SMEFT Renormalisation

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SMEFT Renormalisation Status (2023)!

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{\leq 4}$ (bosonic)			✓						✓		This talk
$d_{\leq 4}$ (fermionic)			✓						X		X
d_5	✓				✓	✓					
d_6 (bosonic)		✓	✓					X	✓	X	This talk
d_6 (fermionic)		✓	✓					X	X	X	X
d_7				✓	✓	✓					
d_8 (bosonic)							This talk	This talk	✓	This talk	This talk
d_8 (fermionic)							X	X	X	X	✓

Blank entries vanish; ✓ $\to$ known; ✓ $\to$ substantially known (not complete); X $\to$ nothing, or very little, is known. The contribution discussed in this talk is marked by ▬.

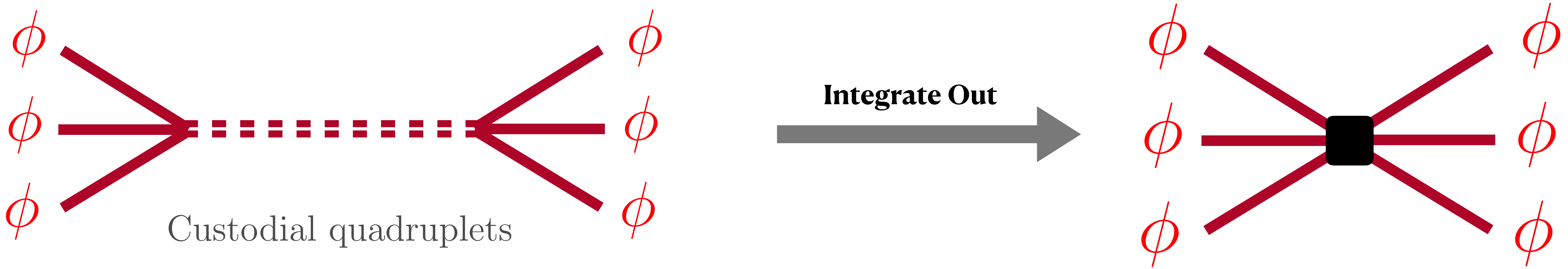
Table compiled from: arxiv:2106.05291, 2205.03301, 2301.07151 . Check these out for Refs.

Motivations :

- **RGEs reveal restrictions on Dim-8 WC space imposed from positivity bounds.**

Chala, Santiago (2110.01624); Chala (2301.09995), …

- **Custodial symmetry violation absent at tree-level dim-6, dim-8, and 1-loop dim-6.**



Chala, Krause, Nardini (1802.02168); Durieux, McCullough, Salvioni (2209.00666)

- **Also, Dim-8 RGEs computed using multiple computational tools, with agreement in results. These establish validation among these tools.**

SMEFT Dim-8 RGEs

Λ = EFT cut-off scale

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_i c_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)} O_i^{(6)} + \frac{1}{\Lambda^3}\sum_j c_j^{(7)} O_j^{(7)} + \frac{1}{\Lambda^4}\sum_j c_j^{(8)} O_j^{(8)} + \cdots$$

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu}\frac{dc_i^{(8)}}{d\tilde{\mu}} = \boxed{\gamma_{ij} c_j^{(8)}} + \gamma'_{ijk} c_j^{(7)} c_k^{(5)} + \gamma''_{ijklm} c_j^{(5)} c_k^{(5)} c_l^{(5)} c_m^{(5)} + \ldots$$

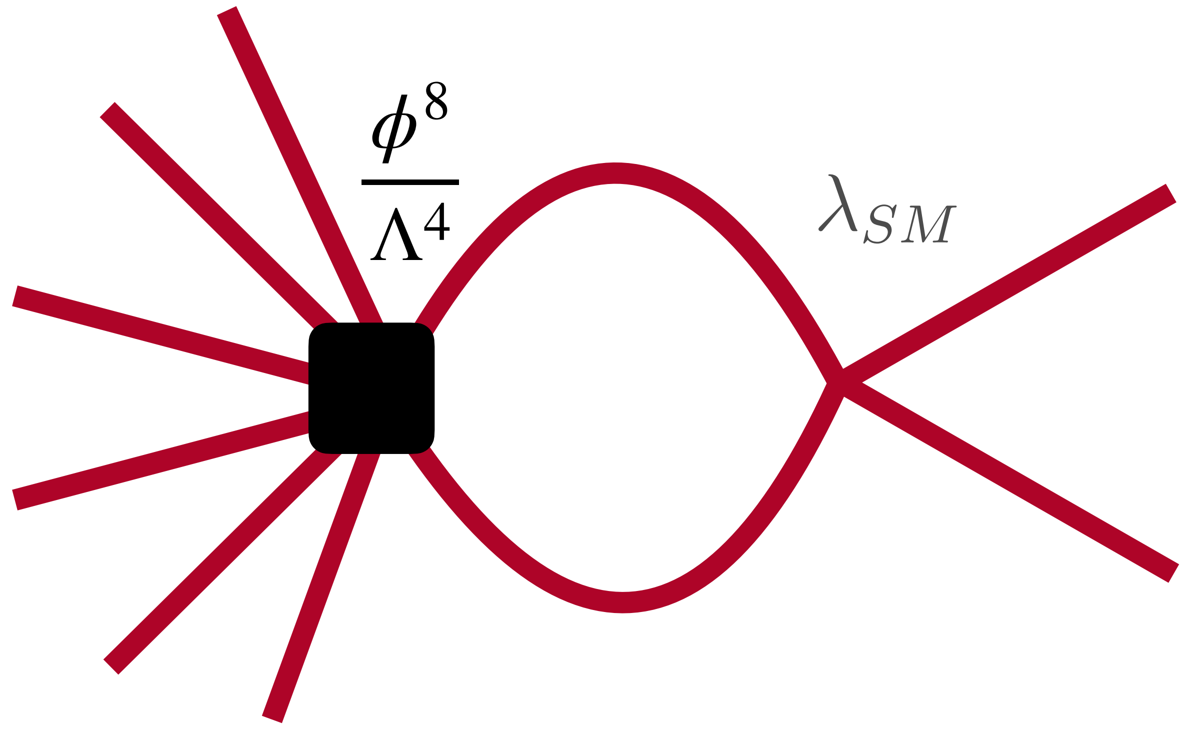
- ❖ One dim-8 operator insertion.

