Analog Hawking radiation in Bose-Einstein condensates

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Black Holes

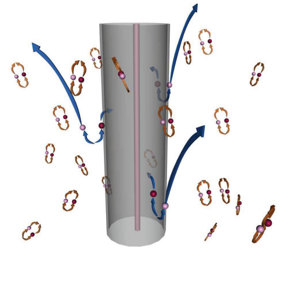
In General Relativity black holes are 'black':

- light gets trapped inside the event horizon $(R_S = 2GM/c^2)$
- they form in the gravitational collapse of massive stars $(M \geq 2 - 3M_{Sun})$

Black Holes and Quantum Mechanics

Hawking '74:

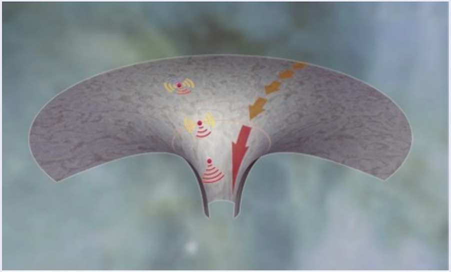
- BHs are not 'black', but emit thermal radiation with a characteristic temperature $T_H = \frac{\hbar\kappa}{2\pi k_B}$ (κ is the horizon's surface gravity)
- Basic mechanism: conversion of quantum vacuum fluctuations into on shell particles due to horizon formation



- Limits on the experimental search: for BHs formed by gravitational collapse $T_H \sim 10^{-7} \frac{M_{Sun}}{M}\ K \ll T_{CMB} \sim 3\ K$...

Analog Hawking radiation (Unruh '81):

- A stationary fluid undergoing transition from subsonic to supersonic motion is for sound what a black hole is for light (acoustic black hole)



- A process identical to that found by Hawking for BHs works for acoustic bhs: they emit a thermal flux of phonons from their acoustic horizon with

$$T_H = \frac{1}{4\pi k_B c_s} \frac{d(c_s^2 - \vec{v}_0^2)}{dn}\Big|_{hor}$$

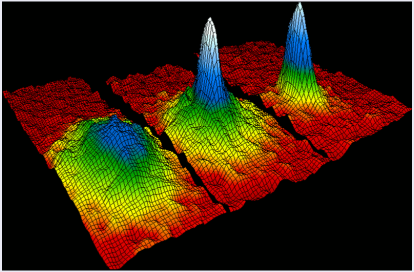
Domain of application and advantages wrt gravity

- Domain of application: fluids and other Condensed Matter systems in the long wavelength (hydrodynamic) approximation
  Barceló, Liberati and Visser 2005
- They allow to test experimentally the existence of Hawking radiation
- The existence of HR can be studied from first principles, overcoming the transplanckian problem in gravity Jacobson '91

Different avenues are explored

- Water tanks experiments (stimulated HR from white hole flows)
  Weinfurtner et al 2011
  (the results have recently been reinterpreted by Michel and Parentani 2014)
- Quantum optics experiments with laser pulse filaments Faccio et al 2010 (the results are controversial Schutzhold and Unruh 2011)
- Experimental realization of an acoustic black hole in a BEC

Bose-Einstein condensates



- Ultracold bosonic systems in which (almost) all constituents occupy the same quantum state Bose and Einstein 1924
- They have been discovered experimentally in 1995 (atomic gases of rubidium and sodium) Cornell, Wieman and Ketterle

Perfect condensates do not exist, not even at zero temperature (quantum depletion): $\hat{\Psi} = \Psi_0(1+\hat{\phi})$

- Gross-Pitaevski equation for the condensate

$$i\hbar\partial_t\Psi_0 = \left(-\frac{\hbar^2\vec{\nabla}^2}{2m} + V_{ext} + g|\Psi_0|^2\right)\Psi_0 \ ,$$

- Bogoliubov-de Gennes equation for the (linear) fluctuations $\left(c_s = \sqrt{\frac{gn_0}{m}},\ n_0 = |\Psi_0|^2\ \right)$

$$i\hbar\partial_t\hat{\phi} = -\left(\frac{\hbar^2\vec{\nabla}^2}{2m} + \frac{\hbar^2}{m}\frac{\vec{\nabla}\Psi_0}{\Psi_0}\vec{\nabla}\right)\hat{\phi} + mc_s^2(\hat{\phi}+\hat{\phi}^\dagger)\ ,$$

