A Universal Approach for Matching at Dimension-eight EFT

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EFT Foundations and Tools 2023
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CoDEx : A package to compute Wilson Coefficients for SMEFT operators

CoDEx: Wilson coefficient calculator connecting SMEFT to UV theory

S Das Bakshi, J Chakrabortty, S K Patra

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What it offers:

- computes WCs in terms of BSM parameters at mass dimension-6 up to one-loop
- can provide output in both SILH and Warsaw bases
- can perform the RG evolution of the operators in Warsaw basis

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What we aim to do:

- automating integrating out at mass dimension-8 up to one-loop for spin-0 and 1/2
- efficiently remove redundancy and find complete matching contribution to dimension-8 WCs
- provide output in a complete and non-redundant basis

Towards one-loop matching at dimension-8 with CoDEx

Computation of effective action up to d_8

1. **One-loop Effective Action up to Dimension Eight: Integrating out Heavy Scalar(s)**
   Upalaparna Banerjee, Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar
   e-Print: 2306.09103 [hep-ph].

2. **One-loop Effective Action up to Dimension Eight: Integrating out Heavy Fermion(s)**
   Joydeep Chakrabortty, Shakeel Ur Rahaman, Kaanapuli Ramkumar
   e-Print: 2308.03849 [hep-ph].

Removing redundancies from d_6 and d_8 structures

3. **Integrating out heavy scalars with modified equations of motion: Matching computation of dimension-eight SMEFT coefficients**
   Upalaparna Banerjee, Joydeep Chakrabortty, Christoph Englert, Shakeel Ur Rahaman, Michael Spannowsky
   Phys.Rev.D 107 (2023) 5, 055007; e-Print: 2210.14761 [hep-ph].

Top-Down EFT: Integrating out heavy fields

Consider Φ to be a heavy scalar that we wish to integrate out

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Action can be expanded around the minima , $$S[\phi, \Phi_c + \eta] = S[\phi, \Phi_c] + \left.\frac{\delta S[\phi, \Phi]}{\delta \Phi}\right|_{\Phi=\Phi_c} \eta + \frac{1}{2}\left.\frac{\delta^2 S[\phi, \Phi]}{\delta^2 \Phi}\right|_{\Phi=\Phi_c} \eta^2 + \mathcal{O}(\eta^3)$$

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Tree-level piece

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How to obtain tree-level Lagrangian, $\quad D_\mu \frac{\partial}{\partial(D_\mu \Phi)} \mathcal{L}(\phi, \Phi) = \frac{\partial}{\partial \Phi} \mathcal{L}(\phi, \Phi)$ *Euler - Lagrange equation*

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Computation of one-loop effective Lagrangian

Let's start with a UV Lagrangian, $\mathscr{L}\left[\phi, \Phi\right] \supset \Phi^\dagger (P^2 - m^2 - U(\phi))\,\Phi + (\Phi^\dagger\, B(\phi) + \text{h.c.}) + \mathcal{O}(\Phi^3)$

$$S_{\text{eff,1-loop}} \approx i c_s \,\text{Tr}\log\left(-\frac{\delta^2 S}{\delta \Phi^2}\bigg|_{\Phi_c} \right) = i c_s\,\text{Tr}\log\Big(-P^2 + m^2 + U(\phi)\Big)$$

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$$\begin{aligned}
\mathcal{L}^{(dim-6)}_{1-loop}[\phi, \Phi_c] = & \frac{c}{(4\pi)^2}\,\text{tr}\Bigg\{ m^2\left(1 + \log\frac{\mu^2}{m^2}\right) U + m^0\left[\frac{1}{12}\left(1 + \log\frac{\mu^2}{m^2}\right) G'^2_{\mu\nu} + \frac{1}{2}\log\frac{\mu^2}{m^2} U^2\right] \\
& + \frac{1}{m^2}\left[-\frac{1}{60}\left(P_\mu G'_{\mu\nu}\right)^2 - \frac{1}{90} G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu} - \frac{1}{12}(P_\mu U)^2 - \frac{1}{6}U^3 \right. \\
& \left. - \frac{1}{12} U G'_{\mu\nu} G'_{\mu\nu}\right] + \frac{1}{m^4}\left[\frac{1}{24}U^4 + \frac{1}{12}U\left(P_\mu U\right)^2 + \frac{1}{120}\left(P^2 U\right)^2 + \frac{1}{24}\left(U^2 G'_{\mu\nu}G'_{\mu\nu}\right)\right. \\
& \left. - \frac{1}{120}\left[(P_\mu U),(P_\nu U)\right]G'_{\mu\nu} - \frac{1}{120}\left[U[U, G'_{\mu\nu}]\right]G'_{\mu\nu}\right] + \frac{1}{m^6}\left[-\frac{1}{60}U^5 - \frac{1}{20}U^2\left(P_\mu U\right)^2 \right. \\
& \left. - \frac{1}{30}\left(U P_\mu U\right)^2\right] + \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned}$$

Henning et. al. JHEP01(2016)023

Drozd et. al. JHEP03(2016)180

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Extend UOLEA upto dimension-8:

- with the application of Heat-Kernel method
- cross-verify the result with covariant-diagram method