arXiv:2205.03301

Towards the renormalisation of the Standard Model effective field theory to dimension eight: Bosonic interactions II

- SDB, M Chala, Á Díaz-Carmona, G Guedes

e.g. :



Bosonic SMEFT Dim-8 RGEs

Λ = EFT cut-off scale

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda}\sum_i c_i^{(5)} O_i^{(5)} + \frac{1}{\Lambda^2}\sum_i c_i^{(6)} O_i^{(6)} + \frac{1}{\Lambda^3}\sum_j c_j^{(7)} O_j^{(7)} + \frac{1}{\Lambda^4}\sum_j c_j^{(8)} O_j^{(8)} + \cdots$$

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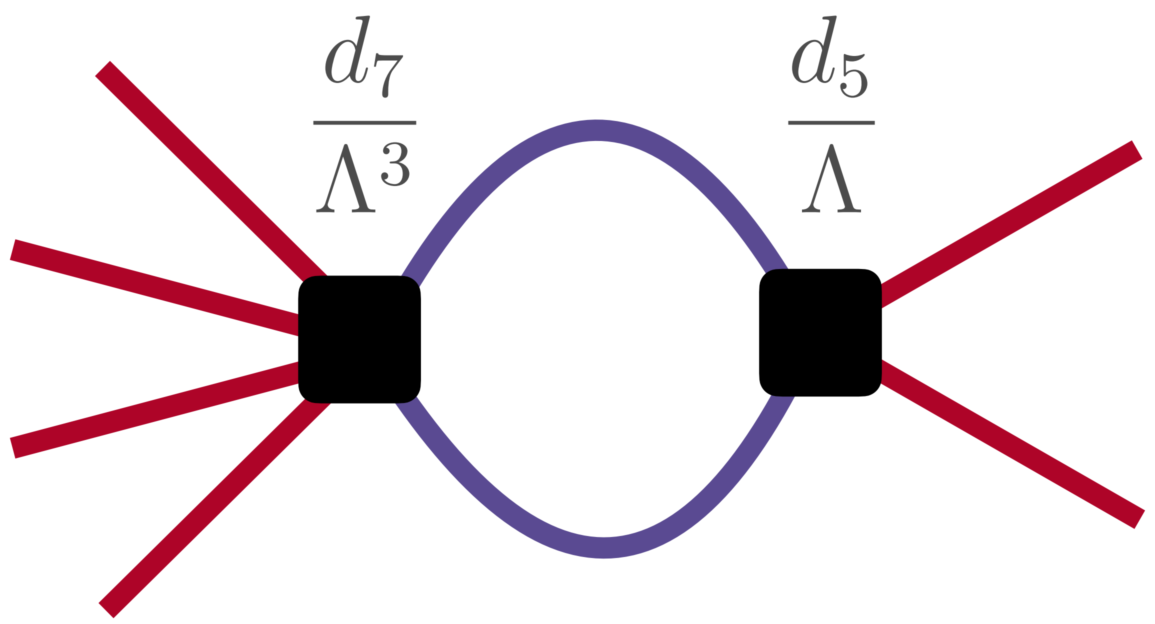
❖ Dim-7 and dim-5 operators insertions.

