Dimensional regularization schemes and Application of BMHV scheme for γ_5 to chiral gauge theories

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Part 1: general remarks, see e.g. review “To d, or not to d” [1705.0182]

Part 2: current long-term project, review: [2303.09120]

with: Bélusca-Maïto, Ilakovac, Kühler, Mađor-Božinović, Weisswange

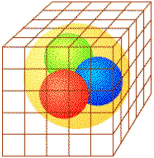
Outline

1. Preliminary remarks on dimensional schemes
2. BMHV treatment of γ_5: Overview, illustrations
3. Explicit computation of symmetry-restoring counterterms

Outline

Motivation

Regularization necessary to define QFT at the quantum level



cutoff-scale Λ

$\int_{|p|<\Lambda} d^4p$

DREG

$\mu^{4-D} \int d^Dp$

Entertaining history: puzzles, problems

- DREG breaks SUSY
- "DRED is mathematically inconsistent [Siegel '80]"
- "DRED has IR factorization problem [van Neerven, Smith, et al '88 and '05][Zerwas et al]"
- "No DRED IR factorization problem found [Kunszt, Signer, Trocsanyi '94; Catani et al '97]"
- "DRED violates unitarity ['t Hooft, van Damme '84]"
- "Some published results therefore wrong [Harlander, Kant, Mihaila, Steinhauser '06; Kilgore '11]"

Precise definitions

4S: ordinary 4-dimensional Minkowski space

metric $\bar{g}^{\mu\nu}$

QDS: [Wilson'73],[Collins] := ∞-dimensional space with D-dim characteristics:

metric $g^{\mu\nu} \equiv g^{\mu\nu}_{(D)}$

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explicit construction of integrals, $g_{\mu\nu}$, γ^{μ} etc
$\Rightarrow$ no contradictions possible

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necessarily 4S $\subset$ QDS

$$g^{\mu\nu}_{(D)} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu}$$

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For dimensional reduction/FDH scheme:

Q2ϵS: "2ϵ-dimensional space" analogous

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Q4S: "quasi-4-dimensional space" Q4S := QDS $\oplus$ Q2ϵS

metric $g^{\mu\nu}_{Q4S} = g^{\mu\nu}_{(D)} + g^{\mu\nu}_{2\epsilon}$

Common formulation of FDH (Bern, Dixon, Freitas; Kilgore, ...): "$4 < D < N_s$, internal gluons are N_s-dim.; at the end $N_s = 4$"

Issue 1: Unitarity, Causality, Equivalence

- unitarity determines imaginary terms, causality determines nonlocal terms uniquely ("causal perturbation theory" [Bogoliubov et al, Epstein, Glaser])

Basic requirement:

any correct regularization must satisfy at the $(n+1)$-loop level:

- it may differ from BPHZ only by real, local terms
- any two correct regularizations may differ only by real, local terms

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Counter example: set all divergent integrals = 0 — yields finite theory that violates causality and unitarity

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Counter example 2: DREG with anticommuting γ_5 treated consequently — some loops will be incorrectly set to zero!!

In practice, check correctness of your calculation!

e.g. 2-loop muon decay [Freitas,Hollik,Walter,Weiglein '02], 2-loop $g-2$ [Heinemeyer,DS,Weiglein '04]

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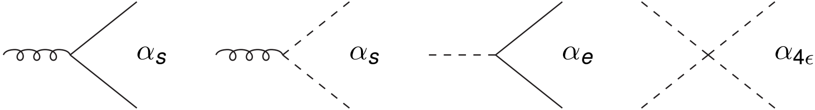
Counter example 3: DRED/FDH "treated naively, without ϵ-scalar coupling renormalization" violates unitarity (see next slide)

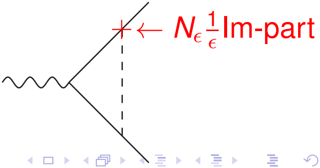
't Hooft, van Damme-problem: unitarity violation in DRED

$D^\mu = \hat\partial^\mu + igA^\mu$	$=$	$\hat\partial^\mu + ig\hat{A}^\mu$	$+$	$ig\tilde{A}^\mu$
4-component Gluon in DRED	$=$	D-component gauge field	$+$	ϵ-scalars
g		$\hat{g}$		$\tilde{g}$

- ϵ-scalars not "protected" by gauge invariance



- Different couplings α_s, α_e, $\alpha_{4\epsilon}$,
  especially $\delta\alpha_s \neq \delta\alpha_e, \beta^s \neq \beta^e, \ldots$
- Distinction required, otherwise divergent/non-unitary results



