Renormalization of the LEFT in the ’t Hooft–Veltman scheme and matching to the gradient flow

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with E. Mereghetti, C. J. Monahan, **Matthew D. Rizik**, A. Shindler: JHEP **04** (2022) 050
with **Jona Bühler**: 2304.00985 [hep-lat], to appear in JHEP
with **Òscar Lara Crosas**, C. J. Monahan, M. D. Rizik, A. Shindler: to appear
with **Luca Naterop**: to appear

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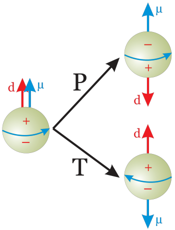
CP violation: a case for new physics

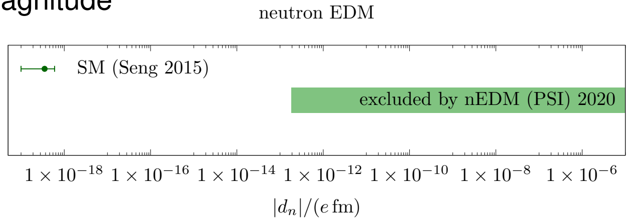
- **baryon asymmetry** in the universe requires more CP violation than Standard Model (SM) can provide
- so far **no direct evidence** of physics beyond the SM
- two options:
  - light new physics is very well hidden (weakly coupled)
  - **new physics is heavy**, with masses well above the electroweak scale $\Rightarrow$ use EFT framework
- focus here on the second option

Electric dipole moments

- **electric dipole moments** (EDMs) are sensitive probes of CP violation
- **SM (CKM) contribution** tiny
- current experimental limit for neutron: $|d_n| < 1.8 \times 10^{-13}\, e\,\text{fm}$

  $\rightarrow$ nEDM Collaboration, PRL **124** (2020) 081803
- n2EDM (PSI) will improve sensitivity by **two orders** of magnitude





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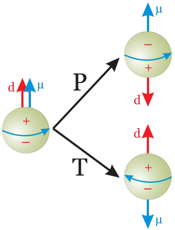
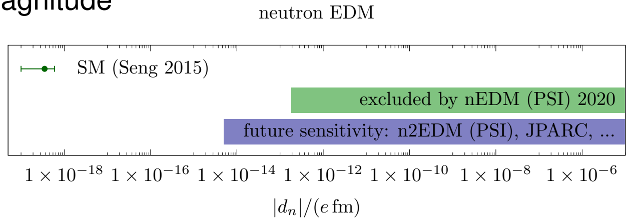




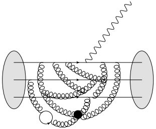
Neutron EDM

- non-observation leads to strong **constraints on CP-violating sources**
- observation would be a clear **signal of physics beyond the SM** or QCD θ-term

Neutron EDM in LEFT

- contribution schematically given as

$$d_N \sim$$



$$= \sum_i L_i \, \langle N|\mathcal{O}_i|N\gamma\rangle$$

Neutron EDM in LEFT

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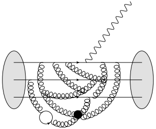

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LEFT operator coefficients

Neutron EDM in LEFT

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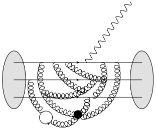
$d_N \sim$  $= \sum_i L_i \langle N|\mathcal{O}_i|N\gamma\rangle$

hadronic matrix element

Neutron EDM in LEFT

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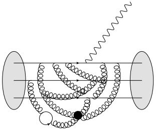
$d_N \sim$  $= \sum_i L_i \, \langle N|\mathcal{O}_i|N\gamma\rangle$

- calculate matrix element in LEFT at a renormalization scale of $\mu \sim 2 \ldots 3$ GeV
- at present, **large uncertainties** on matrix elements dilute experimental sensitivity
- aim for $10 - 25\%$ precision to avoid cancellations

