2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740202, ...

Renato Fonseca
renatofonseca@ugr.es
High-Energy Physics Group, University of Granada

EFT Foundations and Tools, MITP, Mainz, 6 September 2023

Some comments:

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1 = **Number of (real) SMEFT terms of dimension 5, 6, 7, 8, ...**

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1 = **Number of (real) SMEFT terms of dimension 5, 6, 7, 8, ...**

2 ≠

2, 84, 30, 993, 560, 15456, 11962, 261485, . . . :
Higher dimension operators in the SM EFT

Brian Henning,[a] Xiaochuan Lu,[b] Tom Melia[c,d] and Hitoshi Murayama[c,d,e]

Some comments:

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2, 84, 30, 993, 560, 15456, 11962, 261485,. . .: Higher dimension operators in the SM EFT

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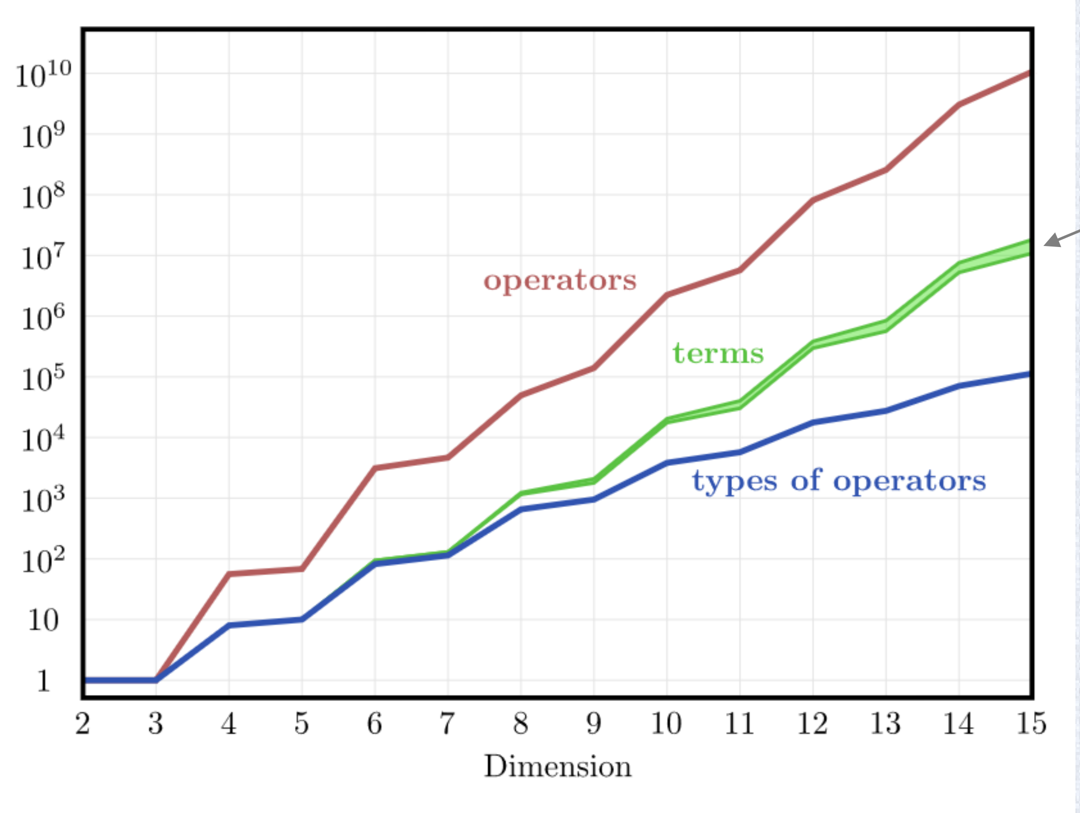
	B	$204 + 5$	$895 + 15$	$895(36971)$, $n_g = 1(3)$
	$\not{B}$	$19 + 3$	$98 + 22$	$98(7836)$, $n_g = 1(3)$
	Total	$223 + 8$	$993 + 37$	$993(44807)$, $n_g = 1(3)$

3 Murphy basis has 1030>1019 terms

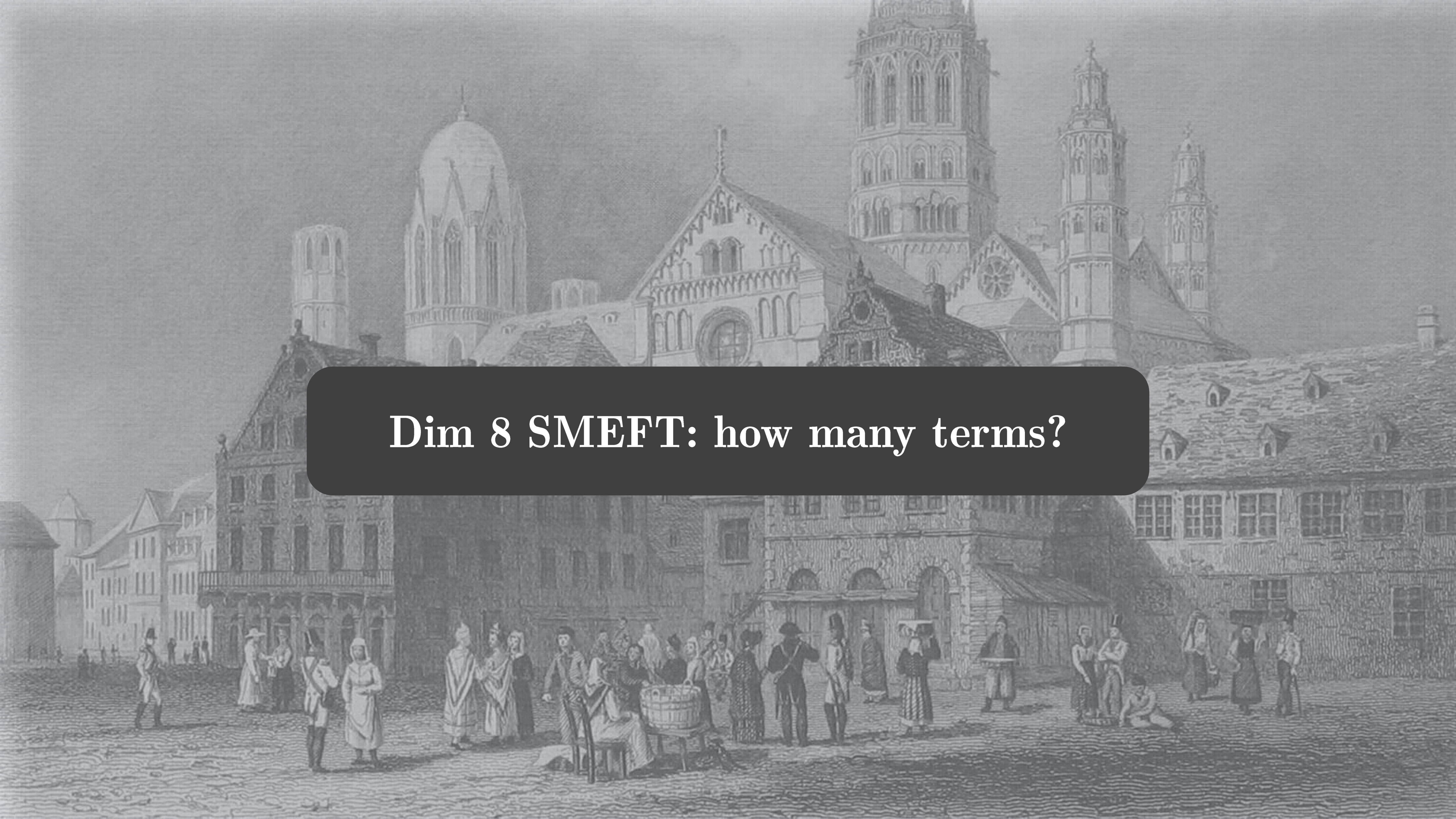
2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740202, ...

4

I have provided bounds on these numbers in [RF 1907.12584]. But they are not quite correct.

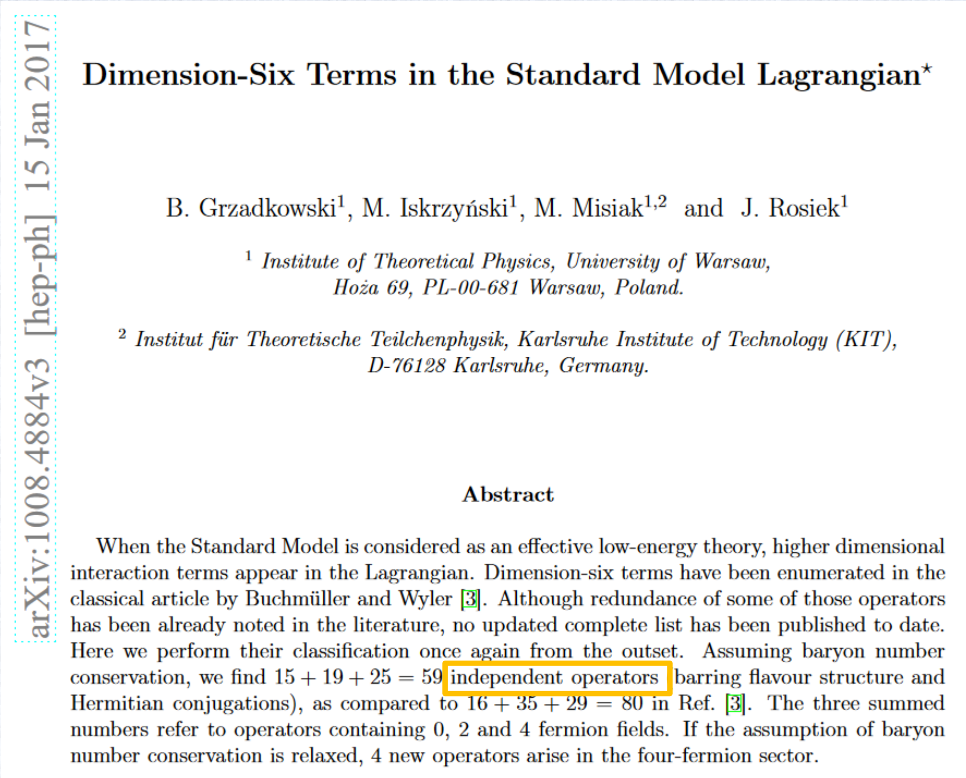


Actual number sometimes falls a bit below this band



Dim 8 SMEFT: how many terms?

Operators, terms, types of operators


arXiv:1008.4884v3 [hep-ph] 15 Jan 2017

Dimension-Six Terms in the Standard Model Lagrangian*

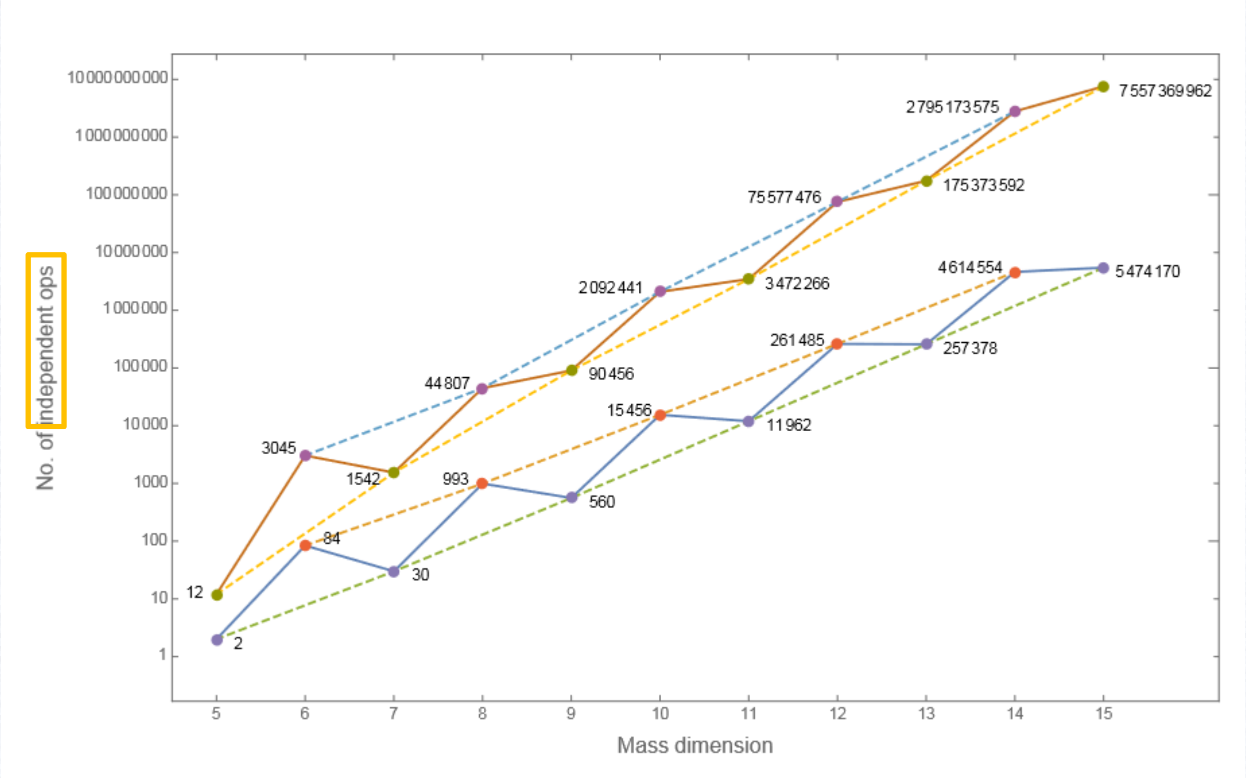
B. Grzadkowski[1], M. Iskrzyński[1], M. Misiak[1,2] and J. Rosiek[1]

[1] *Institute of Theoretical Physics, University of Warsaw, Hoża 69, PL-00-681 Warsaw, Poland.*

[2] *Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT), D-76128 Karlsruhe, Germany.*

Abstract

When the Standard Model is considered as an effective low-energy theory, higher dimensional interaction terms appear in the Lagrangian. Dimension-six terms have been enumerated in the classical article by Buchmüller and Wyler [3]. Although redundance of some of those operators has been already noted in the literature, no updated complete list has been published to date. Here we perform their classification once again from the outset. Assuming baryon number conservation, we find $15 + 19 + 25 = 59$ independent operators barring flavour structure and Hermitian conjugations), as compared to $16 + 35 + 29 = 80$ in Ref. [3]. The three summed numbers refer to operators containing 0, 2 and 4 fermion fields. If the assumption of baryon number conservation is relaxed, 4 new operators arise in the four-fermion sector.



[Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884]

[Henning, Lu, Melia, Murayama 1512.03433]

Different authors call "operator" to different things:
does SMEFT at dimension 6 have 3045 real operators or 84?

Operators, terms, types of operators

In [RF 1907.12584] I suggested using the words
"operator", "(Lagrangian) terms" and "type of operator/term" as follows

$(\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$

$n^2(n^2+1)/2$ operators { 1 term { 1 type of operator ($l^* l^* l\, l$)

$2n^4$ operators { 2 terms { 1 type of operator ($l^* l\, q^* q$)

Counting of operators with Hilbert series (ECO,...) or traditional methods (Sym2Int, BasisGen) works well ✓

Counting of types of operators/terms is quite simple ✓

[RF 1703.05221] [Criado 1901.03501] [RF 1907.12584]
[Marinissen, Rahn, Waalewijn 2004.09521]

Counting terms for is not trivial, although they might arguably be more important to count than operators. E.g.: How many terms in SMEFT at dim 8?

As far as I known, Sym2Int is the only program to count terms of an EFT

History of counting terms

Counting terms is not easy!

Take the QQQL-type of interactions (more on this later)

History of counting terms

Counting terms is not easy!

Take the **QQQL**-type of interactions (more on this later)

$$O^{(3)}_{abcd} = (\bar{q}^{C}_{i\alpha aL} q_{j\beta bL})(\bar{q}^{C}_{k\gamma cL} l_{ldL})\epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{kl}\,, \qquad (3)$$

$$O^{(4)}_{abcd} = (\bar{q}^{C}_{i\alpha aL} q_{j\beta bL})(\bar{q}^{C}_{k\gamma cL} l_{ldL})\epsilon_{\alpha\beta\gamma} \times (\vec{\tau}\epsilon)_{ij} \cdot (\vec{\tau}\epsilon)_{kl}\,, \qquad (4)$$

[Weinberg 1979] [Wilczek, Zee 1979]

History of counting terms

Counting terms is not easy!

Take the QQQL-type of interactions (more on this later)

$$O_{abcd}^{(3)} = (\bar{q}_{i\alpha aL}^{C} q_{j\beta bL})(\bar{q}_{k\gamma cL}^{C} l_{ldL})\epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{kl}, \quad (3)$$

$$O_{abcd}^{(4)} = (\bar{q}_{i\alpha aL}^{C} q_{j\beta bL})(\bar{q}_{k\gamma cL}^{C} l_{ldL})\epsilon_{\alpha\beta\gamma} \times (\vec{\tau}\epsilon)_{ij}\cdot(\vec{\tau}\epsilon)_{kl}, \quad (4)$$

[Weinberg 1979] [Wilczek, Zee 1979]

the operators being considered. The operators $O^{(3)}$ and $O^{(4)}$ can be written as the symmetric and antisymmetric part (in the first two generation indices) of a single operator. We therefore find it most convenient to define an operator

$$\tilde{O}_{abcd}^{(4)} = (q_{\alpha i a L} q_{\beta j b L})(q_{\gamma k c L} l_{l d L})\epsilon_{\alpha\beta\gamma}\epsilon_{il}\epsilon_{jk} \quad (1.7)$$

and note that[6]

$$O_{abcd}^{(3)} = -(\tilde{O}_{abcd}^{(4)} + \tilde{O}_{bacd}^{(4)}) \quad (1.8)$$

and

$$O_{abcd}^{(4)} = -(\tilde{O}_{abcd}^{(4)} - \tilde{O}_{bacd}^{(4)}). \quad (1.9)$$

With the relations (1.8) and (1.9) the effective Hamiltonian for nucleon decay can be expressed in terms of only four types of operators:

$$O_{abcd}^{(1)},\ O_{abcd}^{(2)},\ \tilde{O}_{abcd}^{(4)},\ \text{and}\ O_{abcd}^{(5)}.$$

[Abbott, Wise 1980]

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$$O^{(3)}_{abcd} = (\bar{q}^{C}_{i\alpha aL} q_{j\beta bL})(\bar{q}^{C}_{k\gamma cL} l_{ldL})\epsilon_{\alpha\beta\gamma}\epsilon_{ij}\epsilon_{kl}, \qquad (3)$$

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[Abbott, Wise 1980]

	B-violating
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d^{\alpha}_{p})^{T}Cu^{\beta}_{r}\right]\left[(q^{\gamma j}_{s})^{T}Cl^{k}_{t}\right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q^{\alpha j}_{p})^{T}Cq^{\beta k}_{r}\right]\left[(u^{\gamma}_{s})^{T}Ce_{t}\right]$
$Q^{(1)}_{qqq}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q^{\alpha j}_{p})^{T}Cq^{\beta k}_{r}\right]\left[(q^{\gamma m}_{s})^{T}Cl^{n}_{t}\right]$
$Q^{(3)}_{qqq}$	$\varepsilon^{\alpha\beta\gamma}(\tau^{I}\varepsilon)_{jk}(\tau^{I}\varepsilon)_{mn}\left[(q^{\alpha j}_{p})^{T}Cq^{\beta k}_{r}\right]\left[(q^{\gamma m}_{s})^{T}Cl^{n}_{t}\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d^{\alpha}_{p})^{T}Cu^{\beta}_{r}\right]\left[(u^{\gamma}_{s})^{T}Ce_{t}\right]$

[Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884 – v1 2010]

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Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(q^{\gamma j}_s)^T C l^k_t\right]$
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Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q^{\alpha j}_p)^T C q^{\beta k}_r\right]\left[(q^{\gamma m}_s)^T C l^n_t\right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d^\alpha_p)^T C u^\beta_r\right]\left[(u^\gamma_s)^T C e_t\right]$

[Grzadkowski, Iskrzynski, Misiak, Rosiek 1008.4884 – v3 2017]

The numbers for SMEFT

* Kinetic terms were not included in the counting

Dimension	# operators	# terms	# types of operators
2 *	1	1	1
3	0	0	0
4	55	7	7
5	12	2	2
6	3045	84	72
7	1542	36	32
8	44807	1025 to 1102	541
9	90456	628 to 852	296
10	2092441	15769 to 18345	2868
11	3472266	12726 to 19666	1898
12	75577476	266031 to 343511	11942
13	175373592	266802 to 457898	9824
14	2795173575	4669533 to 6717444	43158
15	7557369962	5599846 to 10567408	42206

[RF 1907.12584]

2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740202, ...

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Dimension	# operators	# terms		# types of operators
2*	1	1		1
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4	55	[illegible]		[illegible]
5	12	[illegible]		[illegible]
6	3045	[illegible]		[illegible]
7	1542	36		32
8	44807	1025 to 1102	1019	[illegible]
9	90456	628 to 852	624	[illegible]
10	2092441	15769 to 18345	15666	[illegible]
11	3472266	12726 to 19666	12620	[illegible]
12	75577476	266031 to 343511	264389	[illegible]
13	175373592	266802 to 457898	269026	[illegible]
14	2795173575	4669533 to 6717444	4669553	[illegible]
15	7557369962	5599846 to 10567408	5740202	[illegible]

I now think these are the correct numbers which sometimes fall outside the ranges in [RF 1907.12584]

[RF 1907.12584]

[Murphy 2005.00059] has 1030 real terms, inside the range but in excess of 1019.
The extra terms might have been included due to the mistake in [RF 1907.12584]

Dim 8 SMEFT: excess terms

Sym2Int code (delegates to GroupMath the group theory computations): [RF 2011.01764]

```
gaugeGroup[SM] ^= {SU3, SU2, U1};

fld1 = {"u", {3, 1, 2/3}, "R", "C", n};
fld2 = {"d", {3, 1, -1/3}, "R", "C", n};
fld3 = {"Q", {3, 2, 1/6}, "L", "C", n};
fld4 = {"e", {1, 1, -1}, "R", "C", n};
fld5 = {"L", {1, 2, -1/2}, "L", "C", n};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, fld6};

savedResults = GenerateListOfCouplings[SM, MaxOrder → 8, Verbose → False];
```

Define the model (SMEFT)

```
resultsMod = Cases[savedResults, x_ /; x[[3]] == 8];
resultsMod = Join[#, {Null, Null}] & /@ resultsMod;
resultsMod[[All, 10]] = Sort /@ (resultsMod[[All, 2]] /. x_Integer :→ Which[x == 0, "D", Abs[x] == 6, "H", Abs[x] ≤ 5, "ψ", Abs[x] ≤ 10, "X"]);
resultsMod[[All, 11]] = Total /@ (resultsMod[[All, 2]] /. x_Integer :→ Sign[x] Which[0 < Abs[x] ≤ 3, 1/3, True, 0]);

data =
  {Times @@ #[[1, 10]], #[[1, 11]], #[[All, 6]].(#[[All, 4]] /. {True → 1, False → 2}),
    Simplify[#[[All, 5]].(#[[All, 4]] /. {True → 1, False → 2})]} & /@ SortBy[GatherBy[resultsMod, #[[{10, 11}]] &], #[[1, {10, 11}]] &];
Grid[Prepend[data, {"Op Class", "ΔB", "Terms", "Operators"}], Frame → All, FrameStyle → LightGray]
```

Some code to parse and compile the results in a table

Dim 8 SMEFT: excess terms

Sym2Int code (delegates to GroupMath the group theory computations): [RF 2011.01764]

```
gaugeGroup[SM] ^= {SU3, SU2, U1};

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fld3 = {"Q", {3, 2, 1/6}, "L", "C", n};
fld4 = {"e", {1, 1, -1}, "R", "C", n};
fld5 = {"L", {1, 2, -1/2}, "L", "C", n};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, fld6};

savedResults = GenerateListOfCouplings[SM, MaxOrder → 8, Verbose → False];
```

```
resultsMod = Cases[savedResults, x_ /; x[[3]] == 8];
resultsMod = Join[#, {Null, Null}] & /@ resultsMod;
resultsMod[[All, 10]] = Sort /@ (resultsMod[[All, 2]] /. x_Integer :> Which[x == 0, "D", Abs[x] == 6, "H", Abs[x] ≤ 5, "ψ", Abs[x] ≤ 10, "X"]);
resultsMod[[All, 11]] = Total /@ (resultsMod[[All, 2]] /. x_Integer :> Sign[x] Which[0 < Abs[x] ≤ 3, 1/3, True, 0]);

data =
  {Times @@ #[[1, 10]], #[[1, 11]], #[[All, 6]].(#[[All, 4]] /. {True → 1, False → 2}),
     Simplify[#[[All, 5]].(#[[All, 4]] /. {True → 1, False → 2})]} & /@ SortBy[GatherBy[resultsMod, #[[{10, 11}]] &], #[[1, {10, 11}]] &];
Grid[Prepend[data, {"Op Class", "ΔB", "Terms", "Operators"}], Frame → All, FrameStyle → LightGray]
```

