How to Match Effective Field Theories with MATCHETE

Felix Wilsch — University of Zurich

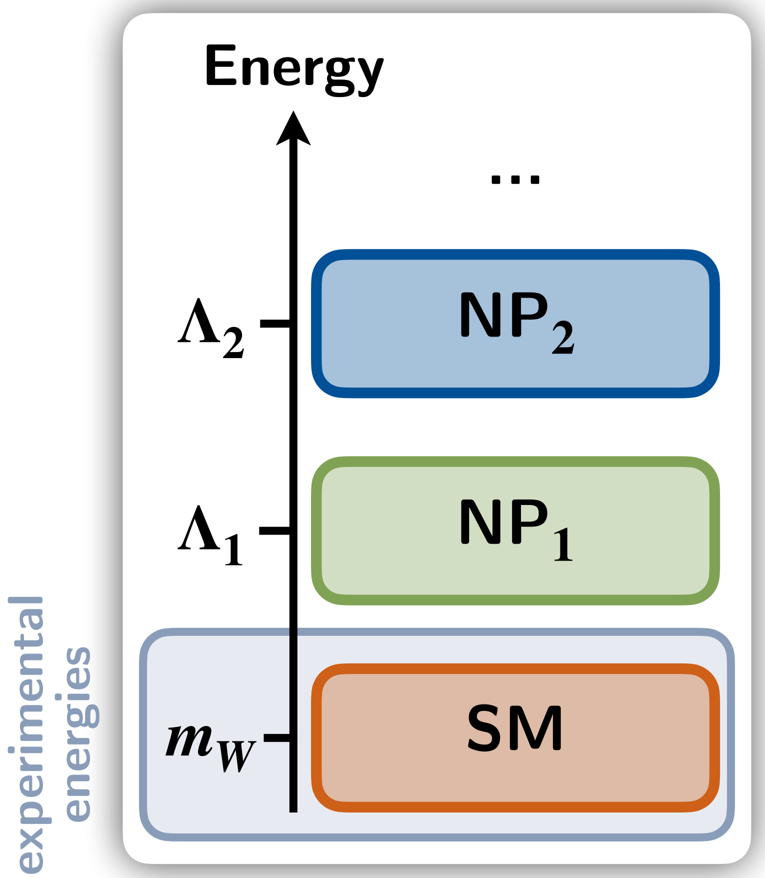
<u>In collaboration with:</u>

Javier Fuentes-Martín, Matthias König, Julie Pagès, Anders Eller Thomsen

Based on arXiv: [2212.04510], [2211.09144], [2012.08506]

MITP — EFT Foundations and Tools 2023 August 31, 2023

- Heavy BSM particles not directly produced in experiments
- Probe heavy states indirectly through imprints on low-energy observables $\mathcal{O}_{\text{exp}} \simeq \mathcal{O}_{\text{SM}} + \delta\mathcal{O}_{\text{NP}}$



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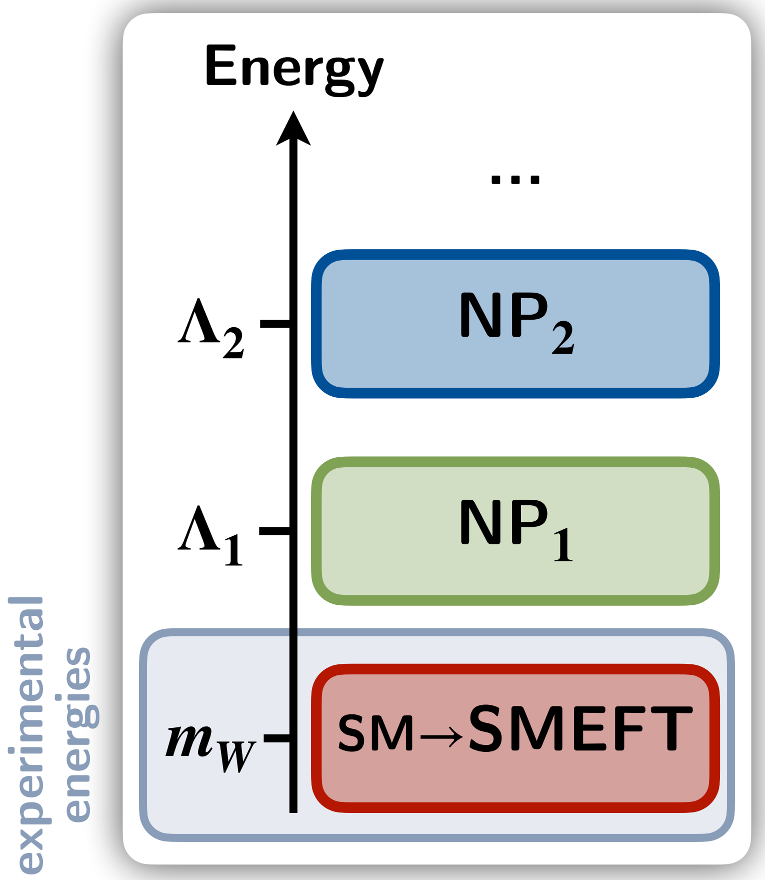
- **Effective Field Theory (EFT):**
  - Consider $\mathscr{L}_{\text{NP}}(\eta_H, \eta_L)$ with fields η_H and η_L with masses $\Lambda_1 \sim m_H \gg m_L \sim m_W$
  - Construct effective description $\mathscr{L}_{\text{EFT}}(\eta_L)$ containing only SM particles η_L
  - Effects η_H incorporated through new small interactions Q_i

$$\mathscr{L}_{\text{EFT}}(\eta_L) = \mathscr{L}_{d=4}(\eta_L) + \sum_{d=5}^{\infty} \frac{1}{m_H^{d-4}} \sum_i C_i^{(d)} Q_i^{(d)}(\eta_L)$$

  - Only finite number of operators Q_i allowed (for fixed d)

➡ Model independent



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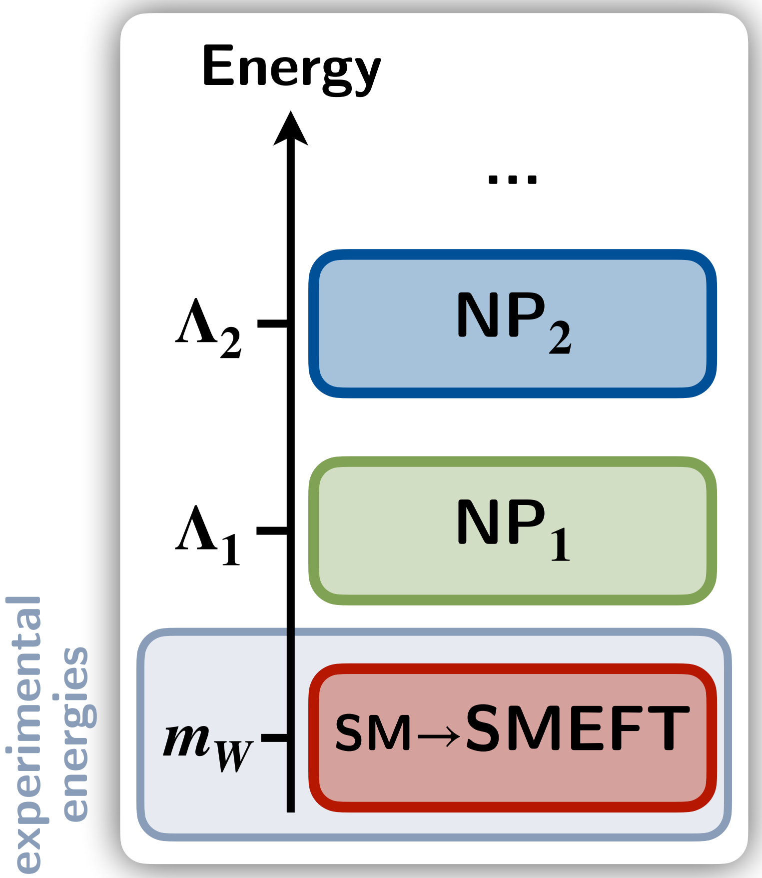
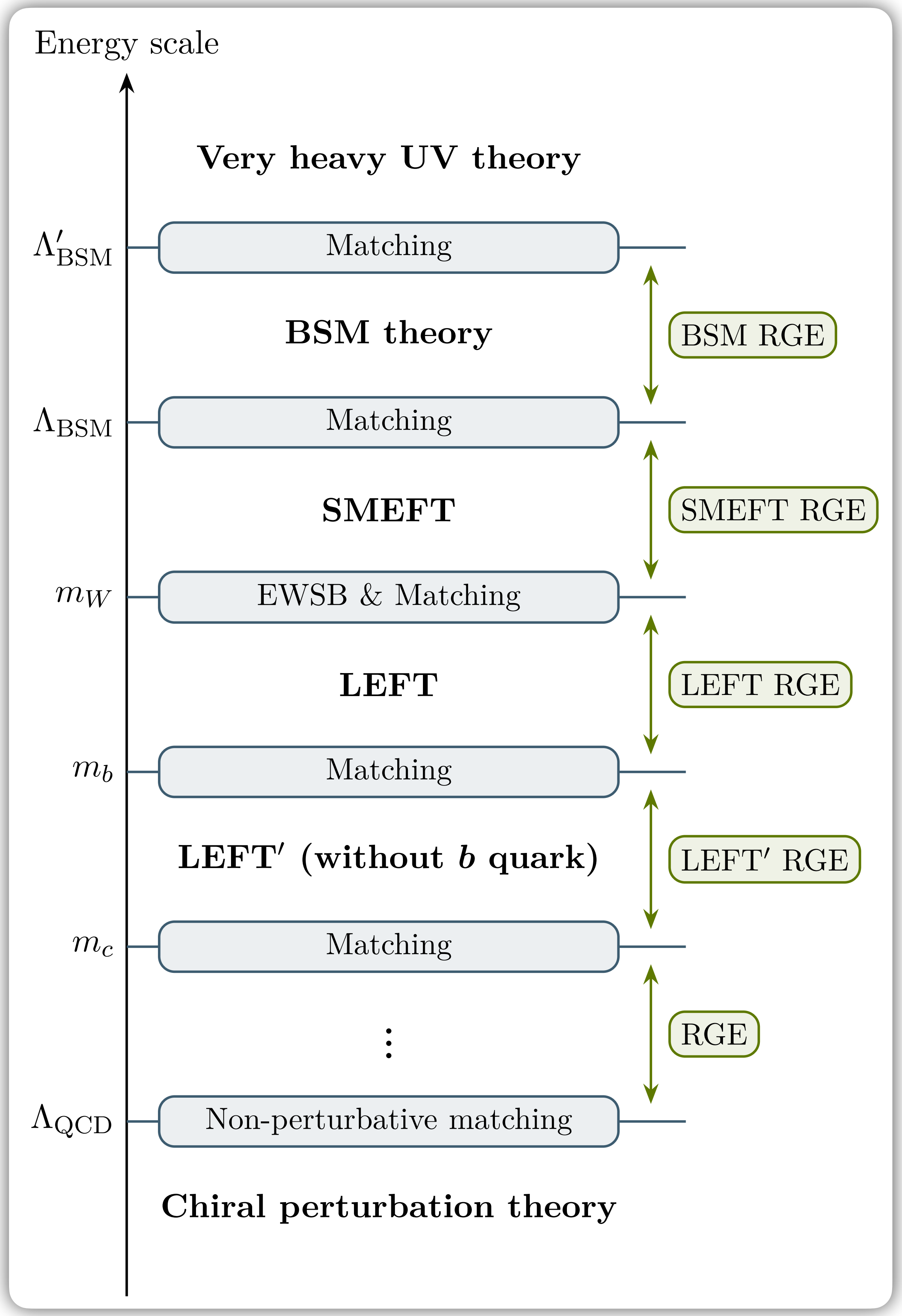


- Challenge:
  - Relate Wilson coefficients C_i to explicit BSM theories

Phenomenological SMEFT Analyses

- Tower of EFTs valid at different energies
  - Different ETFs related by:
    - Matching calculations
    - Renormalization group evolution



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  - **Automation: SMEFT—LEFT**

    `DsixTools`: Celis, Fuentes-Martin, Vicente, Virto [1704.04504]
    Fuentes-Martin, Ruiz-Femenia, Vicente, Virto [2010.16341]
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    based on theory work:
    Jenkins, Manohar, Trott [1308.2627], [1310.4838]
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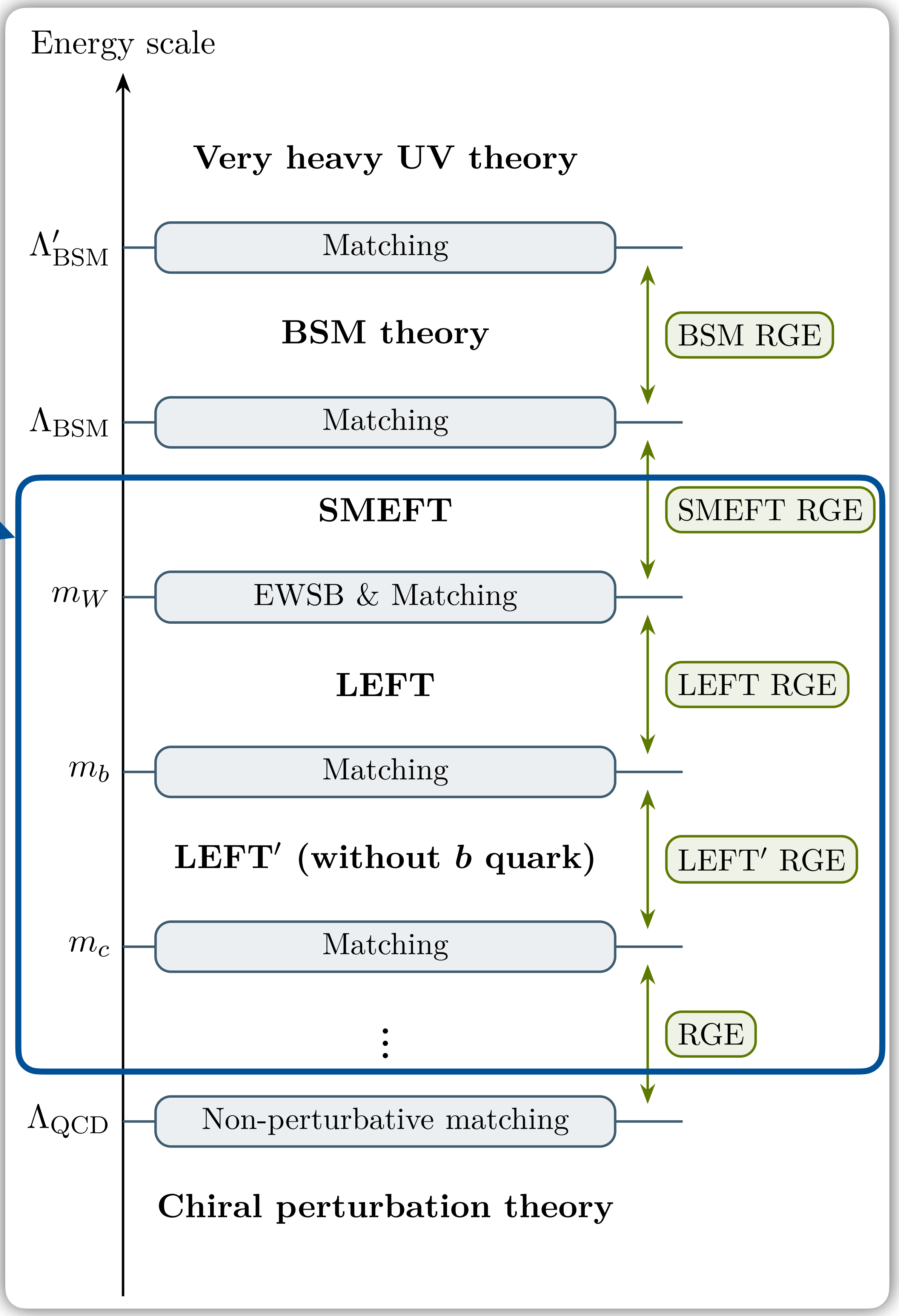