$\rightarrow$ Alarcon et al., arXiv:2203.08103

Neutron EDM in LEFT

- hadronic EDMs (nEDM) complicated: **QCD is non-perturbative** at low energies
- any P-odd, CP-odd flavor-conserving operator **contributes non-perturbatively** to nEDM:
  - QCD θ-term
  - dimension-five quark (C)EDM operators
  - dimension-six three-gluon operator
  - dimension-six P/CP-odd four-fermion operators

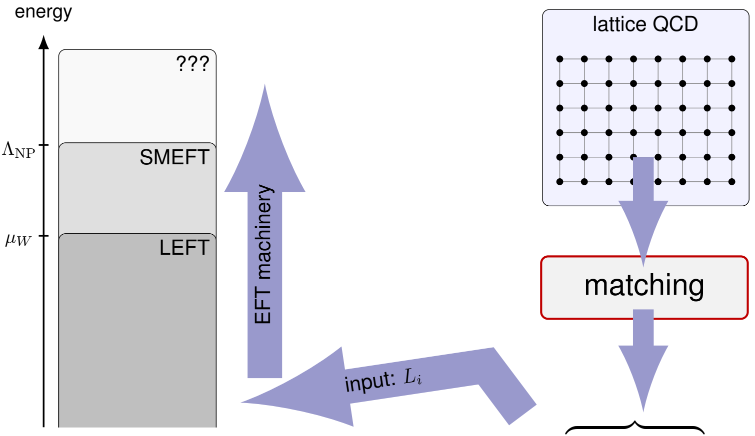
Neutron EDM in LEFT

$$
\begin{aligned}
d_N = &- (1.5 \pm 0.7) \times 10^{-3}\, \bar{\theta}\, e \,\mathrm{fm} \\
&- (0.20 \pm 0.01) d_u + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s \\
&- (0.55 \pm 0.28) e\, \tilde{d}_u - (1.1 \pm 0.55) e\, \tilde{d}_d + (??) e\, \tilde{d}_s \\
&+ (50 \pm 40) \mathrm{MeV}\, e\, \tilde{d}_G + (??) \text{ four-quark}
\end{aligned}
$$

$\to$ Alarcon et al., arXiv:2203.08103

- ideally use **lattice QCD** to compute matrix elements
- problem with lattice and EFT: $d_N \sim \sum_i L_i(\mu) \langle N | \mathcal{O}_i^{\overline{\mathrm{MS}}} | N\gamma \rangle$
  $\overline{\mathrm{MS}}$ cannot be implemented on the lattice!
- requires a **matching calculation**

Neutron EDM in the LEFT



$$\underbrace{\sum_i L_i(\mu) \overbrace{\langle N|\mathcal{O}_i^{\overline{\rm MS}}|N\gamma\rangle}}$$

experimental nEDM constraint

General procedure

- **non-perturbative definition** of renormalized operators in a scheme amenable to lattice computations
- define non-perturbative subtraction scheme of **power divergences** $\rightarrow$ Maiani, Martinelli, Sachrajda, NPB **368** (1992) 281
- compute matrix elements in lattice QCD
- for operators without power divergences: calculate relation between $\overline{\mathrm{MS}}$ and lattice scheme at $\mu \sim 2 \ldots 3\,$GeV
- use this matching to derive matrix elements of $\overline{\mathrm{MS}}$ operators

RI schemes

- **R**egularization-**I**ndependent (**S**ymmetric) **MOM**entum-subtraction scheme

  → Martinelli et al. (1995), Sturm et al. (2010)
- impose renormalization conditions on truncated **off-shell Green's functions** for Euclidean momenta
- RI-SMOM: **insert momentum** into operator to suppress unwanted IR effects
- calculation in a **fixed R_ξ gauge**

Matching $\overline{\mathrm{MS}}$ and RI-SMOM

- dimension 5 electric & chromo-electric dipoles:

  $\rightarrow$ Bhattacharya, Cirigliano, Gupta, Mereghetti, Yoon, PRD **92** (2015) 11, 114026

- dimension-6 three-gluon operator $GG\widetilde{G}$:

