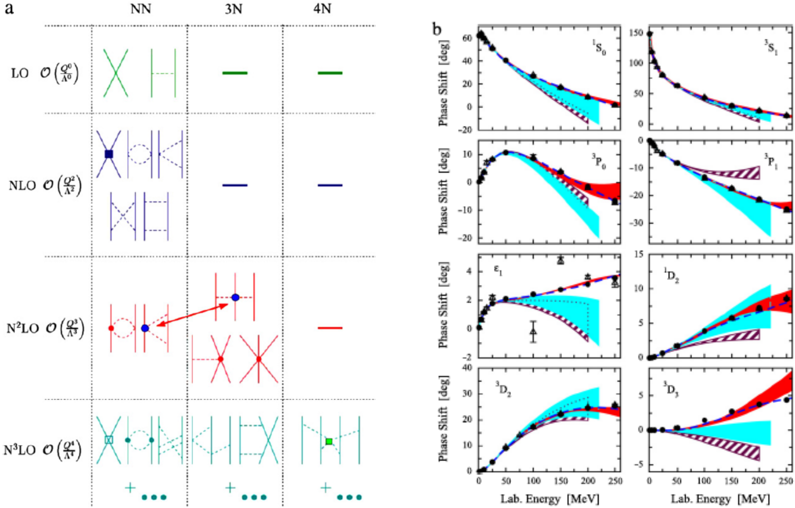
EFT provides the nuclear force with a systematic expansion in Q/Λ

- gives an estimate of theoretical uncertainty
- naturally includes many-body forces



[see A. Schwenk's talk on Tuesday morning]

Expansion of the EFT force



Solving the Schrödinger equation

$$H\,|\Psi_n\rangle = E_n|\Psi_n\rangle$$

Ψ usually developed on a basis $\{|\Phi_{[\nu]}\rangle\}$:

$$|\Psi_n\rangle = \sum_{[\nu]} \langle\Phi_{[\nu]}|\Psi_n\rangle\ |\Phi_{[\nu]}\rangle$$

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Solving the Schrödinger equation reduces to matrix diagonalisation

$$\begin{aligned}\langle\Phi_{[\mu]}|H|\Psi_n\rangle &= \sum_{[\nu]}\langle\Phi_{[\mu]}|H|\Phi_{[\nu]}\rangle\langle\Phi_{[\nu]}|\Psi_n\rangle \\ &= E_n\,\langle\Phi_{[\mu]}|\Psi_n\rangle\end{aligned}$$

No-core shell model

Slater determinants of 1-body mean-field wave functions ϕ_{ν_i}

$$\langle \xi_1 \xi_2 \ldots \xi_A | \Phi_{[\nu]} \rangle = \mathcal{A}\, \phi_{\nu_1}(\xi_1)\, \phi_{\nu_2}(\xi_2) \ldots \phi_{\nu_A}(\xi_A)$$

But short-range correlations couple low and high momenta

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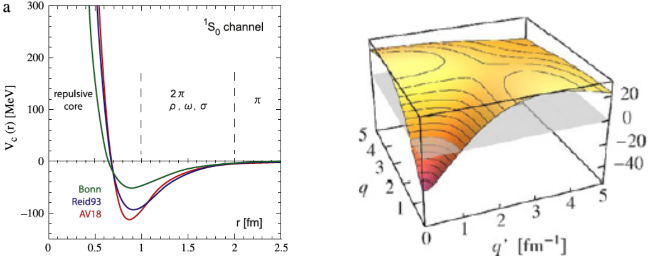
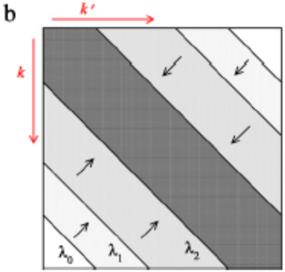


⇒ requires large basis $|\Phi_{[\nu]}\rangle$

Similarity Renormalisation Group

Idea : apply a unitary transformation

$$\begin{aligned} |\widetilde{\Phi}_{[\nu]}\rangle &= U|\Phi_{[\nu]}\rangle \\ \Leftrightarrow H_{\mathrm{eff}} &= U^{\dagger}HU \end{aligned}$$

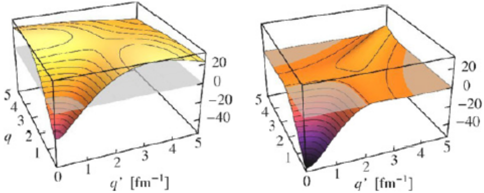


- keeps the same spectrum (unitary)
- keeps the same on-shell properties (phaseshifts)
- removes the short-range correlations

This has a costs : induces “unphysical” three-body forces

SRG : example on ^{4}He

SRG lowers correlations

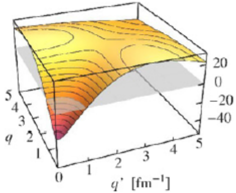


(a) V_{AVIS}. (b) V_{SRG}.

