One-Loop Corrections as Shifts to Fierz Identities*

05.09.2023, Mainz

Marko Pesut
University of Zürich

*in collaboration with Jason Aebischer & Zach Polonsky

Universität Zürich[UZH]

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Travelling Beyond 4-Dimensions**

10:00	**TBD**	*Marko Pesut*
	2413/2-430 - MITP Seminar Room, MITP - Mainz Institute for Theoretical Physics, Johannes Gutenberg University Mainz	
	10:00 - 11:00	

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**Thanks Ajdin ☺

Outline of the talk

- Evanescent operators : definitions, prescription & scheme

- The shift paradigm:

  - *One-Loop Fierz Identities* ([2208.10513 : J.Aebischer & M.P]),

  - *Dipole Operators in Fierz Identities* ([2211.01379 : J.Aebischer, M.P, Z.Polonsky])

- Shift paradigm in the context of change of bases of NLO ADMs:

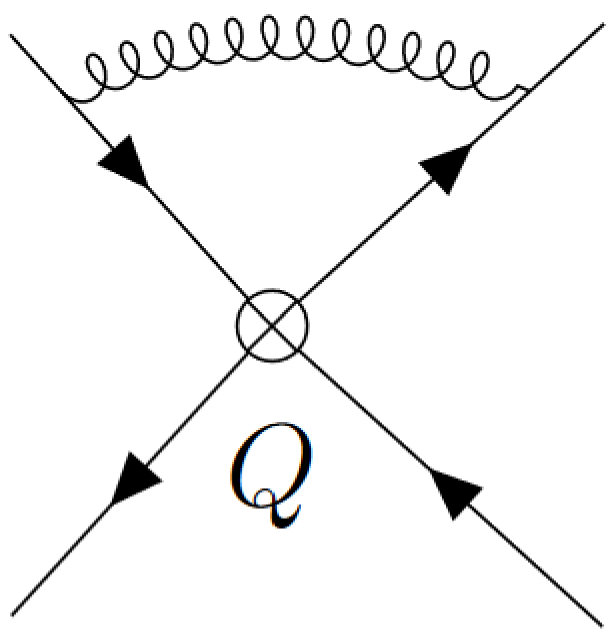
  - *Renormalization Scheme Factorization of one-loop Fierz Identities* ([2306.16449 : J.Aebischer, M.P, Z.Polonsky])

Plenty of Literature on ev. ops. / scheme / γ5 / LO and NLO ADMS & operator bases:

[Buras & Weisz (1990)], [Buras, Misiak, Urban (2000)], [Jenkins, Manohar, Stoffer (2018)], [Dugan & Grinstein (1991)], [Herrlich & Nierste (1995)], [Aebischer, Bobeth, Buras, Kumar (2020-2021)], [Bélusca-Maïto, Ilakovac, Mađor-Božinović, Stöckinger (2020)], ['t Hooft and Veltman (1972)], [Buras & Girrbach (2012)], [Chetyrkin, Misiak, Munz (1998)], [Dekens & Stoffer (2019)], [Grzadkowski, Iskrzynski, Misiak (2010)]...

General Context

Consider a set of physical operators $\{Q\}$, and compute one-loop matrix elements $\langle Q \rangle^{(1)}$:



$\longrightarrow$ Dirac Structure $\{\mathcal{E}\}$. In d=4, they map back to the physical basis :

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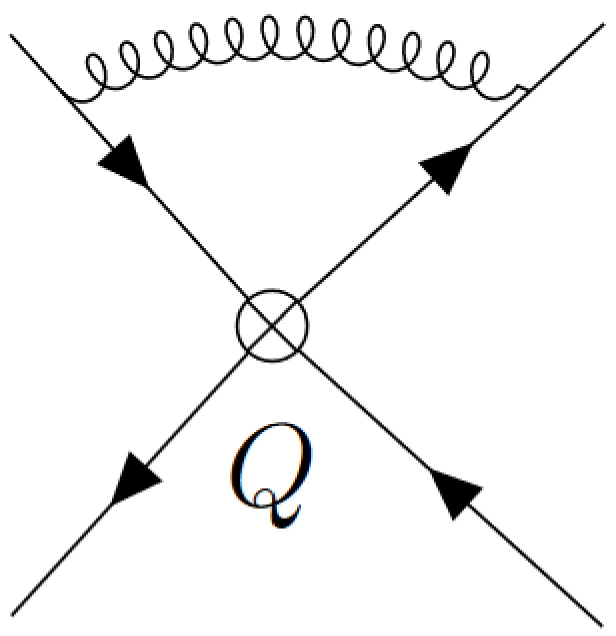

$\longrightarrow$ Dirac Structure $\{\mathcal{E}\}$. In d=4, they map back to the physical basis :

$$\mathcal{E} \stackrel{d=4}{=} \mathcal{F}_4 Q$$

➡ In dim. reg (d=4-2ε), no unambiguous way to continue Dirac Algebra !

[Talks by Stoffer, Stöckinger & Wilsch]

Evanescent operators, Prescription and Scheme dependence

To account for this, we must :

- specify a *prescription* (i.e. how to *treat* Dirac Algebra: NDR, HV, Larin,…),

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[Z.Polonsky’s talk]

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$$E = \mathcal{E} - \mathcal{F}Q \qquad \mathcal{F} = \mathcal{F}_4 + \sum_{n=1}^{\infty} \epsilon^n \sigma_n$$

The d=4 part is fixed.

Constants fix the *scheme dependence*.

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The d=4 part is fixed.

$$\Rightarrow \quad E\,\Big|_{d=4} = 0$$

$$\Rightarrow \quad \langle E \rangle^{(1)} \neq 0$$

Constants fix the *scheme dependence*.

Finite pieces to be taken into account

One-Loop Fierz Identities [2208.10513]

- Evanescent operators : definitions, prescription & scheme ✓

- The shift paradigm :

  - ***One-Loop Fierz Identities*** **([2208.10513 : J.Aebischer & M.P])***

Interpret the finite contributions of the Fierz-Ev. Ops.
as one-loop corrections (*shifts*) to Fierz identities.

*Four-fermion operators in the LEFT

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$$\vec{\tilde{Q}} = \mathcal{F}_4 \vec{Q} + \vec{E} \quad \Rightarrow \quad \vec{\tilde{Q}} = \left(\mathcal{F}_4 + \frac{\alpha}{4\pi} R_1 \right) \vec{Q}$$

One-Loop Fierz Identities [2208.10513]

- Evanescent operators : definitions, prescription & scheme ✓

- The shift paradigm :

  - ***One-Loop Fierz Identities*** (**[2208.10513 : J.Aebischer & M.P]**)

Interpret the finite contributions of the Fierz-Ev. Ops. as one-loop corrections (*shifts*) to Fierz identities.

→ Only relates physical operators of the two bases !

- NLO Matching
- Change of bases for ADMs

One-Loop Fierz Identities [2208.10513]

Fierz Identities* are relations, valid in d=4, between four-fermion operators :

$$\mathcal{F}_4 \circ \left(\bar{f}_1 \gamma_\mu P_X f_2\right) \left(\bar{f}_3 \gamma^\mu P_X f_4\right) \equiv \left(\bar{f}_1 \gamma_\mu P_X f_4\right) \left(\bar{f}_3 \gamma^\mu P_X f_2\right)$$

*Fierz (1939)

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$$\vdots$$

$$\mathcal{O} - \mathcal{F}_4 \circ \mathcal{O} = 0 \quad (d = 4)$$

One-Loop Fierz Identities [2208.10513]

These relations do not hold at the loop-level and lead to the introduction of Evanescent Operators.

$$\langle \mathcal{O} \rangle^{(1)} - \langle \mathcal{F}_4 \circ \mathcal{O} \rangle^{(1)} = \langle E \rangle^{(1)} \quad (d = 4 - 2\epsilon)$$

