The Neutron Star Mass-Radius Relationship and the Dense Matter Equation of State

Bob Rutledge
McGill University

Outline

Three approaches for measuring the Dense Matter Equation of State using Neutron Stars

Why we believe we are measuring neutron star radii from qLMXBs

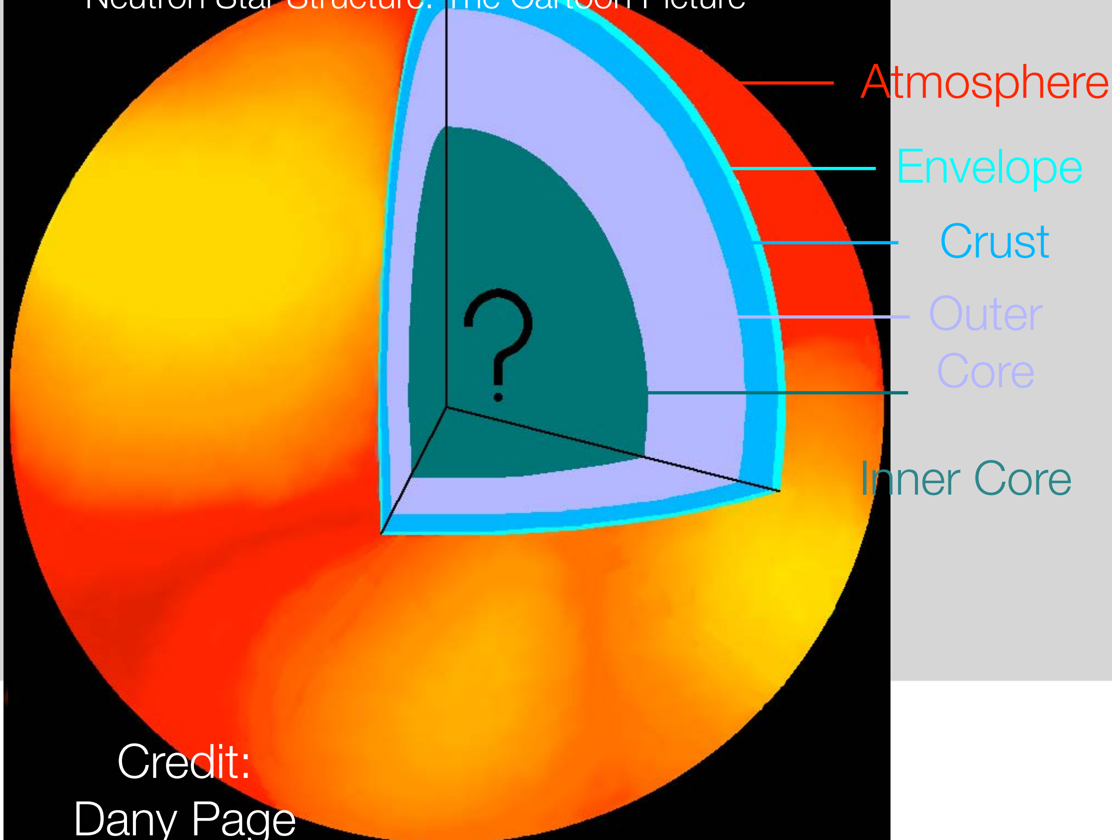
Current Measurement using H atmosphere neutron stars (model dependent)

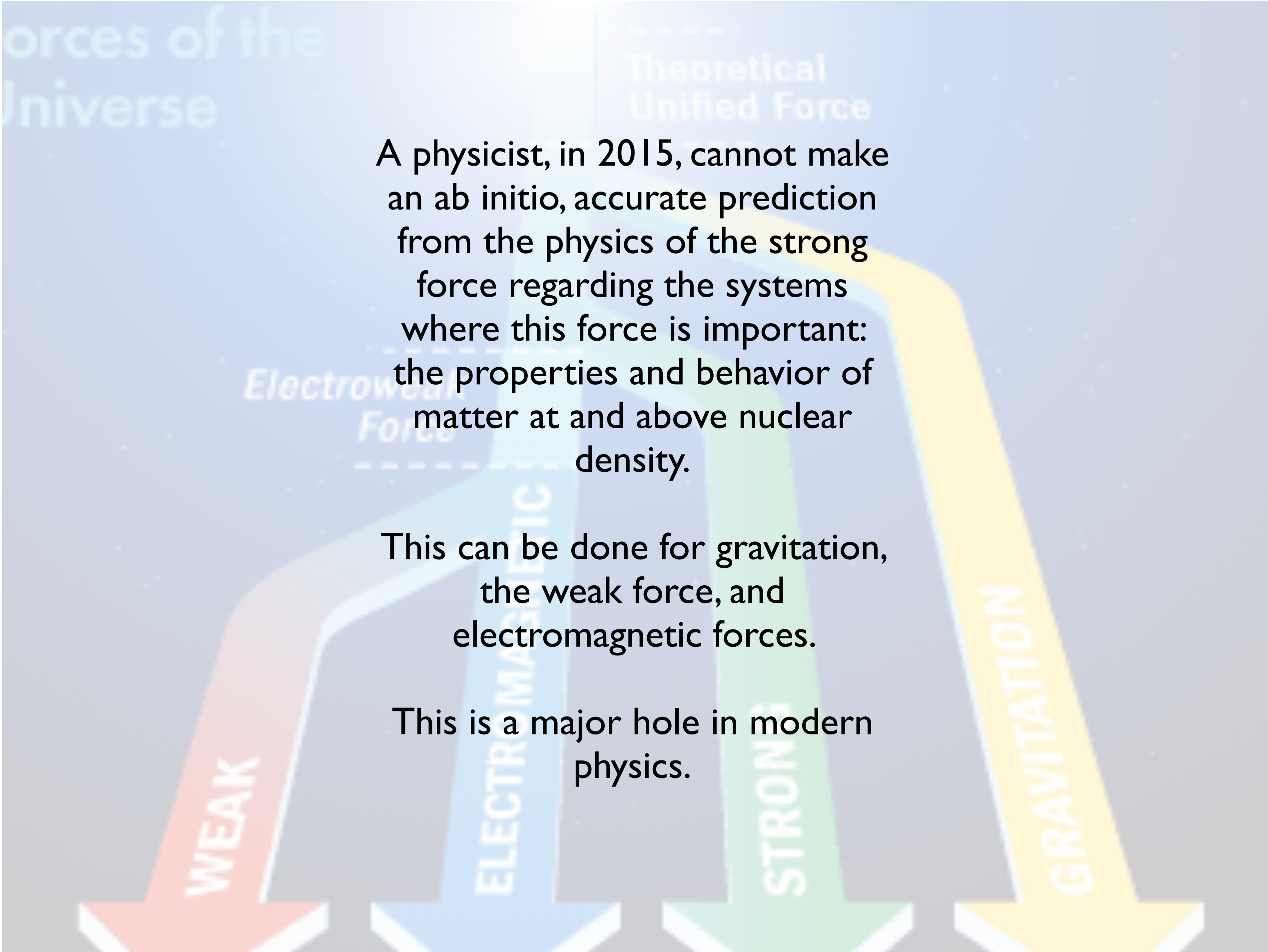
Future determination of the Dense Matter Equation of State

Collaborators: **Sebastien Guillot (McGill).** Mathieu Servillat (Saclay) Natalie Webb (Toulouse). Ed Brown (MSU) Lars Bildsten (UCSB/KITP) George Pavlov (PSU) Vyacheslav Zavlin (MSFC).

#53iwmnp

Neutron Star Structure: The Cartoon Picture





A physicist, in 2015, cannot make an ab initio, accurate prediction from the physics of the strong force regarding the systems where this force is important: the properties and behavior of matter at and above nuclear density.

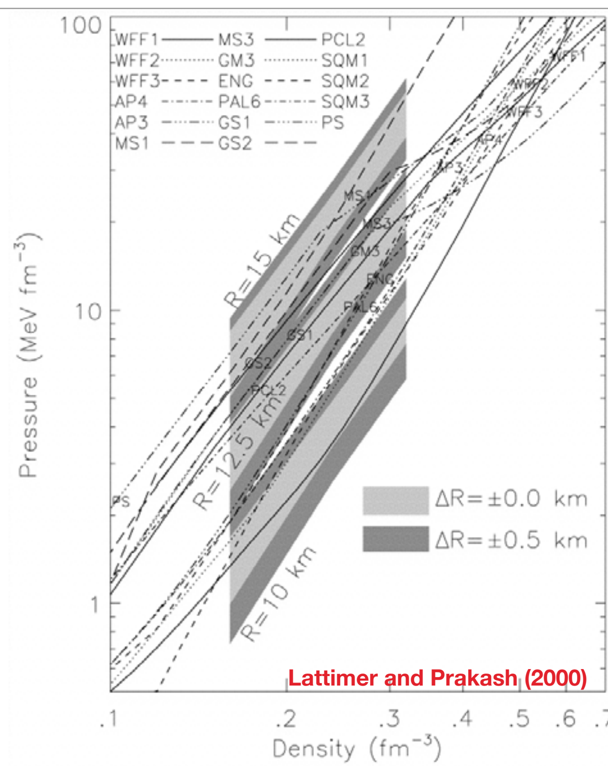
This can be done for gravitation, the weak force, and electromagnetic forces.