[see A. Schwenk's talk on Tuesday morning]

SRG : example on ^{4}He

SRG lowers correlations ⇒ fastens convergence



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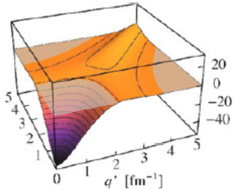
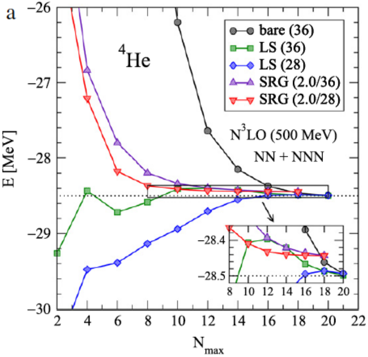


(b) V_{SRG}.



[see A. Schwenk's talk on Tuesday morning]

What happens far from stability ?

Liquid-drop and shell models are fair models of stable nuclei

What happens away from stability ?

Are there superheavy nuclei ? [see M. Bloch's talk on Tuesday]

In 80s Radioactive-Ion Beams were developed
Enable study of nuclear structure [see P. Egelhof's talk on Tuesday]

- are radioactive nuclei compact ?
- are shells conserved far from stability ?

Study of reactions involving radioactive nuclei
useful for astrophysics

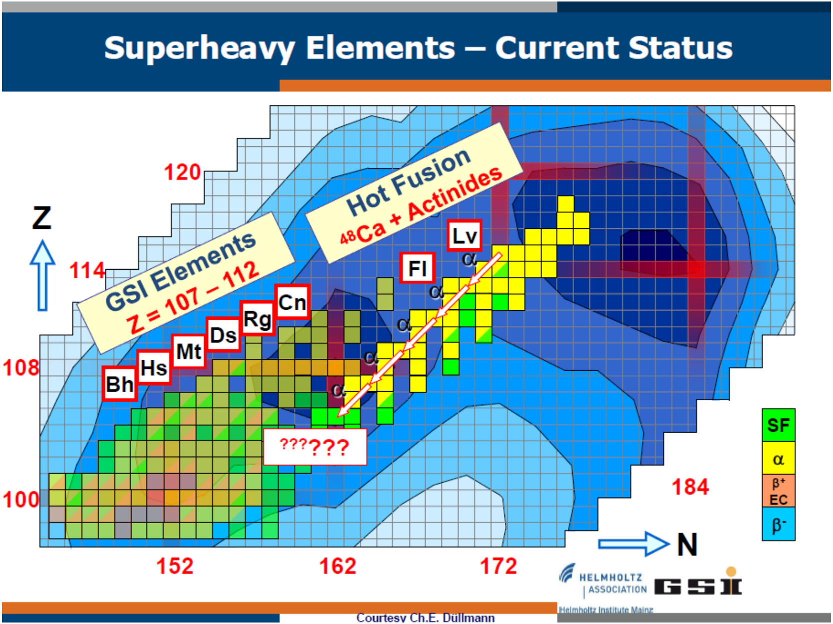
Superheavy nuclei

Does the stability end with U ?
Or is there an island of stability ?
Is $Z \sim 114 - 126$ a new magic number ?

Search elements heavier than U has started in the 40’s
Pu produced by U+d and U+n (identified by Seaborg in 1941)

Nowadays, use ^{48}Ca fusion on actinide target
Recently, element $Z = 117$ has been confirmed at GSI
using ^{48}Ca+^{249}Bk [PRL 112, 172501 (2014)]
Element identified by α cascade
[see M. Bloch’s talk on Tuesday morning]

$Z = 117$



Superheavy Elements – Current Status

Courtesy Ch.E. Düllmann

How ?