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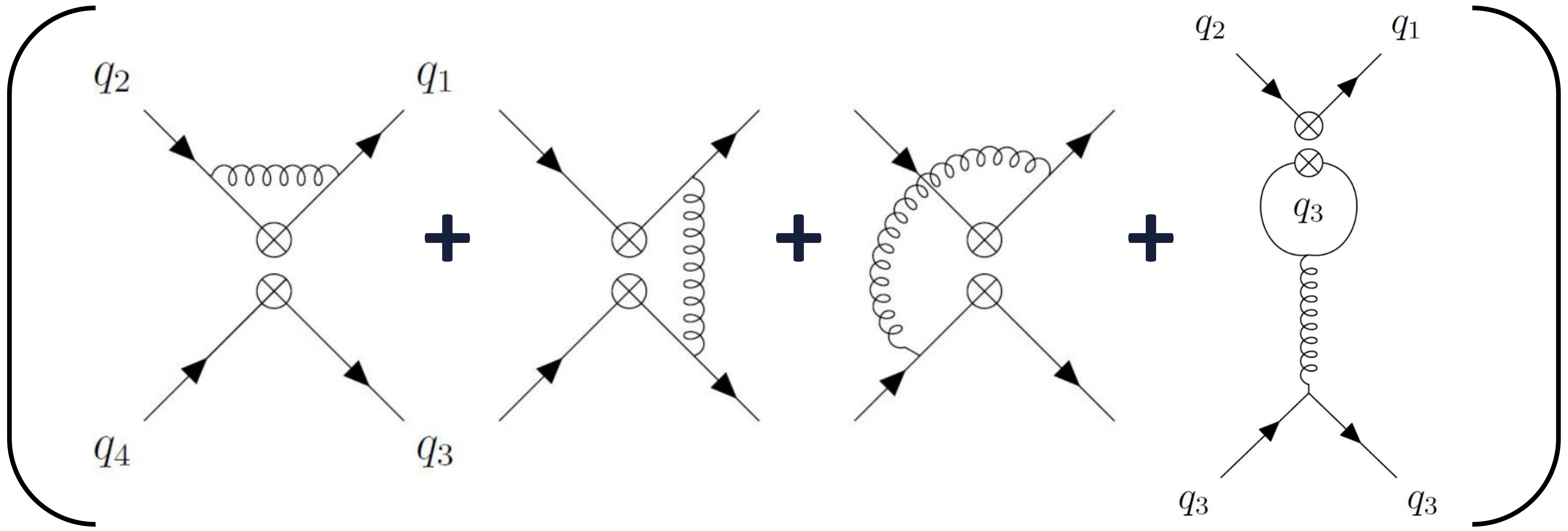
(QCD)



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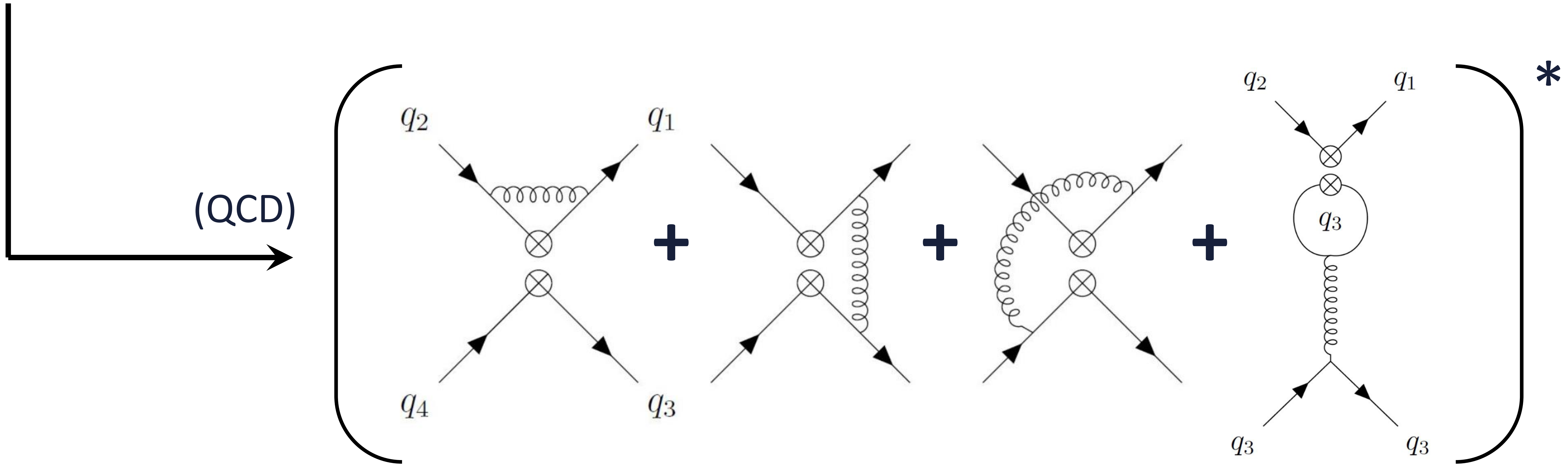


At one-loop, **finite, local, scheme-dependent** shifts are generated !

*Only UV poles need to be computed

One-Loop Fierz Identities [2208.10513]

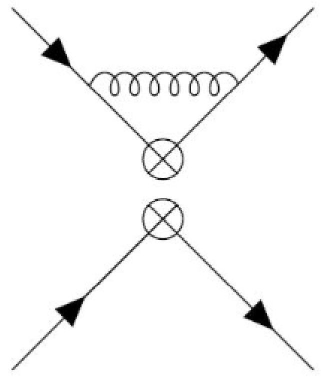
One-loop QED and QCD shifts to four-fermion ops. in the (generalized) BMU scheme* :

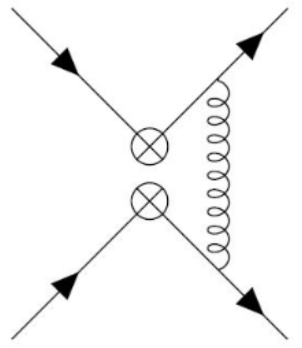
$$\mathcal{O} = \left(\bar{f}_1 \Gamma_A f_2\right)\left(\bar{f}_3 \Gamma_B f_4\right) \text{ with } f_i = \{q, \ell\} \text{ and } \Gamma_X = \{P_X,\, \gamma^\mu P_X,\, \sigma^{\mu\nu} P_X\} \quad X = L \text{ or } R$$

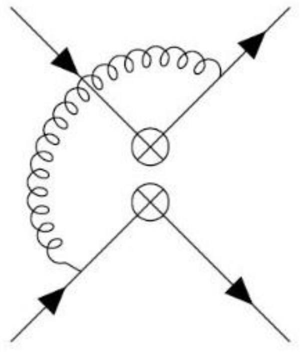
*The **Greek Method** was used to reduce the Dirac Algebra in D-dim :
Tracas & Vlachos (1982),
Buras, Misiak, Urban (2000)

One-Loop Fierz Identities [2208.10513]

Greek Identities in the (generalized) BMU scheme* :







VLR

$$\gamma_\alpha\gamma_\beta\gamma_\mu\,(1\pm\gamma_5)\,\gamma^\beta\gamma^\alpha\otimes\gamma^\mu\,(1\mp\gamma_5)=4(1-2b_1\epsilon)\gamma_\mu\,(1\pm\gamma_5)\otimes\gamma^\mu\,(1\mp\gamma_5)\;, \tag{A.4}$$

$$\gamma_\mu\,(1\pm\gamma_5)\,\gamma_\alpha\gamma_\beta\otimes\gamma^\mu\,(1\mp\gamma_5)\,\gamma^\alpha\gamma^\beta=4(1+b_2\epsilon)\gamma_\mu\,(1\pm\gamma_5)\otimes\gamma^\mu\,(1\mp\gamma_5)\;, \tag{A.5}$$

$$\gamma_\mu\,(1\pm\gamma_5)\,\gamma_\alpha\gamma_\beta\otimes\gamma^\beta\gamma^\alpha\gamma^\mu\,(1\mp\gamma_5)=16(1-b_3\epsilon)\gamma_\mu\,(1\pm\gamma_5)\otimes\gamma_\mu\,(1\mp\gamma_5)\;. \tag{A.6}$$

SLR

$$\gamma_\nu\gamma_\mu\,(1\mp\gamma_5)\,\gamma^\mu\gamma^\nu\otimes(1\pm\gamma_5)=16(1-c_1\epsilon)\,(1\mp\gamma_5)\otimes(1\pm\gamma_5)\;, \tag{A.7}$$

$$(1\mp\gamma_5)\,\gamma_\mu\gamma_\nu\otimes(1\pm\gamma_5)\,\gamma^\mu\gamma^\nu=4(1+c_2\epsilon)\,(1\mp\gamma_5)\otimes(1\pm\gamma_5)\;, \tag{A.8}$$

$$(1\mp\gamma_5)\,\gamma_\nu\gamma_\mu\otimes\gamma^\mu\gamma^\nu\,(1\pm\gamma_5)=4(1-2c_3\epsilon)\,(1\mp\gamma_5)\otimes(1\pm\gamma_5)\;. \tag{A.9}$$