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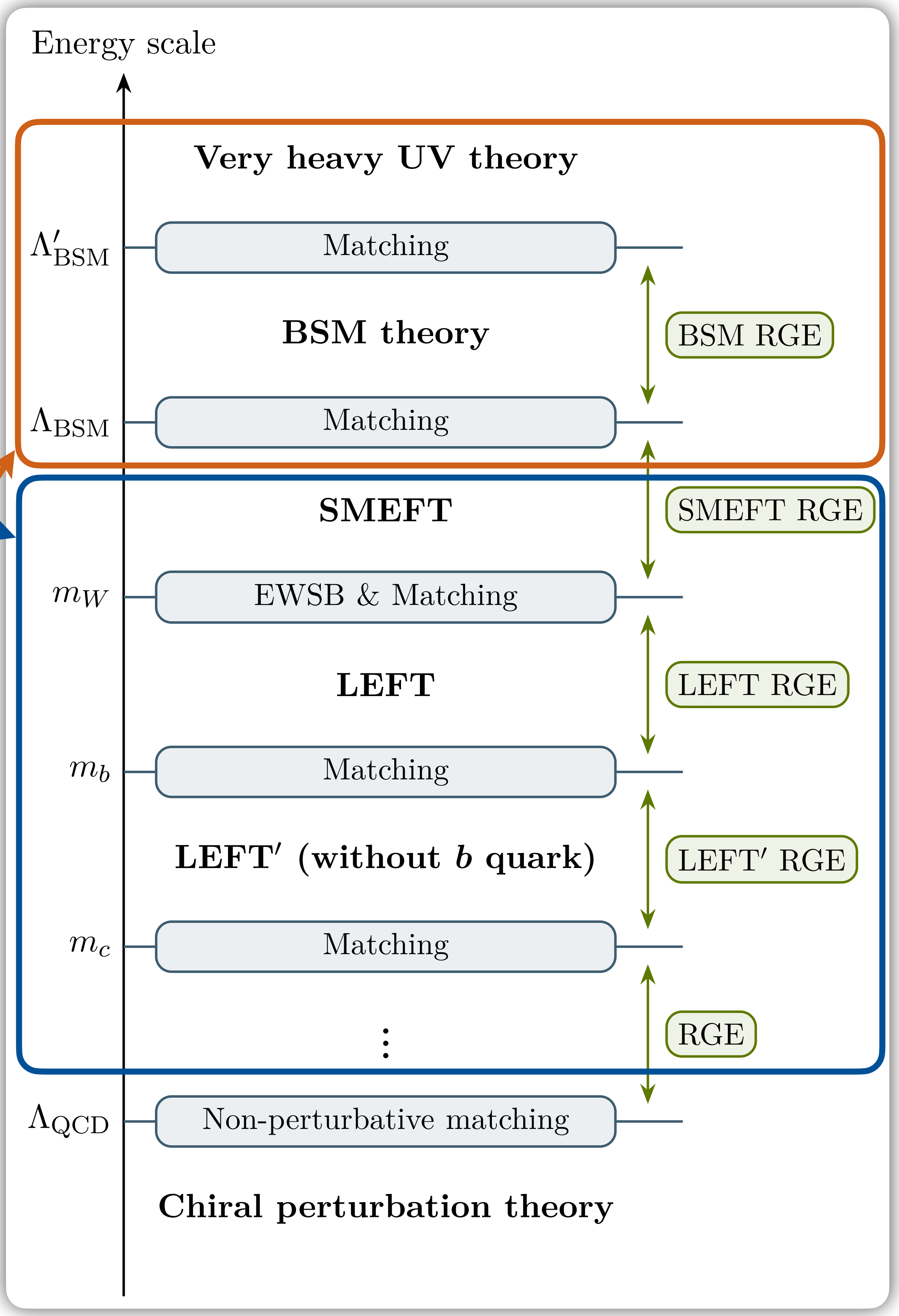

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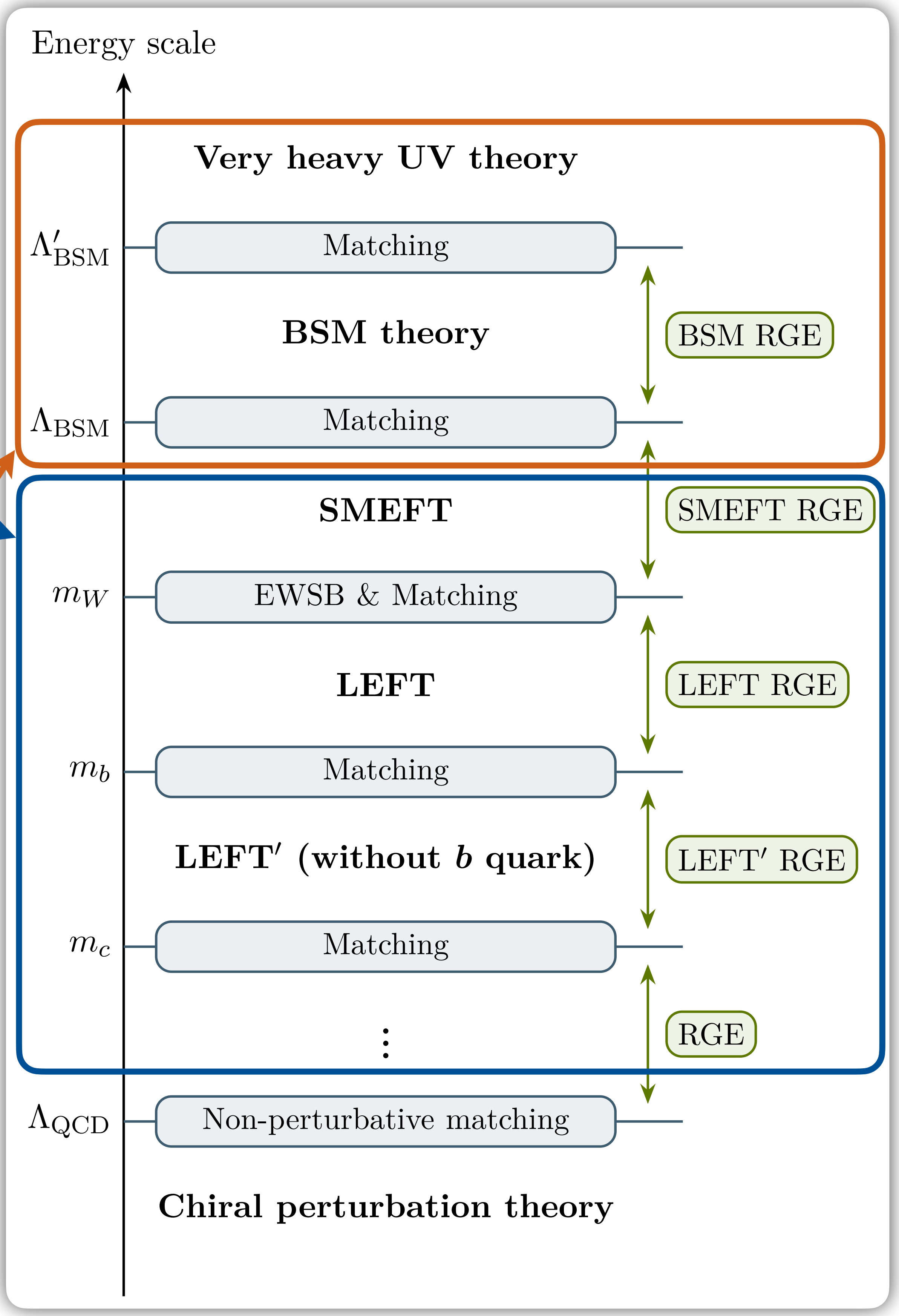
➡ **Goal:** integrate matching, running and phenomenology codes into unified software



Matching Effective Field Theories

- **EFT Matching:** determine the EFT Wilson coefficients in terms of the UV couplings

  - Equating S-matrix elements in both theories: $\langle \eta_L | S_{\text{EFT}} | \eta_L \rangle = \langle \eta_L | S_{\text{UV}} | \eta_L \rangle$

  - Equating the effective action of both theories: $\Gamma_{\text{EFT}}[\eta_L] = \Gamma_{\text{UV}}[\eta_L, \eta_H(\eta_L)]$

  ➡ Expand UV contribution in powers of m_H^{-1} (*operator product expansion*)

  ➡ Matching conditions

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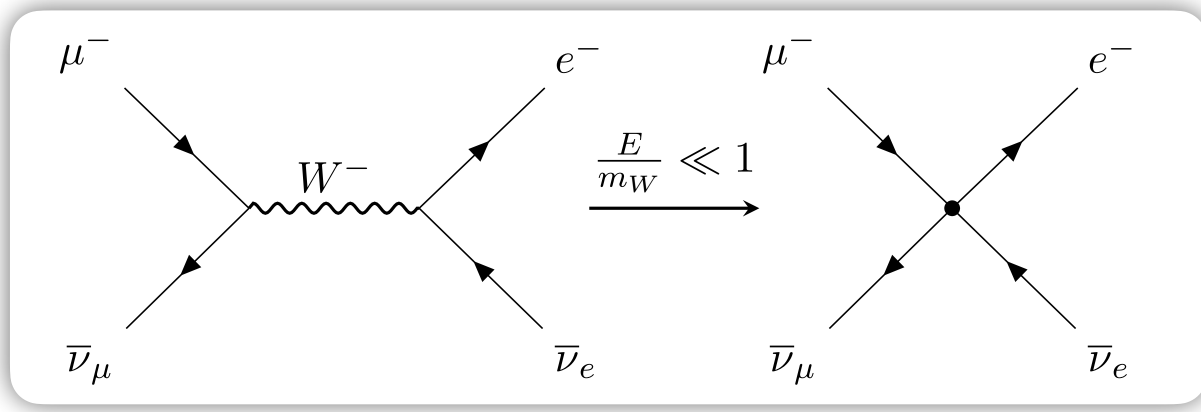
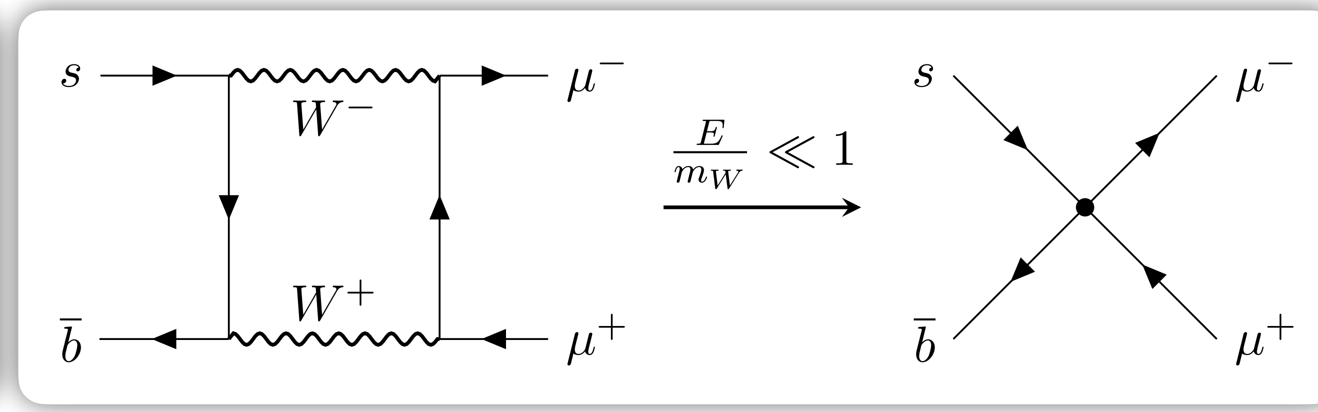


Tree-level matching



One-loop matching

➡ Knowledge of EFT operators required

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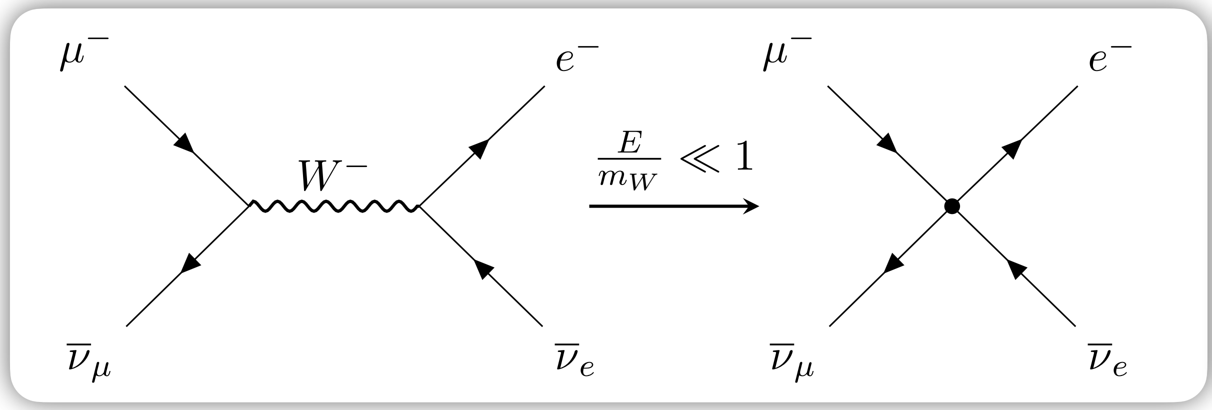

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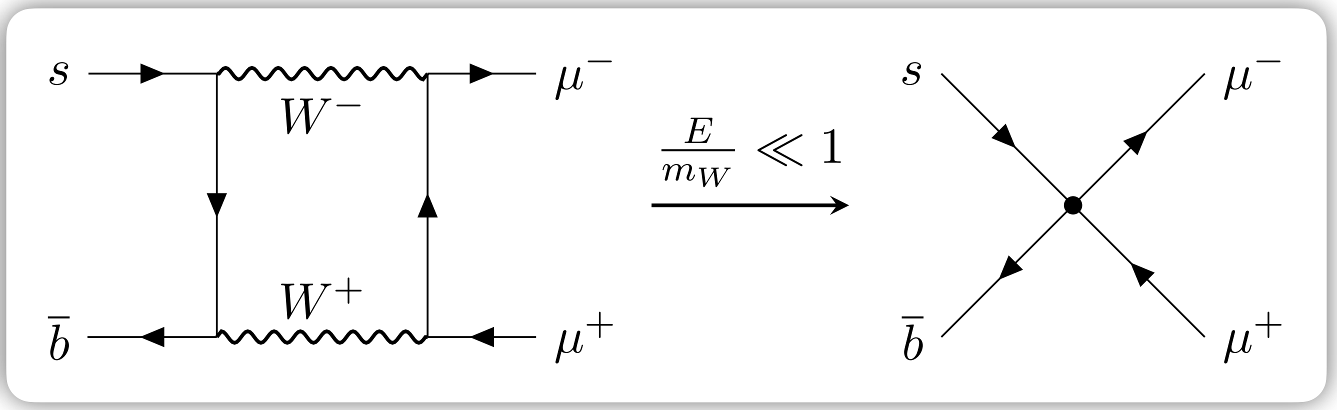

One-loop matching

➡ Knowledge of EFT operators required

- **Functional matching:** compute $\Gamma[\eta_L]$ through its path integral representation

One-Loop Matching

Automation of Functional One-Loop Matching of EFTs

Functional Matching

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- **Lagrangian:** $\mathscr{L}_{\mathrm{UV}}(\eta)$ with fields $\eta = (\eta_H, \eta_L)^\intercal$ and hierarchy $m_H \gg m_L$

- **Background field method:** shift all fields $\eta \to \hat{\eta} + \eta$
  - $\hat{\eta}$: background fields (satisfy classical EOM)
  - η: pure quantum fluctuation

- **Path integral representation of effective quantum action:**

$$\exp\left(i\Gamma_{\mathrm{UV}}(\hat{\eta})\right) = \int \mathscr{D}\eta \, \exp\left(i\int \mathrm{d}^d x \, \mathscr{L}_{\mathrm{UV}}(\eta + \hat{\eta})\right)$$

  - Perform path integral over η_H (*"integrating out"* the heavy states)
  - Expand in powers of m_H^{-1}

➡ Γ_{EFT} containing all higher-dimensional operators and coefficients

Gaillard [*Nucl. Phys. B* 268 (1986) 669-692];

Cheyette [*Nucl. Phys. B* 297 (1988) 183-204];

Dittmaier, Grosse-Knetter [hep-ph/9501285] [hep-ph/9505266];

Henning, Lu, Murayama [1412.1837];

Drozd, Ellis, Quevillon, You [1512.03003];

del Aguila, Kunszt, Santiago [1602.00126];

Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142];

Henning, Lu, Murayama [1604.01019];

Zhang [1610.00710];

Cohen, Lu, Zhang [2011.02484] [2012.07851];

Fuentes-Martín, König, Pagès, Thomsen, FW [2012.08506] [2212.04510];

& many more

- **Expanding the action in η:**

$$S_{\mathrm{UV}}(\eta) \to S_{\mathrm{UV}}(\hat{\eta}+\eta) = \; S_{\mathrm{UV}}(\hat{\eta}) \; + \; \frac{1}{2}\bar{\eta}_i \; \frac{\delta^2 S_{\mathrm{UV}}}{\delta\bar{\eta}_i\,\delta\eta_j}\bigg|_{\eta=\hat{\eta}} \; \eta_j + \mathcal{O}(\eta^3)$$

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- **Tree-level matching:** $\mathscr{L}^{(0)}_{\mathrm{EFT}} = \mathscr{L}_{\mathrm{UV}}\left(\hat{\eta}_L, \hat{\eta}_H(\hat{\eta}_L)\right)$
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  - Gaussian path integral:

$$\Gamma^{(1)}_{\text{UV}} = -\,i\log\left(\text{SDet}\,\mathcal{Q}[\hat{\eta}]\right)^{1/2} = \frac{i}{2}\text{STr}\left(\log \mathcal{Q}[\hat{\eta}]\right)$$

  - Expressed through a superdeterminant (SDet) or supertrace (STr)

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higher loop orders
→ see talk by Ajdin Palavric

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Supertraces

Cohen, Lu, Zhang [2011.02484]; Fuentes-Martín, König, Pagès, Thomsen, FW [2012.08506]

- **Supertraces:** $\Gamma^{(1)}_{\mathrm{UV}} = \frac{i}{2}\mathrm{STr}\left(\ln \mathcal{Q}\right) = \pm\frac{i}{2}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\langle k\,|\,\mathrm{tr}\left(\ln \mathcal{Q}\right)|\,k\rangle$

