Outline

The problem: γ_5 and DReg

Three properties in 4-dimensions:

$$\{\gamma_5, \gamma^\mu\} = 0, \tag{1}$$
$$\text{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4i\epsilon^{\mu\nu\rho\sigma}, \tag{2}$$
$$\text{Tr}(\Gamma_1\Gamma_2) = \text{Tr}(\Gamma_2\Gamma_1)\,. \tag{3}$$

Inconsistent in $D \neq 4$ (can prove that trace=0).
Give up at least one $\Rightarrow$ many proposals!

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Inconsistent in $D \neq 4$ (can prove that trace=0).
Give up at least one $\Rightarrow$ many proposals!

- "Naive" anticommuting? Reading point? ... Many alternatives!
- Often limited range of applicability
- BMHV (non-anticommuting, very complicated, breaks gauge inv. But unitary, consistent)

BMHV scheme — non-anticommuting γ_5

Our idea: No-compromise approach to BMHV — apply it and accept/deal with its difficulties!

- Take seriously, apply to 1-loop, 2-loop ... EW calculations
- Here — Technical task: restore gauge invariance
- Progress will feed back to other schemes

- "*D*-dim space" split into pure 4-dim space $\oplus$ (-2ϵ)-dim space

$$X^\mu = \bar{X}^\mu + \hat{X}^\mu$$
$$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$$
$$\{\gamma_5, \bar{\gamma}^\mu\} = 0$$
$$[\gamma_5, \hat{\gamma}^\mu] = 0$$

Similar study by Cornella, Feruglio, Vecchi '22 for non-abelian 1-loop case

The problem for chiral gauge theories using BMHV

E.g., only $P_R\psi$ should interact! What is $\mathcal{L}$ in D-dim?

$$\mathcal{L}_{\text{kin+int}} = \bar{\psi} i \gamma^\mu \partial_\mu \psi + \bar{\psi} P_L \gamma^\mu P_R A_\mu \psi + \ldots$$

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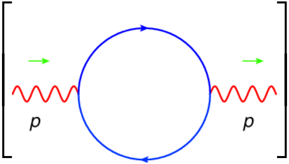
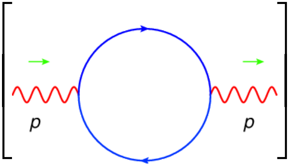
- Leads to breaking of gauge invariance, Ward/Slavnov-Taylor identities, e.g.

p_μ



$\neq 0 \Rightarrow \begin{cases} \text{need symmetry-restoring} \\ \text{counterterms} \end{cases}$

How does the breaking arise and look like?



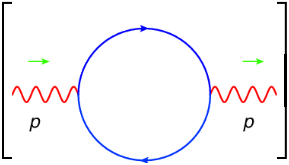
$$= \int d^D k \frac{\mathrm{Tr}(\bar{\not{k}}\bar{\gamma}^\mu(\bar{\not{k}}+\bar{\not{p}})\bar{\gamma}^\nu P_L)}{k^2(k+p)^2}$$

here, simply the numerator is purely 4-dimensional, and the textbook transversality proof (contract with p_μ, shift mom, cancel) does not apply!

$$\widetilde{\Gamma}^{\nu\mu}_{AA}(p)|^1_{\text{fin, }\chi\text{QED}} \sim \frac{1}{3}\left[\left(\frac{10}{3} - 2\ln(-p^2)\right)(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}) - \overline{p}^2\overline{g}^{\mu\nu}\right].$$

Step 1: Regularization breaks transversality! Gauge invariance is broken by div-evan. plus finite, non-evanescent, **local** terms!

How does the breaking arise and look like?



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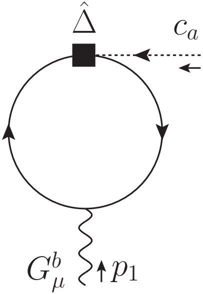
Step 2: restoring c.t. here: $\mathcal{L}^1_{\text{fct},\chi\text{QED}} \sim \frac{-1}{6}\bar{A}_\mu\bar{\partial}^2\bar{A}^\mu + \ldots$

This is how the breaking can look like and how it can be repaired.

But what is a more efficient way to compute the breaking?