SLL

$$\gamma_\nu\gamma_\mu\,(1\pm\gamma_5)\,\gamma^\mu\gamma^\nu\otimes(1\pm\gamma_5)=16(1-d_1\epsilon)\,(1\pm\gamma_5)\otimes(1\pm\gamma_5)\;, \tag{A.10}$$

$$\begin{aligned}&(1\pm\gamma_5)\,\gamma_\mu\gamma_\nu\otimes(1\pm\gamma_5)\,\gamma^\mu\gamma^\nu\\&=(4-2d_2\epsilon)\,(1\pm\gamma_5)\otimes(1\pm\gamma_5)-\sigma_{\mu\nu}\,(1\pm\gamma_5)\otimes\sigma^{\mu\nu}\,(1\pm\gamma_5)\;,\end{aligned} \tag{A.11}$$

$$\begin{aligned}&(1\pm\gamma_5)\,\gamma_\mu\gamma_\nu\otimes\gamma^\nu\gamma^\mu\,(1\pm\gamma_5)\\&=(4-2d_3\epsilon)\,(1\pm\gamma_5)\otimes(1\pm\gamma_5)+\sigma_{\mu\nu}\,(1\pm\gamma_5)\otimes\sigma^{\mu\nu}\,(1\pm\gamma_5)\;.\end{aligned} \tag{A.12}$$

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Operator	Tree-level Fierz
$S^{LL}_{q_1 q_2 q_3 q_4}$	$-\frac{1}{2}\widetilde{S}^{LL}_{q_1 q_4 q_3 q_2} - \frac{1}{8}\widetilde{T}^{LL}_{q_1 q_4 q_3 q_2}$

$$\tilde{S}^{AB}_{f_1 f_2 f_3 f_4} \equiv \left(\bar{f}^\alpha_1 P_A f^\beta_2\right)\left(\bar{f}^\beta_3 P_B f^\alpha_4\right)$$

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Operator	Tree-level Fierz	QCD shift	QED shift
$S^{LL}_{q_1q_2q_3q_4}$	$-\frac{1}{2}\widetilde{S}^{LL}_{q_1q_4q_3q_2} - \frac{1}{8}\widetilde{T}^{LL}_{q_1q_4q_3q_2}$	$-\frac{1}{N_c} S^{LL}_{q_1q_2q_3q_4} + \widetilde{S}^{LL}_{q_1q_2q_3q_4} + \frac{N_c^2-6}{8N_c} T^{LL}_{q_1q_2q_3q_4} + \frac{5}{8}\widetilde{T}^{LL}_{q_1q_2q_3q_4}$	$\frac{1}{2}(Q_1+Q_2)(Q_3+Q_4) S^{LL}_{q_1q_2q_3q_4} + \frac{1}{8}\left(Q_{1234} + 2Q_{1423} + 3Q_{1324}\right) T^{LL}_{q_1q_2q_3q_4}$

(QCD)

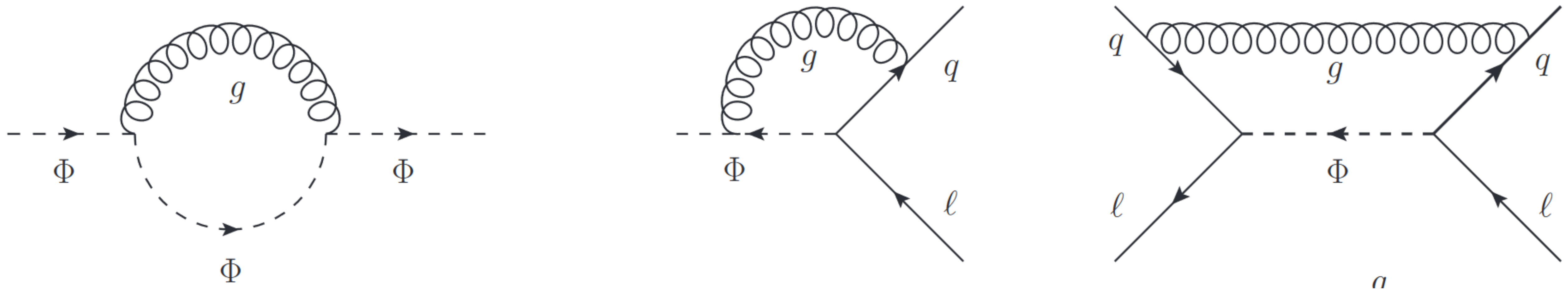
$$\boxed{\left\langle S^{LL}_{q_1q_2q_3q_4}\right\rangle^{(1)} - \left\langle \mathcal{F}_4 \circ S^{LL}_{q_1q_2q_3q_4}\right\rangle^{(1)}}$$

(QED)

One-Loop Fierz Identities [2208.10513]

Example: one-loop matching*

$$L_{q\ell}^{LQ} = \bar{q}\left(\Gamma_L^S P_L + \Gamma_R^S P_R\right)\ell\Phi^* + \text{h.c.}$$



$$\mathcal{L}\big|_{q\ell\ell q} \supset \widetilde{C}_S^{LL}\left(\bar{q}P_L\ell\right)\left(\bar{\ell}P_L q\right) + \widetilde{C}_T^{LL}\left(\bar{q}\sigma_{\mu\nu}P_L\ell\right)\left(\bar{\ell}\sigma^{\mu\nu}P_L q\right)$$

*Aebischer, Crivellin, Greub (2019)

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The two bases are related by Fierz Ids !

*Aebischer, Crivellin, Greub (2019)

One-Loop Fierz Identities [2208.10513]

Example: one-loop matching*

$$\mathcal{L}\big|_{q\ell\ell q} \supset \widetilde{C}_S^{LL}\,(\bar{q}P_L\ell)\,(\bar{\ell}P_Lq) + \widetilde{C}_T^{LL}\,(\bar{q}\sigma_{\mu\nu}P_L\ell)\,(\bar{\ell}\sigma^{\mu\nu}P_Lq)$$

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The two bases are related by Fierz Ids !

Operator	Tree-level Fierz	QCD shift
$T^{LL}_{q_1q_2\ell_1\ell_2}$	$-6S^{LL}_{q_1\ell_2\ell_1q_2} + \frac{1}{2}T^{LL}_{q_1\ell_2\ell_1q_2}$	$\frac{7-7N_c^2}{N_c}S^{LL}_{q_1q_2\ell_1\ell_2}$
$S^{LL}_{q_1q_2\ell_1\ell_2}$	$-\frac{1}{2}S^{LL}_{q_1\ell_2\ell_1q_2} - \frac{1}{8}T^{LL}_{q_1\ell_2\ell_1q_2}$	$\frac{N_c^2-1}{16N_c}T^{LL}_{q_1q_2\ell_1\ell_2}$

$$\begin{pmatrix} C_S^{LL} \\ C_T^{LL} \end{pmatrix} = \left[R_0 + \frac{\alpha_s}{4\pi}R_1R_0\right]^{-T} \begin{pmatrix} \widetilde{C}_S^{LL} \\ \widetilde{C}_T^{LL} \end{pmatrix}$$

$\mathcal{F}_4$

One-Loop Fierz Identities [2208.10513]

Example: one-loop matching*

$$\mathcal{L}\big|_{q\ell\ell q} \supset \widetilde{C}_S^{LL}\,(\bar{q}P_L\ell)\left(\bar{\ell}P_Lq\right) + \widetilde{C}_T^{LL}\,(\bar{q}\sigma_{\mu\nu}P_L\ell)\left(\bar{\ell}\sigma^{\mu\nu}P_Lq\right)$$

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$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{8} \\ -6 & \frac{1}{2} \end{pmatrix} \qquad \begin{pmatrix} 0 & \frac{N_c^2-1}{16N_c} \\ \frac{7-7N_c^2}{N_c} & 0 \end{pmatrix}$$

$$\begin{pmatrix} C_S^{LL} \\ C_T^{LL} \end{pmatrix} = \left[\underbrace{R_0}_{\downarrow} + \frac{\alpha_s}{4\pi}\,R_1 R_0\right]^{-T} \begin{pmatrix} \widetilde{C}_S^{LL} \\ \widetilde{C}_T^{LL} \end{pmatrix}$$

➡ We only related physical operators !

One-Loop Fierz Identities [2208.10513]

Another Example*: basis change

$$\vec{\tilde{Q}} = R_0(\vec{Q} + W\vec{E})$$

Gorbahn & al.