Op Class	ΔB	Terms	Operators
X^4	0	43	43
$D X^2 \psi^2$	0	57	$57 n^2$
$H^2 X^3$	0	6	6
$H X^2 \psi^2$	0	96	$96 n^2$
$X \psi^4$	0	168	$4 n^2 (-1 + 40 n^2)$
$X \psi^4$	1	48	$2 n^3 (1 + 21 n)$
$D^2 H^2 X^2$	0	18	18
$D^2 H X \psi^2$	0	48	$48 n^2$
$D^2 \psi^4$	0	55	$\frac{11}{2} (n^2 + 9 n^4)$
$D^2 \psi^4$	1	12	$n^3 (-1 + 11 n)$
$D H^2 X \psi^2$	0	92	$92 n^2$
$D H \psi^4$	0	136	$n^3 (-1 + 135 n)$
$D H \psi^4$	1	32	$n^3 (3 + 29 n)$
$H^4 X^2$	0	10	10
$H^3 X \psi^2$	0	22	$22 n^2$
$H^2 \psi^4$	0	75	$n^2 (7 + n + 67 n^2)$
$H^2 \psi^4$	1	18	$\frac{1}{3} n^2 (2 - 9 n + 43 n^2)$
$D^3 H^2 \psi^2$	0	16	$16 n^2$
$D^2 H^4 X$	0	6	6
$D^2 H^3 \psi^2$	0	36	$36 n^2$
$D H^4 \psi^2$	0	13	$13 n^2$
$H^5 \psi^2$	0	6	$6 n^2$
$D^4 H^4$	0	3	3
$D^2 H^6$	0	2	2
H^8	0	1	1

So ... the

Dim 8 SMEFT: excess terms

[Murphy 2005.00059]

Sym2Int code (delegates to ... ons): [RF 2011.01764]

```
gaugeGroup[SM] ^= {SU3, SU2, U1};

fld1 = {"u", {3, 1, 2/3}, "R", "C", n};
fld2 = {"d", {3, 1, -1/3}, "R", "C", n};
fld3 = {"Q", {3, 2, 1/6}, "L", "C", n};
fld4 = {"e", {1, 1, -1}, "R", "C", n};
fld5 = {"L", {1, 2, -1/2}, "L", "C", n};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, flc

savedResults = GenerateListOfCouplings[SM, Max
```

```
resultsMod = Cases[savedResults, x_ /; x[[3]] =
resultsMod = Join[#, {Null, Null}] & /@ resultsl
resultsMod[[All, 10]] = Sort /@ (resultsMod[[Al ... Abs[x] ≤ 10, "X"]);
resultsMod[[All, 11]] = Total /@ (resultsMod[[A

data =
  {Times @@ #[[1, 10]], #[[1, 11]], #[[All, 6]
     Simplify[#[[All, 5]].(#[[All, 4]] /. {Tr ... #[[1, {10, 11}]] &];
Grid[Prepend[data, {"Op Class", "ΔB", "Terms",
```

#	Class	N_{type}	N_{term}	N_{op} [10]	Table(s)
1	X^4	7	43	43	2
2	H^8	1	1	1	2
3	H^6D^2	1	2	2	2
4	H^4D^4	1	3	3	2
5	X^3H^2	3	6	6	3
6	X^2H^4	5	10	10	3
7	$X^2H^2D^2$	4	18	18	3
8	XH^4D^2	2	6	6	3
9	ψ^2X^2H	16	96	$96n_g^2$	4
10	ψ^2XH^3	8	22	$22n_g^2$	5
11	$\psi^2H^2D^3$	6	16	$16n_g^2$	5
12	ψ^2H^5	3	6	$6n_g^2$	5
13	ψ^2H^4D	6	13	$13n_g^2$	5
14	ψ^2X^2D	21	57	$57n_g^2$	6, 7
15	ψ^2XH^2D	16	92	$92n_g^2$	7, 8
16	ψ^2XHD^2	8	48	$48n_g^2$	9
17	$\psi^2H^3D^2$	3	36	$36n_g^2$	9
18(B)	ψ^4H^2	19	75 + 1	$n_g^2(67n_g^2+n_g+7)$	10, 11
18($\not{B}$)		4 + 3	12 + 8	$\frac{1}{3}n_g^2(43n_g^2-9n_g+2)$	10
19(B)	ψ^4X	40 + 5	156 + 12	$4n_g^2(40n_g^2-1)$	12, 13, 14
19($\not{B}$)		4	44 + 12	$2n_g^3(21n_g+1)$	15
20(B)	ψ^4HD	16	134 + 2	$n_g^3(135n_g-1)$	16, 17
20($\not{B}$)		7	32	$n_g^3(29n_g+3)$	17
21(B)	ψ^4D^2	18	55	$\frac{11}{2}n_g^2(9n_g^2+1)$	10, 18
21($\not{B}$)		4	10 + 2	$n_g^3(11n_g-1)$	10
	B	204 + 5	895 + 15	895(36971), $n_g=1(3)$	
	$\not{B}$	19 + 3	98 + 22	98(7836), $n_g=1(3)$	
	Total	223 + 8	993 + 37	993(44807), $n_g=1(3)$	

Op Class	ΔB	Terms	Operators
X^4	0	43	43
$D X^2 \psi^2$	0	57	$57 n^2$
$H^2 X^3$	0	6	6
$H X^2 \psi^2$	0	96	$96 n^2$
$X \psi^4$	0	168	$4 n^2 (-1 + 40 n^2)$
$X \psi^4$	1	48	$2 n^3 (1 + 21 n)$
$D^2 H^2 X^2$	0	18	18
$D^2 H X \psi^2$	0	48	$48 n^2$
$D^2 \psi^4$	0	55	$\frac{11}{2} (n^2 + 9 n^4)$
$D^2 \psi^4$	1	12	$n^3 (-1 + 11 n)$
$D H^2 X \psi^2$	0	92	$92 n^2$
$D H \psi^4$	0	136	$n^3 (-1 + 135 n)$
$D H \psi^4$	1	32	$n^3 (3 + 29 n)$
$H^4 X^2$	0	10	10
$H^3 X \psi^2$	0	22	$22 n^2$
$H^2 \psi^4$	0	75	$n^2 (7 + n + 67 n^2)$
$H^2 \psi^4$	1	18	$\frac{1}{3} n^2 (2 - 9 n + 43 n^2)$
$D^3 H^2 \psi^2$	0	16	$16 n^2$
$D^2 H^4 X$	0	6	6
$D^2 H^3 \psi^2$	0	36	$36 n^2$
$D H^4 \psi^2$	0	13	$13 n^2$
$H^5 \psi^2$	0	6	$6 n^2$
$D^4 H^4$	0	3	3
$D^2 H^6$	0	2	2
H^8	0	1	1

So ... the

Dim 8 SMEFT: excess terms

[Murphy 2005.00059]

#	Class	$N_{\rm type}$	$N_{\rm term}$	$N_{\rm op}$ [10]	Table(s)
1	X^4	7	43	43	2
2	H^8	1	1	1	2
3	H^6D^2	1	2	2	2
4	H^4D^4	1	3	3	2
5	X^3H^2	3	6	6	3
6	X^2H^4	5	10	10	3
7	$X^2H^2D^2$	4	18	18	3
8	XH^4D^2	2	6	6	3
9	ψ^2X^2H	16	96	$96n_g^2$	4
10	ψ^2XH^3	8	22	$22n_g^2$	5
11	$\psi^2H^2D^3$	6	16	$16n_g^2$	5
12	ψ^2H^5	3	6	$6n_g^2$	5
13	ψ^2H^4D	6	13	$13n_g^2$	5
14	ψ^2X^2D	21	57	$57n_g^2$	6, 7
15	ψ^2XH^2D	16	92	$92n_g^2$	7, 8
16	ψ^2XHD^2	8	48	$48n_g^2$	9
17	$\psi^2H^3D^2$	3	36	$36n_g^2$	9
18(B)	ψ^4H^2	19	75 + 1	$n_g^2(67n_g^2+n_g+7)$	10, 11
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19($\not B$)		4	44 + 12	$2n_g^3(21n_g+1)$	15
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21($\not B$)		4	10 + 2	$n_g^3(11n_g-1)$	10
	B	204 + 5	895 + 15	895(36971), $n_g=1(3)$	
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	Total	223 + 8	993 + 37	993(44807), $n_g=1(3)$	

Sym2Int code (delegates to… …ons): [RF 2011.01764]

```
gaugeGroup[SM] ^= {SU3, SU2, U1};

fld1 = {"u", {3, 1, 2/3}, "R", "C", n};
fld2 = {"d", {3, 1, -1/3}, "R", "C", n};
fld3 = {"Q", {3, 2, 1/6}, "L", "C", n};
fld4 = {"e", {1, 1, -1}, "R", "C", n};
fld5 = {"L", {1, 2, -1/2}, "L", "C", n};
fld6 = {"H", {1, 2, 1/2}, "S", "C", 1};
fields[SM] ^= {fld1, fld2, fld3, fld4, fld5, flc

savedResults = GenerateListOfCouplings[SM, Max
```

```
resultsMod = Cases[savedResults, x_ /; x[[3]] =
resultsMod = Join[#, {Null, Null}] & /@ results
resultsMod[[All, 10]] = Sort /@ (resultsMod[[Al          Abs[x] ≤ 10, "X"]);
resultsMod[[All, 11]] = Total /@ (resultsMod[[/

data =
  {Times @@ #[[1, 10]], #[[1, 11]], #[[All, 6]              #[[1, {10, 11}]] &];
    Simplify[#[[All, 5]].(#[[All, 4]] /. {Tr
Grid[Prepend[data, {"Op Class", "ΔB", "Terms",
```

So… …the

Op Class	ΔB	Terms	Operators
X^4	0	43	43
$D X^2 \psi^2$	0	57	$57 n^2$
$H^2 X^3$	0	6	6
$H X^2 \psi^2$	0	96	$96 n^2$
$X \psi^4$	0	168	$4 n^2 (-1+40 n^2)$
$X \psi^4$	1	48	$2 n^3 (1+21 n)$
$D^2 H^2 X^2$	0	18	18
$D^2 H X \psi^2$	0	48	$48 n^2$
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$D H^4 \psi^2$	0	13	$13 n^2$
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$D^4 H^4$	0	3	3
$D^2 H^6$	0	2	2
H^8	0	1	1

Dim 8 SMEFT: excess terms

$\mathbf{18 : (\bar{L}L)(\bar{L}L)H^2}$	
$Q^{(1)}_{l^4H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{l}_s\gamma_\mu l_t)(H^\dagger H)$
$Q^{(2)}_{l^4H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{l}_s\gamma_\mu\tau^I l_t)(H^\dagger\tau^I H)$
$Q^{(1)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu q_t)(H^\dagger H)$
$Q^{(2)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger\tau^I H)$
$Q^{(3)}_{q^4H^2}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger H)$
$Q^{(1)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma_\mu q_t)(H^\dagger H)$
$Q^{(2)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma_\mu q_t)(H^\dagger\tau^I H)$
$Q^{(3)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger H)$
$Q^{(4)}_{l^2q^2H^2}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma_\mu\tau^I q_t)(H^\dagger\tau^I H)$
$Q^{(5)}_{l^2q^2H^2}$	$\epsilon^{IJK}(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma_\mu\tau^J q_t)(H^\dagger\tau^K H)$
$Q^{(5)}_{q^4H^2}$	$\epsilon^{IJK}(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{q}_s\gamma_\mu\tau^J q_t)(H^\dagger\tau^K H)$

3 Q*Q*QQH*H terms suffice

$\mathbf{18(\not{B}) : \psi^4H^2 + h.c.}$	
$Q^{(1)}_{lqudH^2}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(d^\alpha_p Cu^\beta_r)(q^{j\gamma}_s Cl^k_t)(H^\dagger H)$
$Q^{(2)}_{lqudH^2}$	$\epsilon_{\alpha\beta\gamma}(\epsilon\tau^I)_{jk}(d^\alpha_p Cu^\beta_r)(q^{j\gamma}_s Cl^k_t)(H^\dagger\tau^I H)$
$Q_{eq^2uH^2}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(q^{j\alpha}_p Cq^{m\beta}_r)(u^\gamma_s Ce_t)(H^\dagger_m H^k)$
$Q^{(1)}_{lq^3H^2}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{mn}\epsilon_{jk}(q^{m\alpha}_p Cq^{j\beta}_r)(q^{k\gamma}_s Cl^n_t)(H^\dagger H)$
$Q^{(2)}_{lq^3H^2}$	$\epsilon_{\alpha\beta\gamma}(\epsilon\tau^I)_{mn}\epsilon_{jk}(q^{m\alpha}_p Cq^{j\beta}_r)(q^{k\gamma}_s Cl^n_t)(H^\dagger\tau^I H)$
$Q_{eu^2dH^2}$	$\epsilon_{\alpha\beta\gamma}(d^\alpha_p Cu^\beta_r)(u^\gamma_s Ce_t)(H^\dagger H)$
$Q^{(3)}_{lq^3H^2}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{mn}(\epsilon\tau^I)_{jk}(q^{m\alpha}_p Cq^{j\beta}_r)(q^{k\gamma}_s Cl^n_t)(H^\dagger\tau^I H)$
$Q_{lqu^2H^2}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}(l^j_p Cq^{m\alpha}_r)(u^\beta_s Cu^\gamma_t)\widetilde{H}^k\widetilde{H}^n$
$Q_{lqd^2H^2}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}(l^j_p q^{m\alpha}_r)(d^\beta_s Cd^\gamma_t)H^k H^n$
$Q_{eq^2dH^2}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}(e_p d^\alpha_r)(q^{j\beta}_s Cq^{m\gamma}_t)H^k H^n$