  $\rightarrow$ Cirigliano, Mereghetti, Stoffer, JHEP **09** (2020) 094

- complications:
  - **large set of operators** (34 for three-gluon operator), including unphysical ones
  - requires calculation of **many matrix elements**
  - **power divergences** in lattice spacing difficult to tackle

A promising scheme: gradient flow

→ Lüscher, JHEP **08** (2010) 071, JHEP **04** (2013) 123

- **gradient flow**: introduce new artificial dimension: flow time t (not related to ordinary time)
- boundary condition: ordinary Euclidean QCD at $t = 0$

$$B_\mu(t=0) = G_\mu \, , \quad \chi(t=0) = \psi$$

- **flow equations**:

$$\partial_t B_\mu = D_\nu G_{\nu\mu} + \alpha_0 D_\mu \partial_\nu B_\nu \, ,$$
$$\partial_t \chi = D^2 \chi - \alpha_0 (\partial_\mu B_\mu) \chi$$

- flow acts as a UV regulator

Gradient flow: advantages

- "flowed operators" are automatically **UV finite**, apart from quark-field (+ coupling & mass) renormalization
- non-perturbative RI renormalization condition for quark fields:

  $\rightarrow$ Makino, Suzuki, PTEP **2014**, 063B02 (2014)

$$\langle 0|\mathring{\bar{\chi}}(x;t)\overleftrightarrow{\not{D}}\mathring{\chi}(x;t)|0\rangle = -\frac{2N_c}{(4\pi)^2 t^2}$$

- **gauge-invariant** results
- on the lattice: **continuum limit** $a \to 0$ for fixed t possible
- **power divergences** no longer in $1/a$, but in $1/t$

  $\Rightarrow$ **disentangled from continuum limit**

Perturbative solution of flow equations

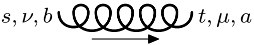
- rewrite flow equations in **integral form**

$$B_\mu(x;t) = \int d^D y \left[K_{\mu\nu}(x-y;t) G_\nu(y) + \int_0^t ds K_{\mu\nu}(x-y;t-s) R_\nu(y;s) \right]$$

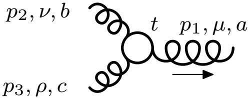
  with the kernel $\tilde{K}_{\mu\nu}(p;t) = \delta_{\mu\nu} e^{-tp^2}$

- **solve perturbatively** as an expansion in g_0
- written as **Feynman rules**, extending QCD:

$$= g_0^2 \delta^{ab} \delta_{\mu\nu} \frac{1}{p^2} e^{-(s+t)p^2} \,,$$



$$= \theta(t-s) \delta^{ab} \delta_{\mu\nu} e^{-(t-s)p^2} \,,$$



$$= -if^{abc} \int_0^\infty dt \left(\delta_{\nu\rho}(p_2 - p_3)_\mu + 2\delta_{\mu\rho} {p_3}_\nu - 2\delta_{\mu\nu} {p_2}_\rho \right)$$

Matching calculations

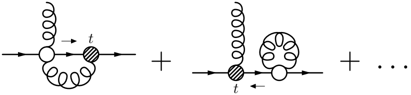
- perform **operator-product expansion** at short flow times

$$\mathcal{O}_i^R(t) = \sum_j C_{ij}(t,\mu)\mathcal{O}_j^{\overline{\mathrm{MS}}}(\mu) + \sum_j C_{ij}^{\mathcal{N}}(t,\mu)\mathcal{N}_j^{\overline{\mathrm{MS}}}(\mu) + \sum_j C_{ij}^{\mathcal{E}}(t,\mu)\mathcal{E}_j^{\overline{\mathrm{MS}}}(\mu)$$

  - $\mathcal{O}_i$: physical operators
  - $\mathcal{N}_i$: nuisance (redundant) operators
  - $\mathcal{E}_i$: evanescent operators

- determine **matching coefficients** in **perturbation theory** by computing insertions in suitable Green's functions

- e.g., for dipole:



Matching calculations

- a priori complicated flow-time integrals simplified by **method of regions**: flow-time t is the only hard scale
- CP-odd flavor-conserving sector:
  one-loop effects $\sim 30\% - 60\%$
- $\mathcal{O}(40\%)$ relative perturbative uncertainty motivates matching at two loops