These equations are valid at all scales

Hydrodynamic limit

- It is more convenient to consider the density-phase representation $\hat{\Psi} = \sqrt{\hat{n}} e^{i\hat{\theta}}$ and $\hat{n} = n_0 + \hat{n}_1, \ \hat{\theta} = \theta_0 + \hat{\theta}_1$
- Considering backgrounds that vary on scales bigger than the healing length $\xi = \frac{\hbar}{mc_s}$ (the analogous of the Planck length in gravity) one obtains the continuity and Euler equations for $n_0, \ \theta_0$
- In the same approximation, for the fluctuations $\hat{n}_1 = n_0(\hat{\phi} + \hat{\phi}^\dagger)$ and $\hat{\theta}_1 = \frac{\hat{\phi} - \hat{\phi}^\dagger}{2i}$ we obtain an algebraic equation for $\hat{n}_1$ ($v_0 = \frac{\hbar \partial_x \theta_0}{m}$)

$$\hat{n}_1 = -\frac{n_0}{mc_s^2}\left[v_0 \partial_x \hat{\theta}_1 + \partial_t \hat{\theta}_1 \right] \ ,$$

while the equation for $\hat{\theta}_1$ decouples..

Gravitational analogy

The equation for θ_1 is mathematically equivalent to the Klein-Gordon equation for a massless and minimally coupled scalar field

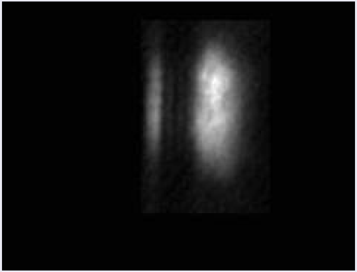
$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\hat{\theta}_1) = 0$$

in the acoustic metric

$$ds^2 = \frac{n_0}{mc_s}\left(-(c_s^2 - v_0^2)dt^2 + 2v_0 dt dx + dx^2 + dy^2 + dz^2\right)$$

The horizon's surface gravity of the acoustic metric $\kappa = \frac{1}{2c_s}\frac{d(c_s^2 - v_0^2)}{dx}|_{hor}$ gives the analog Hawking temperature T_H^{an}

An acoustic black hole in a BEC has already been realised by means of an external step-like potential accelerating the atoms and creating a region of supersonic flow Steinhauer et al 2010

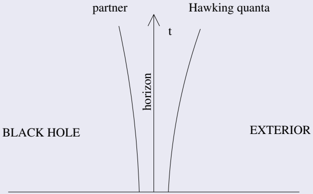


Problems

- The Hawking signal is small and there are competing effects (nonzero background temperature, quantum noise, ..)
- In BECs $T_H \sim 10\ nK < T_C \sim 100\ nK$: much better wrt gravity, but not enough to attempt a direct detection of the Hawking flux

The Hawking effect in acoustic black holes in BECs

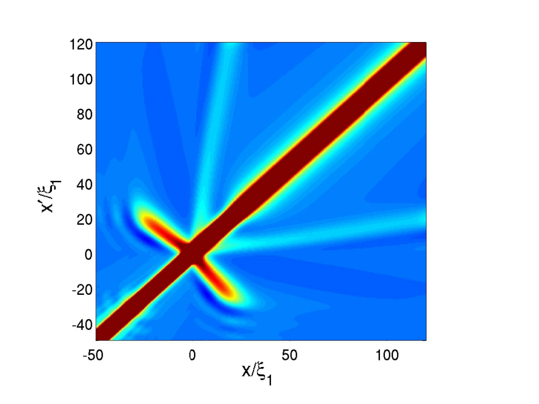
The emission of phonon pairs (Hawking quanta/ partner) on both sides of the acoustic horizon



leads to a characteristic stationary signal in the (equal time) correlation function of the density fluctuations Balbinot et al 2008

$$G^{(2)}_{BH}(t;x,x') \equiv \langle \hat{n}_1(t,x)\hat{n}_1(t,x')\rangle \sim \kappa^2 \cosh^{-2}[\frac{\kappa}{2}(\frac{x}{v+c_l} - \frac{x'}{v+c_r})]$$