BMU**

*Gorbahn, Jäger, Nierste, Trine (2009)
**Buras, Misiak, Urban (2000)

One-Loop Fierz Identities [2208.10513]

Another Example*: basis change

$$\vec{\tilde{Q}} = R_0(\vec{Q} + W\vec{E})$$

[Z.Polonsky's talk]

Change of bases formula for LO and NLO ADMs :

$$Z^{(1,0)}_{\tilde{Q}\tilde{Q}} = R_0 \left[W Z^{(1,0)}_{EQ} \right] R_0^{-1}$$

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[Z^{(1,0)}_{\tilde{Q}\tilde{Q}}, \tilde{\gamma}^{(0)} \right] - 2\beta^{(0)} Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$$

These are our shifts :

$$\boxed{R_1 = -R_0 W Z^{(1,0)}_{EQ}}$$

[Explicit example in Pol's talk]

(Not the full story…more on this later)

*Gorbahn, Jäger, Nierste, Trine (2009)

Dipole Operators in Fierz Identities [2211.01379]

- ➢ Evanescent operators : definitions, prescription & scheme ✓

- ➢ The shift paradigm :

  - *One-Loop Fierz Identities* ([2208.10513 : J.Aebischer & M.P]) ✓

  - ***Dipole Operators in Fierz Identities*** (**[2211.01379 : J.Aebischer, M.P, Z.Polonsky]**)*

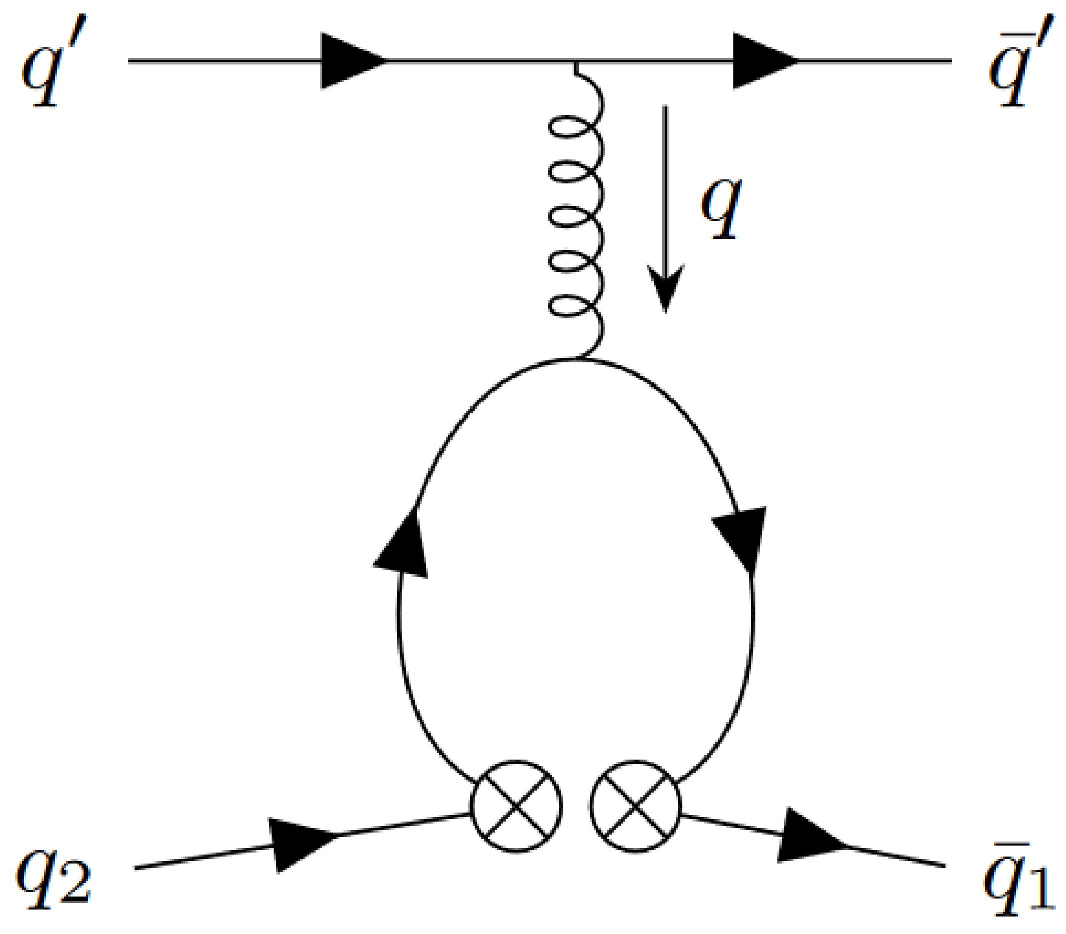
We reported the *shifts* of four-fermion operators that map onto Dipole operators (QED + QCD).

[F.Wilsch's talk]

*See [Fuentes-Martín, König, Pagès, Thomsen, Wilsch (2022)] for dipoles in the SMEFT

Dipole Operators in Fierz Identities [2211.01379]

- Only Penguin contributions to four-fermion operators



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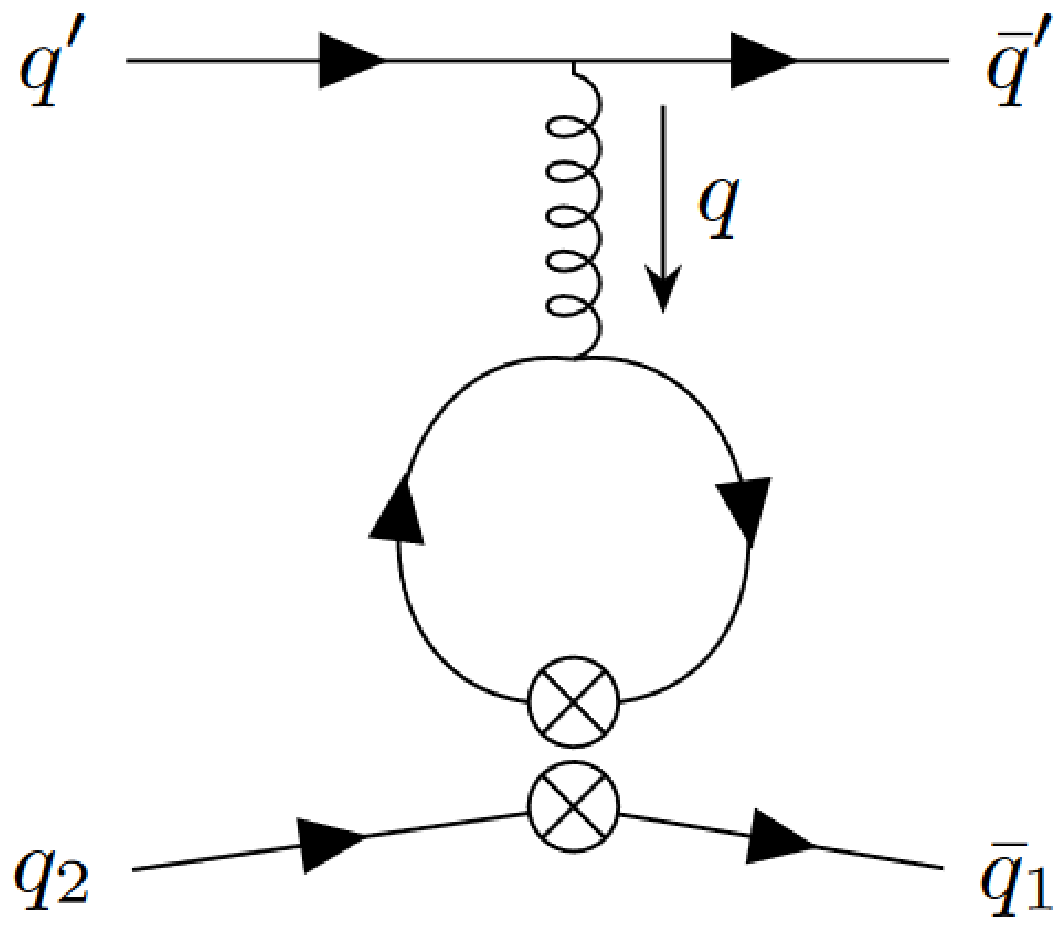

closed

Dipole Operators in Fierz Identities [2211.01379]