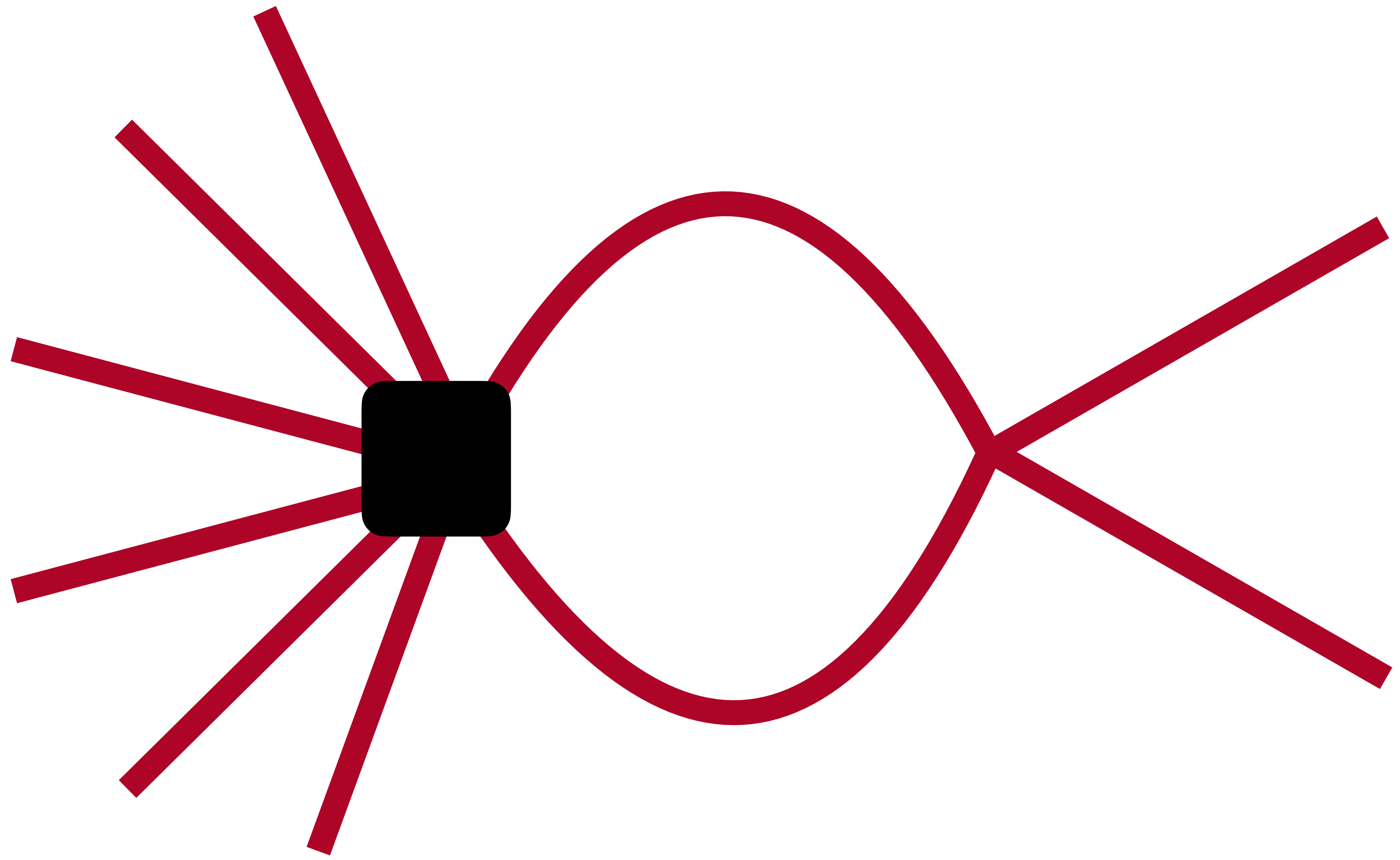
arXiv:2301.07151

Renormalisation of SMEFT bosonic interactions
up to dimension eight by LNV operators

- SDB, Á Díaz-Carmona

e.g. :





Part 1 : Dim-8 renormalisation by Dim-8

arXiv:2301.07151

SMEFT Operator Classes

$$\dot{c}_i^{(8)} \equiv 16\pi^2 \tilde{\mu} \frac{dc_i^{(8)}}{d\tilde{\mu}} = \gamma_{ij} c_j^{(8)} + \gamma'_{ijk} c_j^{(7)} c_k^{(5)} + \gamma''_{ijklm} c_j^{(5)} c_k^{(5)} c_l^{(5)} c_m^{(5)} + \ldots$$

- One **tree-level generated** dim-8 operator in one-loop.

Classes of operator that are tree-level generated:

arXiv:2001.00017
- Craig, Jiang, Li, Sutherland

Bosonic : $\{\phi^8, \phi^6 D^2, \phi^4 D^4, X^2 \phi^4, X \phi^4 D^2, X^2 H^2 D^2, X^3 H^2, X^4\}$

Fermionic : $\{\psi^2 X \phi^3,\ \psi^2 \phi^2 D^3,\ \psi^2 \phi^5,\ \psi^2 \phi^4 D,\ \psi^2 X \phi^2 D,\ \psi^2 \phi^3 D^2, \psi^2 X^2 \phi, \psi^2 X^2 D, \psi^2 X \phi D^2\}$

SMEFT Dim-8 **on-shell basis** : arXiv:2005.00059 — C. W. Murphy

SMEFT Dim-8 **Green's/off-shell basis** : arXiv:2112.12724 — M. Chala, Á Díaz-Carmona, G. Guedes

Divergences to RGEs, some details:

- Compute **1-PI loop diagrams**. Use **FeynRules, FeynARTs,** and **FormCalc** packages.

