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RGEs at NLL “without” Evanescent Operators

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Motivation

- Tremendous progress in fully automated LO + NLO matching for generic models (Matchete, MatchMakerEFT)
- LL LEFT and SMEFT running implemented (DsixTools, wilson)
- Scheme-independent only with NLL running
- Evanescent operators increase size of basis

Evanescent Operators

- Divergent loop integrals $\rightarrow$ work in $d = 4 - 2\epsilon$
- Operator relations in $d = 4$ no longer hold
- Introduce d-dim independent operators to basis:

$$E = \tilde{\mathcal{O}} - \left(\mathcal{F} + \epsilon\, \Sigma\right) Q$$

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- Introduce d-dim independent operators to basis:

$$E = \tilde{\mathcal{O}} - \left(\mathcal{F} + \epsilon\,\Sigma\right) Q$$

- Unpysical operators, but mixing generates physical effects

(Dugan, Grinstein 1991)

Evanescent Operators

- Evanescent insertions generate more evanescents
  - $\rightarrow$ Cascades to infinite number at one-loop

$$C_i \langle \mathcal{O}_i \rangle^{(1)} = \begin{pmatrix} C_Q & C_E \end{pmatrix} \left(\begin{array}{c|c} r^{(1)}_{QQ} & r^{(1)}_{QE} \\ \hline r^{(1)}_{EQ} & r^{(1)}_{EE} \end{array} \right) \begin{pmatrix} \langle Q \rangle^{(0)} \\ \langle E \rangle^{(0)} \end{pmatrix}$$

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- **Need an infinite number of initial conditions!**

Evanescent Operators

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- Finite counterterm $Z_{EQ}^{(1;0)} \rightarrow$ enters physical ADM

Evanescent Operators

- Enlarged basis $\rightarrow$ more computations
- Ev. Op.s are often more complicated than physicals:
  - NDR: $(\bar{s}_L\gamma_\mu\gamma_\nu\gamma_\sigma d_L)(\bar{s}_L\gamma^\mu\gamma^\nu\gamma^\sigma d_L) - (16 - a\epsilon)Q_{S2}$ (Herrlich, Nierste, 1996)
  - Larin:
    $$\left[(\bar{e}\gamma_{[\mu}\gamma_{\nu]}\gamma^{[\rho}\gamma^{\sigma]}e)(\bar{q}\gamma^{[\mu}\gamma^{\nu]}\gamma^{[\tau}\gamma^{\zeta]}q) + (\bar{e}\gamma^{[\rho}\gamma^{\sigma]}\gamma_{[\mu}\gamma_{\nu]}e)(\bar{q}\gamma^{[\tau}\gamma^{\zeta]}\gamma^{[\mu}\gamma^{\nu]}q)\right]\epsilon_{\rho\sigma\tau\zeta}$$
    $$-48(Q_1^{eq} + Q_1^{qe}) + 16Q_2^{eq}$$
    (Brod, Stamou, ZP, 2023)
  - HV: $(\bar{q}_p\hat{\gamma}^\mu\hat{\gamma}^\nu\tilde{\sigma}^{\lambda\sigma}q_p)(\bar{q}_r\hat{\gamma}_\mu\hat{\gamma}_\nu\sigma_{\lambda\sigma}q_r)$ (Bühler, Stoffer, 2023)

Eliminating Evanescents

- Don't use dim. reg. (e.g. RI-(S)MOM)

  (Ciuchini, et al., 1997)

- Change basis for fewer/simpler evanescent operators

  (Brod, Gorbahn, 2010)

- Absorb evanescent contributions elsewhere/modify $\overline{\text{MS}}$

  (Fuentes-Martin, et al. 2022)

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Can we find a way to not work with Ev. Op.s at NLO?