2 complex QQQLH*H terms suffice

$\mathbf{19(\not{B}) : \psi^4X + h.c.}$	
$Q^{(1)}_{eq^2uG}$	$(T^A)^\delta_\gamma\epsilon_{\delta\alpha\beta}\epsilon_{jk}(q^{j\alpha}_p C\sigma^{\mu\nu}q^{k\beta}_r)(u^\gamma_s Ce_t)G^A_{\mu\nu}$
$Q^{(2)}_{eq^2uG}$	$(T^A)^\delta_{(\alpha}\epsilon_{\beta)\gamma\delta}\epsilon_{jk}(q^{j\alpha}_p Cq^{k\beta}_r)(u^\gamma_s C\sigma^{\mu\nu}e_t)G^A_{\mu\nu}$
$Q^{(1)}_{eq^2uW}$	$\epsilon_{\alpha\beta\gamma}(\epsilon\tau^I)_{jk}(q^{j\alpha}_p C\sigma^{\mu\nu}q^{k\beta}_r)(u^\gamma_s Ce_t)W^I_{\mu\nu}$
$Q^{(1)}_{eq^2uB}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{jk}(q^{j\alpha}_p Cq^{k\beta}_r)(u^\gamma_s C\sigma^{\mu\nu}e_t)B_{\mu\nu}$
$Q^{(1)}_{lq^3G}$	$(T^A)^\delta_\gamma\epsilon_{\delta\alpha\beta}\epsilon_{mn}\epsilon_{jk}(q^{m\alpha}_p C\sigma^{\mu\nu}q^{j\beta}_r)(q^{k\gamma}_s Cl^n_t)G^A_{\mu\nu}$
$Q^{(2)}_{lq^3G}$	$(T^A)^\delta_{(\alpha}\epsilon_{\beta)\gamma\delta}\epsilon_{mn}\epsilon_{jk}(q^{m\alpha}_p Cq^{j\beta}_r)(q^{k\gamma}_s C\sigma^{\mu\nu}l^n_t)G^A_{\mu\nu}$
$Q^{(1)}_{lq^3W}$	$\epsilon_{\alpha\beta\gamma}(\epsilon\tau^I)_{mn}\epsilon_{jk}(q^{m\alpha}_p Cq^{j\beta}_r)(q^{k\gamma}_s C\sigma^{\mu\nu}l^n_t)W^I_{\mu\nu}$
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$Q^{(1)}_{eu^2dG}$	$(T^A)^\delta_\gamma\epsilon_{\delta\alpha\beta}(d^\alpha_p C\sigma^{\mu\nu}u^\beta_r)(u^\gamma_s Ce_t)G^A_{\mu\nu}$
$Q^{(2)}_{eu^2dG}$	$(T^A)^\delta_\gamma\epsilon_{\delta\alpha\beta}(u^\alpha_p C\sigma^{\mu\nu}u^\beta_r)(d^\gamma_s Ce_t)G^A_{\mu\nu}$
$Q^{(3)}_{eu^2dG}$	$(T^A)^\delta_{(\alpha}\epsilon_{\beta)\gamma\delta}(u^\alpha_p Cu^\beta_r)(d^\gamma_s C\sigma^{\mu\nu}e_t)G^A_{\mu\nu}$
$Q^{(1)}_{eu^2dB}$	$\epsilon_{\alpha\beta\gamma}(d^\alpha_p C\sigma^{\mu\nu}u^\beta_r)(u^\gamma_s Ce_t)B_{\mu\nu}$
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$Q^{(4)}_{lq^3G}$	$(T^A)^\delta_\gamma\epsilon_{\delta\alpha\beta}\epsilon_{mn}\epsilon_{jk}(q^{m\alpha}_p Cq^{j\beta}_r)(q^{k\gamma}_s C\sigma^{\mu\nu}l^n_t)G^A_{\mu\nu}$
$Q^{(3)}_{lq^3W}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{mn}(\epsilon\tau^I)_{jk}(q^{m\alpha}_p Cq^{j\beta}_r)(q^{k\gamma}_s C\sigma^{\mu\nu}l^n_t)W^I_{\mu\nu}$
$Q^{(2)}_{lq^3B}$	$\epsilon_{\alpha\beta\gamma}\epsilon_{mn}\epsilon_{jk}(q^{m\alpha}_p C\sigma^{\mu\nu}q^{j\beta}_r)(q^{k\gamma}_s Cl^n_t)B_{\mu\nu}$

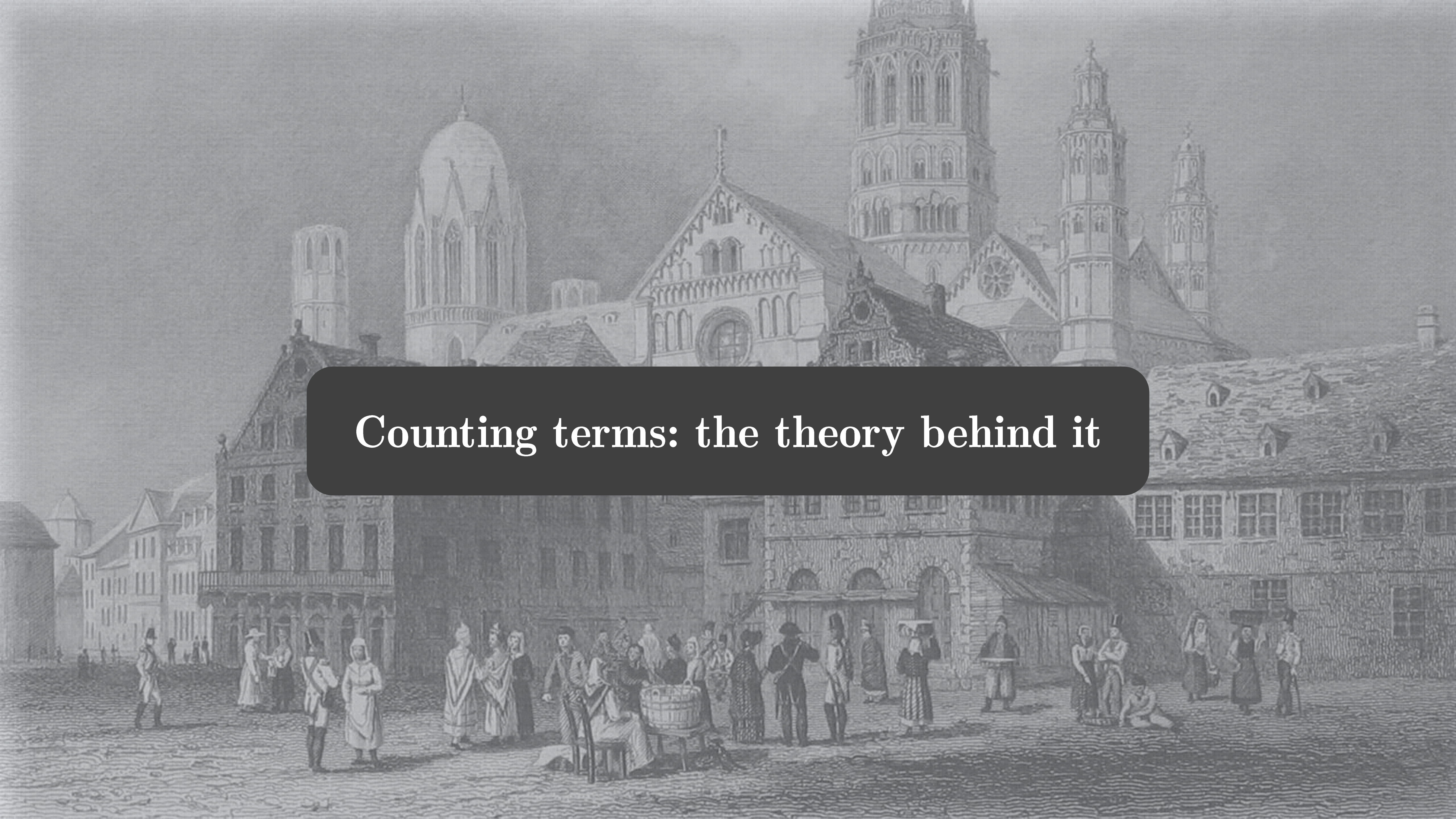
2 complex QQQLG terms suffice

2 complex QQQLW terms suffice

1 complex QQQLB terms suffice

This is just a counting exercise: less terms are possible in these cases. It does not serve as a full check of the validity of this Lagrangian.

Counting terms: the theory behind it



Repeated fields is a complication

When all fields are different

Simple: there must be 1 term for every independent contraction of the Lorentz and gauge indices

For example, *L*LQ*Q*:
- 1 way to contract the Lorentz indices
- 2 ways to contract the *SU(2)* indices of 4 doublets
- 1 way to make the color contractions

$$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$$

$$\left(\bar{l}_p \gamma_\mu \tau^I l_r\right)\left(\bar{q}_s \gamma^\mu \tau^I q_t\right)$$

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We must start thinking about the effect of permutating same fields

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Not part of the Warsaw basis

We must start thinking about the effect of permutating same fields

[more details: Wu-Ki Tung]

Tensors with symmetries

Let's call $|i_1 i_2 \cdots i_n\rangle = |i_1\rangle \cdot |i_2\rangle \cdots |i_n\rangle$ to the basis of the tensor space $\mathrm{V}_m \times \mathrm{V}_m \times \cdots \times \mathrm{V}_m$

V_m is an m-dimensional vector space

We can perform a linear transformation U of some group G each copy V_m :

$$|i_1\rangle \cdot |i_2\rangle \cdots |i_n\rangle \rightarrow U_{j_1 i_1} U_{j_2 i_2} \cdots U_{j_n i_n} |j_1\rangle \cdot |j_2\rangle \cdots |j_n\rangle$$

We can also permute the V_m :

$$|i_1\rangle \cdot |i_2\rangle \cdots |i_n\rangle \rightarrow |i_{\pi^{-1}(1)}\rangle \cdot |i_{\pi^{-1}(2)}\rangle \cdots |i_{\pi^{-1}(n)}\rangle$$

These two transformations commute. Consequence:

$(V_m)^n$ decomposes into irreducible representation of $G \times S_n$ (not just of G)

Under exchange of the 2's

Under exchange of the 3's

SU(2) examples:

$$2 \times 2 \times 3 = 5_S + 1_S + 3_S + 3_A$$

$$2 \times 2 \times 3 \times 3 = 7_{SS} + 2\,(3_{SS}) + 3_{SA} + 3_{AA} + 5_{SS} + 5_{SA} + 5_{AS} + 1_{SA} + 1_{AS}$$

[more details: Wu-Ki Tung]

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$$2 \times 2 \times 2 \times 2 = ???$$

Under exchange of the 2's

Under exchange of the 3's

[more details: Wu-Ki Tung]

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$$2 \times 2 \times 2 \times 2 = 5_{[4]} + 3_{[3,1]} + 1_{[2,2]}$$

Group is *S4*. It has irreps which are no longer just A or S.

[more details: Wu-Ki Tung]

Tensors with symmetries

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V_m is an m-dimensional vector space

We can perform a linear transformation U of some group G each copy V_m :

$$|i_1\rangle \cdot |i_2\rangle \cdots |i_n\rangle \to U_{j_1 i_1} U_{j_2 i_2} \cdots U_{j_n i_n} |j_1\rangle \cdot |j_2\rangle \cdots |j_n\rangle$$

We can also permute the V_m :

$$|i_1\rangle \cdot |i_2\rangle \cdots |i_n\rangle \to |i_{\pi^{-1}(1)}\rangle \cdot |i_{\pi^{-1}(2)}\rangle \cdots |i_{\pi^{-1}(n)}\rangle$$

These two transformations commute. Consequence:

$(V_m)^n$ decomposes into irreducible representation of $G \times S_n$ (not just of G)

SU(2) examples:

Under exchange of the 2's

Under exchange of the 3's

$$2 \times 2 \times 3 = 5_S + 1_S + 3_S + 3_A$$

$$2 \times 2 \times 3 \times 3 = 7_{SS} + 2\,(3_{SS}) + 3_{SA} + 3_{AA} + 5_{SS} + 5_{SA} + 5_{AS} + 1_{SA} + 1_{AS}$$

$$2 \times 2 \times 2 \times 2 = 5_{\square\square\square\square} + 3_{\square\square\square,\square} + 1_{\square\square,\square\square}$$

Group is *S4*. It has irreps which are no longer just A or S.

