Functional Matching and Renormalization Group Equations at Two-Loop Order

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Outline

- **Introduction**
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- **Functional approach to the two-loop computations**
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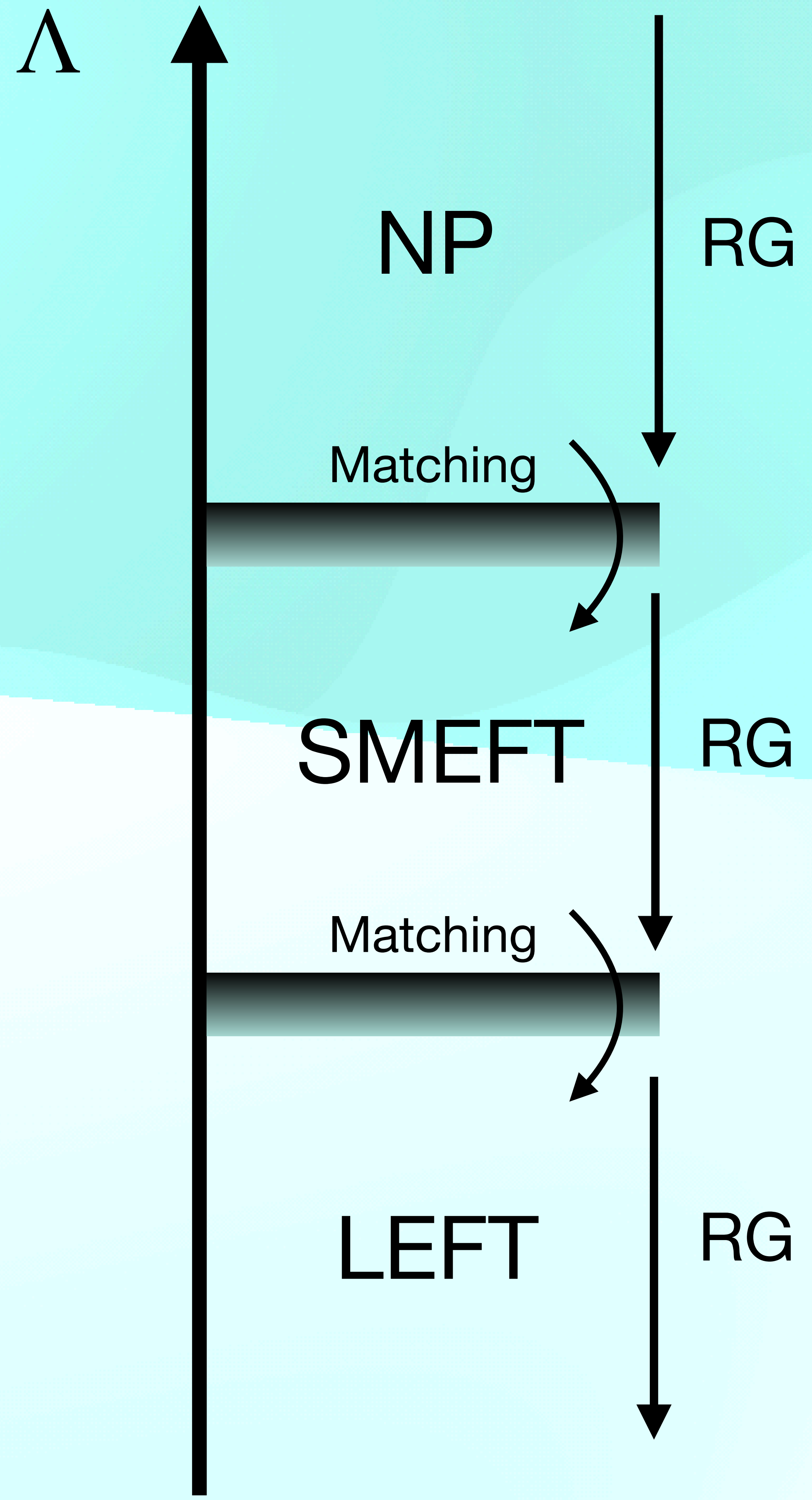
- **Toy model example: scalar theory**
  - One-loop CTs, Evaluation of different topologies, Running in the EFT
- **Matching conditions**
- **Outlook**

Introduction

Introduction: Effective Field Theories

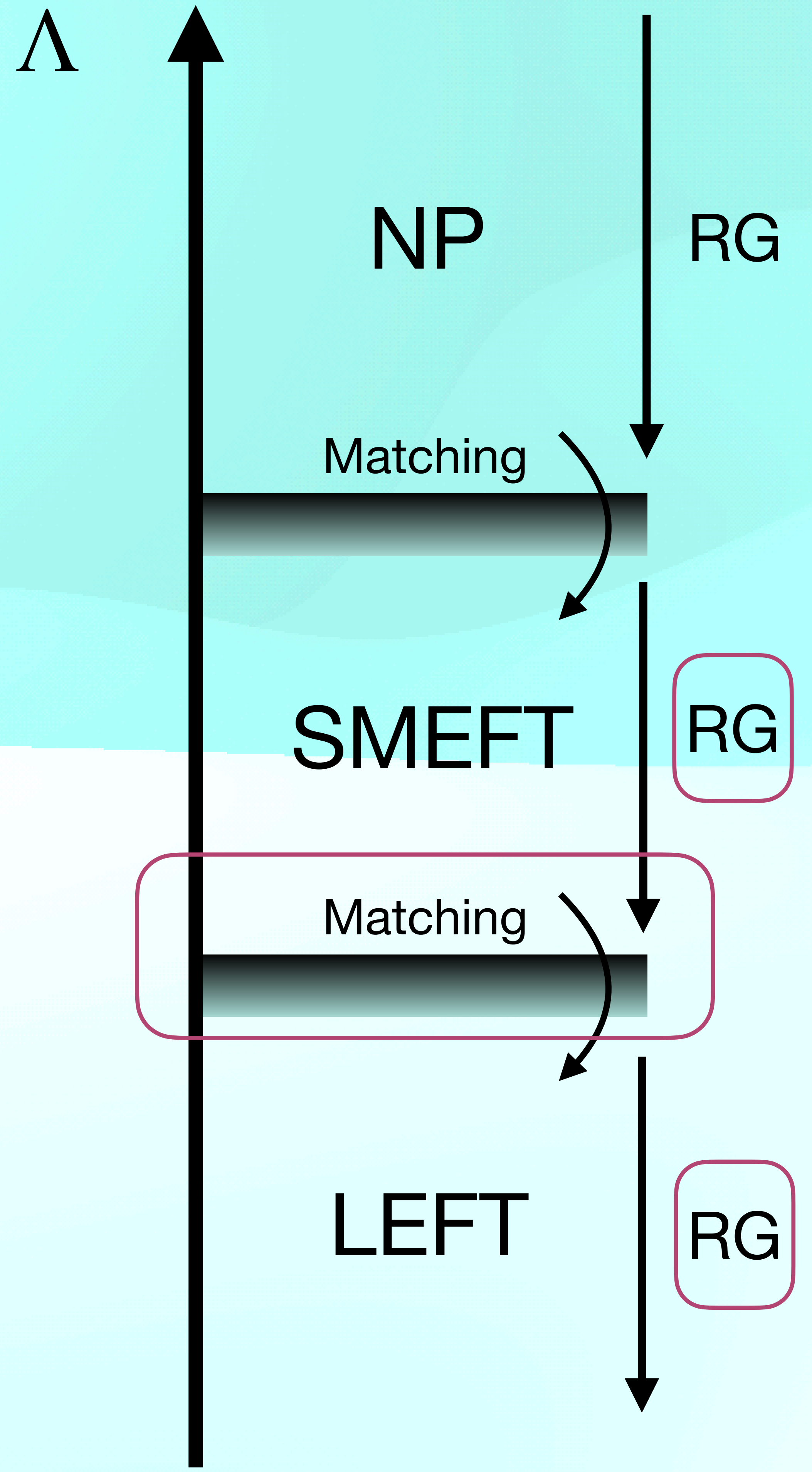


- Inclusion of the effects of the new physics in an EFT description

$$\mathscr{L}_{EFT} = \mathscr{L}^{d=4} + \sum_{d=5}^{\infty} \sum_{k} \frac{C^{(d,k)}}{\Lambda^{d-4}} \mathscr{O}^{(d,k)}$$

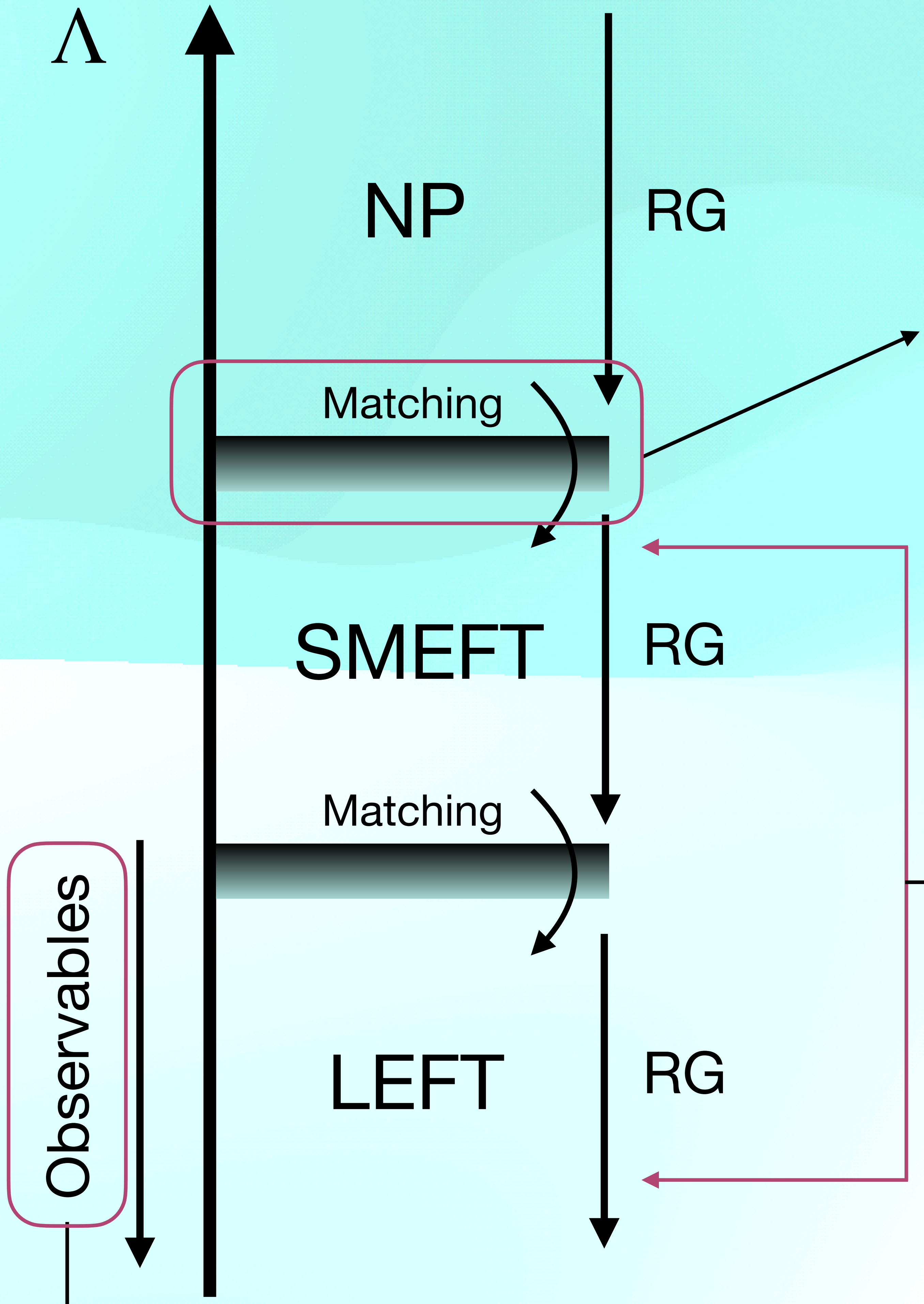
- Wilson coefficients $C^{(d,k)}$ contain the information on the UV physics
- Two distinct approaches in the construction of the EFTs
  - Bottom-up: model-independent analysis with deviations quantified as the E/Λ expansion
  - Top-down: starting from a higher scale, facilitate the **precision computations** in order to move towards the lower scales