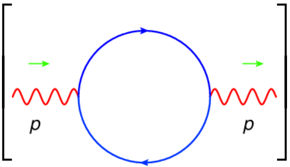
Step 3: More direct calculation of breaking using quantum action principle: breaking of STI is given by insertion of $\widehat{\Delta} = \int \delta\mathcal{L}^{(D)}$

$$i[\widehat{\Delta} \cdot \widetilde{\Gamma}^{\mu}_{Ac}]^{(1)} =$$



The result of this single diagram is equal to the breaking → Backup

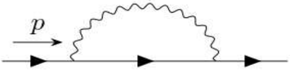
$$p_\mu$$

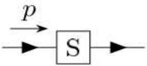


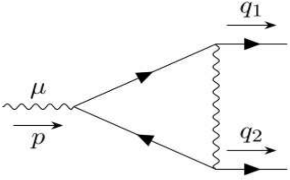
and thus sufficient to determine symmetry-restoring c.t.s
but the calculation is simpler and more direct

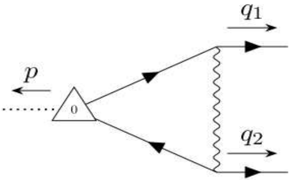
Outline

Preview: Procedure in a nutshell









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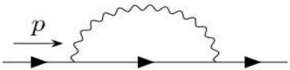

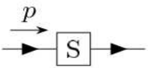

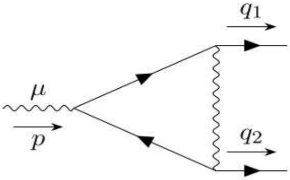

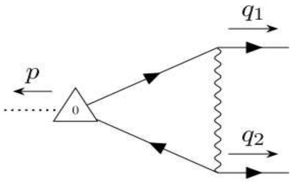

Ward identity
violated

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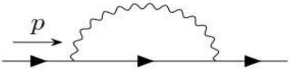
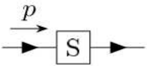
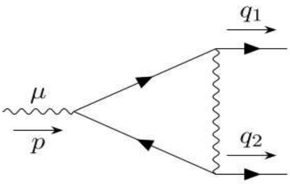
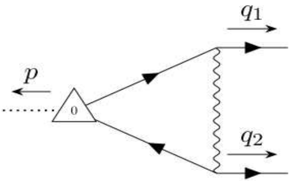






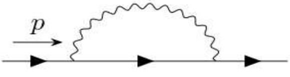


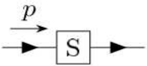
Ward identity violated

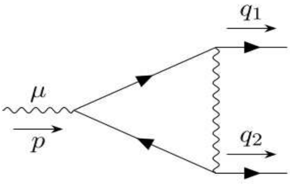
compensated by special c.t.

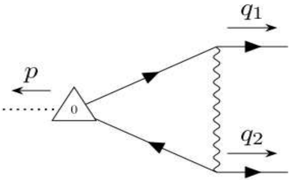
(our main task)

Preview: Procedure in a nutshell









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(guarantee completeness!)

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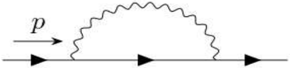

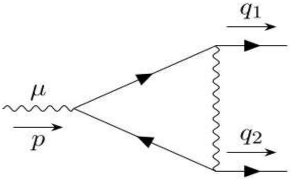

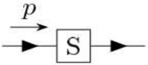

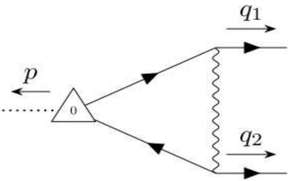

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Alternative: breaking via $\frac{\epsilon}{\epsilon}$ term

(our main task)

(tool)

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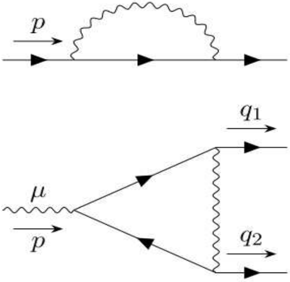

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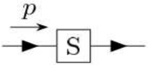

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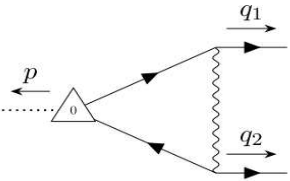

Alternative: breaking via $\frac{\hat{\epsilon}}{\epsilon}$ term

(tool)

see also: SUSY in DRed (3-loop) [DS'05, Hollik,DS'05, DS,Unger'18]

Plan here: chiral "QED" (only $P_R\psi$) at 1-/2-loop

[Bélusca-Maïto, Ilakovac,Kühler Mađor-Božinović, DS '21]

1. Define D-dimensional Lagrangian compute symmetry breaking
2. Determine 1-loop UV divs $\rightsquigarrow \mathcal{L}_{\text{sct}}$
3. Determine 1-loop violation of Slavnov-Taylor identity
4. Determine 1-loop symmetry-restoring counterterms $\rightsquigarrow \mathcal{L}_{\text{fct}}$
5. Repeat at 2-loop new features?