$5 + 3 + 3 + 3 + 1 + 1$ if we remove the S_4 information

Four SU(2) doublets $\phi\phi\phi\phi$

It is quite clear from here that 0 contractions are possible if all doublets are equal

Let us build the two contractions explicitly, assuming that we have 4 distinct doublets

$$c^{(1)} = \epsilon_{ij}\epsilon_{kl}\phi_i\phi'_j\phi''_k\phi'''_l$$
$$c^{(2)} = \epsilon_{ik}\epsilon_{jl}\phi_i\phi'_j\phi''_k\phi'''_l$$

Fact: Any of the m! permutation of *m can be* generated from two permutations: $1 \rightarrow 2 \rightarrow 1$ and $1 \rightarrow 2 \rightarrow 3 \rightarrow \cdots \rightarrow m \rightarrow 1$

$$\begin{pmatrix} c^{(1)} \\ c^{(2)} \end{pmatrix}_{\phi\leftrightarrow\phi'} = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} c^{(1)} \\ c^{(2)} \end{pmatrix}$$
$$\begin{pmatrix} c^{(1)} \\ c^{(2)} \end{pmatrix}_{\substack{\phi\rightarrow\phi'\rightarrow \\ \phi''\rightarrow\phi'''\rightarrow\phi}} = \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \cdot \begin{pmatrix} c^{(1)} \\ c^{(2)} \end{pmatrix}$$

The effect of any other permutation can be obtained from products of these two matrices

The two matrices above cannot be simultaneously diagonalized. Therefore $c^{(1)}$ and $c^{(2)}$ form a 2-dimensional irreducible representation of S_4

Compare this to the completely symmetric (S) and anti-symmetric (A) representations, which are 1-dimensional: it all boils down to a +/- sign. But as we see here in general things get more complicated

The permutation group of n objects (S_n)

Elements

It has $n!$ elements: $\{1, 2, \cdots, n\} \rightarrow \{\pi(1), \pi(2), \cdots, \pi(n)\}$

As you know, they can also be represented with cycle notation ()()()...
E.g.: $(1\,4\,2)\,(3\,5)$ is the same as $\{1,2,3,4,5\} \rightarrow \{4,1,5,2,3\}$

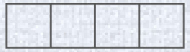
Generators

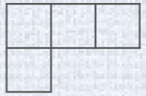
All elements can be generated from just two: $(1\,2)\,(3)\cdots(n)$ and $(1\,2\,3\cdots n)$

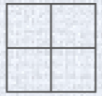
[Extremely useful info in some calculations]

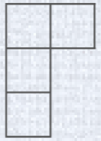
Representations

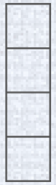
The irreducible representations of Sn can be labelled with partitions λ of n. For n=4:

$\{4\} =$  1D

$\{3,1\} =$  3D

$\{2,2\} =$  2D

$\{2,1,1\} =$  3D

$\{1,1,1,1\} =$  1D

One can also write these representations explicitly (in some basis)

$$\{(1), (1)\}$$

$$\left\{\begin{pmatrix} \frac{1}{2} & -\frac{1}{2\sqrt{3}} & -\sqrt{\frac{2}{3}} \\ -\frac{1}{2\sqrt{3}} & \frac{5}{6} & -\frac{\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{1}{2\sqrt{3}} & -\frac{1}{6} & \frac{2\sqrt{2}}{3} \\ -\sqrt{\frac{2}{3}} & -\frac{\sqrt{2}}{3} & -\frac{1}{3} \end{pmatrix}\right\}$$

etc...

Above I'm just showing the matrices for the two generators: $(1\,2)\,(3)(4)$ and $(1\,2\,3\,4)$

$d(\lambda)$ and $\mathcal{S}(\lambda, n)$

Size of irreps $d(\lambda)$ = Number of standard Young tableaux with a given shape

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array}\,,\quad \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array}\,,\quad \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} \qquad \Rightarrow d(\{3,1\}) = 3$$

But it is quicker to use the following formula:

$$d(\lambda) = \frac{m!}{\prod_u h(u)} \qquad h(u) = \text{Hook length of cell } u$$

$$h(u):\ \begin{array}{|c|c|c|}\hline 4 & 2 & 1 \\ \hline 1 \\ \cline{1-1}\end{array} \Rightarrow d(\{3,1\}) = \frac{4!}{4.2.1.1} = 3$$

$d(\lambda)$ and $\mathcal{S}(\lambda, n)$

Number of semi-simple Young tableaux

$\mathcal{S}(\lambda, n)$

Semi simple Young tableaux = tableaux filled with the numbers 1 to some n (omissions/repetitions allowed) such that the numbers increase along columns and do no decrease along rows

$$\begin{array}{|c|c|}\hline 1&1\\\hline 2&2\\\hline\end{array}\,,\ \begin{array}{|c|c|}\hline 1&1\\\hline 2&3\\\hline\end{array}\,,\ \begin{array}{|c|c|}\hline 1&1\\\hline 3&3\\\hline\end{array}\,,\ \begin{array}{|c|c|}\hline 1&2\\\hline 2&3\\\hline\end{array}\,,\ \begin{array}{|c|c|}\hline 1&2\\\hline 3&3\\\hline\end{array}\,,\ \begin{array}{|c|c|}\hline 2&2\\\hline 3&3\\\hline\end{array} \quad \Rightarrow \mathcal{S}(\{2,2\},3) = 6$$

Quick formula: $\mathcal{S}(\lambda, n) = \prod_u \frac{n + c(u)}{h(u)}$ c(u) = Content of cell u

$$c(u) : \begin{array}{|c|c|c|}\hline 0&1&2\\\hline -1\\\cline{1-1}\end{array} \Rightarrow \mathcal{S}(\{3,1\}, n) = \frac{n(n+1)^2(n+2)}{8}$$

Back to fields and operators

Let's consider *uude*-type: how many terms are needed?

If we consider all types of indices (spinor, color, SU(2)) we arrive at the conclusion that there are two contraction which are Lorentz and gauge invariant

What do we find in the Warsaw basis?

$$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$$

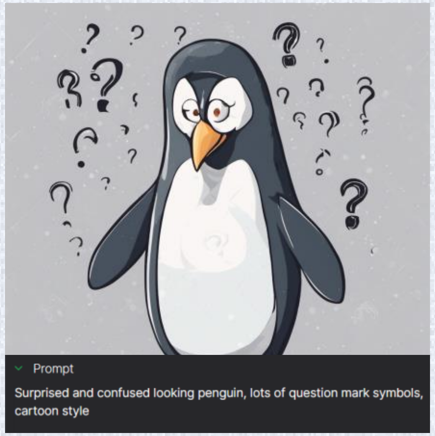
1 term

For *L*L*LL*-type operators we saw that there are two contractions too (of the SU(2) indices) ...

... but only one term in the Warsaw basis

$$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$$

1 term



Back to fields and operators

Consider the simplified case where are two contractions and just two flavor indices (*i,j*):

$$\mathcal{O}^S_{ij} = \mathcal{O}^S_{ji} \quad \text{and} \quad \mathcal{O}^A_{ij} = -\mathcal{O}^A_{ji}$$

We could write a Lagrangian with two terms: $w^S_{ij}\mathcal{O}^S_{ij} + w^A_{ij}\mathcal{O}^A_{ij}$

The w^S and w^A can be arbitrary matrices, but only their symmetric (w^S) and anti-symmetric (w^A) parts matter.
This is generic: the symmetry of the field contractions is propagated to the Wilson coefficients.

But why not just 1 term?

Define $\mathcal{O}_{ij} \equiv c^S\mathcal{O}^S_{ij} + c^A\mathcal{O}^A_{ij}$ with any non-zero *c*-coefficients

Then $w_{ij}\mathcal{O}_{ij}$ with a generic Wilson coefficient matrix encoded the two previous terms

For two repeated fields, this is it: we can merge S+A terms, not S+S nor A+A.

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
46	u u d e	6	False	n^4	1	u	□□ + ⊟

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
24	L* L* L L	6	True	$\frac{1}{2}(n^2+n^4)$	1	{L*, L}	{□□, □□} + {⊟, ⊟}

More complex situations

For two repeated fields (S_2 symmetry) we have the full picture.
With $m+m'$ contractions, m symmetric and m' anti-symmetric, ...

$$m\,\square\!\square + m'\,\begin{smallmatrix}\square\\\square\end{smallmatrix} = \max(m,m') \text{ terms in the Lagragian}$$

From here we can appreciate that it is important to track the permutation symmetry of the gauge/Lorentz index contractions, not just to know that the Wilson coefficients (WC) might have some symmetry but even to decide how many WC (i.e. terms) one needs in the Lagrangian

What happens for more complex symmetry groups (S_3, S_4,...) which appear when there are 3 or more fields of the same type?

More complex situations

Take a quartic coupling between doublets with flavor. Recall that so there are two contraction we a mixed symmetry:

$$\mathbf{2} \times \mathbf{2} \times \mathbf{2} \times \mathbf{2} = \mathbf{5}_{\square\square\square\square} + \mathbf{3}_{\square\square\square,\square} + \mathbf{1}_{\boxplus}$$

One can show (I will not to it here) that only 1 term is needed for the two SU(2) invariant contractions.

If $\left(\mathbf{1}_{\boxplus}\right)_1$ and $\left(\mathbf{1}_{\boxplus}\right)_2$ are the two linearly independent singlet contractions then it is enough to consider a non-zero linear combination of the two:

$$c_1 \left(\mathbf{1}_{\boxplus}\right)_1 + c_2 \left(\mathbf{1}_{\boxplus}\right)_2 \qquad c_1 \neq 0 \quad \text{or} \quad c_2 \neq 0$$

Valid more broadly: for any irreducible representation of the permutation group only one term is needed

Most general case

One can compress into a single term a combination of several irreps λ, as long as the multiplicity of λ does not excess $d(\lambda)$

Recall: this is the dimension of the irrep

Mistake in [RF 1907.12584]:
Wrongly capped the multiplicity at 1

For example, up to 6=3! invariants with the following symmetries can be accounted for with a single term:

$$1\ \ydiagram{3} + 2\ \ydiagram{2,1} + 1\ \ydiagram{1,1,1}$$

1D | 2D | 1D

$$\mathbf{1}_{\ydiagram{3}} \qquad \left(\begin{pmatrix}\left(\mathbf{1}^{(1)}_{\ydiagram{2,1}}\right)_1 \\ \left(\mathbf{1}^{(1)}_{\ydiagram{2,1}}\right)_2\end{pmatrix}\right)\left(\begin{pmatrix}\left(\mathbf{1}^{(2)}_{\ydiagram{2,1}}\right)_1 \\ \left(\mathbf{1}^{(2)}_{\ydiagram{2,1}}\right)_2\end{pmatrix}\right) \qquad \mathbf{1}_{\ydiagram{1,1,1}}$$

A single linear combination

$$c_{\ydiagram{3}}\mathbf{1}_{\ydiagram{3}} + c^{ij}_{\ydiagram{2,1}}\left(\mathbf{1}^{i}_{\ydiagram{2,1}}\right)_j + c_{\ydiagram{1,1,1}}\mathbf{1}_{\ydiagram{1,1,1}}$$

of the 6 Clebsch-Gordan contractions suffices, as long as

$$c_{\ydiagram{3}},\ c_{\ydiagram{1,1,1}},\ \det\begin{pmatrix} c^{11}_{\ydiagram{2,1}} & c^{12}_{\ydiagram{2,1}} \\ c^{21}_{\ydiagram{2,1}} & c^{22}_{\ydiagram{2,1}}\end{pmatrix} \neq 0$$

Up to two copies of the mixed symmetry irrep can be placed in a single term, as long as they are not "aligned"

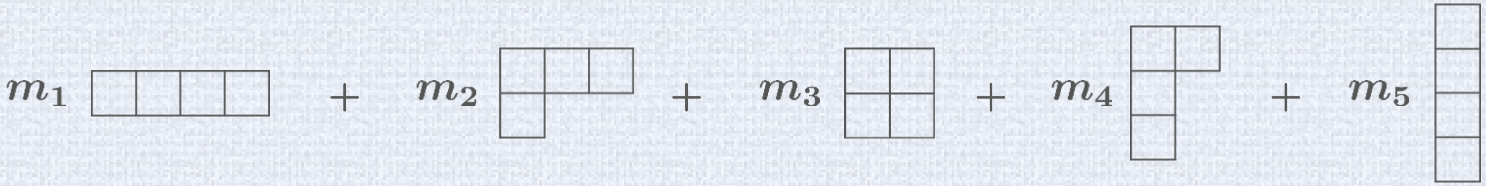
Most general case (one more example)

One more example, just to make it clear. Consider the quartic interactions of some field ϕ_i with flavor (that's the i).

It has some gauge quantum numbers and maybe transforms non-trivially under the Lorentz group. We seek invariants under both groups to build the most general Lagrangian:

$$w^{(1)}_{ijkl}\left(\phi_i\phi_j\phi_k\phi_l\right)_{(1)} + w^{(2)}_{ijkl}\left(\phi_i\phi_j\phi_k\phi_l\right)_{(2)} + \ldots$$

How far can we compact this expression? We must see how the gauge/Lorentz contractions transform under S_4



Minimum number of terms:

$$\left\lceil \max\left(m_1, \frac{m_2}{3}, \frac{m_3}{2}, \frac{m_4}{3}, m_5\right)\right\rceil$$

Round up to the nearest integer
Denominators are the size of the irreps

1

The $\left(\phi_i\phi_j\phi_k\phi_l\right)_{(n)}$ can be combined in these many terms (no less; no more are needed).
As in the last slide, this combination must avoid the 0-measure cases where there are "alignments".