- Divergences are captured by the operators of **off-shell/Green's basis**.

arXiv:2112.12724
- M Chala, Á Díaz-Carmona, G Guedes

$$16\pi^2\epsilon\,\mathcal{L}_{\rm DIV} = \tilde{K}_\phi (D_\mu\phi)^\dagger(D^\mu\phi) + \tilde{\mu}^2|\phi|^2 - \tilde{\lambda}|\phi|^4 + \tilde{c}_i^{(6)}\frac{\mathcal{O}_i^{(6)}}{\Lambda^2} + \tilde{c}_j^{(8)}\frac{\mathcal{O}_j^{(8)}}{\Lambda^4}$$

[on RHS we have Green's basis]

- **Removing redundant operators** using on-shell relations.

arXiv:2106.05291
- M Chala, G Guedes, M Ramos, J Santiago

- **Cross-checks with MatchMakerEFT**. ✓
  H^8 topologies are computed in MM primarily.

arXiv:2112.10787
- A Carmona, A Lazopoulos, P Olgoso, J Santiago

- **Subset of our RGEs cross-validated** with arXiv:2108.03669 (on-shell amplitude methods). ✓

(at linear order in the Wilson coefficients and leading (quadratic) order in the renormalisable couplings)

arXiv:2108.03669
- M A Huber, S De Angelis.

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- **Cross-validated** with arXiv:2108.03669 (geo-SMEFT approach).

arXiv:2108.03669
- A Helset, E Jenkins, A Manohar. ✓

arXiv:2307.03187
- B Assi, A Helset, A Manohar, J Pages, C Shen.

Bosonic-bosonic RGE:

Classes of tree-generated bosonic operators

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	ϕ^8
$B^2\phi^2 D^2$	g_1^2	0	0	0	0	0	0	0	0
$W^2\phi^2 D^2$	g_2^2	0	0	0	0	0	0	0	0
$WB\phi^2 D^2$	$g_1 g_2$	0	0	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0
$W^2B\phi^2$	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	g_2^2	0	0	0	0	0	0	0	0
$B\phi^4 D^2$	$g_1 g_2^2$	λ	0	0	0	0	0	0	0
$W\phi^4 D^2$	g_2^3	0	g_2^2	0	0	0	0	0	0
$B^2\phi^4$	$g_1^2 g_2^2$	$g_1\lambda$	$g_1^2 g_2$	λ	0	$g_1 g_2$	0	0	0
$W^2\phi^4$	g_2^4	$g_1 g_2^2$	g_2^3	0	λ	$g_1 g_2$	0	0	0
$WB\phi^4$	$g_1 g_2^3$	$g_2\lambda$	$g_1\lambda$	$g_1 g_2$	$g_1 g_2$	λ	0	0	0
$G^2\phi^4$	0	0	0	0	0	0	g_3^2	0	0
$\phi^6 D^2$	g_2^4	$g_1\lambda$	$g_2\lambda$	0	0	0	0	λ	0
ϕ^8	λ^3	$g_1\lambda^2$	$g_2\lambda^2$	$g_1^2\lambda$	$g_2^2\lambda$	$g_1 g_2\lambda$	0	λ^2	λ

- **Largest contribution from each operator class is shown.**
- **Loop generated operators that are renormalised by tree-generated operators are in gray.**
- **Blue entries contribute larger than what expected from naive dimensional analysis.**

$$\tilde{\mu}\frac{dc_{\phi^8}}{d\tilde{\mu}} = \frac{1}{16\pi^2}\left(192\lambda - 6(g_1^2 + 3g_2^2) + \ldots\right) c_{\phi^8}$$

Fermionic-bosonic RGE:

Classes of tree-generated fermionic operators

	$\psi^2 B\phi^3$	$\psi^2 W\phi^3$	$\psi^2 G\phi^3$	$\psi^2\phi^2 D^3$	$\psi^2\phi^5$	$\psi^2\phi^4 D$	$\psi^2 B\phi^2 D$	$\psi^2 W\phi^2 D$	$\psi^2 G\phi^2 D$	$\psi^2\phi^3 D^2$
$B^2\phi^2 D^2$	0	0	0	g_1^2	0	0	0	0	0	0
$W^2\phi^2 D^2$	0	0	0	g_2^2	0	0	0	0	0	0
$WB\phi^2 D^2$	0	0	0	$g_1 g_2$	0	0	0	0	0	0
$G^2\phi^2 D^2$	0	0	0	g_3^2	0	0	0	0	0	0
$W^3\phi^2$	0	0	0	0	0	0	0	0	0	0
$W^2 B\phi^2$	0	0	0	0	0	0	0	0	0	0
$G^3\phi^2$	0	0	0	0	0	0	0	0	0	0
$\phi^4 D^4$	0	0	0	$\|y^t\|^2$	0	0	0	0	0	0
$B\phi^4 D^2$	0	0	0	$g_1\|y^t\|^2$	0	0	$\|y^t\|^2$	0	0	$g_1 y^t$
$W\phi^4 D^2$	0	0	0	$g_2\|y^t\|^2$	0	0	0	$\|y^t\|^2$	0	$g_2 y^t$
$B^2\phi^4$	$g_1 y^t$	0	0	$g_1^2\|y^t\|^2$	0	0	$g_1\|y^t\|^2$	0	0	$g_1^2 y^t$
$W^2\phi^4$	0	$g_2 y^t$	0	$g_2^2\|y^t\|^2$	0	g_2^2	0	$g_2\|y^t\|^2$	0	$g_2^2 y^t$
$WB\phi^4$	$g_2 y^t$	$g_1 y^t$	0	$g_1 g_2\|y^t\|^2$	0	$g_1 g_2$	$g_2\|y^t\|^2$	$g_1\|y^t\|^2$	0	$g_1 g_2 y^t$
$G^2\phi^4$	0	0	$g_3 y^t$	0	0	0	0	0	0	0
$\phi^6 D^2$	0	0	0	$g_2^2\|y^t\|^2$	0	$\|y^t\|^2$	$g_1\|y^t\|^2$	$g_2\|y^t\|^2$	0	$y^t\|y^t\|^2$
ϕ^8	0	0	0	$\lambda\|y^t\|^4$	$y^t\|y^t\|^2$	$\lambda\|y^t\|^2$	$g_1\lambda\|y^t\|^2$	$g_2\lambda\|y^t\|^2$	0	$\lambda y^t\|y^t\|^2$