[Jack, Jones, Roberts '94][Harlander, Kant, Mihaila, Steinhauser '06][Kilgore '11]

Issue 2: Mathematical consistency

Basic requirement:

Mathematical consistency is required for any scheme

- Math. inconsistency means: possible to derive e.g. $0 = 1$
- In other words: one initial expression leads to different results, depending on the order of calculational steps

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Counter examples:

- Siegel's inconsistency of DRED — identify 4S $=$ Q4S (next slide)
- inconsistent γ_5, e.g. in

$$\mathrm{Tr}(\gamma^\mu\gamma_5\gamma^\alpha\gamma^\beta\gamma^\gamma\gamma^\delta\gamma_\mu) \propto \begin{cases} \text{either} & D\epsilon^{\alpha\beta\gamma\delta} \\ \text{or} & (8-D)\epsilon^{\alpha\beta\gamma\delta} \end{cases}$$

How does Q4S avoid Siegel's inconsistency?

Siegel: "With

$$\epsilon^{(4)}_{\mu_1\mu_2\mu_3\mu_4}\epsilon^{(4)}_{\nu_1\nu_2\nu_3\nu_4} \propto \det((g^{(4)}_{\mu_i\nu_j}))$$

calculate

$$\epsilon^{(D)\mu\nu\rho\sigma}\,\epsilon^{(\epsilon)}{}_{\alpha\beta\gamma\delta}\,\epsilon^{(D)}{}_{\mu\nu\rho\sigma}\,\epsilon^{(\epsilon)\alpha\beta\gamma\delta}$$

in two different ways

$$\Rightarrow 0 = D(D-1)^2(D-2)^2(D-3)^2(D-4)$$

different calculational steps lead to different results,

mathematical inconsistency!!!" [Siegel'80]

Solution: Don't allow explicit index counting (step one) any more, because $g^{(4)}{}_{\mu\nu}$ $\in$quasi-4-dim space!

Issue 3: Regularized quantum action principle

Basic statement:

Naive result from symmetry variation in path integral:

$$\langle(\delta\phi_1)\phi_2\ldots\rangle + \langle\phi_1(\delta\phi_2)\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$$

- property of regularization scheme, does not necessarily hold (no fundamental QFT requirement)
- if it holds, it is very useful! $\Leftrightarrow$ "P.I. measure is invariant"

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holds on the regularized level in DREG Breitenlohner/Maison '77 and DRED DS '05

Idea of proof: compare all Wick contractions on both sides [review 2303.09120]

Quantum action principle in DREG/DRED vs. other schemes

Intuition of statement: relation loop diagrams $\leftrightarrow$ Lagrangian $\mathcal{L}^{(D)}$

essential point in proof, valid in DREG/DRED:

- D-dim propagator Feynman rule $=$ inverse of diff.-op. in $\mathcal{L}^{(D)}$

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example for opposite: purely 4-dim numerators!

- D-dim Dirac propagator $= \frac{p_\mu \bar{\gamma}^\mu + m}{p^2 - m^2}$

while such a scheme might be (?) consistent, checking symmetries might be difficult

Issue 4: Symmetry properties (from $\mathcal{L}^{(D)}$ and quantum action principle)

- DREG and DRED manifestly preserve gauge invariance in QED and QCD
- DREG (BMHV) breaks gauge invariance in EWSM because of non-anticommuting γ_5 main topic of this talk
- DREG breaks SUSY
- DRED preserves SUSY to large extent but not completely

  breaking via Fierz-evanescent operator which would vanish if Q4S=4S but which does not vanish in consistent DRED

  [Avdeev, Chochia, Vladimirov '81][DS '05]...[DS, Unger '18] $\rightsquigarrow$ relation to γ_5 problem at 3-loop

Summary

- Issue 1: Unitarity, Causality, Equivalence of schemes
  - counter examples: DRED: 't Hooft/v. D.-problem, DREG: γ_5 problem
- Issue 2: Consistent definition, representation independence
  - counter examples: DRED: Siegel's inconsistency, DREG: γ_5 problem
- Issue 3: Quantum action principle $\langle(\delta\phi_1)\phi_2\ldots\rangle + \ldots = -i\langle\phi_1\phi_2\ldots(\int\delta\mathcal{L})\rangle$
  - counter example: purely 4-dim numerators
- Issue 4: Symmetries and symmetry violation

Outline

1. Preliminary remarks on dimensional schemes
2. BMHV treatment of γ_5: Overview, illustrations
   - γ_5 problem of DREG
   - BMHV breaks chiral gauge invariance
   - gauge-invariance restoring counterterm
3. Explicit computation of symmetry-restoring counterterms

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