UNIVERSITÀ DEGLI STUDI DI CATANIA

INFN-LNS

Anisotropic flows and shear viscosity of the Quark-Gluon plasma within a transport approach

S. Plumari, G.L. Guardo, A. Puglisi,

F. Scardina, M. Ruggieri, V. Greco

Outline

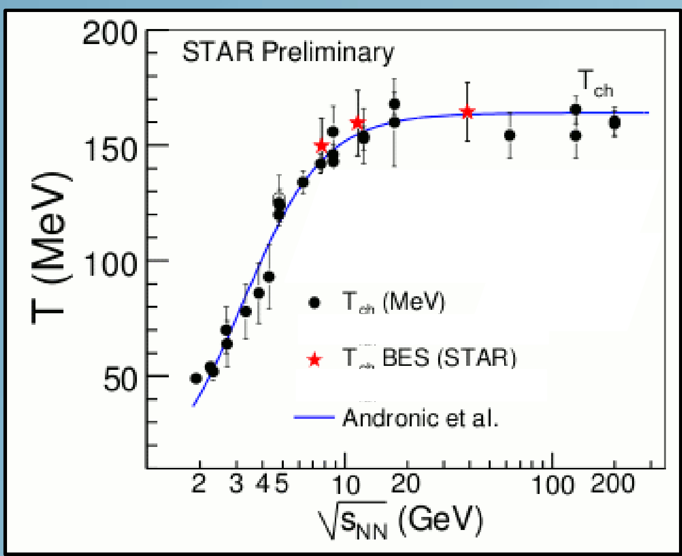
- QGP: nearly perfect fluid
- Transport approach at fixed η/s
- Initial state fluctuations:
  - η/s and generation of $v_n(pT)$: from RHIC to LHC
  - Correlations between ε_n and v_n
- Conclusions

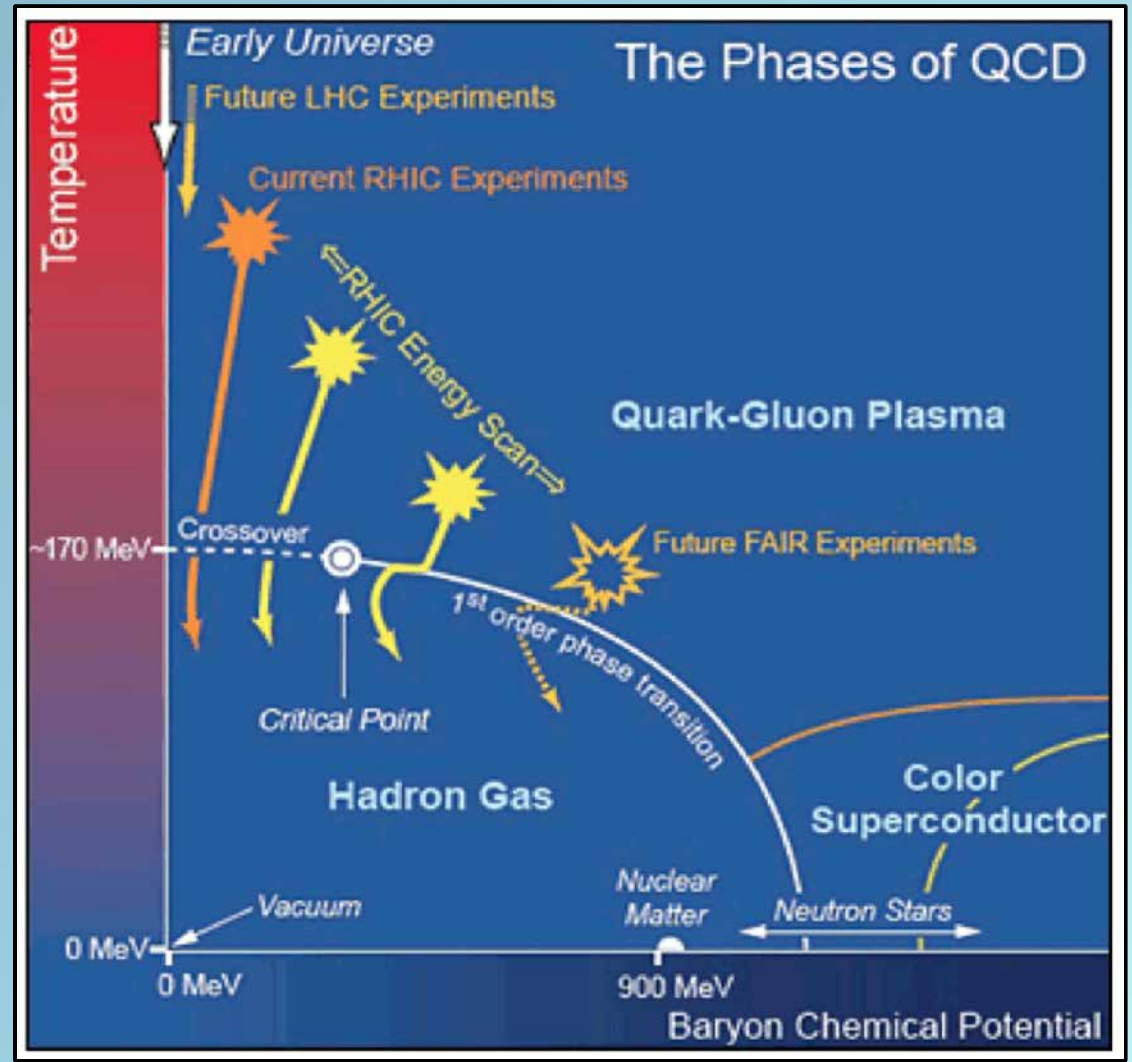
Probing the QCD Phase Diagram

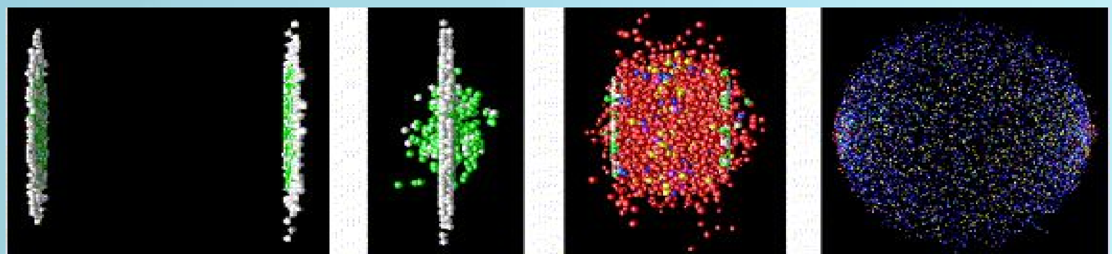
By systematically varying the beam energy, with heavy ion collisions it possible to probe different regions of the QCD phase diagram.

RHIC: $T \sim 1.5 - 2\ T_c$, $\tau \sim 4 - 5$ fm/c

LHC: $T \sim 3 - 3.5\ T_c$, $\tau \sim 8 - 10$ fm/c







η/s(T) around to a phase transition

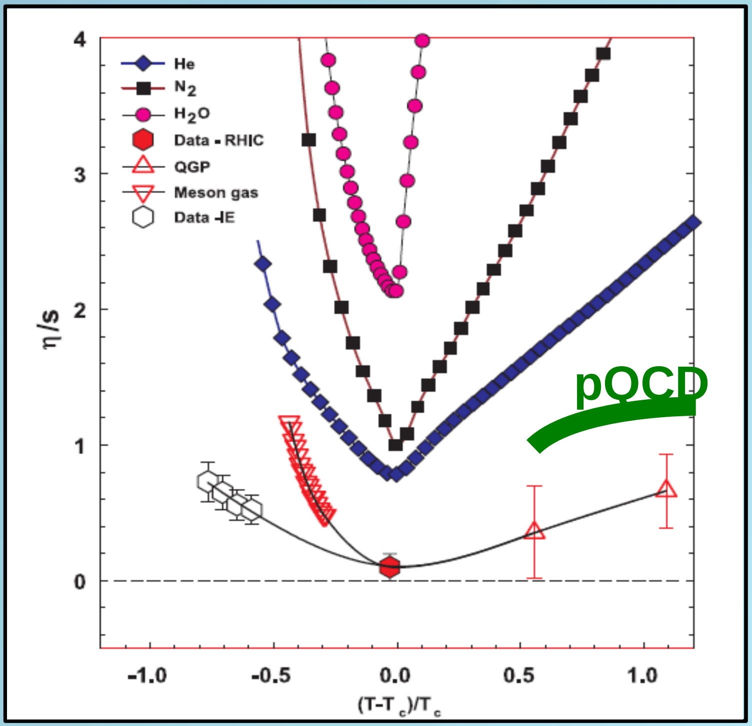
- **Quantum mechanism**

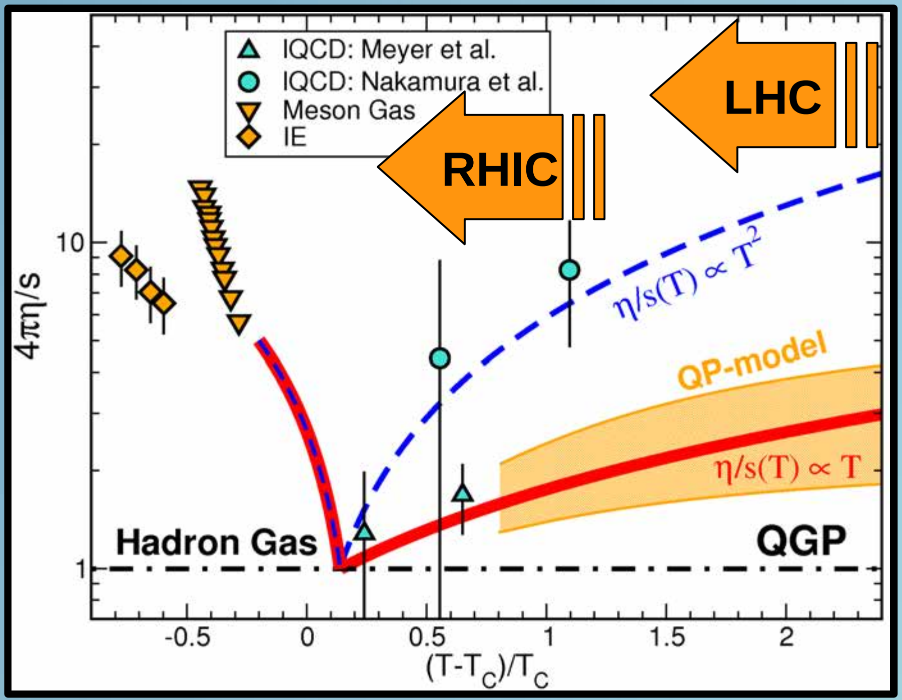
$$\Delta E \cdot \Delta t \geqslant 1 \;\rightarrow\; \eta/s \approx \frac{1}{15}\langle p\rangle \cdot \tau > \frac{1}{15}$$

- **AdS/CFT suggest a lower bound $\eta/s = 1/(4\pi) \sim 0.08$**
- **From pQCD:** $\eta/s \sim \frac{1}{g^4 \ln(1/g)} \;\Rightarrow\; \eta/s \sim 1$

P.Arnold et al., JHEP 0305 (2003) 051.

P. Kovtun et al.,Phys.Rev.Lett. 94 (2005) 111601.
L. P. Csernai et al., Phys.Rev.Lett. 97 (2006) 152303.
R. A. Lacey et al., Phys.Rev.Lett. 98 (2007) 092301.

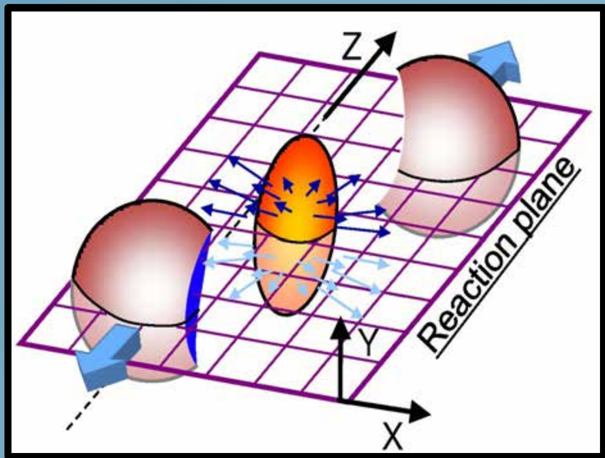




S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.

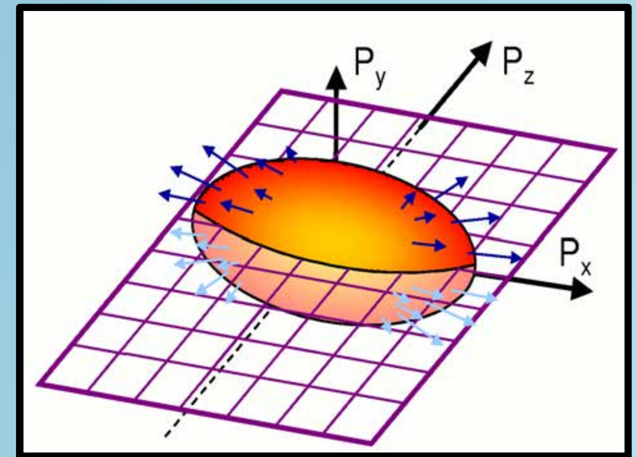
- **LQCD some results for quenched approx. large error bars**
- **Quasi-Particle models seem to suggest a $\eta/s \sim T^{\alpha}$ $\alpha \sim 1 - 1.5$.**
- **Chiral perturbation theory → Meson Gas**
- **Intermediate Energies – IE ($\mu_B > T$)**

Information from non-equilibrium: elliptic flow



$\lambda = (\sigma\rho)^{-1}$ or η/s viscosity

$c^2_s = dP/d\varepsilon$, EoS-lQCD



$$\varepsilon_x = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$$

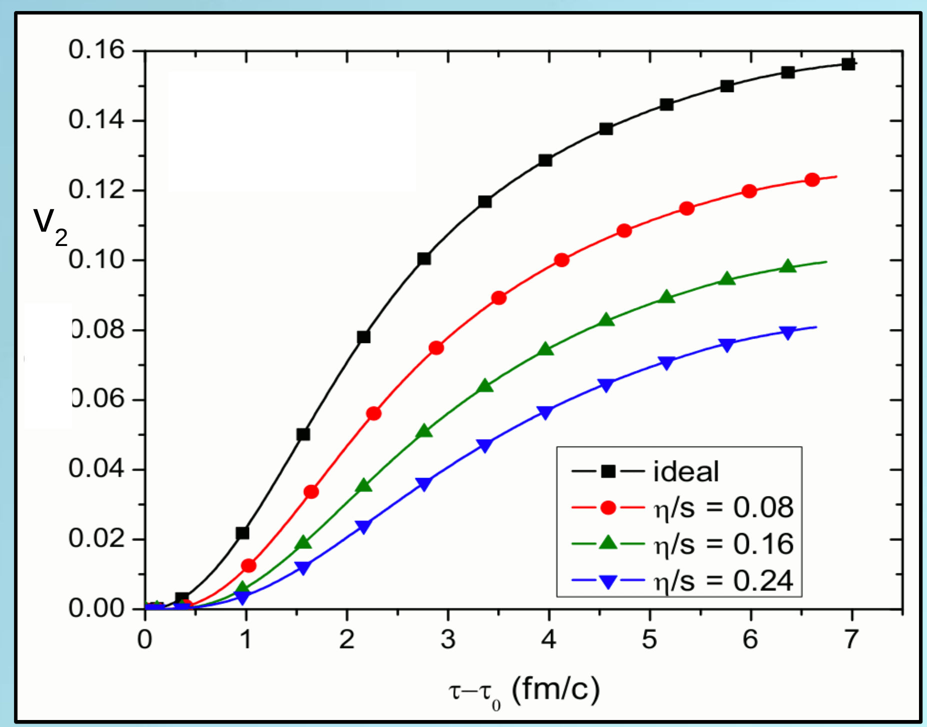
The v_2/ε measures efficiency in converting the eccentricity from Coordinate to Momentum space

$$v_2 = \langle \cos 2\varphi \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

Can be seen also as Fourier expansion

$$\frac{dN}{dp_T d\phi} = \frac{dN}{dp_T}[1 + 2v_2\cos(2\phi) + 2v_4\cos(4\phi) + \dots]$$

by symmetry v_n with n odd expected to be zero … (but event by event fluctuations)



Ideal Hydrodynamics: QGP an almost perfect fluid

$$\begin{cases} \partial_\mu T^{\mu\nu}(x) = 0 \\ \partial_\mu J^\mu_B(x) = 0 \end{cases}$$