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interaction terms (X_{ij})

propagators (Δ_i^{-1})

$$\Delta_i^{-1} = \begin{cases} -(D^2 + M_i^2) \\ i\gamma^\mu D_\mu - M_i \\ g^{\mu\nu}(D^2 + M_i^2) \end{cases}$$

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  - **log-type STr:** depends on Δ → model independent
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- **One-loop EFT Lagrangian:**

$$\int \mathrm{d}^d x\, \mathcal{L}^{(1)}_{\rm EFT} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} = \frac{i}{2}\mathrm{STr}\left(\ln \Delta^{-1}\right)\Big|_{\rm hard} - \frac{i}{2}\sum_{n=1}^{\infty}\mathrm{STr}\left[(\Delta X)^n\right]\Big|_{\rm hard}$$

Obtained with: Beneke, Smirnov [hep-ph/9711391]; Jantzen [1111.2589]

Method of regions

Evaluated with: Chan [PRL 57, 1199]; Cheyette [Nucl. Phys. B 297, 183]; Gaillard [Nucl. Phys. B 268, 669];

Covariant derivative expansion

The Method of Regions

- Expand loop integrands in soft ($k \sim m_L$) and hard ($k \sim m_H$) region before integration
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- Long-distance contributions by one-loop matrix elements with tree-level EFT operators (captured by $C^{(0)}$)

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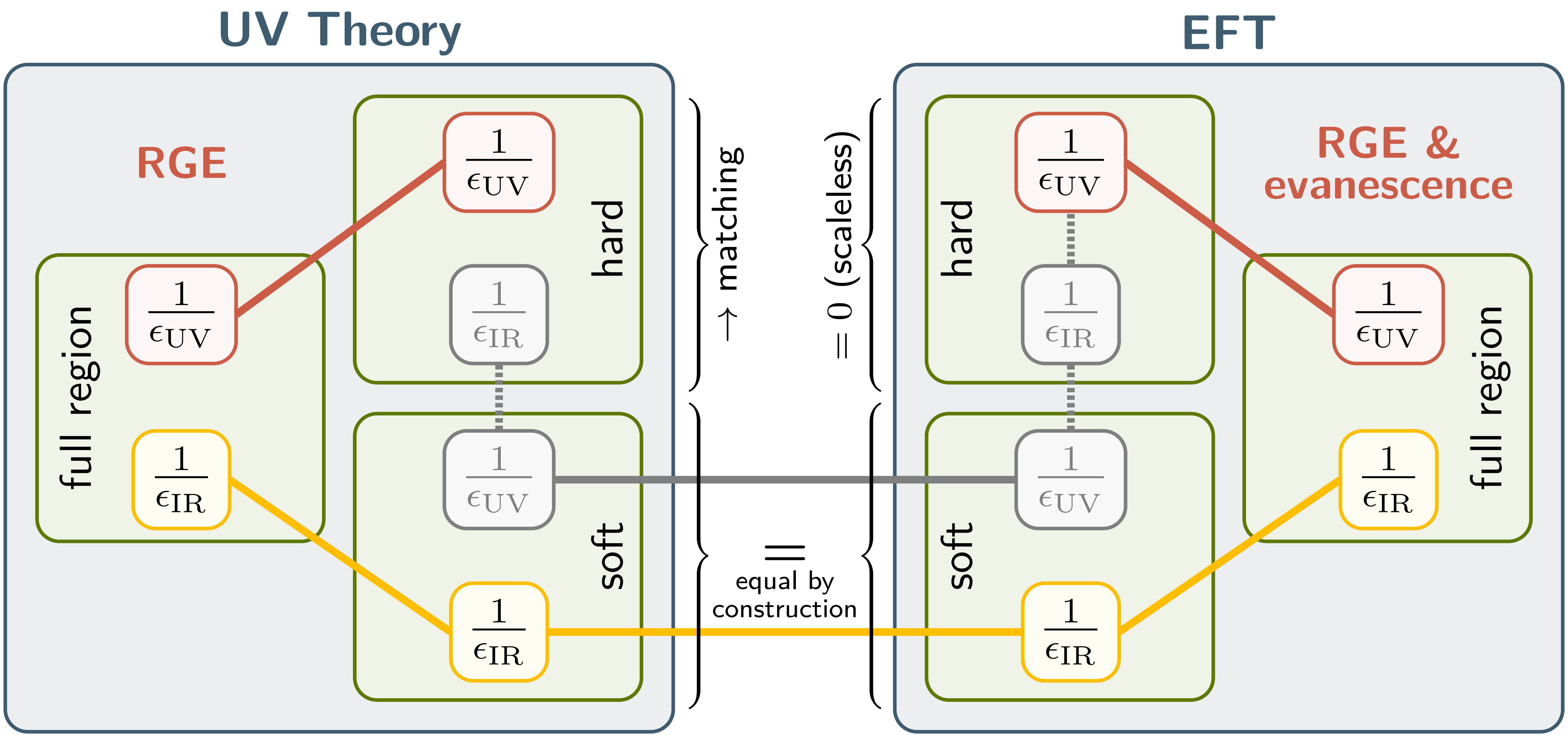
The Method of Regions: Divergences



- The artificial IR poles of the hard region of the UV theory integrals provide the counterterms for the full EFT Lagrangian.

➡ The EFT is automatically renormalized.

Covariant Derivative Expansion

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See also: Henning, Lu, Murayama [1412.1837] [1604.01019];

- Operators $Q(iD_\mu, U_m)$ can depend on:
  Covariant derivatives D_μ and momentum-independent functions U_m
- Supertraces not manifestly covariant (open covariant derivatives $D_\mu \mathbb{1}$)

$$\mathrm{STr}\left(Q(iD_\mu, U_m)\right) = \pm \int \frac{\mathrm{d}^d k}{(2\pi)^d} \langle k \,|\, \mathrm{tr}\left(Q(iD_\mu, U_m)\right) |\, k\rangle = \pm \int \mathrm{d}^d x \int \frac{\mathrm{d}^d k}{(2\pi)^d}\, \mathrm{tr}\left(Q(iD_\mu + k_\mu, U_m)\right) \mathbb{1}$$

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- **Covariant derivative expansion (CDE)**

  Path integral transformation sandwiching the trace between $e^{-iD\cdot\partial_k}$ and $e^{iD\cdot\partial_k}$

  - $e^{\pm iD\cdot\partial_k}$ vanishes when acting to the left/right
  - Pass $e^{-iD\cdot\partial_k}$ through Q to cancel against $e^{iD\cdot\partial_k}$ (using *Baker-Campbell-Hausdorff* formula)
    $\Rightarrow$ Organizes all covariant derivatives D_μ into commutators

➡ Functional matching approach and supertraces are manifestly covariant

Redundant Operators

- Supertrace output $\int \mathrm{d}^d x\, \mathscr{L}_{\text{EFT}}^{(1)} = \Gamma_{\text{UV}}^{(1)}\Big|_{\text{hard}}$ directly **provides EFT operators** (no a priori knowledge required), but $\mathscr{L}_{\text{EFT}}$ is **not** in a **minimal basis**

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  - Field redefinitions (equations of motion)
  - Reduction of Dirac algebra
  - Fierz identities
  - ...

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→ **evanescent operators !!!**

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Evanescent Operators

Buras, Weisz [*Nucl.Phys.B* 333 (1990) 66-99]; Herrlich, Nierste [hep-ph/9412375]

$$\mathscr{L} \supset C_{lqde}^{prst}(\bar{\ell}^p \gamma^\mu q^t)(\bar{d}^s \gamma_\mu e^r) \xrightarrow[d=4]{\textbf{Fierz identity}} \mathscr{L}' \supset -2C_{lqde}^{prst}(\bar{\ell}^p e^r)(\bar{d}^s q^t)$$

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- One-loop: $\mathscr{L}$ & $\mathscr{L}'$ do not lead to same physics (in dimensional regularization $d = 4 - 2\epsilon$)

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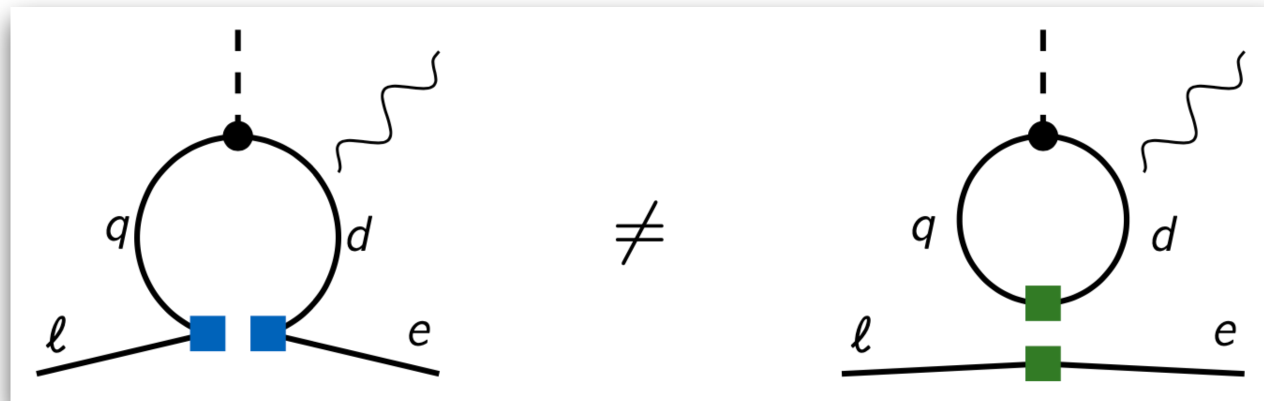

figure by A. Thomsen

The one-loop effective action built from $\mathscr{L}$ and $\mathscr{L}'$ do not agree:

$$\Gamma_{\text{EFT}}^{(1)} \neq \Gamma_{\text{EFT}}^{\prime(1)}$$

- In d dimensions we have: $C_{lqde}^{prst}(\bar{\ell}^p\gamma^\mu q^t)(\bar{d}^s\gamma_\mu e^r) = -2C_{lqde}^{prst}(\bar{\ell}^p e^r)(\bar{d}^s q^t) + C_{lqde}^{prst} E_{lqde}^{prst}$

evanescent operator $\mathcal{O}(\epsilon)$

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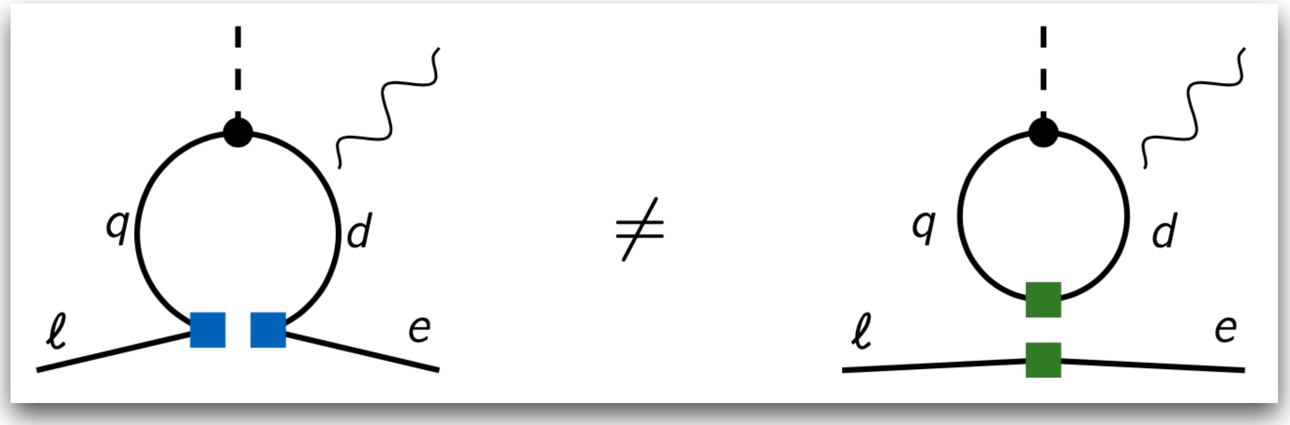

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evanescent operator $\mathcal{O}(\epsilon)$

- Effective one-loop action: $\Gamma^{(1)}_{\text{EFT}} = \Gamma'^{(1)}_{\text{EFT}} + \Delta S_E$
- Absorb physical effect of evanescent operators by finite one-loop shift of action ΔS_E (depends on all UV poles ϵ_{UV} of SMEFT one-loop integrals)
- Computed for the SMEFT in Fuentes-Martin, König, Pages, Thomsen, FW [2211.09144]

→ see talk by Marko Pesut

- For LEFT: Aebischer, Buras, Kumar [2202.01225]; Aebischer, Pesut [2208.10513]; Aebischer, Pesut, Polonsky [2211.01379]

Fuentes-Martín, König, Pagès, Thomsen, FW [2212.04510]

https://gitlab.com/matchete/matchete

- **User input:**
  weakly coupled UV theory
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  (symmetries, fields, couplings)
- **Automation:**
  - **Matching**
    Computation of EOM & $\mathcal{Q}_{ij}$,
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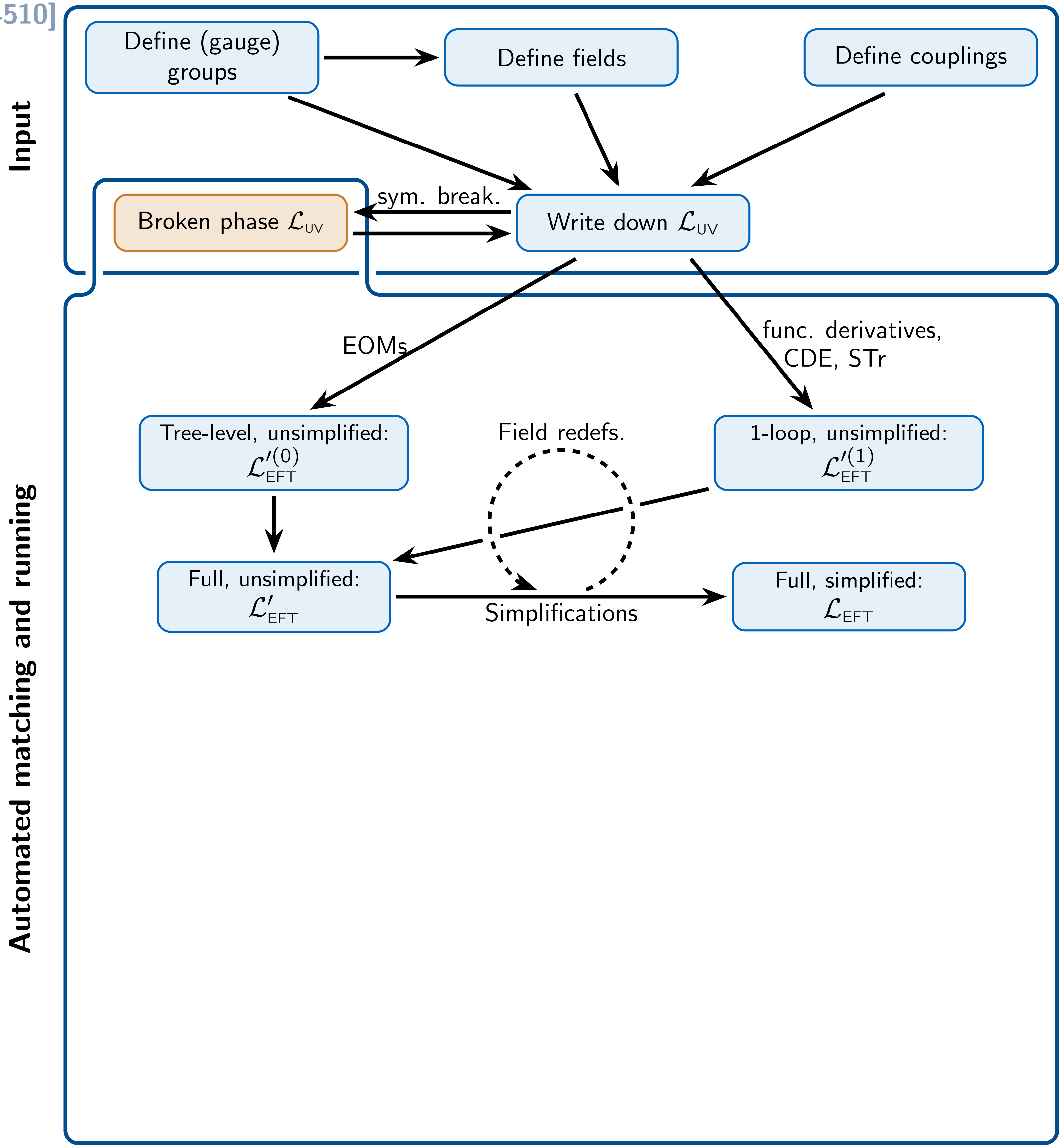
  - **Simplifications**
    Reduction of redundant operators:
    IbP, field redefinitions, (Fierz), ...