Introduction: Top-down approach



- Top-down approach is based upon moving towards lower scales
- Two pivotal concepts of the top-down approach are **matching** and **running**
- Previously, both of these notions have been thoroughly explored
- A comprehensive body of literature is available
  - SMEFT RGE: [1308.2627], [1310.4838], [1312.2014]
  - SMEFT - LEFT Matching: [1709.04486], [1908.05295]
  - LEFT RGE: [1711.05270]

Introduction: Recent development



- In recent years, many aspects of the top-down workflow have been either partly or fully automatized

MATCHETE [2212.04510] | Matchmakereft [2112.10787] | CoDEx [1808.04403] | SuperTracer [2012.08506] | STrEAM [2012.07851]

DsixTools [2010.16341] | wilson [1804.05033]

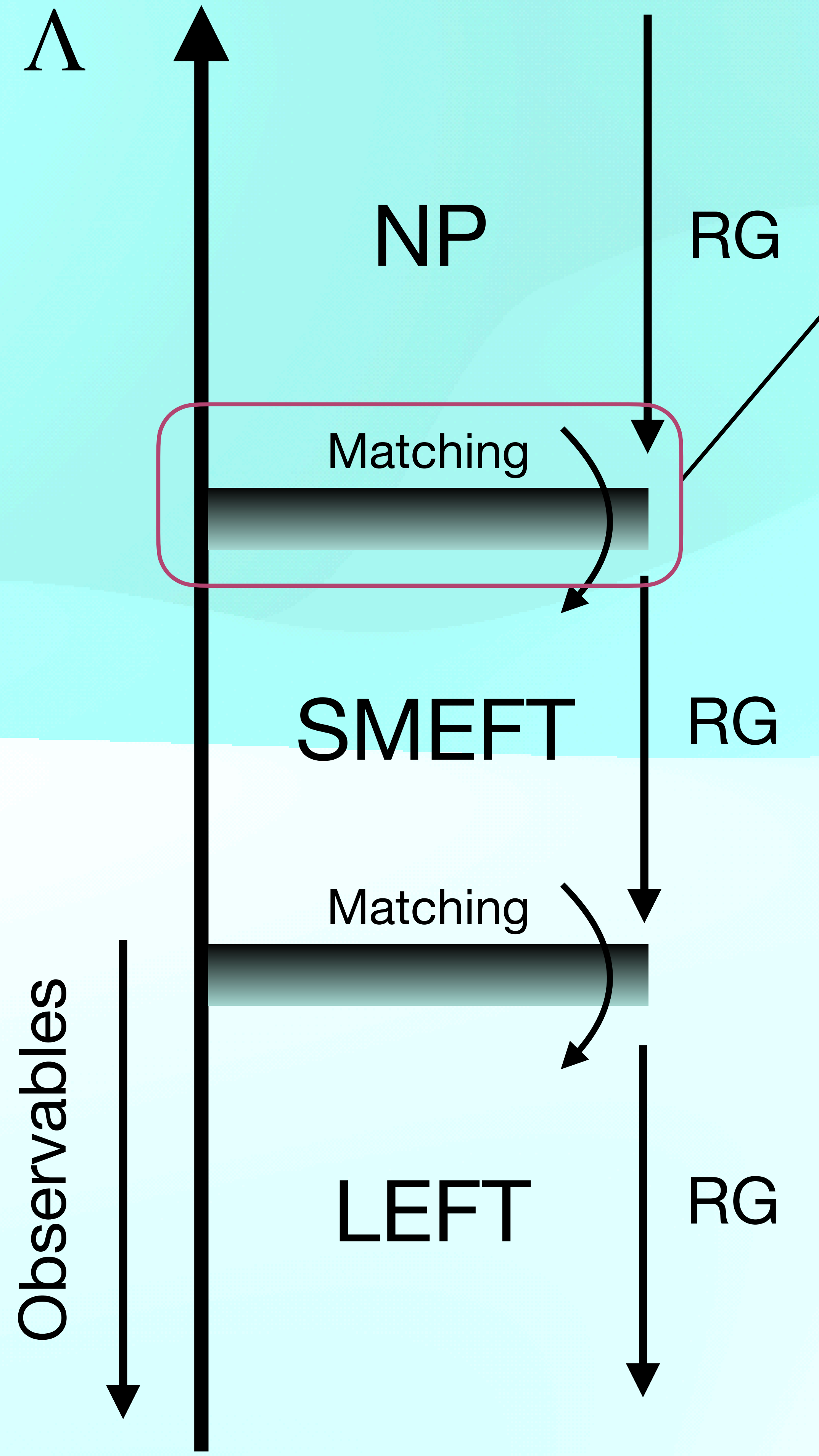
flavio [1810.08132] | HEPfit [1910.14012]

smelli [1810.07698] | SMEFiT [2302.06660]

HighPT [2207.10756] | EOS [2111.15428]

Likelihood generators and fitting tools

Introduction: Functional vs diagrammatic matching



MATCHETE [2212.04510] | [2112.10787] | CoDEx [1808.04403] | SUPER TRACER [2012.08506] | STrEAM [2012.07851]

- Matching between various regimes can be performed using **functional** or **diagrammatic** approach

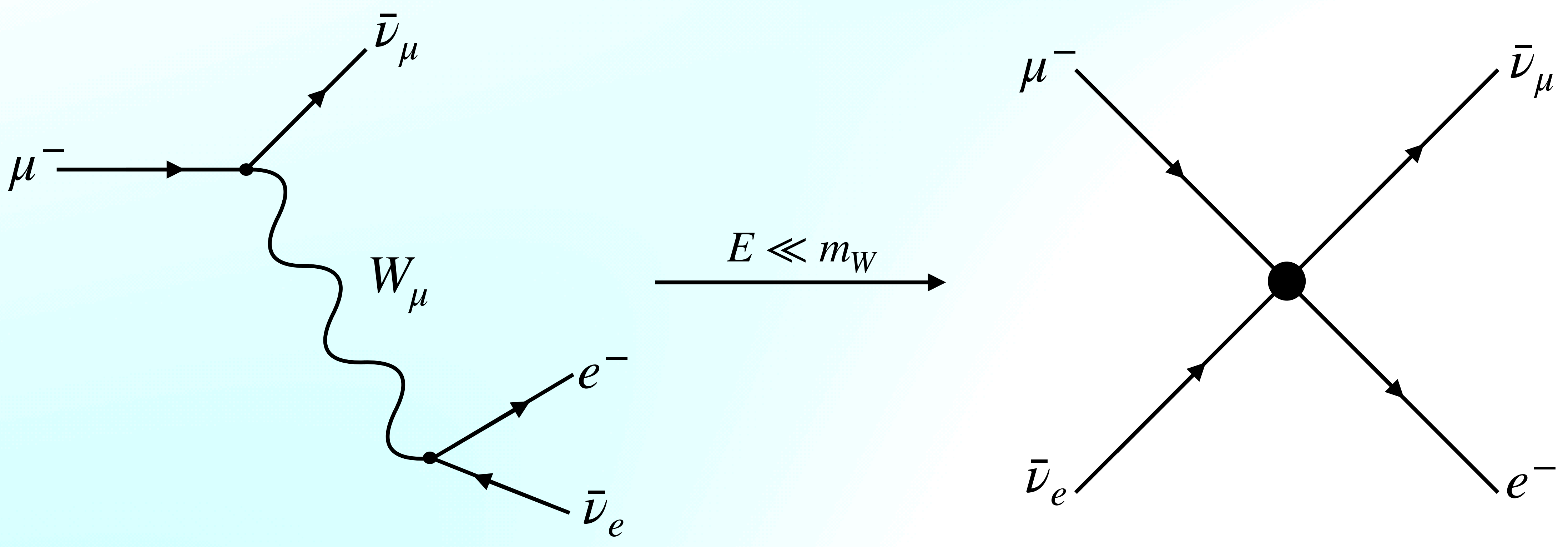
- Common goal of both approaches is to determine the Wilson coefficients

$$\mathscr{L}_{UV}(\Phi_H, \varphi_L) \xrightarrow{E \ll m_H} \mathscr{L}_{EFT}(\varphi_L)$$

- However, both approaches have some distinct features, which are worth mentioning

Introduction: Functional vs diagrammatic matching

- **Diagrammatic matching**
  - Based on equating the amplitudes
  - Valid at any loop order
  - Possible both on- and off-shell
  - Redundancies and EFT basis construction



- **Functional matching**
  - Based on equating the effective actions
  - EFT basis automatically obtained (redundancies)
  - Active and exciting field of research with many recent and upcoming developments

