Janitor's Work on the Two-Loop Anomalous Dimensions in the LEFT

Collection and Transformation of All Results Found in the Literature

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Outline

Overview

Our main goals are:

- Collecting all known results for the two-loop (up to $\mathcal{O}(\alpha_s^2)$) anomalous dimensions in the LEFT, and transforming them into a common operator basis (Jenkins, Manohar, Stoffer).
- Using flavour *tricks* (or other properties of the theory) to extend the known results to as many unknown sectors as possible.
- Prepare a comprehensible list of all sectors that remain unknown, to provide a clear direction toward the completion of the full RGE of the LEFT in the near future.

Overview

$$\text{LEFT}\ (d \leq 6) \begin{cases} L_{\psi_i\psi_j\psi_k\psi_l} \begin{cases} \Delta F = 0,1,2 \begin{cases} \text{Four-Quark} \\ \text{Quark-Lepton} \end{cases} \\ \Delta B \neq 0 \\ \Delta L \neq 0 \begin{cases} \text{Quark-Lepton} \\ \text{Four-Lepton} \end{cases} \end{cases} \\ L_G, L_{\widetilde{G}} \\ L_{\psi\gamma}, L_{\psi G} \\ e, g_s, \theta_{\text{QED}}, \theta_{\text{QCD}} \\ M_u, M_d, M_e, M_\nu \end{cases}$$

The LEFT conserves flavour (QCD + QED), so the four-fermion sector consists on several independent blocks.

Overview

Four-Quark Flavour Sectors in the LEFT

Sector	Example	General Case	Flavors
$\Delta F = 2$	$(\bar{s}\Gamma_1 b)(\bar{s}\Gamma_2 b)$	$(\bar{q}_i\Gamma_1 q_j)(\bar{q}_i\Gamma_2 q_j)$	$ijij$
$\Delta F = 1.5$	$(\bar{s}\Gamma_1 b)(\bar{d}\Gamma_2 b)$	$(\bar{q}_i\Gamma_1 q_j)(\bar{q}_k\Gamma_2 q_j)$	$ijkj$
$\Delta F = 1$	$(\bar{s}\Gamma_1 b)(\bar{u}\Gamma_2 c)$	$(\bar{q}_i\Gamma_1 q_j)(\bar{q}_k\Gamma_2 q_l)$	$ijkl$
$\Delta F = 1^{\bar{q}q}$	$(\bar{s}\Gamma_1 b)(\bar{c}\Gamma_2 c)$	$(\bar{q}_i\Gamma_1 q_j)(\bar{q}_k\Gamma_2 q_k)$	$ijkk$
$\Delta F = 0$	$(\bar{s}\Gamma_1 s)(\bar{c}\Gamma_2 c)$	$(\bar{q}_i\Gamma_1 q_i)(\bar{q}_j\Gamma_2 q_j)$	$iijj$

Completing the four-fermion mega-sector of the LEFT, we also have semileptonic $\Delta F = 1$ and $\Delta F = 0$, and three-quark $\Delta B \neq 0$.

Outline

General Strategy

Let us recall a couple basic concepts...

Example: $b \to s$ transitions with $\Delta F = 2$

BMU		JMS	
Q_1^{VLL}	$= (\bar{s}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L b)$	$O_{dd}^{V,LL}$	$= (\bar{s}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L b)$
Q_1^{LR}	$= (\bar{s}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_R b)$	$O_{dd}^{V1,LR}$	$= (\bar{s}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_R b)$
Q_2^{LR}	$= (\bar{s}P_L b)(\bar{s}P_R b)$	$O_{dd}^{V8,LR}$	$= (\bar{s}\gamma_\mu P_L T^A b)(\bar{s}\gamma^\mu P_R T^A b)$
Q_1^{SRR}	$= (\bar{s}P_R b)(\bar{s}P_R b)$	$O_{dd}^{S1,RR}$	$= (\bar{s}P_R b)(\bar{s}P_R b)$
Q_2^{SRR}	$= (\bar{s}\sigma_{\mu\nu} P_R b)(\bar{s}\sigma^{\mu\nu} P_R b)$	$O_{dd}^{S8,RR}$	$= (\bar{s}P_R T^A b)(\bar{s}P_R T^A b)$

The transformation between these bases involves the usage of Fierz identities, which are only true in d $= 4$ space-time dimensions.

General Strategy

In the context of dimensional regularization ($\mathrm{d} = 4 - 2\epsilon$) Fierz identities instead define nonphysical evanescent operators, up to factors of $\mathcal{O}(\epsilon)$.

The regularization scheme (RS) is usually defined to cancel out evanescent amplitudes, such that everything remains physical.

However, the choice of an evanescent basis will therefore be part of the definition of the scheme, and have physical effects.

In our case, the evanescent basis corresponds to the one in [1], which is also the one in [2] for $a_{\mathrm{EV}}, b_{\mathrm{EV}}, c_{\mathrm{EV}}, \ldots = 1$ (*greek projections*).