➡ (Nearly) **minimal basis**
e.g. Warsaw basis



MATCHETE

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Fuentes-Martín, König, Pagès, Thomsen, FW [2212.04510]

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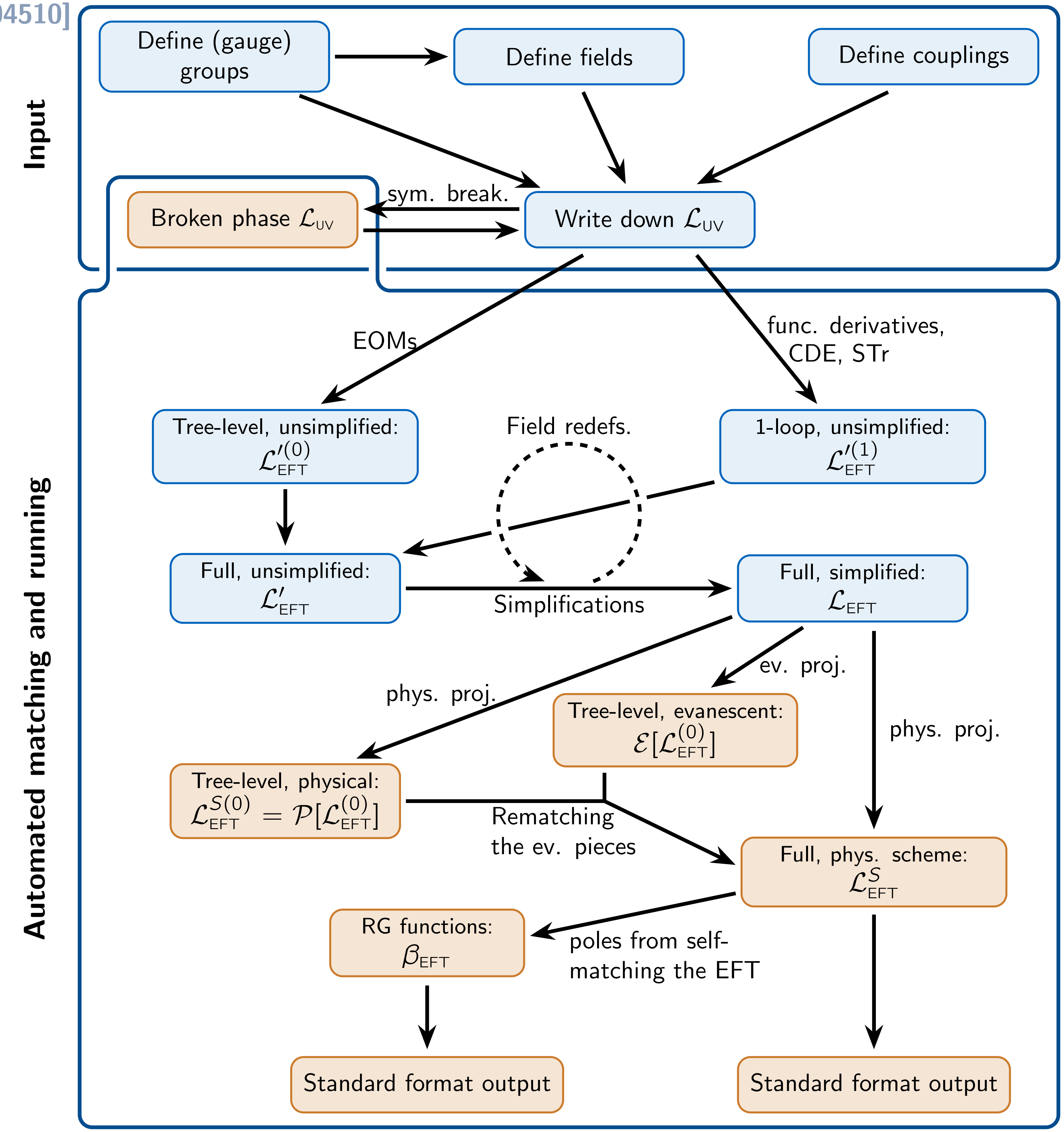
- **Future features:**
  - Fierz identities and evanescent operators
    Fuentes-Martín, König, Pagès, Thomsen, FW [2211.09144]
  - β-functions computation
  - Integrating out heavy vectors



Conclusions

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- Functional methods well-suited for automation in MATCHETE
  - Currently supported UV states: Scalars, Fermions
  - For heavy vectors only tree-level matching is available
- Reduction of $\mathscr{L}_{\text{EFT}}$ to a nearly minimal and *Warsaw like* basis
  - Fierz identities not yet automatically implemented due to evanescent operators
- Functional methods can be extended to computations of β-functions and evanescent operator contributions

MATCHETE

➡ Combination with other tools desirable to constrain landscape of BSM scenarios

Thank you for your attention!

Backup

Functional Matching: Technical Details

The Method of Regions: Example Integral

Beneke, Smirnov [hep-ph/9711391]
Jantzen [1111.2589]

How to evaluate loop integrals in supertraces ?

- Method of regions in dimensional regularization:
  - The loop-integrals contain light m_L and heavy m_H masses ($m_H \gg m_L$)
  - Separate and expand in momentum regions:
    soft-region: $p \sim m_L \quad \leftrightarrow \quad$ *hard-region:* $p \sim m_H$
  - Integrate each region over the full d-dimensional space
  - Summing both integrals gives the full integral without expansion

$$I = \int \mathrm{d}^d p \, \frac{N}{(p^2 - m_L^2)(p^2 - m_H^2)} = I_{\text{soft}} + I_{\text{hard}}$$

$$I_{\text{soft}} = \int \mathrm{d}^d p \, \frac{N}{(p^2 - m_L^2)(-m_H^2)} \left[1 + \frac{p^2}{m_H^2} + \frac{p^4}{m_H^4} + \cdots \right], \qquad I_{\text{hard}} = \int \mathrm{d}^d p \, \frac{N}{p^2(p^2 - m_H^2)} \left[1 + \frac{m_L^2}{p^2} + \frac{m_L^4}{p^4} + \cdots \right]$$

- All the short distance effects we are interested in are encoded in hard region

The Method of Regions: Scalar Toy Model

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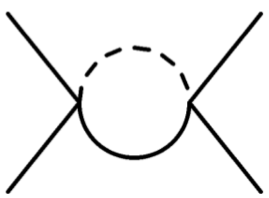
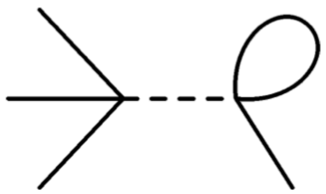
$$\mathcal{L}_{\mathrm{UV}}(\varphi,\Phi) = \frac{1}{2}\left(\partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2\right) + \frac{1}{2}\left(\partial_\mu\Phi\partial^\mu\Phi - M^2\Phi^2\right) - \frac{\kappa}{4!}\varphi^4 - \frac{\lambda}{3!}\varphi^3\Phi$$

UV theory (soft and hard contributions)



$$= \frac{i}{16\pi^2}\lambda^2\left[3 + 3\frac{m^2}{M^2} + \frac{s+t+u}{2M^2}\right]\Bigg|_{\mathrm{hard}}$$

$$+ \frac{i}{16\pi^2}\lambda^2\left[-3\frac{m^2}{M^2} + 3\frac{m^2}{M^2}\ln\left(\frac{m^2}{M^2}\right)\right]\Bigg|_{\mathrm{soft}} + \mathcal{O}(M^{-4}),$$



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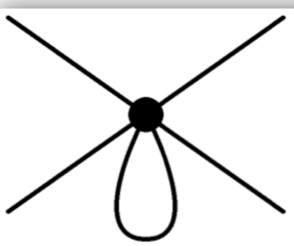
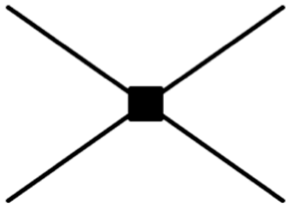
$\Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{hard}}$

$\Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{soft}}$

EFT (only soft contributions)



$$= \frac{i}{16\pi^2}\lambda^2\left[-5\frac{m^2}{M^2} + 5\frac{m^2}{M^2}\log\left(\frac{m^2}{M^2}\right)\right] + \mathcal{O}(M^{-4}),$$



$$= iC_{\varphi^4} - i\frac{C_{\varphi^4\partial^2}}{M^2}(s+t+u)$$

$\mathcal{L}^{(0)}_{\mathrm{EFT}}$

$\mathcal{L}^{(1)}_{\mathrm{EFT}}$

Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

Universität Zürich[UZH]

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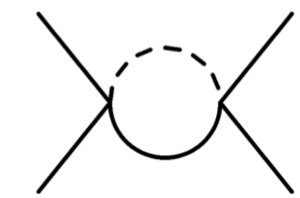

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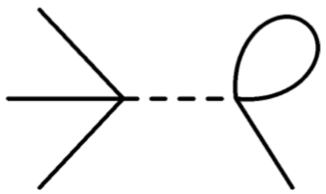

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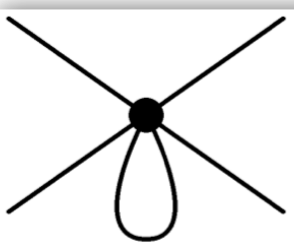
$\Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{hard}}$

$\Gamma^{(1)}_{\mathrm{UV}}\Big|_{\mathrm{soft}}$

cancel exactly

EFT (only soft contributions)



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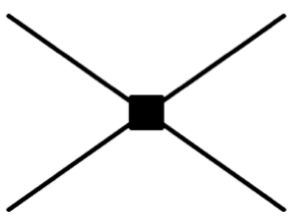

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Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

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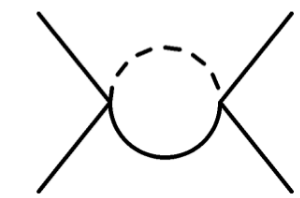

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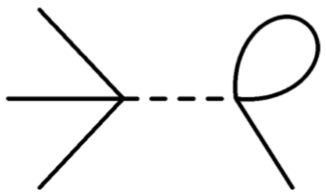

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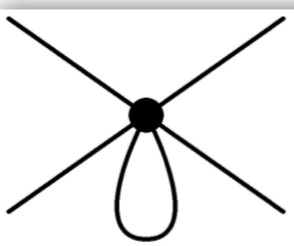

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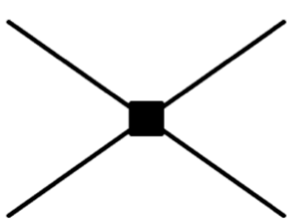

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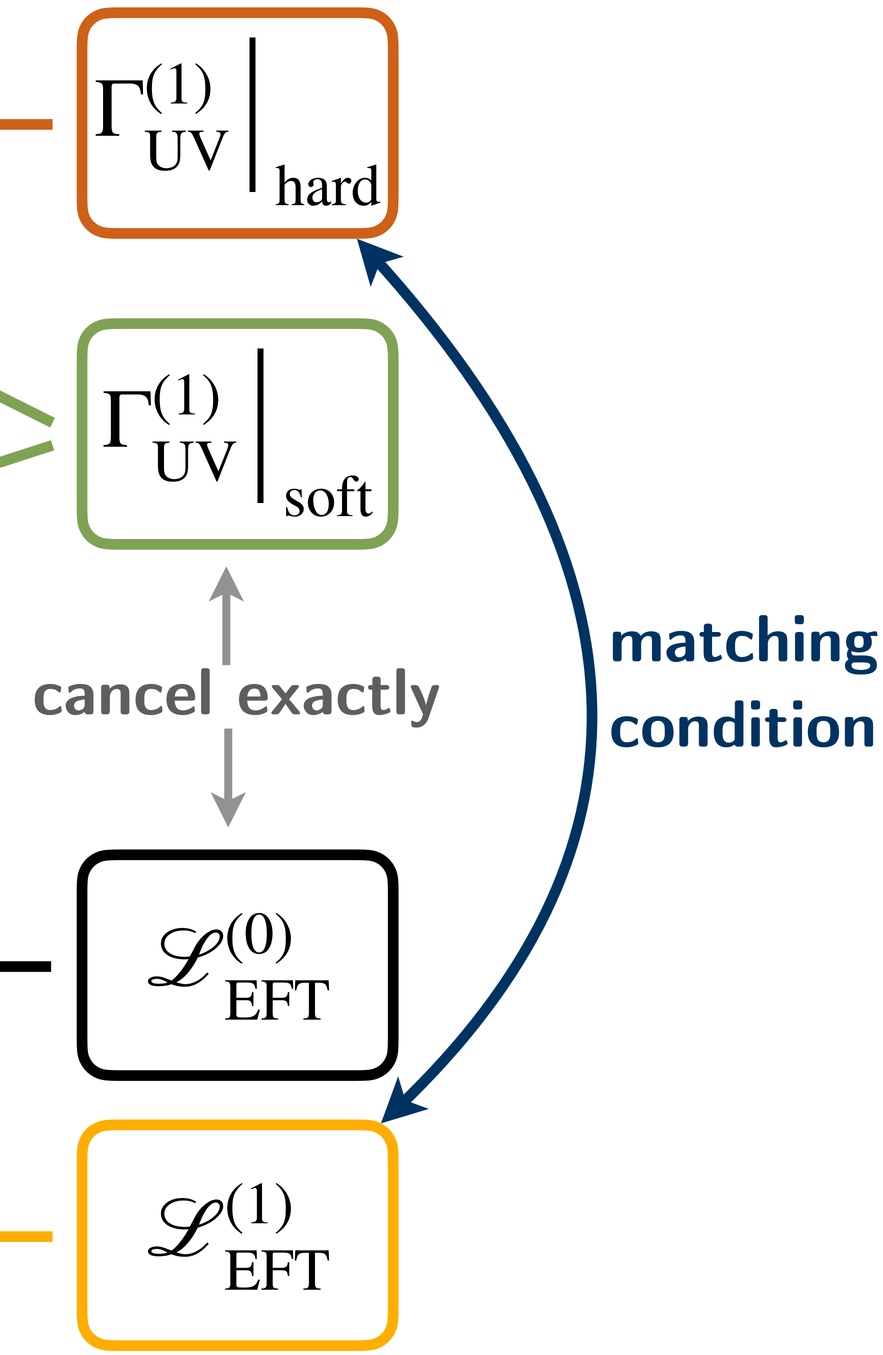


Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

Covariant Derivative Expansion of the supertrace:

$$\mathrm{STr}\Big(Q(iD_\mu, U_m)\Big) = \pm \int \mathrm{d}^d x \int \frac{\mathrm{d}^d k}{(2\pi)^d}\, e^{-iD\cdot\partial_k}\, \mathrm{tr}\Big(Q(iD_\mu + k_\mu, U_m)\Big)\, e^{iD\cdot\partial_k}$$

- Transformation properties:
  - $e^{-iD\cdot\partial_k}(k_\mu + iD_\mu)\, e^{-iD\cdot\partial_k} = k_\mu + i\tilde{G}_{\mu\nu}\partial_k^\nu$
  - $\tilde{G}_{\mu\nu} \equiv \sum_{n=0}^{\infty} \frac{(-i)^n}{(n+2)n!}\,(D_{\{\alpha_1,\ldots,\alpha_n\}}G_{\mu\nu})\,\partial_k^{\alpha_1}\cdots\partial_k^{\alpha_n}$
  - $\tilde{U}_m \equiv e^{-iD\cdot\partial_k}\, U_m\, e^{-iD\cdot\partial_k} = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!}\,(D_{\{\alpha_1,\ldots,\alpha_n\}}U_m)\,\partial_k^{\alpha_1}\cdots\partial_k^{\alpha_n}$
- **The gauge covariant supertrace:**

$$\mathrm{STr}\Big(Q(iD_\mu, U_m)\Big) = \pm \int \mathrm{d}^d x \int \frac{\mathrm{d}^d k}{(2\pi)^d}\, \mathrm{tr}\Big(Q(k_\mu + i\tilde{G}_{\mu\nu}\partial_k^\nu, \tilde{U}_m(x))\Big)$$

Example: Scalar Toy Model

Universität Zürich[UZH]

Fuentes-Martin, Portoles, Ruiz-Femenia [1607.02142]

Two real scalars with mass hierarchy $M \gg m$

$$\mathscr{L}_{\mathrm{UV}}(\varphi, \Phi) = \frac{1}{2}\left(\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2\right) + \frac{1}{2}\left(\partial_\mu \Phi \partial^\mu \Phi - M^2 \Phi^2\right) - \frac{\kappa}{4!}\varphi^4 - \frac{\lambda}{3!}\varphi^3 \Phi$$

- Integrate out Φ applying the functional method up to $d = 6$
- **Tree-level matching:**
  - Equation of motion: $M^2 \hat{\Phi} = -D^2 \hat{\Phi} - \frac{\lambda}{3!}\hat{\varphi}^3$
  - Solution: $\hat{\Phi} = -\frac{\lambda}{6M^2}\hat{\varphi}^3 + \mathcal{O}(M^{-4})$ (**substitute** into Φ in $\mathscr{L}_{\mathrm{UV}}$)
  - Tree-level EFT Lagrangian:

$$\mathscr{L}^{(0)}_{\mathrm{EFT}} = \frac{1}{2}\left(\partial_\mu \hat{\varphi} \partial^\mu \hat{\varphi} - m^2 \hat{\varphi}^2\right) - \frac{\kappa}{4!}\hat{\varphi}^4 + \frac{10\lambda^2}{6!M^2}\hat{\varphi}^6$$