An Example

- Consider a simple UV theory with heavy ϕ:

$$\mathcal{L}_{UV} \supset -y_\ell^L \left(\bar{\ell} P_L \nu_\ell\right)\phi - y_\ell^R \left(\bar{\ell} P_R \nu_\ell\right)\phi + \text{h.c.}$$

- Matches onto LEFT (QED vector-like; only look at LH)

$$Q_s = (\bar{\nu}_\mu P_L \mu)(\bar{e} P_L \nu_e), \quad Q_t = (\bar{\nu}_\mu \sigma^{\mu\nu} P_L \mu)(\bar{e} \sigma_{\mu\nu} P_L \nu_e)$$

- Compute matching and ADM using three prescriptions:
  1. NDR (anticommuting γ_5)
  2. No γ_5 prescription $\rightarrow$ always give Ev. Op. (Misiak 1992)
  3. Naive replacement rules + fixed $O(\epsilon)$ (no Ev. Op.s)

Reducing Structures

- NDR:

$$\left(\bar{\nu}_\mu P_L \gamma^\mu \gamma^\nu \mu\right)\left(\bar{e}\gamma_\nu \gamma_\mu P_L \nu_e\right) = (4-2\epsilon)Q_s + Q_t$$

$$\left(\bar{\nu}_\mu \gamma^{\mu\nu\alpha\beta} P_L \mu\right)\left(\bar{e}\gamma_{\mu\nu\alpha\beta} P_L \nu_e\right) = (64-\epsilon\sigma_s)Q_s + (-16-\epsilon\sigma_t)Q_t + E_{\text{NDR}}$$

$$\left(\bar{\nu}_\mu \gamma^{\mu\nu\alpha\beta\sigma\rho} P_L \mu\right)\left(\bar{e}\gamma_{\mu\nu\alpha\beta\sigma\rho} P_L \nu_e\right) = (1024-\epsilon\sigma'_s)Q_s + (-256-\epsilon\sigma'_t)Q_t + E'_{\text{NDR}}$$

- No γ_5 prescription:

$$\left(\bar{\nu}_\mu P_L \gamma^\mu \gamma^\nu \mu\right)\left(\bar{e}\gamma_\nu \gamma_\mu P_L \nu_e\right) = (4-2\epsilon)Q_s + (1-\epsilon\overline{\sigma}_{t1})Q_t + \overline{E}_1$$

$$\left(\bar{\nu}_\mu \sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta \mu\right)\left(\bar{e}\gamma_\beta \gamma_\alpha \sigma_{\mu\nu} P_L \nu_e\right) = (48-\epsilon\overline{\sigma}_{s2})Q_s + (12-\epsilon\overline{\sigma}_{t2})Q_t + \overline{E}_2$$

$$\left(\bar{\nu}_\mu P_L \gamma^{\mu\nu\alpha\beta} \mu\right)\left(\bar{e}\gamma_{\beta\alpha\nu\mu} P_L \nu_e\right) = (64-\epsilon\overline{\sigma}'_{s1})Q_s + (16-\epsilon\overline{\sigma}'_{t1})Q_t + \overline{E}'_1$$

$$\left(\bar{\nu}_\mu \sigma^{\mu\nu} P_L \gamma^{\alpha\beta\sigma\rho} \mu\right)\left(\bar{e}\gamma_{\rho\sigma\beta\alpha} \sigma_{\mu\nu} P_L \nu_e\right) = (768-\epsilon\overline{\sigma}'_{s2})Q_s + (192-\epsilon\overline{\sigma}'_{t2})Q_t + \overline{E}'_2$$

Reducing Structures

- Fixed Replacement Rules:

$$\left(\bar{\nu}_\mu P_L \gamma^\mu \gamma^\nu \mu\right)\left(\bar{e}\gamma_\nu \gamma_\mu P_L \nu_e\right) = (4 - 2\epsilon)Q_s + Q_t$$

$$\left(\bar{\nu}_\mu \sigma^{\mu\nu} P_L \gamma^\alpha \gamma^\beta \mu\right)\left(\bar{e}\gamma_\beta \gamma_\alpha \sigma_{\mu\nu} P_L \nu_e\right) = (48 - 80\epsilon)Q_s + (12 - 14\epsilon)Q_t$$

$$\left(\bar{\nu}_\mu P_L \gamma^{\mu\nu\alpha\beta} \mu\right)\left(\bar{e}\gamma_{\beta\alpha\nu\mu} P_L \nu_e\right) = (64 - 96\epsilon)Q_s + (16 - 16\epsilon)Q_t$$

$$\left(\bar{\nu}_\mu \sigma^{\mu\nu} P_L \gamma^{\alpha\beta\sigma\rho} \mu\right)\left(\bar{e}\gamma_{\rho\sigma\beta\alpha} \sigma_{\mu\nu} P_L \nu_e\right) = (768 - 2048\epsilon)Q_s + (192 - 416\epsilon)Q_t$$