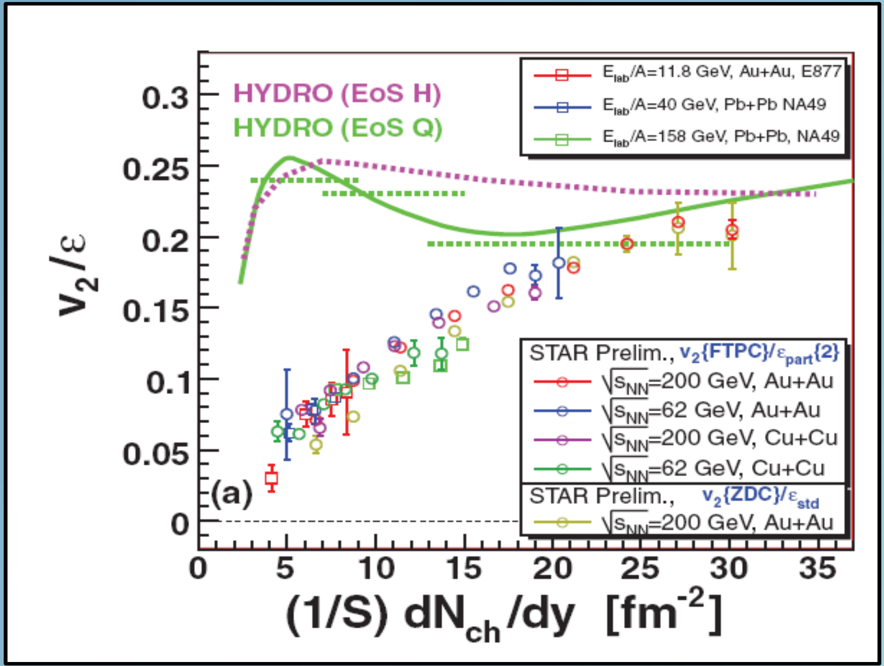
$$T^{\mu\nu}(x) = [\varepsilon + p]\, u^\mu u^\nu - p g^{\mu\nu}$$

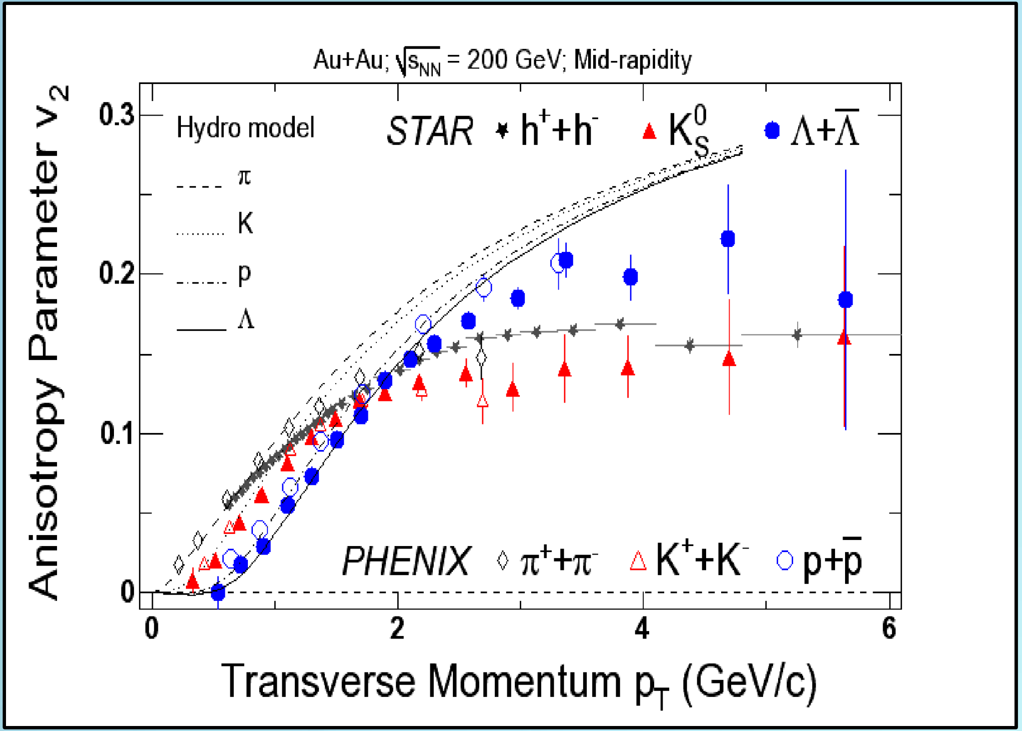
No microscopic description (λ ->0), no dissipation,…only conservation laws!

Mass ordering of v2(pT)

The experimental data close to ideal hydro

Large v_2/ε



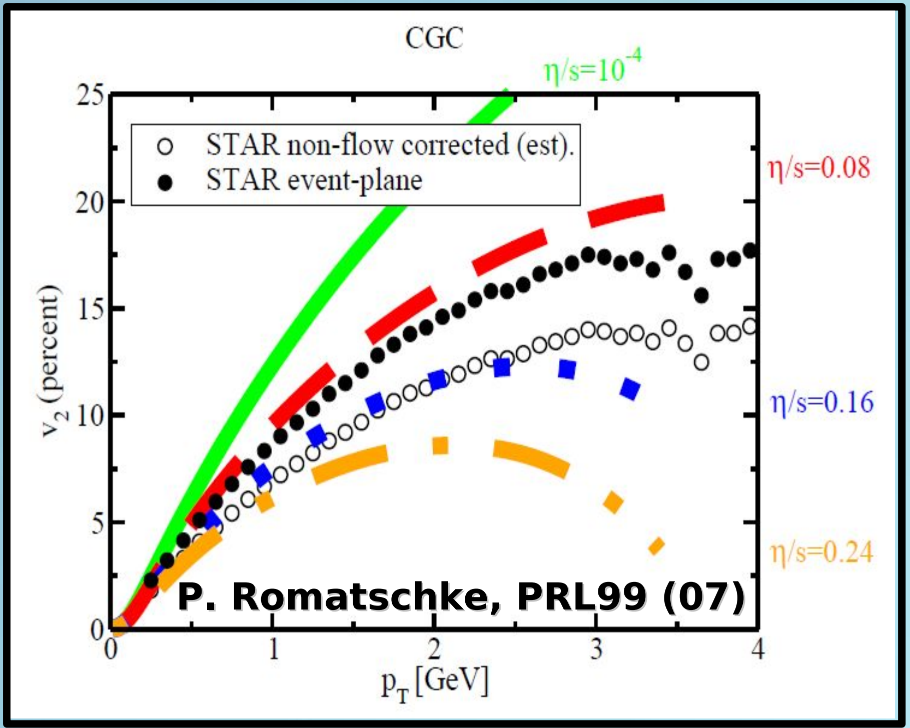


Viscous Hydrodynamics

$$T^{\mu\nu} = \overbrace{T^{\mu\nu}_{ideal}}^{\eta=0} + \overbrace{\Pi^{\mu\nu}_{dissip}}^{\eta\neq 0}$$

$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + \eta(\nabla^{\mu} u^{\nu} + \nabla^{\nu} u^{\mu} - \frac{2}{3}\Delta^{\mu\nu}\partial^{\alpha} u_{\alpha})$$

I^0 order Navier Stokes violates causality.
II^0 order expansion needed (Israel-Stewart)



Viscosity η/s

τ_η, τ_ζ two parameters appears +

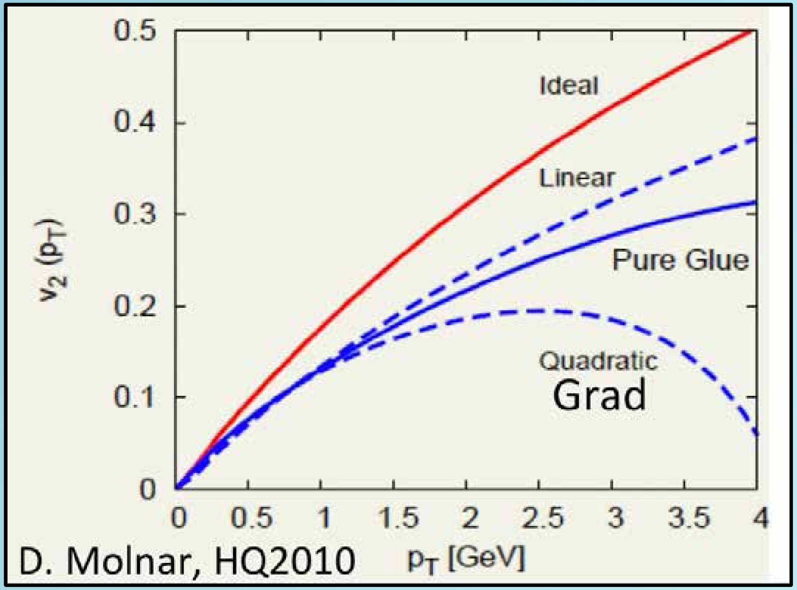
$\delta f \sim f_{eq}$ reduce the p_T validity range

Grad ansantz

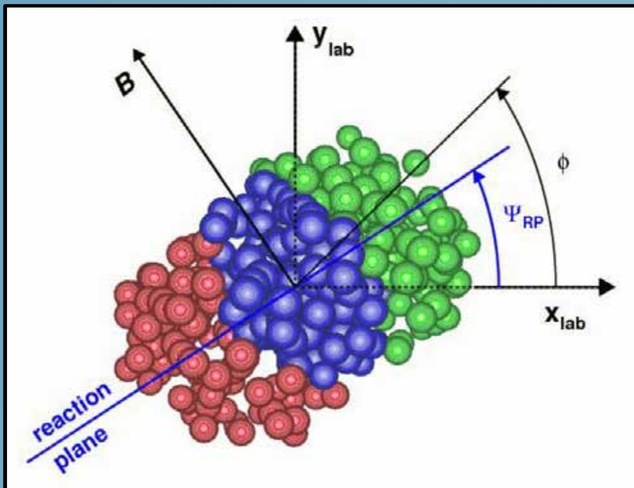
R. Lacey et al., PRC82

$$\delta f = \underbrace{\frac{\pi^{\mu\nu} p_\mu p_\nu}{(\epsilon + p) T^2} f_{eq} \approx \frac{\eta}{3s} \frac{p_T^2}{\tau T^2} f_{eq}}$$

- **inplies η in Relaxation Time Approximation**
  D. Teaney,Phys.Rev. C68 (2003) 034913
- **Hydro is valid up to $p_T \sim 2$ GeV**

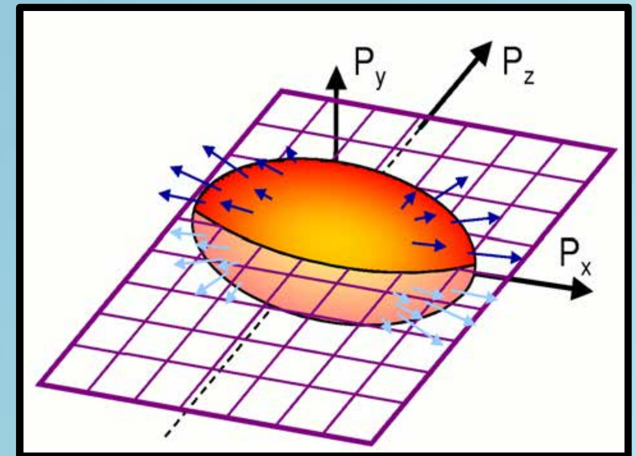


Information from non-equilibrium: $v_n(p_T)$



$\lambda = (\sigma\rho)^{-1}$ or η/s viscosity

$c^2_s = dP/d\varepsilon$, EoS-lQCD



$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\phi - \Phi_n)] \rangle}{\langle r_\perp^n \rangle}$$

The v_2/ε measures efficiency in converting the eccentricity from Coordinate to Momentum space

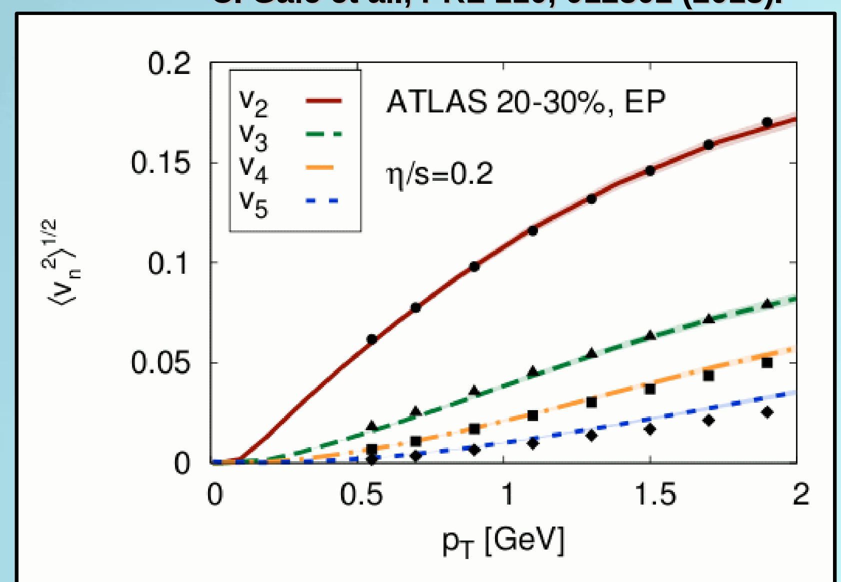
$$\langle v_n \rangle = \langle \cos[n(\phi - \Psi_n)] \rangle$$

Can be seen also as Fourier expansion

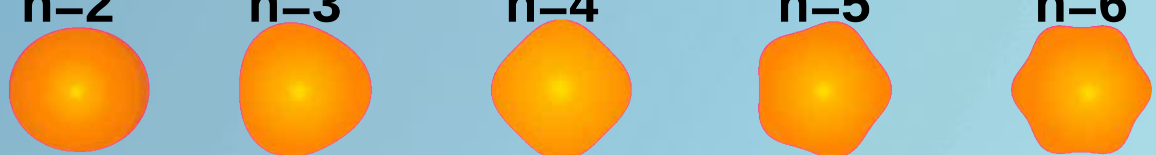
$$\frac{dN}{dp_T\, d\phi} = \frac{dN}{dp_T}[1 + 2 v_2 \cos(2\phi) + 2 v_4 \cos(4\phi) + \ldots]$$

by symmetry v_n with n odd expected to be zero … (but event by event fluctuations)

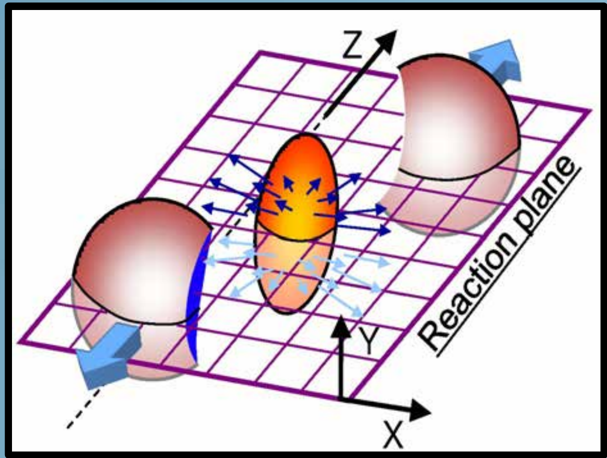
C. Gale et al., PRL 110, 012302 (2013).



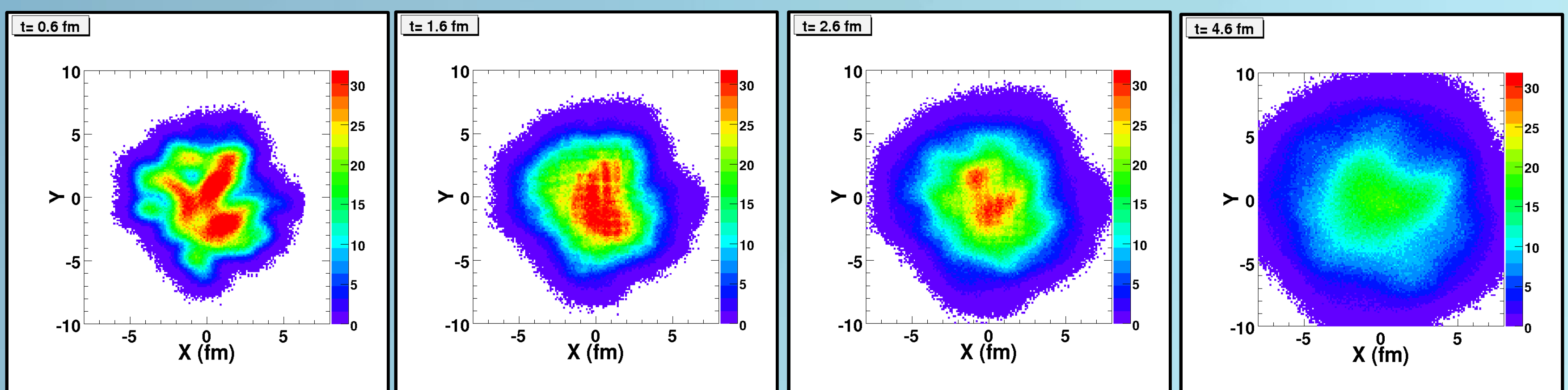
n=2 n=3 n=4 n=5 n=6



Applying kinetic theory to A+A Collisions….



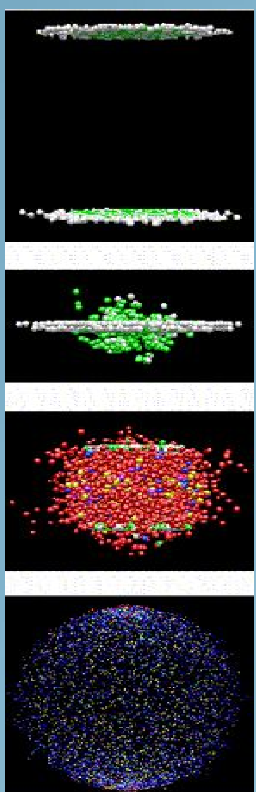
- Impact of $\eta/s(T)$ on the build-up of $v_n(p_T)$ vs. beam energy.
- To include the Initial state fluctuations.