This signature is robust and can be exploited to isolate HR from competing processes and experimental noise Carusotto et al 2008



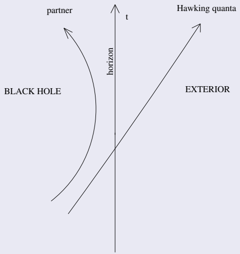
A numerical analysis using QFT in curved space techniques has confirmed the good qualitative and quantitative agreement with the 'ab-initio' CM calculation Anderson, Balbinot, Fabbri, Parentani 2013

Resolution of the transplanckian problem in BECs

- The relativistic dispersion relation for the fluctuations gets modified at large momenta $k \gg k_c = \frac{1}{\xi}$

$$w - vk = \pm c_s k \sqrt{1 + \frac{\xi^2 k^2}{4}}$$

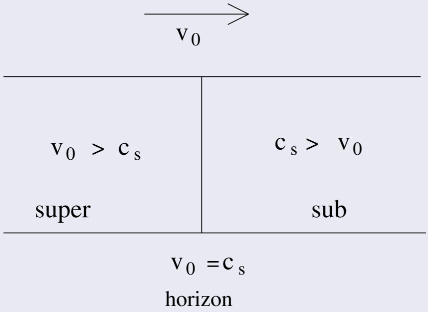
- This makes the Hawking quanta-partner pairs, evolved backwards in time, to emerge both from inside (and not at) the horizon



- Redshift is finite and the emitted flux is approximately thermal until $w_{max} \sim \frac{1}{\xi}$

White holes

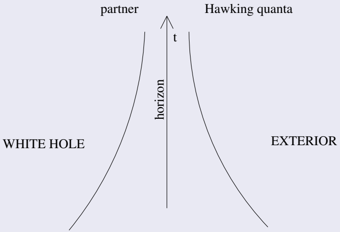
- They are the time reversal of black holes, and their acoustic version is realised with a supersonic fluid that decelerates and becomes subsonic (for acoustic black holes it is the opposite)



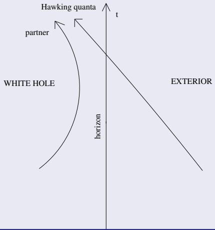
- In gravity, because they need an initial singularity, they have received little attention

Hawking effect in white holes

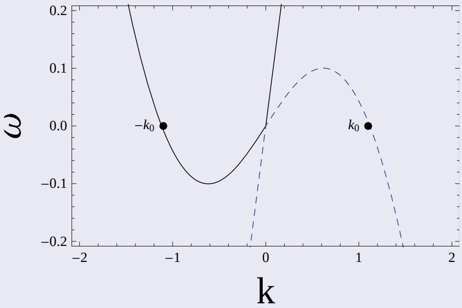
In gravity the pairs of the Hawking effect accumulate at both sides of the horizon with a large (transplanckian) frequency



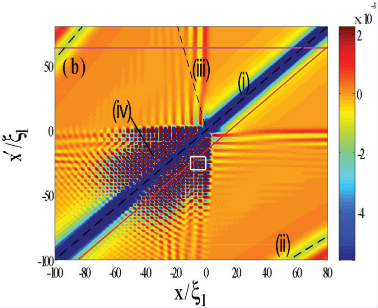
making the horizon unstable, while in BECs they enter the horizon



To understand the correlation signal inside the horizon we have to take into account the presence of a nontrivial outgoing zero mode in the dispersion relation of the supersonic region



The density correlator inside the horizon displays a checkerboard pattern



well approximated, analytically, by

$$G^{(2)}_{WH}(x,x') \sim [A\cos k_0(x+x') + B\sin k_0(x+x') + C\cos k_0(x-x')]\, I_\epsilon,$$

where the overall amplitude $I_\epsilon = \int \frac{dw}{w}$ is infrared divergent (regularised by introducing a IR cutoff $\epsilon = \frac{1}{t}$, where t is time elapsed from horizon formation)
Mayoral et al 2011