[1] A.Buras, M.Misiak, J.Urban, Nucl.Phys. B586 (2000) 397-426

[2] W.Deckens, P.Stoffer, JHEP 1910 (2019) 197

General Strategy

The ADM defines the RG evolution of the Wilson coefficients,

$$\frac{d\vec{L}(\mu)}{d\log\mu} = -\vec{L}(\mu)\cdot\left(\frac{d}{d\log\mu}\hat{\mathcal{Z}}_L\right)\hat{\mathcal{Z}}_L{}^{-1} \equiv \vec{L}(\mu)\cdot\hat{\gamma} \tag{1}$$

$$\hat{\gamma} = \frac{\alpha_s}{4\pi}\hat{\gamma}^{(0)} + \left(\frac{\alpha_s}{4\pi}\right)^2\hat{\gamma}^{(1)} + \mathcal{O}(\alpha_s^3)\,. \tag{2}$$

A change of basis will have the effect one would naively expect...

$$\vec{O}' = \hat{R}\,\vec{O} \quad \longrightarrow \quad \begin{cases} \vec{L}' = \vec{L}\cdot\hat{R}^{-1} \\ \\ \hat{\mathcal{Z}}' = \hat{R}\hat{\mathcal{Z}}\hat{R}^{-1} \qquad (\hat{\gamma}' = \hat{R}\hat{\gamma}\hat{R}^{-1}) \end{cases} \tag{3}$$

...as long as no evanescent operators are involved in the transformation.

General Strategy

However, if the transformation mixes physical and evanescent operators, the picture becomes more complex.

For instance, if we look at the following transformation,

$$\vec{O}' = \hat{R}\left(\vec{O} + \hat{W}\vec{E}\right) \ ,$$

and then attempt to transform $\hat{\mathcal{Z}}$ in the naive way, we get a finite contribution to the physical sector of the renormalization matrix,

$$\delta\hat{\mathcal{Z}}'_{OO} = \hat{R}\hat{W}\hat{\mathcal{Z}}_{EO} \ . \tag{4}$$

This implies that in attempting to preserve simplicity in the transformation we have slipped out of the original scheme!

An analogous finite shift in the physical sector emerges for any type of transformation involving evanescent operators.

General Strategy

To correct the shift in the RS, one can use some scheme-independent expressions to relate the ADM under two different schemes,

$$\vec{L}_0 = \vec{L} \cdot \left[\hat{\mathbf{1}} + \left(\frac{\alpha_s}{4\pi} \right) \hat{r}^{(1)} + \left(\frac{\alpha_s}{4\pi} \right)^2 \hat{r}^{(2)} + \mathcal{O}(\alpha_s^3) \right] , \tag{5}$$

$$\hat{\mathcal{Z}}_0 = \left[\hat{\mathbf{1}} - \left(\frac{\alpha_s}{4\pi} \right) \hat{r}^{(1)} - \left(\frac{\alpha_s}{4\pi} \right)^2 \left\{ \hat{r}^{(2)} - (\hat{r}^{(1)})^2 \right\} + \mathcal{O}(\alpha_s^3) \right] \hat{\mathcal{Z}} . \tag{6}$$

- $\hat{r}^{(k)}$ are the k-loop finite amplitude matrices renormalized by $\hat{\mathcal{Z}}$, under some particular renormalization scheme.
- $\vec{L}_0$ and $\hat{\mathcal{Z}}_0$ correspond to a renormalization scheme by which all $\hat{r}^{(k)} = 0$.

General Strategy

Putting everything together, we obtain the expressions for the full change of basis* (up to NLO) that preserves the RS,

$$\hat{\gamma}_A^{(0)} = \hat{R}\,\hat{\gamma}_B^{(0)}\,\hat{R}^{-1}\,, \tag{7}$$

$$\hat{\gamma}_A^{(1)} = \hat{R}\,\hat{\gamma}_B^{(1)}\,\hat{R}^{-1} - 2\beta_0\Delta\hat{r} - [\Delta\hat{r}, \hat{\gamma}_A^{(0)}]\,. \tag{8}$$

$$\left(\Delta\hat{r} = \hat{R}\,\hat{r}_A^{(1)}\,\hat{R}^{-1} - \hat{r}_B^{(1)}\right) \tag{9}$$

*There are alternative methods to perform the ADM transformations beyond LO, e.g. the one presented in Marko's talk.

General Strategy

Using the previous expressions, we have collected the four-fermion ADMs corresponding to $b \to s$ transitions with