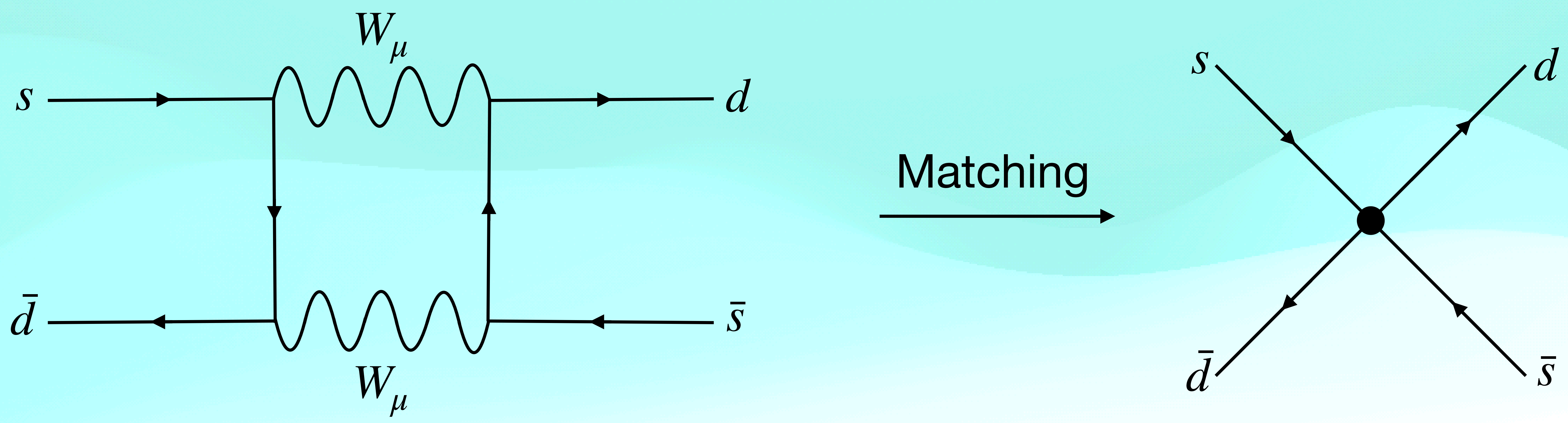
$$\Gamma_{UV}(\Phi_H, \varphi_L) = \Gamma_{EFT}(\varphi_L)$$

  - Our work presented in this talk aligns with this matching approach

Functional approach to the two-loop computations

Motivation: Going beyond tree level (one-loop)

- One-loop effects are often the leading order contribution (e.g. FCNCs in the SM)



- Top-down EFT construction relies on the precise computations in order to move towards the lower scales - combination of this requirement and the functional approach resulted in the significant development in various important aspects
  - Automation tools: [2212.04510], [1808.04403], [2012.08506]
  - UOLEA: [1604.02445], [1706.07765], [1806.05171], [1908.04798], [2006.16260]
  - Matching and running with CDEs [1604.01019], [2301.00821], [2301.00827]

Motivation: Going beyond tree level (two-loop)

- Is the precision offered by the one-loop formalism really enough?
- Following the same reasoning as for one-loop case, depending on the model, some low-energy effects can only be generated at two-loops (e.g. models of neutrino mass generation)
- In addition, both the top Yukawa and strong coupling can give significant contribution to the running effects [2302.11584]
- Functional approach has been fully established only for one-loop
- Method already partially applied
  - Computation of the SM effective potential at two-loops [arXiv:0111190]
  - Renormalization of chiral perturbation theory [arXiv:9907333]

Formalism: Generating functionals and effective action

- Our starting point is the expression for the generating functional

$$e^{i\hbar^{-1}\mathscr{W}[J]} = \int \mathscr{D}\eta \, e^{i\hbar^{-1}\left(S[\eta] + J_I\eta_I\right)}, \quad I = (x, a), \quad J_I\eta_I = \int_x J_a(x)\eta_a(x), \quad \int_x \equiv \int d^d x$$

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- In the next step, we expand the action around the background: $\eta \to \eta + \bar{\eta}$

$$S[\bar{\eta} + \eta] = S[\bar{\eta}] + \eta_I \frac{\delta S}{\delta \eta_I}[\bar{\eta}] + \frac{1}{2}\eta_I\eta_J \frac{\delta^2 S}{\delta\eta_I\delta\eta_J}[\bar{\eta}] + \frac{1}{6}\eta_I\eta_J\eta_K \frac{\delta^3 S}{\delta\eta_I\delta\eta_J\delta\eta_K}[\bar{\eta}] + \frac{1}{24}\eta_I\eta_J\eta_K\eta_L \frac{\delta^4 S}{\delta\eta_I\delta\eta_J\delta\eta_K\delta\eta_L}[\bar{\eta}] + \mathscr{O}(\eta^5)$$

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- We label the derivatives of the action evaluated at $\bar{\eta}$

$$\mathscr{Q}_{IJ} = \frac{\delta^2 S}{\delta\eta_I\delta\eta_J}[\bar{\eta}], \quad \mathscr{B}_{IJK} = \frac{\delta^3 S}{\delta\eta_I\delta\eta_J\delta\eta_K}[\bar{\eta}], \quad \mathscr{D}_{IJKL} = \frac{\delta^4 S}{\delta\eta_I\delta\eta_J\delta\eta_K\delta\eta_L}[\bar{\eta}]$$

Formalism: Generating functionals and effective action

- Action is then rewritten as

$$S[\eta + \bar{\eta}] = S[\bar{\eta}] + \frac{1}{2}\eta_I \eta_J \mathcal{Q}_{IJ}[\bar{\eta}] + \frac{1}{6}\eta_I \eta_J \eta_K \mathcal{B}_{IJK}[\bar{\eta}] + \frac{1}{24}\eta_I \eta_J \eta_K \eta_L \mathcal{D}_{IJKL}[\bar{\eta}] + \mathcal{O}(\eta^5)$$

- Linear term vanishes due to the equations of motion:

$$\frac{\delta S}{\delta \eta_I}[\bar{\eta}] + J_I = 0$$

Formalism: Generating functionals and effective action

- Action is then rewritten as

$$S[\eta + \bar{\eta}] = S[\bar{\eta}] + \frac{1}{2}\eta_I\eta_J\mathcal{Q}_{IJ}[\bar{\eta}] + \frac{1}{6}\eta_I\eta_J\eta_K\mathcal{B}_{IJK}[\bar{\eta}] + \frac{1}{24}\eta_I\eta_J\eta_K\eta_L\mathcal{D}_{IJKL}[\bar{\eta}] + \mathcal{O}(\eta^5)$$

- Linear term vanishes due to the equations of motion:

$$\frac{\delta S}{\delta \eta_I}[\bar{\eta}] + J_I = 0$$

- The generating functional then becomes, using the **saddle point approximation**

$$e^{i\hbar^{-1}\mathcal{W}[J]} = e^{i\hbar^{-1}(S[\bar{\eta}]+J_I\bar{\eta}_I)}\int \mathcal{D}\eta\, e^{\frac{i}{2}\hbar^{-1}\eta_I\eta_J\mathcal{Q}_{IJ}[\bar{\eta}]}\left[1 + \frac{i}{24}\hbar^{-1}\eta_I\eta_J\eta_K\eta_L\mathcal{D}_{IJKL}[\bar{\eta}] + \frac{i^2}{72}\hbar^{-2}\mathcal{B}_{IJK}[\bar{\eta}]\mathcal{B}_{LMN}[\bar{\eta}]\eta_I\eta_J\eta_K\eta_L\eta_M\eta_N\right]$$

- Integrals on the right can be evaluated (related to) using the standard Gaussian integrals

Formalism: Generating functionals and effective action

- More precisely, we take

$$\mathcal{Z}[J] = \int d^n x\, e^{-\frac{1}{2}x^T A x + x^T J} = \frac{(2\pi)^{n/2}}{\sqrt{\det A}} e^{\frac{1}{2}(A^{-1})_{kl} J_k J_l}$$

- The integrals on the previous slide can be expressed as

$$\mathcal{I}_n = \int d^n x\, x_1 x_2 \ldots x_n\, e^{-\frac{1}{2}x^T A x} = \frac{\partial^n \mathcal{Z}[J]}{\partial J_1 \partial J_2 \ldots \partial J_n}\bigg|_{J=0}$$

- E.g. $\mathcal{I}_2$, $\mathcal{I}_3$ and $\mathcal{I}_4$ become

$$\mathcal{I}_2 = \frac{\partial^2 \mathcal{Z}[J]}{\partial J_i \partial J_j}\bigg|_{J=0} = \frac{(2\pi)^{n/2}}{\sqrt{\det A}}(A^{-1})_{ij},$$

$$\mathcal{I}_3 = \frac{\partial^3 \mathcal{Z}[J]}{\partial J_i \partial J_j \partial J_k}\bigg|_{J=0} = 0,$$

$$\mathcal{I}_4 = \frac{\partial^4 \mathcal{Z}[J]}{\partial J_i \partial J_j \partial J_k \partial J_l}\bigg|_{J=0} = \frac{(2\pi)^{n/2}}{\sqrt{\det A}}\left[(A^{-1})_{kl}(A^{-1})_{ij} + (A^{-1})_{ki}(A^{-1})_{lj} + (A^{-1})_{li}(A^{-1})_{kj}\right]$$

Formalism: Generating functionals and effective action

- $\mathscr{W}[J]$ finally becomes (with the labels $\mathscr{Q}_{IJ}[\bar{\eta}] = \overline{\mathscr{Q}}_{IJ}$, $\mathscr{B}_{IJK}[\bar{\eta}] = \overline{\mathscr{B}}_{IJK}$, $\mathscr{D}_{IJKL}[\bar{\eta}] = \overline{\mathscr{D}}_{IJKL}$)
- Expression organises in powers of $\hbar$ (loop order)

$$\mathscr{W}[J] = \bar{S} + J_I \bar{\eta}_I + \frac{i\hbar}{2}\mathrm{sTr}\ln \overline{\mathscr{Q}}$$

One-loop

$$+\frac{\hbar^2}{12}\overline{\mathscr{B}}_{IJK}\overline{\mathscr{B}}_{LMN}(\overline{\mathscr{Q}}^{-1})_{IL}(\overline{\mathscr{Q}}^{-1})_{JM}(\overline{\mathscr{Q}}^{-1})_{KN} - \frac{\hbar^2}{8}\overline{\mathscr{D}}_{IJKL}(\overline{\mathscr{Q}}^{-1})_{IJ}(\overline{\mathscr{Q}}^{-1})_{KL}$$
$$+\frac{\hbar^2}{8}\overline{\mathscr{B}}_{IJK}\overline{\mathscr{B}}_{LMN}(\overline{\mathscr{Q}}^{-1})_{IJ}(\overline{\mathscr{Q}}^{-1})_{KL}(\overline{\mathscr{Q}}^{-1})_{MN}$$
$$+\mathscr{O}(\hbar^3)$$

Two-loop

- In the next step we compute the quantum effective action

Formalism: Generating functionals and effective action

- Effective action is the generating functional for all 1PI diagrams

$$\Gamma[\hat{\eta}] = \mathscr{W}[J] - J_I \hat{\eta}_I, \quad \hat{\eta}_I = \frac{\delta \mathscr{W}[J]}{\delta J_I}$$

Formalism: Generating functionals and effective action

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$$\Gamma[\hat{\eta}] = \mathscr{W}[J] - J_I \hat{\eta}_I, \quad \hat{\eta}_I = \frac{\delta \mathscr{W}[J]}{\delta J_I}$$