Heat Kernel: A Brief Review

Consider the part of the UV Lagrangian that's bi-linear in Φ, $\mathcal{L}^{\Phi} = \Phi^{\dagger}(D^2 + U + M^2)\Phi = \Phi^{\dagger}(\Delta)\Phi,$

The Heat-Kernel for operator Δ can be written as, $K(t, x, y, \Delta) = < y | e^{-t\Delta} | x >$

The Heat-Kernel satisfies the heat equation, $(\partial_t + \Delta)\, K(t, x, y, \Delta) = 0$

with initial condition, $K(0, x, y, \Delta) = \delta(x - y)$

A.A.Bel'kov et al., hep-ph/9506237

I.G.Avramidi, Nuc.Phys.B, 355(1991)

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The HK for the operator Δ, $K(t, x, y, \Delta) = K_0(t, x, y)\, H(t, x, y, \Delta).$

The HK for the free operator $\Delta_0 = \partial_\mu \partial^\mu + M^2$

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with initial condition, $K(0, x, y, \Delta) = \delta(x - y)$

The HK for the operator Δ, $K(t, x, y, \Delta) = K_0(t, x, y) H(t, x, y, \Delta)$

Contains the information about the interaction, U

A.A.Bel'kov et al., hep-ph/9506237

I.G.Avramidi, Nuc.Phys.B, 355(1991)

Heat Kernel: A Brief Review

Consider the part of the UV Lagrangian that's bi-linear in Φ, $\mathcal{L}^{\Phi} = \Phi^{\dagger}(D^2 + U + M^2)\Phi = \Phi^{\dagger}(\Delta)\Phi,$

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$$K_0(t,x,y) = (4\pi t)^{-d/2}\,\mathrm{Exp}\left[\frac{(x-y)^2}{4t} - tM^2\right]$$

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$K_0(t,x,y)$: $(4\pi t)^{-d/2}\, \text{Exp}\left[\frac{(x-y)^2}{4t} - tM^2\right]$

$H(t,x,y,\Delta)$: $\sum_k \frac{(-t)^k}{k!} b_k(x, y)$

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$b_k(x,y)$: Heat-Kernel coefficients

A.A.Bel'kov et al., hep-ph/9506237

I.G.Avramidi, Nuc.Phys.B, 355(1991)

One-loop effective action and Heat-Kernel coefficients

Consider the part of the UV Lagrangian that's bi-linear in Φ, $\mathcal{L}^{\Phi} = \Phi^{\dagger}(D^2 + U + M^2)\Phi = \Phi^{\dagger}(\Delta)\Phi,$

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The HK for the operator Δ, $K(t, x, y, \Delta) = K_0(t, x, y)\ H(t, x, y, \Delta).$

- How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\textit{eff, 1-loop}} = c_s\, \mathrm{tr} \log(-P^2 + U + M^2) = c_s\, \mathrm{tr} \int_0^{\infty} \frac{dt}{t} e^{-t\Delta} = c_s\, \mathrm{tr} \int_0^{\infty} \frac{dt}{t} K(t, x, x, \Delta)$$

Use:

$$\ln \lambda = -\int_0^{\infty} \frac{dt}{t} e^{-t\lambda}$$

$$= c_s \int_0^{\infty} \frac{dt}{t} (4\pi t)^{-d/2} e^{-tM^2} \sum_k \frac{(-t)^k}{k!} \mathrm{tr}[b_k] = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k - d/2]\, \underbrace{\mathrm{tr}[b_k]}_{\text{Heat-Kernel coefficients}}$$

Effective contributions to renormalisable Lagrangian

- How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\text{eff, 1-loop}} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k-d/2]\, tr[b_k]$$

For $k \leq d/2$ the Gamma function has simple poles. Assuming $d = 4 - \epsilon$, $\Gamma[k-d/2] = \frac{(\epsilon/2-3+k)!}{(\epsilon/2-1)!}\Gamma[\epsilon/2]$.

The divergent part of the one-loop effective action, $\mathcal{L}^{(k)}_{div} = \frac{c_s}{(4\pi)^{2-\epsilon/2}} M^{d-2k} \frac{(-1)^k}{k!} \frac{(\epsilon/2-3+k)!}{(\epsilon/2-1)!} \underbrace{\Gamma[\epsilon/2]}_{2/\epsilon - \gamma_E + \mathcal{O}(\epsilon)} \, \text{tr}[b_k]$,

Effective contributions to renormalisable Lagrangian

- How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\text{eff, 1-loop}} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k-d/2]\, \text{tr}[b_k]$$

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$\underline{k = 0}$

$$\begin{aligned}
\mathcal{L}_{div}^{(0)} &= \frac{c_s}{(4\pi)^{2-\epsilon/2}} M^d \frac{(\epsilon/2 - 3)!}{(\epsilon/2 - 1)!} \Gamma[\epsilon/2] \ \text{tr}[b_0] \\
&= \frac{c_s}{(4\pi)^{2-\epsilon/2}} M^{4-\epsilon} \frac{1}{(\epsilon/2 - 1)(\epsilon/2 - 2)} \Gamma[\epsilon/2] \ \text{tr}[b_0] \\
&= c_s \left(\frac{M^2}{4\pi}\right)^2 \left(\frac{4\pi}{M^2}\right)^{\epsilon/2} \frac{1}{(\epsilon/2 - 1)(\epsilon/2 - 2)} \Gamma[\epsilon/2] \ \text{tr}[b_0].
\end{aligned}$$

$$\mathcal{L}_{\text{eff}}^{(0)} = \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2} \left(\ln\left[\frac{M^2}{\mu^2}\right] - 3/2 \right) \text{tr}[b_0] \right]$$

