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Inertia for mirrors in vacuum

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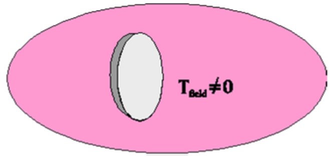
Outline :

- Moving mirrors as analog systems for gravity
- Dynamical Casimir Effect for a mirror in vacuum : Fluctuations-dissipation relations, Dynamical stability, Inertia
- Dynamical Casimir Effect for two mirrors in vacuum : Relative inertia effects, Inertia of Casimir energy

Classical fluctuations-dissipation relations for a mirror in a field at thermal equilibrium

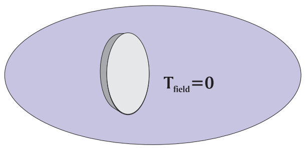


- A mirror at rest in a thermal field experiences fluctuations of the radiation pressure
  → noise spectrum $C_{FF}[\omega]$ (Fourier Transf. of the correlation function)
- A mirror moving in a thermal field experiences a dissipative force
  → mean force given by a linear susceptibility $\langle \delta F \rangle [\omega] = \chi_{FF}[\omega] \delta q[\omega]$
- Classical fluctuations-dissipation relations (Einstein 1909, 1917)
  → for a perfectly reflecting mirror in 1-dimensional (1d) space

$$C_{FF}[\omega] = 2\gamma T \qquad \gamma = \frac{3\pi k_B^2 T^2}{2\hbar c^2}$$
$$\chi_{FF}[\omega] = i\gamma\omega$$

Friction force proportional to the velocity $\langle \delta F \rangle (t) = -\gamma \delta q'(t)$

Quantum fluctuations-dissipation relations for a mirror in vacuum



- Dissipative force in vacuum : linearization of the Fulling-Davies force, proportional to the derivative of the acceleration

$$\langle \delta F \rangle (t) = \frac{\hbar}{6\pi c^2} \delta q'''(t)$$

S.A. Fulling, P.C.W. Davies, PRS **348** (1976) 393

Again the simplest model : perfect mirror in 1d space

- A mirror at rest in vacuum also experiences radiation pressure fluctuations which are related to dissipation by quantum relations

$$\chi_{FF}[\omega] = \frac{i\hbar\omega^3}{6\pi c^2}$$
$$C_{FF}[\omega] = \frac{\hbar^2\omega^3}{3\pi c^2}\theta(\omega)$$
$$\xi_{FF}[\omega] = \frac{C_{FF}[\omega] - C_{FF}[-\omega]}{2\hbar} = \mathrm{Im}\,\chi_{FF}[\omega]$$

M. Jaekel & S. Reynaud, Quantum Optics **4** (1992) 39

Instability for a perfect mirror in vacuum

- Fulling-Davies force shows the same dependence as the radiation reaction force for an electron in 3d electromagnetic vacuum
  → same instability problem
- Equation of motion in response to an applied force F_A modified by the reaction of vacuum

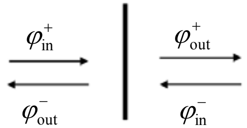
$$kq(t) + m_0 q''(t) = F_A(t) + \frac{\hbar}{6\pi c^2}\delta q'''(t)$$
$$\left(k - m_0\omega^2 - \frac{i\hbar\omega^3}{6\pi c^2}\right) q[\omega] = F_A[\omega]$$
$$-i\omega q[\omega] = \frac{1}{Z[\omega]} F_A[\omega] = Y[\omega] F_A[\omega]$$

- The mechanical admittance *Y* (inverse of the impedance *Z*) has a pole in the upper half of the complex plane
  → instability for a perfect mirror at rest in vacuum $\omega \simeq i\frac{6\pi m_0 c^2}{\hbar}$

This problem is solved below by considering "real mirrors"

"Real mirror" in quantum vacuum on the 1d line

- Real mirrors are described by a scattering matrix



$$\begin{pmatrix} \varphi_{\rm out}^{+}[\omega] \\ \varphi_{\rm out}^{-}[\omega] \end{pmatrix} = S[\omega] \begin{pmatrix} \varphi_{\rm in}^{+}[\omega] \\ \varphi_{\rm in}^{-}[\omega] \end{pmatrix}$$