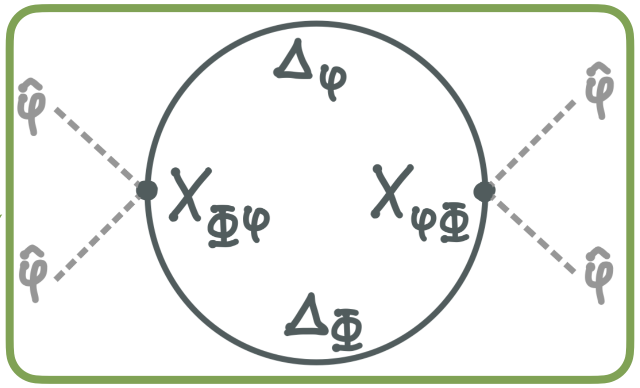
Super Traces of the Scalar Toy Model

- The fluctuation operator $\mathscr{O}_{ij}$

$$\Delta_\Phi^{-1} = -\,\partial^2 - M^2, \qquad X_{\Phi\Phi} = 0, \qquad X^{[2]}_{\varphi\Phi} = (X^{[2]}_{\varphi\Phi})^\dagger = -\frac{\lambda}{2}\hat{\varphi}^2,$$

$$\Delta_\varphi^{-1} = -\,\partial^2, \qquad X^{[2]}_{\varphi\varphi} = -\,m^2 - \frac{\kappa}{2}\hat{\varphi}^2 - \lambda\hat{\varphi}\hat{\Phi} = -\,m^2 - \frac{\kappa}{2}\hat{\varphi}^2 - \frac{\lambda^2}{6M^2}\hat{\varphi}^4$$

- Supertraces to compute with the CDE:
  - Log-type: $\mathrm{STr}\left(\ln \Delta_\Phi^{-1}\right)\Big|_{\mathrm{hard}}$
  - Power-type: $\mathrm{STr}\left(\Delta_\Phi X^{[2]}_{\Phi\varphi}\Delta_\varphi X^{[2]}_{\varphi\Phi}\right)\Big|_{\mathrm{hard}}$, $\mathrm{STr}\left(\Delta_\Phi X^{[2]}_{\Phi\varphi}\Delta_\varphi X^{[2]}_{\varphi\varphi}\Delta_\varphi X^{[2]}_{\varphi\Phi}\right)\Big|_{\mathrm{hard}}$



diagrammatic representation of supertraces

- One-loop EFT Lagrangian from supertrace evaluation:

$$\mathscr{L}^{(1)}_{\mathrm{EFT}} = \frac{1}{16\pi^2}\frac{\lambda^2}{16}\left[2\left(1 + \frac{m^2}{M^2}\right)\hat{\varphi}^4 - \frac{1}{M^2}\hat{\varphi}^2\partial^2\hat{\varphi}^2 + \frac{\kappa}{M^2}\hat{\varphi}^6\right]$$

Field Redefinitions vs. Equations of Motion

Criado, Perez-Victoria [1811.09413]

- LSZ formula: S-matrix invariant under field redefinitions

- Perturbative field redefinition: $$\eta \to \tilde{\eta} = \eta + \frac{1}{\Lambda}\delta\eta$$

- Shifting the EFT Lagrangian:

$$\mathscr{L}[\eta] \to \mathscr{L}[\tilde{\eta}] = \mathscr{L}[\eta] + \frac{1}{\Lambda}\left.\frac{\delta\mathscr{L}[\tilde{\eta}]}{\delta\tilde{\eta}}\right|_{\tilde{\eta}=\eta}\delta\eta$$

EOM

Toy model:

$$\partial^2\hat{\varphi} = -m^2\hat{\varphi} - \left(\frac{\kappa}{3!} - \frac{\lambda^2}{32\pi^2}\right)\hat{\varphi}^3$$

$$\hat{\varphi}^3\partial^2\hat{\varphi} = -m^2\hat{\varphi}^4 - \left(\frac{\kappa}{3!} - \frac{\lambda^2}{32\pi^2}\right)\hat{\varphi}^6$$

- At leading power: field redefinitions are equivalent to EOM for relating redundant operators

Field Redefinitions vs. Equations of Motion

Universität Zürich[UZH]

Criado, Perez-Victoria [1811.09413]

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- Shifting the EFT Lagrangian:

$$\mathscr{L}[\eta] \to \mathscr{L}[\tilde{\eta}] = \mathscr{L}[\eta] + \frac{1}{\Lambda}\left.\frac{\delta\mathscr{L}[\tilde{\eta}]}{\delta\tilde{\eta}}\right|_{\tilde{\eta}=\eta}\delta\eta + \frac{1}{2\Lambda^2}\left.\frac{\delta^2\mathscr{L}[\tilde{\eta}]}{\delta\tilde{\eta}^2}\right|_{\tilde{\eta}=\eta}\delta\eta^2 + \mathcal{O}\left(\Lambda^{-3}\right)$$

EOM

- At leading power: field redefinitions are equivalent to EOM for relating redundant operators
- At sub-leading power: EOMs do not capture the full effect of the field redefinitions.
- **At sub-leading power field redefinitions have to be used!**

MATCHETE Example 1: Vectorlike Fermions

Example: Vectorlike Fermions

In[1]:=
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<< Matchete`
```

MATCHETE v0.1.5

by Javier Fuentes-Martín, Matthias König,
Julie Pagès, Anders Eller Thomsen, and Felix Wilsch
Reference: arXiv:2212.04510
Website: https://gitlab.com/matchete/matchete

Define gauge group

In[2]:=
```
DefineGaugeGroup[U1e, U1, e, A]
```

Define fields

In[3]:=
```
(* heavy vectorlike fermion with charge 1 *)
DefineField[Ψ, Fermion, Charges → {U1e[1]}, Mass → {Heavy, M}]
```

In[4]:=
```
(* light vectorlike fermion with charge 1 *)
DefineField[ψ, Fermion, Charges → {U1e[1]}, Mass → {Light, m}]
```

In[5]:=
```
(* real light scalar *)
DefineField[ϕ, Scalar, Mass → Light, SelfConjugate → True]
```

Define coupling

In[6]:=
```
(* Yukawa coupling *)
DefineCoupling[y, EFTOrder -> 0]
```

Defining models:

Example: Vectorlike Fermions

Universität Zürich[UZH]

In[1]:=
```
<< Matchete`
```

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Defining models:

1) Define gauge groups:
$U(1)_e$ symmetry with gauge field A_μ and gauge coupling e

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2) Define field content
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ψ light fermion with charge 1
ϕ light real scalar

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1) Define gauge groups:
$U(1)_e$ symmetry with gauge field A_μ and gauge coupling e

2) Define field content
Ψ heavy fermion with charge 1
ψ light fermion with charge 1
ϕ light real scalar

3) Define couplings
y Yukawa coupling order $\mathcal{O}(m_L^0)$

Vectorlike Fermions: Lagrangian

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Write Lagrangian

Free Lagrangian

In[7]:=
```
ℒfree = FreeLag[];
ℒfree // NiceForm
```

Out[8]//NiceForm=

$$-\frac{1}{4}A^{\mu\nu 2}+\frac{1}{2}\left(D_\mu\phi\right)^2-\frac{1}{2}m\phi^2\,\phi^2+i\left(\overline{\psi}\cdot\gamma_\mu\cdot D_\mu\psi\right)-m\left(\overline{\psi}\cdot\psi\right)+i\left(\overline{\Psi}\cdot\gamma_\mu\cdot D_\mu\Psi\right)-M\left(\overline{\Psi}\cdot\Psi\right)$$

Write interactions

In[9]:=
```
ℒint = -y[] × Bar[ψ[]] ** PR ** Ψ[] × ϕ[] // PlusHc;
ℒint // NiceForm
```

Out[10]//NiceForm=

$$-y\,\phi\left(\overline{\psi}\cdot P_R\cdot\Psi\right)-\overline{y}\,\phi\left(\overline{\Psi}\cdot P_L\cdot\psi\right)$$

Full UV Lagrangian

In[11]:=
```
ℒUV = ℒfree + ℒint;
ℒUV // NiceForm
```

Out[12]//NiceForm=

$$-\frac{1}{4}A^{\mu\nu 2}+\frac{1}{2}\left(D_\mu\phi\right)^2-\frac{1}{2}m\phi^2\,\phi^2+i\left(\overline{\psi}\cdot\gamma_\mu\cdot D_\mu\psi\right)-m\left(\overline{\psi}\cdot\psi\right)+i\left(\overline{\Psi}\cdot\gamma_\mu\cdot D_\mu\Psi\right)-M\left(\overline{\Psi}\cdot\Psi\right)-y\,\phi\left(\overline{\psi}\cdot P_R\cdot\Psi\right)-\overline{y}\,\phi\left(\overline{\Psi}\cdot P_L\cdot\psi\right)$$

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Tree-level

Matching

In[13]:=
```
ℒEFT0 = Match[ℒUV, LoopOrder → 0, EFTOrder → 6];
ℒEFT0 // NiceForm
```

Out[14]//NiceForm=

$$-\frac{1}{4} A^{\mu\nu 2} + \frac{1}{2} (D_\mu\phi)^2 - \frac{1}{2} m\phi^2 \phi^2 + i (\bar{\psi} \cdot \gamma_\mu \cdot D_\mu\psi) - m (\bar{\psi} \cdot \psi) + i \bar{y} y \frac{1}{M^2} \phi D_\mu\phi (\bar{\psi} \cdot \gamma_\mu P_L \cdot \psi) + i \bar{y} y \frac{1}{M^2} \phi^2 (\bar{\psi} \cdot \gamma_\mu P_L \cdot D_\mu\psi)$$

Removing redundant operators off-shell

In[15]:=
```
ℒEFT0OffShell = ℒEFT0 // GreensSimplify // HcSimplify;
ℒEFT0OffShell // NiceForm
```

Out[16]//NiceForm=

$$-\frac{1}{4} A^{\mu\nu 2} + \frac{1}{2} (D_\mu\phi)^2 - \frac{1}{2} m\phi^2 \phi^2 + i (\bar{\psi} \cdot \gamma_\mu \cdot D_\mu\psi) - m (\bar{\psi} \cdot \psi) + \left(-\frac{i}{2} \bar{y} y \frac{1}{M^2} \phi^2 (D_\mu\bar{\psi} \cdot \gamma_\mu P_L \cdot \psi) + \text{H.c.}\right)$$

Removing redundant operators on-shell

In[17]:=
```
ℒEFT0OnShell = ℒEFT0 // EOMSimplify;
ℒEFT0OnShell // NiceForm
```

Out[18]//NiceForm=

$$-\frac{1}{4} A^{\mu\nu 2} + \frac{1}{2} (D_\mu\phi)^2 - \frac{1}{2} m\phi^2 \phi^2 + i (\bar{\psi} \cdot \gamma_\mu \cdot D_\mu\psi) - m (\bar{\psi} \cdot \psi) + \frac{1}{2} m \bar{y} y \frac{1}{M^2} \left(\phi^2 (\bar{\psi} \cdot P_L \cdot \psi) + \phi^2 (\bar{\psi} \cdot P_R \cdot \psi)\right)$$

One-loop matching

In[19]:=
```
ℒEFT = Match[ℒUV, LoopOrder → 1, EFTOrder → 6] /. ϵ^-1 -> 0;
ℒEFT // NiceForm
```

Out[20]//NiceForm=

$$-\frac{1}{4} A^{\mu\nu 2} - \frac{1}{3}\hbar e^2 A^{\mu\nu 2}\,\text{Log}\left[\frac{\bar\mu^2}{M^2}\right] + \frac{1}{2}(D_\mu\phi)^2 - \frac{1}{2} m\phi^2 \phi^2 - 2\hbar\bar{y} y m^2\phi^2 - 2\hbar \bar{y} y m^4 \frac{1}{M^2}\phi^2 - 2\hbar\bar{y} y M^2\phi^2 - 2\hbar\bar{y} y m^2\phi^2\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] -$$

$$2\hbar\bar{y} y m^4\frac{1}{M^2}\phi^2\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] - 2\hbar \bar{y} y M^2\phi^2\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] - \frac{1}{2} \hbar\bar{y} y \phi D^2\phi + 2\hbar\bar{y} y m^2 \frac{1}{M^2}\phi D^2\phi - \hbar\bar{y} y\phi D^2\phi\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] + i(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi) - m(\bar\psi\cdot\psi) +$$

$$\frac{3i}{8}\hbar\bar{y} y(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi) + \frac{3i}{4}\hbar\bar{y} y m\phi^2\frac{1}{M^2}(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi) + \frac{i}{4}\hbar\bar{y} y(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi)\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] + \frac{i}{2}\hbar\bar{y} y m\phi^2\frac{1}{M^2}(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi)\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] -$$

$$\frac{3i}{8}\hbar\bar{y} y(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi) - \frac{3i}{4}\hbar \bar{y} y m\phi^2 \frac{1}{M^2}(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi) - \frac{i}{4} \hbar\bar{y} y (D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi)\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] - \frac{i}{2} \hbar\bar{y} y m\phi^2\frac{1}{M^2}(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi)\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] +$$

$$\frac{1}{9}\hbar\bar{y} y\frac{1}{M^2}\phi D^2D^2\phi + \frac{1}{9} \hbar\bar{y} y \frac{1}{M^2}\phi D_\mu D_\nu D_\mu D_\nu\phi + \frac{1}{9}\hbar\bar{y} y\frac{1}{M^2}\phi D_\mu D^2 D_\mu\phi + \frac{7}{270}\hbar e^2\frac{1}{M^2}(D_\rho A^{\mu\nu})^2 + \frac{1}{20}\hbar e^2\frac{1}{M^2}A^{\mu\nu}D^2A^{\mu\nu} +$$

$$\frac{13}{240}\hbar e^2\frac{1}{M^2}D_\nu D_\rho A^{\mu\nu}A^{\mu\rho} + \frac{13}{240}\hbar e^2\frac{1}{M^2}D_\rho D_\nu A^{\mu\nu}A^{\mu\rho} + \frac{1}{90}\hbar e^2\frac{1}{M^2}D_\rho A^{\mu\nu}D_\nu A^{\mu\rho} + \frac{7}{270}\hbar e^2\frac{1}{M^2}D_\nu A^{\mu\nu}D_\rho A^{\mu\rho} - \frac{1}{48}\hbar e^2\frac{1}{M^2}A^{\mu\nu}D_\nu D_\rho A^{\mu\rho} -$$

$$\frac{1}{48}\hbar e^2 \frac{1}{M^2}A^{\mu\nu}D_\rho D_\nu A^{\mu\rho} + \frac{1}{120}\hbar e^2\frac{1}{M^2}D_\mu D_\rho A^{\mu\nu}A^{\nu\rho} + \frac{1}{120}\hbar e^2\frac{1}{M^2}D_\rho D_\mu A^{\mu\nu}A^{\nu\rho} - \frac{2}{135}\hbar e^2\frac{1}{M^2}D_\rho A^{\mu\nu}D_\mu A^{\nu\rho} + \frac{1}{27}\hbar e^2\frac{1}{M^2}D_\mu A^{\mu\nu}D_\rho A^{\nu\rho} -$$

$$\frac{1}{40}\hbar e^2\frac{1}{M^2}A^{\mu\nu}D_\mu D_\rho A^{\nu\rho} - \frac{1}{40}\hbar e^2\frac{1}{M^2}A^{\mu\nu}D_\rho D_\mu A^{\nu\rho} - \frac{i}{18}\hbar\bar{y} y\frac{1}{M^2}(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu D^2\psi) - \frac{i}{18}\hbar\bar{y} y\frac{1}{M^2}(\bar\psi\cdot\gamma_\mu P_L\cdot D_\nu D_\mu D_\nu\psi) -$$

$$\frac{i}{18}\hbar\bar{y} y\frac{1}{M^2}(\bar\psi\cdot\gamma_\mu P_L\cdot D^2D_\mu\psi) + \frac{i}{18}\hbar\bar{y} y\frac{1}{M^2}(D^2D_\nu\bar\psi\cdot\gamma_\nu P_L\cdot\psi) + \frac{i}{18}\hbar\bar{y} y\frac{1}{M^2}(D_\mu D_\nu D_\mu\bar\psi\cdot\gamma_\nu P_L\cdot\psi) + \frac{i}{18}\hbar\bar{y} y\frac{1}{M^2}(D_\mu D^2\bar\psi\cdot\gamma_\mu P_L\cdot\psi) -$$

$$\hbar\bar{y}^2y^2\phi^4\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] + 2\hbar m^2\bar{y}^2y^2\frac{1}{M^2}\phi^4 + \frac{1}{3}\hbar\bar{y}^3y^3\frac{1}{M^2}\phi^6 + \frac{13}{12}\hbar\bar{y}^2y^2\frac{1}{M^2}\phi^2(D_\mu\phi)^2 + \frac{13}{12}\hbar\bar{y}^2 y^2\frac{1}{M^2}D^2\phi\,\phi^3 + \frac{1}{3}\hbar\bar{y} y e^2\frac{1}{M^2}\phi^2A^{\mu\nu 2} +$$

$$\frac{5}{12}\hbar e\bar{y} y\frac{1}{M^2}D_\nu A^{\mu\nu}(\bar\psi\cdot\gamma_\mu P_L\cdot\psi) - \frac{1}{12}\hbar e\bar{y} y\frac{1}{M^2}D_\mu A^{\mu\nu}(\bar\psi\cdot\gamma_\nu P_L\cdot\psi) - \frac{1}{8}\hbar e\bar{y} y\frac{1}{M^2}A^{\mu\nu}(\bar\psi\cdot\Gamma_{\mu\nu\rho}P_L\cdot D_\rho\psi) +$$

$$\frac{1}{8}\hbar e\bar{y} y\frac{1}{M^2}A^{\mu\nu}(D_\rho\bar\psi\cdot\Gamma_{\mu\nu\rho}P_L\cdot\psi) + i\bar{y} y\frac{1}{M^2}\phi D_\mu\phi(\bar\psi\cdot\gamma_\mu P_L\cdot\psi) + i\bar{y} y\frac{1}{M^2}\phi^2(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi) - \frac{5i}{4}\hbar\bar{y}^2y^2\frac{1}{M^2}\phi^2(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi) -$$

$$i\hbar\bar{y}^2y^2\frac{1}{M^2}\phi^2(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi)\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right] + \frac{5i}{4}\hbar\bar{y}^2y^2\frac{1}{M^2}\phi^2(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi) + i\hbar\bar{y}^2y^2\frac{1}{M^2}\phi^2(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi)\, \text{Log}\left[\frac{\bar\mu^2}{M^2}\right]$$

Vectorlike Fermions: Operator Simplifications

Removing redundant operators off- and on-shell

In[21]:=
```
ℒEFTOffShell = ℒEFT // GreensSimplify // HcSimplify;
ℒEFTOffShell // NiceForm
```

Out[22]//NiceForm=

$$\left(-\frac{1}{4}-\frac{1}{3}\hbar\, e^2\, \mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) A^{\mu\nu 2} + \left(\frac{1}{2}+\frac{1}{2}\hbar\, \bar y\, y\, \frac{1}{M^2}\left(-4\, m^2 + M^2\left(1+2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\right)(D_\mu\phi)^2 + \left(-\frac{1}{2} m\phi^2 - 2\,\hbar\, \bar y\, y\, \frac{1}{M^2}\left(m^4+m^2 M^2+M^4\right)\left(1+\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\phi^2 +$$

$$i\,(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi) - m\,(\bar\psi\cdot\psi) + \frac{i}{4}\hbar\,\bar y\, y\,\frac{1}{M^2}\left(M^2+2\, m\phi^2\right)\left(3+2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)(\bar\psi\cdot\gamma_\mu P_L\cdot D_\mu\psi) + \frac{1}{3}\hbar\,\bar y\, y\,\frac{1}{M^2} D^2\phi\, D^2\phi - \frac{2}{15}\hbar\, e^2\frac{1}{M^2} D_\nu A^{\mu\nu} D_\rho A^{\mu\rho} +$$

$$\hbar\,\bar y^2 y^2\frac{1}{M^2}\left(2\, m^2 - M^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\phi^4 + \frac{1}{3}\hbar\,\bar y^3 y^3\frac{1}{M^2}\phi^6 + \frac{13}{18}\hbar\,\bar y^2 y^2\frac{1}{M^2} D^2\phi\,\phi^3 + \frac{1}{3}\hbar\,\bar y\, y\, e^2\frac{1}{M^2}\phi^2 A^{\mu\nu 2} + \frac{7}{36}\hbar\, e\,\bar y\, y\,\frac{1}{M^2} D_\nu A^{\mu\nu}(\bar\psi\cdot\gamma_\mu P_L\cdot\psi) +$$

$$\left(-\frac{i}{6}\hbar\,\bar y\, y\,\frac{1}{M^2}(D^2\bar\psi\cdot\gamma_\nu P_L\cdot D_\nu\psi) + \frac{1}{8}\hbar\, e\,\bar y\, y\,\frac{1}{M^2} A^{\mu\nu}(D_\rho\bar\psi\cdot\gamma_\rho\Gamma_{\mu\nu} P_L\cdot\psi) + \left(-\frac{i}{2}\bar y\, y\,\frac{1}{M^2} + \frac{i}{4}\hbar\,\bar y^2 y^2\frac{1}{M^2}\left(5+4\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\phi^2(D_\mu\bar\psi\cdot\gamma_\mu P_L\cdot\psi) + \mathrm{H.c.}\right)$$

In[23]:=
```
ℒEFTOnShell = ℒEFT // EOMSimplify // HcSimplify;
ℒEFTOnShell // NiceForm
```