This is a major hole in modern physics.

Estimated Equations of State for cold, dense nuclear matter

$$P = f(\rho)$$

- Different calculational (approximation) methods
- Different input physics
- Different nuclear parameters (example: nuclear compressibility as a function of fractional neutron excess).
- The diversity of viable EOSs is due to these uncertainties.
- We will address this uncertainty with observations of neutron stars.



From Neutron Star Mass-Radius Relation to the Equation of State

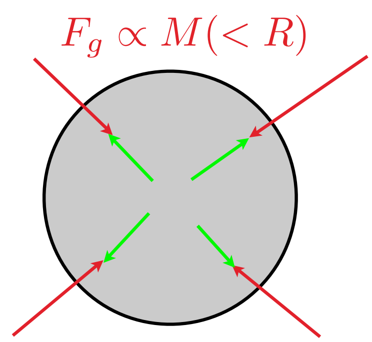
$$P = f(\rho)$$

- Lindblom (1992) showed that each Dense Matter Equation of State maps to a unique Mass-Radius relationship for neutron stars.

- Ozel and Psaltis (2009) demonstrate how to perform the inverse problem: take the mass-radius relationship, and produce an equation of state. Only ~5-7 such objects are needed, but “with different masses”, to derive a new dense matter equation of state.

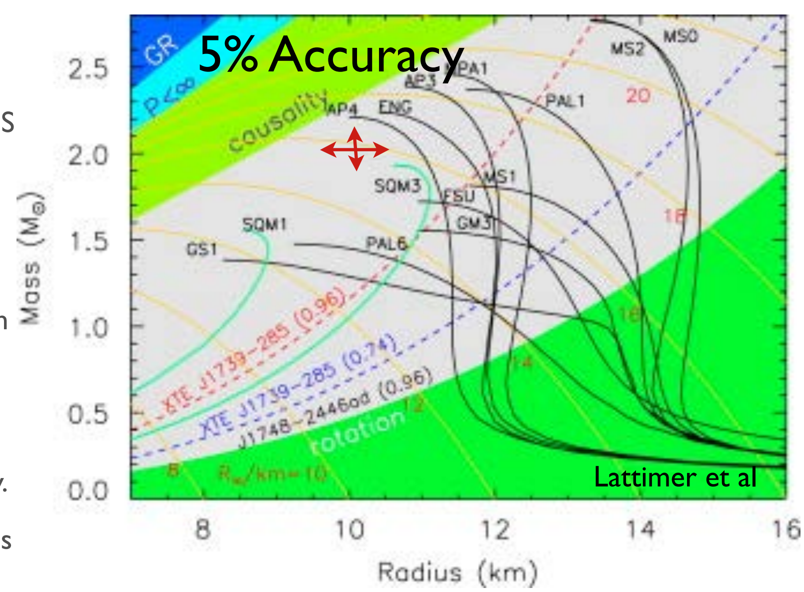
- Thus, measurement of the neutron star mass-radius relationship would implicate a unique dEOS.

Short Course:
Gravity pulls inward
Pressure Pushes Outward
Result: R=f(M)



The Dense Matter Equation of State is an important Strong Force Regime

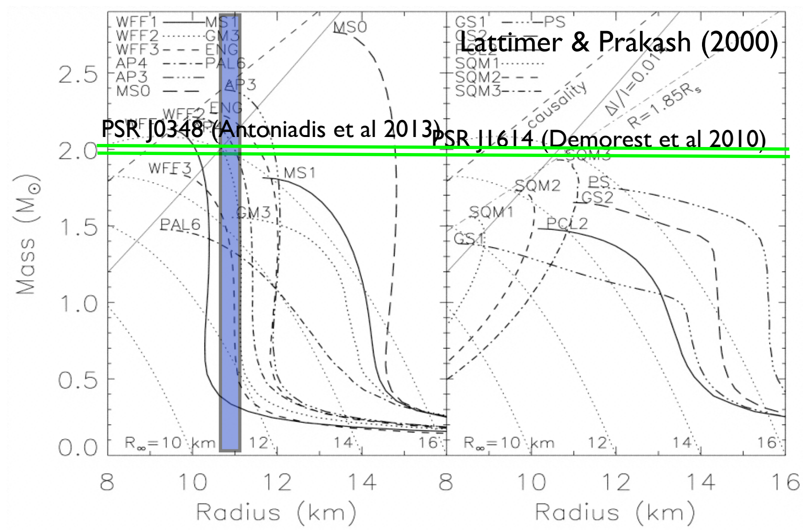
- Each different proposed dEOS produces a different mass-radius relationship for neutron stars.
- Thus, measure the mass-radius relationship of neutron stars, and you have a measurement of the dEOS.
- Precision requirement -- 5% in mass and radius, separately.
- A larger uncertainty is useless to nuclear physics.



Mass-Radius Relation from the Equation of State

Measuring the Mass and Radius simultaneously is difficult.

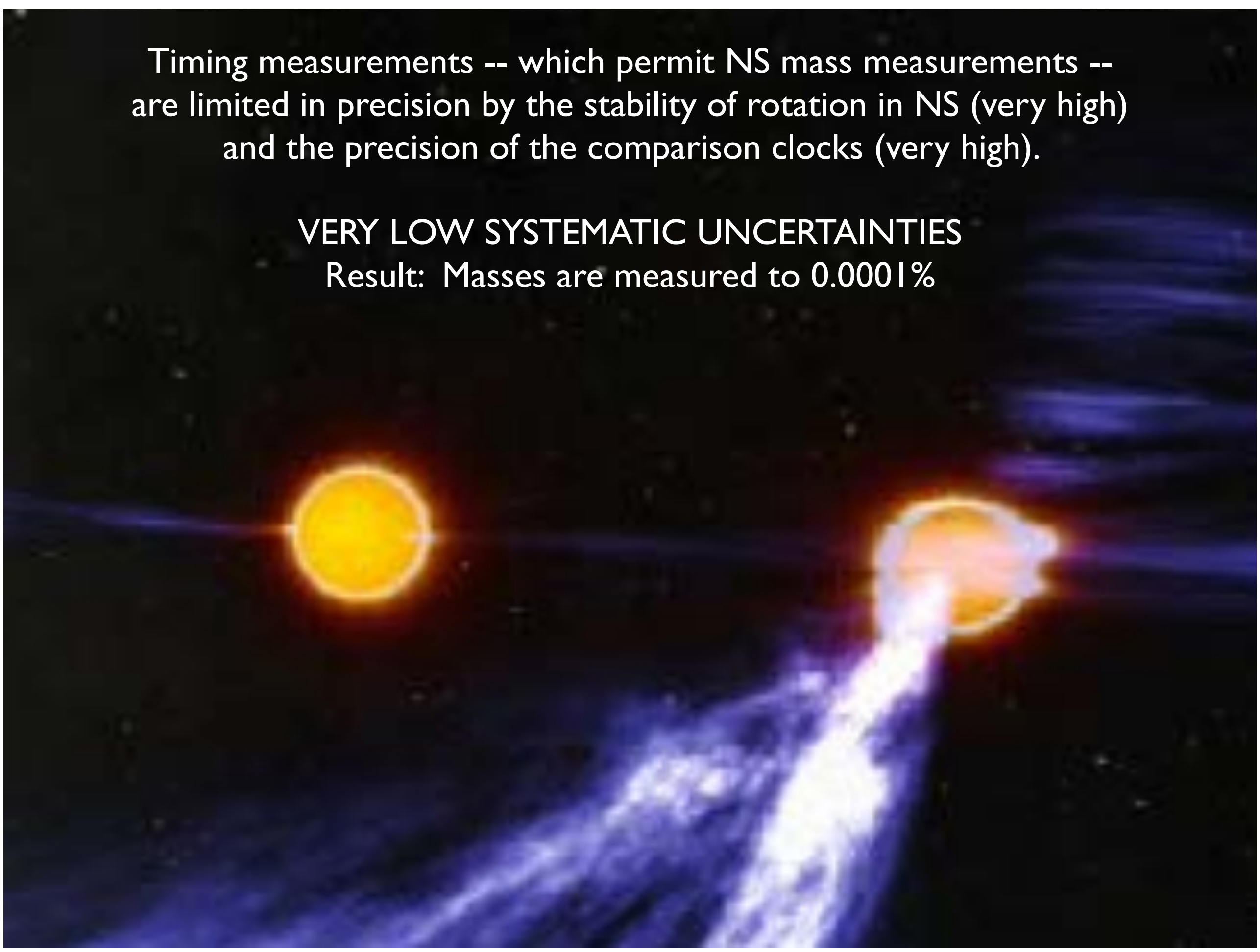
High-mass measurements prefer EOSs which produce a nearly constant radius at astrophysically interesting masses.