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Abelian theory like U(1)$_Y$-part of SM, only ψ_{Ri} interact

Description of symmetry: gauge invariance $\rightarrow$ BRST invariance $\rightarrow$ Slavnov-Taylor identity is required for renormalized theory:

$$S(\Gamma_{\text{ren}}) = 0$$

summarizes all Ward identities such as

$$\begin{aligned} p_\mu \Gamma^{\mu\nu}_{AA} &= 0 \\ p_\mu \Gamma^{\mu\nu\rho\sigma}_{AAAA} &= 0 \\ p_\mu \Gamma^{\mu}_{A\psi\bar\psi} &\propto eQ(\Sigma(p_\psi) - \Sigma(p_{\bar\psi})) \end{aligned}$$

1. Define *D*-dimensional Lagrangian

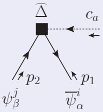
Abelian theory like $\mathrm{U}(1)_Y$-part of SM, only ψ_{Ri} interact

$\mathcal{L}$ has *D*-dim kinetic but 4-dim interaction term!

$$\mathcal{L}_{\text{fermions}} = i\overline{\psi}_i \partial\!\!\!/ \psi_i + e\mathcal{Y}_{Ri}\overline{\psi_{R}}_i A\!\!\!/ \psi_{Ri}\,.$$

$\mathcal{L}$ breaks *D*-dim gauge/BRST invariance
$\Rightarrow$ and leads to breaking of tree-level Slavnov-Taylor identity

$$\mathcal{S}_d(S_0) = \widehat{\Delta} \equiv \int \mathrm{d}^d x\, (e\mathcal{Y}_{Ri}) c \left\{ \overline{\psi}_i \left(\overleftarrow{\widehat{\partial\!\!\!/}} P_\mathrm{R} + \overrightarrow{\widehat{\partial\!\!\!/}} P_\mathrm{L} \right) \psi_i \right\}\,.$$



$$= (e\mathcal{Y}_{Ri}) \left(\widehat{p\!\!\!/}_1 P_\mathrm{R} + \widehat{p\!\!\!/}_2 P_\mathrm{L} \right)_{\alpha\beta}$$

This is the core of the difficulties. Breaking can be written as a local Feynman rule

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

First part as usual

second part is special for BMHV, sym-breaking and “evanescent”

3. Determine 1-loop violation of Slavnov-Taylor id.

Ultimate structure at 1-loop (finite ct to be determined)

$$\Gamma^{(1)}_{\text{DReg}} = \Gamma^{(1)} + S^1_{\text{sct}} + S^1_{\text{fct}}\,,$$

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Evaluate STI at 1-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma^{(1)}_{\mathrm{DReg}}) = \underbrace{\mathcal{S}_d(\Gamma^{(1)})|_{\mathrm{finite}}} + \mathcal{S}_d S^{1}_{\mathrm{fct}}$$

Left term means: breaking of regularized STI; must be computed.

In principle this corresponds to checking all STIs/WIs, e.g. Fermion 2-point/3-point function etc.

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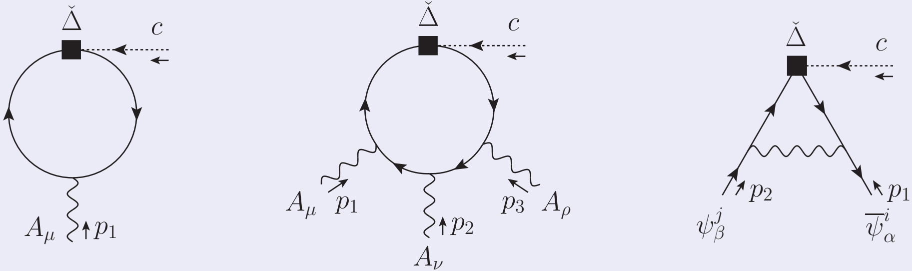
But can be simplified by using quantum action principle (BM)

$$\mathcal{S}_d(\Gamma^{(1)}) = \widehat{\Delta} \cdot \Gamma^{(1)}\,,$$

Bonneau (1980): only power-counting divergent diagrams matter!