```
» The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms
  to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.
```

Out[24]//NiceForm=

$$\left(-\frac{1}{4}-\frac{1}{3}\hbar\, e^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) A^{\mu\nu 2} + \frac{1}{2}(D_\mu\phi)^2 + \left(c\phi\phi + \frac{1}{3}\hbar\,\bar y\, y\, c\phi\phi\,\frac{1}{M^2}\left(4\, c\phi\phi + 12\, m^2 - 3\, M^2\left(1+2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\right)\phi^2 +$$

$$i\,(\bar\psi\cdot\gamma_\mu\cdot D_\mu\psi) - m\,(\bar\psi\cdot\psi) + \frac{1}{9}\hbar\,\bar y^2 y^2\frac{1}{M^2}\left(13\, c\phi\phi + 18\, m^2 - 9\, M^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\phi^4 + \frac{1}{3}\hbar\,\bar y^3 y^3\frac{1}{M^2}\phi^6 + \frac{1}{3}\hbar\,\bar y\, y\, e^2\frac{1}{M^2}\phi^2 A^{\mu\nu 2} +$$

$$\left(\frac{1}{24}\hbar\, m\,\bar y\, y\,\frac{1}{M^2}\left(4\, m^2 - 12\, c\phi\phi\left(3+2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right) + 3\, M^2\left(3+2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)(\bar\psi\cdot P_R\cdot\psi) - \frac{i}{24}\hbar\, e\, m\,\bar y\, y\,\frac{1}{M^2} A^{\mu\nu}(\bar\psi\cdot\Gamma_{\nu\mu} P_R\cdot\psi) + \right.$$

$$\left.\left(\frac{1}{2} m\,\bar y\, y\,\frac{1}{M^2} - \frac{1}{16}\hbar\, m\,\bar y^2 y^2\frac{1}{M^2}\left(37+38\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]\right)\right)\phi^2(\bar\psi\cdot P_R\cdot\psi) + \mathrm{H.c.}\right) - \frac{2}{15}\hbar\, e^4\frac{1}{M^2}(\bar\psi\cdot\gamma_\mu\cdot\psi)^2 + \frac{7}{36}\hbar\,\bar y\, y\, e^2\frac{1}{M^2}(\bar\psi\cdot\gamma_\mu\cdot\psi)(\bar\psi\cdot\gamma_\mu P_L\cdot\psi)$$

In[25]:=
```
PrintEffectiveCouplings[ℒEFTOnShell]
```

$$c\phi\phi = -\frac{1}{2} m\phi^2 - 2\,\hbar\,\bar y\, y\, m^2 - 2\,\hbar\,\bar y\, y\, m^4\frac{1}{M^2} - 2\,\hbar\,\bar y\, y\, M^2 - 2\,\hbar\,\bar y\, y\, m^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] - 2\,\hbar\,\bar y\, y\, m^4\frac{1}{M^2}\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right] - 2\,\hbar\,\bar y\, y\, M^2\,\mathrm{Log}\left[\frac{\bar\mu^2}{M^2}\right]$$

MATCHETE Example 2: S_1 Leptoquark

Example: Defining the S_1 Leptoquark

Defining the $S_1 \sim (\bar{3},1)_{1/3}$ leptoquark model:

$$\mathscr{L}_{\rm UV} = \mathscr{L}_{\rm SM} + (D_\mu S_1)^\dagger (D^\mu S_1) - M^2\, S_1^\dagger S_1 - \Big[\, \lambda^L_{pr}\, (\bar{q}^c_p \varepsilon \ell_r)\, S_1 + \lambda^R_{pr}\, (\bar{u}^c_p e_r)\, S_1 + \mathrm{h.c.} \Big]$$

Example: Defining the S_1 Leptoquark

Defining the $S_1 \sim (\bar{3},1)_{1/3}$ leptoquark model:

$$\mathscr{L}_{\rm UV} = \mathscr{L}_{\rm SM} + (D_\mu S_1)^\dagger (D^\mu S_1) - M^2 S_1^\dagger S_1 - \left[\lambda^L_{pr} \, (\bar{q}^c_p \varepsilon \ell_r) \, S_1 + \lambda^R_{pr} \, (\bar{u}^c_p e_r) \, S_1 + \mathrm{h.c.} \right]$$

Define the SM:

```
LSM = LoadModel["SM"];
```

Example: Defining the S_1 Leptoquark

Defining the $S_1 \sim (\bar{3},1)_{1/3}$ leptoquark model:

$$\mathscr{L}_{\mathrm{UV}} = \mathscr{L}_{\mathrm{SM}} + (D_\mu S_1)^\dagger (D^\mu S_1) - M^2 S_1^\dagger S_1 - \Big[\lambda^L_{pr}\, (\bar{q}^c_p \varepsilon \ell_r)\, S_1 + \lambda^R_{pr}\, (\bar{u}^c_p e_r)\, S_1 + \mathrm{h.c.} \Big]$$

Define the SM:

```
LSM = LoadModel["SM"];
```

Define the S_1 field:

```
DefineField[S1, Scalar, SelfConjugate -> False, Mass -> {Heavy, M},
    Indices -> {Bar[SU3c[fund]]}, Charges -> {U1Y[1/3]}]
```

Example: Defining the S_1 Leptoquark

Defining the $S_1 \sim (\bar{3},1)_{1/3}$ leptoquark model:

$$\mathscr{L}_{\rm UV} = \mathscr{L}_{\rm SM} + (D_\mu S_1)^\dagger (D^\mu S_1) - M^2 S_1^\dagger S_1 - \left[\lambda^L_{pr} (\bar{q}^c_p \varepsilon \ell_r) S_1 + \lambda^R_{pr} (\bar{u}^c_p e_r) S_1 + \mathrm{h.c.} \right]$$

Define the SM:

```
LSM = LoadModel["SM"];
```

Define the S_1 field:

```
DefineField[S1, Scalar, SelfConjugate -> False, Mass -> {Heavy, M},
    Indices -> {Bar[SU3c[fund]]}, Charges -> {U1Y[1/3]}]
```

Define the S_1 couplings:

```
DefineCoupling[λL, Indices -> {Flavor, Flavor}]
DefineCoupling[λR, Indices -> {Flavor, Flavor}]
```

Example: Defining the S_1 Leptoquark

Defining the $S_1 \sim (\bar{3},1)_{1/3}$ leptoquark model:

$$\mathscr{L}_{\mathrm{UV}} = \mathscr{L}_{\mathrm{SM}} + (D_\mu S_1)^\dagger (D^\mu S_1) - M^2 S_1^\dagger S_1 - \left[\lambda^L_{pr} (\bar{q}^c_p \varepsilon \ell_r) S_1 + \lambda^R_{pr} (\bar{u}^c_p e_r) S_1 + \mathrm{h.c.} \right]$$

Define the SM:

```
LSM = LoadModel["SM"];
```

Define the S_1 field:

```
DefineField[S1, Scalar, SelfConjugate -> False, Mass -> {Heavy, M},
    Indices -> {Bar[SU3c[fund]]}, Charges -> {U1Y[1/3]}]
```

Define the S_1 couplings:

```
DefineCoupling[λL, Indices -> {Flavor, Flavor}]
DefineCoupling[λR, Indices -> {Flavor, Flavor}]
```

Define the NP interactions:

```
Lint = λL[p, r] × CConj[Bar[q[a, n, p]]] ** l[m, r] × Bar[CG[eps[SU2L], {n, m}]] × S1[a] +
λR[p, r] × CConj[Bar[u[a, p]]] ** e[r] × S1[a];
```

Example: Defining the S_1 Leptoquark

Universität Zürich[UZH]

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λR[p, r] × CConj[Bar[u[a, p]]] ** e[r] × S1[a];
```

Define the NP Lagrangian:

```
LS1 = FreeLag[S1] - PlusHc[Lint] // RelabelIndices;
% // HcSimplify // NiceForm
```

$$D_\mu \overline{S1}^a D_\mu S1_a - M^2 \overline{S1}^a S1_a + \left(-\lambda R^{rp} S1_a \left(e^{pT} \cdot C P_R \cdot u^{ar} \right) - \lambda L^{rp} S1_a \left(l^{jpT} \cdot C P_L \cdot q^{air} \right) \bar{\varepsilon}_{ij} + \mathrm{H.c.} \right)$$

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$$D_\mu \overline{S1}^a D_\mu S1_a - M^2 \overline{S1}^a S1_a + \left(-\lambda R^{rp} S1_a \left(e^{pT} \cdot C P_R \cdot u^{ar} \right) - \lambda L^{rp} S1_a \left(l^{jpT} \cdot C P_L \cdot q^{air} \right) \bar{\varepsilon}_{ij} + \mathrm{H.c.} \right)$$

Define $\mathscr{L}_{\mathrm{UV}}$:

```
LUV = LSM + LS1;
```

Integrating out the S_1 Leptoquark

Tree-level matching:

```
LSMEFT0 = Match[LUV, EFTOrder -> 6, LoopOrder -> 0];
LSMEFT0 - LSM // GreensSimplify // HcSimplify // NiceForm
```

$$\left(\overline{\lambda L}^{ts}\,\lambda R^{rp}\,\frac{1}{M^2}\,\left(\bar{l}^{s}_{j}\cdot C\,P_R\cdot \bar{q}^{t\ T}_{ai}\right)\left(e^{pT}\cdot C\,P_R\cdot u^{ar}\right)\varepsilon^{ij}+\text{H.c.}\right)+\overline{\lambda R}^{ts}\,\lambda R^{rp}\,\frac{1}{M^2}\,\left(\bar{e}^{s}\cdot C\,P_L\cdot \bar{u}^{tT}_{a}\right)\left(e^{pT}\cdot C\,P_R\cdot u^{ar}\right)+$$

$$\overline{\lambda L}^{ts}\,\lambda L^{rp}\,\frac{1}{M^2}\,\left(\bar{l}^{s}_{i}\cdot C\,P_R\cdot \bar{q}^{t\ T}_{aj}\right)\left(l^{ipT}\cdot C\,P_L\cdot q^{ajr}\right)-\overline{\lambda L}^{ts}\,\lambda L^{rp}\,\frac{1}{M^2}\,\left(\bar{l}^{s}_{i}\cdot C\,P_R\cdot \bar{q}^{t\ T}_{aj}\right)\left(l^{jpT}\cdot C\,P_L\cdot q^{air}\right)$$

Integrating out the S_1 Leptoquark

Tree-level matching:

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LSMEFT0 = Match[LUV, EFTOrder -> 6, LoopOrder -> 0];
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```

$$\left(\bar{\lambda}L^{ts}\,\lambda R^{rp}\,\frac{1}{M^2}\,\left(\bar{l}^{s}_{j}\cdot C\,P_R\cdot \bar{q}^{t\,T}_{ai}\right)\left(e^{pT}\cdot C\,P_R\cdot u^{ar}\right)\varepsilon^{ij}+\mathrm{H.c.}\right)+\bar{\lambda}R^{ts}\,\lambda R^{rp}\,\frac{1}{M^2}\,\left(\bar{e}^{s}\cdot C\,P_L\cdot \bar{u}^{t\,T}_{a}\right)\left(e^{pT}\cdot C\,P_R\cdot u^{ar}\right)+$$
$$\bar{\lambda}L^{ts}\,\lambda L^{rp}\,\frac{1}{M^2}\,\left(\bar{l}^{s}_{i}\cdot C\,P_R\cdot \bar{q}^{t\,T}_{aj}\right)\left(l^{ipT}\cdot C\,P_L\cdot q^{ajr}\right)-\bar{\lambda}L^{ts}\,\lambda L^{rp}\,\frac{1}{M^2}\,\left(\bar{l}^{s}_{i}\cdot C\,P_R\cdot \bar{q}^{t\,T}_{aj}\right)\left(l^{jpT}\cdot C\,P_L\cdot q^{air}\right)$$

Fierz identities* → Warsaw basis

(*lead to evanescent operators at one loop

Fuentes-Martín, König, Pagès, Thomsen, FW [2211.09144])

setting: $\Lambda = M$

$$\begin{aligned}
Q_{lq}^{(1)} &= (\bar{\ell}_p\gamma_\mu\ell_r)(\bar{q}_s\gamma^\mu q_t) &\rightarrow\quad C_{lq}^{(1)} &= \tfrac{1}{4}\lambda^L_{pr}\lambda^{L*}_{ts}\\
Q_{lq}^{(3)} &= (\bar{\ell}_p\gamma_\mu\tau^I\ell_r)(\bar{q}_s\gamma^\mu\tau^I q_t) &\rightarrow\quad C_{lq}^{(3)} &= -\tfrac{1}{4}\lambda^L_{pr}\lambda^{L*}_{ts}\\
Q_{eu} &= (\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t) &\rightarrow\quad C_{eq} &= \tfrac{1}{2}\lambda^R_{rp}\lambda^{R*}_{st}\\
Q_{lequ}^{(1)} &= (\bar{\ell}^i_p e_r)\varepsilon_{ij}(\bar{q}^j_s u_t) &\rightarrow\quad C_{lequ}^{(1)} &= \tfrac{1}{2}\lambda^R_{pr}\lambda^{L*}_{ts}\\
Q_{lequ}^{(3)} &= (\bar{\ell}^i_p\sigma_{\mu\nu}e_r)\varepsilon_{ij}(\bar{q}^j_s\sigma^{\mu\nu}u_t) &\rightarrow\quad C_{lequ}^{(3)} &= -2\lambda^R_{pr}\lambda^{L*}_{ts}
\end{aligned}$$