Effective contributions to renormalisable Lagrangian

- How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

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Effective contributions to renormalisable Lagrangian

✤ How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\text{eff, 1-loop}} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k-d/2]\, \mathrm{tr}[b_k]$$

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Renormalised one-loop effective action ,

$$\mathcal{L}^{\text{ren}}_{\text{eff}} = \frac{c_s}{(4\pi)^2} \mathrm{tr}\left\{ M^4 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right] - 3/2 \right) [b_0] \right] + M^2 \left[-\left(\ln\left[\frac{M^2}{\mu^2}\right] - 1 \right) [b_1] \right] + M^0 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right] \right) [b_2] \right] \right\}. \quad ($$

One-loop effective action and Heat-Kernel coefficients

- How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\text{eff, 1-loop}} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k - d/2]\, \mathrm{tr}[b_k]$$

Operators of the form $\mathcal{O}(D^r U^s)$ appear in the HKC $b_n(x,x)$ where $n = r/2 + s$.

Start with the initial condition, $b_0(x,x) = I$

Use recursive relation, $\mathcal{O}(D^r U^s) \equiv [b_{r/2+s}][[U^s]] = \sum_{k=0}^{n=r/2+s} \frac{n!\,(n-1)!}{k!(2n-k)!} \{k\, D^{2(n-k)}\{U b_{k-1}[[U^{s-1}]]\} - T_{2(n-k)}\, b_k[[U^s]]\}_{z=0}$

$$\Rightarrow T_{\mu_1\mu_2\ldots\mu_m} = D_{\mu_1} T_{\mu_2\ldots\mu_m} + R_{\mu_2\ldots\mu_m,\mu_1},$$

in the above equation, $R_{\mu_2\ldots\mu_m,\mu_1} = [D_{\mu_2}\ldots D_{\mu_m}, D_{\mu_1}]$,

$$\Rightarrow R_{\mu_2\mu_3\ldots\mu_m,\mu_1} = G_{\mu_2\mu_1} D_{\mu_3}\ldots D_{\mu_m} + D_{\mu_2} D_{\mu_3\ldots\mu_m,\mu_1},$$

One-loop effective action and Heat-Kernel coefficients

- How to obtain one-loop effective action in terms of Heat-Kernel coefficients?

$$\mathscr{L}_{\text{eff, 1-loop}} = \frac{c_s}{(4\pi)^{d/2}} \sum_{k=0}^{\infty} M^{d-2k} \frac{(-1)^k}{k!} \Gamma[k - d/2]\, \text{tr}[b_k]$$

Relevant coefficients upto D8,

$$\begin{aligned}
\text{tr}[b_0] &= \text{tr}\, I, \\
\text{tr}[b_1] &= \text{tr}\, U, \\
\text{tr}[b_2] &= \text{tr}\left[U^2 + \frac{1}{6}(G_{\mu\nu})^2\right], \\
\text{tr}[b_3] &= \text{tr}\left[U^3 - \frac{1}{2}(U_{;\mu})^2 + \frac{1}{2}U\, G_{\mu\nu}G_{\mu\nu} - \frac{1}{10}(J_\nu)^2 + \frac{1}{15}G_{\mu\nu}\, G_{\nu\rho}\, G_{\rho\mu}\right], \\
\text{tr}[b_4] &= \text{tr}\Big[U^4 + U^2 U_{;\mu\mu} + \frac{4}{5}U^2(G_{\mu\nu})^2 + \frac{1}{5}(U\, G_{\mu\nu})^2 + \frac{1}{5}(U_{;\mu\mu})^2 - \frac{2}{5}U\, U_{;\nu}\, J_\nu \\
&\quad - \frac{2}{5}U(J_\mu)^2 + \frac{2}{15}U_{;\mu\mu}(G_{\rho\sigma})^2 + \frac{4}{15}U\, G_{\mu\nu}G_{\nu\rho}G_{\rho\mu} + \frac{8}{15}U_{;\nu\mu}\, G_{\rho\mu}G_{\rho\nu} \\
&\quad + \frac{1}{35}(J_{\mu;\nu})^2 + \frac{16}{105}G_{\mu\nu}J_\mu J_\nu + \frac{1}{420}(G_{\mu\nu}G_{\rho\sigma})^2 + \frac{17}{210}(G_{\mu\nu})^2(G_{\rho\sigma})^2 \\
&\quad + \frac{1}{105}G_{\mu\nu}G_{\nu\rho}G_{\rho\sigma}G_{\sigma\mu} + \frac{2}{35}(G_{\mu\nu}G_{\nu\rho})^2 + \frac{16}{105}J_{\nu;\mu}G_{\nu\sigma}G_{\sigma\mu}\Big],
\end{aligned}$$