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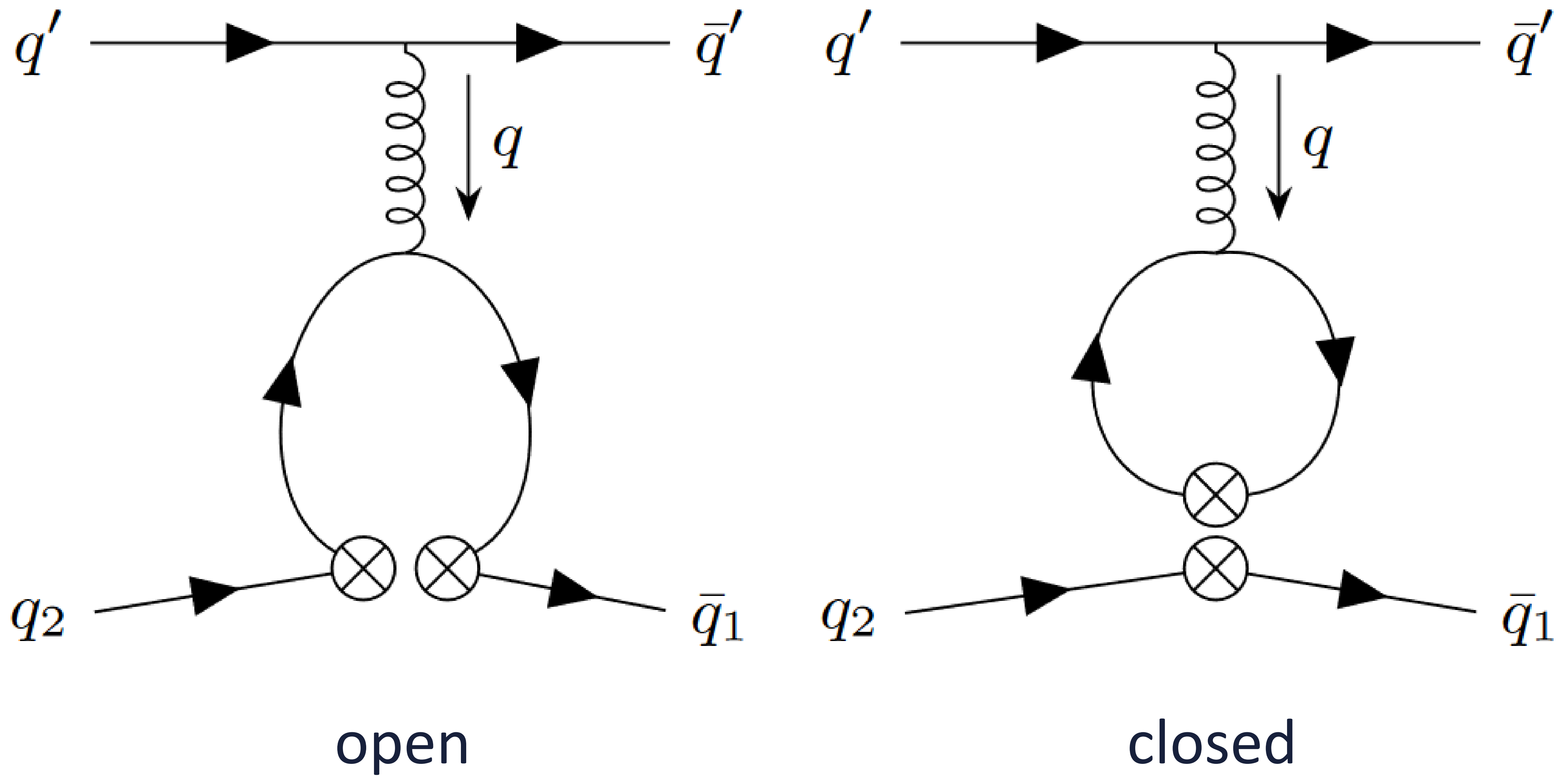
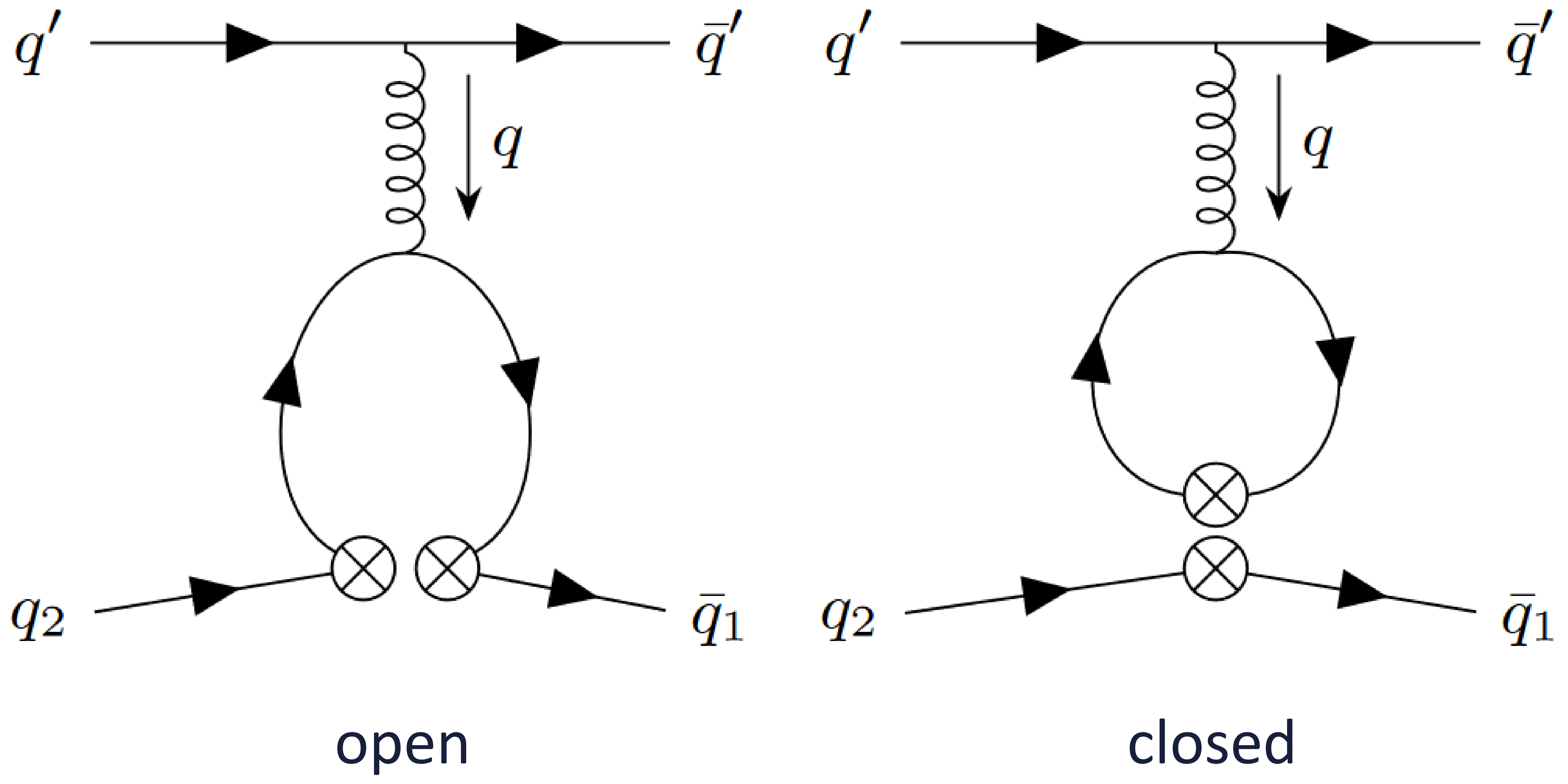


- One complication* : $\mathrm{Tr}\left[\gamma_\mu\gamma_\nu\gamma_\sigma\gamma_\rho\gamma_5\right] \longrightarrow$ inconsistent in NDR !

*Only closed penguins with tensors insertions

Dipole Operators in Fierz Identities [2211.01379]

- Only Penguin contributions to four-fermion operators



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- Prescription : **modified** Naive Dimensional Regularization (NDR)**

*Only closed penguins with tensors insertions
**Misiak (1993)
**Chetyrkin, Zoller (2012)
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Dipole Operators in Fierz Identities [2211.01379]

- Only Penguin contributions to four-fermion operators

- Prescription : **modified** Naive Dimensional Regularization (NDR)*

Operator	QCD shift	QED shift
$V^{LR}_{q_1 q_3 q_3 q_2}$	$\frac{m_{q_3}}{m_q}\mathcal{D}^R_{q_1 q_2 G}$	$A_{q_3}\mathcal{D}^R_{q_1 q_2 \gamma}$

$$V^{AB}_{f_1 f_2 f_3 f_4} \equiv (\bar{f}_1^{\alpha}\gamma^{\mu}P_A f_2^{\alpha})\,(\bar{f}_3^{\beta}\gamma_{\mu}P_B f_4^{\beta})$$

$$A_{f'} \equiv \frac{m_{f'}}{m_f}Q_{f'}$$

$$D^B_{q_1 q_2 G} = \frac{1}{g_s}m_q(\bar{q}_1\sigma^{\mu\nu}P_B T^A q_2)G^A_{\mu\nu}$$

$$D^B_{f_1 f_2 \gamma} = \frac{1}{e}m_f(\bar{f}_1\sigma^{\mu\nu}P_B f_2)F_{\mu\nu}$$

*Misiak (1993)
*Chetyrkin, Zoller (2012)
*Mihaila, Salomon, Steinhauser (2012)

Dipole Operators in Fierz Identities [2211.01379]