- $\hat{\eta}_I$ and $\bar{\eta}_I$ are related through

$$\hat{\eta}_I = \frac{\delta \mathscr{W}[J]}{\delta J_I} = \bar{\eta}_I - \frac{i\hbar}{2} \overline{\mathcal{Q}}^{-1}_{IJ} \overline{\mathscr{B}}_{JKL} \overline{\mathcal{Q}}^{-1}_{LK} + \mathcal{O}(\hbar^2)$$

Formalism: Generating functionals and effective action

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- The effective action becomes

$$\Gamma[\hat{\eta}] = \hat{S} + \underbrace{\frac{i\hbar}{2} \mathrm{sTr} \ln \hat{\mathscr{Q}}}_{\text{One-loop}} + \underbrace{\frac{\hbar^2}{12} \hat{\mathscr{B}}_{IJK} \hat{\mathscr{B}}_{LMN} \hat{\mathscr{Q}}^{-1}_{IL} \hat{\mathscr{Q}}^{-1}_{JM} \hat{\mathscr{Q}}^{-1}_{KN} - \frac{\hbar^2}{8} \hat{\mathscr{D}}_{IJKL} \hat{\mathscr{Q}}^{-1}_{IJ} \hat{\mathscr{Q}}^{-1}_{KL}}_{\text{Two-loop}}$$

Formalism: Generating functionals and effective action

- Inclusion of the one-loop action $S^{(1)}$

$$e^{i\hbar^{-1}\mathscr{W}[J]} = \int \mathscr{D}\eta\, e^{i\hbar^{-1}\left(S[\eta] + J_I\eta_I\right)} = \int \mathscr{D}\eta\, e^{i\hbar^{-1}\left(S^{(0)}[\eta] + \hbar S^{(1)}[\eta] + \hbar^2 S^{(2)}[\eta] + J_I\eta_I\right)}$$

- With this split, the final form of the effective action is

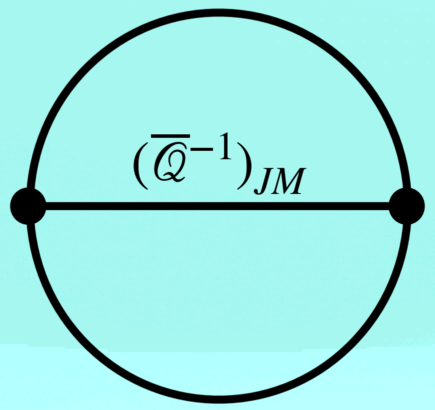
Additional terms from $S[\eta]$ loop expansion

$$\Gamma[\hat{\eta}] = (\hat{S}^{(0)} + \hbar\hat{S}^{(1)} + \hbar^2\hat{S}^{(2)}) + \frac{i\hbar}{2}\mathrm{sTr}\ln\hat{\mathscr{Q}}^{(0)} + \frac{i\hbar^2}{2}(\hat{\mathscr{Q}}^{(0)})^{-1}_{IJ}(\hat{\mathscr{Q}}^{(1)})_{IJ}$$

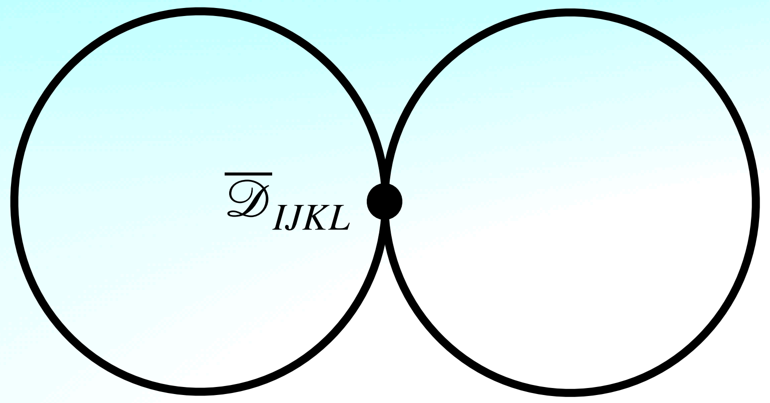
$$-\frac{\hbar^2}{8}\hat{\mathscr{D}}^{(0)}_{IJKL}(\hat{\mathscr{Q}}^{(0)})^{-1}_{IJ}(\hat{\mathscr{Q}}^{(0)})^{-1}_{KL} + \frac{\hbar^2}{12}\hat{\mathscr{B}}^{(0)}_{IJK}\hat{\mathscr{B}}^{(0)}_{LMN}(\hat{\mathscr{Q}}^{(0)})^{-1}_{IL}(\hat{\mathscr{Q}}^{(0)})^{-1}_{JM}(\hat{\mathscr{Q}}^{(0)})^{-1}_{KN} + \mathscr{O}(\hbar^3)$$

Formalism: $\mathcal{O}(\hbar^2)$ topologies contained in $\mathscr{W}[J]$ and $\Gamma[\hat{\eta}]$

$$\mathscr{W}[J] \supset \frac{\hbar^2}{12}\overline{\mathscr{B}}_{IJK}\overline{\mathscr{B}}_{LMN}(\overline{\mathscr{Q}}^{-1})_{IL}(\overline{\mathscr{Q}}^{-1})_{JM}(\overline{\mathscr{Q}}^{-1})_{KN}$$



$$\mathscr{W}[J] \supset -\frac{\hbar^2}{8}\overline{\mathscr{D}}_{IJKL}(\overline{\mathscr{Q}}^{-1})_{IJ}(\overline{\mathscr{Q}}^{-1})_{KL}$$



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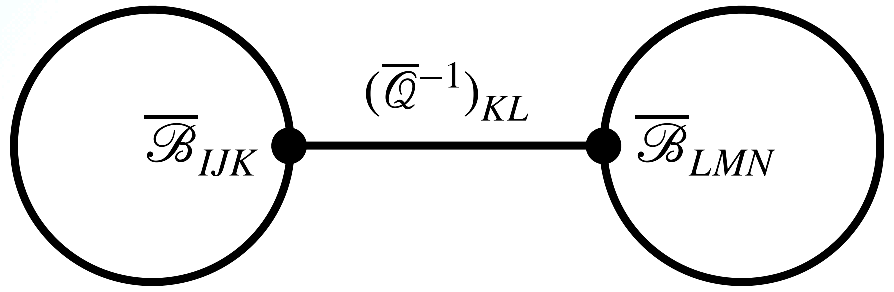
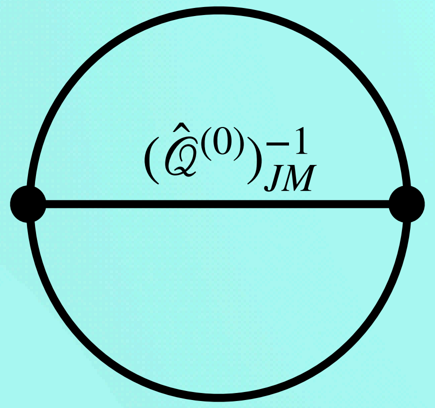


$$\Gamma[\hat{\eta}] \supset \frac{\hbar^2}{12}\hat{\mathscr{B}}^{(0)}_{IJK}\hat{\mathscr{B}}^{(0)}_{LMN}(\hat{\mathscr{Q}}^{(0)})^{-1}_{IL}(\hat{\mathscr{Q}}^{(0)})^{-1}_{JM}(\hat{\mathscr{Q}}^{(0)})^{-1}_{KN}$$



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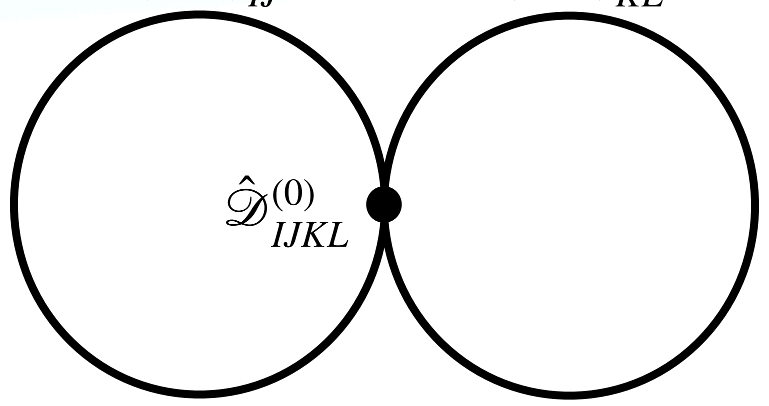
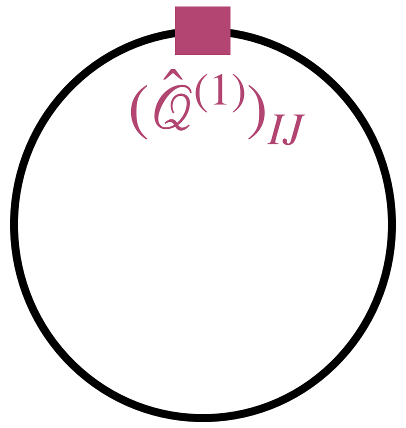


$$\Gamma[\hat{\eta}] \supset \frac{i\hbar^2}{2}(\hat{\mathscr{Q}}^{(0)})^{-1}_{IJ}(\hat{\mathscr{Q}}^{(1)})_{IJ}$$



Non-covariant evaluations

- We present the preliminary results for the non-covariant objects

- Terms in the effective action containing $\mathscr{Q}$, $\mathscr{B}$ and $\mathscr{D}$ are converted into the integral form

$$\mathscr{Q}_{IJ} \equiv Q_{ab}(x, P_x)\delta(x - y)$$

$$\mathscr{B}_{IJK} \equiv \left[\sum_{m,n=0}^{\infty} B_{\underline{abc}}^{(\underline{m},\underline{n})}(z) P_{\underline{x}}^{\underline{m}} P_{\underline{y}}^{\underline{n}} \right] \delta(x - z)\delta(y - z)$$

$$\mathscr{D}_{IJKL} \equiv \left[\sum_{m,n,r=0}^{\infty} D_{\underline{abcd}}^{(\underline{m},\underline{n},\underline{r})}(w) P_{\underline{x}}^{\underline{m}} P_{\underline{y}}^{\underline{n}} P_{\underline{z}}^{\underline{r}} \right] \delta(x - w)\delta(y - w)\delta(z - w)$$

- We analyze the two-loop terms in the effective action

Non-covariant evaluations: $\mathcal{O}(\hbar^2)$ terms

- One-loop counterterm insertion

$$\Gamma \supset \frac{i\hbar^2}{2}\mathcal{Q}^{-1}_{IJ}\mathcal{Q}^{(1)}_{JI} = \frac{i\hbar^2}{2}\int_x\int_k Q^{-1}_{ab}(x, P_x + k)Q^{(1)}_{ba}(x, P_x + k)$$