- The scattering matrix has to depend on frequency (causality and transparency at high frequencies)
- Model of a local source responding linearly to the local field (Ω response function for the source, r reflection amplitude)

$$S[\omega] = \begin{pmatrix} 1+r[\omega] & r[\omega] \\ r[\omega] & 1+r[\omega] \end{pmatrix} \qquad r[\omega] = -\frac{\Omega[\omega]}{\Omega[\omega]-i\omega}$$

Unitarity and causality properties

- It follows from unitarity

$$R[\omega] = |r[\omega]|^2 = \frac{\Omega^2[\omega]}{\Omega^2[\omega]+\omega^2} = -Re r[\omega]$$

and then from causality

$$r[\omega] = -\int \frac{\mathrm{d}x}{\pi}\frac{R[x]}{\varepsilon - i\omega + ix}$$

- Asymptotic behaviour deduced

$$\lim_{\omega\to\infty}(i\omega r[\omega]) = \int_{-\infty}^{\infty} R[x]\frac{\mathrm{d}x}{\pi} = \lim_{\omega\to\infty}\Omega[\omega] \equiv \Omega_\infty$$

- There still remains freedom to change the variation with frequency of the reflection amplitude $r[\omega]$ by specifying the response function $\Omega[\omega]$ for the source

Ω constant : M. Jaekel & S. Reynaud, Phys. Lett. **A180** (1993) 9

Ω resonant : Q. Wang & W.G. Unruh, Phys. Rev. **D89** (2014) 085009

Motional susceptibility and inertia

- Motional susceptibility deduced from scattering amplitudes
- Fluctuations-dissipation relations still valid

$$\xi_{FF}[\omega] = \mathrm{Im}\chi_{FF}[\omega] = \frac{\chi_{FF}[\omega]-\chi_{FF}[-\omega]}{2i}$$
$$C_{FF}[\omega] = 2\hbar\xi_{FF}[\omega]\theta(\omega)$$

- Thanks to high-frequency transparency of mirror, the linear susceptibility varies less rapidly at high frequencies
- This cures the instability problem associated with perfect mirrors
- There is also a correction of the inertial mass.
- Agreement with the results in

G. Barton & A. Calogeracos, Ann. Phys. **238** (1995) 227
A. Calogeracos & G. Barton, Ann. Phys. **238** (1995) 268
J. Haro & E. Elizalde PRL **97** (2006) 130401
J. Haro & E. Elizalde Phys. Rev. **D76** (2007) 065001

Dissipation at low frequencies

- Dominant term for dissipation at low frequencies

$$\xi_{FF}[\omega] \simeq \frac{\hbar\omega^3}{6\pi c^2} r_0^2 \quad , \quad r_0 \equiv r[0]$$

- The dissipative force remains null for a mirror with constant velocity (no term proportional to ω)
- This is a consequence of Lorentz invariance of vacuum
- When Ω is constant, $r[\omega] = -\frac{\Omega_\infty}{\Omega_\infty - i\omega} \quad\to\quad r_0 = -1$
the dissipative part of the susceptibility remains proportional to the cube of the frequency at low frequencies
- Otherwise, the force is smaller (higher index in the power law)
- Fluctuations at low frequencies also remain the same as for a perfect mirror when $r_0^2 = 1$

Mechanical impedance and effective mass

- Equation of motion accounting for the reaction of vacuum written in terms of a mechanical impedance *Z* or admittance *Y*

$$-i\omega q[\omega] = Y[\omega] F_A[\omega]$$

$$\frac{1}{Y[\omega]} = Z[\omega] = \frac{k}{-i\omega} - i\omega m_0 + \frac{\chi_{FF}[\omega]}{i\omega}$$