The complete set of power-counting divergent 1-loop diagrams with insertion of $\widehat{\Delta}$:



Results mean: breaking of three concrete WI/STIs.

They have the form $\frac{\epsilon/\text{evanescent}}{\epsilon} \times$ (local)

$\rightsquigarrow$ local counterterms can repair the symmetry!

(There is an additional diagram corresponding to the fermion triangle loop and the true anomaly (assumed absent))

4. Determine symmetry-restoring counterterms

$$\mathcal{S}_d(\Gamma^{(1)})|_{\text{finite}} + \mathcal{S}_d S^1_{\text{fct}} \stackrel{!}{=} 0$$

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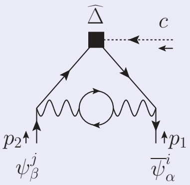
Requiring this renormalized STI to hold leads to the result

$$S^1_{\text{fct}} = \frac{e^2}{16\pi^2} \int d^4 x \left\{ \frac{-\operatorname{Tr}(\mathcal{Y}_R^2)}{6} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + \frac{e^2 \operatorname{Tr}(\mathcal{Y}_R^4)}{12} (\bar{A}^2)^2 + \left(\frac{5+\xi}{6}\right) (\mathcal{Y}_R^j)^2 \left(\bar{\psi}_j i \bar{\partial\!\!\!/} P_R \psi_j\right) \right\}.$$

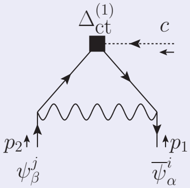
This is the full 1-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Modify both self-energies and A^4 interaction

5. Repeat at 2-loop (subrenormalization!)

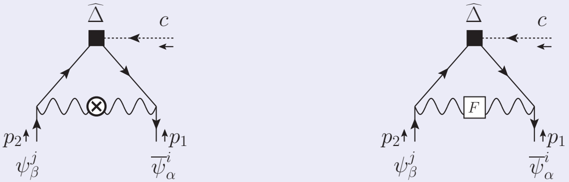
2-loop Slavnov-Taylor breaking — many diagrams of four types:



2-loop insertion of $\widehat{\Delta}$



1-loop insertion of $\Delta^1_{\rm ct}$



insertion of $\widehat{\Delta}$ into 1-loop diagram with 1-loop ct insertion

Sum gives $\mathcal{S}_d(\Gamma^{(2)})|_{\rm finite}$ =local. Can cancel by local counterterms

Determine sym-restoring counterterms at 2-loop

$$\mathcal{S}_d(\Gamma^{(2)})|_{\text{finite}} + \mathcal{S}_d S^2_{\text{fct}} \stackrel{!}{=} 0$$

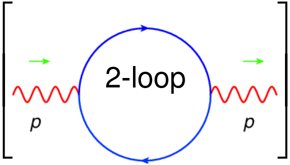
Requiring this renormalized STI to hold leads to the result

$$S^2_{\text{fct}} = \frac{e^4}{(16\pi^2)^2} \int d^4x \left\{ \text{Tr}(\mathcal{Y}_R^4) \frac{11}{48} \bar{A}_\mu \bar{\partial}^2 \bar{A}^\mu + e^2 \frac{\text{Tr}(\mathcal{Y}_R^6)}{8} (\bar{A}^2)^2 \right.$$
$$\left. -(\mathcal{Y}_R^j)^2 \left(\frac{127}{36} (\mathcal{Y}_R^j)^2 - \frac{1}{27} \text{Tr}(\mathcal{Y}_R^2) \right) \left(\bar{\psi}_j i \bar{\partial\!\!\!/} P_R \psi_j \right) \right\}$$

This is the full 2-loop result of symmetry-restoring counterterms for this chiral QED in BMHV scheme for our Lagrangian.
Finite, NON-evanescent counterterms. Not gauge invariant!
Same structure as at 1-loop

Application: restoration of 2-loop photon self energy

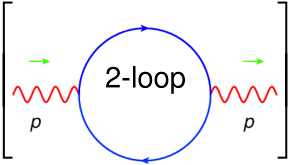
This is how the breaking looks like:



$$\overset{\text{fin-part}}{\propto} \frac{ie^4}{3 \cdot 256\pi^4} \left[\left(\frac{673}{23} - 6\,\log(-\overline{p}^2) - 24\zeta(3) \right) (\overline{p}^\mu \overline{p}^\nu - \overline{p}^2 \overline{g}^{\mu\nu}) + \frac{11}{8}\, \overline{p}^\mu \overline{p}^\nu \right],$$