- Sunset diagram

$$\Gamma \supset \frac{\hbar^2}{12}\mathcal{B}_{IJK}\mathcal{Q}^{-1}_{IL}\mathcal{Q}^{-1}_{JM}\mathcal{Q}^{-1}_{KN}\mathcal{B}_{LMN} = \frac{\hbar^2}{12}\int_x\int_{k,l}\sum_{m,n,m',n'}(-1)^{m+n}\left[B^{(\underline{m},\underline{n})}_{abc}(x)\sum_r \frac{(-i)^r}{r!}\partial^{\underline{r}}_x B^{(\underline{m}',\underline{n}')}_{def}(x)\right]$$
$$\times\left[(P_x + k)^{\underline{m}}Q^{-1}_{ad}(x, P_x + k)(P_x + k)^{\underline{m}'}\right]\left[(P_x + l)^{\underline{n}}Q^{-1}_{be}(x, P_x + l)(P_x + l)^{\underline{n}'}\right]$$
$$\times\left[\partial^{\underline{r}}_k Q^{-1}_{cf}(x, P_x - k - l)\right]$$

- Figure-8 diagram

$$\Gamma \supset -\frac{\hbar^2}{8}\mathcal{D}_{IJKL}\mathcal{Q}^{-1}_{IJ}\mathcal{Q}^{-1}_{KL} = -\frac{\hbar^2}{8}\int_x\int_{k,l}\sum_{m,n,r}(-1)^{m+n}D^{(\underline{m},\underline{n},\underline{r})}_{abcd}(x)\left[(P_x + k)^{\underline{m}}Q^{-1}_{ab}(x, P_x + k)(P_x + k)^{\underline{r}}\right]\left[(P_x + l)^{\underline{n}}Q^{-1}_{cd}(x, P_x + l)\right]$$

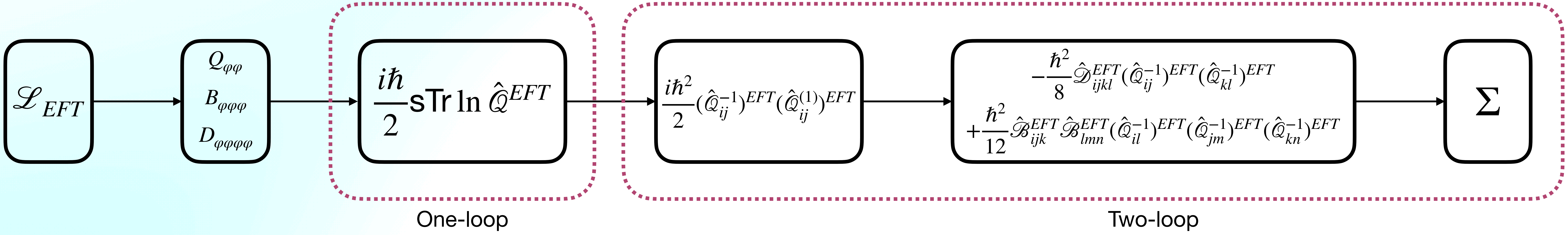
Toy model example: scalar theory

Toy model example

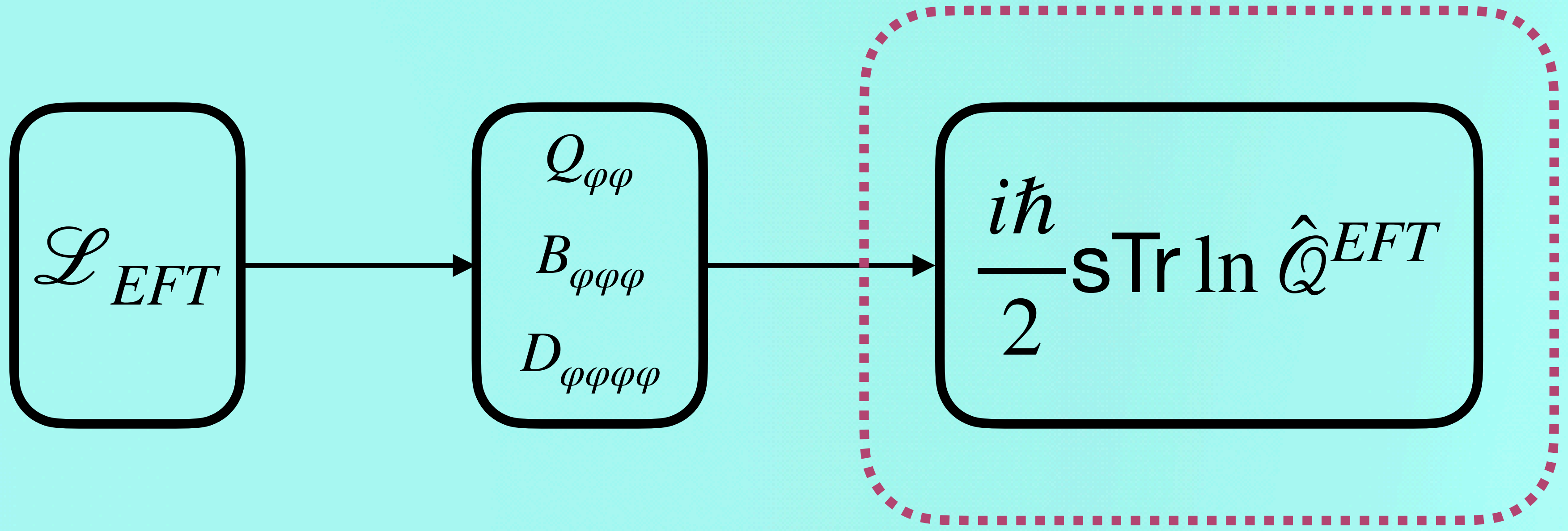
- We use the effective action and the non-covariant evaluations for simple scalar toy model
- Expressions for $\mathscr{Q}$, $\mathscr{B}$ and $\mathscr{D}$ reduce to simpler expressions with propagator derivatives
- We first present the results for the RGE of the toy model EFT

$$\mathscr{L}_{EFT}(\varphi) = \frac{1}{2}(\partial_\mu \varphi)(\partial^\mu \varphi) - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!}\varphi^4 - \frac{C}{6!}\varphi^6$$

- Sequence of the computations



Toy model RGE - one-loop CTs



- $Q_{\varphi\varphi}$, $B_{\varphi\varphi\varphi}$ and $D_{\varphi\varphi\varphi\varphi}$

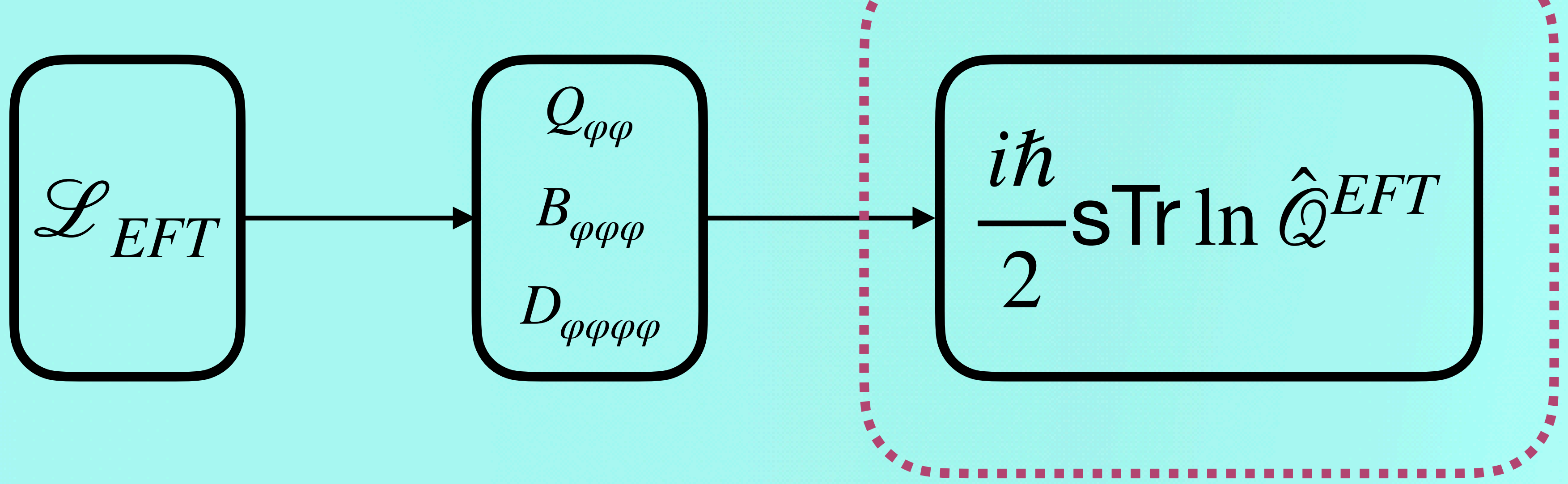
$$Q_{\varphi\varphi} = -\partial^2 - m^2 - \frac{\lambda}{2}\varphi^2 - \frac{C}{4!}\varphi^4, \quad B_{\varphi\varphi\varphi} = -\lambda\varphi - \frac{C}{6}\varphi^3, \quad D_{\varphi\varphi\varphi\varphi} = -\lambda - \frac{C}{2}\varphi^2$$

- One-loop CTs extracted from

$$\int d^dx\, \mathscr{L}_{EFT} = -\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int d^dx \int \frac{d^dp}{(2\pi)^d}\mathrm{tr}[\Delta\tilde{X}]^n, \quad \Delta^{-1} = -\partial^2 - m^2, \quad X = \frac{\lambda}{2}\varphi^2 + \frac{C}{4!}\varphi^4$$

[2012.08506]

Toy model RGE - one-loop CTs



- $Q_{\varphi\varphi}$, $B_{\varphi\varphi\varphi}$ and $D_{\varphi\varphi\varphi\varphi}$

$$Q_{\varphi\varphi} = -\partial^2 - m^2 - \frac{\lambda}{2}\varphi^2 - \frac{C}{4!}\varphi^4, \quad B_{\varphi\varphi\varphi} = -\lambda\varphi - \frac{C}{6}\varphi^3, \quad D_{\varphi\varphi\varphi\varphi} = -\lambda - \frac{C}{2}\varphi^2$$

- One-loop CTs extracted from

$$\int d^d x\, \mathscr{L}_{EFT} = -\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\int d^d x \int \frac{d^d p}{(2\pi)^d}\text{tr}[\Delta\tilde{X}]^n, \quad \Delta^{-1} = -\partial^2 - m^2, \quad X = \frac{\lambda}{2}\varphi^2 + \frac{C}{4!}\varphi^4$$