- Equivalent description by an effective mass

$$Z[\omega] = \frac{k}{-i\omega} - i\omega\left(m_\infty + \mu[\omega]\right) \qquad \mu_0 \equiv \mu[0] = \frac{\hbar\Omega_\infty}{2\pi}$$

$$\lim_{\omega\to\infty} \mu[\omega] = 0$$

- The effective mass varies from $m_0 = m_\infty + \mu_0$ at low frequencies to m_∞ at high frequencies

- The difference is a mass correction corresponding to an energy bound to the mirror's motion

Barton & Calogeracos (1995), Haro & Elizalde (2006-2007)

Stability for a real mirror in vacuum

- Stability of motion of a real mirror in vacuum is ensured as soon as $m_\infty > 0$

- This follows from the "passivity property" obeyed by the model of real mirrors

$$\mathrm{Im}\omega > 0 \;\rightarrow\; \mathrm{Im}\left(\omega\mu[\omega]\right) > 0 \;\rightarrow\; \mathrm{Re}Z[\omega] > 0$$

→ no pole in the upper half of the complex plane → stability

- This condition means that the energy radiated in vacuum is positive : vacuum damps the mirror's motion but cannot excite it ; vacuum cannot sustain runaway solutions !

M. Jaekel & S. Reynaud, Phys. Lett. **A167** (1992) 227

- Similar radiated energy argument in

J. Haro & E. Elizalde PRL **97** (2006) 130401
J. Haro & E. Elizalde Phys. Rev. **D76** (2007) 065001

Fluctuations of position of a real mirror in vacuum

- As the instability problem is solved, the question of equilibrium fluctuations of position of the mirror coupled to vacuum fluctuations makes sense

- The ground state of the coupled system is described by the already studied functions

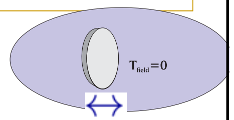
$$C_{qq}[\omega] = 2\hbar\xi_{qq}[\omega]\theta(\omega) \qquad \xi_{qq}[\omega] = \mathrm{Re}\frac{Y[\omega]}{\omega}$$

$$\xi_{qq}[\omega] = \mathrm{Im}\frac{1}{k - \omega^2\left(m_\infty + \mu[\omega]\right)}$$

- This spectrum contains narrow peaks of noise at the eigenvalues of the suspension system, corresponding to standard quantum fluctuations, above a small and broad background induced by vacuum radiation pressure fluctuations

M. Jaekel & S. Reynaud, J. Physique **I-3** (1993) 1

Mirrors in 3d electromagnetic vacuum



- Fluctuations-dissipation relation still valid for mirrors in electromagnetic vacuum (3d space)
- Calculations done for a few geometries

- Disk shape (radius a) vibrating at frequency ω
  - In the geometrical limit $(a\omega \gg c)$

$$\xi_{FF}[\omega] = \frac{A\hbar\omega^5}{60\pi^2c^4} \qquad (A = \pi a^2)$$

  - In the point-like limit $(a\omega \ll c)$

$$\xi_{FF}[\omega] = \frac{A^2\hbar\omega^7}{840\pi^3c^6}$$

- Sphere (radius a) vibrating at frequency ω
  - In the geometrical limit

$$\xi_{FF}[\omega] = \frac{A\hbar\omega^5}{45\pi^2c^4}$$

  - In the point-like limit

$$\xi_{FF}[\omega] = \frac{A^3\hbar\omega^9}{648\pi^4c^8}$$

G. Barton, J. Phys. **A24** (1991) 991 & 5533
C. Eberlein, J. Phys. **A25** (1992) 3015 & 3039
P.A. Maia Neto & S. Reynaud, Phys. Rev. **A47** (1993) 1639
P.A. Maia Neto, J. Phys. **A27** (1994) 2167

Two mirrors in vacuum

- Calculation done for two mirrors in vacuum (1d line, distance *L*)
  - Fluctuations-dissipation relation still hold
  - Each mirror influenced by the other mirror → presence of crossed effects

$$\langle \delta F_i \rangle [\omega] = \sum_j \chi_{ij}[\omega] \delta q_j[\omega]$$