$\text{tr}[b_5][[U^5, U^4, U^3, U^2]]$, $\text{tr}[b_6][[U^6, U^5, U^4]]$, $\text{tr}[b_7][[U^7, U^6]]$, $\text{tr}[b_8][[U^8]]$

Universal One-loop Effective Action upto D8

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}^{d\leq 8} =\; & \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right]-\frac{3}{2}\right)\right] + \frac{c_s}{(4\pi)^2}\,\mathrm{tr}\Bigg\{ M^2\left[-\left(\ln\left[\frac{M^2}{\mu^2}\right]-1\right)U\right] \\
& + M^0\,\frac{1}{2}\left[-\ln\left[\frac{M^2}{\mu^2}\right]U^2 - \frac{1}{6}\ln\left[\frac{M^2}{\mu^2}\right](G_{\mu\nu})^2\right] \\
& + \frac{1}{M^2}\frac{1}{6}\left[-U^3 - \frac{1}{2}(P_\mu U)^2 - \frac{1}{2}U\,(G_{\mu\nu})^2 - \frac{1}{10}(J_\nu)^2 + \frac{1}{15}G_{\mu\nu}G_{\nu\rho}G_{\rho\mu}\right] \\
& + \frac{1}{M^4}\frac{1}{24}\Bigg[U^4 - U^2(P^2U) + \frac{4}{5}U^2(G_{\mu\nu})^2 + \frac{1}{5}(U\,G_{\mu\nu})^2 + \frac{1}{5}(P^2U)^2 \\
& \quad - \frac{2}{5}U\,(P_\mu U)\,J_\mu + \frac{2}{5}U(J_\mu)^2 - \frac{2}{15}(P^2U)(G_{\rho\sigma})^2 + \frac{1}{35}(P_\nu J_\mu)^2 \\
& \quad - \frac{4}{15}U\,G_{\mu\nu}G_{\nu\rho}G_{\rho\mu} - \frac{8}{15}(P_\mu P_\nu U)\,G_{\rho\mu}G_{\rho\nu} + \frac{16}{105}G_{\mu\nu}J_\mu J_\nu \\
& \quad + \frac{1}{420}(G_{\mu\nu}G_{\rho\sigma})^2 + \frac{17}{210}(G_{\mu\nu})^2(G_{\rho\sigma})^2 + \frac{2}{35}(G_{\mu\nu}G_{\nu\rho})^2 \\
& \quad + \frac{1}{105}G_{\mu\nu}G_{\nu\rho}G_{\rho\sigma}G_{\sigma\mu} + \frac{16}{105}(P_\mu J_\nu)G_{\nu\sigma}G_{\sigma\mu}\Bigg] \\
& + \frac{1}{M^6}\frac{1}{60}\Bigg[-U^5 + 2U^3(P^2U) + U^2(P_\mu U)^2 - \frac{2}{3}U^2G_{\mu\nu}U\,G_{\mu\nu} - U^3(G_{\mu\nu})^2 \\
& \quad + \frac{1}{3}U^2(P_\mu U)J_\mu - \frac{1}{3}U\,(P_\mu U)(P_\nu U)\,G_{\mu\nu} - \frac{1}{3}U^2J_\mu(P_\mu U) \\
& \quad - \frac{1}{3}U\,G_{\mu\nu}(P_\mu U)(P_\nu U) - U\,(P^2U)^2 - \frac{2}{3}(P^2U)(P_\nu U)^2 - \frac{1}{7}((P_\mu U)G_{\mu\alpha})^2 \\
& \quad + \frac{2}{7}U^2G_{\mu\nu}G_{\nu\alpha}G_{\alpha\mu} + \frac{8}{21}U\,G_{\mu\nu}U\,G_{\nu\alpha}G_{\alpha\mu} - \frac{4}{7}U^2(J_\mu)^2 - \frac{3}{7}(U\,J_\mu)^2 \\
& \quad + \frac{4}{7}U\,(P^2U)(G_{\mu\nu})^2 + \frac{4}{7}(P^2U)U(G_{\mu\nu})^2 - \frac{2}{7}U\,(P_\mu U)J_\nu G_{\mu\nu} \\
& \quad - \frac{2}{7}(P_\mu U)U\,G_{\mu\nu}J_\nu - \frac{4}{7}U\,(P_\mu U)G_{\mu\nu}J_\nu - \frac{4}{7}(P_\mu U)U\,J_\nu G_{\mu\nu} \\
& \quad + \frac{4}{21}U\,G_{\mu\nu}(P^2U)G_{\mu\nu} + \frac{11}{21}(P_\alpha U)^2(G_{\mu\nu})^2 - \frac{10}{21}(P_\mu U)J_\nu U\,G_{\mu\nu} \\
& \quad - \frac{10}{21}(P_\mu U)G_{\mu\nu}U\,J_\nu - \frac{2}{21}(P_\mu U)(P_\nu U)G_{\mu\alpha}G_{\alpha\nu} + \frac{10}{21}(P_\nu U)(P_\mu U)G_{\mu\alpha}G_{\alpha\nu} \\
& \quad - \frac{1}{7}(G_{\alpha\mu}(P_\mu U))^2 - \frac{1}{42}((P_\alpha U)G_{\mu\nu})^2 - \frac{1}{14}(P_\mu P^2U)^2 - \frac{4}{21}(P^2U)(P_\mu U)J_\mu \\
& \quad + \frac{4}{21}(P_\mu U)(P^2U)J_\mu + \frac{2}{21}(P_\mu U)(P_\nu U)(P_\mu J_\nu) - \frac{2}{21}(P_\nu U)(P_\mu U)(P_\mu J_\nu)\Bigg] \\
& + \frac{1}{M^8}\frac{1}{120}\Bigg[U^6 - 3U^4(P^2U) - 2U^3(P_\nu U)^2 + \frac{12}{7}U^2(P_\mu P_\nu U)(P_\nu P_\mu U) \\
& \quad + \frac{26}{7}(P_\mu P_\nu U)U\,(P_\mu U)(P_\nu U) + \frac{26}{7}(P_\mu P_\nu U)(P_\mu U)(P_\nu U)U + \frac{9}{7}(P_\mu U)^2(P_\nu U)^2 \\
& \quad + \frac{9}{7}U\,(P_\mu P_\nu U)U\,(P_\nu P_\mu U) + \frac{17}{14}((P_\mu U)(P_\nu U))^2 + \frac{8}{7}U^3G_{\mu\nu}U\,G_{\mu\nu} \\
& \quad + \frac{5}{7}U^4(G_{\mu\nu})^2 + \frac{18}{7}G_{\mu\nu}(P_\mu U)U^2(P_\nu U) + \frac{9}{14}(U^2G_{\mu\nu})^2 \\
& \quad + \frac{18}{7}G_{\mu\nu}U\,(P_\mu U)(P_\nu U)U + \frac{18}{7}(P_\mu P_\nu U)(P_\mu U)U\,(P_\nu U) \\
& \quad + \left(\frac{8}{7}G_{\mu\nu}U\,(P_\mu U)U\,(P_\nu U) + \frac{26}{7}G_{\mu\nu}(P_\mu U)U\,(P_\nu U)U\right) \\
& \quad + \left(\frac{24}{7}G_{\mu\nu}(P_\mu U)(P_\nu U)U^2 - \frac{2}{7}G_{\mu\nu}U^2(P_\mu U)(P_\nu U)\right)\Bigg] \\
& + \frac{1}{M^{10}}\frac{1}{210}\left[-U^7 - 5\,U^4(P_\nu U)^2 - 8\,U^3(P_\mu U)U(P_\mu U) - \frac{9}{2}(U^2(P_\mu U))^2\right] \\
& + \frac{1}{M^{12}}\frac{1}{336}\left[U^8\right]\Bigg\}. \qquad (7.1)
\end{aligned}
$$