Precision Radius Measurements (<5%) may be they key to measuring the dEOS.

Timing measurements -- which permit NS mass measurements -- are limited in precision by the stability of rotation in NS (very high) and the precision of the comparison clocks (very high).

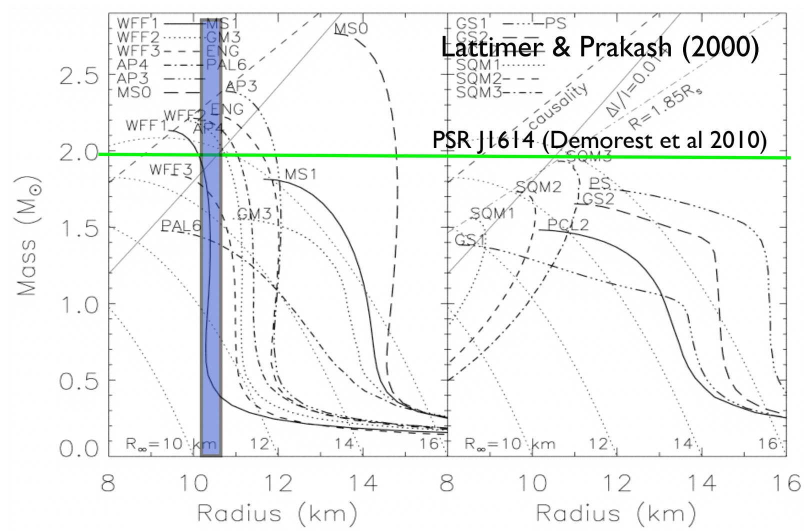
VERY LOW SYSTEMATIC UNCERTAINTIES
Result: Masses are measured to 0.0001%



Mass-Radius Relation from the Equation of State

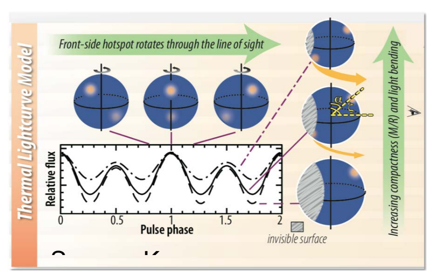
Measuring the Mass and Radius simultaneously is difficult.

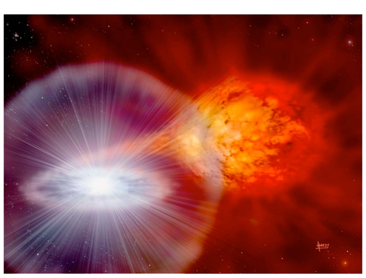
High Mass measurements implicate a possibly nearly constant radius.



Three Observational Approaches to Measure the Neutron Star Mass+Radius Relation

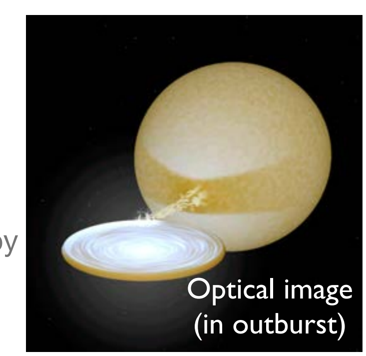
- Millisecond X-ray Pulsar Phase-Resolved Spectroscopy



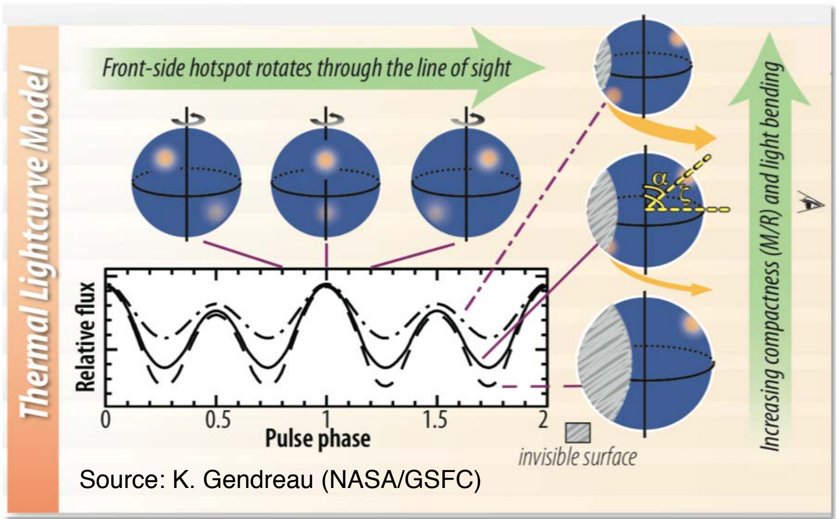


- Type I X-ray Bursts (Radius Expansion)

- Quiescent Transient Low-Mass X-ray Binary Spectroscopy



Millisecond Pulsars: X-ray Pulse Shape



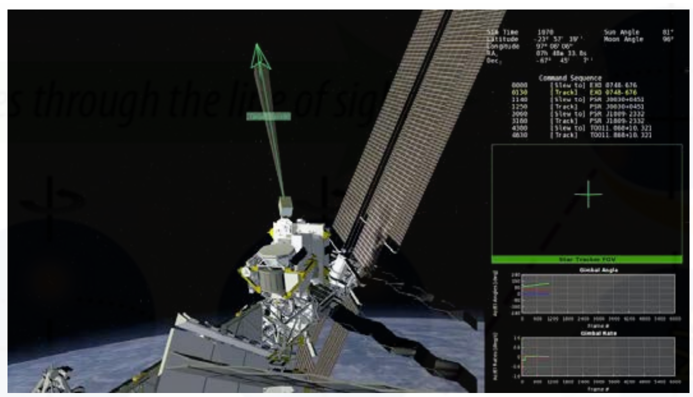
Source: K. Gendreau (NASA/GSFC)

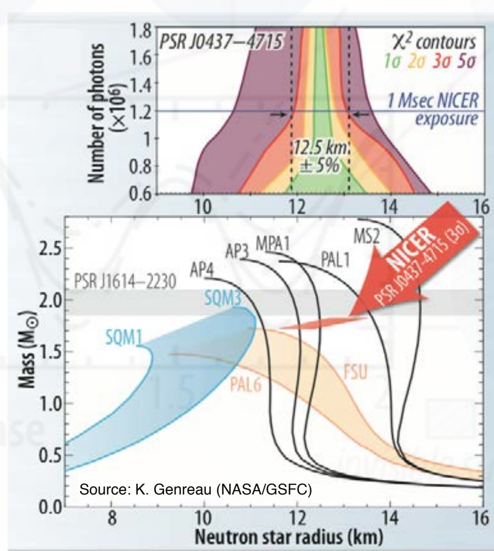
The more compact (higher M/R) the NS, the more "washed out" the pulse shape is.

See Work by Bogdanov (2007, 2013), Psaltis et al (2014).

Neutron Star Interior Composition ExploreR (NICER)

- Will be mounted on International Space Station (late 2016; NASA).
- Part of Primary Science: Use Pulsar-Phase Intensity Modelling to constrain the neutron star M/R for PSR J0437-4715.
- Combining this with phase resolved spectroscopy, the group claims they can place the shown constraint on the neutron star mass and radius for PSR J0437-415.





Source: K. Genreau (NASA/GSFC)

See Work by Bogdanov (2007, 2013), Psaltis et al (2014).