- A variety of results in the quasi-static limit
  - Stiffness terms (variation of the static Casimir force)
  - No friction forces for uniform velocities
  - Inertial forces depending on *L* for uniform accelerations

$$\langle \delta F_i \rangle (t) \simeq -\sum_j \left(\kappa_{ij} \delta q_j(t) + \mu_{ij} \delta q_j''(t) \right)$$

- Resonant enhancement when the frequency of motion is the sum of two cavity resonance frequencies
  - Opto-mechanical coupling of mirrors to vacuum fields

M. Jaekel & S. Reynaud, J. Physique **I-2** (1992) 149

Simple case with two perfect mirrors

- Closed expressions obtained in this particular case
- Susceptibilities written as sums of dissipative and dispersive parts

$$\chi_{ij}[\omega] = i\xi_{ij}[\omega] + \widetilde{\chi_{ij}}[\omega]$$

$$\xi_{ij}[\omega] = \frac{\hbar\omega^3}{12\pi c^2}\delta_{ij}$$

- Dissipative parts are similar to the case of one perfect mirror
- Dispersive parts much richer than for one mirror

$$\widetilde{\chi_{11}}[\omega] = \widetilde{\chi_{22}}[\omega] = -\frac{\hbar\omega}{12\pi c^2}\frac{\omega^2 - \pi^2/\tau^2}{\tan(\omega\tau)}$$

$$\widetilde{\chi_{12}}[\omega] = \widetilde{\chi_{21}}[\omega] = \frac{\hbar\omega}{12\pi c^2}\frac{\omega^2 - \pi^2/\tau^2}{\sin(\omega\tau)}$$

- Relative inertia properties
  - Mass corrections for each mirror depending on the distance of the second mirror
  - Force on one mirror when the other one is accelerated

$$\mu_{11} = \mu_{22} = -\frac{\hbar}{12\pi c L}\left(\frac{\pi^2}{3} + 1\right)$$

$$\mu_{12} = \mu_{21} = -\frac{\hbar}{12\pi c L}\left(\frac{\pi^2}{6} - 1\right)$$

Inertia of Casimir energy

- For a global motion of the cavity "moving as a whole"

$$\langle \delta F_1 + \delta F_2 \rangle [\omega] = \chi[\omega]\delta q[\omega] \quad , \quad \chi[\omega] = \sum_{ij} \chi_{ij}[\omega] = i\xi[\omega] + \widetilde{\chi}[\omega]$$

- Dissipation is the same as for one perfect mirror

$$\xi[\omega] = \frac{\hbar\omega^3}{6\pi c^2}$$

- Dispersive parts different from the one-mirror case

$$\widetilde{\chi}[\omega] = -\frac{\hbar\omega}{6\pi c^2}\left(\omega^2 - \frac{\pi^2}{\tau^2}\right)\tan\left(\frac{\omega\tau}{2}\right)$$

- Lowest order in the quasi-static expansion is a global mass correction

$$\mu = \sum_{ij} \mu_{ij} = -\frac{\hbar\pi}{12cL}$$

- It matches the expectation from the law of inertia of energy for accelerated motion of a stressed body (Einstein, 1907)

$$\mu c^2 = E_{\mathrm{Cas}} - \frac{\mathrm{d}E_{\mathrm{Cas}}}{\mathrm{d}L}L = 2 \times E_{\mathrm{Cas}} \quad , \quad E_{\mathrm{Cas}} \propto \frac{1}{L}$$

M. Jaekel & S. Reynaud, J. Physique **I-3** (1993) 1093

Cavity in 3d electromagnetic vacuum

- Inertial forces have also been calculated for the Casimir configuration : two perfectly reflecting plates in electromagnetic vacuum in 3d space
- Results similar to the 1d case
  - Stiffness terms (variation of the static Casimir force)
  - No friction for uniform velocities
  - Relative inertia depending on *L* for uniform accelerations
- For the motion of the cavity as a whole, lowest-order term in quasi-static expansion is a global mass correction

$$\mu = \sum_{ij} \mu_{ij}$$

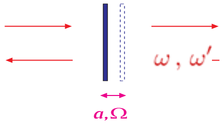
- It matches the expectation from the law of inertia of energy for accelerated motion of a stressed body (Einstein, 1907)

$$\mu c^2 = E_{\mathrm{Cas}} - \frac{\mathrm{d}E_{\mathrm{Cas}}}{\mathrm{d}L}L = 4 \times E_{\mathrm{Cas}} \quad , \quad E_{\mathrm{Cas}} \propto \frac{1}{L^3}$$