Reducing Structures

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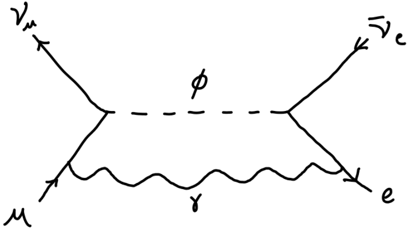
- Can obtain these replacements with

$$\sigma_s = 96, \quad \sigma_t = -8, \quad \sigma'_s = 2944, \quad \sigma'_t = -352$$

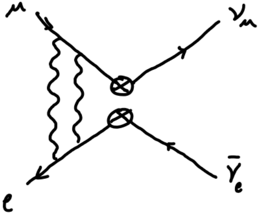
$$\overline{\sigma}_{t1} = 0, \quad \overline{\sigma}_{s2} = 80, \quad \overline{\sigma}_{t2} = 14, \quad \overline{\sigma}'_{s1} = 96, \quad \overline{\sigma}'_{t1} = 16, \quad \overline{\sigma}'_{s2} = 2048, \quad \overline{\sigma}'_{t2} = 416$$

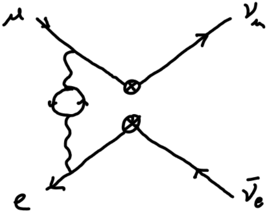
NLO Matching



$$C_t^{(1)}(\mu_0) = \tilde{C}_t^{(1)}(\mu_0) = \frac{y_\mu^R y_e^L}{4M_\phi^2}\left(\frac{3}{2} + \log\frac{\mu_0^2}{M_\phi^2}\right), \quad \overline{C}_t^{(1)}(\mu_0) = \frac{y_\mu^R y_e^L}{4M_\phi^2}\left(\frac{3}{2} - \overline{\sigma}_{t1} + \log\frac{\mu_0^2}{M_\phi^2}\right)$$

Two-Loop ADM





$$\gamma^{(1)} = \begin{pmatrix} -21 - \frac{\sigma_s}{8} & -\frac{23}{18} - \frac{\sigma_t}{8} \\ \frac{520}{3} + \frac{\sigma_s}{3} + 6\sigma_t & \frac{103}{9} + \frac{\sigma_s}{8} + \frac{4\sigma_t}{3} \end{pmatrix}, \quad \tilde{\gamma}^{(1)} = \begin{pmatrix} -33 & -\frac{5}{18} \\ \frac{472}{3} & \frac{115}{9} \end{pmatrix}$$

$$\overline{\gamma}^{(1)} = \begin{pmatrix} -23 + \overline{\sigma}_{t1} - \frac{\overline{\sigma}_{s2}}{8} & \frac{53}{36} + \frac{7\overline{\sigma}_{t1}}{3} - \frac{\overline{\sigma}_{t2}}{8} \\ \frac{140}{3} + \frac{\overline{\sigma}_{s2}}{3} + 6\overline{\sigma}_{t2} & -\frac{143}{9} - 6\overline{\sigma}_{t1} + \frac{\overline{\sigma}_{s2}}{8} + \frac{4\overline{\sigma}_{t2}}{3} \end{pmatrix}$$

NLO Matching and ADM

$$C_t^{(1)}(\mu_0) = \tilde{C}_t^{(1)}(\mu_0) = \frac{y_\mu^R y_e^L}{4M_\phi^2}\left(\frac{3}{2} + \log\frac{\mu_0^2}{M_\phi^2}\right), \quad \overline{C}_t^{(1)}(\mu_0) = \frac{y_\mu^R y_e^L}{4M_\phi^2}\left(\frac{3}{2} - \overline{\sigma}_{t1} + \log\frac{\mu_0^2}{M_\phi^2}\right)$$

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All coincide for “proper” choices of σ!