Motivation for a kinetic approach:

$$\{p^{\mu}\partial_{\mu} + [p_{\nu}F^{\mu\nu} + M\,\partial^{\mu}M]\partial^{p}_{\mu}\}\, f(x,p) = C_{22} + C_{23} + \dots$$

Free streaming — **Field Interaction → $\varepsilon \neq 3P$** — **Collisions → $\eta \neq 0$**



- **Starting from 1-body distribution function and not from $T^{\mu\nu}$:**

  - possible to include f(x,p) out of equilibrium.

  M. Ruggieri et.al, PLB 727 (2013) 177

  - extract information about the viscous correction δf to f(x,p)

- **It is not a gradient expansion in η/s.**
- **Valid at intermediate p_T out of equilibrium.**
- **Valid at high η/s (cross over region): + self consistent kinetic freeze-out**

Parton Cascade model

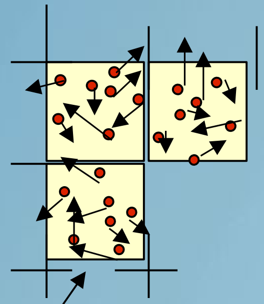
$$p^{\mu}\partial_{\mu} f(X,p) = C = C_{22} + C_{23} + \dots$$

Collisions → $\varepsilon - 3p = 0$, $\eta \neq 0$

$$C_{22} = \frac{1}{2E_1}\int \frac{d^3 p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3 p'_1}{(2\pi)^3 2E'_1}\frac{d^3 p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 \left|\boldsymbol{M}_{1'2'\to 12}\right|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

For the numerical implementation of the collision integral we use the stochastic algorithm. (Z. Xu and C. Greiner, PRC 71 064901 (2005))

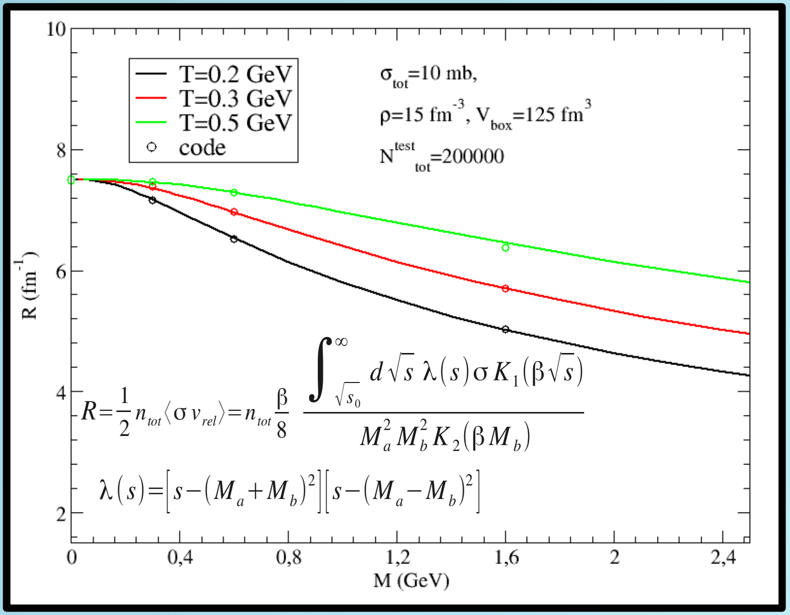
$$P_{22} = \frac{\Delta N_{coll}^{2\to 2}}{\Delta N_1 \Delta N_2} = v_{rel}\,\sigma_{22}\frac{\Delta t}{\Delta^3 x}$$



$\Delta t \to 0$

$\Delta^3 x \to 0$

⇒ **right solution**



Simulating a constant η/s

For the general case of anisotropic cross section and massless particles:

$$\eta(\vec{x},t)/s = \frac{1}{15}\langle p\rangle\,\tau_\eta \quad \longrightarrow \quad \sigma_{tot}^{\eta/s} = \frac{1}{15}\frac{\langle p\rangle}{g(m_D/2T)\,n}\frac{1}{\eta/s}$$

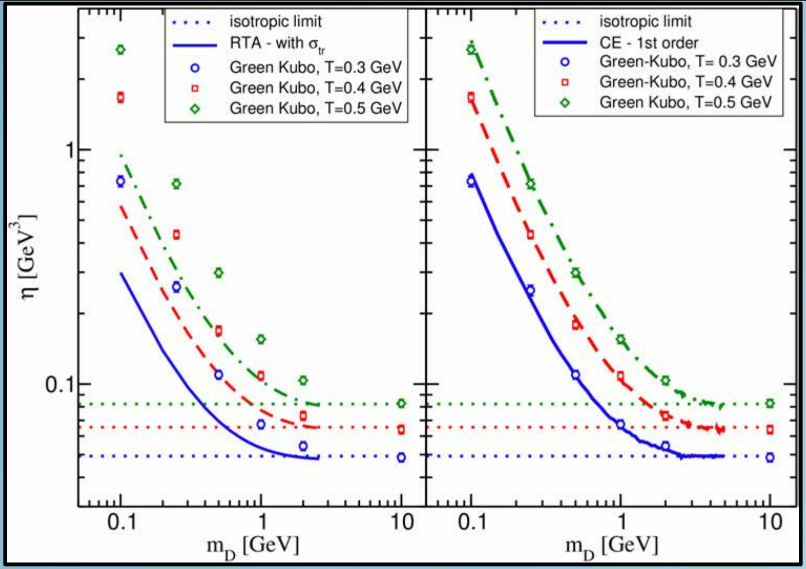
σ is evaluated in such way to keep fixed the η/s during the dynamics according the Chapman-Enskog equation. (similar to D. Molnar, arXiv:0806.0026[nucl-th] but our approach is local.)

- We know how to fix locally η/s(T)

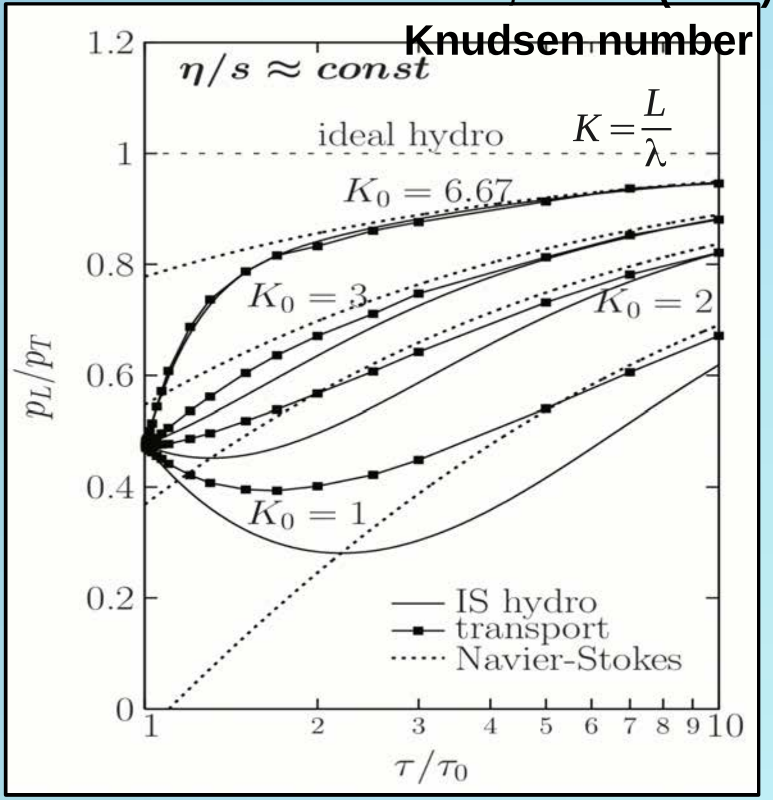
- We have checked the Chapmann-Enskog (CE):
  - CE good already at 1st order ≈ *5%*

  - Relaxation Time Approx. severely understimates η

    S. Plumari et al.,PRC86 (2012) 054902.

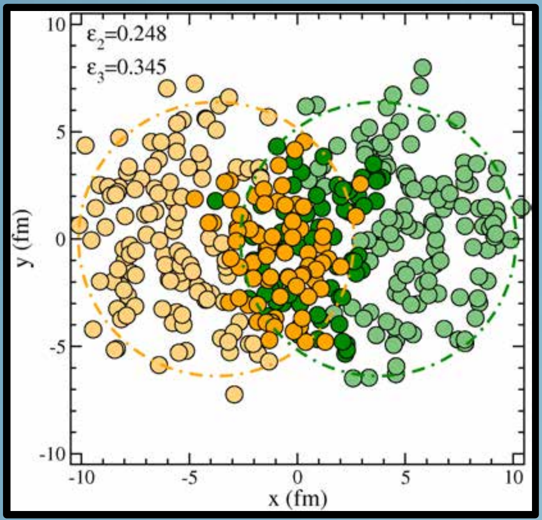


P. Huovinen and D. Molnar, PRC79(2009)



In the limit of small η/s (<0.16) and for small pT equivalent viscous hydro

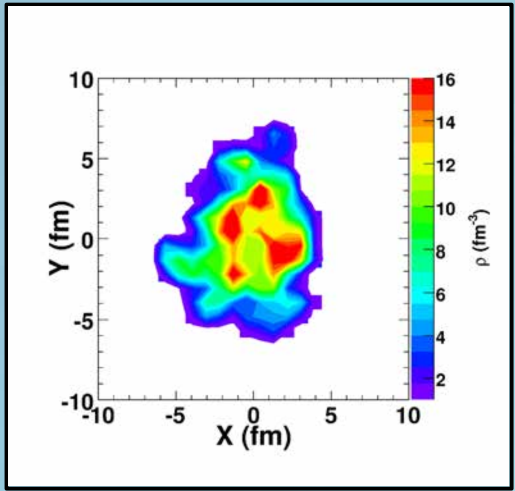
Initial State Fluctuations

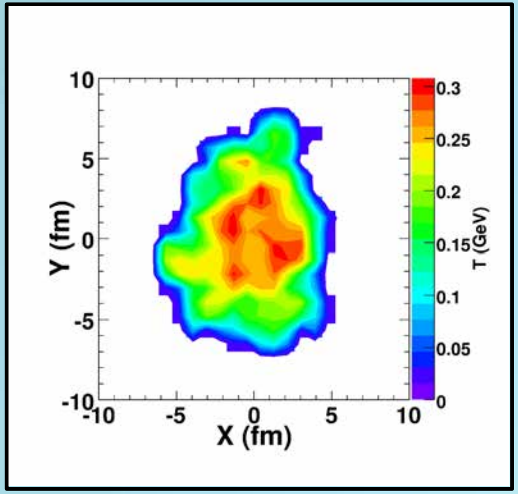


smooth distribution

Monte Carlo Glauber

$$\rho_\perp(x,y) \propto \sum_{i=1}^{N_{part}} \exp\{-[(x-x_i)^2+(y-y_i)^2]/(2\sigma^2)\}$$

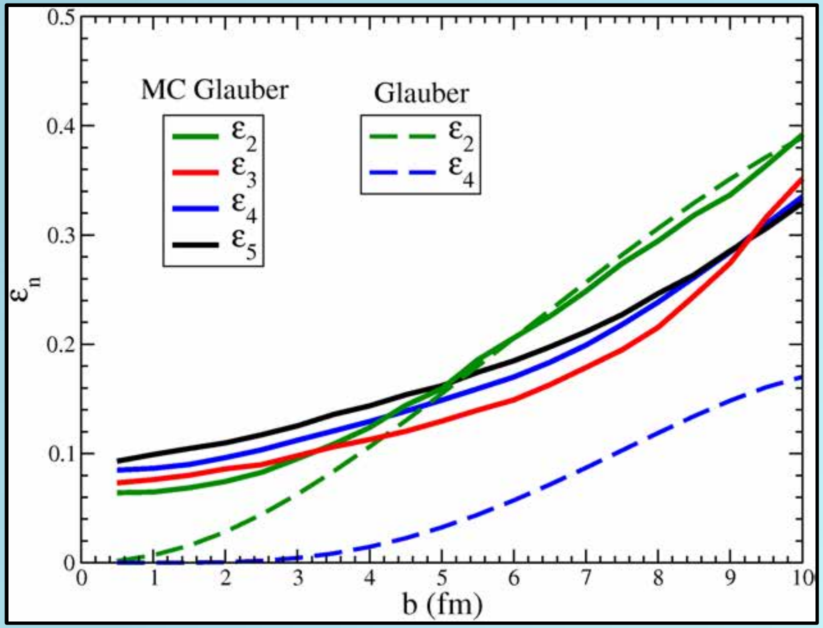




Characterization of the initial profile in terms of Fourier coefficients

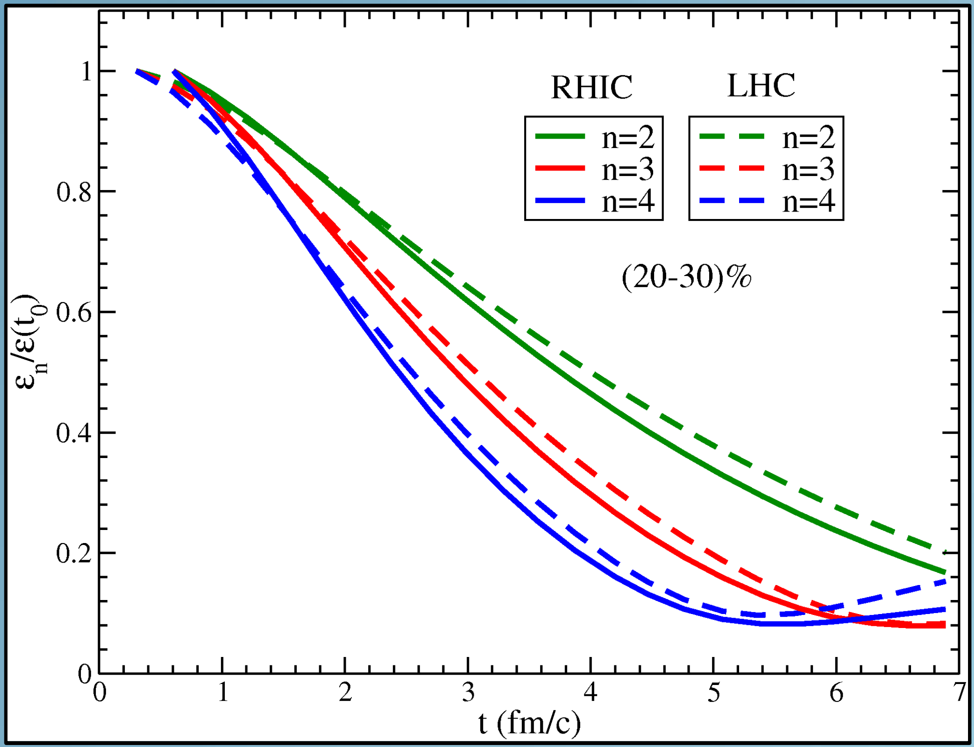
$$\epsilon_n = \frac{\langle r_\perp^n \cos[n(\phi-\Phi_n)]\rangle}{\langle r_\perp^n\rangle} \qquad \Phi_n = \frac{1}{n}\arctan\frac{\langle r_\perp^n \sin(n\phi)\rangle}{\langle r_\perp^n \cos(n\phi)\rangle}$$

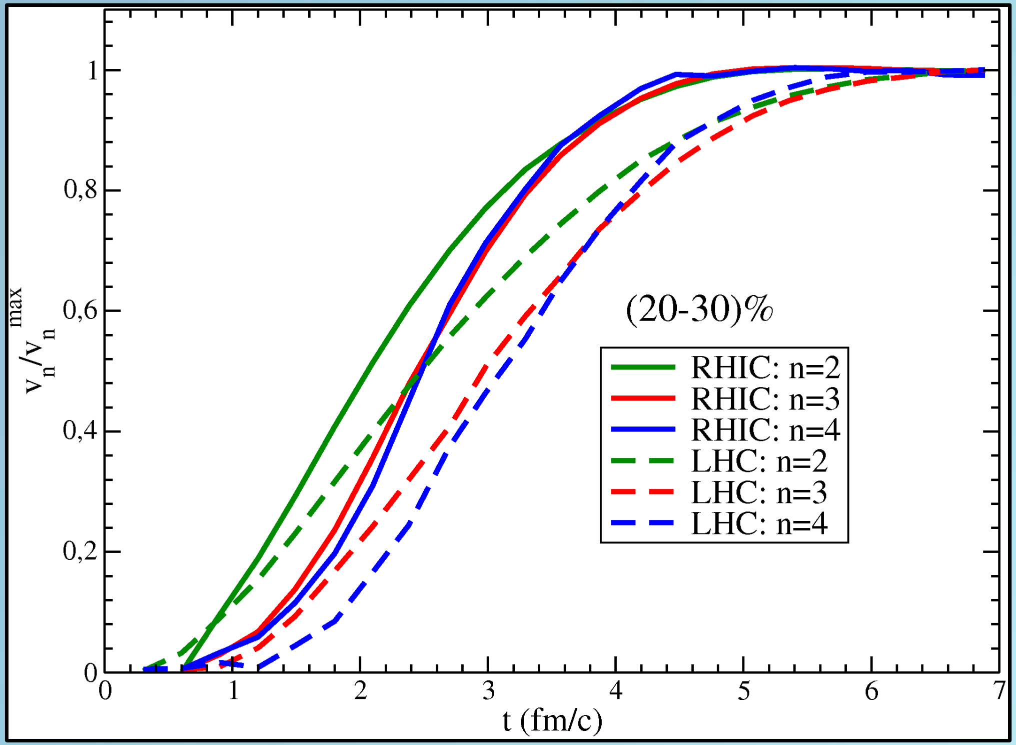
$$r_\perp = \sqrt{x^2+y^2}, \qquad \phi = \arctan(y/x)$$



G-Y. Qin, H. Petersen, S.A. Bass and B. Muller, PRC82,064903 (2010).
H.Holopainen, H. Niemi and K.J. Eskola, PRC83, 034901 (2011).