Example* : One-loop contributions to the muon magnetic moment in the LEFT

$$a_\ell = \frac{\alpha \mathsf{q}_e^2}{2\pi} - 4\frac{m_\ell}{e\mathsf{q}_e}\mathrm{Re}\, L_{\substack{e\gamma\\ \ell\ell}}(\mu)\left\{1 - \frac{\alpha \mathsf{q}_e^2}{4\pi}\left[2 + 5\log\left(\frac{\mu^2}{m_\ell^2}\right)\right]\right\} + a_\ell^{4\ell} + \boxed{a_\ell^{2\ell 2q}} + \mathcal{O}(L_{e\gamma}^2)$$

Semileptonic tensor contribution

$$O^{T,RR}_{\substack{eu\\ ijkl}} = (\overline{e}^i \sigma_{\mu\nu} P_R e^j)(\overline{u}^k \sigma^{\mu\nu} P_R u^l)$$

*Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer (2021),
*Dekens, Stoffer (2022)

Dipole Operators in Fierz Identities [2211.01379]

Example* : One-loop contributions to the muon magnetic moment in the LEFT

$$O^{T,RR}_{\substack{eu\\ijkl}} = (\overline{e}^i \sigma_{\mu\nu} P_R e^j)(\overline{u}^k \sigma^{\mu\nu} P_R u^l)$$

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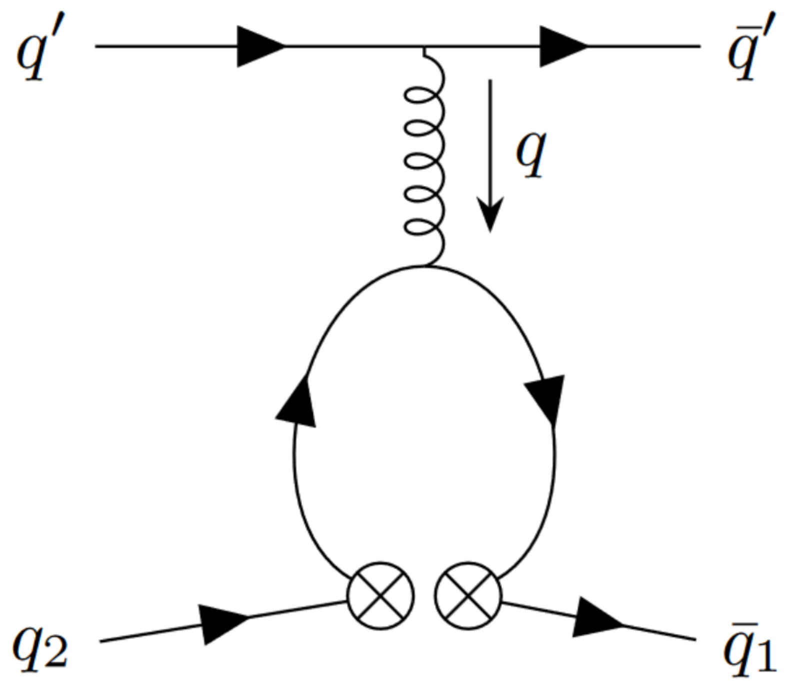
*Dekens, Stoffer (2022)

Example* : One-loop contributions to the muon magnetic moment in the LEFT

$$O^{T,RR}_{\substack{eu\\ ijkl}} = (\bar{e}^i \sigma_{\mu\nu} P_R e^j)(\bar{u}^k \sigma^{\mu\nu} P_R u^l) \xrightarrow{\mathcal{F}_4}$$



$$\cancel{\mathrm{Tr}\left[\gamma_\mu \gamma_\nu \gamma_\sigma \gamma_\rho \gamma_5\right]}$$

Use NDR + our shifts to recover the result

$$\boxed{\text{Illustration of how Fierz + shifts allow to go to a simpler basis to compute and convert the result back into the original basis}}$$

*Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer (2021),
*Dekens, Stoffer (2022)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

- Evanescent operators : definitions, prescription & scheme ✓

- The shift paradigm ✓

- Shift paradigm in the context of change of bases of NLO ADMs:

  - *Renormalization Scheme Factorization of one-loop Fierz Identities* ([2306.16449 : J.Aebischer, M.P, Z.Polonsky])

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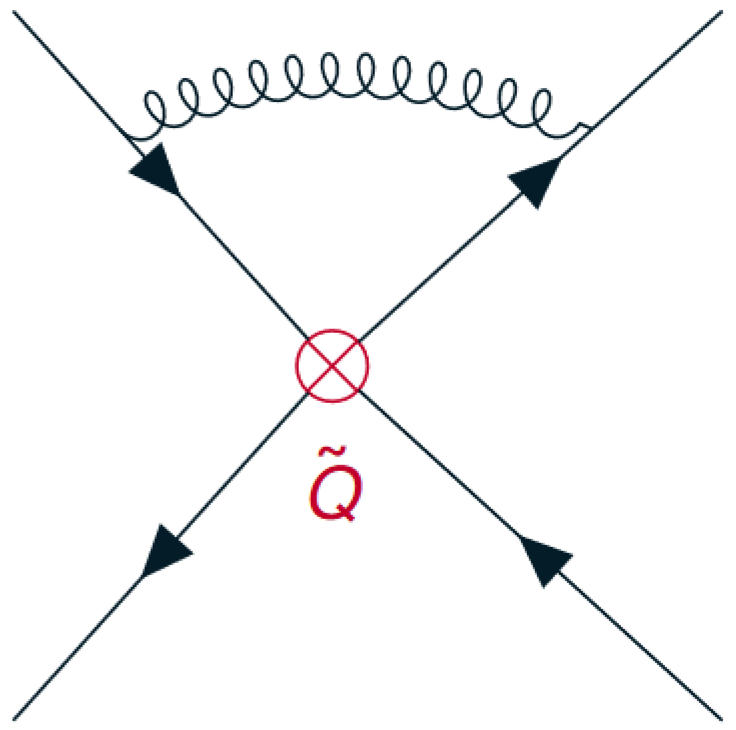
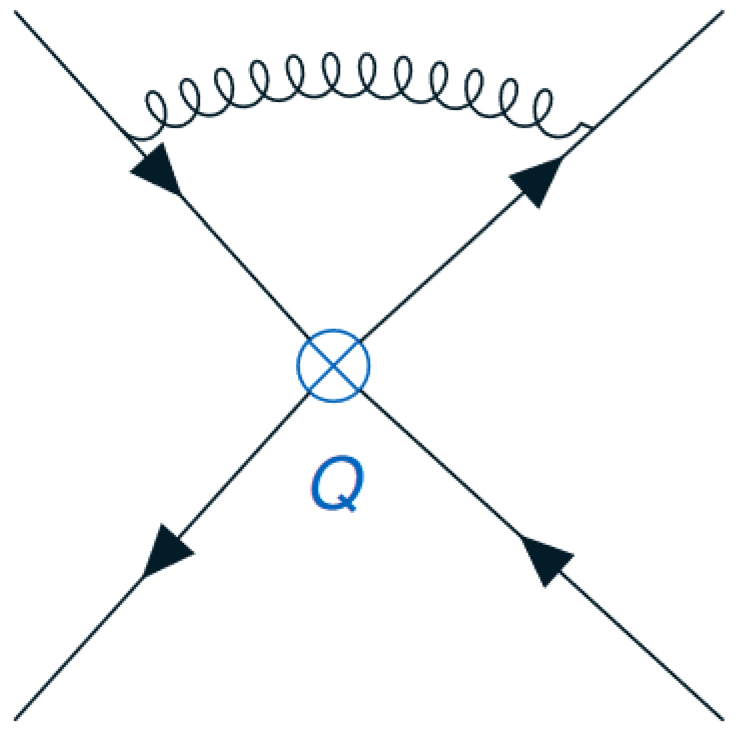
  - ***Renormalization Scheme Factorization of one-loop Fierz Identities*** (**[2306.16449 : J.Aebischer, M.P, Z.Polonsky]**)

- Basis $\tilde{Q}$, Scheme $\tilde{S}$
- Better suited for NLO matching



- Basis Q, Scheme S
- Easier to compute two-loop ADM



[Zach’s slides]

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

- Evanescent operators : definitions, prescription & scheme ✓

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- Shift paradigm in the context of change of bases of NLO ADMs:

  - ***Renormalization Scheme Factorization of one-loop Fierz Identities*** **([2306.16449 : J.Aebischer, M.P, Z.Polonsky])**

> - Shift « method » allows to simultaneously change basis and scheme in a simple way !
> - The double scheme-dependence appearing in the shifts factorizes.