Why Did Fixed-Replacement Prescription Work?

- Four-dimensional relations:

$$\big(\Gamma_{\mathcal{E}_i}\big) \otimes \big(\Gamma_{\mathcal{E}_j}\big) - C_{ij}^{k\ell}\big(\Gamma_{Q_k}\big) \otimes \big(\Gamma_{Q_\ell}\big) = 0$$

- Insert into one-loop diagram:

$$\Big\langle \big(\Gamma_{\mathcal{E}_i}\big) \otimes \big(\Gamma_{\mathcal{E}_j}\big)\Big\rangle^{(1)} - C_{ij}^{k\ell} \Big\langle \big(\Gamma_{Q_k}\big) \otimes \big(\Gamma_{Q_\ell}\big)\Big\rangle^{(1)} \neq 0$$

- Difference is Rank(ϵ) $\Rightarrow$ **Finite and local**

- Loop effects break four-dimensional relations; introduce finite, local counterterm to restore them $\rightarrow$ evanescnet operator

Why Did Fixed-Replacement Prescription Work?

- In matching, Ev. Op. is equivalent to replacement rule ($C_E(\mu_0)$ not relevant by construction)

- In ADM, only enters via $Z_{QE}^{(1;1)} Z_{EQ}^{(1;0)}$

$$Z_{\overline{E}Q}^{(1;0)} = \begin{pmatrix} -2\overline{\sigma}_{s1} + \frac{\overline{\sigma}'_{s1}}{4} - \frac{\overline{\sigma}_{s2}}{4} - 12\overline{\sigma}_{t1} & -\frac{\overline{\sigma}_{s1}}{4} - 4\overline{\sigma}_{t1} + \frac{\overline{\sigma}'_{t1}}{4} - \frac{\overline{\sigma}_{t2}}{4} \\ -12\overline{\sigma}_{s1} - 4\overline{\sigma}_{s2} + \frac{\overline{\sigma}'_{s2}}{4} - 12\overline{\sigma}_{t2} & -\frac{\overline{\sigma}_{s2}}{4} - 12\overline{\sigma}_{t1} - 6\overline{\sigma}_{t2} + \frac{\overline{\sigma}'_{t2}}{4} \end{pmatrix}$$

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- **Vanishes for Fixed-Replacement Prescription choice of $\overline{\sigma}$'s!**

This is a Bad Solution!!!!

1. Method of fixing scheme not guaranteed to give correct cancellations

2. Need to insert Ev. Op.s anyway to ensure/fix cancellation

3. Can feasibly have more elements of Z_{EQ} than scheme constants

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Can we find a way to guarantee the preservation of four-dimensional relations at the one-loop level?

A Trick

- Algebra from loop is “blind” to structure of operator insertion
- Consistent treatment of renormalizable sector + replacement rules for irreducible bilinears

$$\left(\Gamma_{\mathcal{E}_i}\right) \otimes \left(\Gamma_{\mathcal{E}_j}\right) \to C_{ij}^{k\ell}\left(\Gamma_{Q_k}\right) \otimes \left(\Gamma_{Q_\ell}\right)$$

$$\left(\Gamma_1\Gamma_{\mathcal{E}_i}\Gamma_2\right) \otimes \left(\Gamma_3\Gamma_{\mathcal{E}_j}\Gamma_4\right) - C_{ij}^{k\ell}\left(\Gamma_1\Gamma_{Q_i}\Gamma_2\right) \otimes \left(\Gamma_3\Gamma_{Q_j}\Gamma_4\right) = 0$$

The Method

1. Insert physical operators into one-loop diagrams
2. Assign replacement rules; general scheme unless fixed by d-dimensional Dirac algebra
3. Insert physical operators into two-loop diagrams
4. Use renormalizable scheme within sub-lines (Γ_1, Γ_2, Γ_3, Γ_4)
5. Project $1/\epsilon^2$ poles using (2) (cancellation of subdivergences)
6. Assign new projection rules for $1/\epsilon$ poles