Idea : break a heavy nuclei into pieces to produce exotic isotopes

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- ISOL : Fire a proton at a heavy nucleus

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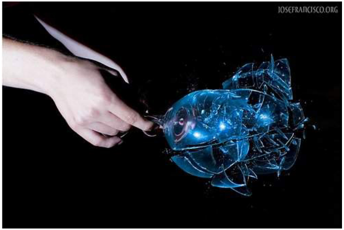
Idea : break a heavy nuclei into pieces to produce exotic isotopes

- ISOL : Fire a proton at a heavy nucleus

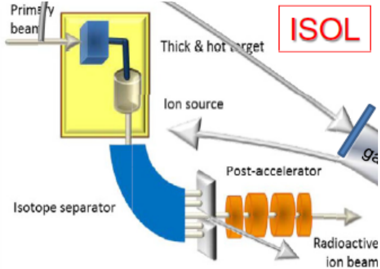
- In-flight : Smash a heavy nucleus on a target



Where ?


ARCTIC OCEAN
Greenland
CANADA
UNITED STATES
NORTH PACIFIC OCEAN
NORTH ATLANTIC OCEAN
RUSSIA
SIBERIA
KAZAKHSTAN
MONGOLIA
CHINA
IRAN
INDIA
ALGERIA
LIBYA
EGYPT
SAHARA
MALI
NIGER
CHAD
SUDAN
BRAZIL
SOUTH PACIFIC OCEAN
SOUTH ATLANTIC OCEAN
INDIAN OCEAN
INDONESIA
AUSTRALIA
SOUTHERN OCEAN
Antarctica

ISOL : Isotope Separation On Line



high-energy/intensity primary beam of light nuclei (e.g. protons) on thick target of heavy elements (Ta or UC_x)
⇒ spallation/fragmentation produces exotic fragments

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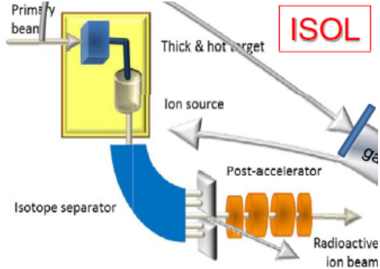

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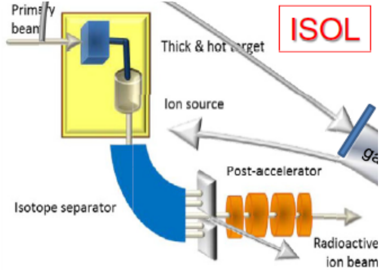

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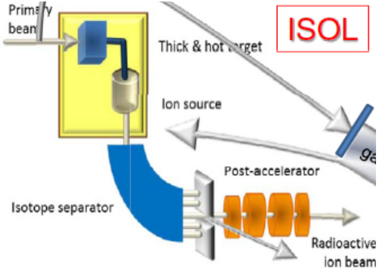

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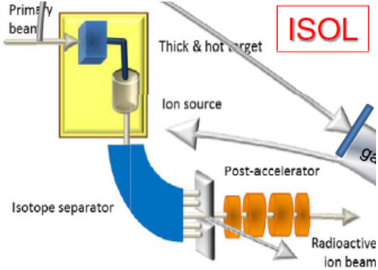