Application: restoration of 2-loop photon self energy

This is how the breaking looks like:



$$\stackrel{\text{fin-part}}{\propto} \frac{ie^4}{3\cdot 256\pi^4}\left[\left(\frac{673}{23} - 6\log(-\overline{p}^2) - 24\zeta(3)\right)(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu}) + \frac{11}{8}(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{g}^{\mu\nu})\right],$$

The breaking is compensated by the counterterm of the previous slide:

$$\mathcal{L}_{\text{fin-ct}} \propto -\frac{e^4}{3\cdot 256\pi^4}\frac{11}{16}\overline{A}_\mu\overline{\partial}^2\overline{A}^\mu$$

Generalization to YM

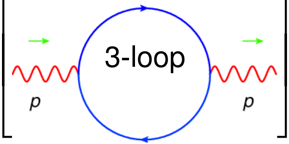
[Bélusca-Maïto, Ilakovac, Mađor-Božinović, DS, 2020, see also Cornella, Feruglio, Vecchi '22]

symmetry-restoring counterterm for YM+fermions+scalars (1-loop)

$$
\begin{aligned}
S^1_{\text{fct,restore}} = \\
\frac{\hbar}{16\pi^2}\Bigg\{ & g^2 \frac{S_2(R)}{6}\left(5S_{GG} + S_{GGG} - \int d^4x\, G^{a\mu}\partial^2 G^a_\mu\right) + \frac{Y_2(S)}{3}\overline{S_{\Phi\Phi}} \\
& + g^2 \frac{(T_R)^{abcd}}{3} \int d^4x\, \frac{g^2}{4} G^a_\mu G^{b\,\mu} G^c_\nu G^{d\,\nu} - \frac{(\mathcal{C}_R)^{ab}_{mn}}{3} \int d^4x\, \frac{g^2}{2} G^a_\mu G^{b\,\mu} \Phi^m \Phi^n \\
& + g^2 \left(1 + \frac{\xi - 1}{6}\right) C_2(R) S_{\overline{\psi}\psi} - \frac{\left((Y^m_R)^* T_{\overline{R}}{}^a Y^m_R\right)_{ij}}{2} \int d^4x\, g\overline{\psi}_i \not{G}^a P_R \psi_j \\
& - g^2 \frac{\xi C_2(G)}{4} \left(S_{\bar{R}c\psi_R} + S_{Rc\overline{\psi_R}}\right)\Bigg\},
\end{aligned}
$$

Finite, NON-evanescent counterterms. Not gauge invariant!
Modify all self-energies and some interactions!
But rather compact, universal, can be/is implemented e.g. in FeynArts

3-loop outlook — photon self energy breaks transversality (preliminary) [Matthias Weisswange]

$$i\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{3}_{\mathrm{div}} = \Bigg[$$  $$\Bigg]$$

$$\begin{aligned}
&\propto \frac{i\,e^6}{(16\pi^2)^3}\Bigg[\left(\frac{10}{81}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) - \frac{2}{27}\mathrm{Tr}(\mathcal{Y}_R^6)\right)\frac{1}{\epsilon^2} \\
&\qquad + \left(\frac{61}{1620}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \frac{638}{405}\mathrm{Tr}(\mathcal{Y}_R^6)\right)\frac{1}{\epsilon}\Bigg]\left(\overline{p}^{\mu}\overline{p}^{\nu} - \overline{p}^2\overline{g}^{\mu\nu}\right) \\
&+ \frac{i\,e^6}{(16\pi^2)^3}\Bigg[-\frac{1}{18}\mathrm{Tr}(\mathcal{Y}_R^6)\frac{1}{\epsilon^3} + \left(\frac{61}{540}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \frac{529}{1080}\mathrm{Tr}(\mathcal{Y}_R^6)\right)\frac{1}{\epsilon^2} \\
&\qquad + \left(\frac{4187}{32400}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \left(\frac{49427}{64800} - \frac{544}{225}\zeta_3\right)\mathrm{Tr}(\mathcal{Y}_R^6)\right)\frac{1}{\epsilon}\Bigg]\widehat{p}^2\overline{g}^{\mu\nu} \\
&+ \frac{i\,e^6}{(16\pi^2)^3}\left(\frac{79}{1080}\mathrm{Tr}(\mathcal{Y}_R^2)\,\mathrm{Tr}(\mathcal{Y}_R^4) + \frac{1}{60}\mathrm{Tr}(\mathcal{Y}_R^6)\right)\frac{1}{\epsilon}\,\overline{p}^2\overline{g}^{\mu\nu}
\end{aligned}$$