$$U_{ij} = \frac{\delta^2\mathcal{L}_{UV}}{\delta\Phi_i\delta\Phi_j}$$

$$G_{\mu\nu} = [P_\mu, P_\nu],$$

$$J_\mu = P_\nu G_{\nu\mu} = [P_\nu,[P_\nu,P_\mu]],$$

Eff. action : DR + MS-bar,

μ is the matching scale,

Method of Covariant Diagram

$$\mathscr{L}_{\text{eff,1-loop}} = ic_s \int \frac{d^d q}{(2\pi)^d} \, \mathrm{tr} \log(-P^2 + M^2 + U)_{P \to P-q}$$

$$= ic_s \int \frac{d^d q}{(2\pi)^d} \, \mathrm{tr} \log(-P^2 - q^2 + 2q \, . \, P + M^2 + U)$$

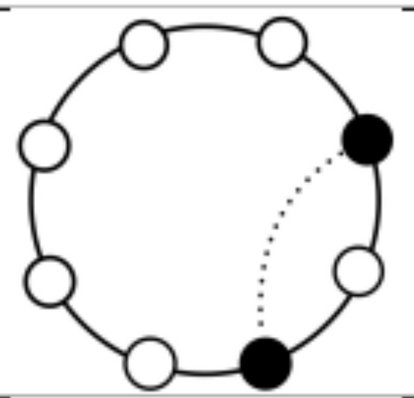
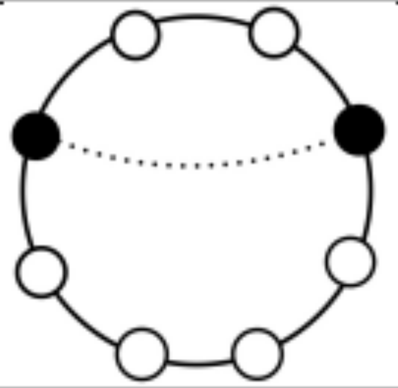
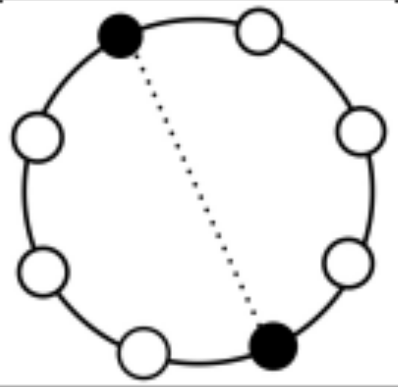
$$\approx -ic_s \, \mathrm{tr} \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d q}{(2\pi)^d} [(q^2 - M^2)^{-1}(2q \, . \, P - P^2 + U)]^n$$