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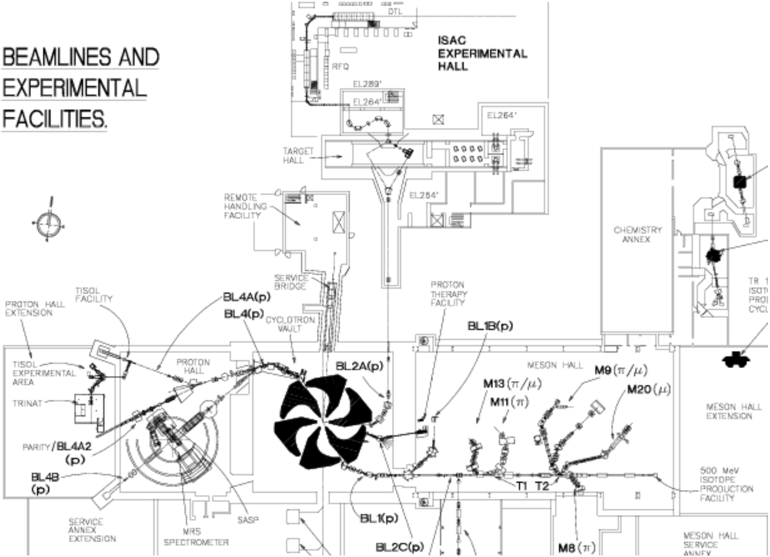
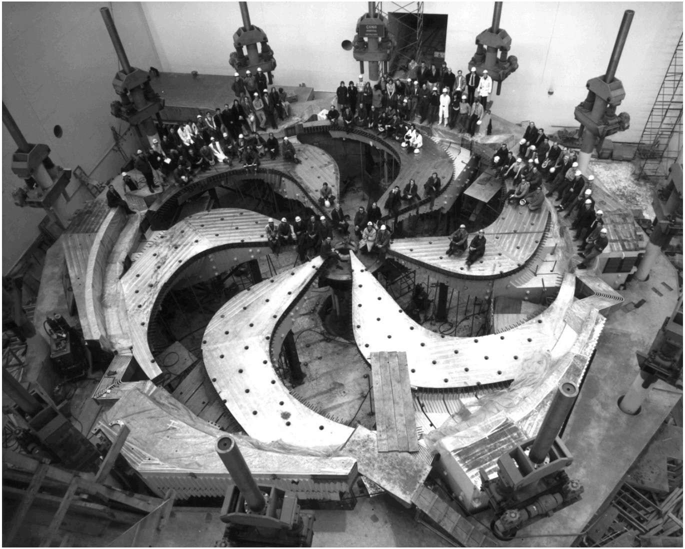
Examples : TRIUMF, ISOLDE (CERN), SPIRAL (GANIL)

ISAC@TRIUMF


BEAMLINES AND EXPERIMENTAL FACILITIES.
DTL
ISAC EXPERIMENTAL HALL
RFQ
EL289'
EL264'
EL264'
TARGET HALL
EL264'
REMOTE HANDLING FACILITY
CHEMISTRY ANNEX
SERVICE BRIDGE
PROTON THERAPY FACILITY
TISOL FACILITY
PROTON HALL EXTENSION
BL4A(p)
BL4(p)
CYCLOTRON VAULT
BL1B(p)
TR 1
TISOL EXPERIMENTAL AREA
PROTON HALL
BL2A(p)
MESON HALL
M9 (π/μ)
M13 (π/μ)
M20 (μ)
M11 (π)
TRINAT
MESON HALL EXTENSION
PARITY/BL4A2 (p)
BL4B (p)
T1
T2
500 MeV ISOTOPE PRODUCTION FACILITY
SERVICE ANNEX EXTENSION
SASP
BL1(p)
MR5 SPECTROMETER
BL2C(p)
M8 (π)
MESON HALL SERVICE ANNEX

World largest cyclotron



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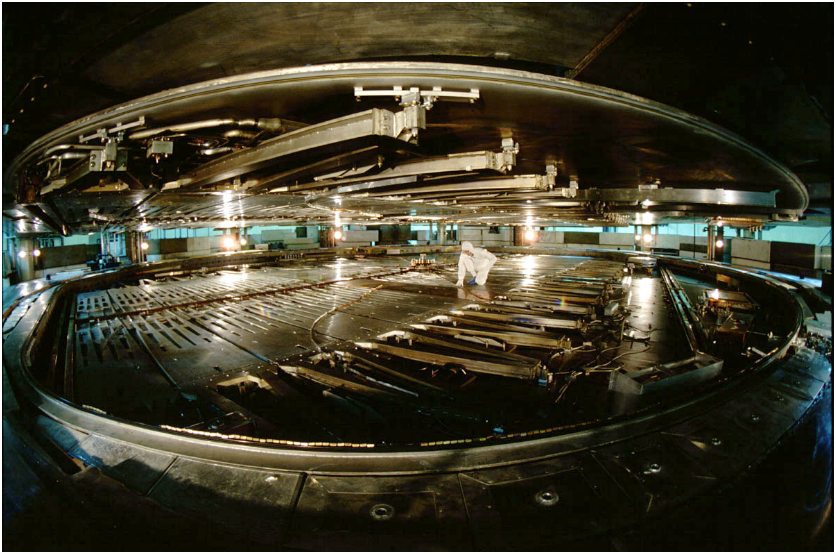
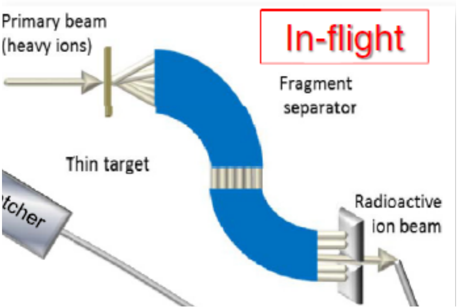