QQQL

A troublemaker over the last decades

$QQQL$

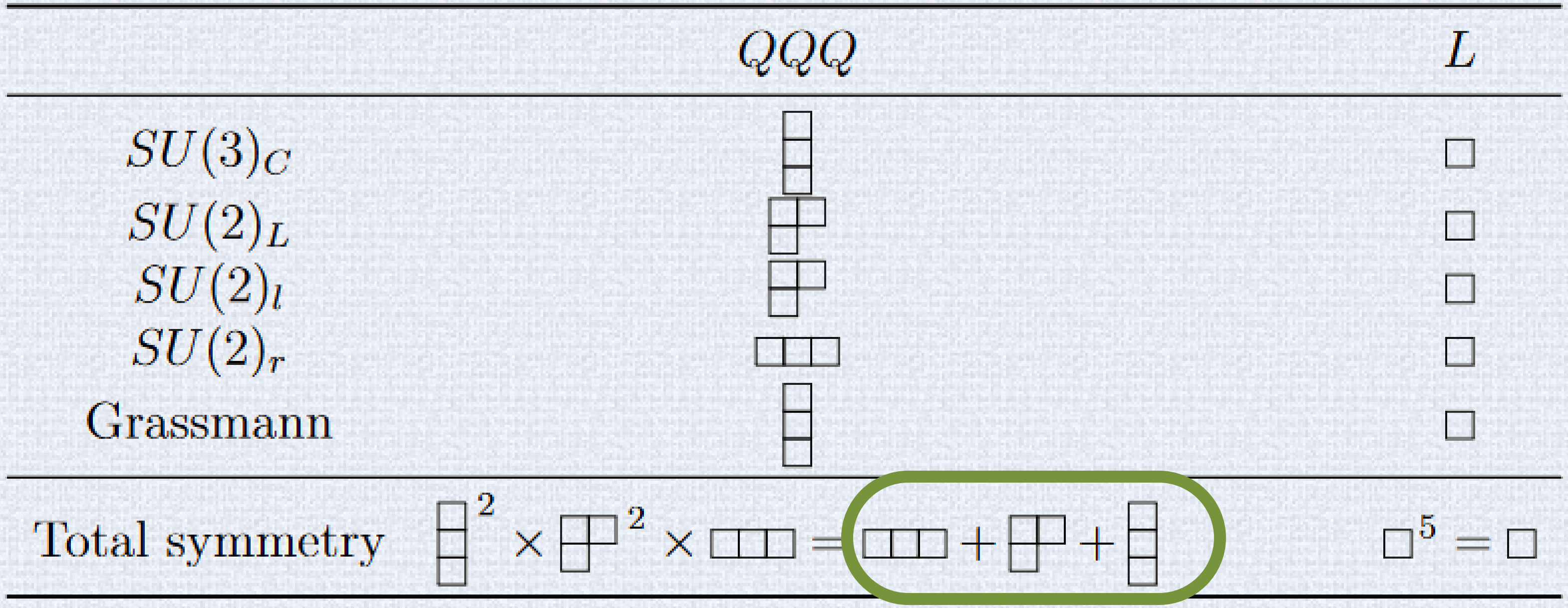
A troublemaker over the last decades

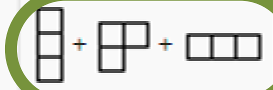
	QQQ	L
$SU(3)_C$	$\ydiagram{1,1,1}$	$\ydiagram{1}$
$SU(2)_L$	$\ydiagram{2,1}$	$\ydiagram{1}$
$SU(2)_l$	$\ydiagram{2,1}$	$\ydiagram{1}$
$SU(2)_r$	$\ydiagram{3}$	$\ydiagram{1}$
Grassmann	$\ydiagram{1,1,1}$	$\ydiagram{1}$
Total symmetry	$\ydiagram{1,1}^{\,2} \times \ydiagram{2,1}^{\,2} \times \ydiagram{3} = \ydiagram{3} + \ydiagram{2,1} + \ydiagram{1,1,1}$	$\ydiagram{1}^{\,5} = \ydiagram{1}$

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
49	Q Q Q L	6	False	$\frac{1}{3}\left(n^2 + 2\,n^4\right)$	1	Q	$\ydiagram{1,1,1} + \ydiagram{2,1} + \ydiagram{3}$

$QQQL$

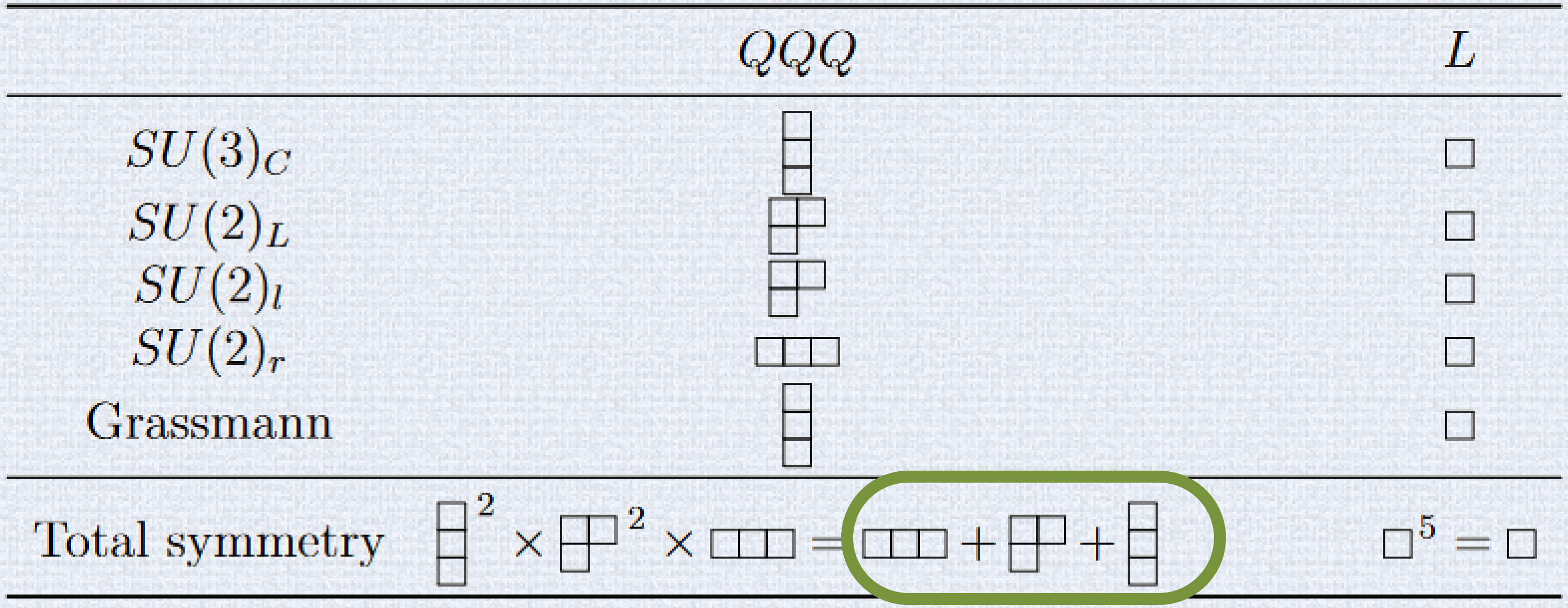
A troublemaker over the last decades

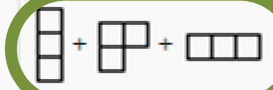


#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
49	Q Q Q L	6	False	$\frac{1}{3}\left(n^2 + 2\, n^4\right)$	1	Q	

QQQL

A troublemaker over the last decades



#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
49	Q Q Q L	6	False	$\frac{1}{3}\left(n^2+2\,n^4\right)$	1	Q	

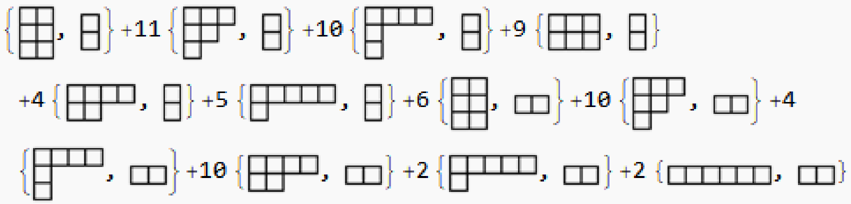
Computation becomes straightforward.
Does not say what exact form to use; only that a single term is needed

Sphaleron

$QQQQQQLL$ & $QQQQQQQQQLLL$

Sphaleron

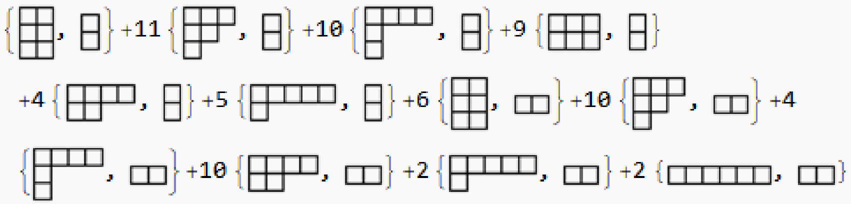
QQQQQQLL & QQQQQQQQQLLL

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	Q Q Q Q Q Q L L	12	False	4818	2	{Q, L}	

5 x 14 x 14 = 980 independent contractions of gauge/spinor quantum numbers. But we need only 2 terms. Only possible because fields are repeated in this interaction.

Sphaleron

QQQQQQLL & QQQQQQQQQLLL

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	Q Q Q Q Q Q L L	12	False	4818	2	{Q, L}	

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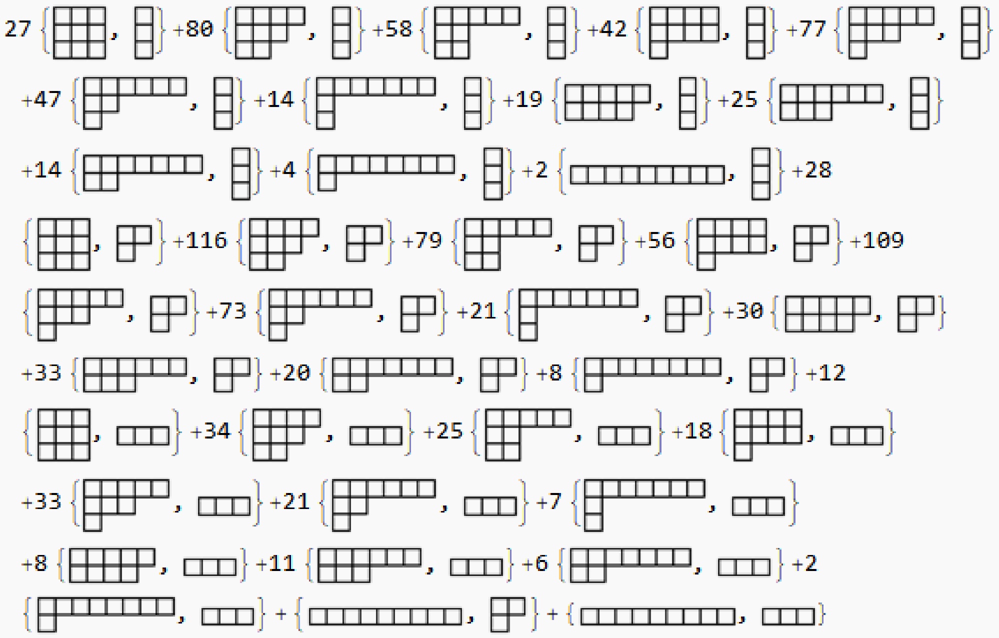
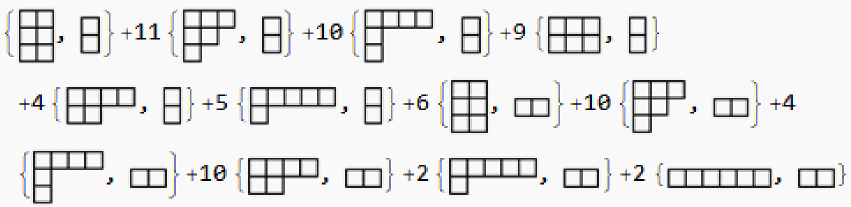
#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	Q Q Q Q Q Q Q Q Q L L L	18	False	162 774	2	{Q, L}	

132 x 132 x 42 = 731808 (!) independent contractions

2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740202, ...