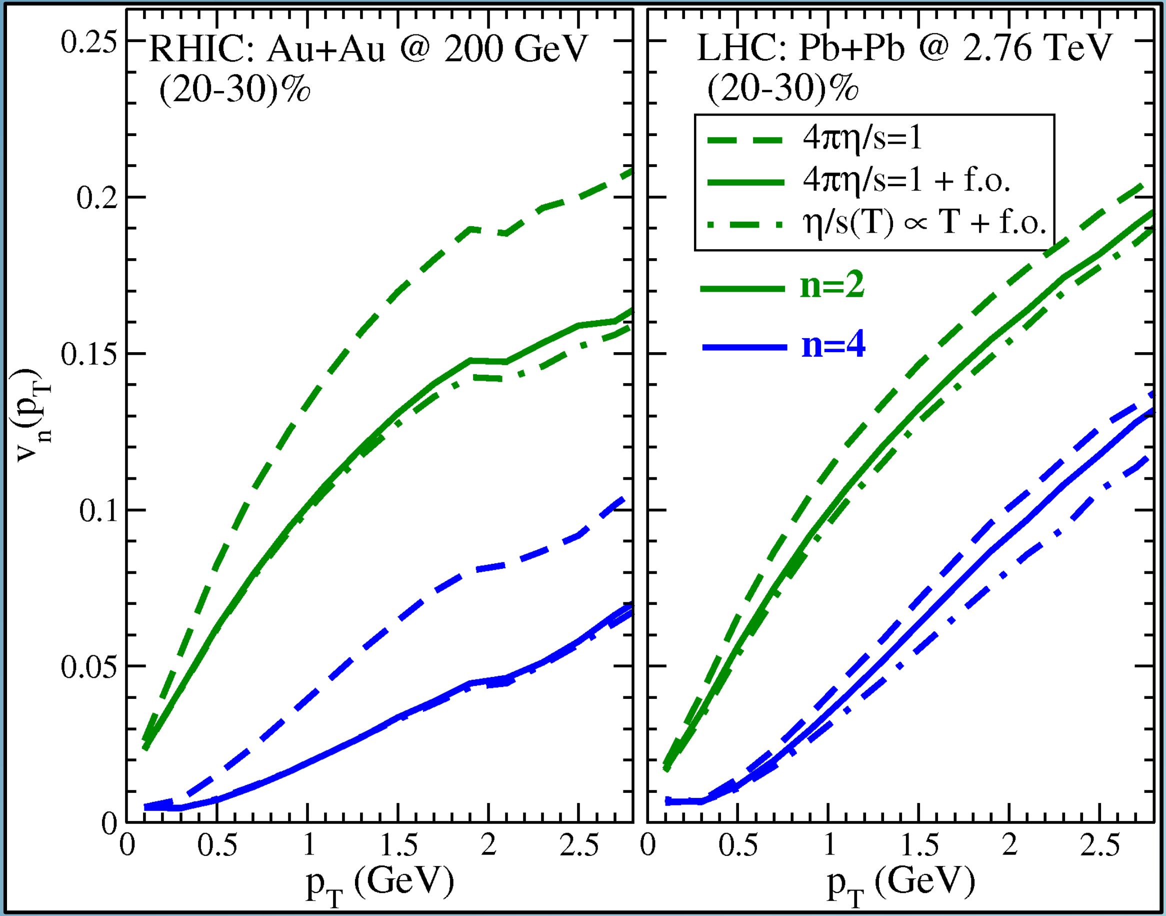
Initial State Fluctuations: time evolution of $\langle v_n \rangle$ and ε_n

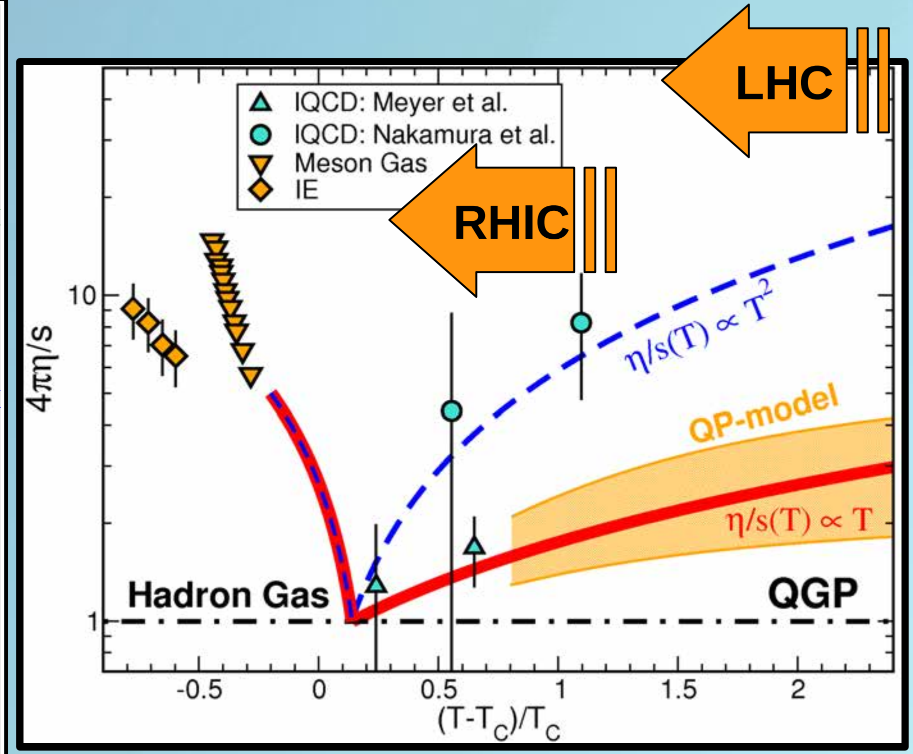




- **The time evolution for ε_n is faster for large n. At very early times $\varepsilon_n(t)=\varepsilon_n(t_0)-\alpha_n t^{n-2}$.**
- **$\langle v_n \rangle$ shows an opposite behaviour: $\langle v_n \rangle$ develops later for large n. At very early times $\langle v_n \rangle \propto t^{n+1}$.**
- **Different v_n can probes different value of $\eta/s(T)$ during the expansion of the fireball.**

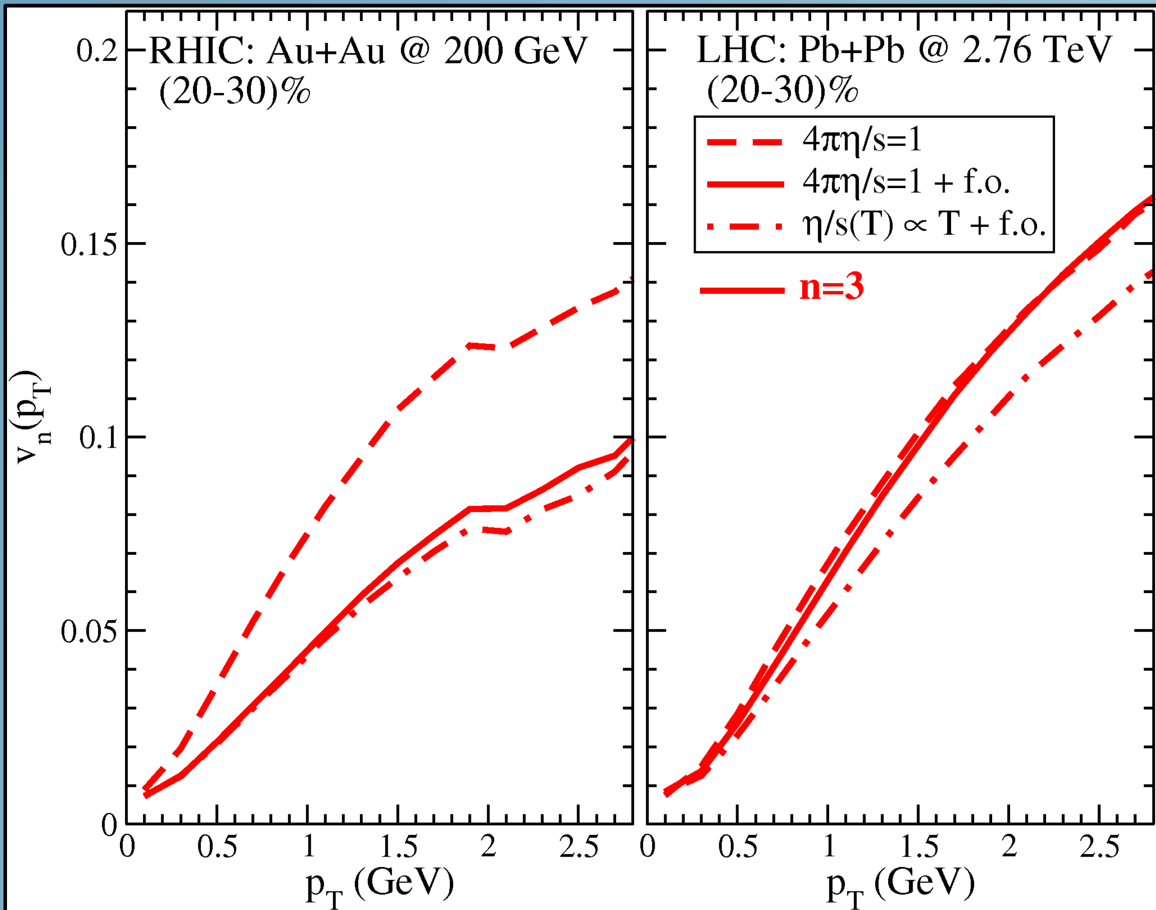
Initial State Fluctuations: $v_n(p_T)$ and η/s

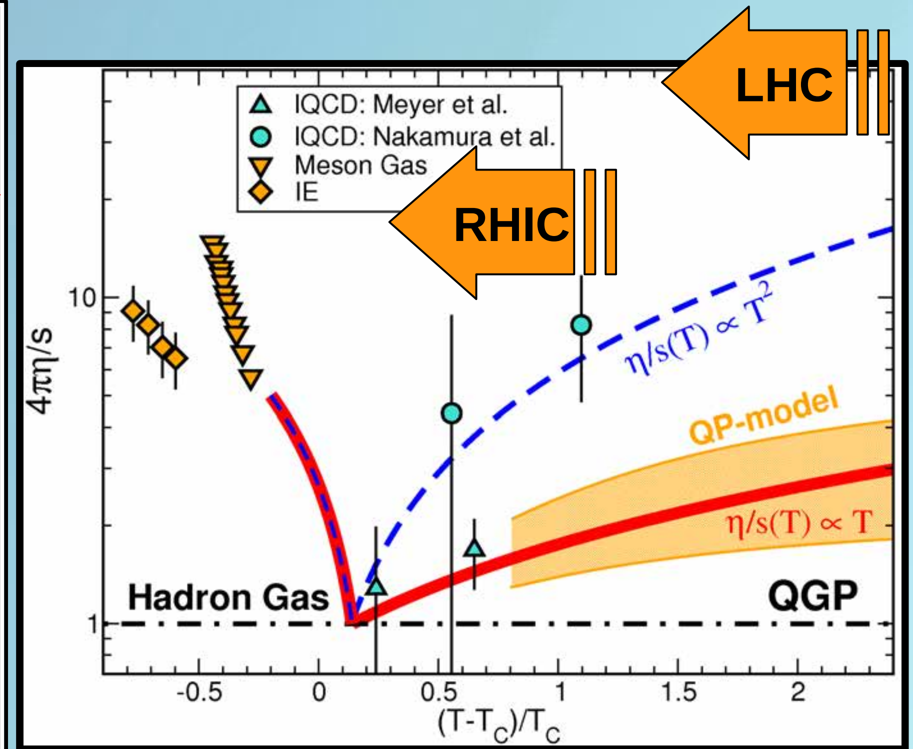




- $v_n(p_T)$ at RHIC is more sensitive to the value of the η/s at low temperature. $v_4(p_T)$ and $v_3(p_T)$ are more sensitive to the value of η/s than the $v_2(p_T)$.
- At LHC energies $v_n(p_T)$ is more sensitive to the value of η/s in the QGP phase (compare solid and dot-dashed lines).

Initial State Fluctuations: $v_n(p_T)$ and η/s





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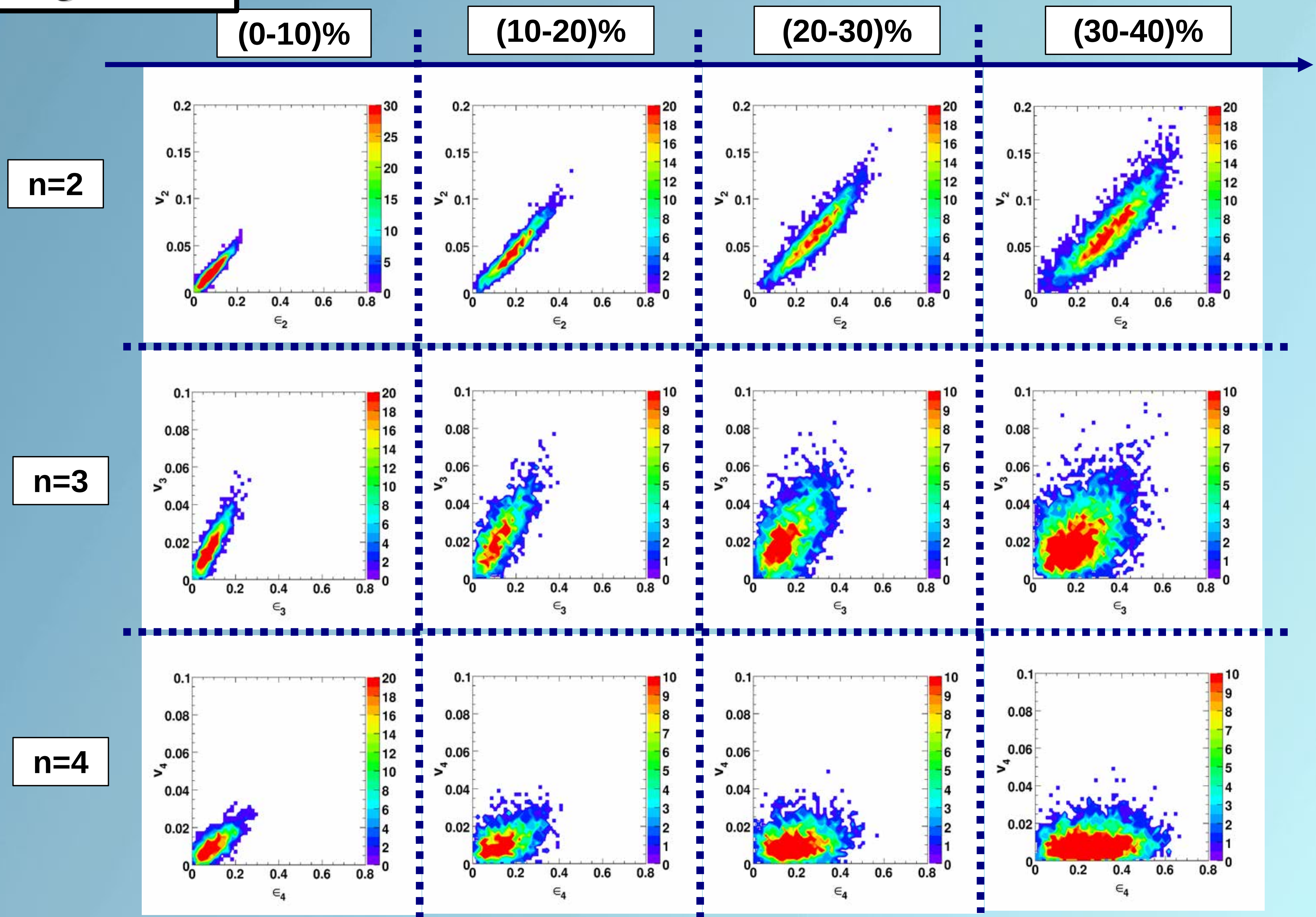
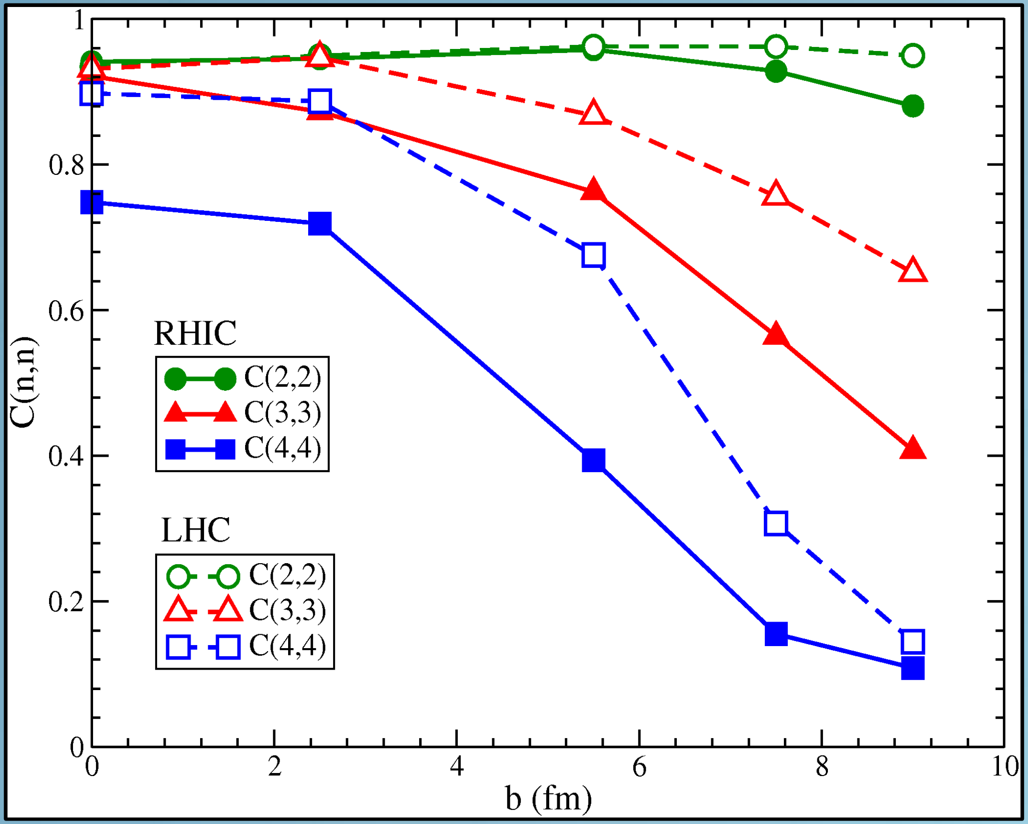
- At LHC energies $v_n(p_T)$ is more sensitive to the value of **η**/s in the QGP phase (compare solid and dot-dashed lines).