Undulations

- The low-frequency dominant contribution to the 2pt function factorizes Coutant et al 2012

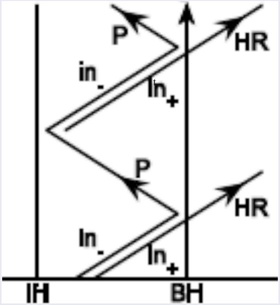
$$G_{WH} \sim \int dw |\beta_w^H|^2 \psi(x)\psi(x')$$

describing the emission of a classical zero-frequency wave propagating away from the horizon

- Its macroscopic growing amplitude $\propto \ln(t)$ ($\sim t$ if the initial state is thermal) is fixed by $|\beta_w^H|^2 \sim \frac{\kappa}{2\pi w}$, making the system (weakly) unstable (nonlinear effects are expected to saturate this growth Busch, Michel and Parentani 2014)
- Excitations with transverse momentum (massive, in 1D) exhibit a (small) undulation in black holes, whose (linear) amplitude now saturates (no IR div) Coutant, Fabbri, Parentani, Balbinot, Anderson 2012

Laser effect

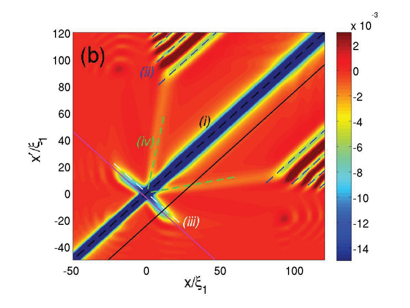
In the presence of a second horizon (BH-WH system) the Hawking effect gets (exponentially) amplified by a laser-type/dynamical instability (Corley and Jacobson 1999, Coutant and Parentani 2010, Finazzi and Parentani 2010)



A growing (negative energy) standing wave as well as a growing pattern in the density correlator in the supersonic region was observed by Steinhauer 2014

Momentum correlators

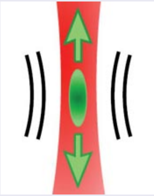
In our numerical simulations in acoustic black holes we observed, at early times, the transient feature (ii) (due to the time-dependent formation of the bh)



that we interpreted to be due to dynamical Casimir effect
Carusotto, Balbinot, Fabbri, Recati 2010

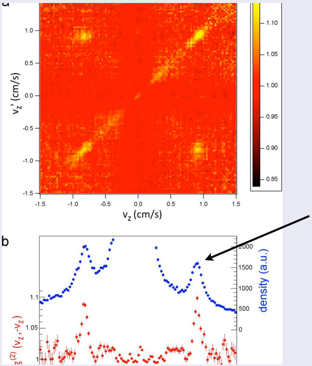
Inspired by our work, Westbrook et al 2012 considered an experiment in which:

- They varied rapidly in time the confining potential of a homogeneous 1D condensate
- They subsequently (adiabatically) opened the trap (during which excitations are converted into particles expelled from both ends of the condensate)



- They measured the velocity of the particles from their time of arrival at the detector (time of flight measurements)

- They measured the correlations between particles' (vertical) velocities v and v' and found a peak along $v = -v'$, indicating the creation of correlated excitations with opposite momentum $k = -k'$ (typical in homogeneous configurations)