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Relating two bases with different schemes :

$$\underbrace{\vec{\tilde{Q}}_{\tilde{\Sigma};\tilde{S}}}_{\text{Scheme \& Prescription}} = (R_0 + \Delta)\vec{Q}_{\Sigma;S}$$

Shifts (« Generalized » R_1)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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Shifts (« Generalized » R_1)

Scheme dependence of E

$$\Delta\langle\vec{Q}\rangle^{(0)} = P_{Q;\Sigma}\left(\langle\vec{\tilde{Q}}\rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}\right) - (R_0 + \epsilon\Sigma)\langle\vec{Q}\rangle^{(1)}_{\Sigma;S}$$

Projects the matrix elements on the Q-basis using the Σ-scheme

The double scheme-dependence appearing in the shifts factorizes

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Another example* : basis change

$$\vec{\tilde{Q}} = R_0(\vec{Q} + W\vec{E})$$

Change of bases formula for LO and NLO ADMs :

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

$$\tilde{\gamma}^{(1)} = R_0 \gamma^{(1)} R_0^{-1} - \left[Z^{(1,0)}_{\tilde{Q}\tilde{Q}}, \tilde{\gamma}^{(0)} \right] - 2\beta^{(0)} Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$$

$$Z^{(1,0)}_{\tilde{Q}\tilde{Q}} = R_0 \left[W Z^{(1,0)}_{EQ} \right] R_0^{-1}$$

These are our shifts :

$$\boxed{R_1 = -R_0 W Z^{(1,0)}_{EQ}}$$

*Gorbahn, Jäger, Nierste, Trine (2009)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Another example* : basis change

$$\cancel{\vec{\tilde{Q}} = R_0(\vec{Q} + W\vec{E})} \Rightarrow \vec{\tilde{Q}} = R_0(\vec{Q} + W\vec{E}) \quad \& \quad \vec{\tilde{E}} = M(\epsilon U\vec{Q} + (\mathbb{1} + \epsilon UW)\vec{E})$$

Change of bases formula for LO and NLO ADMs :

$$Z^{(1;0)}_{\tilde{Q}\tilde{Q}} = R_0 \left[W Z^{(1;0)}_{EQ} - \underbrace{\left(Z^{(1;1)}_{QE} + W Z^{(1;1)}_{EE} - Z^{(1;1)}_{QQ} W \right) U}_{\text{ev-to-ev}} \right] R_0^{-1}$$

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

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$\Sigma; S$

$\tilde{\Sigma}; \tilde{S}$

*Gorbahn, Jäger, Nierste, Trine (2009)
*Chetyrkin, Misiak, Münz (1998)
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Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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➡ Need to **relate** ev. ops. of the two bases (M and U matrices)

➡ Need to **compute** 1-loop matrix elements ev. ops.

*Gorbahn, Jäger, Nierste, Trine (2009)
*Chetyrkin, Misiak, Münz (1998)
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Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Instead : use shifts !

$$\tilde{\gamma}^{(0)} = R_0 \gamma^{(0)} R_0^{-1}$$

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Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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- No need to relate anything accross different bases,
- 1-loop matrix elements of physical operators only,
- the shift **factorises** the schemes : erases S - dependence & restores $\tilde{S}$ - dependence

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

In our paper, we checked :

➡ The full equivalence* with the « traditional » method a.k.a $Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$

*Very soon in V2 !

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

In our paper, we checked :

- The <u>full equivalence</u> with the « traditional » method a.k.a $Z^{(1,0)}_{\tilde{Q}\tilde{Q}}$
- The shift « method » in an explicit example* :

*Brod, Polonsky, and Stamou (2023)

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- The shift « method » in an explicit example* :

$$\mathcal{L}_{\text{Lar}} = -\sqrt{2}G_F\left(C_1^{eb}\mathcal{O}_1^{eb} + C_1^{be}\mathcal{O}_1^{be} + C_2\mathcal{O}_2 + C_3\mathcal{O}_3\right)$$

Larin's scheme [Larin, 1993]

$$\gamma_5 = \frac{i}{4!}\epsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

CP-odd operators: induce electron EDM.
(Explicit form in backup slides)

*Brod, Polonsky, and Stamou (2023)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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$$\Big\downarrow \mathcal{F}_4$$

$$\mathcal{L}_{\text{NDR}} = -\sqrt{2}G_F\left(\tilde{C}_s\tilde{\mathcal{O}}_s + \tilde{C}_v\tilde{\mathcal{O}}_v + \tilde{C}_t\tilde{\mathcal{O}}_t + \tilde{C}_3\tilde{\mathcal{O}}_3\right)$$

- Computed** LO and NLO ADMs + Shifts using NDR

*Brod, Polonsky, and Stamou (2023)

**Chetyrkin, Misiak, Munz (1998)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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$$\tilde{\gamma}^{(1)} = \tilde{\gamma}^{(1),\text{SI}} + a_s\tilde{\gamma}^{(1),s} + a_v\tilde{\gamma}^{(1),v} + a_t\tilde{\gamma}^{(1),t}$$

NDR scheme-dependence

- Computed** LO and NLO ADMs + Shifts using NDR

*Brod, Polonsky, and Stamou (2023)

**Chetyrkin, Misiak, Munz (1998)

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

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- Computed** LO and NLO ADMs + Shifts using NDR

Explicitely checked the ADMs and the scheme factorization via the shifts Δ (+ NDR is much simpler…)

*Brod, Polonsky, and Stamou (2023)
**Chetyrkin, Misiak, Munz (1998)

Conclusion

Interpreting the Fierz-evanescent contribution as shifts allows to express finite, ev. scheme-dependent contributions purely in terms of physical operators,

- *One-Loop Fierz Identities* ([2208.10513 : J.Aebischer & M.P]),

- *Dipole Operators in Fierz Identities* ([2211.01379 : J.Aebischer, M.P, Z.Polonsky])

The « shift paradigm » provides a useful and more transparent picture on how to relate different evanescent schemes and different bases of operators :

- 1-loop matrix elements of physical operators only + no need to relate ev.ops. across bases,
- Scheme factorization

- *Renormalization scheme factorization of one-loop Fierz identities* ([2306.16449 : J.Aebischer, M.P, Z.Polonsky])

Thank you for your attention !

Backup Slides

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

$$\mathcal{L}_{\rm Lar} = -\sqrt{2}G_F\Big(C_1^{eb}\mathcal{O}_1^{eb} + C_1^{be}\mathcal{O}_1^{be} + C_2\mathcal{O}_2 + C_3\mathcal{O}_3\Big)$$

$$\mathcal{O}_1^{ij} = \big(\bar\psi_i\psi_i\big)\big(\bar\psi_j\, i\gamma_5\,\psi_j\big), \quad \mathcal{O}_2 = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\big(\bar e\sigma_{\mu\nu}e\big)\big(\bar b\sigma_{\rho\sigma}b\big), \quad \mathcal{O}_3 = \frac{Q_e m_b}{2e}\big(\bar e\sigma^{\mu\nu}e\big)\tilde F_{\mu\nu}$$

$$\Bigg\downarrow \quad \mathcal{F}_{ij} = \begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{8} & 0 \\ 1 & -1 & 0 & 0 \\ -3 & -3 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\tilde{\mathcal{O}}_i = \Big[\mathcal{F}_{ij} + \sum_n \Big(\frac{\alpha_s}{4\pi}\Big)^n \Delta_{ij}^{(n)}\Big]\mathcal{O}_j$$

$$\mathcal{L}_{\rm NDR} = -\sqrt{2}G_F\Big(\tilde C_s\tilde{\mathcal{O}}_s + \tilde C_v\tilde{\mathcal{O}}_v + \tilde C_t\tilde{\mathcal{O}}_t + \tilde C_3\tilde{\mathcal{O}}_3\Big)$$

$$\tilde{\mathcal{O}}_s = \frac{1}{2}\Big[\big(\bar b i\gamma_5 e\big)\big(\bar e b\big) + \big(\bar b e\big)\big(\bar e i\gamma_5 b\big)\Big], \quad \tilde{\mathcal{O}}_v = \frac{1}{2}\Big[\big(\bar b i\gamma^\mu\gamma_5 e\big)\big(\bar e\gamma_\mu b\big) - \big(\bar b\gamma^\mu e\big)\big(\bar e i\gamma_\mu\gamma_5 b\big)\Big]$$

$$\tilde{\mathcal{O}}_t = \frac{1}{2}\Big[\big(\bar b i\sigma_{\mu\nu}\gamma_5 e\big)\big(\bar e\sigma^{\mu\nu}b\big) + \big(\bar b\sigma_{\mu\nu}e\big)\big(\bar e i\sigma^{\mu\nu}\gamma_5 b\big)\Big], \quad \tilde{\mathcal{O}}_3 = \frac{Q_e m_b}{2e}\big(\bar e i\sigma_{\mu\nu}\gamma_5 e\big)F^{\mu\nu}.$$