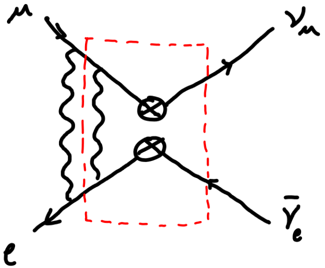
Example One: Scalar-Mediated μ-decay

- Two replacement rules from one-loop insertions:

$$(P_L\gamma^\mu\gamma^\nu)\otimes(\gamma_\nu\gamma_\mu P_L) = (4-2\epsilon)(P_L)\otimes(P_L) + (1-\epsilon\overline{\sigma}_{t1})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

$$(\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L) = (48-\epsilon\overline{\sigma}_{s2})(P_L)\otimes(P_L) + (12-\epsilon\overline{\sigma}_{t2})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

- Recursive reduction of two-loop structures (subdivergences):



$$\sim \qquad \frac{1}{\epsilon^2}\left(\Gamma_{Q_1}\gamma^{\mu\nu\alpha\beta}\right)\otimes\left(\gamma_{\beta\alpha\nu\mu}\Gamma_{Q_2}\right)$$

Example Two: Charged-Current $|\Delta F| = 1$

- Four replacement rules generated at one-loop:

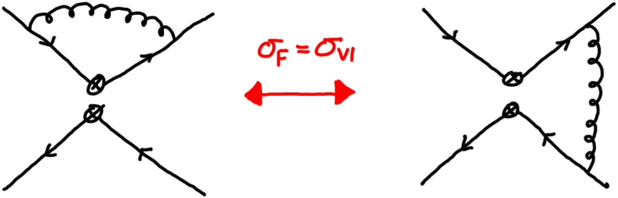
$$(\gamma^\mu\gamma^\nu\gamma^\alpha P_L\gamma_\nu\gamma_\mu) \otimes (\gamma_\alpha P_L) = (4 - \epsilon\sigma_F)(\gamma^\alpha P_L) \otimes (\gamma_\alpha P_L)$$

$$(\gamma^\alpha P_L\gamma^\mu\gamma^\nu) \otimes (\gamma_\nu\gamma_\mu\gamma_\alpha P_L) = (4 - \epsilon\sigma_{V1})(\gamma^\alpha P_L) \otimes (\gamma_\alpha P_L)$$

$$(\gamma^\mu\gamma^\nu\gamma^\alpha P_L) \otimes (\gamma_\nu\gamma_\mu\gamma_\alpha P_L) = (-8 - \epsilon\sigma_{V2})(\gamma^\alpha P_L) \otimes (\gamma_\alpha P_L)$$

$$(\gamma^\alpha P_L\gamma^\mu\gamma^\nu) \otimes (\gamma_\alpha P_L\gamma_\nu\gamma_\mu) = (-8 - \epsilon\sigma_{V3})(\gamma^\alpha P_L) \otimes (\gamma_\alpha P_L)$$

- Choosing $\sigma_F = \sigma_{V1}$ preserves Fierz (diagonal two-loop ADM)

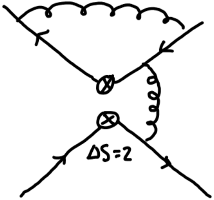


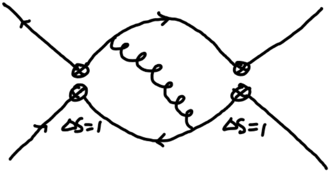
Example Three: $|\Delta S| = 2$

- One additional replacement from double insertion:

$$\left(\gamma^\alpha P_L \gamma^\mu \gamma^\beta P_L\right) \otimes \left(\gamma_\beta P_L \gamma_\mu \gamma_\alpha P_L\right) = (4 - \epsilon\sigma_{V4})\left(\gamma^\alpha P_L\right) \otimes \left(\gamma_\alpha P_L\right)$$

- In principle, need Fierz-evanescent operators; $\sigma_F = \sigma_{V1}$ ensures these contributions vanish





Are Evanescents Really Gone?