Radius Expansion Type-I X-ray Bursts

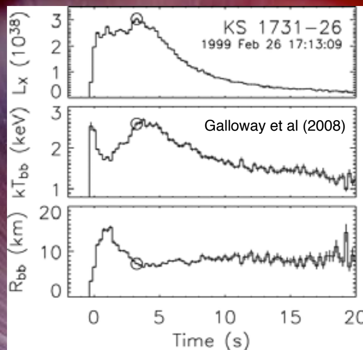
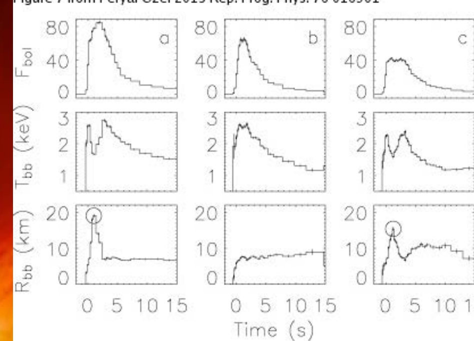
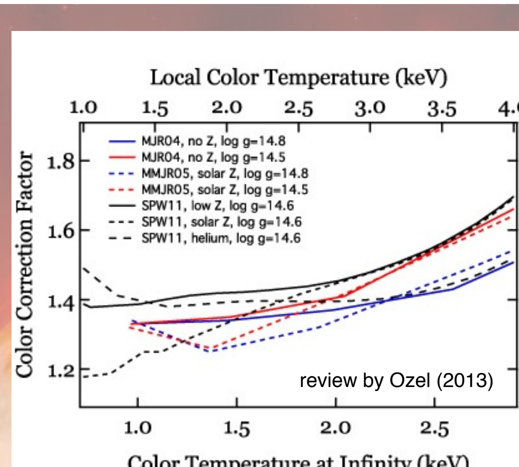


Figure 7 from Feryal Özel 2013 Rep. Prog. Phys. 76 016901

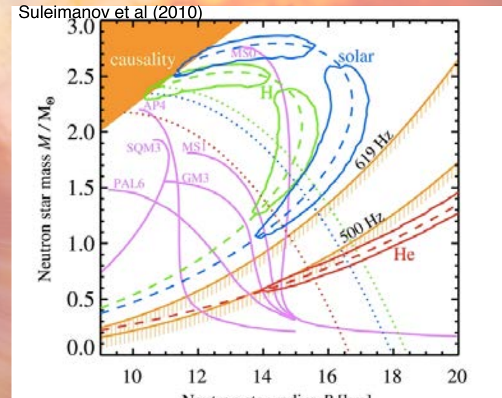


- Due to thermonuclear He flashes under a dense pile, in accreting neutron stars.
- Radius Expansion bursts are about 1% of all Type-I X-ray busts.
- Combined with spectral modelling of surface emission, permit extraction of NS radius and mass.

- The major advantage of Radius expansion bursts is the are significantly higher flux (and so, S/N) then other methods.
- A disadvantage is that theoretical interpretation of the spectra is ambiguous: some observers use *ab initio* atmospheric model calculations, finding limitation from theoretical uncertainties. Others collect these uncertainties in a Color Correction Factor (a model free parameter), with the idea that statistical characterization of this CCF will permit measurements.
- If these theoretical ambiguities can be overcome, this will likely be the best way to measure neutron star masses and radii, due to the very high fluxes of type I X-ray bursts.



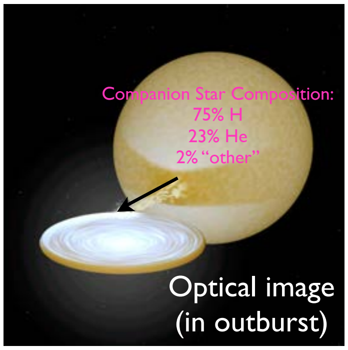
Suleimanov et al (2010)

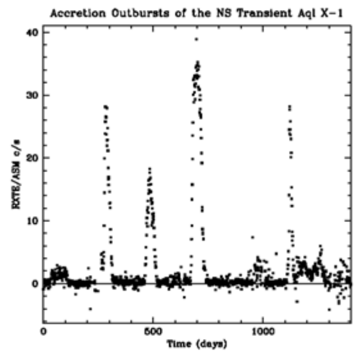


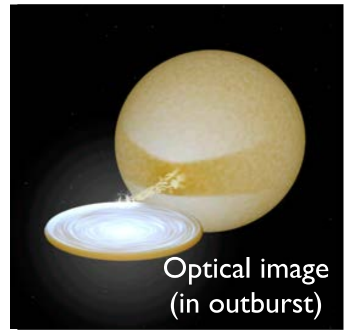
See additional work by Suleimanov and Poutanen

Quiescent Low Mass X-ray Binaries (qLMXB)

Outburst







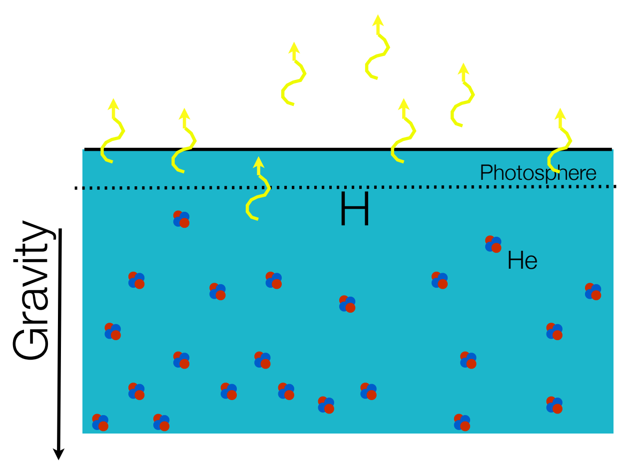
Quiescence

- Transient LMXBs in quiescence are H atmosphere neutron stars, powered by a core heated through equilibrium nuclear reactions in the crust.

Brown, Bildsten & RR (1998)

qLMXBs, in this scenario, have pure Hydrogen atmospheres

- When accretion stops, the He (and heavier elements, gravitationally settle on a timescale of ~10s of seconds (like rocks in water), leaving the photosphere to be pure Hydrogen (Alcock & Illarionov 1980, Bildsten et al 1992).



Brown, Bildsten & RR (1998)

Emergent Spectrum of a Neutron Star Hydrogen Atmosphere

•H atmosphere calculated Spectra are ab initio radiative transfer calculations using the Eddington equations.

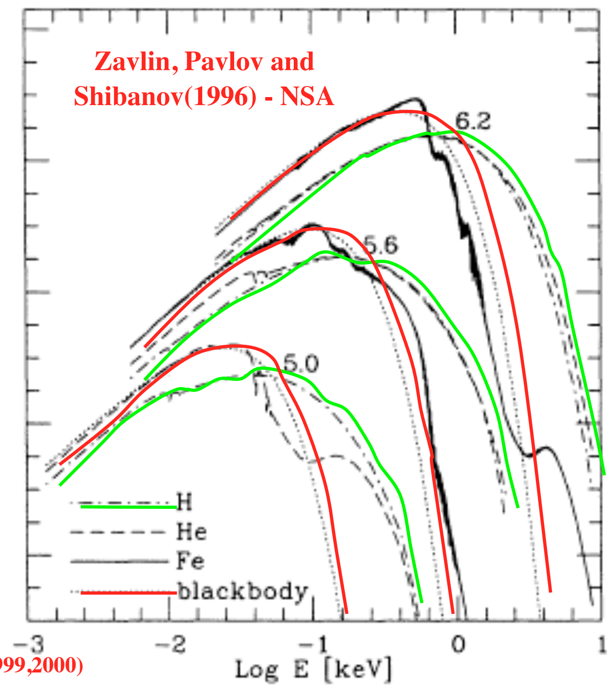
• Rajagopal and Romani (1996); Zavlin et al (1996); Pons et al (2002; Heinke et al (2006) -- NSATMOS; Gaensicke, Braje & Romani (2001); Haakonsen et al (2012)

All comparisons show consistency within ~few % (e.g. Webb et al 2007, Haakonsen 2012).