$$\dot{c}^{(1)}_{W\phi^4 D^2} = 44 g_2 (c^{(4)}_{q^2\phi^2 D^3})_{\alpha_1,\alpha_2}\, y^u_{\alpha_2,\alpha_3} (y^u)^*_{\alpha_3,\alpha_1} + 48 (c^{(11)}_{q^2 W H^2 D})_{\alpha_1,\alpha_2}\, y^u_{\alpha_2,\alpha_3} (y^u)^*_{\alpha_1,\alpha_3} + \cdots$$

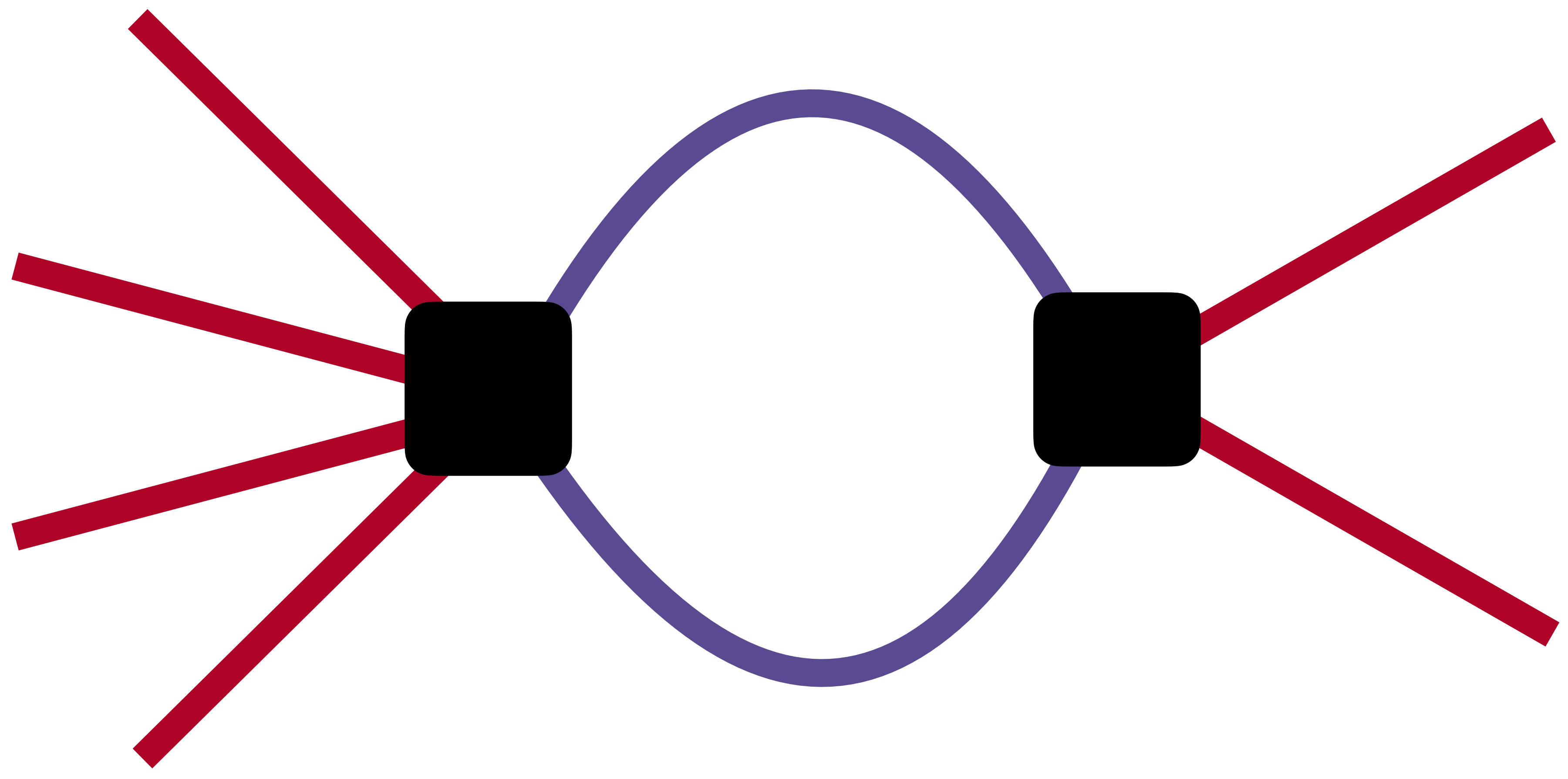
RGEs of Dim-6,4,2

μ^2 **is the squared Higgs mass in the SMEFT.**

- Dim-8 operators also induce running of dim-6, dim-4, dim-2 operators.