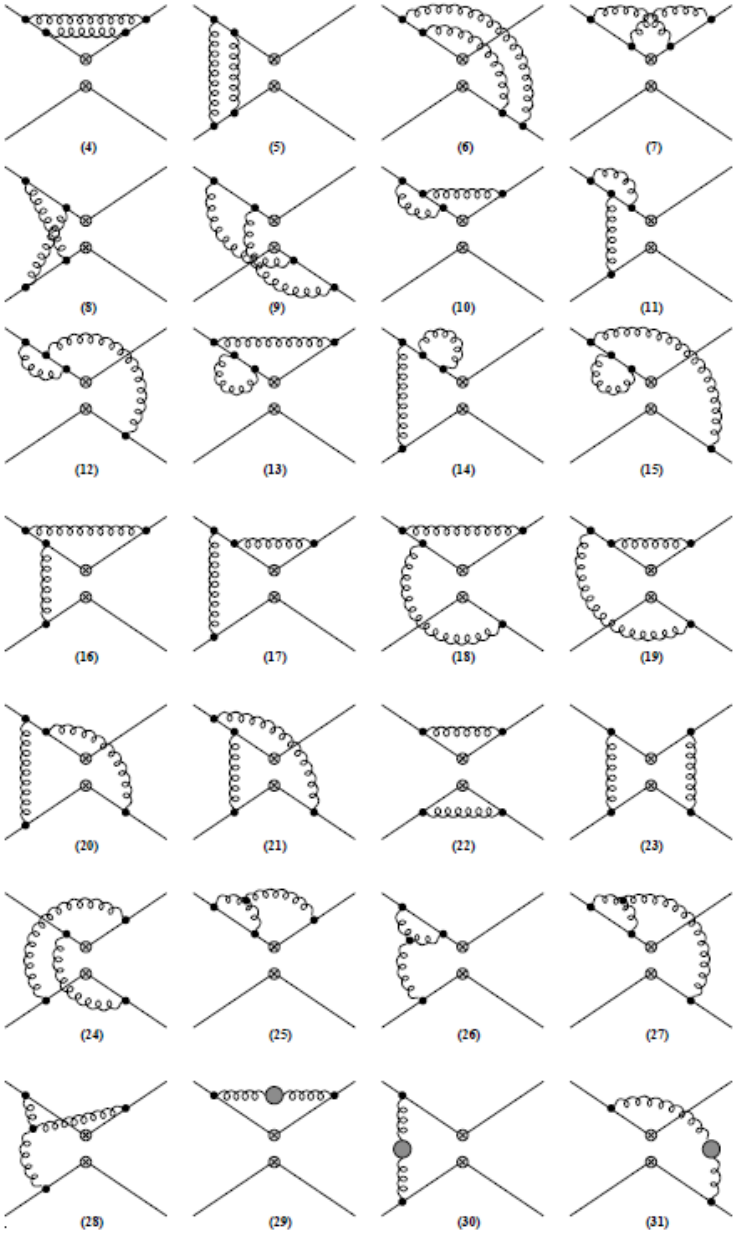
- $\Delta F = 2 \quad \Longrightarrow \quad O_i \sim (\bar{s}\Gamma b)(\bar{s}\Gamma b)$
- $\Delta F = 1 \quad \Longrightarrow \quad O_i \sim (\bar{s}\Gamma b)(\bar{q}\Gamma q') \text{ with } q \neq q'$
- $\Delta F = 1^{\bar{q}q} \quad \Longrightarrow \quad O_i \sim (\bar{s}\Gamma b)(\bar{q}\Gamma q)$

All three results we have taken from [1], where they are written in the BMU basis (Buras, Misiak, Urban), and transformed them to JMS.

Then, using flavor universality (no masses participate in the ADM), we can generalize these anomalous dimensions to any transition type $q_j \to q_i$.

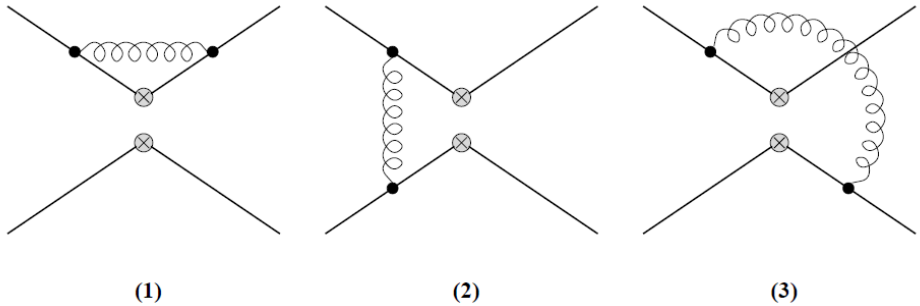
[1] A.Buras, M.Misiak, J.Urban, Nucl.Phys. B586 (2000) 397-426

General Strategy



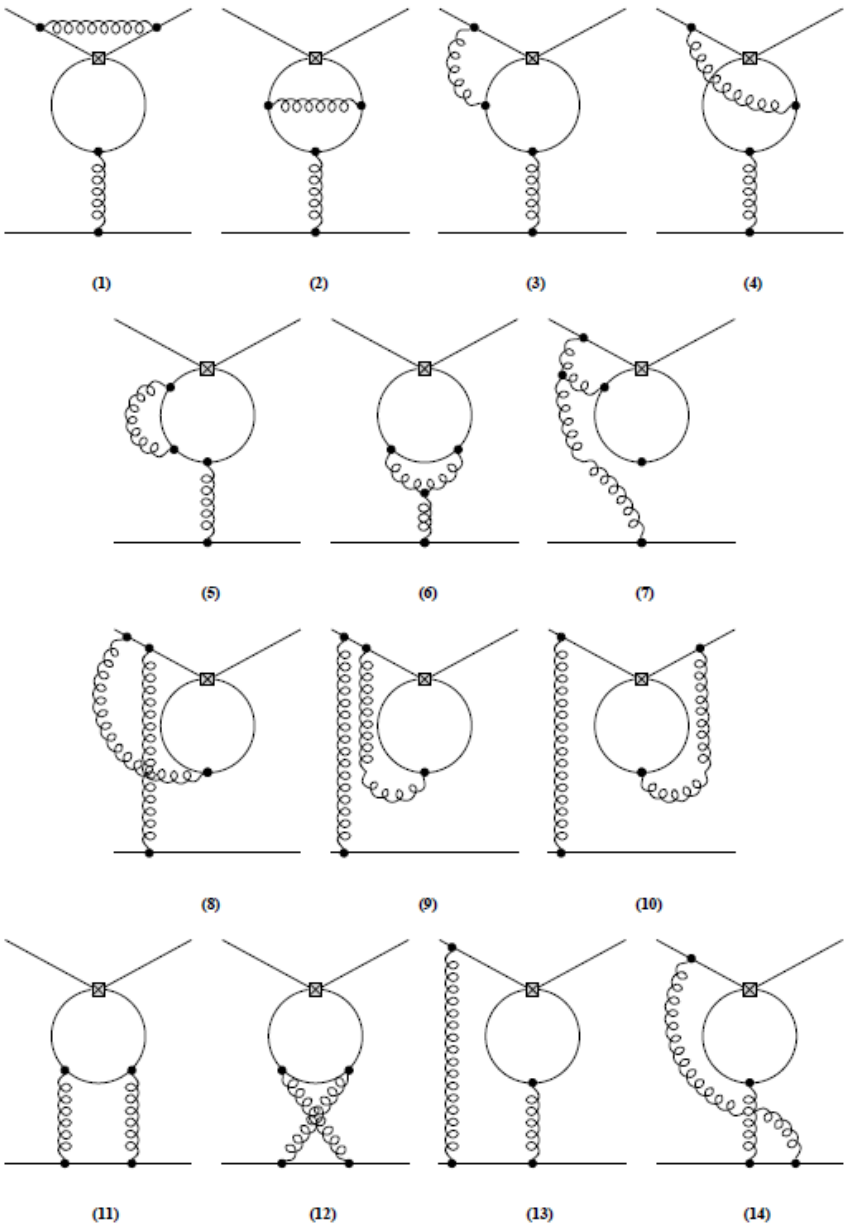
For the sectors of $\Delta F = 2$ $(ijij)$ and $\Delta F = 1$ $(ijkl)$, only current-current diagrams contribute.