"Vetted": X-ray spectra of Zavlin, Heinke together have been used in several dozen analyses by several different groups.

$$F = 4\pi\sigma_{SB}T^4_{\mathrm{eff},\infty}\left(\frac{R_\infty}{D}\right)^2$$

$$R_\infty = \frac{R}{\sqrt{1-\frac{2GM}{c^2R}}}$$



RR et al (1999,2000)

Non-Equilibrium Processes in the Outer Crust

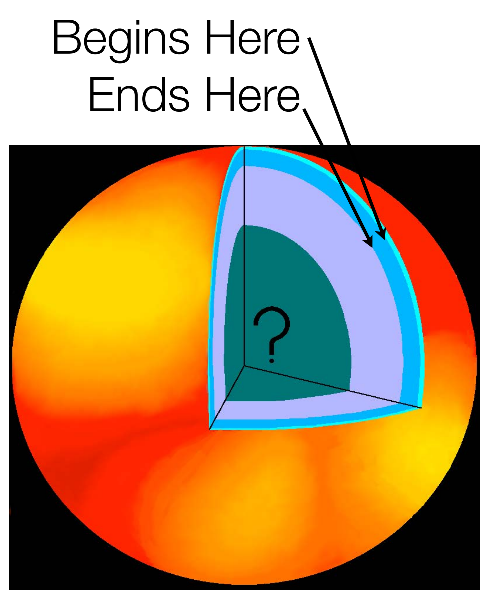
Beginning with

ρ (g cm	Reaction	Δρ/ρ	Q (Mev/np)
1.5	^{56}Fe	0.08	0.01
1.1	^{56}Cr	0.09	0.01
7.8	^{56}Ti	0.1	0.01
2.5	^{56}Ca	0.11	0.01
6.1	^{56}Ar	0.12	0.01

Deep Crustal Heating

Non-Equilibrium Processes in the Inner Crust

ρ (g cm	Reaction	X	Q (Mev/np)
9.1	^{52}S	0.07	0.09
1.1	^{46}Si	0.07	0.09
1.5	^{40}Mg		
	^{34}Ne+	0.29	0.47
1.8	^{68}Ca	0.39	0.05
2.1	^{62}Ar	0.45	0.05
2.6	^{56}S	0.5	0.06
3.3	^{50}Si	0.55	0.07
4.4	^{44}Mg		
	^{36}Ne+		
	^{68}Ca	0.61	0.28
5.8	^{62}Ar	0.7	0.02
7.0	^{60}S	0.73	0.02
9.0	^{54}Si	0.76	0.03
1.1	^{48}Mg+	0.79	[illegible]
1.1	^{96}Cr	0.8	0.01



1.47 Mev per np

Brown, Bildsten & RR (1998)

How to Measure a Neutron Star Radius.
The Assumptions: The Systematic Uncertainties.

- **H atmosphere neutron stars.** Expected from a Hydrogen companion LMXB; can be supported through optical observations of a H companion. Strongly justified on theoretical grounds.

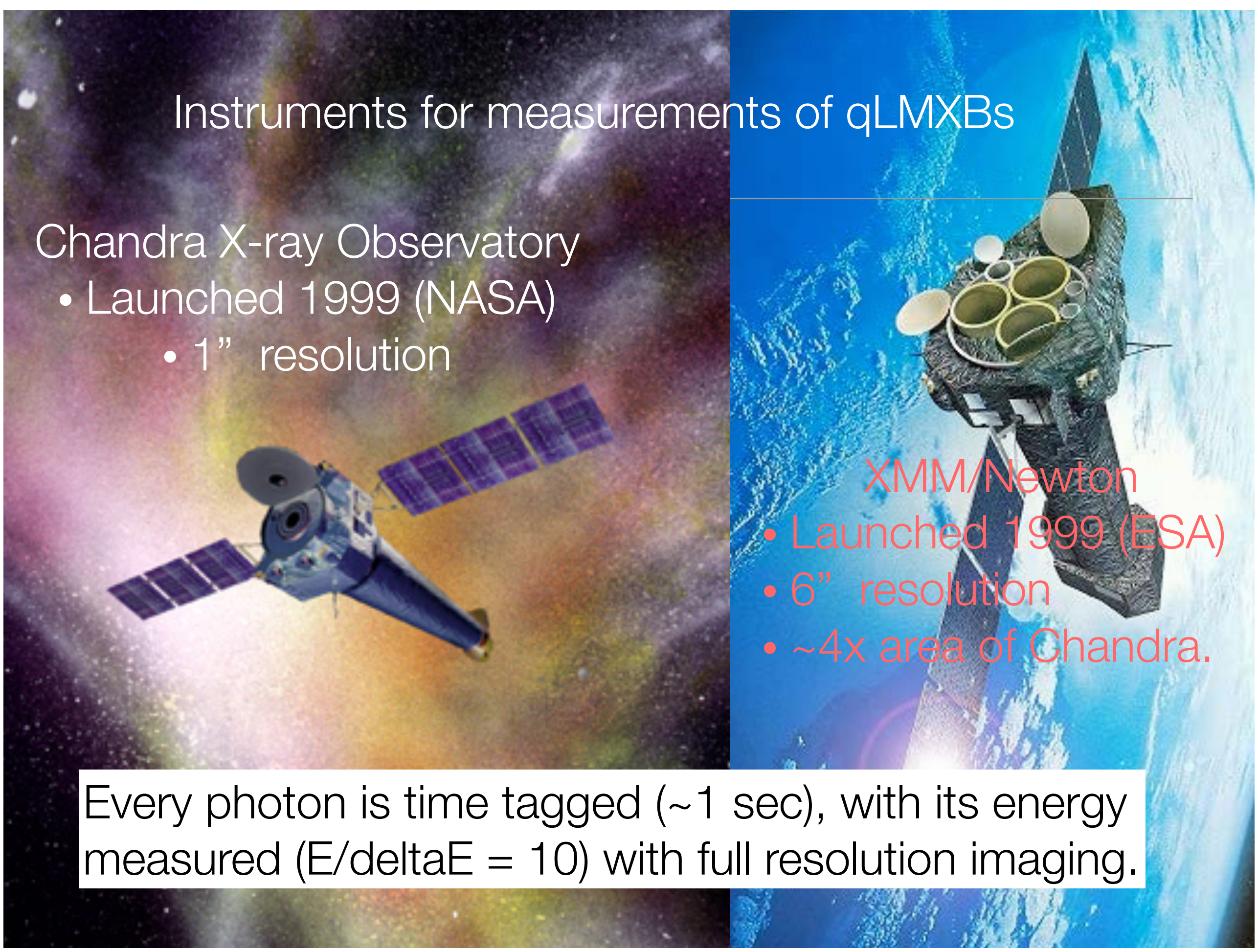
- **Low B-field (<10^{10} G) neutron stars.** This is true for 'standard' LMXBs as a class, but difficult to prove on a case-by-case basis.

- **Emitting isotropically.** Occurs naturally when powered by a hot core.

- **Non-Rotating neutron stars.** qLMXBs are observed to rotate at 100-600 Hz. This can be a significant fraction of the speed of light. Doppler boosting and deviation from NS spheroidal geometry are not included in emission models.

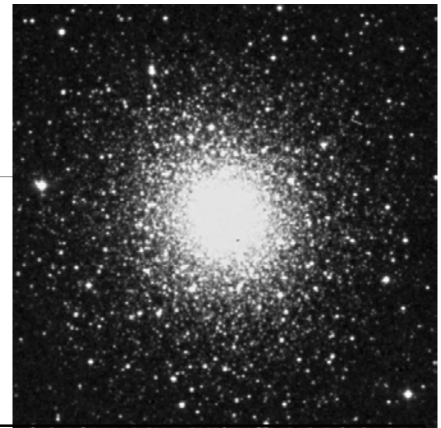
- **Consider neutron star masses >0.5 solar mass, only.**

If you don't like these assumptions:
"We find the assumptions not strongly supported and therefore ignore this result."