In-flight projectile fragmentation



high-energy primary beam of heavy ions (e.g. ^{18}O, ^{48}Ca, U. . .)
on thin target of light elements (Be or C)
⇒ fragmentation/fission produces many exotic fragments at $\approx v_{\text{beam}}$

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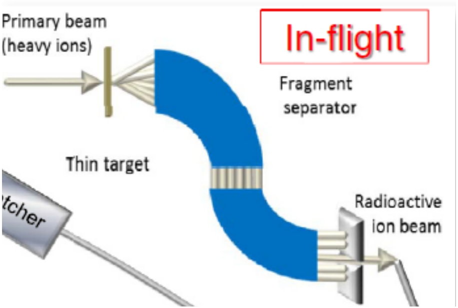

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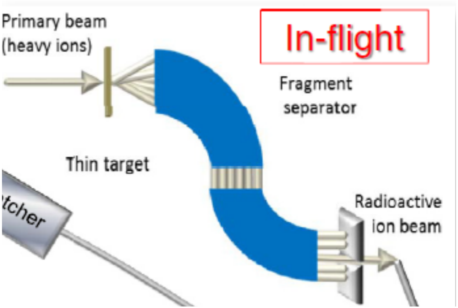

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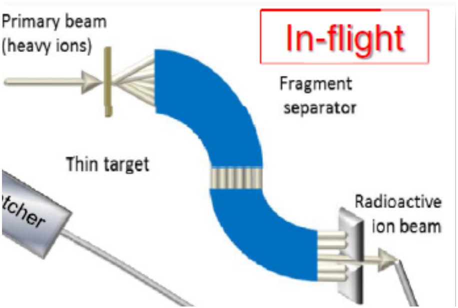

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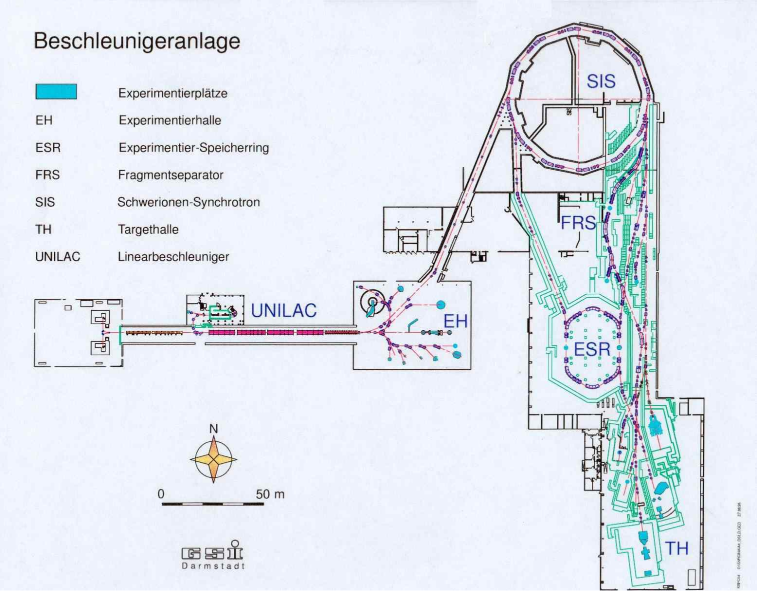
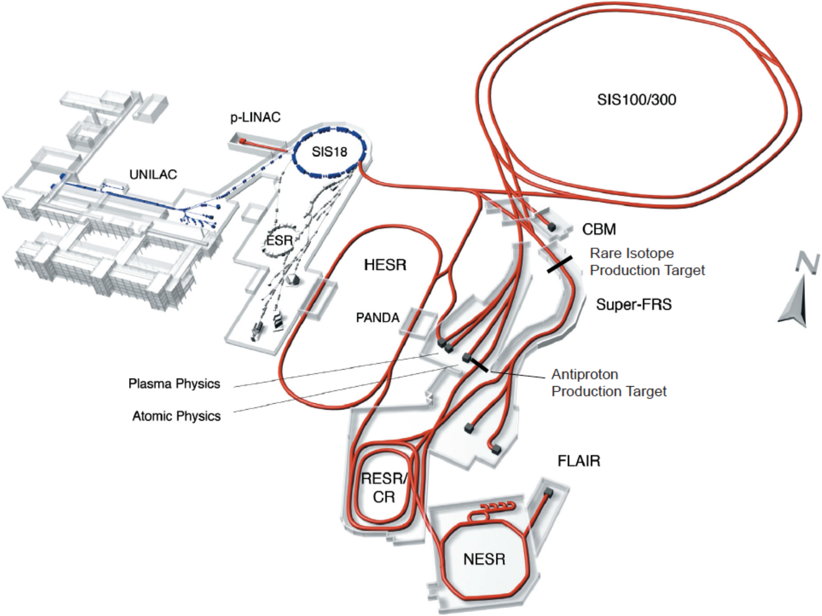
Examples : NSCL (MSU), GSI, RIKEN, GANIL