Sphaleron

QQQQQQLL & QQQQQQQQQLLL

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	Q Q Q Q Q Q L L	12	False	4818	2	{Q, L}	

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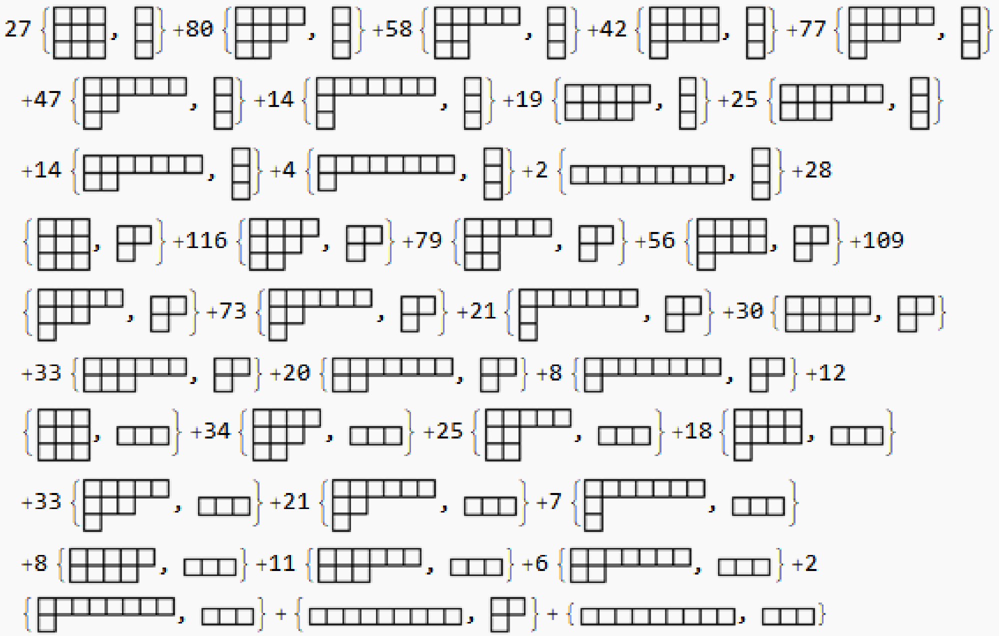
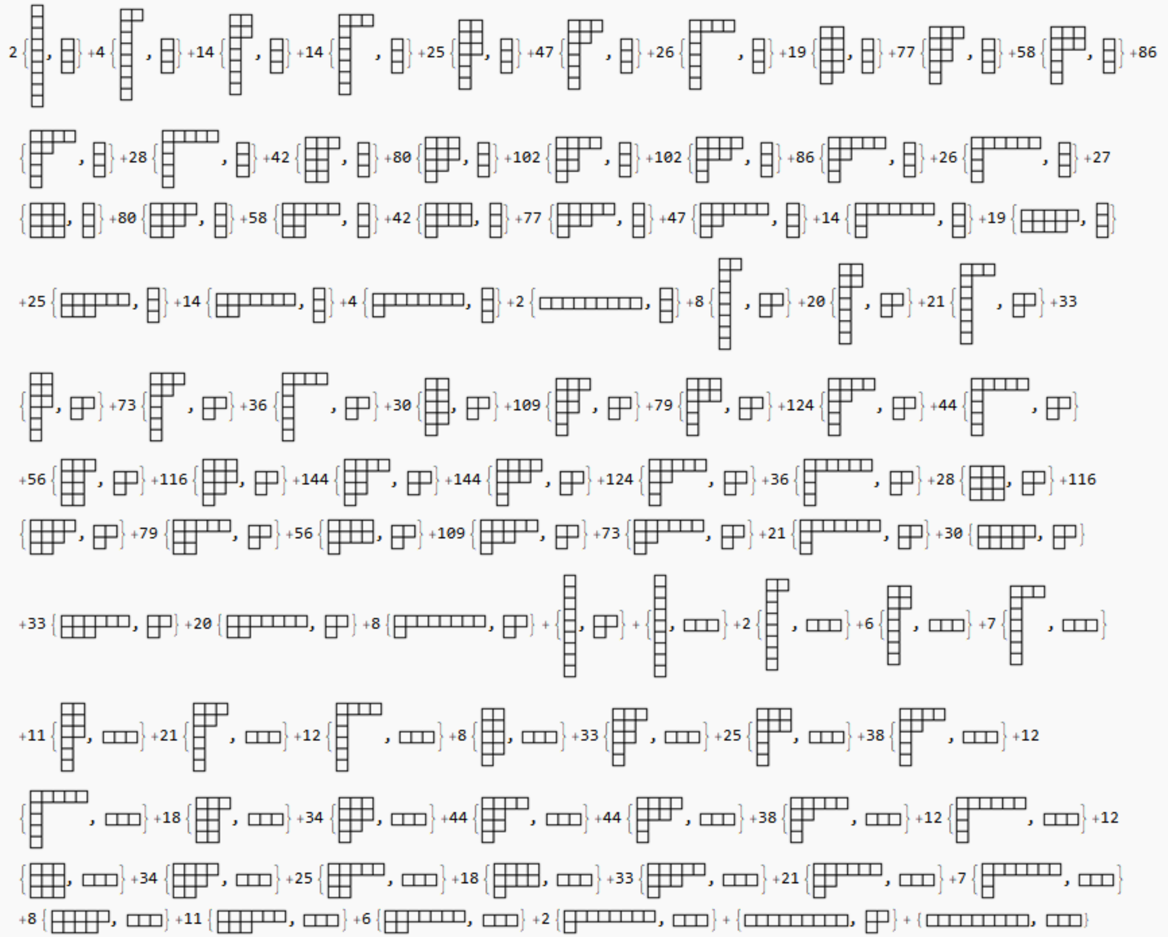
#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	Q Q Q Q Q Q Q Q Q L L L	18	False	162 774	2	{Q, L}	

132 x 132 x 42 = 731808 (!) independent contractions

n=3 generations was used (observed that Young tableaux have at most 3 rows)

Changing n can affect the number of terms

$QQQQQQQQLLL$ (for n flavors)

#	Operator type	Dim.	Self conj.?	Number of operators	Number of terms	Repeated fields	Permutation symmetry
1	Q Q Q Q Q Q Q Q L L L	18	False	$\frac{1}{720} n^4 (472 - 168 n + 181 n^2 - 84 n^3 + 96 n^4 - 12 n^5 + 89 n^6 - 96 n^7 + 242 n^8)$	2	{Q, L}	

In this example, even in the most conservative scenario where n is left unspecified (i.e. n could be arbitrarily large) the answer stays the same)

$(QQQL)^1 \rightarrow 1$ $(QQQL)^2 \rightarrow 2$ $(QQQL)^3 \rightarrow 2$ $(QQQL)^4 \rightarrow 3$ $(QQQL)^5 \rightarrow 3$ $(QQQL)^6 \rightarrow 4$

2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740202, ...

Field redefinitions

To include derivatives, one must apply them in all possible ways to the fields.

For every field $X = \phi, \psi, \mathcal{F}_{\mu\nu}$ add a tower of fields
$\partial X, \partial^2 X$, ... which are independent of X

These objects ($\partial^n X$) are not irreducible representations of the Lorentz group. But after (1) symmetrizing the derivatives (2) using field redefinitions/EOMs and (3) Bianchi identities they are

Retain only the highest spin part of $\partial^n X$

[Lehman Martin 1510.00372]

To be specific, consider a scalar ϕ, a left Weyl fermion ψ
and a field strength tensor F

$$\phi = (0,0)$$
$$\psi = \left(\tfrac{1}{2},0\right)$$
$$F = (1,0)$$
$$\partial = \left(\tfrac{1}{2},\tfrac{1}{2}\right)$$

(j_L, j_R)

$$\partial^n \phi = \left(\tfrac{n}{2},\tfrac{n}{2}\right) + \text{EOM-redundant bits}$$
$$\partial^n \psi = \left(\tfrac{n+1}{2},\tfrac{n}{2}\right) + \text{EOM-redundant bits}$$
$$\partial^n F = \left(\tfrac{n+2}{2},\tfrac{n}{2}\right) + \text{EOM-redundant bits}$$

Recall that the Lorentz group $\sim SU(2) \times SU(2)$

Total derivatives

Integration by parts

For some operators, $\int_{\mathcal{M}} \mathcal{O}\, d^4x = 0$

In the language of differential forms, these redundant operators are associated exact to exact differential forms

(by assumption)

$$\omega^{(4),\text{red}} = d\omega^{(3)} \longrightarrow \int_{\mathcal{M}} \omega^{(4),\text{red}} = \int_{\text{Boundary}(\mathcal{M})} \omega^{(3)} = 0$$

[Henning, Lu, Melia, Murayama 1512.03433]

but ...

we need to be careful: for some 3-forms, $d\omega^{(3),\text{red}} = 0$ and we shouldn't consider them because *dd=0*, so these account for identically null 4-forms

Which are these 3-forms? $\omega^{(3),\text{red}} = d\omega^{(2)}$

We have a recursive process, which ends when we reach 0-forms

Translation into language of operators:

The total number of non-redundant operators up to dimension *d* is:

$$\left(\#\mathcal{O}^{dim\leq d}\right) - \left(\#\mathcal{O}_{\mu}^{dim\leq d-1}\right) + \left(\#\mathcal{O}_{[\mu\nu]}^{dim\leq d-2}\right) - \left(\#\mathcal{O}_{[\mu\nu\rho]}^{dim\leq d-3}\right) + \left(\#\mathcal{O}_{[\mu\nu\rho\sigma]}^{dim\leq d-4}\right)$$

μ, ν, ρ, σ are Lorentz completely anti-symmetrized indices

Total derivatives

Performing the computation in the previous slide is fine
(we just have to count also operators which are not Lorentz invariants)

Alternative

Add a dummy field $\mathcal{D}$, with the quantum numbers of a derivative but which is a Grassman field

$$\sum_{i=0}^{4}(-1)^{i}\left(\#\mathcal{O}^{dim\leq d}\text{ with } i\ \mathcal{D}'s\right)$$

Example for a scalar singlet S ($\partial^4 S^4$ interactions)

$\#\mathcal{D}$'s	Operator type	Symmetry of the fields $(S,\partial S,\partial^2 S)$
0	$SS\left(\partial^2S\right)\left(\partial^2S\right)$	$(\square\square, -, \square\square)$
	$S\left(\partial S\right)\left(\partial S\right)\left(\partial^2S\right)$	$(\square, \square\square, \square)$
	$\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)$	$(-, \square\square\square\square, -) + (-, \begin{smallmatrix}\square&\square\\\square&\square\end{smallmatrix}, -) + (-, \begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}, -)$
1	$\mathcal{D}SS\left(\partial S\right)\left(\partial^2S\right)$	$(\square\square, \square, \square)$
	$\mathcal{D}S\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)$	$(\square, \square\square\square, -) + (\square, \begin{smallmatrix}\square&\square\\\square\end{smallmatrix}, -) + (\square, \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}, -)$
2	$\mathcal{D}\mathcal{D}SS\left(\partial S\right)\left(\partial S\right)$	$2\,(\square\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}, -)$
3	$\mathcal{D}\mathcal{D}\mathcal{D}SSS\left(\partial S\right)$	$(\square\square\square, \square, -)$
4	$\mathcal{D}\mathcal{D}\mathcal{D}\mathcal{D}SSSS$	$(\square\square\square\square, -, -)$

Total derivatives

Performing the computation in the previous slide is fine
(we just have to count also operators which are not Lorentz invariants)

Alternative

Add a dummy field $\mathcal{D}$, with the quantum numbers of a derivative but which is a Grassman field

$$\sum_{i=0}^{4} (-1)^i \left(\#\mathcal{O}^{dim \leq d} \text{ with } i\ \mathcal{D}'s \right)$$

Example for a scalar singlet S ($\partial^4 S^4$ interactions)

Add

The list of operators to include

$\#\mathcal{D}$'s	Operator type	Symmetry of the fields $(S, \partial S, \partial^2 S)$
0	$SS\left(\partial^2 S\right)\left(\partial^2 S\right)$	$(\square\square, -, \square\square)$
	$S\left(\partial S\right)\left(\partial S\right)\left(\partial^2 S\right)$	$(\square, \square\square, \square)$
	$\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)$	$(-, \square\square\square\square, -) + \left(-, \begin{smallmatrix}\square\square\\\square\square\end{smallmatrix}, -\right) + \left(-, \begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}, -\right)$
1	$\mathcal{D}SS\left(\partial S\right)\left(\partial^2 S\right)$	$(\square\square, \square, \square)$
	$\mathcal{D}S\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)$	$(\square, \square\square\square, -) + \left(\square, \begin{smallmatrix}\square\square\\\square\end{smallmatrix}, -\right) + \left(\square, \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}, -\right)$
2	$\mathcal{D}\mathcal{D}SS\left(\partial S\right)\left(\partial S\right)$	$2\left(\square\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}, -\right)$
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Add — The list of operators to include