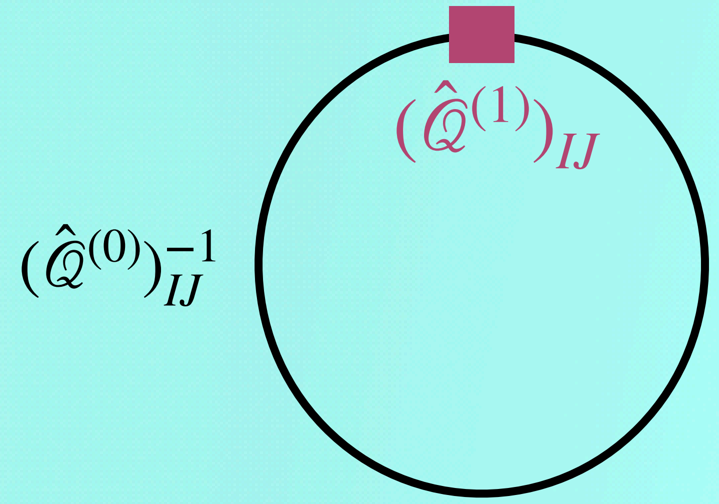
[2012.08506]

- CTs become

$$\delta_\varphi^{(1)} = 0, \quad \delta_m^{(1)} = \frac{m^2\lambda}{32\pi^2}\frac{1}{\varepsilon}, \quad \delta_\lambda^{(1)} = \frac{3\lambda^2 + m^2 C}{32\pi^2}\frac{1}{\varepsilon}, \quad \delta_C^{(1)} = \frac{15\lambda C}{32\pi^2}\frac{1}{\varepsilon}$$

- Next, we evaluate the two-loop contribution from the CTs insertion

Toy model RGE - one-loop CTs insertions



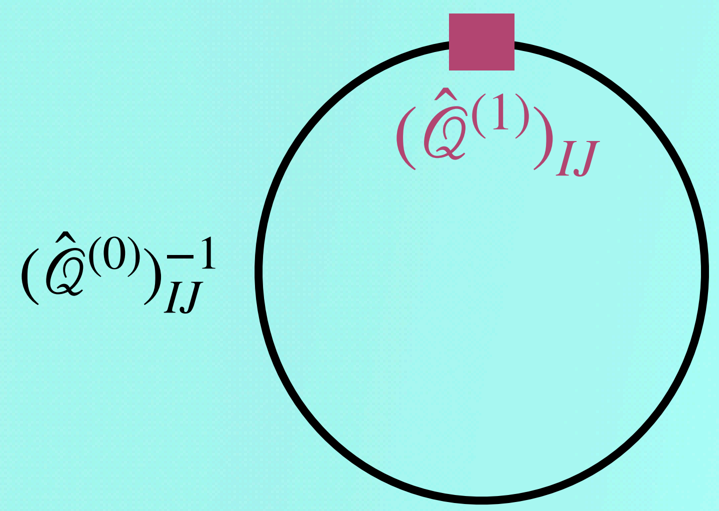
- One-loop CT insertion diagram is given by

$$\Gamma_{EFT} \supset \frac{i\hbar^2}{2} \int_{x,k} Q^{-1}_{\varphi\varphi}(x, P_x + k)\, Q^{(1)}_{\varphi\varphi}(x, P_x + k)$$

$$\frac{\delta^2 \mathcal{L}^{(ct)}_{EFT}}{\delta \varphi^2} = -\delta^{(1)}_m - \frac{\delta^{(1)}_\lambda}{2}\varphi^2 - \frac{\delta^{(1)}_C}{4!}\varphi^4$$

$$\frac{1}{k^2 - m^2} \sum_{n=0}^{\infty} \left(\frac{\partial^2 + 2ik \cdot \partial + \frac{\lambda}{2}\varphi^2 + \frac{C}{4!}\varphi^4}{k^2 - m^2} \right)^n$$

Toy model RGE - one-loop CTs insertions



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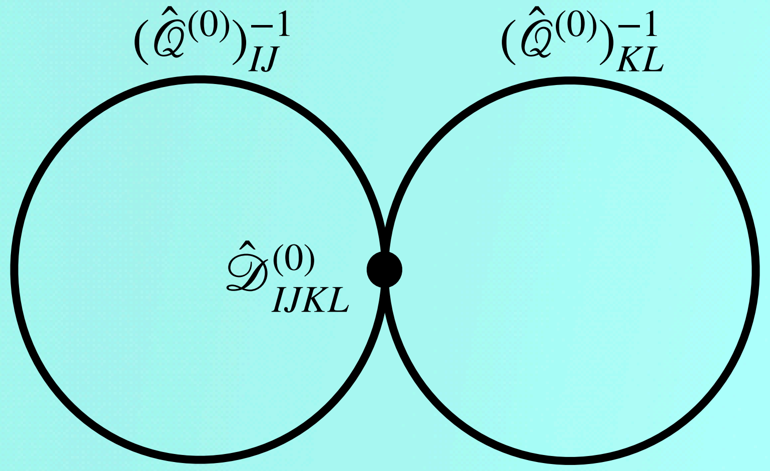
- Result

$$\Gamma_{EFT} \supset \int_x \frac{1}{(16\pi^2)^2} \left(\frac{1}{\varepsilon^2} - \frac{\ln m^2}{\varepsilon} \right) \left(\frac{4\lambda^2 + m^2 C}{4} \frac{m^2\varphi^2}{2} + \frac{9\lambda^2 + 11 m^2 C}{2} \frac{\lambda\varphi^4}{4!} + \frac{135\lambda^2}{2} \frac{C\varphi^6}{6!} \right)$$

$$+ \int_x \frac{1}{(16\pi^2)^2} \frac{1}{\varepsilon} \left(\frac{3\lambda^2 + m^2 C}{4} \frac{m^2\varphi^2}{2} - \frac{3\lambda^2 - 15 m^2 C}{4} \frac{\lambda\varphi^4}{4!} - \frac{30\lambda^4 + 15\lambda^2 m^2 C}{m^2} \frac{\varphi^6}{6!} - \frac{5\lambda^3 + 2\lambda m^2 C}{192 m^2} \varphi^2 \partial^2 \varphi^2 \right)$$

Toy model RGE - figure-8

$$-\frac{\hbar^2}{8}\hat{\mathcal{D}}^{EFT}_{ijkl}(\hat{\mathcal{Q}}^{-1}_{ij})^{EFT}(\hat{\mathcal{Q}}^{-1}_{kl})^{EFT}$$



- Figure-8 diagram becomes

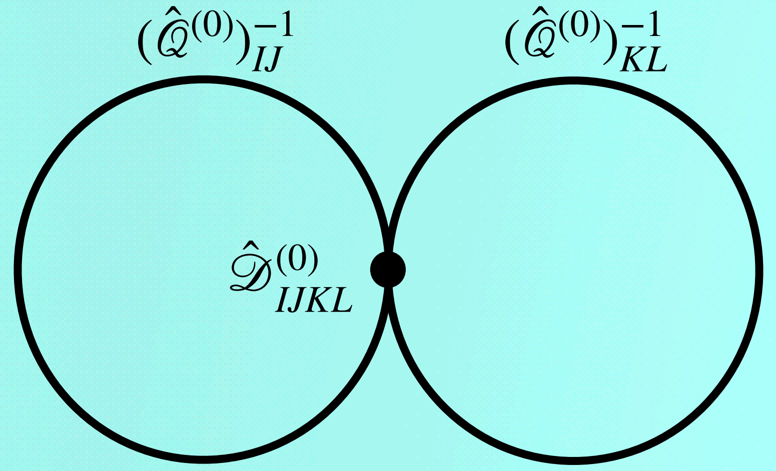
$$\Gamma_{EFT} \supset -\frac{\hbar^2}{8}\int_x\int_{k,l} D_{\varphi\varphi\varphi\varphi}\left[Q^{-1}_{\varphi\varphi}(x, P_x + k)\right]\left[Q^{-1}_{\varphi\varphi}(x, P_x + l)\right]$$

$$\left[Q^{-1}_{\varphi\varphi}(x, P_x + l)\right] \rightarrow \frac{1}{l^2 - m^2}\sum_{n=0}^{\infty}\left(\frac{\partial^2 + 2il\cdot\partial + \frac{\lambda}{2}\varphi^2 + \frac{C}{4!}\varphi^4}{l^2 - m^2}\right)^n$$

$$D_{\varphi\varphi\varphi\varphi} \rightarrow -\lambda - \frac{C}{2}\varphi^2$$

Toy model RGE - figure-8

$$-\frac{\hbar^2}{8}\hat{\mathcal{D}}^{EFT}_{ijkl}(\hat{\mathcal{Q}}^{-1}_{ij})^{EFT}(\hat{\mathcal{Q}}^{-1}_{kl})^{EFT}$$



- Figure-8 diagram becomes

$$\Gamma_{EFT} \supset -\frac{\hbar^2}{8}\int_x\int_{k,l} D_{\varphi\varphi\varphi\varphi}\left[Q^{-1}_{\varphi\varphi}(x, P_x + k)\right]\left[Q^{-1}_{\varphi\varphi}(x, P_x + l)\right]$$

$$D_{\varphi\varphi\varphi\varphi} \rightarrow -\lambda - \frac{C}{2}\varphi^2$$

$$Q^{-1}_{\varphi\varphi}(x, P_x + l) \rightarrow \frac{1}{l^2 - m^2}\sum_{n=0}^{\infty}\left(\frac{\partial^2 + 2il\cdot\partial + \frac{\lambda}{2}\varphi^2 + \frac{C}{4!}\varphi^4}{l^2 - m^2}\right)^n$$

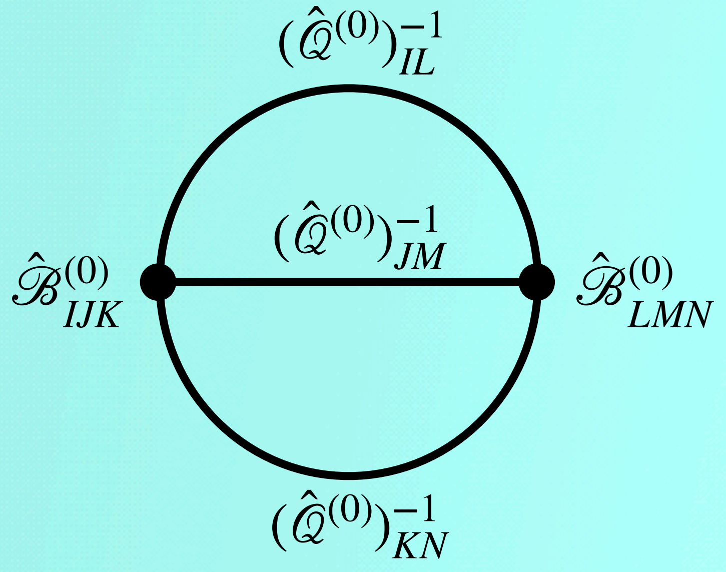
- Result

$$\Gamma_{EFT} \supset -\int_x \frac{1}{(16\pi^2)^2}\left(\frac{1}{\varepsilon^2} - \frac{2\ln m^2}{\varepsilon}\right)\left(\frac{2\lambda^2 + m^2C}{8}\frac{m^2\varphi^2}{2} + \frac{3\lambda^2 + 7m^2C}{4}\frac{\lambda\varphi^4}{4!} + 15\lambda^2\frac{C\varphi^2}{6!}\right)$$

$$+\int_x \frac{1}{(16\pi^2)^2}\frac{1}{\varepsilon}\left(-\frac{\lambda^2 + m^2C}{4}\frac{m^2\varphi^2}{2} + \frac{3\lambda^2 - 7m^2C}{4}\frac{\lambda\varphi^4}{4!} + \frac{15\lambda^4 + 30\lambda^2m^2C}{2m^2}\frac{\varphi^6}{6!} + \frac{\lambda^3 + 2m^2C}{192m^2}\varphi^2\partial^2\varphi^2\right)$$