Zhang, JHEP 05(2017)

- Each integral of order n can be mapped into a number of diagrams consisting n number of heavy propagators
- Treat $2q \, . \, P, -P^2$ and U as vertex insertions, each diagram should consist of different combinations of vertices
  - For example, at $n = 8$, we can have operator structures of $\mathcal{O}(P^8), \mathcal{O}(P^6U^2), \mathcal{O}(P^2U^6)$ etc.
  - The most generic form of the loop integral at order n with $2n_c$ number of $2q \, . \, P$ vertices

$$\int \frac{d^d q}{(2\pi)^d} \frac{q^{\mu^1} \cdots q^{\mu^{2n_c}}}{(q^2 - M^2)^n} \equiv g^{\mu^1 \cdots \mu^{2n_c}} \mathscr{I}[q^{2n_c}]^n$$

Method of Covariant Diagram: a detailed case of $\mathcal{O}(P^2U^6)$

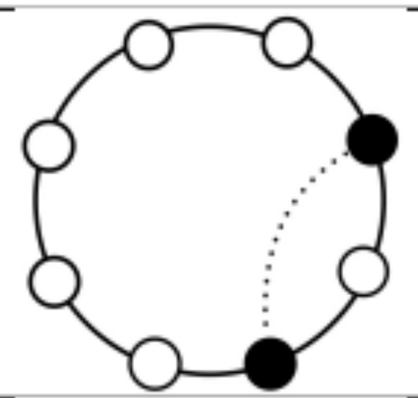
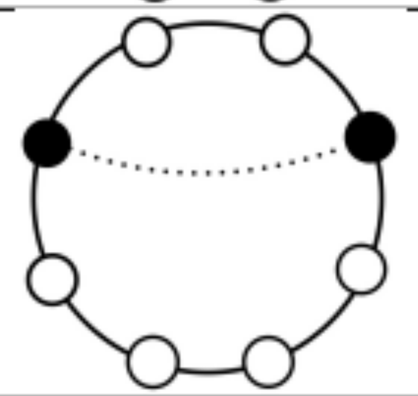
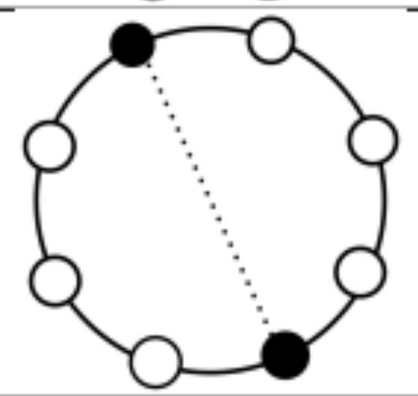
Diagram	$O(P^2U^6)$ structure	Values
	$\mathrm{tr}\big(P_\mu U P_\mu UUUUU\big)$	$-ic_s\, 2^2\, \mathcal{I}[q^2]^8$
	$\mathrm{tr}\big(P_\mu UU P_\mu UUUU\big)$	$-ic_s\, 2^2\, \mathcal{I}[q^2]^8$
	$\mathrm{tr}\big(P_\mu UUU P_\mu UUU\big)$	$-i\frac{c_s}{2}\, 2^2\, \mathcal{I}[q^2]^8$

Black blob $\to 2q\,.\,P$ insertion, white blob $\to U$ insertion

Rules:

- Draw one-loop diagrams with all possible arrangements of black and white blobs
- Two black blobs should not be placed side by side
- The structure for each diagram can be read off starting from one particular blob and going clockwise until we exhaust all the vertices present in the diagram
- The value for each diagram: $-ic_s\, 2^{2n_c}\, \mathcal{I}[q^{2n_c}]^n$, if the diagram has a N-fold rotational symmetry, then we divide the value with a factor of N

Method of Covariant Diagram: a detailed case of $\mathcal{O}(P^2U^6)$

Diagram	$O(P^2U^6)$ structure	Values
	$\mathrm{tr}\big(P_\mu U P_\mu UUUUU\big)$	$-ic_s\, 2^2\, \mathcal{I}[q^2]^8$
	$\mathrm{tr}\big(P_\mu UU P_\mu UUUU\big)$	$-ic_s\, 2^2\, \mathcal{I}[q^2]^8$
	$\mathrm{tr}\big(P_\mu UUU P_\mu UUU\big)$	$-i\frac{c_s}{2}\, 2^2\, \mathcal{I}[q^2]^8$