General Summary

Regularizations:

- Issue 1: Unitarity, Causality, Equivalence of schemes
- Issue 2: Consistent definition, representation independence
- Issue 3: Quantum action principle
- Issue 4: Symmetries and symmetry violation

γ_5 problem:

- γ_5 is problematic in DReg, BMHV scheme is rigorous
- gauge invariance broken already in $\mathcal{L}_D$ and at loop level

Renormalization in general: $\Gamma_{\text{ren}} = \Gamma_{\text{reg}} + \Gamma_{\text{ct}}$

- Γ_{ren} should be finite and satisfy $\mathcal{S}(\Gamma_{\text{ren}}) = 0$
- requires symmetry-restoring counterterms in BMHV

General Summary

Results:

- Symmetry-restoring counterterms: 1-loop YM, 2-loop abelian
- Method established: determine violation of Ward/Slavnov-Taylor identities from $\widehat{\Delta}$-diagrams
- Result has compact simple structure

Outlook:

- 2-loop YM, 2-loop EWSM, 3-loop
- alternative $\mathcal{L}_D$, schemes (FDH, DRed, etc, other γ_5 schemes)
- RGEs [Bélusca-Maïto '22]
- automatize, implement in FeynArts, FeynRules etc

Symmetry identities (Ward, Slavnov-Taylor) are crucial:

$$\text{“}S(\Gamma^{\rm reg} + \Gamma^{\rm ct}) = 0\text{”}$$

- *Unphysical states/negative norm*
- *Unitary and gauge independent physical S-matrix*

They are usually manifestly valid in DReg

Construction of Slavnov-Taylor identity

- Ghosts for all generators $\longrightarrow$ BRST:

$$s\varphi = c_a \delta_{\text{gauge},a} \varphi$$

- BRS transformations of ghosts $\leftrightarrow s^2 = 0$:

$$s c_a = \frac{1}{2} g f_{abc} c_b c_c$$

- Slavnov–Taylor operator ($S(\Gamma) = 0$ aka "Lee identities/Zinn-Justin identity")

$$S(\Gamma) = \int d^4x \underbrace{\langle s\varphi_i(x) \rangle}_{} \frac{\delta\Gamma}{\delta\varphi_i(x)}$$

- Add sources $\mathcal{L}_{\text{ext}} = K_{\varphi_i} s\varphi_i$ for composite operators

$$S(\Gamma) = \int d^4x \frac{\delta\Gamma}{\delta K_{\varphi_i}(x)} \frac{\delta\Gamma}{\delta\varphi_i(x)}$$

Concrete identities

In QED-like theories: Slavnov-Taylor identity $S(\Gamma) = 0 \rightsquigarrow$ "Ward identities"

$$p_\mu \Gamma^{\mu\nu}_{AA} = 0$$
$$p_\mu \Gamma^{\mu\nu\rho\sigma}_{AAAA} = 0$$
$$p_\mu \Gamma^{\mu}_{A\psi\bar{\psi}} \propto eQ(\Sigma(p_\psi) - \Sigma(p_{\bar{\psi}}))$$

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\end{aligned}
$$

- Case 1: regularization preserves identities

  $\rightsquigarrow$field/parameter renormalization transformation

- Case 2: regularization breaks identities

Concrete identities

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- Case 1: regularization preserves identities

- Case 2: regularization breaks identities

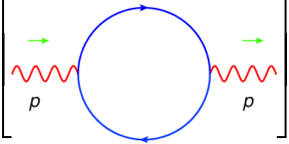
$\rightsquigarrow$add also special counterterms which satisfy "$S(\Gamma^{\mathrm{ct}}) = -\Delta$"

Example: QED Ward identity valid in DReg

$$p_\mu \Gamma^{\mu\nu}_{AA} = 0??$$

Check QED transversality of photon self energy

$$p_\mu \left[\right]$$



$$= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$$

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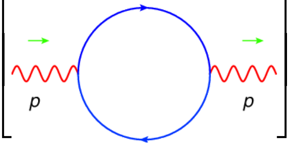

$$= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k}\gamma^\mu(\not{k}+\not{p})\gamma^\nu)}{k^2(k+p)^2}$$

using $\not{p} = (\not{k}+\not{p}) - \not{k}$ gives zero:

$$= \int d^D k \frac{(k+p)^2}{(k+p)^2} \frac{\mathrm{Tr}(\not{k}\gamma^\nu)}{k^2} - \int d^D k \frac{k^2}{k^2} \frac{\mathrm{Tr}((\not{k}+\not{p})\gamma^\nu)}{(k+p)^2} = 0$$

Symmetry identities (Ward, Slavnov-Taylor) are usually manifestly valid in DReg!