Operators with different flavour compositions will not mix.



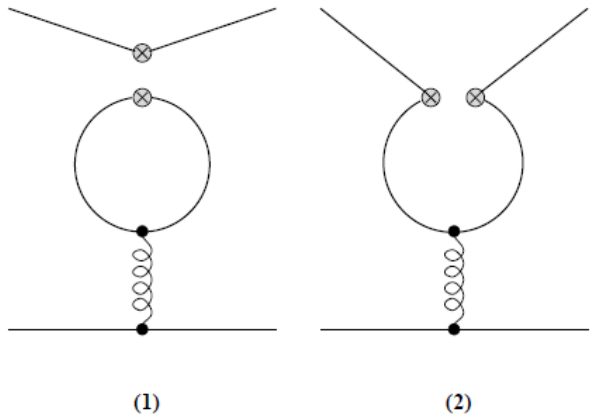
[3] A.Buras, M.Jamin, M.Lautenbacher and P.Weisz, Nucl.Phys. B400 (1993) 37-74

General Strategy



For the $\Delta F = 1$ $(ijkk)$ sector, both current-current and penguin diagrams contribute.

Operators with different flavour compositions will now mix.



[3] A.Buras, M.Jamin, M.Lautenbacher and P.Weisz, Nucl.Phys. B400 (1993) 37-74

General Strategy

A couple things worth mentioning for the big $\Delta F = 1^{\bar{q}q}$ sector...

Our shift matrix $\Delta\hat{r}$ differs from the one previously computed by J.Aebischer, C.Bobeth, A.Buras, J.Kumar and M.Misiak, J. High Energ. Phys. 2021, 227 (2021).

One has to beware of the origin of the N_f factors featured in the two-loop ADM when changing from BMU to JMS.

Outline

Flavour Tricks: Symmetrization of the Basis

In [1] they propose a method for recovering the $\Delta F = 2$ ADM from the results for $\Delta F = 1(ijkl)$, based on the flavor-symmetrization of the basis.

$\underline{\Delta F = 1}$	$\underline{\Delta F = 2}$
$(\bar{q}_i \Gamma_1 q_j)(\bar{q}_k \Gamma_2 q_l)$	$(\bar{q}_i \Gamma_1 q_j)(\bar{q}_i \Gamma_2 q_j)$

Given the flavor universality provided by the lack of mass insertions, these operators only differ in their symmetry properties.

We should recover the $\Delta F = 2$ if we symmetrize $\Delta F = 1$ under the exchange $q_i \leftrightarrow q_k$ and $q_j \leftrightarrow q_l$, and then use flavor universality to change $k, l \to i, j$.

[1] A.Buras, M.Misiak, J.Urban, Nucl.Phys. B586 (2000) 397-426

Flavour Tricks: Symmetrization of the Basis

These exchanges $q_i \leftrightarrow q_k$ and $q_j \leftrightarrow q_l$ are effected by evanescent operators, so one may implement the symmetrization as a linear transformation ($\pm$ basis).

Example of the $\pm$ basis

$$\text{BMU} \left\{ \begin{array}{rcl} Q_1^{\text{VLL}} &=& (\bar{q}_i^\alpha \gamma_\mu P_L q_j^\beta)(\bar{q}_k^\beta \gamma^\mu P_L q_l^\alpha) \\ Q_2^{\text{VLL}} &=& (\bar{q}_i^\alpha \gamma_\mu P_L q_j^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_l^\beta) \end{array} \right.$$

$$\text{Sym.} \left\{ \begin{array}{rcl} Q_+^{\text{VLL}} &=& \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_L q_j^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_l^\beta) + \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_L q_l^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_j^\beta) \\ Q_-^{\text{VLL}} &=& \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_L q_j^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_l^\beta) - \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_L q_l^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_j^\beta) \end{array} \right.$$

Then the $+$ part of the basis agrees completely with $\Delta F = 2$ after relabelling $k, l \to i, j$.