Start with covariant structures with unknown coefficients:

$$\begin{aligned} &C_1^{(2,6)}\,\mathrm{tr}\big(U^4\,[P_\mu,U]\,[P_\mu,U]\big) + C_2^{(2,6)}\,\mathrm{tr}\big(U^3\,[P_\mu,U]\,U\,[P_\mu,U]\big) \\ &\quad + C_3^{(2,6)}\,\mathrm{tr}\big(U^2\,[P_\mu,U]\,U^2\,[P_\mu,U]\big) = \big(2C_1^{(2,6)} - C_2^{(2,6)}\big)\,\mathrm{tr}\,\big(P_\mu U P_\mu UUUUU\big) \\ &\qquad + \big(2C_2^{(2,6)} - 2C_3^{(2,6)} - C_1^{(2,6)}\big)\,\mathrm{tr}\,\big(P_\mu UU P_\mu UUUU\big) \\ &\qquad + \big(2C_3^{(2,6)} - C_2^{(2,6)}\big)\,\mathrm{tr}\,\big(P_\mu UUU P_\mu UUU\big) + \mathrm{tr}\big(\cdots P^2 \cdots\big)\text{terms.} \end{aligned}$$

Equate the structure coefficients with values corresponding to loops:

$$\begin{aligned} 2C_1^{(2,6)} - C_2^{(2,6)} &= -\frac{c_s}{16\pi^2}\frac{1}{M^{10}}\frac{48}{7!}, \\ 2C_2^{(2,6)} - C_1^{(2,6)} - 2C_3^{(2,6)} &= -\frac{c_s}{16\pi^2}\frac{1}{M^{10}}\frac{48}{7!}, \\ 2C_3^{(2,6)} - C_2^{(2,6)} &= -\frac{c_s}{16\pi^2}\frac{1}{M^{10}}\frac{24}{7!}. \end{aligned}$$

Solve for the coefficients:

$$C_1^{(2,6)} = -\frac{c_s}{16\pi^2}\frac{1}{M^{10}}\frac{1}{42}, \qquad C_2^{(2,6)} = -\frac{c_s}{16\pi^2}\frac{1}{M^{10}}\frac{4}{105}, \qquad C_3^{(2,6)} = -\frac{c_s}{16\pi^2}\frac{1}{M^{10}}\frac{3}{140},$$