  $\rightarrow$ Harlander, Kluth, Lange, EPJC **78** (2018) 11, 944
  $\rightarrow$ Harlander, Lange, PRD **105** (2022) 7, L071504

CP-odd flavor-conserving operators: completed at 1 loop

- quark bilinears:

  → Hieda, Suzuki, MPLA **31** (2016) 1650214

- dimension-5 quark (C)EDM:

  → Mereghetti, Monahan, **Matthew D. Rizik**, Shindler, Stoffer, JHEP **04** (2022) 050

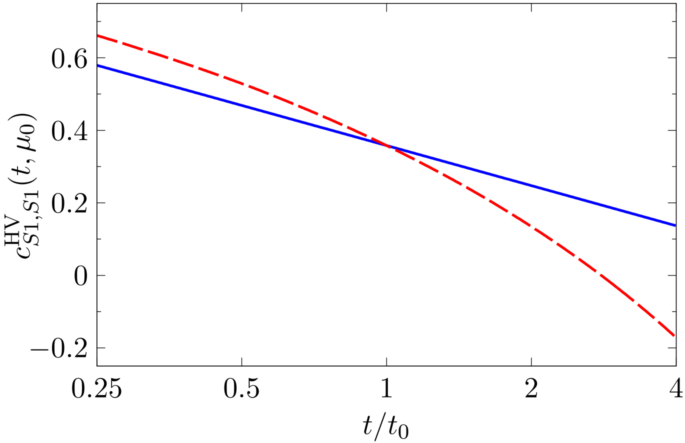
- four-quark operators:

  → **Jona Bühler**, Stoffer, arXiv:2304.00985 [hep-lat], to appear in JHEP

- dimension-6 CP-odd three-gluon operator ($\not{C}\not{P}$-3GO):

  → **Òscar Lara Crosas**, Monahan, Rizik, Shindler, Stoffer, to appear

Example: scalar singlet four-quark matching coefficient



perturbative uncertainty ∼ maximum difference between curves

Scheme choice

- CP-odd operators contain γ_5 or Levi-Civita symbol: **intrinsically four-dimensional**
- only dim.reg. scheme that is proven to be **consistent to all orders**: original HV scheme

  → 't Hooft, Veltman (1972); Breitenlohner, Maison (1977)

$$g^{\mu\nu} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu}\,, \quad \bar{g}^{\mu\nu}\bar{g}_{\nu\mu} = 4\,, \quad \hat{g}^{\mu\nu}\hat{g}_{\nu\mu} = -2\varepsilon\,,$$
$$\{\gamma_5, \bar{\gamma}_\mu\} = 0\,, \quad [\gamma_5, \hat{\gamma}_\mu] = 0$$

- in EFT: further scheme dependence due to **operator definition**

Example: $\not{CP}$-3GO scheme dependence

→ **Òscar Lara Crosas**, Monahan, Rizik, Shindler, Stoffer, to appear

- **scheme 1**: QCD in D dimensions, $\epsilon_{\mu\nu\lambda\sigma}$ only source of dimensional splitting → Cirigliano, Mereghetti, Stoffer, JHEP **09** (2020) 094

$$\mathcal{O}^R_{\widetilde{G}}(t) = \mathrm{Tr}[G_{\bar\mu\nu}(t) G_{\nu\bar\lambda}(t) \widetilde{G}_{\bar\lambda\bar\mu}(t)] \,,$$
$$\mathcal{O}_{\widetilde{G}} = \mathrm{Tr}[G_{\bar\mu\nu} G_{\nu\bar\lambda} \widetilde{G}_{\bar\lambda\bar\mu}]$$

- **scheme 2**: QCD in D dimensions, higher-dim. physical operators in 4 dimensions → **Luca Naterop**, Stoffer, to appear

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Example: $\not{C}\not{P}$-3GO scheme dependence

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gradient-flow matching:

- **scheme 1**: $C_{\widetilde{G}} = 1 + \boldsymbol{\frac{\alpha_s C_A}{12\pi}} + \frac{3\alpha_s C_A \log(8\pi\mu^2 t)}{2\pi}$
- **scheme 2**: $\bar{C}_{\widetilde{G}} = 1 + \frac{3\alpha_s C_A \log(8\pi\mu^2 t)}{2\pi}$
- dependence on $\overline{\mathrm{MS}}$ **scheme** expected
- what happens if we match $\mathcal{O}^R_{\widetilde{G}}(t)$ to scheme 2?
- intuition: **flowed operator UV finite**: $\mathcal{O}^R_{\widetilde{G}}(t)$ and $\bar{\mathcal{O}}^R_{\widetilde{G}}(t)$ should give the same matching
- change on flowed side only affects evanescents on $\overline{\mathrm{MS}}$ side
  $\Rightarrow$ naively obtain scheme 1 result

Example: ~~CP~~-3GO scheme dependence

→ **Òscar Lara Crosas**, Monahan, Rizik, Shindler, Stoffer, to appear

solution: need to renormalize evanescent operators

- **separation of evanescent sector** requires $\langle \mathcal{E}_i^{\overline{\text{MS}}} \rangle|_{\text{phys}} = 0$

  → Dugan, Grinstein (1991), Herrlich, Nierste (1995)
  → Aebischer, Pesut (2022), Fuentes-Martín, König, Pagès, Thomsen, Wilsch (2023)

- fixes **finite renormalization $\Delta_{ij}^{\mathcal{EO}}$**:

$$\mathcal{E}_i^{\overline{\text{MS}}} = (\delta_{ij} + \Delta_{ij}^{\mathcal{EE}})\mathcal{E}_j^{\text{bare}} + \Delta_{ij}^{\mathcal{EO}} \mathcal{O}_j^{\text{bare}}$$

- method of regions leads to

$$\langle \mathcal{O}_{\widetilde{G}}^R(t) \rangle \Big|_{\text{hard,finite}}^{1L} = \left(\bar{C}_i^{1L} + \Delta_{\widetilde{G},i}^{\mathcal{EO}} \right) \langle \mathcal{O}_i^{\text{bare}} \rangle \Big|^{\text{tree}} + \bar{C}_{\mathcal{E}_i}^{1L} \langle \mathcal{E}_i^{\text{bare}} \rangle \Big|^{\text{tree}}$$

- $\Delta_{\widetilde{G},i}^{\mathcal{EO}}$ exactly corresponds to difference between scheme-1/2 results ⇒ $\bar{C}_i$ independent of evanescence in flowed operator

LEFT renormalization including evanescent operators

→ **Luca Naterop**, Stoffer, to appear

- finite counterterms can be determined **once and for all**
- define **complete LEFT operator basis in HV scheme**, including evanescents (four-fermi: only one-loop generated):

$$\mathcal{L}_{\mathrm{LEFT}} = \mathcal{L}_{\mathrm{QCD+QED}} + \sum_i L_i \mathcal{O}_i + \sum_i L_i^{\mathrm{red}} \mathcal{O}_i^{\mathrm{red}} + \sum_i K_i \mathcal{E}_i$$

- avoid gauge-variant nuisance operators by using **background-field method**
- keep $\mathcal{O}_i$, $\mathcal{O}_i^{\mathrm{red}}$ in 4-dimensional sub-space
- remove $\mathcal{O}_i^{\mathrm{red}}$ via **field redefinitions**