Flavour Tricks: Symmetrization of the Basis

Another example of the $\pm$ basis

$$\text{BMU} \begin{cases} Q_1^{\text{VLR}} = (\bar{q}_i^\alpha \gamma_\mu P_L q_j^\beta)(\bar{q}_k^\beta \gamma^\mu P_L q_l^\alpha) \\ Q_2^{\text{VLR}} = (\bar{q}_i^\alpha \gamma_\mu P_L q_j^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_l^\beta) \\ Q_1^{\text{SLR}} = (\bar{q}_i^\alpha P_L q_j^\beta)(\bar{q}_k^\beta P_L q_l^\alpha) \\ Q_2^{\text{SLR}} = (\bar{q}_i^\alpha P_L q_j^\alpha)(\bar{q}_k^\beta P_L q_l^\beta) \end{cases}$$

$$\text{Sym.} \begin{cases} Q_+^{\text{VLR}} = \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_L q_j^\alpha)(\bar{q}_k^\beta \gamma^\mu P_R q_l^\beta) + \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_R q_l^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_j^\beta) \\ Q_-^{\text{VLR}} = \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_L q_j^\alpha)(\bar{q}_k^\beta \gamma^\mu P_R q_l^\beta) - \frac{1}{2}(\bar{q}_i^\alpha \gamma_\mu P_R q_l^\alpha)(\bar{q}_k^\beta \gamma^\mu P_L q_j^\beta) \\ Q_+^{\text{SLR}} = \frac{1}{2}(\bar{q}_i^\alpha P_L q_j^\alpha)(\bar{q}_k^\beta P_R q_l^\beta) + \frac{1}{2}(\bar{q}_i^\alpha P_R q_l^\alpha)(\bar{q}_k^\beta P_L q_j^\beta) \\ Q_-^{\text{SLR}} = \frac{1}{2}(\bar{q}_i^\alpha P_L q_j^\alpha)(\bar{q}_k^\beta P_R q_l^\beta) - \frac{1}{2}(\bar{q}_i^\alpha P_R q_l^\alpha)(\bar{q}_k^\beta P_L q_j^\beta) \end{cases}$$

Flavour Tricks: Symmetrization of the Basis

Now, we could think of using this method in a half-step, symmetrizing $\Delta F = 1\ (ijkl)$ in only one pair of indices, $q_j \leftrightarrow q_l$.
This is very simply done, as the transformation only changes for LR operators.

Doing so, we can mimick the flavour symmetry of the operators in the flavor sector "$\Delta F = 1.5\ (ijkj)$",

$$\underline{\Delta F = 1.5}$$

$$(\bar{q}_i \Gamma_1 q_j)(\bar{q}_k \Gamma_2 q_j)\ ,$$

and thus we obtain the NLO ADM for this sector of the LEFT, which has never been presented before at NLO.

Outline

Flavour Tricks: Penguin/Current Decomposition

Knowing the previous NLO ADMx, we can derive results for $\Delta F = 0$.