Existing GSI



Future : FAIR


SIS100/300
p-LINAC
SIS18
UNILAC
ESR
HESR
PANDA
CBM
Rare Isotope Production Target
Super-FRS
N
Plasma Physics
Atomic Physics
Antiproton Production Target
FLAIR
RESR/CR
NESR

Properties

ISOL

- Low beam energy
  may require post-acceleration
- Low beam intensity
- Not all elements produced
  - Slow
  - Chemically limited
- Good beam quality :
  can use chemistry to select fragments

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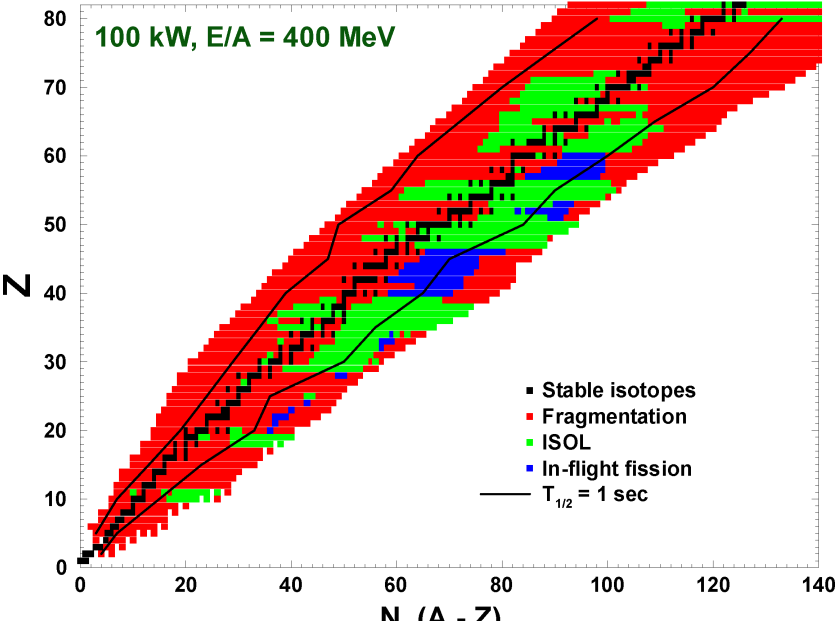
In-flight

- High beam energy
  $v_{\text{fragments}} \approx v_{\text{beam}}$
- High beam intensity
- Efficient production
  - Fast
  - Chemically independent
- Many fragments in beam
  ⇒ need ion ID

Choose according what you want to measure



Halo structure

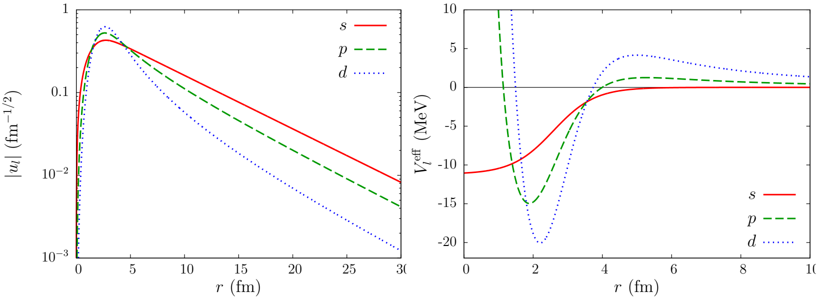
Seen as core + one or two neutrons at large distance

[P. G. Hansen and B. Jonson, Europhys. Lett. 4, 409 (1987)]

Peculiar structure of nuclei due to small S_{n} or $S_{2\mathrm{n}}$

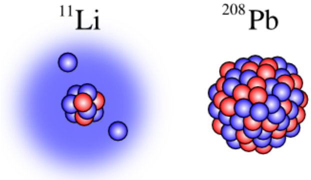
$\Rightarrow$ neutrons tunnel far from the core to form a halo