The qLMXB Factories: Globular Clusters

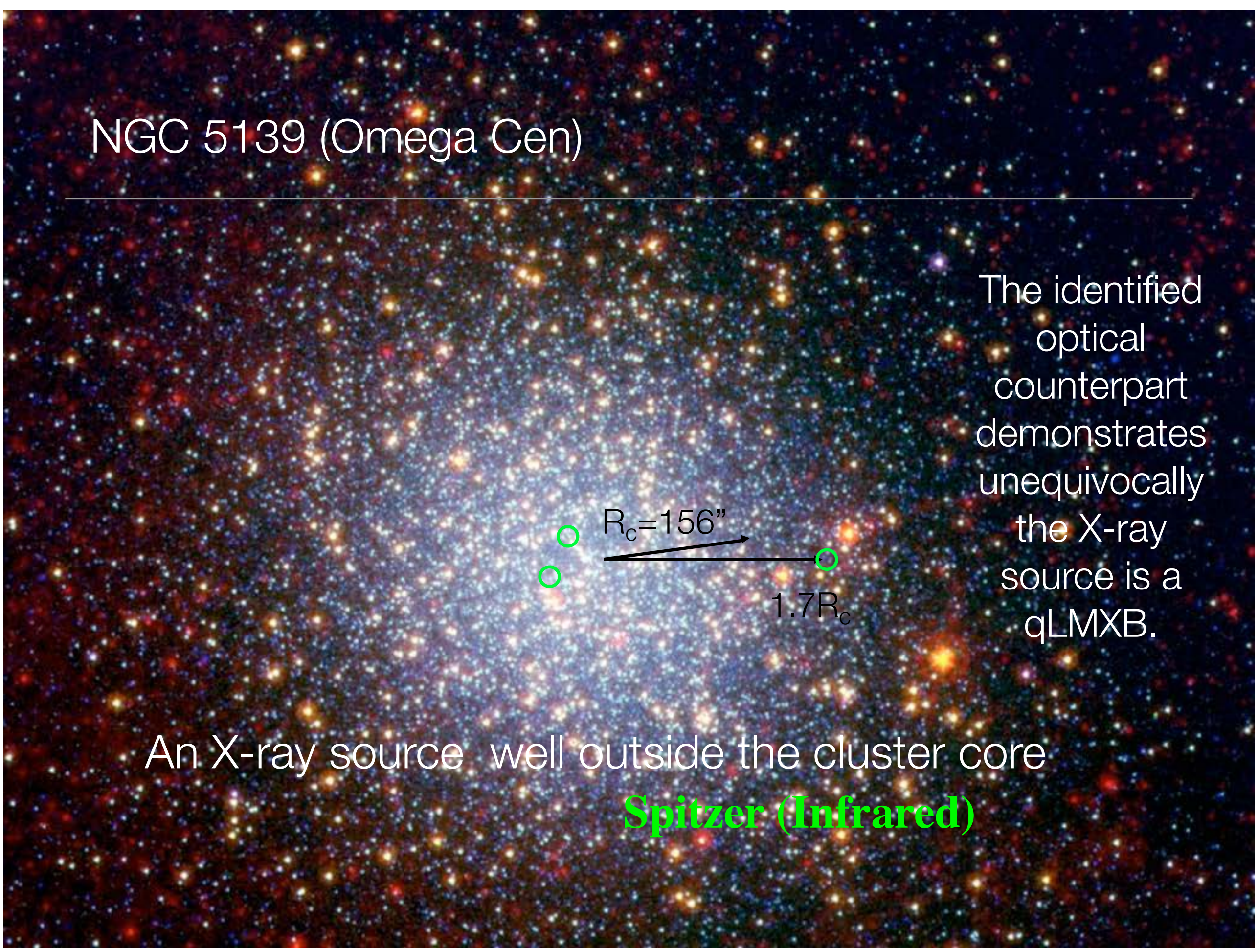
- GCs : overproduce LMXBs by 1000x vs. field stars
- Many have accurate distances measured.



qLMXBs can be identified by their soft X-ray spectra, and confirmed with optical counterparts.

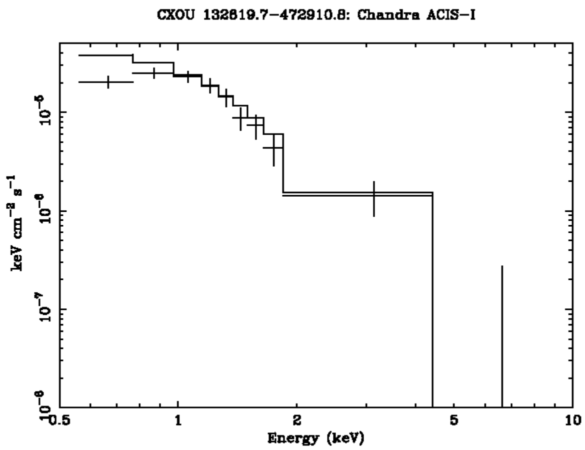
NGC	D (kpc)	+/-(%)
104	5.13	4
288	9.77	3
362	10	3
4590	11.22	3
5904	8.28	3
7099	9.46	2
6025	7.73	2
6341	8.79	3
6752	4.61	2

Carretta et al (2000)



NGC 5139 (Omega Cen)

X-ray Spectrum is inconsistent with any other type of known GC source (pulsars, CVs, coronal sources).



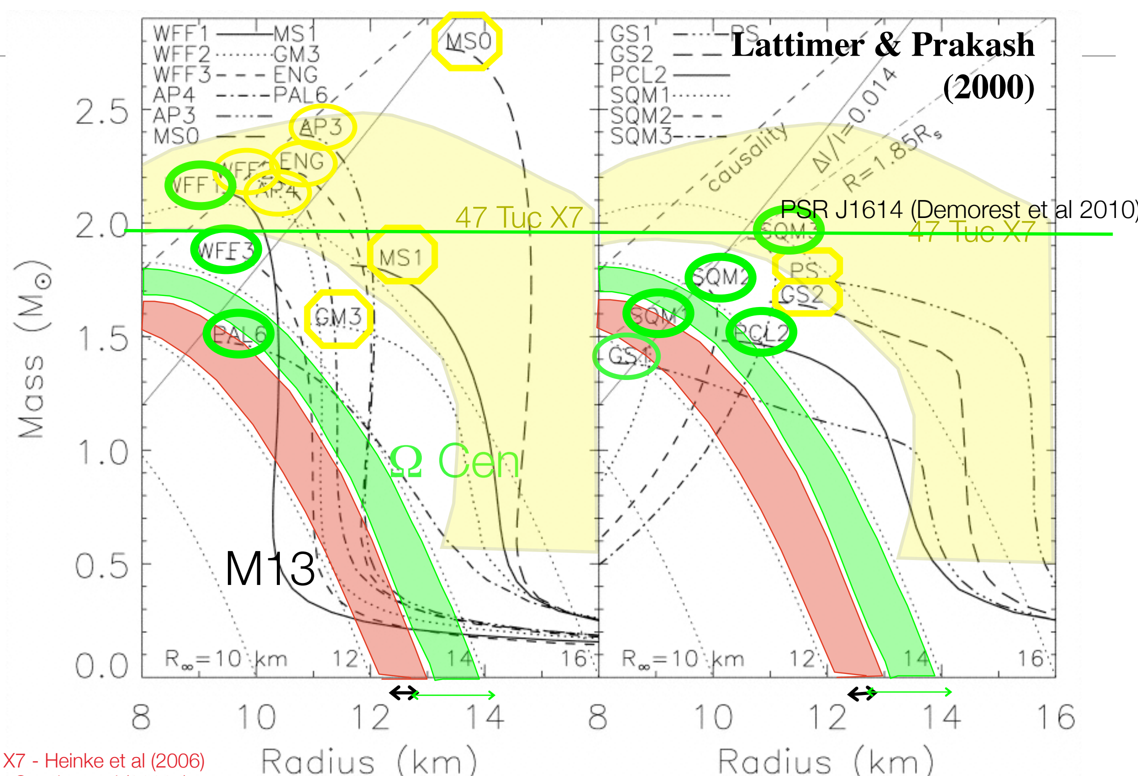
Full confirmation as LMXB requires Hubble photometry

R	kT	N (1e20 cm
14.3 ± 2.1 km	66^{+4}_{-5} eV	(9)

RR et al (2002)

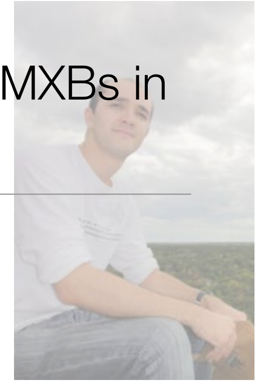
Mass-Radius Constraints on the Equation of State from qLMXBs in GCs

$$R_{\infty} = \frac{R_{\rm NS}}{\sqrt{1 - \frac{2GM_{\rm NS}}{c^2 R_{\rm ns}}}}$$

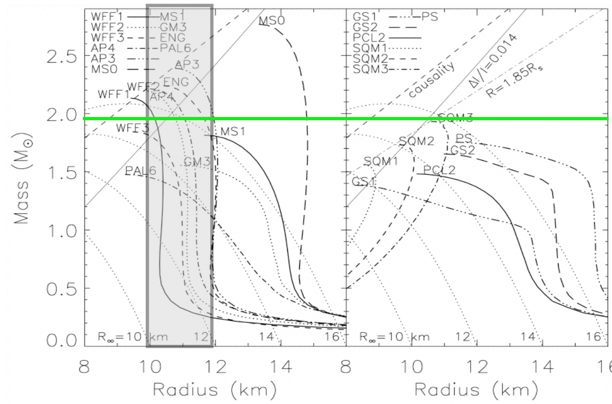


47 Tuc X7 - Heinke et al (2006)
M13 - Gendre et al (2002a)
Omega Cen - Gendre et al (2002b)

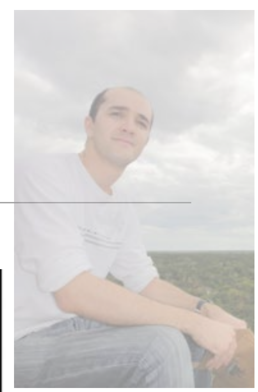
Measuring the Radius of Neutron Stars from qLMXBs in Globular Clusters