Remove — Remove total derivatives

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0	$SS(\partial^2 S)(\partial^2 S)$	$(\yng(2), -, \yng(2))$
	$S(\partial S)(\partial S)(\partial^2 S)$	$(\yng(1), \yng(2), \yng(1))$
	$(\partial S)(\partial S)(\partial S)(\partial S)$	$(-, \yng(4), -) + (-, \yng(2,2), -) + (-, \yng(1,1,1,1), -)$
1	$\mathcal{D}SS(\partial S)(\partial^2 S)$	$(\yng(2), \yng(1), \yng(1))$
	$\mathcal{D}S(\partial S)(\partial S)(\partial S)$	$(\yng(1), \yng(3), -) + (\yng(1), \yng(2,1), -) + (\yng(1), \yng(1,1,1), -)$
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Add — Removed too much; add these ("redundancies of redundancies")

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	$S\left(\partial S\right)\left(\partial S\right)\left(\partial^2S\right)$	$(\square,\square\square,\square)$
	$\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)$	$(-,\square\square\square\square,-)+\left(-,\begin{smallmatrix}\square&\square\\\square&\square\end{smallmatrix},-\right)+\left(-,\begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix},-\right)$
1	$\mathcal{D}SS\left(\partial S\right)\left(\partial^2S\right)$	$(\square\square,\square,\square)$
	$\mathcal{D}S\left(\partial S\right)\left(\partial S\right)\left(\partial S\right)$	$(\square,\square\square\square,-)+\left(\square,\begin{smallmatrix}\square&\square\\\square&\end{smallmatrix},-\right)+\left(\square,\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix},-\right)$
2	$\mathcal{D}\mathcal{D}SS\left(\partial S\right)\left(\partial S\right)$	$2\left(\square\square,\begin{smallmatrix}\square\\\square\end{smallmatrix},-\right)$
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	$(\partial S)(\partial S)(\partial S)(\partial S)$	$(-,\yng(4),-)+(-,\yng(2,2),-)+(-,\yng(1,1,1,1),-)$
1	$\mathcal{D}SS(\partial S)(\partial^2 S)$	$(\yng(2),\yng(1),\yng(1))$
	$\mathcal{D}S(\partial S)(\partial S)(\partial S)$	$(\yng(1),\yng(3),-)+(\yng(1),\yng(2,1),-)+(\yng(1),\yng(1,1,1),-)$
2	$\mathcal{D}\mathcal{D}SS(\partial S)(\partial S)$	$2\,(\yng(2),\yng(1,1),-)$
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Example for a scalar singlet S ($\partial^4 S^4$ interactions)

		$\#\mathcal{D}$'s	Operator type	Symmetry of the fields $(S, \partial S, \partial^2 S)$
Add	The list of operators to include	0	$SS(\partial^2 S)(\partial^2 S)$ $S(\partial S)(\partial S)(\partial^2 S)$ $(\partial S)(\partial S)(\partial S)(\partial S)$	$(\ydiagram{2}, -, \ydiagram{2})$ $(\ydiagram{1}, \ydiagram{2}, \ydiagram{1})$ $(-, \ydiagram{4}, -) + (-, \ydiagram{2,2}, -) + (-, \ydiagram{1,1,1,1}, -)$
Remove	Remove total derivatives	1	$\mathcal{D}SS(\partial S)(\partial^2 S)$ $\mathcal{D}S(\partial S)(\partial S)(\partial S)$	$(\ydiagram{2}, \ydiagram{1}, \ydiagram{1})$ $(\ydiagram{1}, \ydiagram{3}, -) + (\ydiagram{1}, \ydiagram{2,1}, -) + (\ydiagram{1}, \ydiagram{1,1,1}, -)$
Add	Removed too much; add these ("redundancies of redundancies")	2	$\mathcal{D}\mathcal{D}SS(\partial S)(\partial S)$	$2\,(\ydiagram{2}, \ydiagram{1,1}, -)$
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Example for a scalar singlet S ($\partial^4 S^4$ interac

Does not even refer to a common group; spoil the algorithm to count terms

		$\#\mathcal{D}$'s	Operator type	Symmetry of the fields $(S, \partial S, \partial^2 S)$
Add	The list of operators to include	0	$SS(\partial^2 S)(\partial^2 S)$	$(\yng(2), -, \yng(2))$
			$S(\partial S)(\partial S)(\partial^2 S)$	$(\yng(1), \yng(2), \yng(1))$
			$(\partial S)(\partial S)(\partial S)(\partial S)$	$(-, \yng(4), -) + (-, \yng(2,2), -) + (-, \yng(1,1,1,1), -)$
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			$\mathcal{D}S(\partial S)(\partial S)(\partial S)$	$(\yng(1), \yng(3), -)+(\yng(1), \yng(2,1), -)+(\yng(1), \yng(1,1,1), -)$
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Add	"Redundancies of redundancies of redundancies of redundancies"	4	$\mathcal{D}\mathcal{D}\mathcal{D}\mathcal{D}SSSS$	$(\yng(4), -, -)$

Use Littlewood–Richardson rule

A 2-index tensor with no symmetry can still be seen as a mixture of parts with an S_2 symmetry: a symmetric and an anti-symmetric part

$$\yng(1) \times \yng(1) = \yng(2) + \yng(1,1)$$

The same logic applies to more complicated situations. E.g.:

$$SS\,(\partial^2 S)\,(\partial^2 S) \qquad (\yng(2), -, \yng(2))$$

$$\underbrace{\yng(2) \times \yng(2)}_{S_2 \times S_2} = \underbrace{\yng(4) + \yng(2,2) + \yng(3,1)}_{S_4}$$

$\#\mathcal{D}$'s	Operator type	Symmetry of the fields $(S, \partial S, \partial^2 S)$
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	$\mathcal{D}S\,(\partial S)\,(\partial S)\,(\partial S)$	$(\yng(1), \yng(3), -) + (\yng(1), \yng(2,1), -) + (\yng(1), \yng(1,1,1), -)$
2	$\mathcal{D}\mathcal{D}SS\,(\partial S)\,(\partial S)$	$2\,(\yng(2), \yng(1,1), -)$
3	$\mathcal{D}\mathcal{D}\mathcal{D}SSS\,(\partial S)$	$(\yng(3), \yng(1), -)$
4	$\mathcal{D}\mathcal{D}\mathcal{D}\mathcal{D}SSSS$	$(\yng(4), -, -)$

$$\yng(4) + \yng(2,2)$$ after all additions and subtractions

So 1 term is enough

More generally we see that derivatives are not a problem

Important caveat

Consider some scalar doublets with flavor ϕ_i plus a scalar singlet S and triplet Δ (both with no flavor).
We can have the following trilinear interactions:

$$\mathcal{O}_{ij}^{S} \equiv \phi_i \phi_j S \qquad \mathcal{O}_{ij}^{\Delta} \equiv \phi_i \phi_j \Delta$$

Important caveat

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The first is anti-symmetric in the flavor indices; the second is symmetric

We can therefore make a linear combination of both expressions and write a single term in the Lagrangian!

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Most of us will probably find this (very) unuseful/distastful, as we are mixing different types of operators

If we commit not to doing this, then the numbers given by `Sym2Int` are the lowest possible terms.
In the case of SMEFT: numbers are given in the title of this talk.

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Consider some scalar doublets with flavor ϕ_i plus a scalar singlet S and triplet Δ (both with no flavor).
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If we commit not to doing this, then the numbers given by `Sym2Int` are the lowest possible terms.
In the case of SMEFT: numbers are given in the title of this talk.

However, in the case of field strength tensors, we probably are willing to consider these mixtures. That's because the program uses $F^{\mu\nu}_{L,R} = 1/2(F^{\mu\nu} \mp i\widetilde{F}^{\mu\nu})$ as the basic objects. If we were to write the operators as a function of $F^{\mu\nu}$ and $\widetilde{F}^{\mu\nu}$ we must combine interactions with F_L and $F_R = F_L^*$

Using instead $F^{\mu\nu}$ and $\widetilde{F}^{\mu\nu}$ might lead to a (small) difference in the number of terms

d space-time dimensions

Remember this example?

	QQQ	L
$SU(3)_C$	$\yng(1,1,1)$	$\yng(1)$
$SU(2)_L$	$\yng(2,1)$	$\yng(1)$
$SU(2)_l$	$\yng(2,1)$	$\yng(1)$
$SU(2)_r$	$\yng(3)$	$\yng(1)$
Grassmann	$\yng(1,1,1)$	$\yng(1)$
Total symmetry	$\yng(1,1,1)^2 \times \yng(2,1)^2 \times \yng(3) = \yng(3) + \yng(2,1) + \yng(1,1,1)$	$\yng(1)^5 = \yng(1)$

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Total symmetry	$\ydiagram{1,1,1}^2 \times \ydiagram{2,1}^2 \times \ydiagram{3} = \ydiagram{3} + \ydiagram{2,1} + \ydiagram{1,1,1}$	$\ydiagram{1}^5 = \ydiagram{1}$

Lorentz group is not treated in any special way (it is just another group...)

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Fierz identities are taken into account in a trivial way (6-fermion, 8-fermion ones ... all the same)

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Lorentz group is not treated in any special way (it is just another group...)

Fierz identities are taken into account in a trivial way (6-fermion, 8-fermion ones ... all the same)

However, as you know better than me, we may need to insert *d*-dimensional operators in loops. For divergent ones, the *4-d* difference gives finite contributions.

It is unfortunately far from obvious to me what happens to the *SO(1,3)* ~ *SU(2)* x *SU(2)* group.
In any case, it is clear that the relations which depended on *SO(1,3)* can no longer be used.

A seemingly infinite amount of extra terms are needed, and many of you have focused on reducing them back to the 4-dimensional basis. However, it seems to me that on top of that, the number of operators associated to a term may also increase. For example in *4d* $(\bar{e}_i\gamma^\mu e_j)(\bar{e}_k\gamma_\mu e_l)$ is symmetric in $i \leftrightarrow k$ and $j \leftrightarrow l$.

31	e⋆ e⋆ e e	6	True	$\frac{1}{4} n^2 (1+n)^2$	1	{e⋆, e}	{$\yng(2)$, $\yng(2)$}

Does not need to be so in *d*-dimensions (*); so more parameters in the WC; shift them?

(*) Still symmetric under $(ij) \leftrightarrow (kl)$

Summary

Summary

Knowing in advance the number of terms of an EFT is useful (e.g., to build them explicitly).

Terms are harder to count than operators. One needs to study the effect of permutations of equal fields; becoming a standard approach in building operators explicitly.

I've shown how to count terms systematically. This has been implements in `Sym2Int`

In the case of SMEFT, at dimension 8, one needs 1019 real terms [assuming we use $\boldsymbol{F_{L,R}^{\mu\nu} = 1/2(F^{\mu\nu} \mp i\widetilde{F}^{\mu\nu})}$ for gauge bosons].

Thank you

Summary

As for the title of this talk:

THE ON–LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740 Search Hints

(Greetings from The On-Line Encyclopedia of Integer Sequences!)

Search: **seq:2,84,36,1019,624,15666,12620,264389,269026,4669553,5740202**

Sorry, but the terms do not match anything in the table.

If your sequence is of general interest, please submit it using the form provided and it will (probably) be added to the OEIS! Include a brief description and if possible enough terms to fill 3 lines on the screen. We need at least 4 terms.

Summary

As for the title of this talk:

```
FindSequenceFunction[{2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740202},
 n - 4]

FindSequenceFunction[{2, 84, 36, 1019, 624, 15666, 12620, 264389, 269026, 4669553, 5740202}, -4 + n]
```

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Summary

As for the title of this talk:

```
FindSequenceFunction[{2, 84, 36, 1019, 624, 15 666, 12 620, 264 389, 269 026, 4 669 553, 5 740 202},
 n - 4]

FindSequenceFunction[{2, 84, 36, 1019, 624, 15 666, 12 620, 264 389, 269 026, 4 669 553, 5 740 202}, -4 + n]
```

```
FindGeneratingFunction[{2, 84, 36, 1019, 624, 15 666, 12 620, 264 389, 269 026, 4 669 553, 5 740 202},
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FindGeneratingFunction[{2, 84, 36, 1019, 624, 15 666, 12 620, 264 389, 269 026, 4 669 553, 5 740 202}, -4 + n]
```

```
InterpolatingPolynomial[{2, 84, 36, 1019, 624, 15 666, 12 620, 264 389, 269 026, 4 669 553, 5 740 202}, n - 4] //
 Expand
```

$$-72\,370\,049\,540 + \frac{210\,859\,941\,489\,299\, n}{2520} - \frac{103\,024\,089\,008\,519\, n^2}{2400} + \frac{58\,387\,989\,570\,335\, n^3}{4536} - \frac{129\,563\,807\,925\,959\, n^4}{51\,840} + \frac{5\,675\,316\,502\,229\, n^5}{17\,280} - \frac{1\,278\,345\,625\,591\, n^6}{43\,200} + \frac{21\,840\,065\,365\, n^7}{12\,096} - \frac{1\,234\,207\,637\, n^8}{17\,280} + \frac{120\,078\,781\, n^9}{72\,576} - \frac{1\,104\,713\, n^{10}}{64\,800}$$

Thank you