	$\phi^4 D^4$	$B\phi^4 D^2$	$W\phi^4 D^2$	$B^2\phi^4$	$W^2\phi^4$	$WB\phi^4$	$G^2\phi^4$	$\phi^6 D^2$	ϕ^8
ϕ^2	μ^6	0	0	0	0	0	0	0	0
ϕ^4	$\lambda\mu^4$	$g_1\mu^4$	$g_2\mu^4$	0	0	0	0	μ^4	0
$B^2\phi^2$	$g_1^2\mu^2$	$g_1\mu^2$	0	μ^2	0	0	0	0	0
$W^2\phi^2$	$g_2^2\mu^2$	0	$g_2\mu^2$	0	μ^2	0	0	0	0
$WB\phi^2$	$g_1 g_2\mu^2$	$g_2\mu^2$	$g_1\mu^2$	0	0	μ^2	0	0	0
$G^2\phi^2$	0	0	0	0	0	0	μ^2	0	0
$\phi^4 D^2$	$\lambda\mu^2$	$g_1\mu^2$	$g_2\mu^2$	0	0	0	0	μ^2	0
ϕ^6	$\lambda^2\mu^2$	$\lambda g_1\mu^2$	$\lambda g_2\mu^2$	$g_1^2\mu^2$	$g_2^2\mu^2$	$g_1 g_2\mu^2$	0	$\lambda\mu^2$	μ^2

Lower dim. classes renormalised by the bosonic dim-8 operators.
Similar contributions from two-fermionic dim-8 operators are also present.



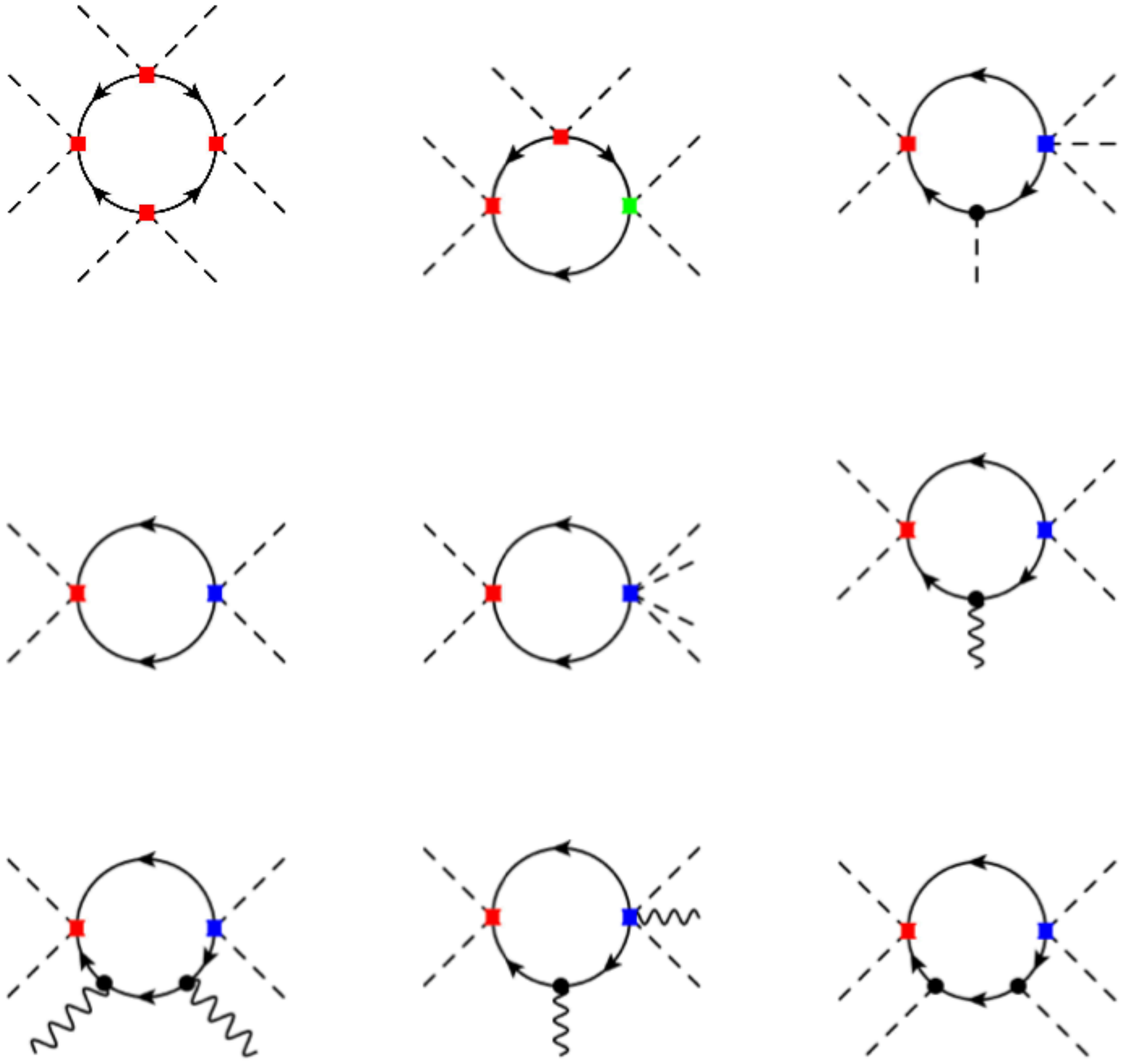
Part 2 : Dim-8 renormalisation by Dim-5 & Dim-7

arXiv:2301.07151

LNV contributions (1-loop) to bosonic SMEFT: D5, D6, D7 insertions

$1/\Lambda^4$	d_5^4	$d_5^2 \times d_6$	$d_5 \times d_7$
8-Higgs	✓	-	-
6-Higgs	-	✓	✓
4-Higgs	-	-	✓
2-Higgs	-	-	-
0-Higgs	-	-	-