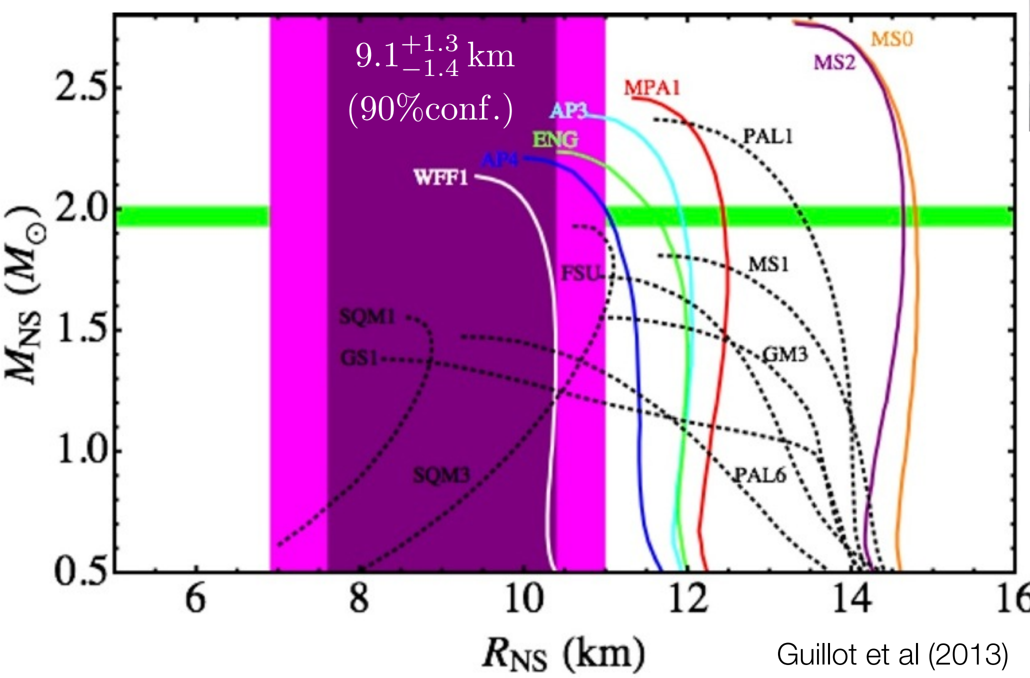
- The 2.0 solar mass neutron stars favor hadronic dEOSs over quark and phase-transition dEOSs. These have the property of a quasi-constant neutron star radius.
- Analysis goal: Using all suitable qLMXB X-ray data sets of targets (there are five) provide the most reliable neutron star radius measurement possible.
- Assume the radius of neutron stars is **quasi-constant** (a constant, at astrophysically important masses, within measurement error).
- Perform a Markoff-Chain-Monte-Carlo (MCMC) and include all known uncertainties and use conservative assumptions.



The Neutron Star Radius



<11 km (99% conf).



Guillot et al (2013)

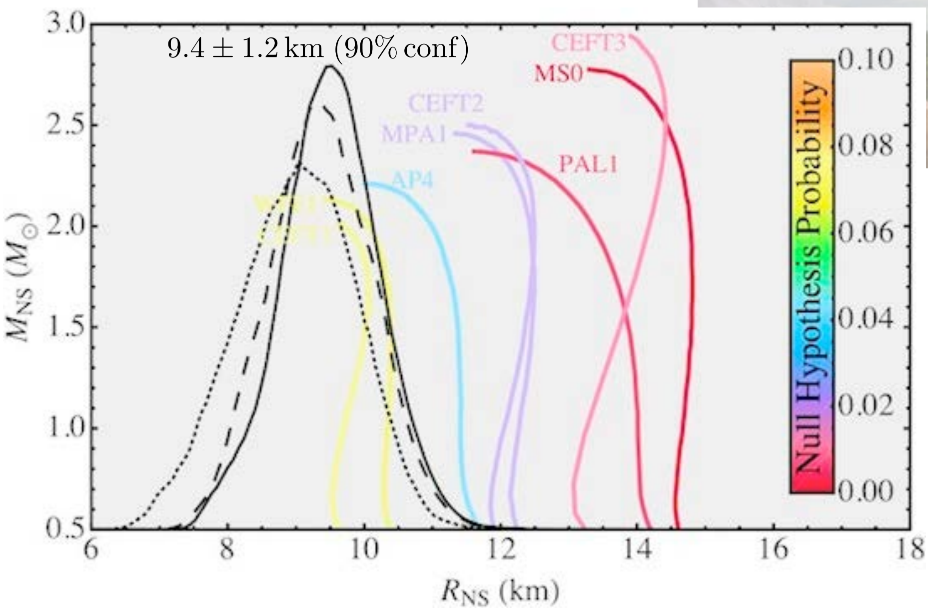
M-R by J. Lattimer

WFF1= Wiring, Fiks and Fabrocini (1988)

Contains uncertainties from:
Distance
All spectral parameters
Calibration

New Result: Guillot & RR (2014)

- PAL1 (Prakash et al 1988), MS0 (Müller & Serot 1996) both excluded with >99% confidence. (Chisqr analysis)
- CEFT1 is favored over CEFT2,3 (Hebeler et al 2013). (Chisqr analysis)
- Assuming it is constant (independent of mass), the Bayesian constraint is:



$$\boxed{R_{NS} = 9.4 \pm 1.2\,\mathrm{km}\ (90\%\ \mathrm{conf})}$$

THE FUTURE

Mass Measurements with Continuum Spectra

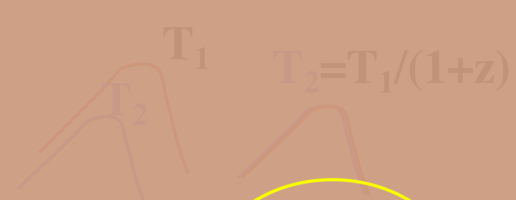
$$R_\infty \rightarrow \quad \text{Normalization}$$

$$T_\infty \rightarrow \quad \text{Peak of the spectrum}$$

$$z = \frac{G\,M_{NS}}{c^2 R_{\rm NS}} \rightarrow \quad \text{Second Derivative at the Peak of the Spectrum}$$

You cannot measure a redshift from blackbody emission due to photon energy (E) temperature (kT) degeneracy.

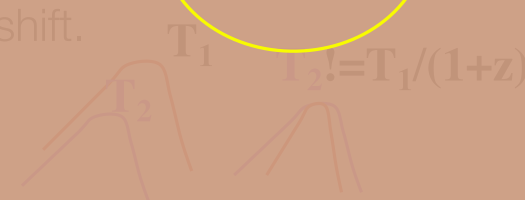
$$I(E_\gamma) \propto \left(\frac{E_\gamma}{kT}\right)^3 \frac{1}{e^{\frac{E_\gamma}{kT}} - 1}$$



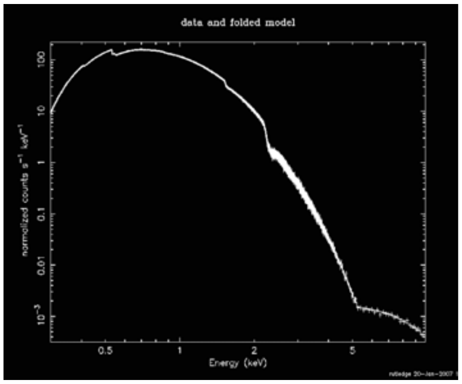
$$\left(R_\infty = \frac{R_{\rm NS}}{\sqrt{1 - \frac{2GM_{\rm NS}}{c^2 R_{\rm ns}}}}, z = \frac{GM_{\rm NS}}{c^2 R_{\rm NS}}\right) \rightarrow (R_{\rm NS}, M_{\rm NS})$$