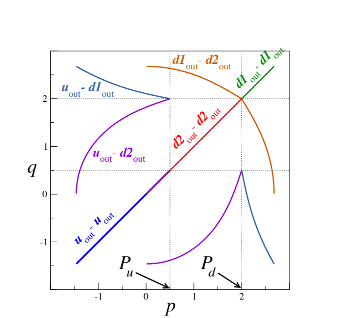


- The same phenomena is responsible for particle creation in the early Universe

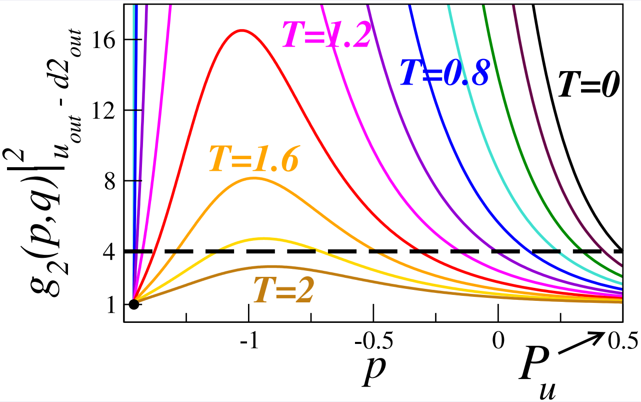
Westbrook and his group are now moving to acoustic black holes. We joined forces to adapt our original proposal to study the features of the Hawking effect in the momentum correlation function

$$g_2(k,q) = \frac{\langle : \hat{n}(k)\hat{n}(q) : \rangle}{\langle \hat{n}(k)\rangle \langle \hat{n}(q)\rangle}$$

to be later compared with the experimental data.



This observable gives a clear signature of the quantum nature of the Hawking signal ($g_2(p,q)^2 > 4$) via the violation of the Cauchy-Schwarz inequality even at $T > 10\ T_H$ (here $T_H = 0.134$)



Boiron, Fabbri, Larré, Pavloff, Westbrook and Zin 2015

Graybody factor and infrared divergences

Black holes do not emit as perfect black bodies, indeed particles' emission

$$N_\omega = |\beta_w^H|^2 = \frac{\Gamma(\omega)}{e^{\frac{\hbar\omega}{k_B T_H}} - 1}$$

is modulated by the gray-body factor $\Gamma = |T|^2$, T being the transmission factor for modes propagating between the horizon and infinity

- In gravity, for BHs in asymptotically flat space (Schwarzschild) we have, for small ω, $\Gamma \sim A_H \omega^2$, where A_H is the area of the horizon: this regulates the IR divergence of the Planckian distribution
- Surprisingly, for 1D acoustic black holes $\Gamma \to const.$ at low frequency: the analog Hawking emission is dominated by an infinite number $(\frac{1}{\omega})$ of soft phonons. A similar behaviour is found for BHs immersed in an expanding universe (Schwarzschild - de Sitter)
  Anderson, Balbinot, Fabbri and Parentani 2014
- Despite this fact, however, the density correlation function is IR finite
  Anderson, Fabbri and Balbinot 2015

Backreaction

- In gravity we study the evolution of black holes due to Hawking radiation by solving the semiclassical Einstein equations

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$$

which are valid until the BH reaches the Planck scale

- In BECs we need to solve the modified Gross-Pitaevski equation

$$i\hbar\partial_t \Psi_0 = \left(-\frac{\hbar^2 \vec{\nabla}^2}{2m} + V_{ext} + g|\Psi_0|^2 + 2\langle \hat{\phi}^\dagger \hat{\phi} \rangle \right) \Psi_0 + \langle \hat{\phi}\hat{\phi} \rangle \Psi_0^* \ ,$$

where $\langle \hat{\phi}^\dagger \hat{\phi} \rangle$ and $\langle \hat{\phi}\hat{\phi} \rangle$ are, respectively, the depletion and the anomalous density

- The search of analytical solutions in the near-horizon region will tell us how an evaporating acoustic black hole evolves and will guide the numerical resolution of the full problem

Information loss paradox

- We do not know whether or not black holes violate the rules of Quantum Mechanics, as predicted by Hawking '76
- Almeheiri, Marolf, Polchinski, Sully 2013 conjectured that the Hawking quanta - partner correlation across the horizon will be destroyed at some (Page) time if correlations between early time and late time Hawking radiation are such that unitarity is preserved
- It is interesting to address these issues in the context of BEC black holes, where concrete calculations can be performed

Conclusions

1. The study of Hawking radiation in BECs allows us to better understand its origin (negative energy states and energy conservation, the role played by the horizon)
2. The existence of Hawking radiation in BECs makes us confident that it exists also in gravity
3. Its experimental detection has allowed to establish a bridge between different fields in Physics