Integrating out the S_1 Leptoquark

Universität Zürich[UZH]

Tree-level matching:

```
LSMEFT0 = Match[LUV, EFTOrder -> 6, LoopOrder -> 0];
LSMEFT0 - LSM // GreensSimplify // HcSimplify // NiceForm
```

$$\left(\overline{\lambda L}^{ts}\lambda R^{rp}\frac{1}{M^2}\left(\bar{l}^s_j\cdot C P_R\cdot \bar{q}^{t\,T}_{ai}\right)\left(e^{pT}\cdot C P_R\cdot u^{ar}\right)\varepsilon^{ij}+\text{H.c.}\right)+\overline{\lambda R}^{ts}\lambda R^{rp}\frac{1}{M^2}\left(\bar{e}^s\cdot C P_L\cdot \bar{u}^{t\,T}_a\right)\left(e^{pT}\cdot C P_R\cdot u^{ar}\right)+$$
$$\overline{\lambda L}^{ts}\lambda L^{rp}\frac{1}{M^2}\left(\bar{l}^s_i\cdot C P_R\cdot \bar{q}^{t\,T}_{aj}\right)\left(l^{ipT}\cdot C P_L\cdot q^{ajr}\right)-\overline{\lambda L}^{ts}\lambda L^{rp}\frac{1}{M^2}\left(\bar{l}^s_i\cdot C P_R\cdot \bar{q}^{t\,T}_{aj}\right)\left(l^{jpT}\cdot C P_L\cdot q^{air}\right)$$

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setting: $\Lambda = M$

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Q^{(1)}_{lq} &= (\bar{\ell}_p\gamma_\mu\ell_r)(\bar{q}_s\gamma^\mu q_t) &\rightarrow\quad C^{(1)}_{lq} &= \tfrac{1}{4}\lambda^L_{pr}\lambda^{L*}_{ts}\\
Q^{(3)}_{lq} &= (\bar{\ell}_p\gamma_\mu\tau^I\ell_r)(\bar{q}_s\gamma^\mu\tau^I q_t) &\rightarrow\quad C^{(3)}_{lq} &= -\tfrac{1}{4}\lambda^L_{pr}\lambda^{L*}_{ts}\\
Q_{eu} &= (\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t) &\rightarrow\quad C_{eq} &= \tfrac{1}{2}\lambda^R_{rp}\lambda^{R*}_{st}\\
Q^{(1)}_{lequ} &= (\bar{\ell}^i_p e_r)\varepsilon_{ij}(\bar{q}^j_s u_t) &\rightarrow\quad C^{(1)}_{lequ} &= \tfrac{1}{2}\lambda^R_{pr}\lambda^{L*}_{ts}\\
Q^{(3)}_{lequ} &= (\bar{\ell}^i_p\sigma_{\mu\nu}e_r)\varepsilon_{ij}(\bar{q}^j_s\sigma^{\mu\nu}u_t) &\rightarrow\quad C^{(3)}_{lequ} &= -2\lambda^R_{pr}\lambda^{L*}_{ts}
\end{aligned}$$

One-loop matching for leptonic dipoles:

For full one-loop matching results see: Gherardi, Marzocca, Venturini [2003.12525]

```
LSMEFT = Match[LUV, EFTOrder -> 6, LoopOrder -> 1];
LSMEFTsimplified = EOMSimplify[LSMEFT];
SelectOperatorClass[LSMEFTsimplified, {Bar@l, e, H}, 2] /. {1/ϵ -> 0} // HcSimplify // NiceForm
```

$$\left(-\frac{i}{8}\hbar\, gL\,\overline{\lambda L}^{sr}\frac{1}{M^2}\left(2\,Ye^{tp}\lambda L^{st}-3\,\overline{Yu}^{st}\lambda R^{tp}\left(3+2\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{M^2}\right]\right)\right)H^j W^{\mu\nu I}\left(\bar{l}^r_i\cdot\Gamma_{\nu\mu}P_R\cdot e^p\right)T^{Ii}_j+\right.$$
$$\left.\frac{i}{16}\hbar\, gY\frac{1}{M^2}\left(2\,Ye^{rt}\,\overline{\lambda R}^{st}\lambda R^{sp}-\overline{Yu}^{st}\,\overline{\lambda L}^{sr}\lambda R^{tp}\left(19+10\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{M^2}\right]\right)\right)H^i B^{\mu\nu}\left(\bar{l}^r_i\cdot\Gamma_{\nu\mu}P_R\cdot e^p\right)+\text{H.c.}\right)$$

$$[Q_{eW}]_{pr} = (\bar{\ell}_p\sigma^{\mu\nu}e_r)\tau^I H W^I_{\mu\nu}$$
$$[Q_{eB}]_{pr} = (\bar{\ell}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$$

$$[C_{eB}]_{pr} = \frac{1}{16\pi^2}\frac{g_1}{8}\left\{-[Y_e]_{pt}\lambda^{R*}_{st}\lambda^R_{sr}+\lambda^{L*}_{sp}[Y^*_u]_{st}\lambda^R_{tr}\left[\frac{19}{2}+5\log\left(\frac{\mu^2_M}{M^2_S}\right)\right]\right\}$$

$$[C_{eW}]_{pr} = \frac{1}{16\pi^2}\frac{g_2}{8}\left\{\lambda^{L*}_{sp}\lambda^L_{st}[Y_e]_{tr}-3\lambda^{L*}_{sp}[Y^*_u]_{st}\lambda^R_{tr}\left[\frac{3}{2}+\log\left(\frac{\mu^2_M}{M^2_S}\right)\right]\right\}$$

- **Combine:**
  - S_1-to-SMEFT matching conditions
  - SMEFT renormalization group equations
    Jenkins, Manohar, Trott [1308.2627], [1310.4838]; Alonso, Jenkins, Manohar, Trott [1312.2014];
  - SMEFT-to-LEFT matching conditions
    Jenkins, Manohar, Stoffer [1709.04486]; Dekens, Stoffer [1908.05295];
  - Experimental constraints from measurements of: $(g-2)_\mu$ and $\mu \to e\gamma$
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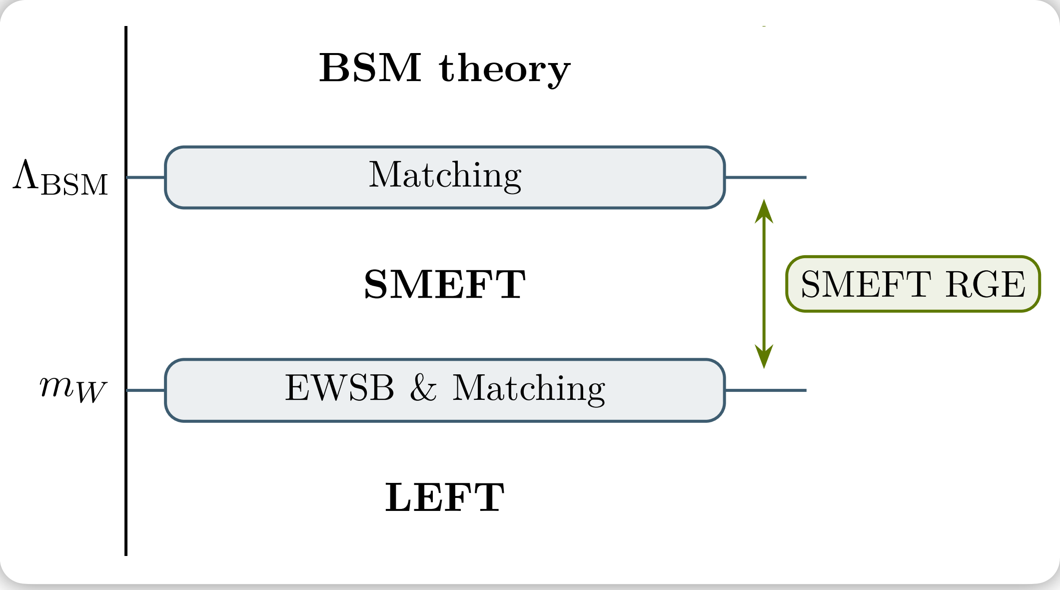

- **Combine:**
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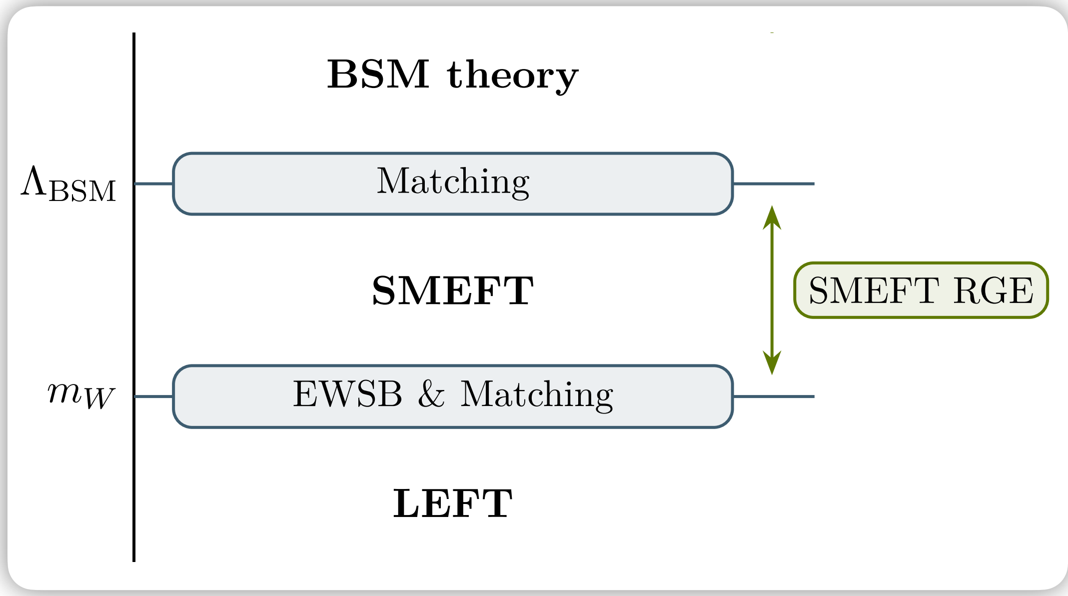
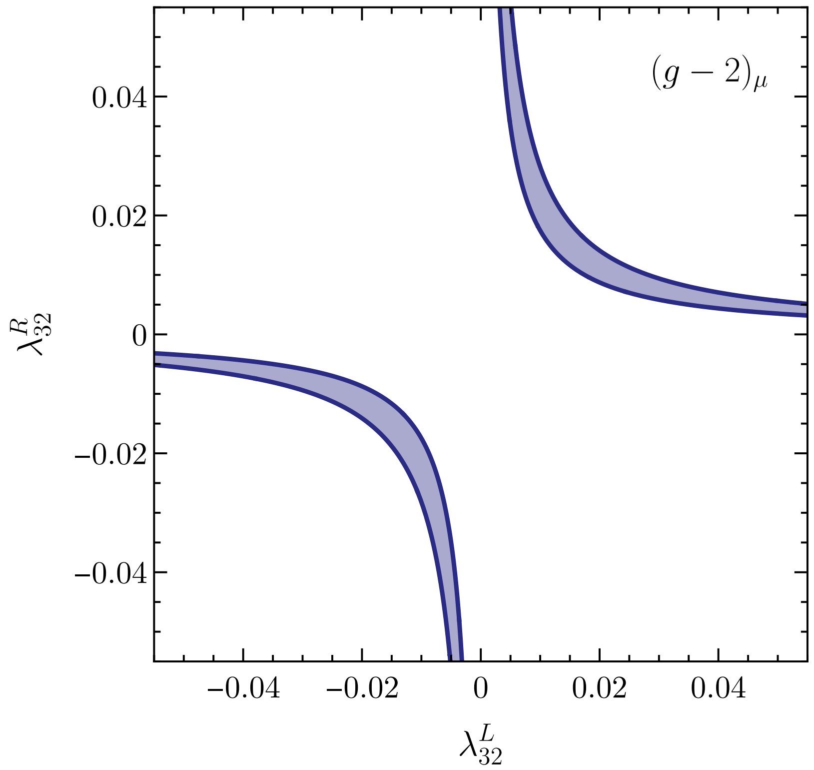
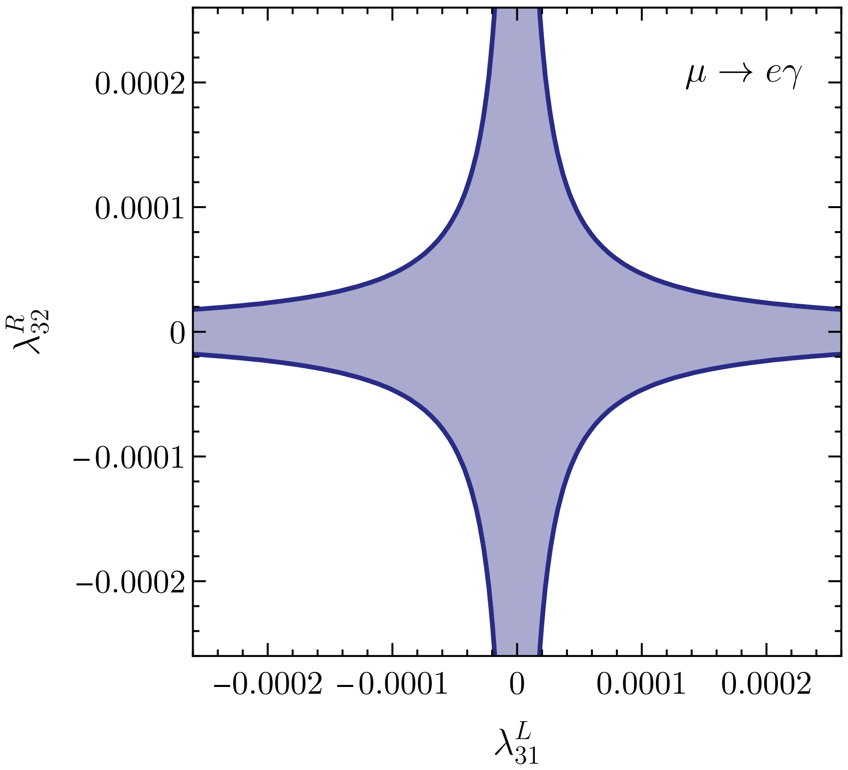


$\Delta a_\mu = (251 \pm 59) \times 10^{-11}$



$\mathcal{B}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13}$



➡ Peculiar flavor structure implied: Isidori, Pagès, FW [2111.13724]; Aebischer, Dekens, Jenkins, Manohar, Sengupta, Stoffer [2102.08954]

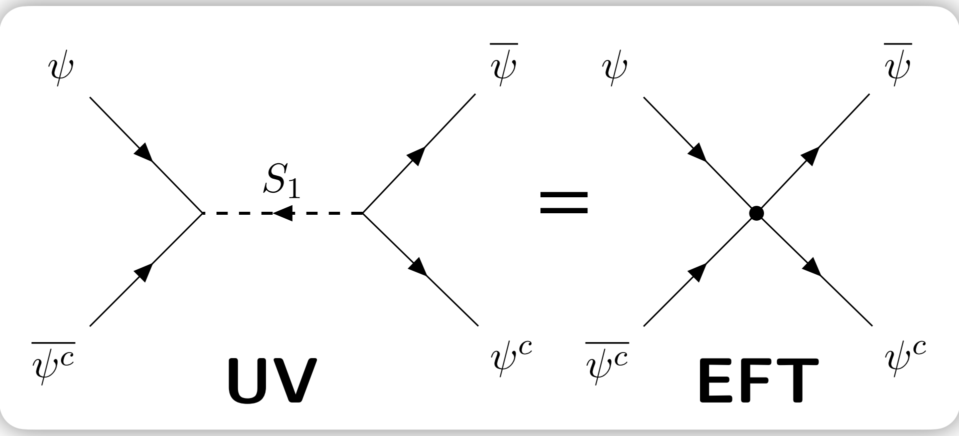
Diagrammatic Matching

- Most common method in the literature
- **UV theory:** $\mathscr{L}_{\rm UV}(\eta_H, \eta_L)$ with heavy η_H and light η_L fields
- **Construct EFT Lagrangian $\mathscr{L}_{\rm EFT}(\eta_L)$:**
  Find all higher-dimensional operators built out of η_L and respecting the symmetries of $\mathscr{L}_{\rm UV}$
- **Matching the UV theory to the EFT (off-shell):**
  - Off-shell matching:
    - Matching conditions: $Z_{\rm UV}[J_{\eta_L}, 0] = Z_{\rm EFT}[J_{\eta_L}]$ or $\Gamma_{\rm UV}(\eta_L) = \Gamma_{\rm EFT}(\eta_L)$
    - Compute all 1LPI off-shell Green's functions with light external particles η_L
    - Requires knowledge of an off-shell basis (called *Green's basis*)
  - On-shell matching:
    - Matching conditions: $\langle \eta_L | S_{\rm EFT} | \eta_L \rangle = \langle \eta_L | S_{\rm UV} | \eta_L \rangle$
    - Compute all amputated on-shell Green's functions with light external particles η_L
    - Requires knowledge of a minimal on-shell basis (such as the *Warsaw basis*)