The hyphen stands for the vanishing contributions from LNVs upto Λ^{-3} at 1-loop.



Bosonic SMEFT RGEs from LNVs :

	$(\alpha_{\ell\phi})^4$	$(\alpha_{\ell\phi})^2\,\beta_{\phi D}$	$(\alpha_{\ell\phi})^2\,\beta^{(1)}_{\phi\ell}$	$(\alpha_{\ell\phi})^2\,\beta^{(3)}_{\phi\ell}$	$\alpha_{\ell\phi}\omega_{\ell\phi}$	$\alpha_{\ell\phi}\omega^{(1)}_{\ell\phi D}$	$\alpha_{\ell\phi}\omega^{(2)}_{\ell\phi D}$	$\alpha_{\ell\phi}\omega_{\ell\phi De}$	$\alpha_{\ell\phi}\omega_{\ell\phi W}$
γ_{ϕ^8}	16	8λ	32λ	32λ	16λ	$2\lambda g_2^2$	λg_2^2	0	0
$\gamma^{(1)}_{\phi^6}$	0	4	48	64	16	$\frac{14}{3}g_2^2$	32λ	$4y^e$	0
$\gamma^{(2)}_{\phi^6}$	0	8	32	16	8	$\frac{1}{6}g_2^2$	16λ	$4y^e$	0
$\gamma^{(2)}_{\phi^4}$	0	0	0	0	0	$\emptyset$	8	0	0
$\gamma^{(1)}_{W\phi^4D^2}$	0	0	0	0	0	$8g_2$	$4g_2$	0	$\emptyset$
$\gamma^{(2)}_{W\phi^4D^2}$	0	0	0	0	0	$8g_2$	$4g_2$	0	$\emptyset$
$\gamma^{(3)}_{W\phi^4D^2}$	0	0	0	0	0	$4g_2$	$2g_2$	0	$\emptyset$
$\gamma^{(4)}_{W\phi^4D^2}$	0	0	0	0	0	$4g_2$	$2g_2$	0	$\emptyset$
$\gamma^{(1)}_{W^2\phi^4}$	0	0	0	0	0	$\frac{1}{2}g_2^2$	$\frac{1}{4}g_2^2$	0	$4g_2$
$\gamma^{(2)}_{W^2\phi^4}$	0	0	0	0	0	$\frac{1}{2}g_2^2$	$\frac{1}{4}g_2^2$	0	$4g_2$
$\gamma^{(3)}_{W^2\phi^4}$	0	0	0	0	0	$\frac{1}{2}g_2^2$	$\frac{1}{4}g_2^2$	0	$4g_2$
$\gamma^{(4)}_{W^2\phi^4}$	0	0	0	0	0	$\frac{1}{2}g_2^2$	$\frac{1}{4}g_2^2$	0	$4g_2$
$\gamma^{(1)}_{WB\phi^4}$	0	0	0	0	0	g_1g_2	$\frac{1}{2}g_1g_2$	0	$\emptyset$
$\gamma^{(2)}_{WB\phi^4}$	0	0	0	0	0	g_1g_2	$\frac{1}{2}g_1g_2$	0	$\emptyset$
γ_{ϕ}	0	$4\mu^2$	$16\mu^2$	$16\mu^2$	$8\mu^2$	$\mu^2 g_2^2$	$16\mu^2\lambda$	0	0
$\gamma_{\phi\Box}$	0	0	0	0	0	0	$16\mu^2$	0	0
$\gamma_{\phi D}$	0	0	0	0	0	0	$16\mu^2$	0	0
γ_{λ}	0	0	0	0	0	0	$8\mu^4$	0	0

Simple Application (in context of LNVs) :

$$T = -\frac{1}{2\alpha}\frac{v^2}{\Lambda^2}\left(c_{\phi D} + c^{(2)}_{\phi^6}\frac{v^2}{\Lambda^2}\right)$$