But sometimes not!

- *How can DReg break a symmetry?* γ_5*-problem!*
- *What does the breaking look like?*
- *How can we repair it?*

Warm-up exercise:

simple divergent one-loop integral

$$\int d^D k \frac{1}{k^2(k+p)^2} = \frac{1}{\epsilon} + \text{ finite}$$

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$$\int d^D k \frac{1}{k^2(k+p)^2} = \frac{1}{\epsilon} + \text{ finite}$$

simple tensor integral

$$\int d^D k \frac{k^\mu k^\nu}{k^2(k+p)^2} = \frac{1}{3\epsilon} p^\mu p^\nu - \frac{1}{12\epsilon} p^2 g^{\mu\nu} + \text{ finite}$$

Warm-up exercise:

simple divergent one-loop integral

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simple tensor integral

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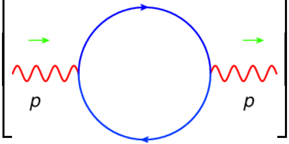
integral with "evanescent numerator" (multiply with $\hat{g}_{\mu\nu}$!)

$$\int d^D k \frac{\hat{k}^2}{k^2(k+p)^2} = \frac{1}{3\epsilon} \hat{p}^2 + \frac{1}{6} p^2 + \text{ finite}$$

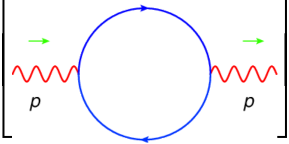
... produces div-evanescent AND finite, non-evanescent terms!

Check transversality of photon self energy in “chiral QED”

p_μ []

$$= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k} P_R \gamma^\mu P_L (\not{k} + \not{p}) P_R \gamma^\nu P_L)}{k^2 (k+p)^2}$$

Check transversality of photon self energy in “chiral QED”

p_μ []

$$= p_\mu \int d^D k \frac{\mathrm{Tr}(\not{k} P_R \gamma^\mu P_L (\not{k} + \not{p}) P_R \gamma^\nu P_L)}{k^2 (k+p)^2}$$

extracts purely 4-dim parts in numerator!

2. Compute Green functions to determine UV divs

Many 1-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

$$S^1_{\text{sct}} = S^1_{\text{sct,inv}} + S^1_{\text{sct,break}}\,,$$

First part as usual

$$S^1_{\text{ct,inv}} = \frac{\delta Z^1_A}{2} L_A + \frac{\delta Z^1_c}{2} L_c + \frac{\delta Z^1_{\psi_R}}{2} \overline{L_{\psi_R}} + \frac{\delta e^1_A}{e_A} L_{e_A}\,,$$

second part is special for BMHV, sym-breaking and “evanescent”

$$S^1_{\text{sct,break}} = \frac{-\hbar\, e_A^2}{16\pi^2\epsilon} \frac{\text{Tr}(\mathcal{Y}_R^2)}{3} \left(2(\overline{S}_{AA} - S_{AA}) + \int \mathrm{d}^d x\, \frac{1}{2} \bar{A}^\mu \widehat{\partial}^2 \bar{A}_\mu \right)\,.$$

Divergences for evanescent operators with independent coefficients, beyond the usual field/parameter renormalization

well-known in DRed: needed for unitarity/finiteness at higher orders

2. Determine UV divs at 2-loop

Many 2-loop diagrams (not shown) $\rightsquigarrow$ divergent counterterms:

$$S^2_{\text{sct}} = S^2_{\text{sct,inv}} + S^2_{\text{sct,break}} \,,$$

First part as usual $\sim$ field and parameter renormalization
second part is special for BMHV, "sym-breaking" (partially non-evan.)

$$S^2_{\text{sct,break}} = -\frac{e^4}{256\pi^4\epsilon}\frac{\text{Tr}(\mathcal{Y}_R^4)}{3}\left(2(\overline{S}_{AA} - S_{AA}) + \left(\frac{1}{2\epsilon} - \frac{17}{24}\right)\int \text{d}^d x\, \frac{1}{2}\bar{A}^\mu \widehat{\partial}^2\right.$$
$$\left. - \frac{e^4}{256\pi^4}\frac{(\mathcal{Y}_R^j)^2}{3\epsilon}\left(\frac{5}{2}(\mathcal{Y}_R^j)^2 - \frac{2}{3}\text{Tr}(\mathcal{Y}_R^2)\right)\overline{S^j_{\bar{\psi}\psi_R}}\right.$$