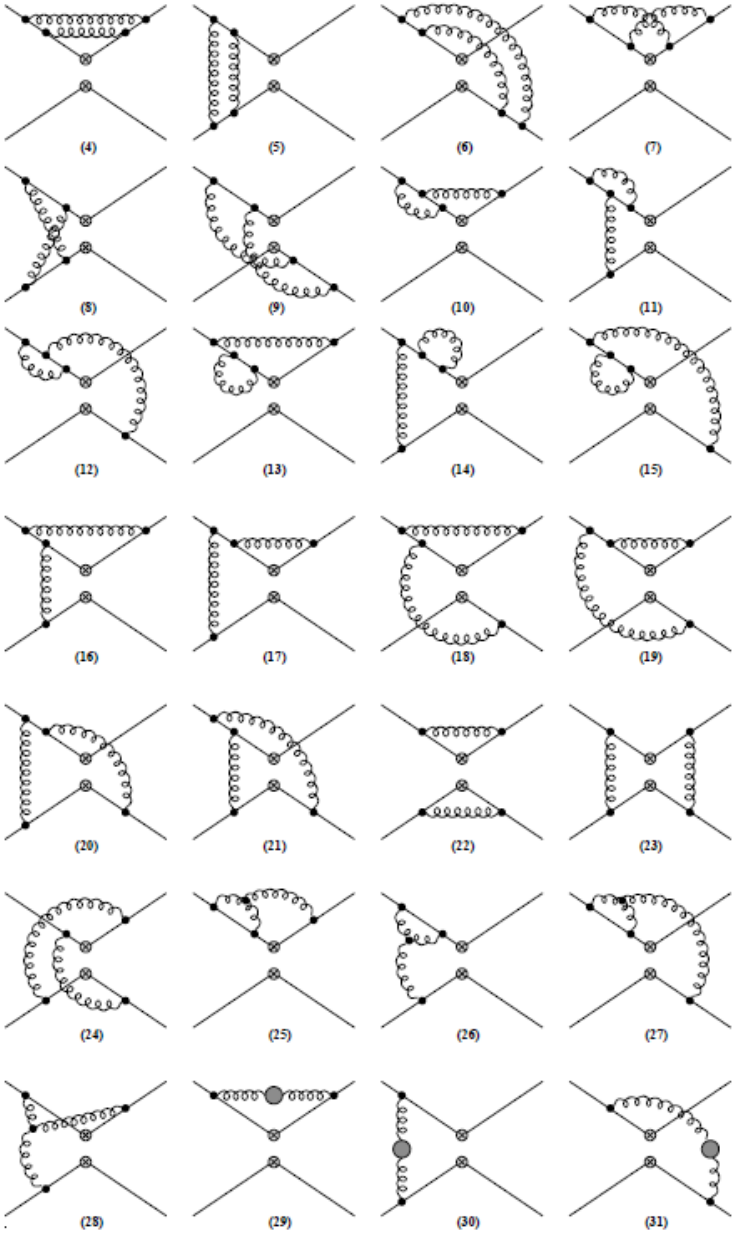
$$\underline{\Delta F = 0}$$

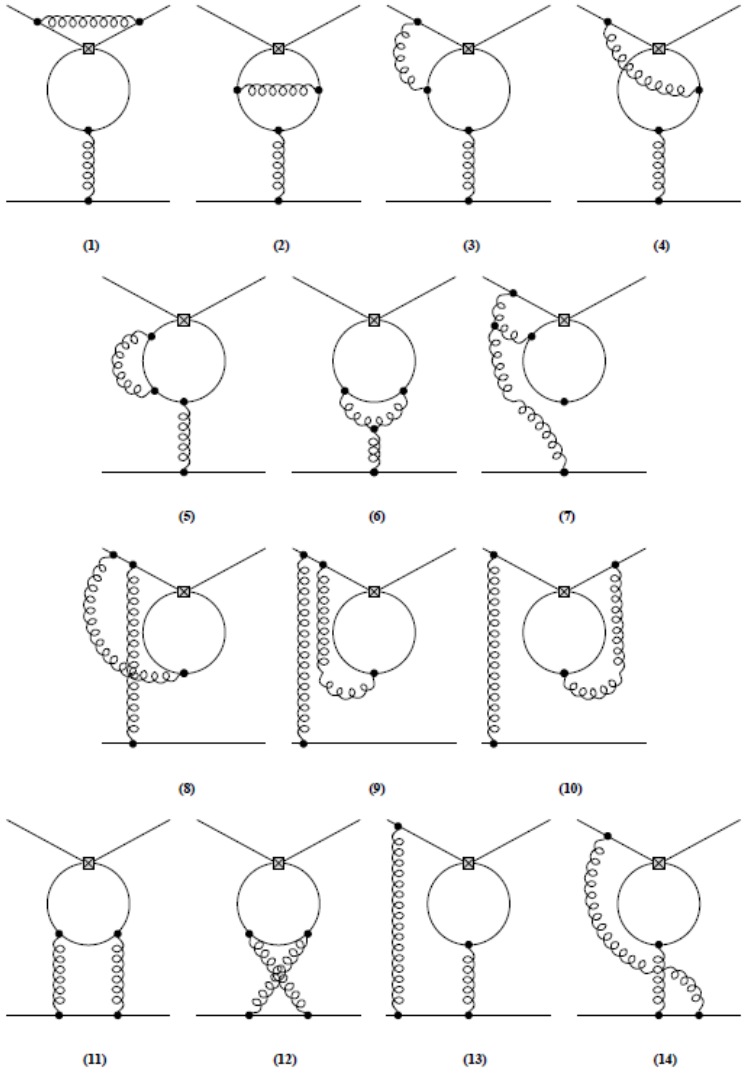
$$(\bar{q}_i \Gamma_1 q_i)(\bar{q}_j \Gamma_2 q_j)$$

The ADM here (both at LO and NLO) gets contributions from insertions into three different types of diagrams: current-current, penguins for external q_i and penguins for external q_j.

For $i \neq j$, in the current-current diagrams, there is virtually no difference between this case and $\Delta F = 1$, as $\Delta F = 0$ has no particular flavor symmetry that may change the physical or evanescent pictures.

Flavor Tricks: Penguin/Current Decomposition





[3] A.Buras, M.Jamin, M.Lautenbacher and P.Weisz, Nucl.Phys. B400 (1993) 37-74

Flavour Tricks: Penguin/Current Decomposition

Penguin contributions at LO and NLO can also be compared to different cases of $\Delta F = 1^{\bar{q}q}$. Penguins with external q_i lines resemble $ijkk$, while penguins with external q_j resemble $iijk$.
Again, no special flavour symmetries stop us from drawing this comparison.

In case of having $i = j$, the current-current diagrams now become identical to $\Delta F = 2$ (i.e. having the maximum possible flavor symmetry).

Meanwhile, penguin diagrams for $i = j$ are indistinguishable from the $j = k$ case in $\Delta F = 1$ $(ijkk)$ in terms of flavor symmetries.

Putting together all these relations, and then using flavor universality to relabel the flavors, we can derive the full $\Delta F = 0$ NLO ADM.

Flavour Tricks: Penguin/Current Decomposition

This being said, it is more simply written in terms of beta functions:

$$\dot{L}_{iijj} = \dot{L}_{ikjj}\Big|_{k\to i} + \dot{L}_{iikj}\Big|_{k\to j} + (\delta_{ij}-1)\,\dot{L}_{kjli}\Big|^{k\to j}_{l\to i} - \delta_{ij}\,\dot{L}_{kili}\Big|_{k,l\to i}$$

$$\dot{L}_{ijji} = \dot{L}_{ijjk}\Big|_{k\to i} + \dot{L}_{ijki}\Big|_{k\to j} + (\delta_{ij}-1)\,\dot{L}_{kjli}\Big|^{k\to i}_{l\to j} - \delta_{ij}\,\dot{L}_{kili}\Big|_{k,l\to i}$$

This expression can be applied for any particular Wilson coefficient, as long as all contributions refer to the same structure (e.g. $L^{V1,LR}$)*.