Toy model RGE - sunset

$$\frac{\hbar^2}{12}\hat{\mathscr{B}}^{EFT}_{ijk}\hat{\mathscr{B}}^{EFT}_{lmn}(\hat{\mathcal{Q}}^{-1}_{il})^{EFT}(\hat{\mathcal{Q}}^{-1}_{jm})^{EFT}(\hat{\mathcal{Q}}^{-1}_{kn})^{EFT}$$



- Sunset diagram becomes

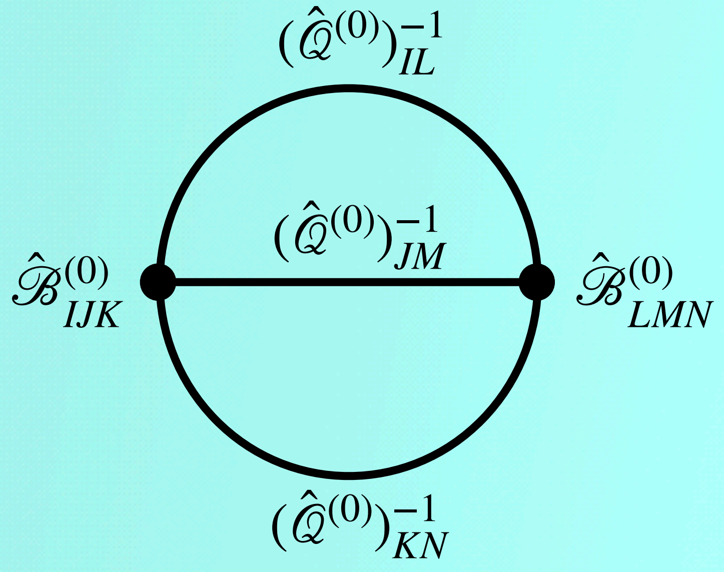
$$\Gamma_{EFT} \supset \frac{\hbar^2}{12}\int_x\int_{k,l}\sum_r \frac{(-i)^r}{r!}\left[B_{\varphi\varphi\varphi}\partial^r_x B_{\varphi\varphi\varphi}\right]\left[Q^{-1}_{\varphi\varphi}(x,P_x+k)\right]\left[Q^{-1}_{\varphi\varphi}(x,P_x+l)\right]\left[\partial^r_k Q^{-1}_{\varphi\varphi}(x,P_x-k-l)\right]$$

$$\lambda^2\varphi\partial^r_x\varphi + \frac{\lambda C}{6}\varphi\partial^r_x\varphi^3 + \frac{\lambda C}{6}\varphi^3\partial^r_x\varphi + \frac{C^2}{36}\varphi^3\partial^r_x\varphi^3$$

$$\frac{1}{k^2-m^2}\sum_{n=0}^{\infty}\left(\frac{\partial^2 + 2ik\cdot\partial + \frac{\lambda}{2}\varphi^2 + \frac{C}{4!}\varphi^4}{k^2-m^2}\right)^n$$

Toy model RGE - sunset

$$\frac{\hbar^2}{12}\hat{\mathscr{B}}^{EFT}_{ijk}\hat{\mathscr{B}}^{EFT}_{lmn}(\hat{\mathcal{Q}}^{-1}_{il})^{EFT}(\hat{\mathcal{Q}}^{-1}_{jm})^{EFT}(\hat{\mathcal{Q}}^{-1}_{kn})^{EFT}$$



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$$\Gamma_{EFT} \supset \frac{\hbar^2}{12}\int_x\int_{k,l}\sum_r \frac{(-i)^r}{r!}\left[B_{\varphi\varphi\varphi}\partial_x^r B_{\varphi\varphi\varphi}\right]\left[Q^{-1}_{\varphi\varphi}(x,P_x+k)\right]\left[Q^{-1}_{\varphi\varphi}(x,P_x+l)\right]\left[\partial_k^r Q^{-1}_{\varphi\varphi}(x,P_x-k-l)\right]$$

$$\lambda^2\varphi\partial_x^r\varphi + \frac{\lambda C}{6}\varphi\partial_x^r\varphi^3 + \frac{\lambda C}{6}\varphi^3\partial_x^r\varphi + \frac{C^2}{36}\varphi^3\partial_x^r\varphi^3$$

$$\frac{1}{k^2-m^2}\sum_{n=0}^{\infty}\left(\frac{\partial^2 + 2ik\cdot\partial + \frac{\lambda}{2}\varphi^2 + \frac{C}{4!}\varphi^4}{k^2-m^2}\right)^n$$

- Sunset integrals treated recursively

$$\mathcal{I}^{(2)}_{(n_1+1)n_2n_3}(m,m,m) = -\frac{1}{3m^2n_1}\left[3n_1 - d + n_2\mathbf{2}^+(\mathbf{1}^- - \mathbf{3}^-) + n_3\mathbf{3}^+(\mathbf{1}^- - \mathbf{2}^-)\right]\mathcal{I}^{(2)}_{n_1n_2n_3}(m,m,m),$$

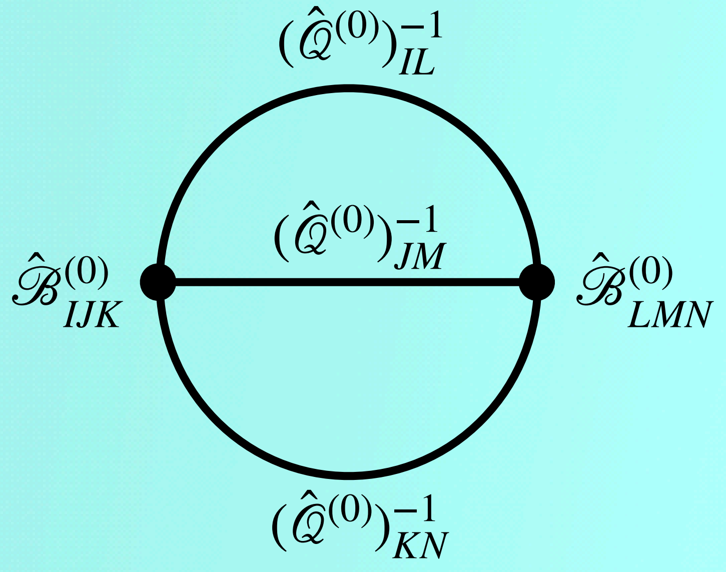
$$\mathcal{I}^{(2)}_{111}(m,m,m) = -\frac{m^2}{(16\pi^2)^2}\frac{3}{2}\left[\frac{1}{\varepsilon^2} + \frac{3-2\,\overline{\ln}\,m^2}{\varepsilon} + 2\,\overline{\ln}^2 m^2 - 6\,\overline{\ln}\,m^2 + 7 + \frac{\pi^2}{6} - 2\sqrt{3}\,\mathrm{Ls}_2 + \mathcal{O}(\varepsilon)\right]$$

$$\ln\left(\frac{m^2}{\bar{\mu}^2}\right)$$

[hep-ph/9711266]

Toy model RGE - sunset

$$\frac{\hbar^2}{12}\hat{\mathscr{B}}^{EFT}_{ijk}\hat{\mathscr{B}}^{EFT}_{lmn}(\hat{\mathcal{Q}}^{-1}_{il})^{EFT}(\hat{\mathcal{Q}}^{-1}_{jm})^{EFT}(\hat{\mathcal{Q}}^{-1}_{kn})^{EFT}$$



- Sunset diagram becomes

$$\Gamma_{EFT} \supset \frac{\hbar^2}{12}\int_x\int_{k,l}\sum_r \frac{(-i)^r}{r!}\left[B_{\varphi\varphi\varphi}\partial_x^r B_{\varphi\varphi\varphi}\right]\left[Q^{-1}_{\varphi\varphi}(x,P_x+k)\right]\left[Q^{-1}_{\varphi\varphi}(x,P_x+l)\right]\left[\partial_k^r Q^{-1}_{\varphi\varphi}(x,P_x-k-l)\right]$$

$$\lambda^2\varphi\partial_x^r\varphi + \frac{\lambda C}{6}\varphi\partial_x^r\varphi^3 + \frac{\lambda C}{6}\varphi^3\partial_x^r\varphi + \frac{C^2}{36}\varphi^3\partial_x^r\varphi^3$$

$$\frac{1}{k^2-m^2}\sum_{n=0}^{\infty}\left(\frac{\partial^2+2ik\cdot\partial+\frac{\lambda}{2}\varphi^2+\frac{C}{4!}\varphi^4}{k^2-m^2}\right)^n$$

- Result

$$\Gamma_{EFT} \supset \int_x \frac{1}{(16\pi^2)^2}\left(\frac{1}{\varepsilon^2}-\frac{2\ln m^2}{\varepsilon}\right)\left(-\frac{\lambda^2}{4}\frac{m^2\varphi^2}{2}-\frac{3\lambda^2+2m^2C}{2}\frac{\lambda\varphi^4}{4!}-\frac{75\lambda^2}{4}\frac{C\varphi^6}{6!}\right)$$

$$+\int_x \frac{1}{(16\pi^2)^2}\frac{1}{\varepsilon}\left(-\frac{3\lambda^2}{4}\frac{m^2\varphi^2}{2}-\frac{3\lambda^2+6m^2C}{2}\frac{\lambda\varphi^4}{4!}+\frac{90\lambda^4-75\lambda^2m^2C}{4m^2}\frac{\varphi^6}{6!}+\frac{\lambda^2}{24}\frac{(\partial_\mu\varphi)^2}{2}+\frac{4\lambda^3-\lambda m^2C}{192m^2}\varphi^2\partial^2\varphi^2\right)$$

Toy model RGE - two-loop CTs

Σ

- Summing all of the results we get

$$\Gamma_{EFT}\Big|_{\text{div.}} \supset \int_x \frac{1}{(16\pi^2)^2}\frac{1}{\varepsilon^2}\left(\frac{4\lambda^2+m^2C}{8}\frac{m^2\varphi^2}{2}+\frac{9\lambda^2+11m^2C}{4}\frac{\lambda\varphi^4}{4!}+\frac{135\lambda^2}{4}\frac{C\varphi^6}{6!}\right)$$

$$+\int_x \frac{1}{(16\pi^2)^2}\frac{1}{\varepsilon}\left(\frac{\lambda^2}{24}\frac{(\partial_\mu\varphi)^2}{2}-\frac{\lambda^2}{4}\frac{m^2\varphi^2}{2}-\frac{9\lambda^2+5m^2C}{6}\frac{\lambda\phi^4}{4!}-\frac{215\lambda^2}{12}\frac{C\varphi^6}{6!}\right)$$