RHIC:
Au+Au @ 200 GeV

Initial State Fluctuations: v_n vs ε_n



Initial State Fluctuations: v_n vs ε_n



$$C(n,m)=\left\langle \frac{(v_n-\langle v_n\rangle)(\epsilon_m-\langle \epsilon_m\rangle)}{\sigma_{v_n}\sigma_{\epsilon_m}} \right\rangle$$

B.H. Alver, C. Gombeaud, M. Luzum and J.-Y. Ollitrault, Phys.Rev. C82 (2010) 034913.

H. Petersen, G.-Y. Qin, S.A. Bass and B. Muller, Phys.Rev. C82 (2010) 041901.

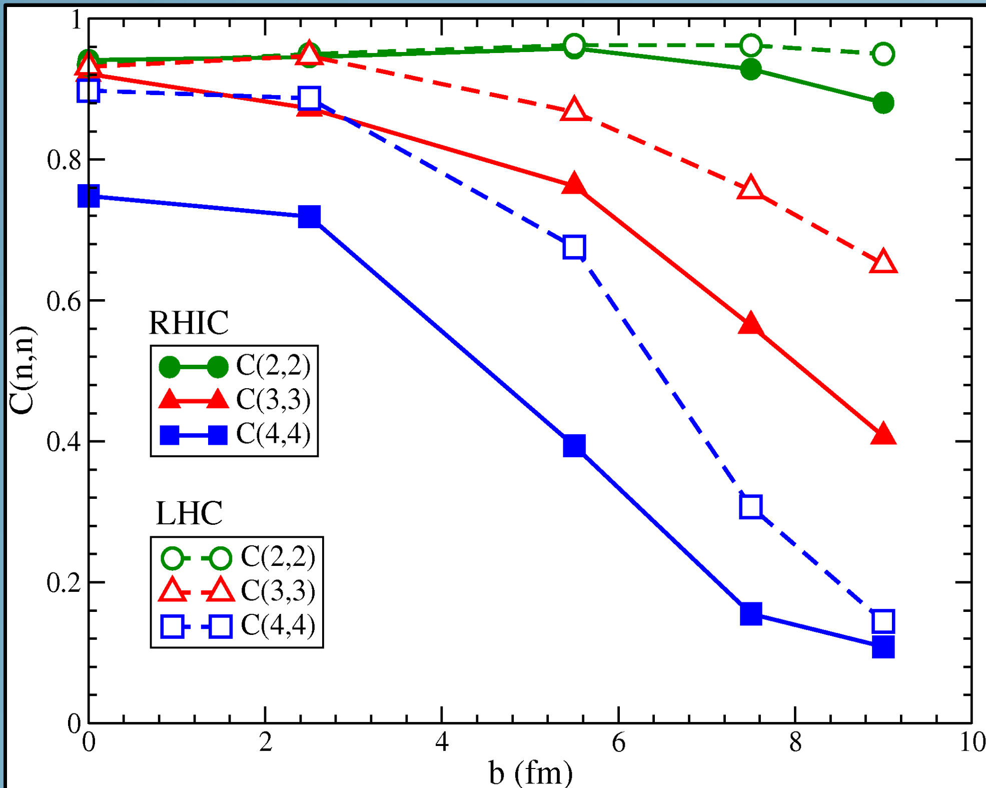
Z. Qiu and U. W. Heinz, Phys.Rev. C84 (2011) 024911.

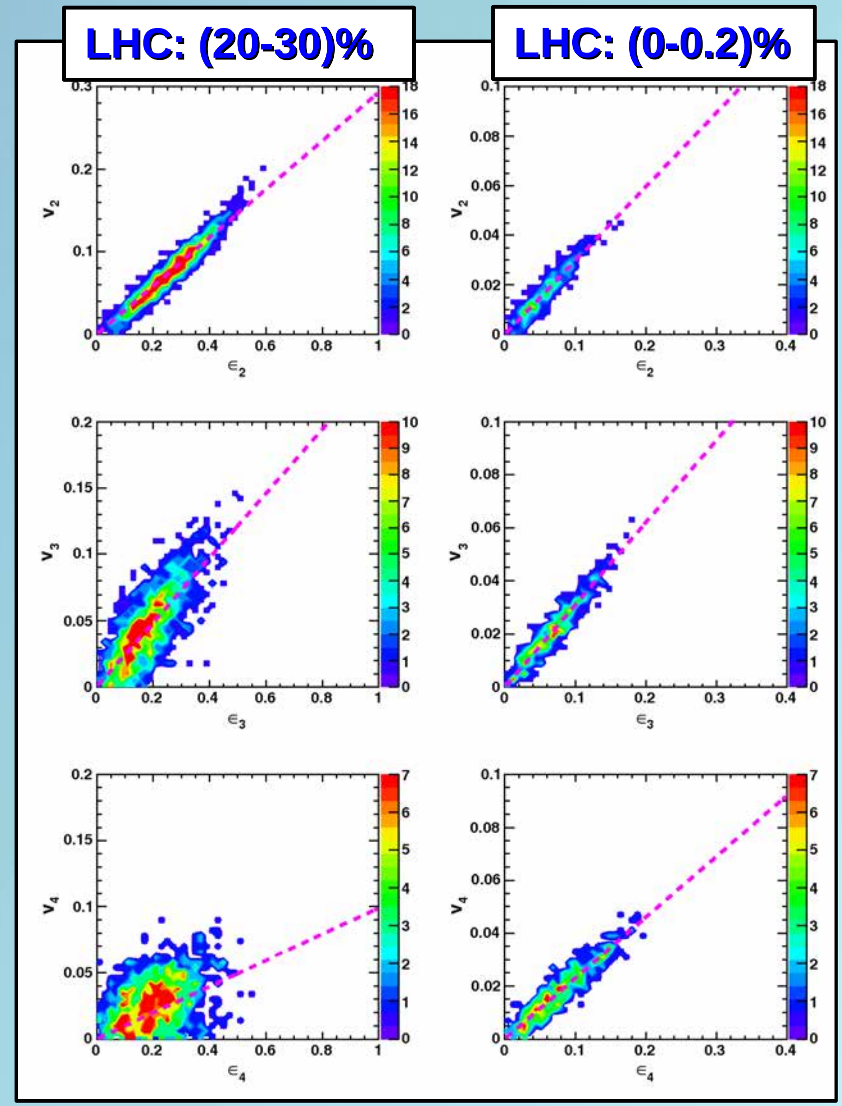
H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen, Phys.Rev. C87 (2013) 5, 054901.

- **At LHC v_n are more correlated correlated to ε_n than at RHIC.**
- **v_2 and v_3 linearly correlated to the corresponding eccentricities ε_2 and ε_3 rispectively.**
- **C(4,4) < C(2,2) for all centralities. v_4 and ε_4 weak correlated similar to hydro calculations:**

  F.G. Gardim, F. Grassi, M. Luzum and J.Y. Ollitrault NPA904 (2013) 503.
  H. Niemi, G.S. Denicol, H. Holopainen and P. Huovinen PRC87(2013) 054901.
- **For central collisions v_n are strongly correlated to ε_n: $v_n \propto \varepsilon_n$ for n=2,3,4.**

Initial State Fluctuations: v_n vs ε_n





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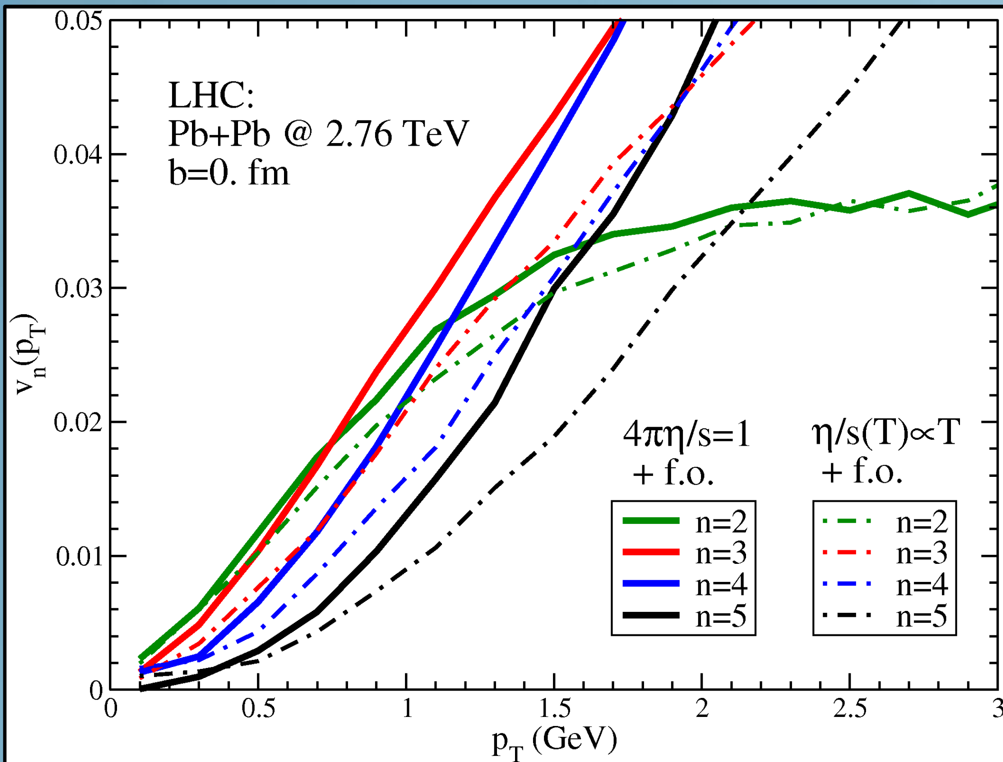
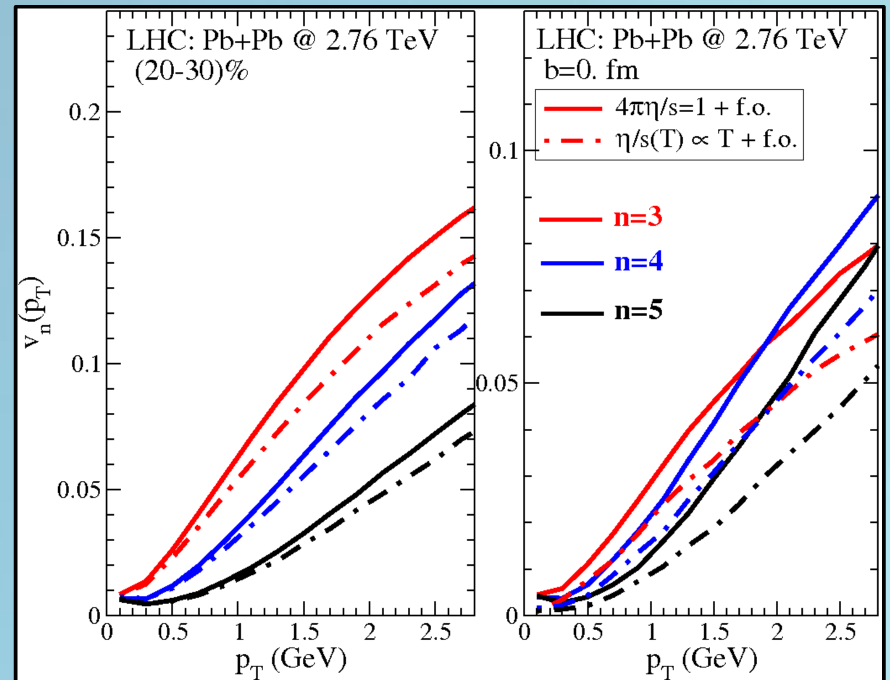
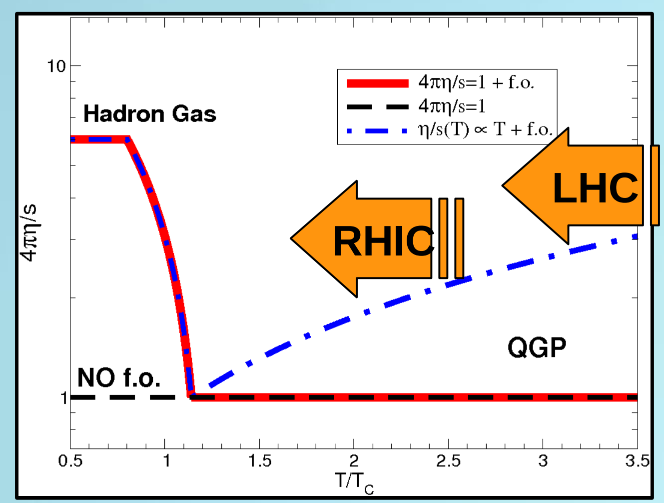
Initial State Fluctuations: $v_n(p_T)$ for central collisions