3. Determine 2-loop violation of Slavnov-Taylor id.

Ultimate structure at 2-loop (fct to be determined)

$$\Gamma^{(2)}_{\mathrm{DReg}} = \Gamma^{(2)} + S^2_{\mathrm{sct}} + S^2_{\mathrm{fct}}\,,$$

Evaluate STI at 2-loop order, div-parts cancel, fin-parts t.b.d.

$$\mathcal{S}_d(\Gamma^{(2)}_{\mathrm{DReg}}) = \mathcal{S}_d(\Gamma^{(2)})|_{\mathrm{finite}} + \mathcal{S}_d S^2_{\mathrm{fct}}$$

Left term (breaking of regularized STI) must be computed, use q.a.p.

$$\mathcal{S}_d(\Gamma^{(2)}) = \widehat{\Delta} \cdot \Gamma^{(2)} + \Delta^1_{\mathrm{ct}} \cdot \Gamma^{(1)}$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

How do Green functions behave?

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

Path integral:

Symmetry transformations of Green functions

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Path integral:

$$Z(J) = \int \mathcal{D}\phi \; e^{i\int \mathcal{L}+J\phi}$$

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(measure invariant)

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(1st order in δ) $$= \int \mathcal{D}\phi \; (1 + i\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

Symmetry transformations of Green functions

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Formal "derivation" for $\delta\mathcal{L} = 0$ gives form of ST identities

$$\langle(\delta\phi_1)\phi_2 \ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = 0$$

Symmetry transformations of Green functions

$$\phi_i(x) \to \phi_i(x) + \delta\phi_i(x), \qquad \mathcal{L}(x) \to \mathcal{L}(x) + \delta\mathcal{L}(x)$$

$$Z(J) = \int \mathcal{D}\phi \; e^{i \int \mathcal{L} + J\phi}$$

(measure invariant) $$= \int \mathcal{D}\phi \; e^{i \int \mathcal{L} + \delta\mathcal{L} + J\phi + J\delta\phi}$$

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result: $$0 = \int \mathcal{D}\phi \; (i \int \delta\mathcal{L} + J\delta\phi) e^{i \int \mathcal{L} + J\phi}$$

"derivation" is valid in DReg and gives breaking DREG: [Breitenlohner, Maison '77], DRED: [DS '05],review[2303.09120]

$$\langle (\delta\phi_1)\phi_2 \ldots \rangle + \langle \phi_1 (\delta\phi_2) \ldots \rangle + \ldots = -i \langle \phi_1 \phi_2 \ldots (\int \delta\mathcal{L}) \rangle$$

Symmetry transformations of Green functions

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result: $$0 = \int \mathcal{D}\phi \; (i\textstyle\int \delta\mathcal{L} + J\delta\phi) e^{i\int \mathcal{L}+J\phi}$$

This is exactly true in DREG (where $\delta\mathcal{L}$ might be $\neq 0$)

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\textstyle\int\delta\mathcal{L})\rangle$$

Symmetry transformations of Green functions — really

Regularized quantum action principle

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

Interpret this as an identity between regularized Feynman diagrams

- becomes a property of regularization scheme, does not necessarily hold (no fundamental QFT requirement)
- if desired, must be proven for each regularization
- valid in BPHZ: [Lowenstein et al '71], DREG: [Breitenlohner, Maison '77], DRED: [DS '05]

Symmetry transformations of Green functions — really

Regularized quantum action principle

$$\langle(\delta\phi_1)\phi_2\ldots\rangle+\langle\phi_1(\delta\phi_2)\ldots\rangle+\ldots=-i\langle\phi_1\phi_2\ldots(\textstyle\int\delta\mathcal{L})\rangle$$

Interpret this as an identity between regularized Feynman diagrams

Idea of proof in DREG/DRED: look at possible Wick contractions

- $\delta\mathcal{L}=\delta\mathcal{L}_{\text{quadratic}}+\delta\mathcal{L}_{\text{int}}, \qquad \delta\mathcal{L}_{\text{quadratic}}=(\delta\phi_i)D_{ij}\phi_j$
- Use properties of DREG/DRED: *D* is inverse propagator even on regularized level, scaleless integrals vanish
- then, combinatorics leads to above identity