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

$$\mathcal{L}_{\rm NDR} = -\sqrt{2}G_F\Big(\tilde{C}_s\tilde{\mathcal{O}}_s + \tilde{C}_v\tilde{\mathcal{O}}_v + \tilde{C}_t\tilde{\mathcal{O}}_t + \tilde{C}_3\tilde{\mathcal{O}}_3\Big)$$

$$\tilde{\mathcal{O}}_s = \frac{1}{2}\Big[\big(\bar{b}i\gamma_5 e\big)\big(\bar{e}b\big) + \big(\bar{b}e\big)\big(\bar{e}i\gamma_5 b\big)\Big], \quad \tilde{\mathcal{O}}_v = \frac{1}{2}\Big[\big(\bar{b}i\gamma^\mu\gamma_5 e\big)\big(\bar{e}\gamma_\mu b\big) - \big(\bar{b}\gamma^\mu e\big)\big(\bar{e}i\gamma_\mu\gamma_5 b\big)\Big]$$

$$\tilde{\mathcal{O}}_t = \frac{1}{2}\Big[\big(\bar{b}i\sigma_{\mu\nu}\gamma_5 e\big)\big(\bar{e}\sigma^{\mu\nu}b\big) + \big(\bar{b}\sigma_{\mu\nu}e\big)\big(\bar{e}i\sigma^{\mu\nu}\gamma_5 b\big)\Big], \quad \tilde{\mathcal{O}}_3 = \frac{Q_e m_b}{2e}\big(\bar{e}i\sigma_{\mu\nu}\gamma_5 e\big)F^{\mu\nu}\,.$$

$$\tilde{\gamma}^{(1),\rm SI} = \begin{pmatrix} \frac{284}{9} & 0 & -\frac{275}{27} & 0 \\ 0 & -92 & 0 & 0 \\ 48 & 0 & -\frac{3340}{27} & 0 \\ 0 & 0 & 0 & \frac{1012}{9} \end{pmatrix}, \quad \tilde{\gamma}^{(1),s} = \begin{pmatrix} \frac{2}{9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{62}{9} & 0 & -\frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\Delta^{(1),\rm SI} = \begin{pmatrix} \frac{3}{2} & -\frac{7}{6} & \frac{1}{12} & 0 \\ -\frac{28}{3} & -\frac{4}{3} & 0 & 0 \\ \frac{110}{3} & \frac{14}{3} & -\frac{13}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Delta^{(1),s} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{12} & -\frac{1}{12} & -\frac{1}{24} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\tilde{\gamma}^{(1),v} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{46}{9} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{\gamma}^{(1),t} = \begin{pmatrix} 0 & 0 & -\frac{2}{9} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{32}{3} & 0 & -\frac{46}{9} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\Delta^{(1),v} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \Delta^{(1),t} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -\frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

Relating two bases with different schemes :

$$\vec{\tilde{Q}}_{\underbrace{\tilde{\Sigma};\tilde{S}}_{\text{Scheme \& Prescription}}} = (R_0 + \Delta)\vec{Q}_{\Sigma;S}$$

Shifts (« Generalized » R_1)

$$P_{Q;\Sigma}\,\mathcal{M}\left[\langle\vec{\tilde{Q}}\rangle^{(0)}\right] = \mathcal{M}\left[(\mathcal{F}-\epsilon\Sigma)\langle\vec{Q}\rangle^{(0)}\right]$$

$$\vec{E} = K\left(\vec{\tilde{Q}} - (\mathcal{F}+\epsilon\Sigma)\vec{Q}\right)$$

$$\Delta\langle\vec{Q}\rangle^{(0)} = P_{Q;\Sigma}\left(\langle\vec{\tilde{Q}}\rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}\right) - (R_0+\epsilon\Sigma)\langle\vec{Q}\rangle^{(1)}_{\Sigma;S}$$

Projects the matrix elements on the Q-basis using the Σ-scheme

$$\tilde{\mathcal{O}}_i = \left[\mathcal{F}_{ij} + \sum_n \left(\frac{\alpha_s}{4\pi}\right)^n \Delta^{(n)}_{ij}\right]\mathcal{O}_j$$

Renormalization scheme factorization of one-loop Fierz identities [2306.16449]

- NDR: $(\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma d_L)(\bar{s}_L\gamma^\mu\gamma^\nu\gamma^\sigma d_L) - (16 - a\epsilon)Q_{S2}$ (Herrlich, Nierste, 1996)

- Larin:
$$[(\bar{e}\gamma_{[\mu}\gamma_{\nu]}\gamma^{[\rho}\gamma^{\sigma]}e)(\bar{q}\gamma^{[\mu}\gamma^{\nu]}\gamma^{[\tau}\gamma^{\zeta]}q) + (\bar{e}\gamma^{[\rho}\gamma^{\sigma]}\gamma_{[\mu}\gamma_{\nu]}e)(\bar{q}\gamma^{[\tau}\gamma^{\zeta]}\gamma^{[\mu}\gamma^{\nu]}q)]\epsilon_{\rho\sigma\tau\zeta}$$
$$-48(Q_1^{eq} + Q_1^{qe}) + 16Q_2^{eq}$$
(Brod, Stamou, ZP, 2023)

- HV: $(\bar{q}_p\hat{\gamma}^\mu\hat{\gamma}^\nu\tilde{\sigma}^{\lambda\sigma}q_p)(\bar{q}_r\hat{\gamma}_\mu\hat{\gamma}_\nu\sigma_{\lambda\sigma}q_r)$ (Bühler, Stoffer, 2023)

[Zach's slides]

$$\langle\vec{\tilde{Q}}\rangle^{(1)}_{\Sigma;S} = \left(r^{(1)}_{\tilde{Q}\tilde{Q}}\mathcal{F} + r^{(1)}_{\tilde{Q}Q} + \epsilon r^{(1)}_{\tilde{Q}\tilde{Q}}\Sigma\right)\langle\vec{Q}\rangle^{(0)},$$

$$\langle\vec{Q}\rangle^{(1)}_{\Sigma;S} = \left(r^{(1)}_{QQ} + r^{(1)}_{Q\tilde{Q}}\mathcal{F} + \epsilon r^{(1)}_{Q\tilde{Q}}\Sigma\right)\langle\vec{Q}\rangle^{(0)}.$$

$$P_{Q;\Sigma}\left(\langle\vec{\tilde{Q}}\rangle^{(1)}_{\tilde{\Sigma};\tilde{S}}\right) = \left(\tilde{r}^{(1)}_{\tilde{Q}\tilde{Q}}\mathcal{F} + \tilde{r}^{(1)}_{\tilde{Q}Q} + \epsilon\tilde{r}_{\tilde{Q}\tilde{Q}}\Sigma + \epsilon\tilde{r}^{(1)}_{\tilde{Q}Q}\mathcal{F}^{-1}\Sigma + \epsilon\tilde{r}^{(1)}_{\tilde{Q}Q}\tilde{\Sigma}\mathcal{F}\right)\langle\vec{Q}\rangle^{(0)}$$

$$r^{(1)}_{ij} = \sum_n \frac{1}{\epsilon^n} r^{(1;n)}_{ij}$$

1-loop amplitudes

$$\vec{E} = K\left(\vec{\tilde{Q}} - (\mathcal{F} + \epsilon\Sigma)\vec{Q}\right), \quad \text{or} \quad \vec{\tilde{E}} = \tilde{K}\left(\vec{Q} - (\mathcal{F}^{-1} + \epsilon\tilde{\Sigma})\vec{\tilde{Q}}\right),$$

$$R = \mathcal{F}, \quad W = \mathcal{F}^{-1}K^{-1}, \quad M = -\tilde{K}\mathcal{F}^{-1}K^{-1}, \quad U = K\Sigma + K\mathcal{F}\tilde{\Sigma}\mathcal{F}$$

$$Z^{(1;1)}_{QQ} = -r^{(1;1)}_{QQ} - r^{(1;1)}_{Q\tilde{Q}}\mathcal{F}, \quad Z^{(1;1)}_{QE} = -r^{(1;1)}_{Q\tilde{Q}}K^{-1},$$

$$Z^{(1;1)}_{EE} = -Kr^{(1;1)}_{\tilde{Q}\tilde{Q}}K^{-1} + K\mathcal{F}r_{Q\tilde{Q}}K^{-1},$$

$$Z^{(1;0)}_{EQ} = -Kr^{(1;0)}_{\tilde{Q}Q} - Kr^{(q;0)}_{\tilde{Q}\tilde{Q}}\mathcal{F} + K\mathcal{F}r^{(1;0)}_{QQ} + K\mathcal{F}r^{(1;0)}_{Q\tilde{Q}}\mathcal{F}$$
$$- Kr^{(1;1)}_{\tilde{Q}\tilde{Q}}\Sigma + K\mathcal{F}r^{(1;1)}_{Q\tilde{Q}}\Sigma + K\Sigma r^{(1;1;)}_{Q\tilde{Q}}\mathcal{F} + K\Sigma r^{(1;1)}_{QQ}$$