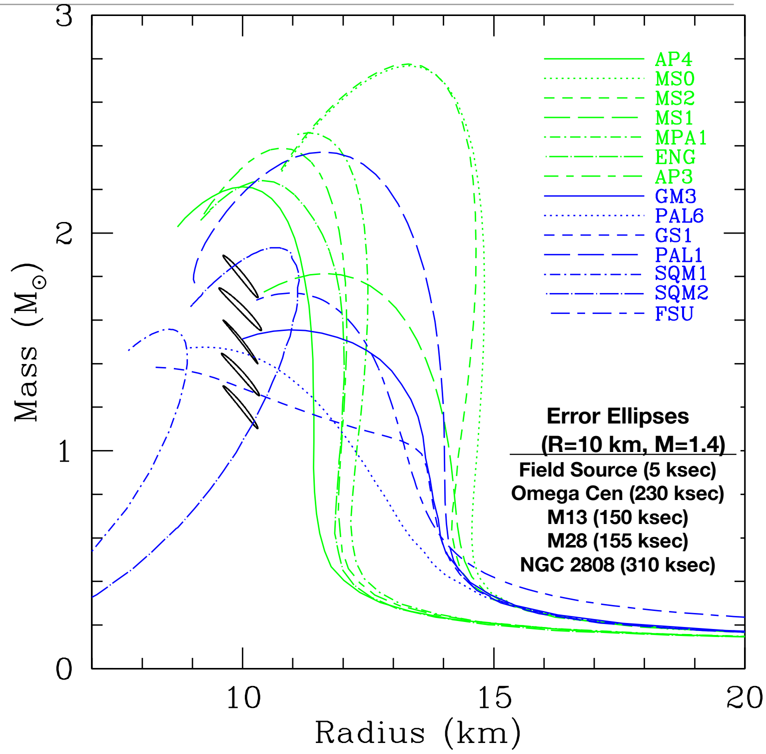
- But the free-free opacity breaks this degeneracy. This spectrum, redshifted, permits (in principle) determination of the redshift.

$$I(E_\gamma) \propto \left(\frac{E_\gamma}{kT}\right)^3 \frac{1}{e^{\frac{E_\gamma}{kT}} - 1} \kappa_{ff,o} \left(\frac{E_o}{E_\gamma}\right)^3$$

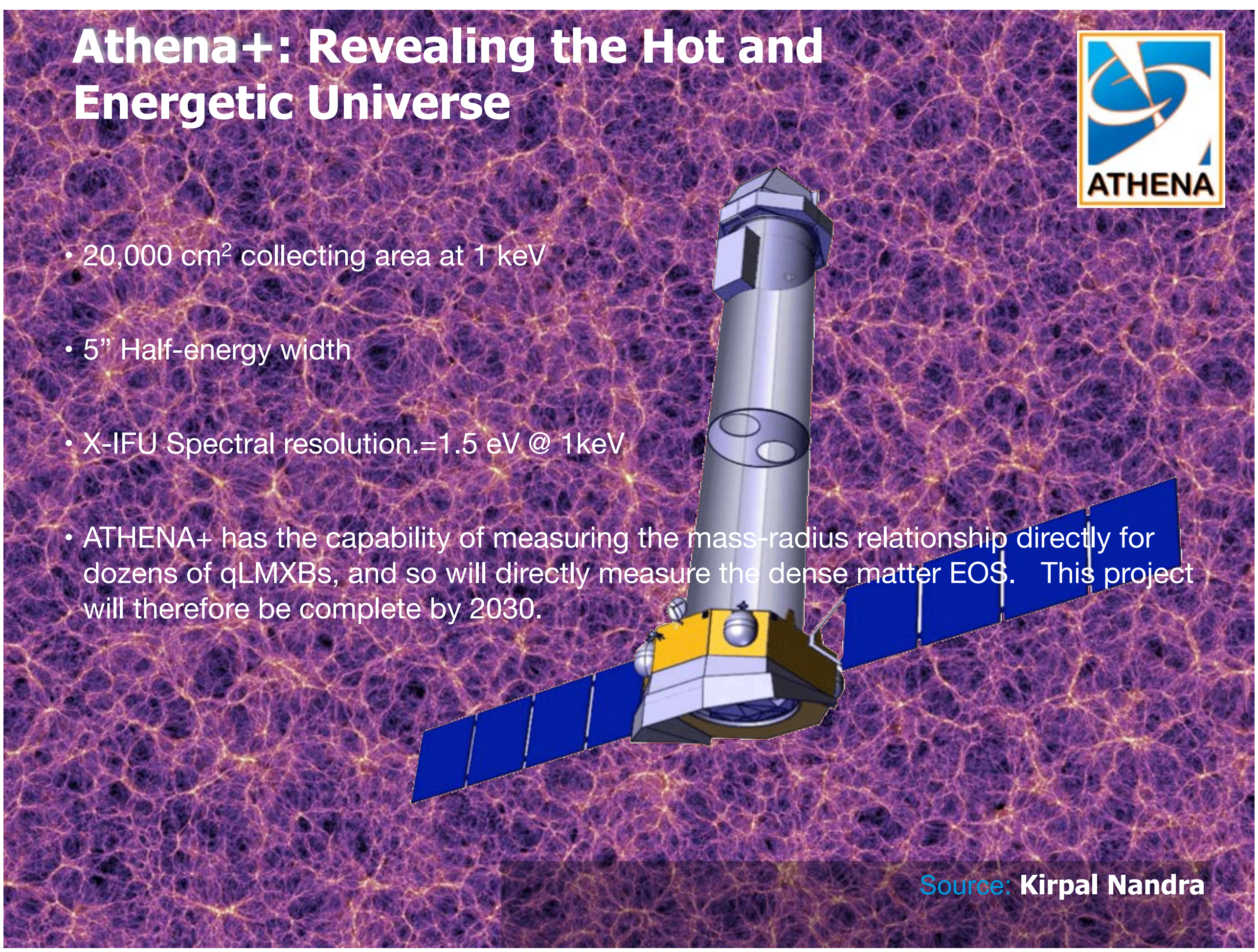


Calorimeter response curves
Simultaneous Mass and Radius Measurement





Requirement: 250k-300k counts with calorimeter (2.5 eV) energy resolution.



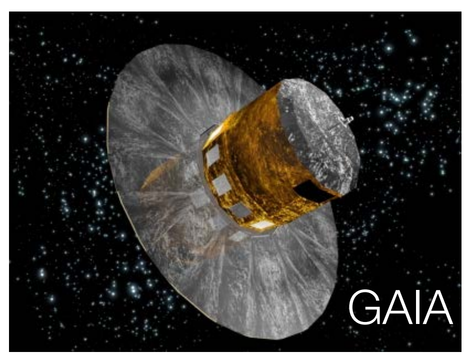
Mission	Collecting Area (cm2) at 1 keV	Energy Resolution (E/dE)	Status
Chandra (NASA-USA)	750	10	Launched 1999. Operating nominally
XMM (ESA-Europe)	4650	10	Launched 1999. Operating nominally
Astro-H (JAXA-Japan)	180	500	Launch Dec 2015
Athena+ (ESA-Europe)	20,000	667	Planned Launch 2028

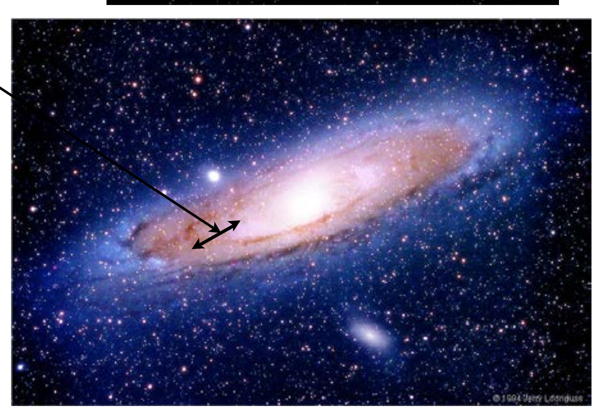
Distances will be measured using GAIA, before 2020.

- Launched (to L2) 2013, now taking data. 5 years, all-sky-survey.

V	# (millions)	$\sigma_{\mu-arcsec}$	3% Distance (kpc)
10	0.34	7	4.2
15	26	22	1.4
20	1000	250	0.12

Estimate for # of qLMXBs within 1.4 kpc
= 2000/galaxy *(1.4 kpc/10 kpc)^2 = 40





Conclusions

- In a model dependent way, we (Guillot et al 2013, Guillot & RR 2014) have measured the radius for neutron stars to be

$$\boxed{R_{NS} = 9.4 \pm 1.2\,\mathrm{km}\ (90\%\,\mathrm{conf})}$$

- There are three methods being undertaken — with different neutron star source classes, different assumptions, and different sources of uncertainties — over the next years - decades.

- The future X-ray mission, ATHENA+ (launch 2028), combined with accurate distance measurements provided by GAIA, will measure the masses and radii of neutron stars simultaneously, for several-dozens of NSs. **The dense matter equation of state will be measured by 2030.**