Halo only appears for low centrifugal barrier (low ℓ)



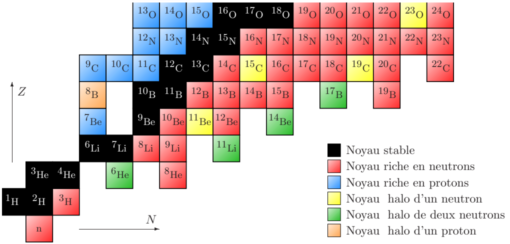
Halo nuclei

- Light, neutron-rich nuclei
- small S_{n} or $S_{2\mathrm{n}}$
- low-ℓ orbital



One-neutron halo
^{11}Be ≡ ^{10}Be + n
^{15}C ≡ ^{14}C + n

Two-neutron halo
^{6}He ≡ ^{4}He + n + n
^{11}Li ≡ ^{9}Li + n + n



⬛ Noyau stable
🟥 Noyau riche en neutrons
🟦 Noyau riche en protons
🟨 Noyau halo d'un neutron
🟩 Noyau halo de deux neutrons
🟧 Noyau halo d'un proton

Proton halœs are possible but less probable : ^{8}B, ^{17}F

Two-neutron halo nuclei are Borromean. . .

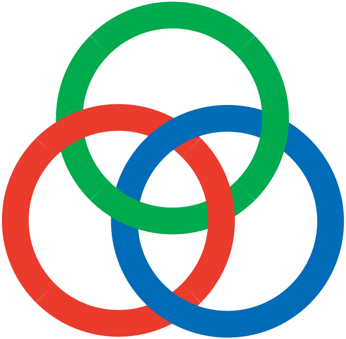
c+n+n is bound but not two-body subsystems

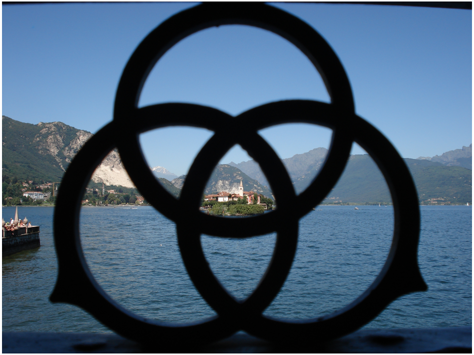
e.g. ^{6}He bound but not ^{5}He or ^{2}n

Borromean nuclei

Named after the Borromean rings…

[M. V. Zhukov *et al.* Phys. Rep. 231, 151 (1993)]





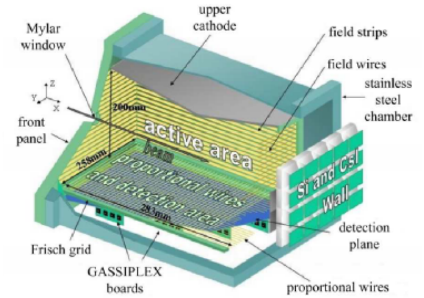
Active target detectors

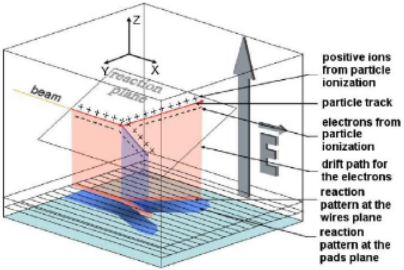
Intensity of RIB much lower than stable beams

⇒ difficult to study reactions

⇒ idea of active target ≡ target and detector

[see P. Egelhof's talk on Tuesday Morning]





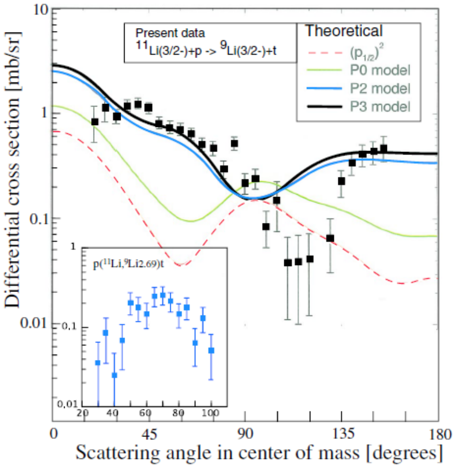
Applications

Using active targets various reactions can be performed (inverse kinematics)