Evanescent operators in HV scheme

→ **Luca Naterop**, Stoffer, to appear

two types of evanescent operators up to dimension six

- operators explicitly containing **summed evanescent Lorentz indices**: appearing at dimension 4, 5, 6
- **Fierz-evanescent** operators at dimension six
- note: Fierz relations **do not hold** in HV scheme:

$$(P_R\bar{\gamma}^\mu P_L) \otimes [P_R\bar{\gamma}_\mu P_L] = -(P_R\bar{\gamma}^\mu P_L] \otimes [P_R\bar{\gamma}_\mu P_L) + E_{LL}^{(F1)} ,$$
$$(P_R\bar{\gamma}^\mu P_L) \otimes [P_L\bar{\gamma}_\mu P_R] = 2(P_R] \otimes [P_L) + E_{LR}^{(F1)} ,$$
$$(P_L\bar{\sigma}^{\mu\nu} P_L) \otimes [P_L\bar{\sigma}_{\mu\nu} P_L] = 8(P_L] \otimes [P_L) - 4(P_L) \otimes [P_L] + E_{LL}^{(F2)}$$

- reason: $\bar{\gamma}_\mu$ are **not** 4×4 matrices, Dirac algebra still ∞-dimensional

LEFT renormalization including evanescent operators

→ **Luca Naterop**, Stoffer, to appear

- double insertions of dim. 5 operators: **renormalize coefficients**, not operators
- perform one-loop renormalization up to dimension-six:

$$L_i = \mu^{n_i \varepsilon}(L_i^r(\mu) + L_i^{\text{ct}}) \,, \quad L_i^{\text{ct}} = \sum_{\ell=1}^{\infty} \sum_{n=0}^{\ell} \frac{1}{\varepsilon^n} \frac{1}{(16\pi^2)^\ell} L_i^{(\ell,n)} \,,$$

$$K_i = \mu^{n_i \varepsilon}(K_i^r(\mu) + K_i^{\text{ct}}) \,, \quad K_i^{\text{ct}} = \sum_{\ell=1}^{\infty} \sum_{n=1}^{\ell} \frac{1}{\varepsilon^n} \frac{1}{(16\pi^2)^\ell} K_i^{(\ell,n)}$$

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LEFT renormalization including evanescent operators

→ **Luca Naterop**, Stoffer, to appear

- tracing over evanescent structures produces factor of ε
  $\Rightarrow$ $\boldsymbol{L_i^{(\ell,\ell)}}$ **independent of** $\boldsymbol{K_i}$
- $L_i^{(1,1)}$ scheme independent, reproduce LEFT RGE
  → Jenkins, Manohar, Stoffer, JHEP **01** (2018) 084, JHEP **03** (2018) 016
- **physical effects** of evanescent insertions arise from one-loop $1/\varepsilon$ divergence $\times$ $\hat{g}^{\mu}{}_{\mu}$
  $\Rightarrow$ **local**, compensated by counterterms $L_i^{(1,0)}$
- evanescents neither $\mathcal{O}(\varepsilon)$, nor $\mathcal{O}(\alpha)$:
  double insertions of dimension-5 evanescents at one loop
  $\Rightarrow$ contribution to $L^{(1,0)}$

Symmetry breaking by regulator

→ **Luca Naterop**, Stoffer, to appear

- LEFT is **vector-like gauge theory**: define fermion kinetic terms in D dimensions:

$$\mathcal{L}_{\text{kin}} = \bar\psi i \not{D} \psi$$

  ⇒ fully gauge invariant in D dimensions

- but: HV scheme **breaks chiral symmetry**:

$$\mathcal{L}_{\text{kin}} \supset \bar\psi_L i \hat{\not{D}} \psi_R + \bar\psi_R i \hat{\not{D}} \psi_L \,,$$

- only **global symmetry** ⇒ less severe than in chiral gauge theories → talk by D. Stöckinger

- symmetry breaking due to evanescent kinetic term ⇒ stems from **local UV divergence**, can be restored by finite counterterms

Symmetry breaking by regulator

→ **Luca Naterop**, Stoffer, to appear

- chiral symmetry **explicitly broken** in LEFT: mass terms, higher-dim. operators

$$\mathcal{L}_{\mathrm{LEFT}} \supset -\bar{\psi}_R M_\psi \psi_L - \bar{\psi}_L M_\psi^\dagger \psi_R$$

- disentangle from spurious breaking by regulator by promoting M_ψ, $M_\psi^\dagger$ (and Wilson coeffs.) to **spurions**, transforming as:

$$M_\psi \mapsto U_R M_\psi U_L^\dagger \,, \quad M_\psi^\dagger \mapsto U_L M_\psi^\dagger U_R^\dagger$$