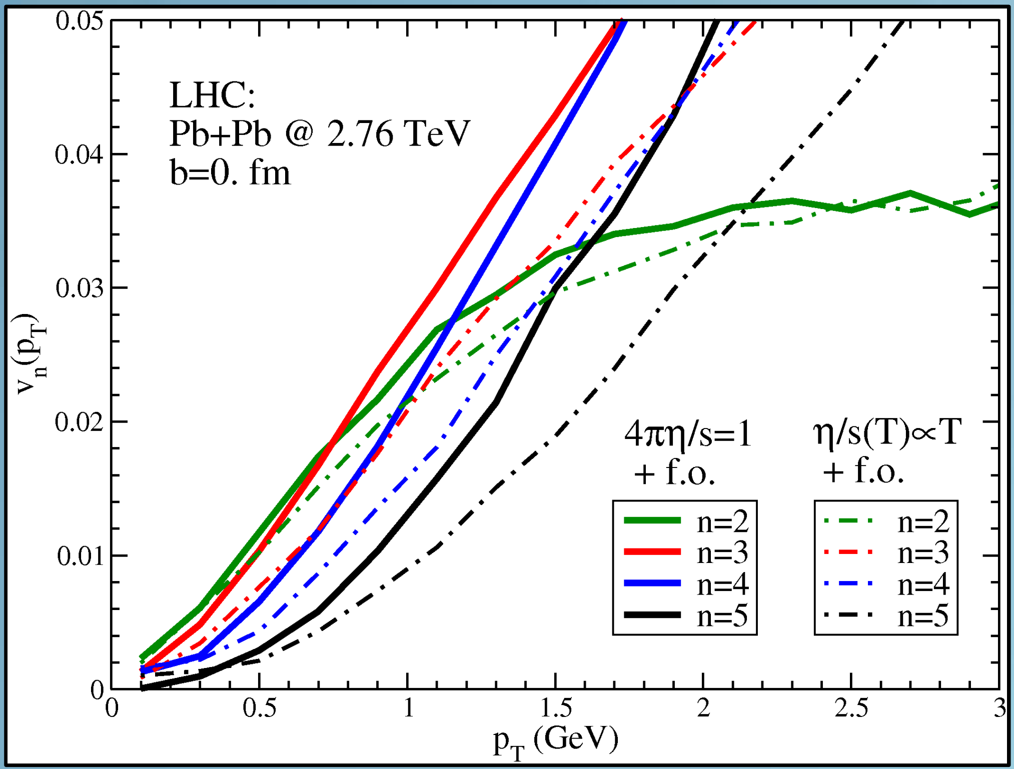


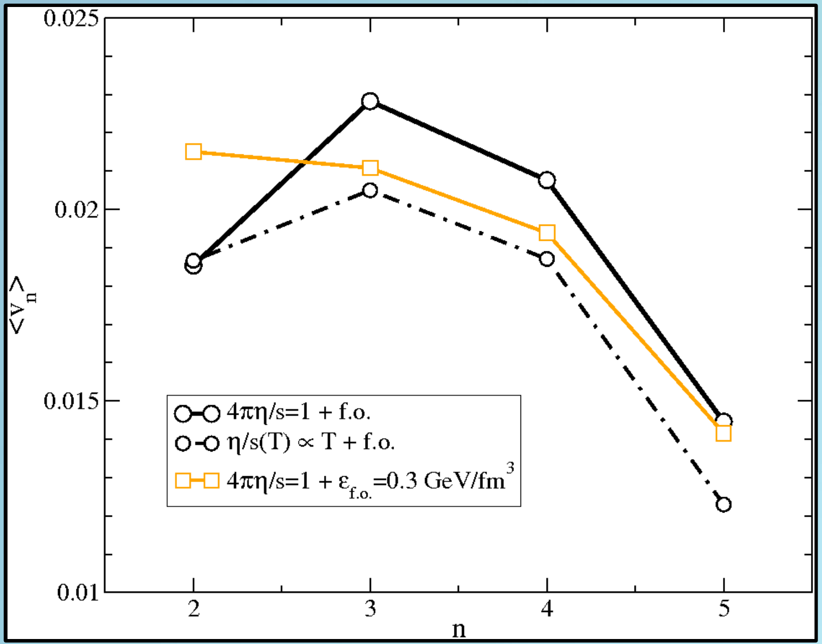
- At low p_T $v_n(p_T) \propto p_T^n$. v_2 for higher p_T saturates while v_n for n>3 increase linearly with p_T.
- For central collisions viscous effect are more relevant. For n>2 the $v_n(p_T)$ are more sensitive

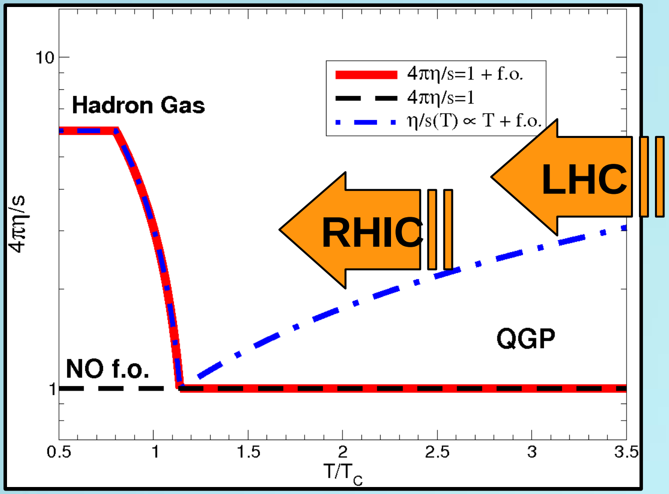
to the η/s ratio in the QGP phase.



Initial State Fluctuations: $v_n(p_T)$ for central collisions







- At low p_T $v_n(p_T) \propto p_T^n$. v_2 for higher p_T saturates while v_n for n>3 increase linearly with p_T.
- For central collisions viscous effect are more relevant. For n>2 the $v_n(p_T)$ are more sensitivity to η/s in the QGP phase.

Conclusions

Transport at fixed η/s:

- Enhancement of η/s(T) in the cross-over region affect differently the expanding QGP from RHIC to LHC.

LHC nearly all the v_n from the QGP phase.

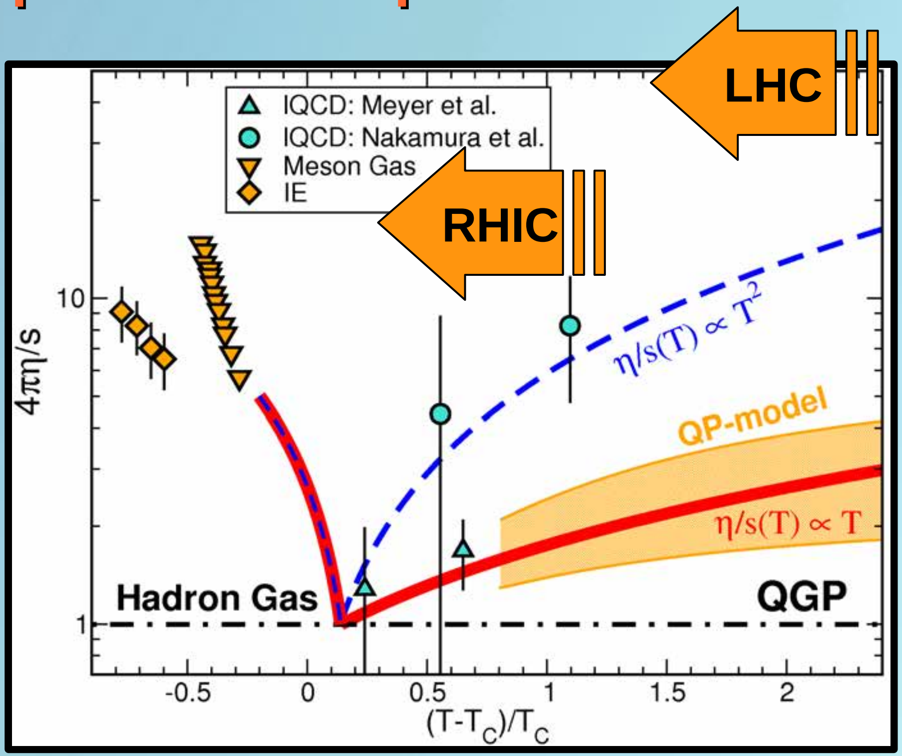
- At LHC there is a stronger correlation between v_n and ε_n than at RHIC for all n.

- Ultra central collisions:
  - $v_n \propto \varepsilon_n$ for n=2,3,4 strong correlation C(n,n)≈1
  - $v_n(p_T)$ much more sensitive to η/s(T)

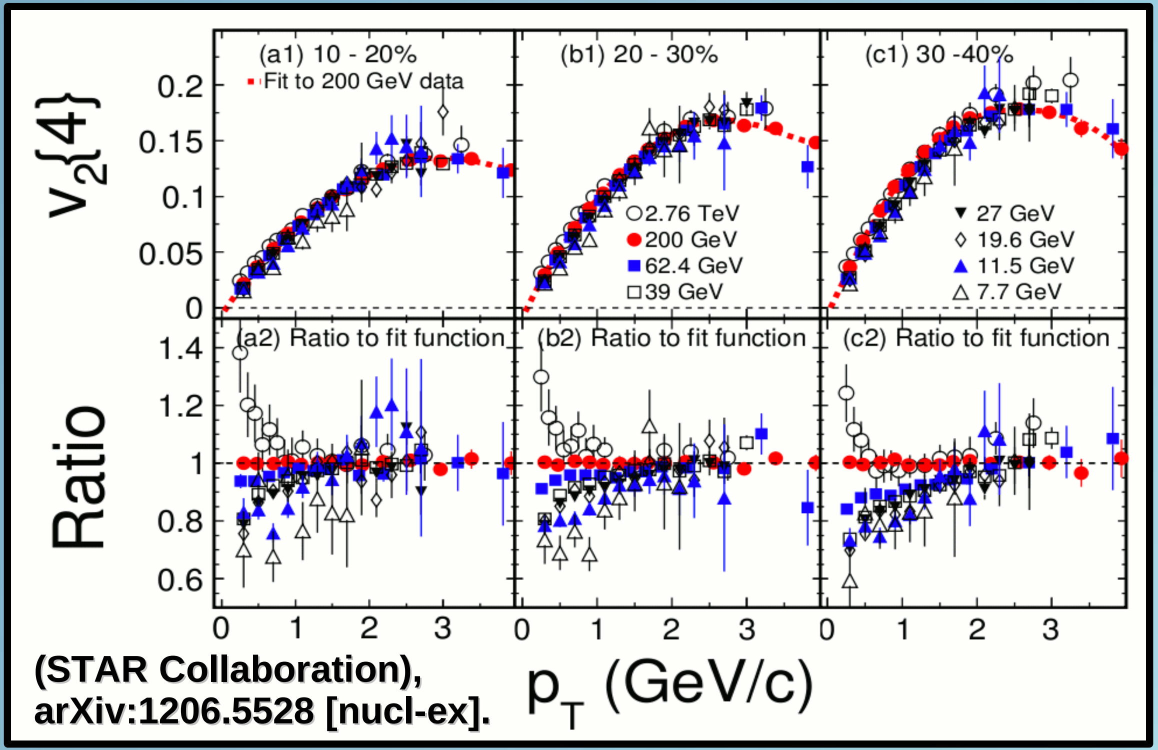
Temperature dependent η/s(T)

Phase transition physic suggest a T dependence of η/s also in the QGP phase

- LQCD some results for quenched approx. large error bars
- Quasi-particle models seem to suggest a η/s~T$^{\alpha}$ α ~ 1 – 1.5.

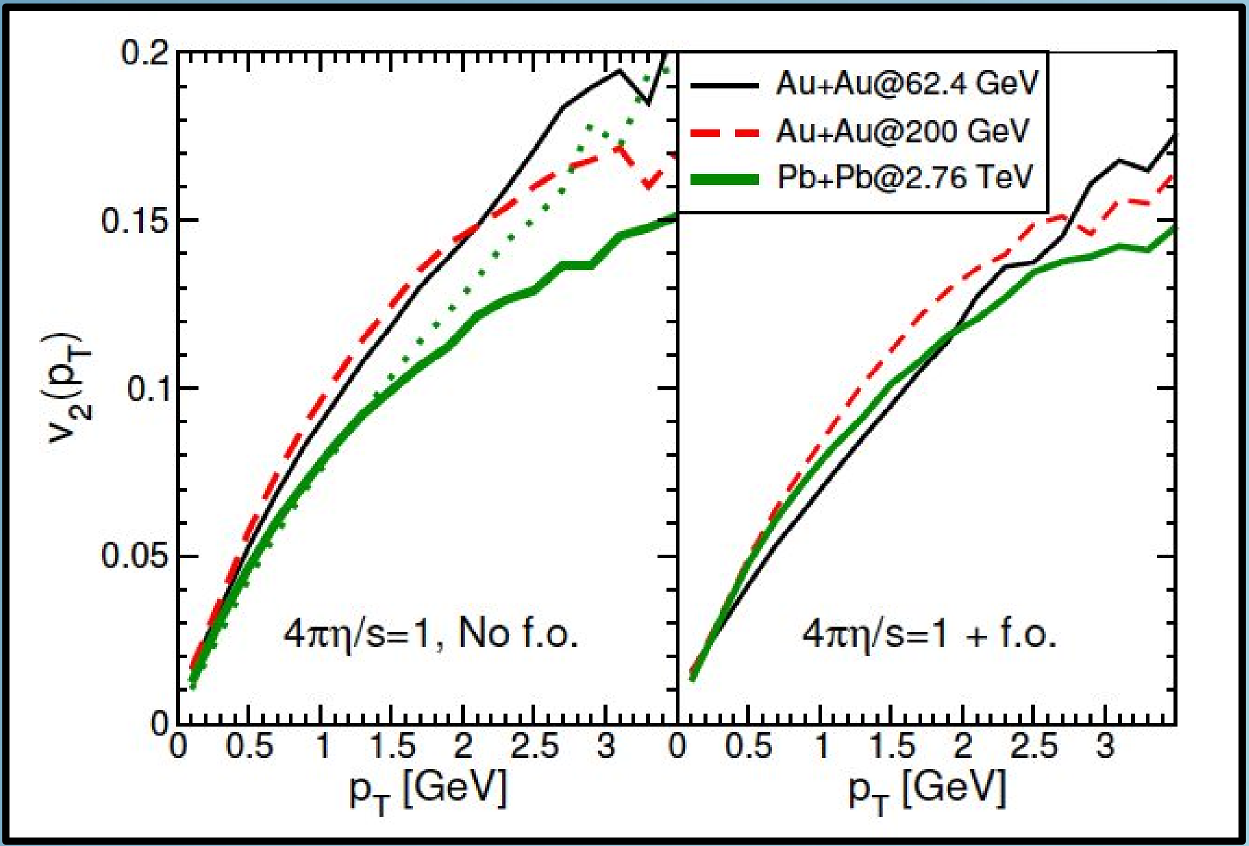


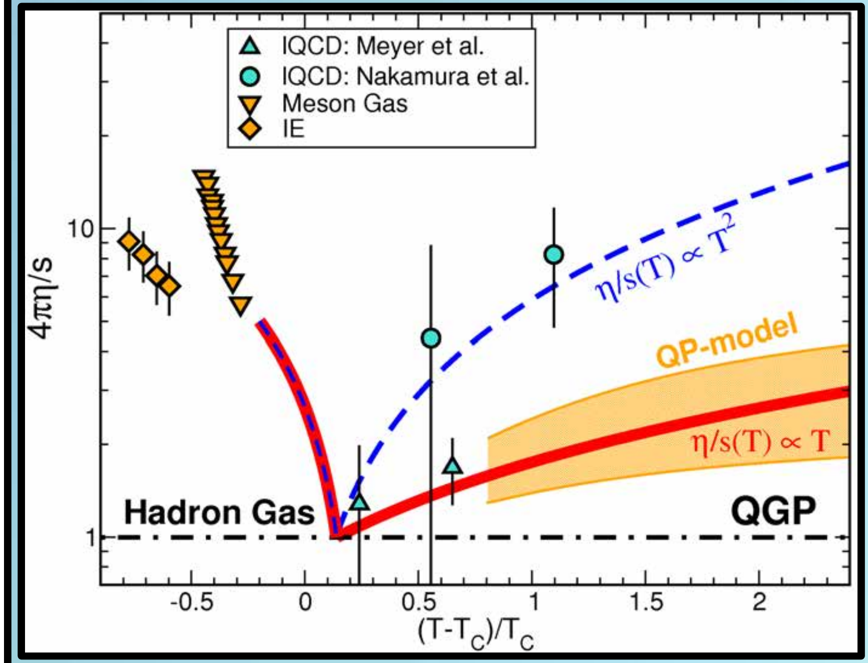
S. Plumari et al., J. Phys.: Conf. Ser. 420 012029 (2013). arXiv:1209.0601.



(STAR Collaboration), arXiv:1206.5528 [nucl-ex].