- elastic scattering → mater distribution
- inelastic scattering → giant resonances, $B(\mathrm{E2})$,...
- charge exchange → GT strengths
- transfer → single-particle structure, N correlations,...
- knock-out → single-particle structure

[see P. Egelhof's talk on Tuesday Morning]

p(^{11}Li, ^{9}Li)t @ $3A$MeV measured at TRIUMF with MAYA



$(p_{1/2})^2$: pure $(0p_{1/2})^2$
$P0$: 3% $(1s_{1/2})^2$
$P1$: 31% $(1s_{1/2})^2$
$P2$: 45% $(1s_{1/2})^2$

$\Rightarrow$ disentangle structure models

[I. Tanihata PRL 100, 192502 (2008)]

Electron scattering

Hadronic probes are not clean :

- V_{NN} not well known
- N are not elementary

Electron scattering is much better [see H. Simon's talk on Tuesday]

- Coulomb force is well known
- point-like particle ⇒ excellent spatial resolution
- elastic scattering → charge distribution
- inelastic scattering → spectrum, resonances,. . .
- knockout → nucleon correlations

But requires a nuclear target. . .

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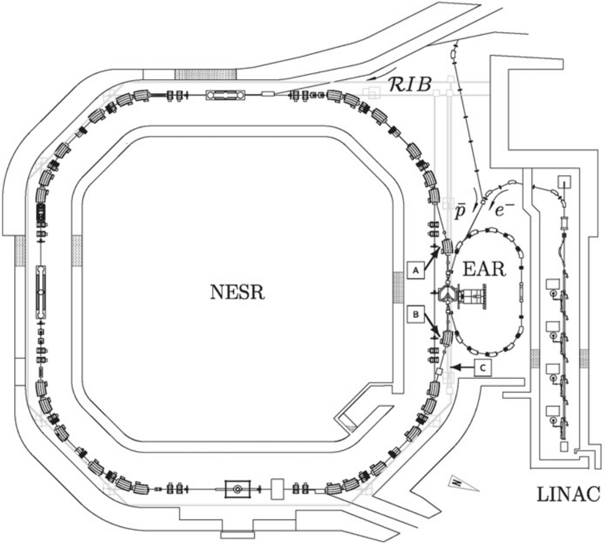
. . . or an e-ion collider : ELectron-Ion Scattering experiment @ FAIR

ELISe



[see H. Simon's talk on Tuesday] [Antonov *et al.* NIMA 637, 60]

Summary

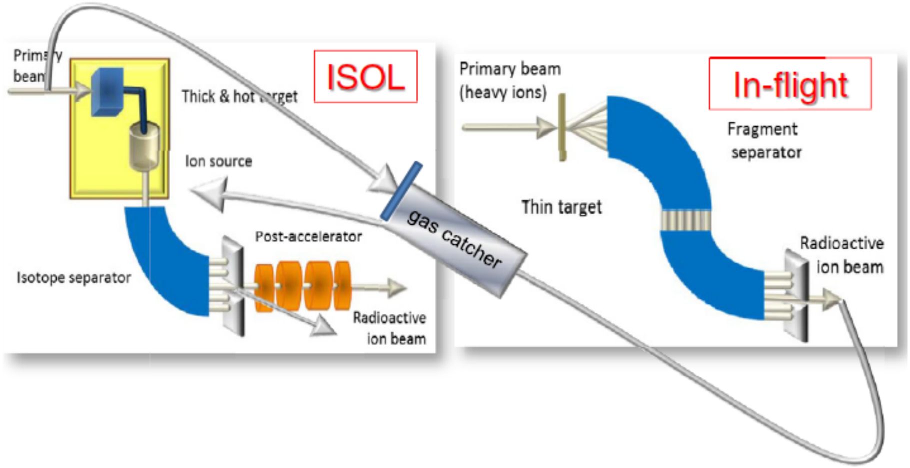
Liquid-drop and shell model describe qualitatively stable nuclei
Nowadays ab-initio nuclear-structure models

RIB enable study nuclear structure far from stability
Low intensities require new experimental techniques :
active target, reactions,. . .

- discovery of halo nuclei
  diffuse halo around a compact core
- shell inversions or shell collapse

RIB can be used to study reactions of astrophysical interest

Combined with a gas stopper



- can use thin target in ISOL
- can study low-energy reaction with in-flight fragmentation