Diagrammatic Matching of S_1 Model

$$\mathscr{L}_{\mathrm{UV}} = \mathscr{L}_{\mathrm{SM}} + (D_\mu S_1)^\dagger (D^\mu S_1) - M^2 S_1^\dagger S_1 - \left[\lambda^L_{pr}\, (\bar{q}^c_p \varepsilon \ell_r)\, S_1 + \lambda^R_{pr}\, (\bar{u}^c_p e_r)\, S_1 + \mathrm{h.c.} \right]$$

- **Tree-level matching:**



EFT operators:

$$[R_{q^c l}]_{prst} = (\overline{q}^c_{ip} \ell_{jr})(\overline{\ell}^j_s q^{ci}_t)\,,$$
$$[R'_{q^c l}]_{prst} = (\overline{q}^c_{ip} \ell_{jr})(\overline{\ell}^i_s q^{cj}_t)\,,$$
$$[R_{e^c u}]_{prst} = (\overline{e}^c_p u_r)(\overline{u}_s e^c_t)\,,$$
$$[R_{u^c e l q^c}]_{prst} = (\overline{u}^c_p e_r)\varepsilon_{ij}(\overline{\ell}^i_s q^{cj}_t)$$

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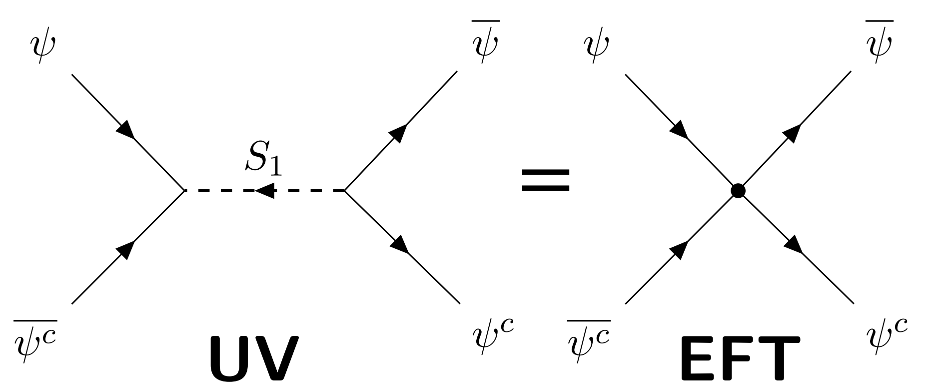

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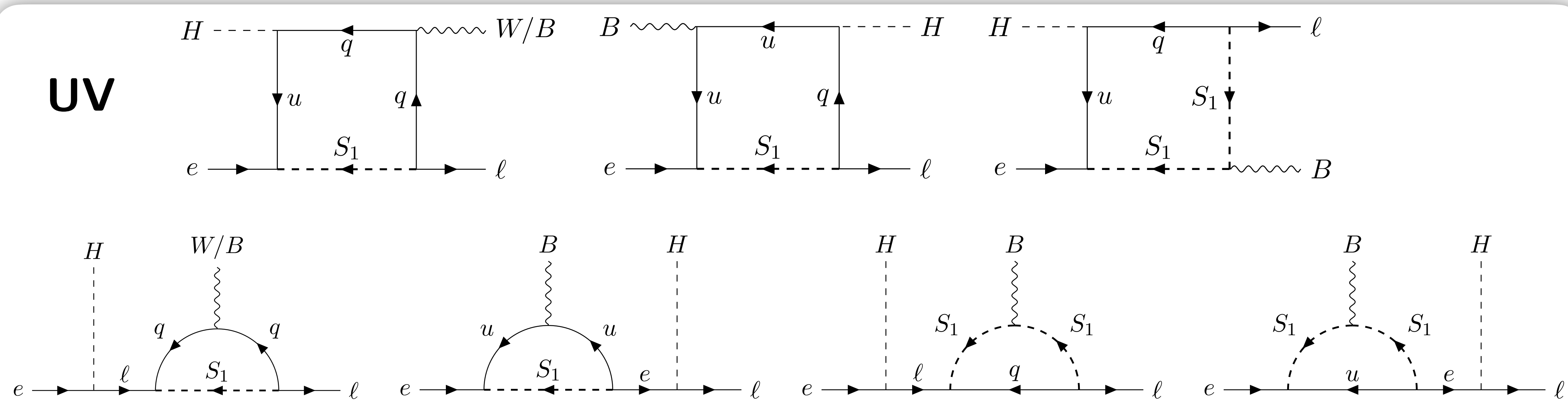
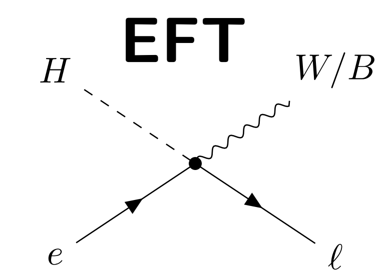
$$[R_{u^c e l q^c}]_{prst} = (\bar{u}^c_p e_r)\varepsilon_{ij}(\bar{\ell}^i_s q^{cj}_t)$$

- **One-loop (on-shell) matching for leptonic dipole operators:**





*method of regions

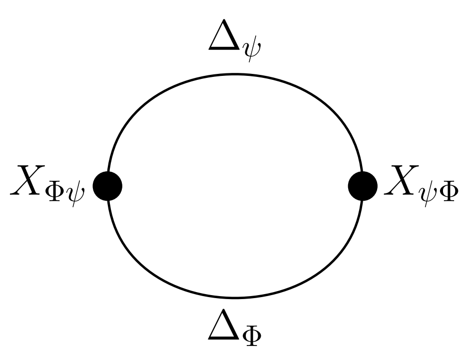
→ scaleless integrals

$$[Q_{eW}]_{pr} = (\bar{\ell}_p \sigma^{\mu\nu} e_r)\tau^I H W^I_{\mu\nu}$$
$$[Q_{eB}]_{pr} = (\bar{\ell}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$$

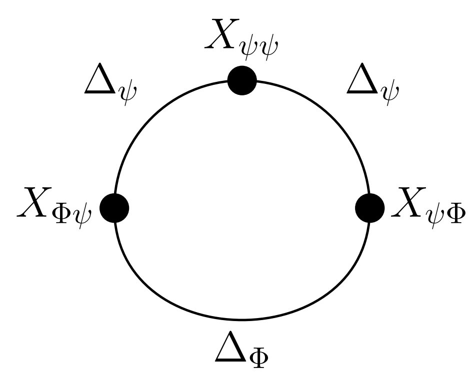
$$[C_{eB}]_{pr} = \frac{1}{16\pi^2}\frac{g_1}{8}\left\{ -[Y_e]_{pt}\lambda^{R*}_{st}\lambda^R_{sr} + \lambda^{L*}_{sp}[Y^*_u]_{st}\lambda^R_{tr}\left[\frac{19}{2} + 5\log\left(\frac{\mu^2_M}{M^2_S}\right)\right]\right\}$$

$$[C_{eW}]_{pr} = \frac{1}{16\pi^2}\frac{g_2}{8}\left\{ \lambda^{L*}_{sp}\lambda^L_{st}[Y_e]_{tr} - 3\lambda^{L*}_{sp}[Y^*_u]_{st}\lambda^R_{tr}\left[\frac{3}{2} + \log\left(\frac{\mu^2_M}{M^2_S}\right)\right]\right\}$$

- **Covariant loop diagrams:** sum of Feynman diagrams that is gauge invariant
- Used to graphically represent supertraces
- Supertraces in S_1 model matching onto leptonic dipole operators:



$\mathrm{STr}[\Delta_\Phi\, X_{\Phi\psi}\, \Delta_\psi\, X_{\psi\Phi}]$



$\mathrm{STr}[\Delta_\Phi\, X_{\Phi\psi}\, \Delta_\psi\, X_{\psi\psi}\, \Delta_\psi\, X_{\psi\Phi}]$

$$\mathcal{Q}_{ij} \equiv \left.\frac{\delta^2 S_{\mathrm{UV}}}{\delta\bar{\eta}_i\,\delta\eta_j}\right|_{\eta=\hat{\eta}} = \delta_{ij}\Delta_i^{-1} - X_{ij} = \Delta_i^{-1}(\delta_{ij} - \Delta_i X_{ij})$$

$$\Delta_i^{-1} = \begin{cases} -(D^2 + M_i^2) \\ i\gamma^\mu D_\mu - M_i \\ g^{\mu\nu}(D^2 + M_i^2) \end{cases}$$

Here:

$\Phi = S_1$

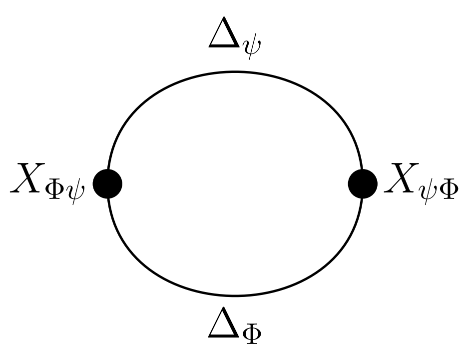
$\psi = q, u$

  - Δ_i : Propagators running in the loop
  - X_{ij} : Interaction terms connecting loop to external fields

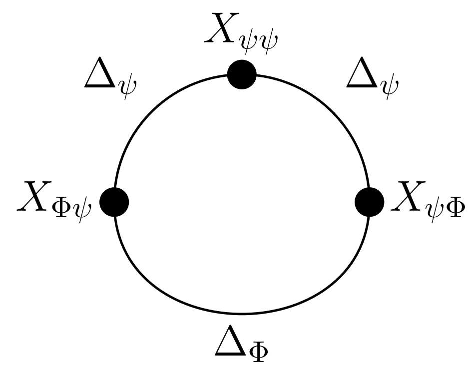
- CDE dresses all Δ_i and X_{ij} with gauge boson emissions → gauge invariant diagrams
- Evaluating hard region of these STr yields one-loop matching condition for dipole operators

Covariant Loops for Dipoles in the S_1 Model

- **Covariant loop diagrams:** sum of Feynman diagrams that is gauge invariant
- Used to graphically represent supertraces
- Supertraces in S_1 model matching onto leptonic dipole operators:



$\mathrm{STr}\left[\Delta_\Phi\, X_{\Phi\psi}\, \Delta_\psi\, X_{\psi\Phi}\right]$



$\mathrm{STr}\left[\Delta_\Phi\, X_{\Phi\psi}\, \Delta_\psi\, X_{\psi\psi}\, \Delta_\psi\, X_{\psi\Phi}\right]$

$$\mathcal{Q}_{ij} \equiv \left.\frac{\delta^2 S_{\mathrm{UV}}}{\delta\bar{\eta}_i\,\delta\eta_j}\right|_{\eta=\hat{\eta}} = \delta_{ij}\Delta_i^{-1} - X_{ij} = \Delta_i^{-1}(\delta_{ij} - \Delta_i X_{ij})$$

$$\Delta_i^{-1} = \begin{cases} -(D^2 + M_i^2) \\ i\gamma^\mu D_\mu - M_i \\ g^{\mu\nu}(D^2 + M_i^2) \end{cases}$$

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$\Phi = S_1$
$\psi = q, u$

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- CDE dresses all Δ_i and X_{ij} with gauge boson emissions → gauge invariant diagrams
- Evaluating hard region of these STr yields one-loop matching condition for dipole operators

$$[C_{eB}]_{pr} = \frac{1}{16\pi^2}\frac{g_1}{8}\left\{ - [Y_e]_{pt}\lambda^{R*}_{st}\lambda^R_{sr} + \lambda^{L*}_{sp}[Y_u^*]_{st}\lambda^R_{tr}\left[\frac{19}{2} + 5\log\left(\frac{\mu_M^2}{M_S^2}\right)\right]\right\}$$

$$[C_{eW}]_{pr} = \frac{1}{16\pi^2}\frac{g_2}{8}\left\{\lambda^{L*}_{sp}\lambda^L_{st}[Y_e]_{tr} - 3\lambda^{L*}_{sp}[Y_u^*]_{st}\lambda^R_{tr}\left[\frac{3}{2} + \log\left(\frac{\mu_M^2}{M_S^2}\right)\right]\right\}$$

SM Definition in MATCHETE

Defining the SM: Symmetries

Define gauge groups

```
In[2]:= DefineGaugeGroup[SU3c, SU[3], gs, G,
          FundAlphabet -> {"a","b","c","d","e","f"},
          AdjAlphabet -> {"A","B","C","D","E","F"}]
        DefineGaugeGroup[SU2L, SU[2], gL, W,
          FundAlphabet -> {"i","j","k","l","m","n"},
          AdjAlphabet -> {"I","J","K","L","M","N"}]
        DefineGaugeGroup[U1Y, U1, gY, B]
```

labels used for printing

label, gauge group, gauge coupling, gauge field

Define flavor indices

```
In[3]:= DefineFlavorIndex[Flavor,3,IndexAlphabet->{"p","r","s","t","u","v"}]
```

Defining the SM: Fields

Fermions

```
In[4]:= DefineField[q, Fermion, Indices -> {SU3c[fund],SU2L[fund],Flavor},
          Charges -> {U1Y[1/6]}, Chiral -> LeftHanded, Mass -> 0]
        DefineField[u, Fermion, Indices -> {SU3c[fund], Flavor},
          Charges -> {U1Y[2/3]}, Chiral -> RightHanded, Mass -> 0]
        DefineField[d, Fermion, Indices -> {SU3c[fund], Flavor},
          Charges -> {U1Y[-1/3]}, Chiral -> RightHanded, Mass -> 0]
        DefineField[l, Fermion, Indices -> {SU2L[fund], Flavor},
          Charges -> {U1Y[-1/2]}, Chiral -> LeftHanded, Mass -> 0]
        DefineField[e, Fermion, Indices -> {Flavor},
          Charges -> {U1Y[-1]}, Chiral -> RightHanded, Mass -> 0]
```

Higgs

```
In[5]:= DefineField[H, Scalar, Indices -> {SU2L[fund]},
          Charges -> {U1Y[1/2]}, Mass -> 0];
```

Defining the SM: Couplings

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Yukawa couplings

```
In[6]:= DefineCoupling[Yu, Indices -> {Flavor, Flavor}]
        DefineCoupling[Yd, Indices -> {Flavor, Flavor}]
        DefineCoupling[Ye, Indices -> {Flavor, Flavor}]
```

Higgs mass and coupling

```
In[7]:= DefineCoupling[μ, SelfConjugate -> True, EFTOrder -> 1]
        DefineCoupling[λ, SelfConjugate -> True, EFTOrder -> 0]
```

Defining the SM: Lagrangian

Yukawa interactions

```
In[8]:= YukawaL = Ye[p,r] Bar[l[i,p]]**e[r] H[i]
        + Yd[p,r] Bar[q[a,i,p]]**d[a,r] H[i]
        + Yu[p,r] Bar[q[a,i,p]]**u[a,r] CG[eps[SU2L], {i,j}] Bar[H[j]];
```

Scalar potential

In[9]:= HiggsPotential = $-\mu[]^2$ Bar[H[i]]H[i] + $\frac{\lambda[]}{2}$ Bar[H[i]]H[i]Bar[H[j]]H[j];

Full SM Lagrangian

```
In[10]:= LSM = FreeLag[q, u, d, l, e, H, G, W, B]
         - PlusHc[YukawaL] - HiggsPotential //RelabelIndices;
         LSM //HcSimplify //NiceForm
```

Out[10]= $-\frac{1}{4}B^{\mu\nu 2} -\frac{1}{4}G^{\mu\nu A2} -\frac{1}{4}W^{\mu\nu I2} +D_\mu\bar{H}_i D^\mu H^i +\mu^2\bar{H}_i H^i -\frac{1}{2}\lambda\bar{H}_i\bar{H}_j H^i H^j +i(\bar{d}^p_a\cdot\gamma_\mu P_R\cdot D_\mu d^{ap})$

$+i(\bar{e}^p\cdot\gamma_\mu P_R\cdot D_\mu e^p) +i(\bar{l}^p_i\cdot\gamma_\mu P_L\cdot D_\mu l^{ip}) +i(\bar{q}^p_{ai}\cdot\gamma_\mu P_L\cdot D_\mu q^{aip}) +i(\bar{u}^p_a\cdot\gamma_\mu P_R\cdot D_\mu u^{ap})$

$+\Big(-Ye^{rp}H^i(\bar{l}^r_i\cdot P_R\cdot e^p) -Yd^{rp}H^i(\bar{q}^r_{ai}\cdot P_R\cdot d^{ap}) -Yu^{rp}\bar{H}_i(\bar{q}^r_{aj}\cdot P_R\cdot u^{ap})\varepsilon^{ji} + \mathrm{H.c.}\Big)$