Temperature dependent η/s(T)

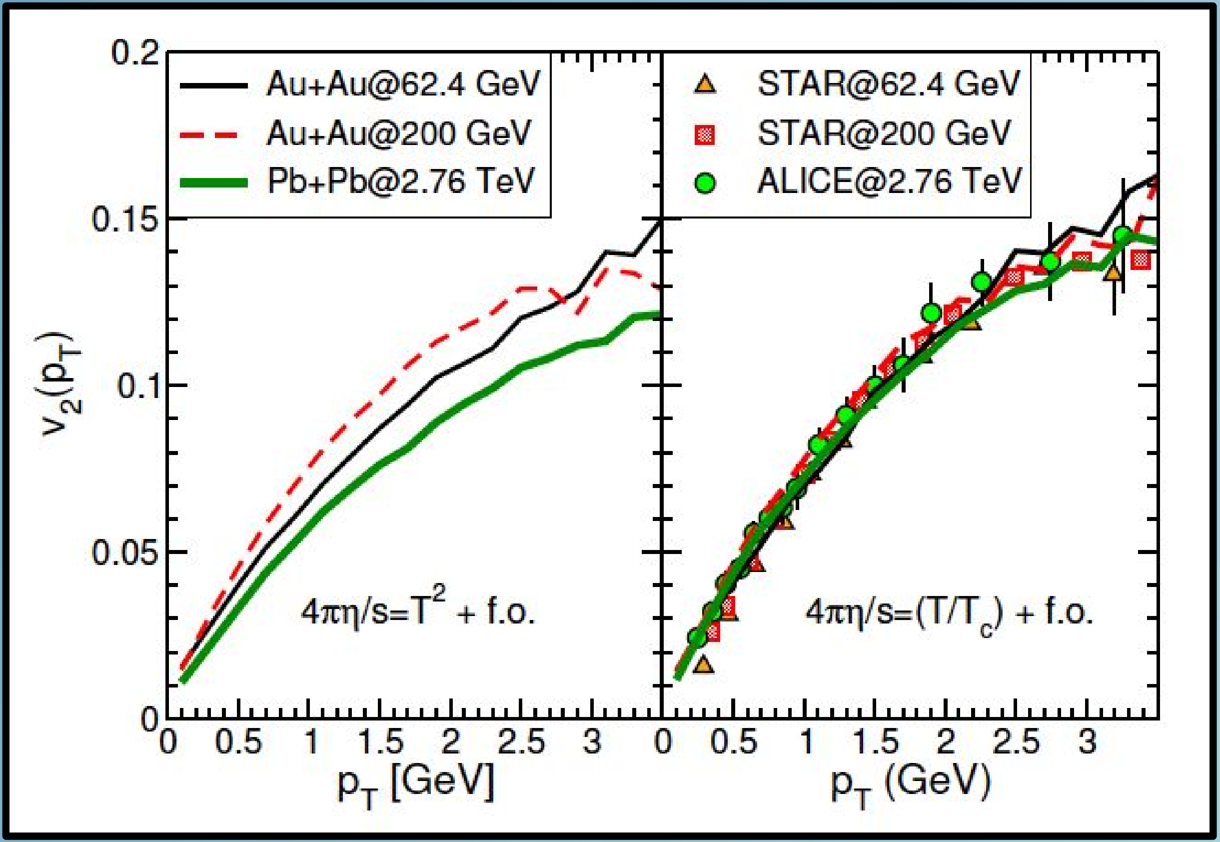


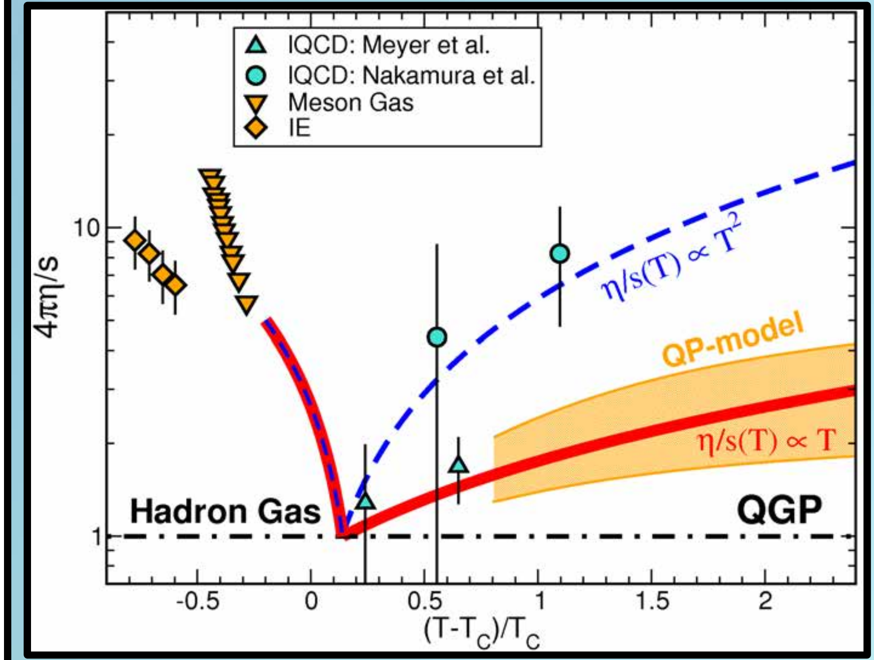


Plumari, Greco, Csernai, arXiv:1304.6566

- **For 4πη/s=1 during all the evolution of the fireball we get a discrepancy for the $v_2(p_T)$, in particular we observe a smaller $v_2(p_T)$ at LHC.**
- **Similar results for $\eta/s \propto T^2$ → a discrepancy about 20%.**
- **Notice only with RHIC → scaling for 4πη/s=1 LHC data play a key role**

Temperature dependent η/s(T)





Plumari, Greco, Csernai, arXiv:1304.6566

- **Invariance of $v_2(p_T)$ in BES suggest that the system goes through a phase transition.**
- **Hope: v_n, n>3 with an event-by-event analysis will put even stronger consstraint**
- **Implementation of local fluctuation under development**

- **Similar results: R. A. Lacey et al., arXiv:1305.3341 [nucl-ex].**

Extraction of the Shear Viscosity: Box calculation

$$\eta^{IS}_{relax}/s = \frac{1}{15}\langle p\rangle \tau_r = \frac{1}{15}\frac{\langle p\rangle}{\sigma_{tot}\langle f(a) v_{rel}\rangle \rho}$$

$$\sigma_{tr} = \int d\Omega \sin^2(\theta_{cm}) \frac{d\sigma}{d\Omega_{cm}} = \sigma_{tot} f(a) \leqslant \frac{2}{3}\sigma_{tot}$$

Employed also for non-isotropic cross section:

G.Ferini, PLB(2009); D. Molnar, JPG35(2008); V.Greco, PPNP(2009);

For the standard pQCD-like cross section

$$\frac{d\sigma}{d\Omega_{cm}} = \frac{9\pi\alpha_S^2}{2}\frac{1}{(q^2+m_D^2)^2}\left(1+\frac{m_D^2}{s}\right)$$

m_D regulates the anisotropy of collision
$m_D \rightarrow \infty$ we recover the isotropic limit

$$f(a) = 4a(1+a)[(2a+1)\ln(1+a^{-1})-2], \quad a = m_D^2/s$$

1st Chapman-Enskog approximation

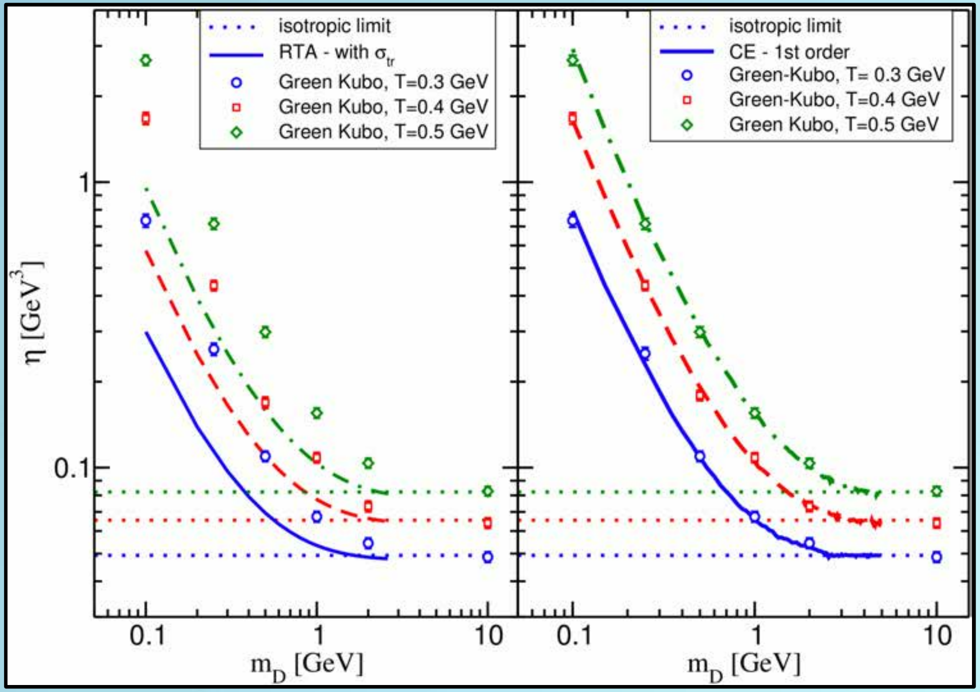
$$[\eta]_{1st}/s = \frac{1}{15}\langle p\rangle \tau_\eta = \frac{1}{15}\frac{\langle p\rangle}{\sigma_{tot}\, g(a)\rho}$$

$$g(a) = \frac{1}{50}\int_0^\infty dy\, y^6 \left[\left(y^2+\frac{1}{3}\right)K_3(2y) - y K_2(2y)\right] f(a), \quad a = \frac{m_D}{2T}$$

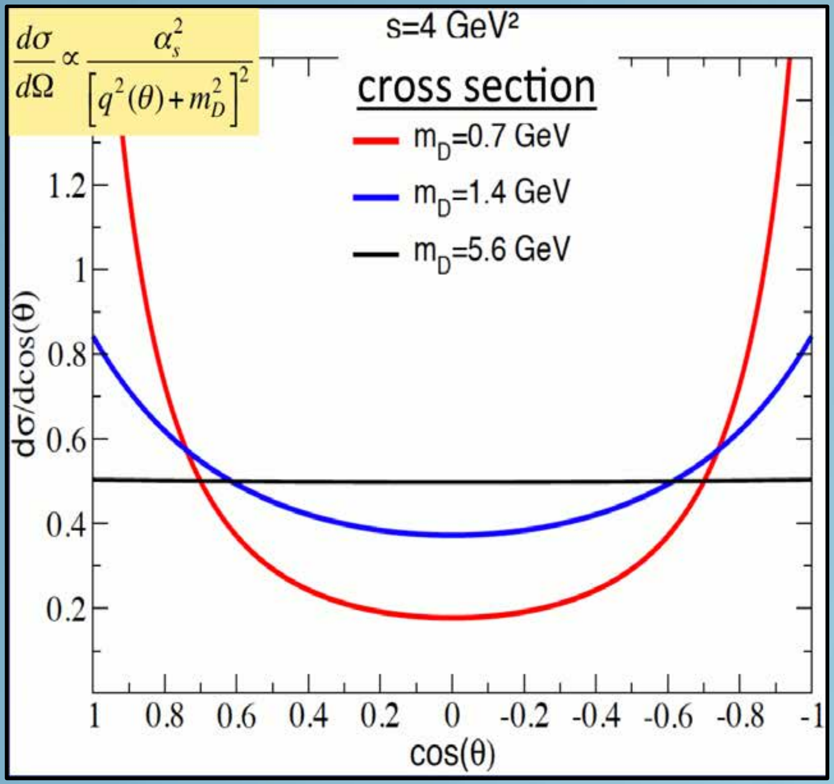
- **CE and RTA can differ by a factor of 2**
- **Green-Kubo agree with CE (< 5%)**

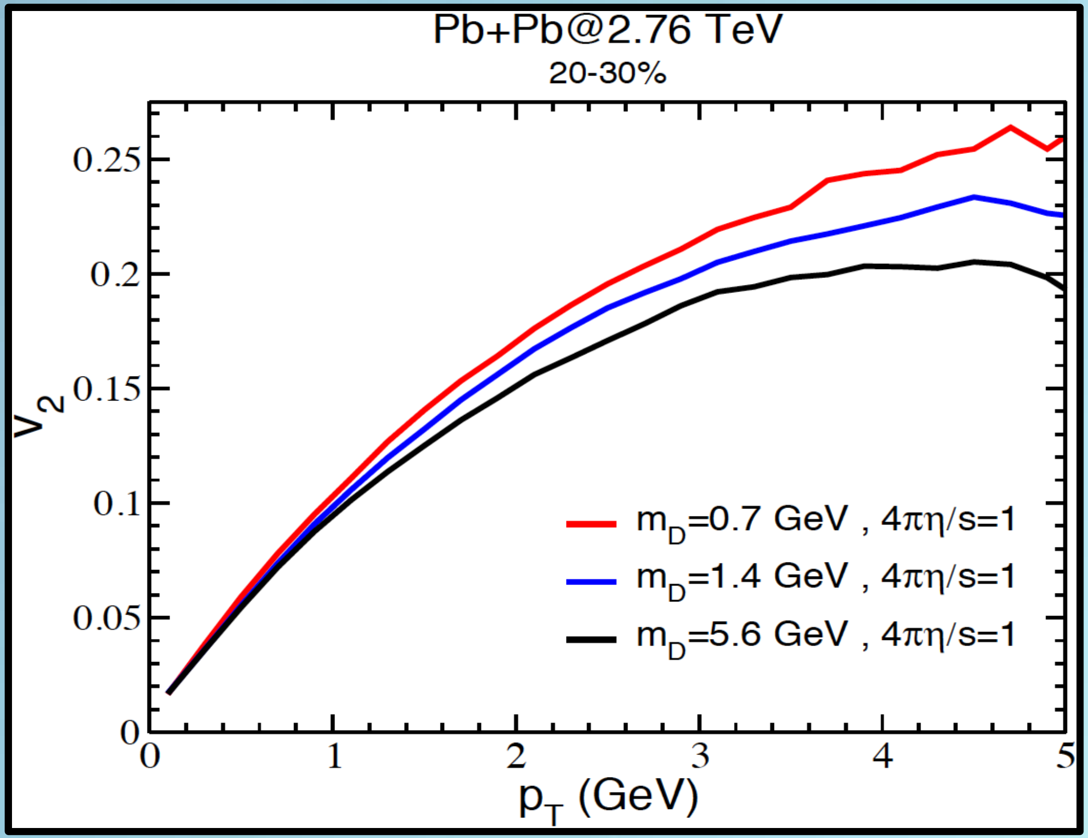
A. Wiranata, M. Prakash, PRC85 (2012) 054908.
O. N. Moroz, arXiv:1112.0277 [hep-ph].

S. Plumari et al.,PRC86 (2012) 054902.



η/s or detail of the corss section



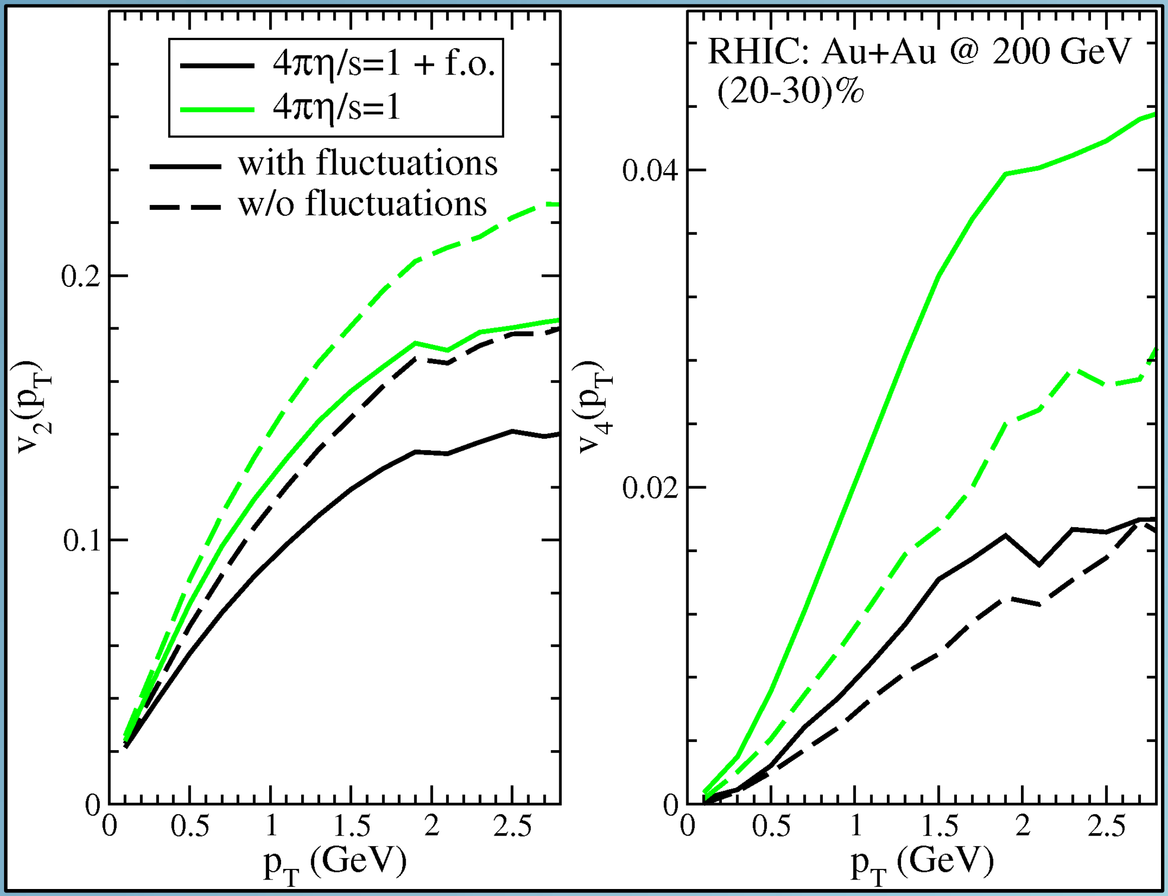


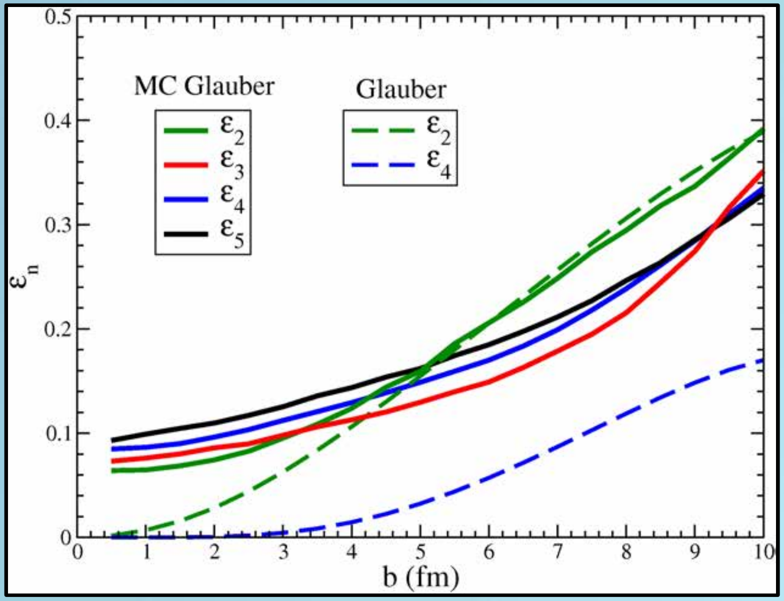
$$\frac{\eta}{s} = \frac{1}{15} \langle p \rangle \tau_\eta$$

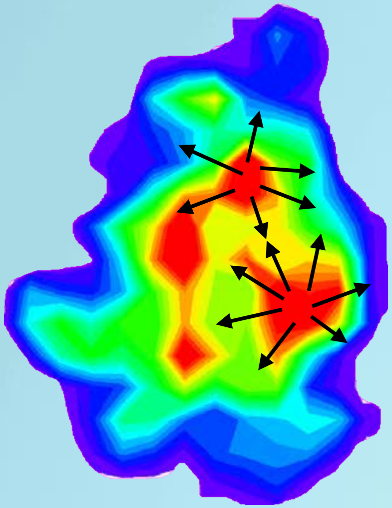
$$\tau_\eta = \frac{1}{\sigma_{tot}\, g(a)\, \rho}$$

- **η/s is the physical parameter determining the v_2 at least up to p_T 1.5 -2 GeV.**
- **microscopic details becomes important at higher p_T.**

Initial State Fluctuations: $v_n(p_T)$ and η/s





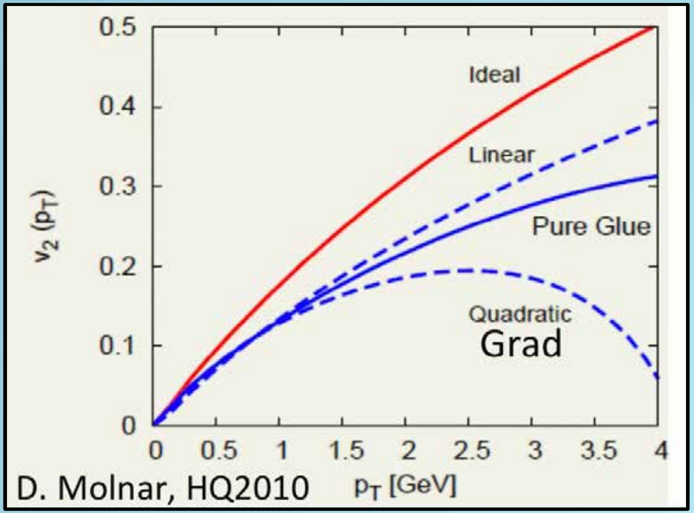


- The initial state fluctuations reduce the $v_2(p_T)$.
- $v_4(p_T)$ increase by the initial state fluctuations and it becomes more sensitive to the viscosity of the QGP. Larger ε_4 gives larger v_4.