- perform **finite renormalization** $L_i^{(1,0)} = L_{i,\mathrm{ev}}^{(1,0)} + L_{i,\chi}^{(1,0)}$, where $L_{i,\chi}^{(1,0)}$ **compensates** breaking of **spurion chiral symmetry**

Theta terms

→ **Luca Naterop**, Stoffer, to appear

- theta parameters **require renormalization** in presence of higher-dim. operators

$$\mu \frac{d}{d\mu} \theta = -\frac{8}{g} M_q \, \mathrm{Im}(L_{qG})$$

→ Jenkins, Manohar, Stoffer, JHEP **01** (2018) 084

- can be calculated perturbatively

→ Georgi, Tomaras, Pais, PRD **23** (1981) 469

Theta terms

→ **Luca Naterop**, Stoffer, to appear

procedure to extract counterterms: → Georgi, Tomaras, Pais (1981)

- multiply CP**-odd sources** by ζ and supplement Lagrangian

$$\mathcal{L}_{\mathrm{LEFT}}(\zeta) \mapsto \mathcal{L}_{\mathrm{LEFT}}(\zeta) + \frac{\partial \mathcal{L}_{\mathrm{LEFT}}(\zeta)}{\partial \zeta} \delta\zeta$$

  with **scalar dummy field** $\delta\zeta$

- compute counterterms to

$$\theta_{\mathrm{QCD}}^{\mathrm{ct}}(\zeta) \delta\zeta \frac{g^2}{32\pi^2} G_{\mu\nu}^A \widetilde{G}^{A\mu\nu}$$

- obtain theta counterterms via integration

$$\theta_{\mathrm{QCD}}^{\mathrm{ct}} = \int_0^1 d\zeta \, \theta_{\mathrm{QCD}}^{\mathrm{ct}}(\zeta)$$

  (for single-operator insertions equivalent to momentum insertion)

Chiral anomaly

→ **Luca Naterop**, Stoffer, to appear

- theta parameters shift under (anomalous) **axial field redefinitions**
- usually derived using Fujikawa method: **path-integral measure** not invariant
- this is **not** what happens in **dim.reg.**: determinant always trivial

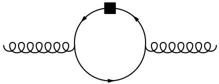
Chiral anomaly

→ **Luca Naterop**, Stoffer, to appear

- in dim.reg.: anomalous rotation generates a **shift in evanescent operators**

$$\mathcal{E}_{\psi D} = \bar{\psi}_L i \hat{\not{D}} \psi_R \,, \quad \mathcal{E}^\dagger_{\psi D} = \bar{\psi}_R i \hat{\not{D}} \psi_L$$

- insertions of evanescents induce **finite shift** in theta parameters



- spurion transformation

$$\theta_{\mathrm{QCD}} \mapsto \theta_{\mathrm{QCD}} + \sum_{\psi=u,d} \arg\det(U_R^{\psi\dagger} U_L^\psi)$$

compensates anomalous shift

- HV scheme correctly produces **chiral anomaly**

Summary

- low-energy precision measurements: need to control **non-perturbative hadronic effects**
- if using lattice QCD for matrix elements
  $\Rightarrow$ **matching calculation** to appropriate scheme
- traditional RI-SMOM schemes very challenging
- recent progress with **gradient flow**: dimension-6 matching completed at one loop for CP-odd flavor-conserving sector
- in some cases, two-loop coefficients would be useful

Summary

- CP-odd sector: **HV scheme** natural choice
- general treatment of the **entire LEFT** in HV scheme at one loop up to dimension 6:
  - full classification of **operator basis**, including HV evanescents
  - **separation of evanescent sector** through finite renormalizations
  - **restoration of chiral symmetry** through finite renormalizations:
    chiral symmetry maintained in intermediate steps of calculations (matching, running, matrix elements)
- **fully consistent** scheme, no tricks as in NDR
- ready to use, e.g., in gradient-flow matching