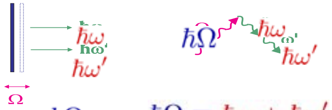
L.A.S. Machado & P.A. Maia Neto, Phys. Rev. **D65** (2002) 125005

Dynamic Casimir effect and experiments

- Dynamic Casimir effect may be interpreted as squeezing of vacuum fluctuations due to optomechanical coupling
- Back to the one-mirror case
  - For an harmonic motion at frequency Ω, optomechanical coupling to fields ω, ω'



$$q = 2a\cos(\Omega t)$$



$$\leftrightarrow \hbar\Omega \qquad \hbar\Omega \equiv \hbar\omega + \hbar\omega'$$

- Analog DCE observed with superconducting microwave circuit techniques in Delsing's group at Chalmers

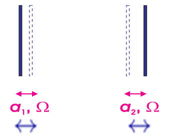
C. M. Wilson, G. Johansson, A. Pourkabirian, M. Simonen, J. R. Johansson, T. Duty, F. Nori, and P. Delsing, Nature **479** (2011) 376

Resonant enhancement in a cavity

- Dynamical Casimir effects - force and radiation - enhanced at optomechanical resonances of the cavity
- For an harmonic motion of the two mirrors at frequency Ω, optomechanical couplings show resonances when

$$\omega = \frac{k\pi}{\tau} \qquad \omega' = \frac{k'\pi}{\tau} \qquad \tau = \frac{L}{c}$$

$$\Omega = \omega + \omega' = \frac{K\pi}{\tau} \qquad K = k + k'$$



$$q_i = 2a_i\cos(\Omega t)$$

- Enhancement of DCE by the cavity

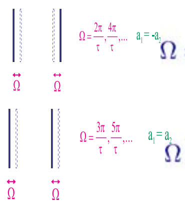
$$N = \frac{\Omega T}{3\pi}\frac{v^2}{c^2}\frac{1}{1-r^2} \text{ at resonance}$$

$\frac{\Omega T}{3\pi}\frac{v^2}{c^2}$ for one mirror
$\times$ finesse of the cavity

A. Lambrecht, M. Jaekel & S. Reynaud, Phys. Rev. Lett. **77** (1996) 615

Special features

- Two kinds of resonances
  - Breathing modes : K even
    $a_2 = -a_1$
  - Global motion : K odd
    $a_2 = a_1$

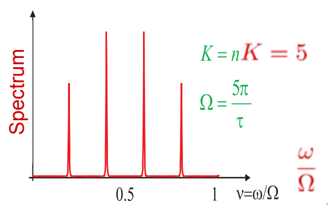


$$\Omega = \frac{2\pi}{\tau}, \frac{4\pi}{\tau}, \frac{6\pi}{\tau}, \ldots$$

$$\Omega = \frac{3\pi}{\tau}, \frac{5\pi}{\tau}, \frac{7\pi}{\tau}, \ldots$$

- Specific spectrum of DCE, useful for discriminating from spurious radiation

A. Lambrecht, J. Optics **B7** (2005) S3



- Ongoing experiment in Delsing's group at Chalmers
- Results expected soon

Conclusions

- Quantum vacuum may be considered as defining a natural reference frame for motions
- Physical effects (dissipation, radiation, inertia…) arise as consequences of motion of objects with no other reference than vacuum fluctuations
- Orders of magnitude of such effects are small
- But experimental confirmation of theoretical predictions now begins to come
- The Casimir energy contributes to the inertia of the cavity as expected from the law of inertia of energy (accelerated motion of a stressed body)
- This also means that it obeys the principle of equivalence

Thanks for your attention