Furthermore, for it to be functional, one must keep track of the particular flavor indices (i,j,k,l) to be able to substitute them.

* In some cases this might require for an intermediate change of basis.

Outline

Flavour Tricks: Tables of Divergences

In [1], [3] and [4] they provide a series of tables containing the $1/\epsilon^2$ and $1/\epsilon$ poles that emerge from insertions of various operators into all four-fermion diagrams featured up to NLO.

These poles already take into account the LO counterterms, as well as the insertion of evanescent operators. However, they don't include SU(N_c) factors.

Supplementing the tables with the SU(N_c) factors from each diagram, and summing over diagrams, we directly get contributions to the ADM.

Using them, one can reproduce the complete ADM for the $\Delta F = 1$ sector ($ijkl$ and $ijkk$). By extension, with a bit of extra manipulation these tables can also reproduce all previous results.

[1] A.Buras, M.Misiak, J.Urban, Nucl.Phys. B586 (2000) 397-426.

[3] A.Buras, M.Jamin, M.Lautenbacher and P.Weisz, Nucl.Phys. B400 (1993) 37-74.

[4] A.Buras and P.Weisz, Nucl.Phys. B333 (1990) 66-99.

		$Q_1^{\rm SLL}$				$Q_3^{\rm SLL}$			
D	M	$Q_1^{\rm SLL}$ and $Q_2^{\rm SLL}$		$Q_3^{\rm SLL}$ and $Q_4^{\rm SLL}$		$Q_1^{\rm SLL}$ and $Q_2^{\rm SLL}$		$Q_3^{\rm SLL}$ and $Q_4^{\rm SLL}$	
		$1/\epsilon^2$	$1/\epsilon$	$1/\epsilon^2$	$1/\epsilon$	$1/\epsilon^2$	$1/\epsilon$	$1/\epsilon^2$	$1/\epsilon$
1	2	–	8	–	0	–	0	–	0
2	2	–	-2	–	$-1/2$	–	-24	–	-6
3	2	–	2	–	$-1/2$	–	-24	–	6
4	2	-16	16	0	0	0	0	0	0
5	2	-4	9	-1	$5/4$	-48	76	-12	7
6	2	-4	9	1	$-7/4$	48	-52	-12	7
7	2	0	-4	0	0	0	0	0	4
8	2	0	2	0	$-1/2$	0	-24	0	-2
9	2	0	2	0	$1/2$	0	24	0	-2
10	4	-8	-8	0	0	0	0	0	4
11	4	2	0	$1/2$	$5/4$	24	20	6	8
12	4	-2	0	$1/2$	$5/4$	24	20	-6	-4
13	4	8	-4	0	0	0	0	0	0
14	4	-2	0	$-1/2$	$-1/4$	-24	28	-6	0
15	4	2	0	$-1/2$	$-1/4$	-24	28	6	-4
16	4	8	-4	0	0	96	64	0	0
17	4	8	4	2	2	0	0	0	0
18	4	-8	4	0	0	96	64	0	0
19	4	-8	-4	2	2	0	0	0	0
20	4	-4	10	-1	1	48	-64	12	2
21	4	-4	10	1	-2	-48	112	12	-22
22	1	-16	0	0	0	0	0	0	0
23	1	-4	5	-1	$1/4$	-48	28	-12	-5
24	1	-4	5	1	$-3/4$	48	-4	-12	-5
25	4	24	-20	0	0	0	0	0	0
26	4	-6	2	$-3/2$	$-1/4$	-72	108	-18	6
27	4	6	-2	$-3/2$	$-1/4$	-72	108	18	-18
28	4	0	0	0	3	0	-144	0	0
29	2	$-5N+2f$	$\frac{26N}{3}-\frac{8f}{3}$	0	0	0	0	$\frac{5N}{3}-\frac{2f}{3}$	$-\frac{16N}{9}+\frac{4f}{9}$
30	2	0	0	$\frac{5N}{12}-\frac{f}{6}$	$-\frac{17N}{72}+\frac{f}{36}$	$20N-8f$	$-\frac{134N}{3}+\frac{44f}{3}$	$\frac{10N}{3}-\frac{4f}{3}$	$-\frac{32N}{9}+\frac{8f}{9}$
31	2	0	0	$\frac{5N}{12}-\frac{f}{6}$	$-\frac{17N}{72}+\frac{f}{36}$	$20N-8f$	$-\frac{134N}{3}+\frac{44f}{3}$	$-\frac{10N}{3}+\frac{4f}{3}$	$\frac{62N}{9}-\frac{20f}{9}$

Flavour Tricks: Tables of Divergences

The tables contain all information about the divergences in the two-loop diagrams, but they don't know about particular flavors. They can be used to probe arbitrary four-fermion diagrams!

Choosing the appropriate diagrams, and computing a new set of SU(N_c) factors, we can also use the tables to explore...

- The "semileptonic" sector of the LEFT (quark-lepton).
- The baryon-number violating sector of the LEFT.

(Which are the two sectors left to collect from the full four-fermion mixing ADM.)

Flavour Tricks: Tables of Divergences

We start by taking only diagrams that leave some particular fermion line disconnected from any gluon, and computing the corresponding SU(N_c) factors for singlet-only operators. We are then left with a setup identical to the computation of the semileptonic anomalous dimensions (up to flavor relabelling).