Universal One-loop Effective Action upto D8

$$
\begin{aligned}
\mathcal{L}_{\text{eff}}^{d\leq 8} =& \frac{c_s}{(4\pi)^2} M^4 \left[-\frac{1}{2}\left(\ln\left[\frac{M^2}{\mu^2}\right]-\frac{3}{2}\right)\right] + \frac{c_s}{(4\pi)^2}\,\text{tr}\left\{ M^2\left[-\left(\ln\left[\frac{M^2}{\mu^2}\right]-1\right)U\right]\right.\\
&+ M^0\,\frac{1}{2}\left[-\ln\left[\frac{M^2}{\mu^2}\right]U^2 - \frac{1}{6}\ln\left[\frac{M^2}{\mu^2}\right](G_{\mu\nu})^2\right]\\
&+ \frac{1}{M^2}\frac{1}{6}\left[-U^3 - \frac{1}{2}(P_\mu U)^2 - \frac{1}{2}U\,(G_{\mu\nu})^2 - \frac{1}{10}(J_\nu)^2 + \frac{1}{15}G_{\mu\nu}\,G_{\nu\rho}\,G_{\rho\mu}\right]\\
&+ \frac{1}{M^4}\frac{1}{24}\left[U^4 - U^2(P^2U) + \frac{4}{5}U^2(G_{\mu\nu})^2 + \frac{1}{5}(U\,G_{\mu\nu})^2 + \frac{1}{5}(P^2U)^2\right.\\
&\qquad - \frac{2}{5}U\,(P_\mu U)\,J_\mu + \frac{2}{5}U(J_\mu)^2 - \frac{2}{15}(P^2U)(G_{\rho\sigma})^2 + \frac{1}{35}(P_\nu J_\mu)^2\\
&\qquad - \frac{4}{15}U\,G_{\mu\nu}G_{\nu\rho}G_{\rho\mu} - \frac{8}{15}(P_\mu P_\nu U)\,G_{\rho\mu}G_{\rho\nu} + \frac{16}{105}G_{\mu\nu}J_\mu J_\nu\\
&\qquad + \frac{1}{420}(G_{\mu\nu}G_{\rho\sigma})^2 + \frac{17}{210}(G_{\mu\nu})^2(G_{\rho\sigma})^2 + \frac{2}{35}(G_{\mu\nu}G_{\nu\rho})^2\\
&\qquad \left. + \frac{1}{105}G_{\mu\nu}G_{\nu\rho}G_{\rho\sigma}G_{\sigma\mu} + \frac{16}{105}(P_\mu J_\nu)G_{\nu\sigma}G_{\sigma\mu}\right]\\
&+ \frac{1}{M^6}\frac{1}{60}\left[-U^5 + 2\,U^3(P^2U) + U^2(P_\mu U)^2 - \frac{2}{3}U^2G_{\mu\nu}U\,G_{\mu\nu} - U^3(G_{\mu\nu})^2\right.\\
&\qquad + \frac{1}{3}U^2(P_\mu U)J_\mu - \frac{1}{3}U\,(P_\mu U)(P_\nu U)\,G_{\mu\nu} - \frac{1}{3}U^2J_\mu(P_\mu U)\\
&\qquad - \frac{1}{3}U\,G_{\mu\nu}(P_\mu U)(P_\nu U) - U\,(P^2U)^2 - \frac{2}{3}(P^2U)(P_\nu U)^2 - \frac{1}{7}((P_\mu U)G_{\mu\alpha})^2\\
&\qquad + \frac{2}{7}U^2G_{\mu\nu}G_{\nu\alpha}G_{\alpha\mu} + \frac{8}{21}U\,G_{\mu\nu}U\,G_{\nu\alpha}G_{\alpha\mu} - \frac{4}{7}U^2(J_\mu)^2 - \frac{3}{7}(U\,J_\mu)^2\\
&\qquad + \frac{4}{7}U\,(P^2U)(G_{\mu\nu})^2 + \frac{4}{7}(P^2U)U(G_{\mu\nu})^2 - \frac{2}{7}U\,(P_\mu U)J_\nu G_{\mu\nu}\\
&\qquad - \frac{2}{7}(P_\mu U)U\,G_{\mu\nu}J_\nu - \frac{4}{7}U\,(P_\mu U)G_{\mu\nu}J_\nu - \frac{4}{7}(P_\mu U)U\,J_\nu G_{\mu\nu}\\
&\qquad + \frac{4}{21}U\,G_{\mu\nu}(P^2U)G_{\mu\nu} + \frac{11}{21}(P_\alpha U)^2(G_{\mu\nu})^2 - \frac{10}{21}(P_\mu U)J_\nu U\,G_{\mu\nu}\\
&\qquad - \frac{10}{21}(P_\mu U)G_{\mu\nu}U\,J_\nu - \frac{2}{21}(P_\mu U)(P_\nu U)G_{\mu\alpha}G_{\alpha\nu} + \frac{10}{21}(P_\nu U)(P_\mu U)G_{\mu\alpha}G_{\alpha\nu}\\
&\qquad - \frac{1}{7}(G_{\alpha\mu}(P_\mu U))^2 - \frac{1}{42}((P_\alpha U)G_{\mu\nu})^2 - \frac{1}{14}(P_\mu P^2U)^2 - \frac{4}{21}(P^2U)(P_\mu U)J_\mu\\
&\qquad \left. + \frac{4}{21}(P_\mu U)(P^2U)J_\mu + \frac{2}{21}(P_\mu U)(P_\nu U)(P_\mu J_\nu) - \frac{2}{21}(P_\nu U)(P_\mu U)(P_\mu J_\nu)\right]\\
&+ \frac{1}{M^8}\frac{1}{120}\left[U^6 - 3\,U^4(P^2U) - 2\,U^3(P_\nu U)^2 + \frac{12}{7}U^2(P_\mu P_\nu U)(P_\nu P_\mu U)\right.\\
&\quad + \frac{26}{7}(P_\mu P_\nu U)U\,(P_\mu U)(P_\nu U) + \frac{26}{7}(P_\mu P_\nu U)(P_\mu U)(P_\nu U)U + \frac{9}{7}(P_\mu U)^2(P_\nu U)^2\\
&\qquad + \frac{9}{7}U\,(P_\mu P_\nu U)U\,(P_\nu P_\mu U) + \frac{17}{14}((P_\mu U)(P_\nu U))^2 + \frac{8}{7}U^3G_{\mu\nu}U\,G_{\mu\nu}\\
&\qquad + \frac{5}{7}U^4(G_{\mu\nu})^2 + \frac{18}{7}G_{\mu\nu}(P_\mu U)U^2(P_\nu U) + \frac{9}{14}(U^2G_{\mu\nu})^2\\
&\qquad + \frac{18}{7}G_{\mu\nu}U\,(P_\mu U)(P_\nu U)U + \frac{18}{7}(P_\mu P_\nu U)(P_\mu U)U\,(P_\nu U)\\
&\qquad + \left(\frac{8}{7}G_{\mu\nu}U\,(P_\mu U)U\,(P_\nu U) + \frac{26}{7}G_{\mu\nu}(P_\mu U)U\,(P_\nu U)U\right)\\
&\qquad \left. + \left(\frac{24}{7}G_{\mu\nu}(P_\mu U)(P_\nu U)U^2 - \frac{2}{7}G_{\mu\nu}U^2(P_\mu U)(P_\nu U)\right)\right]\\
&+ \frac{1}{M^{10}}\frac{1}{210}\left[-U^7 - 5\,U^4(P_\nu U)^2 - 8\,U^3(P_\mu U)U(P_\mu U) - \frac{9}{2}(U^2(P_\mu U))^2\right]\\
&\left. + \frac{1}{M^{12}}\frac{1}{336}\left[U^8\right]\right\}. \qquad (7.1)
\end{aligned}
$$

$$U_{ij} = \frac{\delta^2\mathcal{L}_{UV}}{\delta\Phi_i\delta\Phi_j}$$

$$G_{\mu\nu} = [P_\mu, P_\nu],$$

$$J_\mu = P_\nu G_{\nu\mu} = [P_\nu, [P_\nu, P_\mu]],$$

Eff. action : DR + MS-bar,

μ is the matching scale,

Some nice features of this formulation

1. It’s universal, i.e., does not depend on the specific form of the UV theory as well as IR DoFs

2. Equally applicable for LEFT, SMEFT or any other effective theory at any scale

3. Can be easily implemented in matching tools like CoDEx, or any other to get the WCs

Thanks for your attention!