Toy model RGE - two-loop CTs

Σ

- Summing all of the results we get

$$\Gamma_{EFT}\Big|_{\text{div.}} \supset \int_x \frac{1}{(16\pi^2)^2}\frac{1}{\varepsilon^2}\left(\frac{4\lambda^2+m^2C}{8}\frac{m^2\varphi^2}{2}+\frac{9\lambda^2+11m^2C}{4}\frac{\lambda\varphi^4}{4!}+\frac{135\lambda^2}{4}\frac{C\varphi^6}{6!}\right)$$

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- Two-loop CTs $\delta_i^{(2)} = \frac{1}{\varepsilon}\delta_{i,1}^{(2)} + \frac{1}{\varepsilon^2}\delta_{i,2}^{(2)}$

$$\delta_{m,1}^{(2)} = -\frac{m^2}{(16\pi^2)^2}\frac{\lambda^2}{4}, \qquad \delta_{\lambda,1}^{(2)} = -\frac{\lambda}{(16\pi^2)^2}\frac{9\lambda^2+5m^2C}{6}, \qquad \delta_{C,1}^{(2)} = -\frac{C}{(16\pi^2)^2}\frac{215\lambda^2}{12}, \qquad \delta_{Z_\varphi,1}^{(2)} = -\frac{1}{(16\pi^2)^2}\frac{\lambda^2}{24},$$

$$\delta_{m,2}^{(2)} = \frac{m^2}{(16\pi^2)^2}\frac{4\lambda^2+m^2C}{8}, \qquad \delta_{\lambda,2}^{(2)} = \frac{\lambda}{(16\pi^2)^2}\frac{9\lambda^2+11m^2C}{4}, \qquad \delta_{C,2}^{(2)} = \frac{C}{(16\pi^2)^2}\frac{135\lambda^2}{4}, \qquad \delta_{Z_\varphi,2}^{(2)} = 0$$

Toy model RGE - checks

- Two-loop counterterms satisfy the consistency conditions [2104.07037]

$$4\delta^{(2)}_{i,2} - 2\delta^{(1)}_j \partial_j \delta^{(1)}_i = 0$$

- β and γ functions

$$\gamma_\varphi = \frac{1}{2}\frac{\partial}{\partial \ln \mu} \ln Z_\varphi = \frac{1}{(16\pi^2)^2}\frac{\lambda^2}{12},$$

$$\beta_{m^2} = \frac{m^2\lambda}{16\pi^2} - \frac{5}{6}\frac{m^2\lambda^2}{(16\pi^2)^2},$$

$$\beta_\lambda = \frac{3\lambda^2 + m^2 C}{16\pi^2} - \frac{17\lambda^3 + 10\lambda m^2 C}{3(16\pi^2)^2},$$

→ Results checked using RGBeta [2101.08265]

$$\beta_C = \frac{15\lambda C}{16\pi^2} - \frac{427\lambda^2 C}{6(16\pi^2)^2}$$

→ New result

Matching conditions

Two-loop matching formula, UV and EFT effective actions

- Requirement: determine EFT such that all of the low-energy full theory amplitudes are reproduced

$$\mathscr{W}_{EFT}[J_\varphi] = \mathscr{W}_{UV}[0, J_\varphi]$$

- Using quantum effective action, this requirement translates to

$$\Gamma_{EFT}[\hat{\varphi}] = \Gamma_{UV}[\hat{\Phi}[\hat{\varphi}], \hat{\varphi}], \quad 0 = \frac{\delta \Gamma_{UV}}{\delta \Phi_\alpha}[\hat{\Phi}[\hat{\varphi}], \hat{\varphi}]$$

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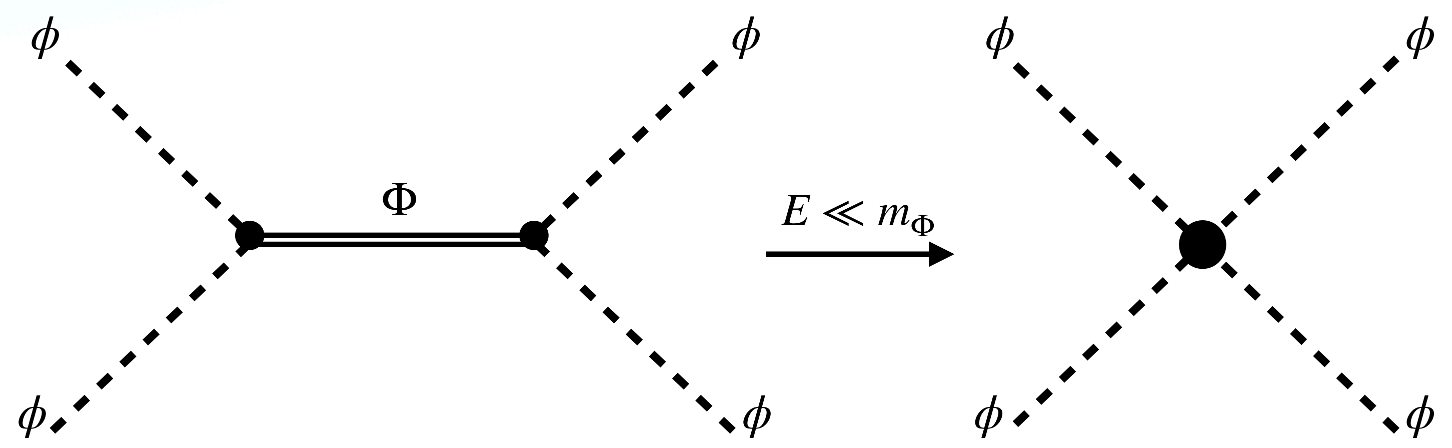
- Φ_α (unsourced) fields are no longer independent
  - E.g. tree-level matching $\mathscr{L}_{UV} \supset -\frac{1}{2} m_\Phi^2 \Phi^2 - \frac{\lambda_{\Phi\phi}}{2}\phi^2\Phi$



- Explicitly, UV and the EFT actions become

$$\Gamma^\phi[\hat{\phi}] = \hat{S} + \frac{i\hbar}{2}\text{sTr}\ln\hat{\mathscr{Q}} + \frac{i\hbar^2}{2}\hat{\mathscr{Q}}^{-1}_{IJ}\hat{\mathscr{Q}}^{(1)}_{IJ} - \frac{\hbar^2}{8}\hat{\mathscr{D}}_{IJKL}\hat{\mathscr{Q}}^{-1}_{IJ}\hat{\mathscr{Q}}^{-1}_{KL} + \frac{\hbar^2}{12}\hat{\mathscr{B}}_{IJK}\hat{\mathscr{B}}_{LMN}\hat{\mathscr{Q}}^{-1}_{IL}\hat{\mathscr{Q}}^{-1}_{JM}\hat{\mathscr{Q}}^{-1}_{KN} + \frac{\hbar^2}{8}\hat{\mathscr{Q}}^{-1}_{IJ}\hat{\mathscr{B}}_{IJ\alpha}\hat{\mathscr{Q}}^{-1}_{\alpha\beta}\hat{\mathscr{B}}_{\beta MN}\hat{\mathscr{Q}}^{-1}_{MN} + \mathscr{O}(\hbar^3),$$

$$\Gamma^{EFT}[\hat{\phi}] = \hat{S}_{EFT} + \frac{i\hbar}{2}\text{sTr}\ln\hat{\mathscr{Q}}^{EFT} + \frac{i\hbar^2}{2}(\hat{\mathscr{Q}}^{-1}_{ij})^{EFT}(\hat{\mathscr{Q}}^{(1)}_{ij})^{EFT} - \frac{\hbar^2}{8}\hat{\mathscr{D}}^{EFT}_{ijkl}(\hat{\mathscr{Q}}^{-1}_{ij})^{EFT}(\hat{\mathscr{Q}}^{-1}_{kl})^{EFT} + \frac{\hbar^2}{12}\hat{\mathscr{B}}^{EFT}_{ijk}\hat{\mathscr{B}}^{EFT}_{lmn}(\hat{\mathscr{Q}}^{-1}_{il})^{EFT}(\hat{\mathscr{Q}}^{-1}_{jm})^{EFT}(\hat{\mathscr{Q}}^{-1}_{kn})^{EFT} + \mathscr{O}(\hbar^3)$$

Matching conditions

- Applying the method of regions, the final two-loop matching conditions can be derived

[hep-ph/9711391], [1111.2589]

- Soft and hard momentum modes are explicitly distinguished
  - [Soft] $I, \alpha, i: \quad p_I, p_\alpha, p_i \ll \Lambda$
  - [Hard] $I, \alpha, i: \quad p_I, p_\alpha, p_i \gtrsim \Lambda$
- Decomposition into a sum of hard and soft regions is

$$A^{[1]}_{I_1 I_2 J_1 \ldots} A^{[2]}_{I_2 I_3 \ldots} \ldots A^{[n]}_{I_n I_1 \ldots} = A^{[1]}_{I_1 I_2 J_1 \ldots} A^{[2]}_{I_2 I_3 \ldots} \ldots A^{[n]}_{I_n I_1 \ldots} + A^{[1]}_{I_1 I_2 J_1 \ldots} A^{[2]}_{I_2 I_3 \ldots} \ldots A^{[n]}_{I_n I_1 \ldots}$$

- Two-loop matching condition becomes

$$S^{(2)}_{EFT}[\hat{\phi}] = S^{(2)s} + \frac{i}{2} \mathcal{Q}^{-1s}_{IJ} \mathcal{Q}^{(1)s}_{JI} - \frac{1}{8} \mathcal{D}^{s}_{IJKL} \mathcal{Q}^{-1s}_{IJ} \mathcal{Q}^{-1s}_{KL} + \frac{1}{12} \mathcal{B}^{s}_{IJK} \mathcal{Q}^{-1s}_{IL} \mathcal{Q}^{-1s}_{JM} \mathcal{Q}^{-1s}_{KN} B^{s}_{LMN} + \frac{1}{8} \mathcal{Q}^{-1s}_{IJ} B^{s}_{IJ\alpha} \mathcal{Q}^{-1s}_{\alpha\beta} B^{s}_{\beta MN} \mathcal{Q}^{-1s}_{MN}$$

Outlook

Future directions

- Inclusion of the fermions into the functional approach (mixed-statistics)
- Inclusion of the covariant derivatives into the formalism
- Matching conditions and the all-orders matching formula
- Applications:
  - SMEFT RG
  - LEFT RG
  - Automation
- Many ideas and directions for interesting upcoming work - stay tuned!