$$\alpha = \frac{1}{137}$$

Blas et. al. 2017
arxiv:1611.05354

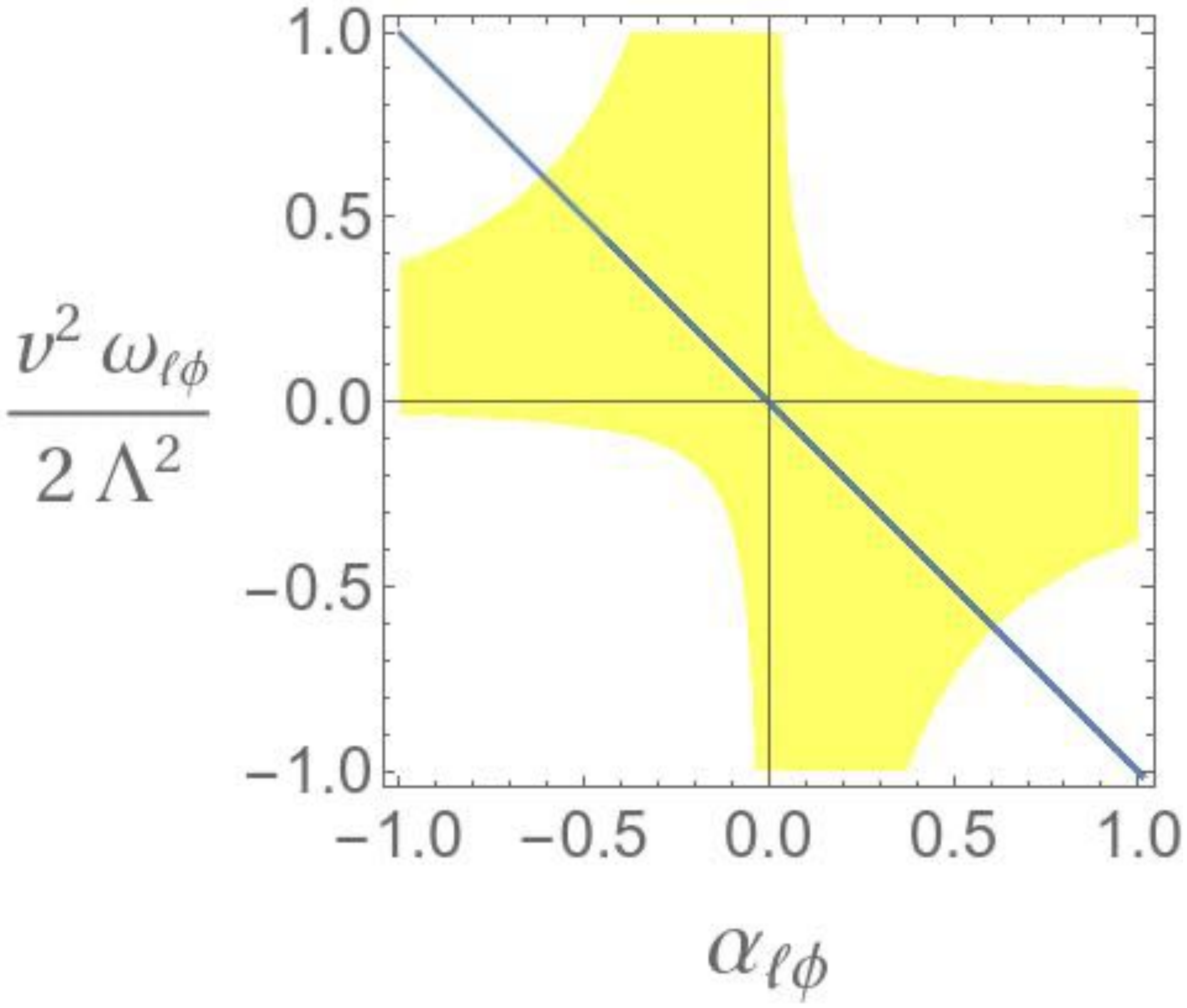
$$\dot{c}^{(2)}_{\phi^6} \supset -8\,\mathrm{Re}\left(\mathrm{Tr}\left[\alpha^{\dagger}_{\ell\phi}\omega_{\ell\phi}\right]\right)$$

$$\therefore \quad T = -\frac{1}{4\pi^2\alpha}\frac{v^4}{\Lambda^4}\ln\left[\frac{\Lambda}{\mu}\right]\sum_{mn}^{3}\alpha_{\ell\phi,mn}\omega_{\ell\phi,mn}$$

Simple Application :

Neutrino mass bounds: $$(M_N)_{mn} = -\frac{v^2}{\Lambda}\left(\alpha_{\ell\phi,mn} + \frac{v^2}{2\Lambda^2}\omega_{\ell\phi,mn}\right)$$

For single non-vanishing flavor direction :



$$\alpha_{\ell\phi} \to \epsilon_{ij}\epsilon_{mn}(\ell^i C \ell^m)(\phi^j \phi^n)$$

$$\omega_{\ell\phi} \to \epsilon_{ij}\epsilon_{mn}(\ell^i C \ell^m)(\phi^j \phi^n)(\phi^\dagger \phi)$$

$$\Lambda = 1 \text{ TeV, and } \mu = 246 \text{ GeV,}$$
$$T = 0.10 \pm 0.12 \text{ , and } M_N < 0.081 \text{ eV}$$

Blas et. al. 2017
arxiv:1611.05354

Loureiro et. al. 2018
Arxiv:1811.02578

Summary

- Renormalisation of bosonic SMEFT dim-8 operators discussed.
  - By Tree-level generated dim-8 operators.
  - By dim-5 & dim-7 LNV operators.
- These operators also contribute to the running of lower dimensional operators.
- Certain elements contribute larger than what expected from naive dimensional analysis.
- T-parameter example: RGEs translate bounds to blind direction of LNV spaces.

Thanks for your attention!