This way we obtain an ADM for the semileptonic $\Delta F = 1$, featuring only current-current diagrams, and mixing only scalar operators*.
These results agree with [5].

$$\underline{\Delta F = 1}$$

$$(\bar{e}_i \Gamma_1 e_j)(\bar{q}_k \Gamma_2 q_l) \ , \ (\bar{\nu}_i \Gamma_1 \nu_j)(\bar{q}_k \Gamma_2 q_l) \ , \ (\bar{\nu}_i \Gamma_1 e_j)(\bar{d}_k \Gamma_2 u_l)$$

*For vectors this is the renormalization of the vector current.

[5] M.González-Alonso, J.Camalich, K.Mimouni, Phys. Lett. B, 772, (2017) 777-785.

Flavour Tricks: Tables of Divergences

We can also obtain an ADM for the semileptonic $\Delta F = 0$, with both current-current and penguin diagrams.
Penguins do mix vector currents in their separated L, R form.

$$\underline{\Delta F = 0}$$

$$(\bar{e}_i \Gamma_1 e_j)(\bar{q}_k \Gamma_2 q_k) \ , \ (\bar{\nu}_i \Gamma_1 \nu_j)(\bar{q}_k \Gamma_2 q_k)$$

This sector of the NLO ADM had not been adressed before.

Flavour Tricks: Tables of Divergences

For the baryon-number violating operators, we proceed in a similar way.

$\underline{\Delta B = \Delta L = 1}$	$\underline{\Delta B = -\Delta L = 1}$
$\epsilon_{\alpha\beta\gamma}(\overline{q^c}_i^{\alpha}\,\Gamma_1\,q_j^{\beta})(\overline{q^c}_k^{\gamma}\,\Gamma_2\,\ell_l)$	$\epsilon_{\alpha\beta\gamma}(\overline{q^c}_i^{\alpha}\,\Gamma_1\,q_j^{\beta})(\overline{\ell}_k\,\Gamma_2\,q_l^{\gamma})$

We pick only the compatible diagrams* (one leg must be gluon-free), and properly change the color factors (this case is fixed to $N_c = 3$).

Furthermore, we must take into account the charge-conjugate fields, which can add extra factors of -1,

$$\overline{\psi}^{\alpha}\gamma^{\mu}\,T^A_{\alpha\beta}\,\xi^{\beta} = -\overline{\psi^c}^{\beta}\gamma^{\mu}\,T^A_{\alpha\beta}\,\xi^{c\,\alpha}$$

The resulting ADM does not mix these operators, it is purely diagonal.

*Baryon-number violating operators can only be inserted in current-current diagrams.

Outline

Emergence of Flavor Symmetries

This is a lateral comment, unrelated to the calculation of the ADMs.

The structure of the JMS basis, without sums over flavours, allows for certain symmetries to be easily picked out of the ADMs. For instance,

$$\{L^{V1,LL}_{\substack{ud\\1123}}, L^{V8,LL}_{\substack{ud\\1123}}\} \quad \longleftrightarrow \quad \{L^{V1,LL}_{\substack{ud\\2223}}, L^{V8,LL}_{\substack{ud\\2223}}\}$$

the operators corresponding to these two coefficients are identical up to $u \leftrightarrow c$.

The evanescent basis also has identical copies of the same operators for u and c quarks, also with the $V1/V8$ structure.

This leads to $u-$related and $c-$related entries in the ADM being identical (as both quarks are indistinguishable without masses).

Emergence of Flavor Symmetries

There is another sector, though, where another flavor symmetry does not hold.

$$L^{V,LL}_{\substack{dd \\ 1123}} + L^{V,LL}_{\substack{dd \\ 1321}} \quad \longleftrightarrow \quad \{L^{V,LL}_{\substack{dd \\ 2223}}, L^{V,LL}_{\substack{dd \\ 3323}}\}$$

In this case we would expect all three elements to be identical (as they differ only in flavor), and it works at LO.
However, the sum in the LHS is quite different than the other two terms.

This is quite weird, as the ADM of the LHS gets contributions from the same diagrams as any of the RHS coefficients.

The answer, as always, lies in the evanescent basis...

Emergence of Flavor Symmetries

At LO, the evanescents do not interfere in the ADM, and thus the "naive" flavor symmetry holds.

Beyond LO, flavor symmetries also depend on the structure of the evanescent operators. Then, symmetries will only emerge if they are present both in the physical and the evanescent basis.

In the u/c case, both bases had a copy of the same operators with the same structures ($V1/V8$).
In the dd case, however, our evanescent basis is still defined in terms of $V1/V8$, while the physical one is defined in terms of open/closed color-singlets.

Furthermore, changing the evanescent basis can get an ADM where the dd symmetry works, and u/c symmetry does not hold.

Outline

Conclusions

So, what do we know about the NLO ADM in the LEFT (up to dim 6)?

- Four-fermion mixing into four-fermions:
  - $\Delta F = 2$ (Four quark)
  - $\Delta F = 1.5$ (Four quark)
  - $\Delta F = 1$ (Four quark and semileptonic)
  - $\Delta F = 0$ (Four quark and semileptonic)
  - $\Delta B \neq 0$
- Four-fermion mixing into $\Delta F = 1$ dipoles:
  - Vector operators (including the SM)
- Three-gluon self-mixing.
- Dipole ($\Delta F = 1$) self-mixing.

Thank You!