- "Replacement rules" are equivalent to evanescent operators

$$E_1 = (P_L\gamma^\mu\gamma^\nu) \otimes (\gamma_\nu\gamma_\mu P_L) - (4-2\epsilon)(P_L)\otimes(P_L) - (1-\epsilon\overline{\sigma}_{t1})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

$$E_2 = (\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L) - (48-\epsilon\overline{\sigma}_{s2})(P_L)\otimes(P_L)$$
$$- (12-\epsilon\overline{\sigma}_{t2})(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

$$E_1' = (P_L\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta)\otimes(\gamma_\beta\gamma_\alpha\gamma_\nu\gamma_\mu P_L) - \left(64 - \{16+\overline{\sigma}_{s2}+48\overline{\sigma}_{t1}\}\epsilon\right)(P_L)\otimes(P_L)$$
$$- \left(16 - \{2+16\overline{\sigma}_{t1}+\overline{\sigma}_{t2}\}\epsilon\right)(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

$$E_2' = (\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta\gamma^\sigma\gamma^\rho)\otimes(\gamma_\rho\gamma_\sigma\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L) - \left(768 - \{96+16\overline{\sigma}_{s2}-48\overline{\sigma}_{t2}\}\epsilon\right)(P_L)\otimes(P_L)$$
$$+ \left(192 - \{\overline{\sigma}_{s2}+48\overline{\sigma}_{t1}+24\overline{\sigma}_{t2}\}\epsilon\right)(\sigma^{\mu\nu}P_L)\otimes(\sigma_{\mu\nu}P_L)$$

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- "Replacement rules" are equivalent to evanescent operators

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$$E_2 = (\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta) \otimes (\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L) - (48-\epsilon\overline{\sigma}_{s2})(P_L) \otimes (P_L) - (12-\epsilon\overline{\sigma}_{t2})(\sigma^{\mu\nu}P_L) \otimes (\sigma_{\mu\nu}P_L)$$

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$$E_2' = (\sigma^{\mu\nu}P_L\gamma^\alpha\gamma^\beta\gamma^\sigma\gamma^\rho) \otimes (\gamma_\rho\gamma_\sigma\gamma_\beta\gamma_\alpha\sigma_{\mu\nu}P_L) - \left(768 - \{96+16\overline{\sigma}_{s2}-48\overline{\sigma}_{t2}\}\epsilon\right)(P_L) \otimes (P_L) + \left(192 - \{\overline{\sigma}_{s2}+48\overline{\sigma}_{t1}+24\overline{\sigma}_{t2}\}\epsilon\right)(\sigma^{\mu\nu}P_L) \otimes (\sigma_{\mu\nu}P_L)$$

Method for choosing Ev. Op.s *a priori* to guarantee $Z_{EQ} = 0$.

Pros and Cons

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Pros

- Eliminates need to re-insert many evanescent operators (simplifies automation)
- Recursive reductions - straightforward to code
- Only relies on products of bilinears - open line or trace
- Makes subdivergences/Fierz relations explicit

Pros and Cons

Pros

- Eliminates need to re-insert many evanescent operators (simplifies automation)
- Recursive reductions - straightforward to code
- Only relies on products of bilinears - open line or trace
- Makes subdivergences/Fierz relations explicit

Cons

- Doesn't know about external states (Fierz)
- Can generate many replacement rules
- **Not** a solution to γ_5 problem - traces with γ_5 still problematic
- Generalizability?

Open Questions

- Checks with chiral gauge theories

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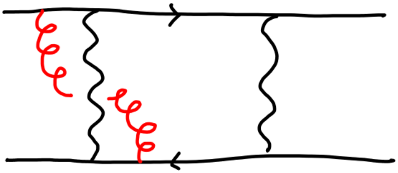

Open Questions

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- Can Fierz-preserving properties be made general?
- How to treat traces with γ_5 (Compatibility with HV?)?
- Loop-level LO matching (scheme change?)
- Ensuring gauge invariance

$$(P_L\gamma^\mu\gamma^\nu) \otimes (\gamma_\nu\gamma_\mu P_L) = (4 - 2\epsilon)(P_L) \otimes (P_L) + (1 - \epsilon\overline{\sigma}_{t1})(\sigma^{\mu\nu} P_L) \otimes (\sigma_{\mu\nu} P_L)$$

Summary

- Presented method to ensure (most) evanescent insertions do not enter computation of NLL ADMs
- Particularly geared toward automating RGE computations
- Still many open questions to be answered