From Transport to Hydro: extraction of viscous corrections to $f(x,p)$ and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

$$f(x,p) = f^{(0)}(x,p) + \delta f(x,p)$$

$$T^{\mu\nu} = T^{(0)\mu\nu} + \delta T^{\mu\nu} \leftarrow f^{(0)} + \delta f$$



A common choice for δf – the Grad ansatz

$$\delta f \propto \Gamma_s f^{(0)} p^\alpha p^\beta \langle \nabla_\alpha u_\beta \rangle \propto p_T^2$$

BUT it doesn't care about the microscopic dynamics

In general in the limit σ→∞, f(σ) can be expanded in power of 1/σ.

$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right) \Rightarrow v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$

PURPOSE: evaluate the ideal hydrodynamics limit $f^{(0)}$, $v_n^{(0)}$ and the viscous corrections δf and δv_n solving the Relativistic Boltzmann eq for large values of the cross section σ

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

Coodinate space (x,y)

- We start with an initial azimuthally symmetric profile (optical Glauber model).
- Then we deform the initial distribution (α<<1)

$$z = x + iy \rightarrow z + \delta z \equiv z - \alpha \bar{z}^{n-1}$$

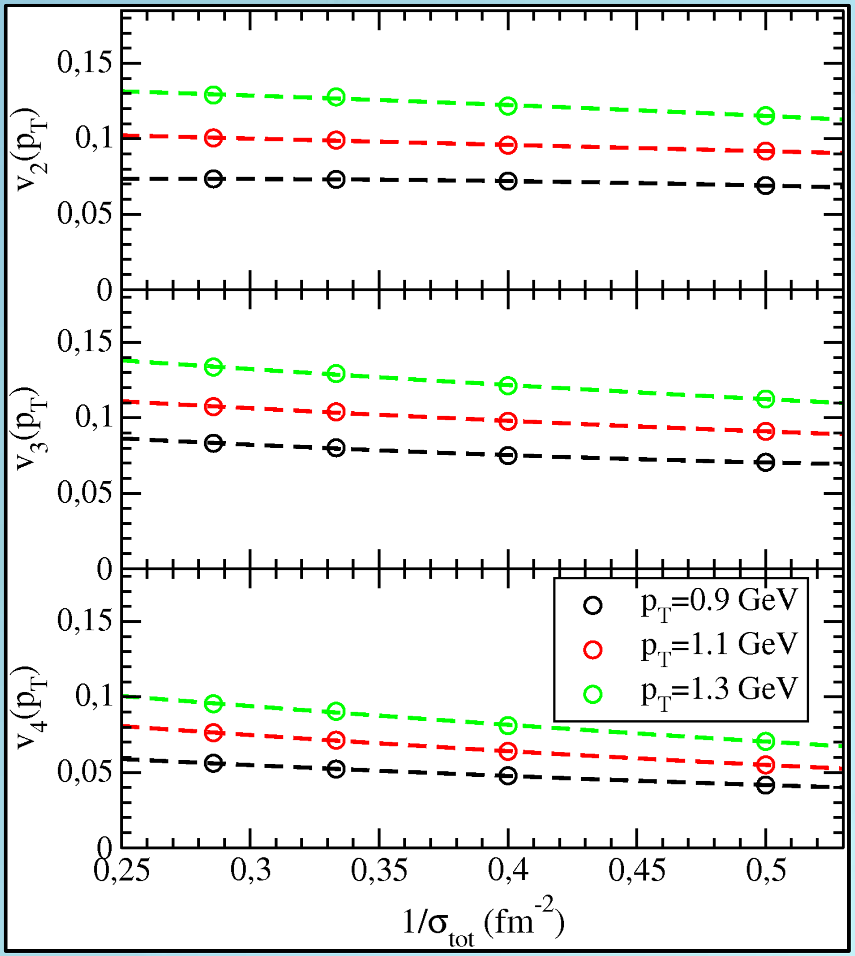
2π/n symmetry

This Creates only

$$\epsilon_n \equiv \frac{-\sum_j (z_j + \delta z_j)^n}{\sum_j |z_j + \delta z_j|^n} \simeq n\alpha \frac{\langle r^{2(n-1)} \rangle}{\langle r^n \rangle}$$

$$f(\sigma) \underset{\sigma\to\infty}{\approx} f^{(0)} + \frac{1}{\sigma}\delta f + O\left(\frac{1}{\sigma^2}\right)$$

$$v_n(p_T) \underset{\sigma\to\infty}{\approx} v_n^{(0)}(p_T) + \frac{1}{\sigma}\delta v_n + O\left(\frac{1}{\sigma^2}\right)$$



Momentum space

- Thermal distribution:

$$dN/d^3p \propto \exp(-p/T)$$

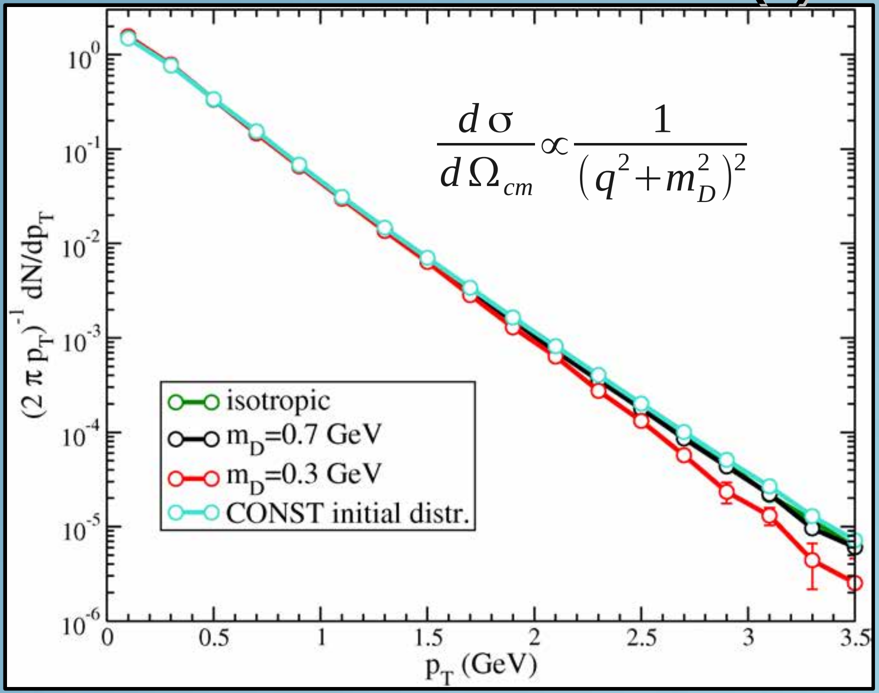
- Constant distribution:

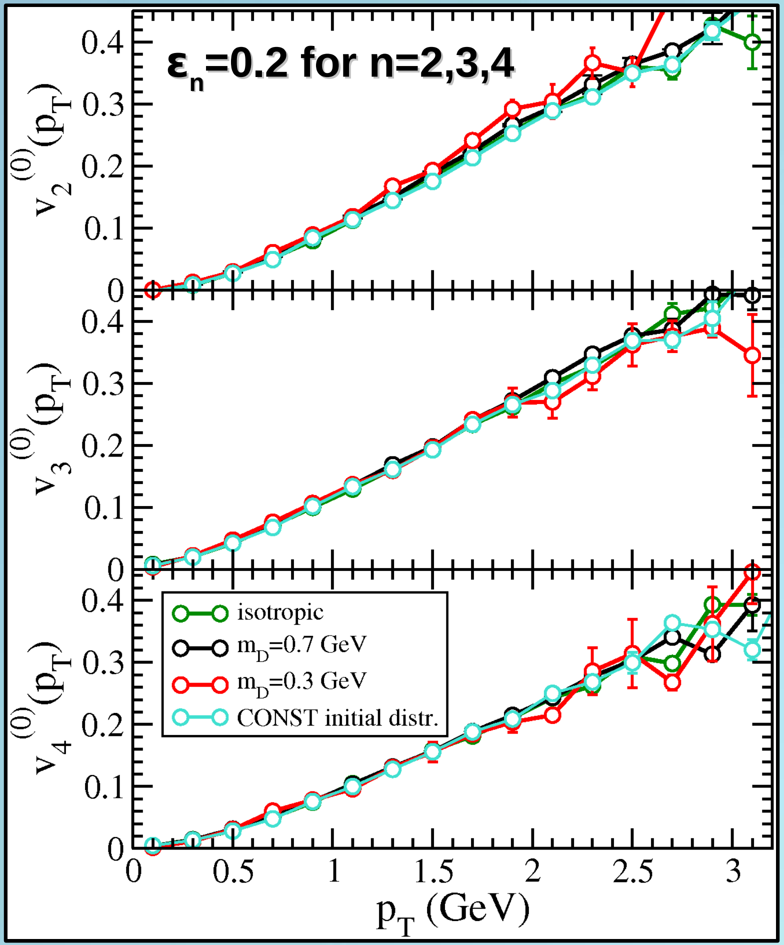
$$dN/d^3p \propto \theta(p_0 - p)$$

We assume initially the same local $T^{\mu\nu}(x)$

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)

For the same initial local $T^{\mu\nu}(x)$:



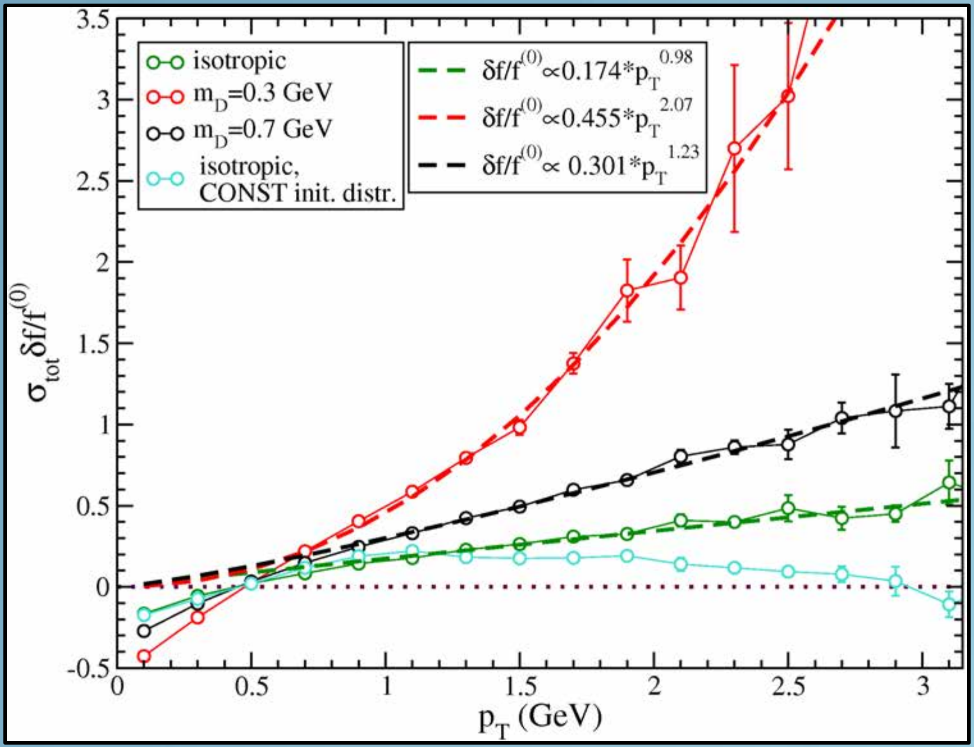


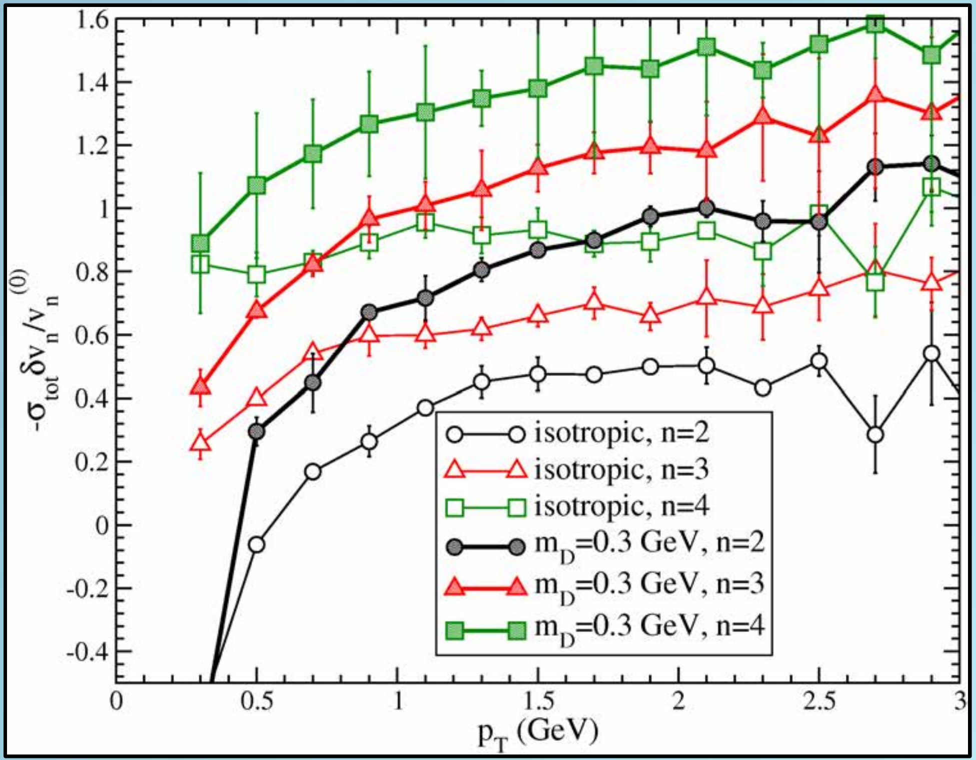
For σ → ∞ we find the ideal Hydro limit:

- $f^{(0)}$ is an exponential decreasing function.
- $f^{(0)}$ doesn't depends on microscopical details (i.e. mD).

- Universal behavior of $v_n^{(0)}(p_T)$
- $v_n^{(0)}(p_T)/\varepsilon_n$ is approximatively the same for all n and p_T.

From Transport to Hydro: extraction of viscous corrections to f(x,p) and $v_n(p_T)$. (work in collaboration with J.Y. Ollitrault)





In δf and δv_n it is encoded the information about the microscopical details

- **$\delta f(p_T)/f^{(0)} \propto p_T^{\alpha}$ with α = 1. - 2. and $\alpha(m_D)$.**

For isotropic σ similar to R.S. Bhalerao et al. PRC 89, 054903 (2014)

- **Larger is n larger is the viscous correction to $v_n(p_T)$**
- **Scaling: for p_T > 1.5 GeV → $-\delta v_n(p_T)/v_n^{(0)} \propto n$**