PARTICLE PRODUCTION, BACKREACTION, AND THE VALIDITY OF THE SEMICLASSICAL APPROXIMATION

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Collaborators

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Topics

- Brief review of original validity criterion and its application to flat space and expanding de Sitter space: Anderson, Molina-París and Mottola
- Validity during the preheating phase of chaotic inflation: Anderson, Molina-París and Sanders
- Validity during the contracting phase of de Sitter space: Anderson and Mottola

Semiclassical approximation for gravity

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- Expansion in $\hbar$: Breaks down if quantum effects are large
- N Identical Fields: Leading order in large N expansion
  - Still breaks down for large quantum fluctuations

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- Anderson, Molina-Parìs, Mottola, 2003: Linear Response Theory
  Has none of the above problems
- Hu, Roura, and Verdaguer, 2004: Stochastic Gravity
  Goes beyond the semiclassical approximation

Linear Response Criterion

- Linear response equations

$$\delta G_{ab} = 8\pi\delta\langle T_{ab}\rangle$$

- Connection with the 2-point correlation function

$$g_{ab} \to g_{ab} + h_{ab}$$

$$\delta\langle T_{ab}\rangle = \frac{1}{4}M_{ab}{}^{cd}(x)h_{cd}(x)$$
$$+\frac{i}{2}\int d^4x'\theta(t,t')\sqrt{-g(x')}\langle[T_{ab}(x), T^{cd}(x')]\rangle h_{cd}(x')$$

- $M_{ab}{}^{cd}$ is the purely local part of the variation

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- A natural way to take two-point correlation function for stress tensor into account
- No state dependent divergences
- Entirely within the semiclassical approximation

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- Hsiang, Ford, Lee, and Yu : Tensor perturbations for conformally invariant free fields are stable below the Planck scale - 2011

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- Particle production during preheating in chaotic inflation- due to parametric amplification
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- Particle production in the contracting part of de Sitter space in spatially closed coordinates
- Universes with future singularities

Particle production during preheating

- Semiclassical equation: $(\Box - m^2 - g^2\langle\psi^2\rangle)\phi = 0$

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- Exponential particle production due to parametric amplification
- Strong backreaction effects damp inflaton field
- An excellent ‘laboratory’ to study linear response: No gauge issues and no higher derivative terms

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- Work in a flat space background
- Assume homogeneity so that $\phi = \phi(t)$
- Study backreaction due to particle production, neglecting scattering effects which are important at late times

Equations for the model

- After scaling out both N and m one finds the exact set of equations describing backreaction are

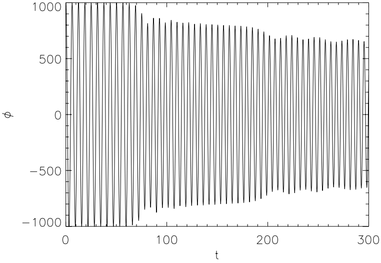
$$\ddot{\phi}(t) + (1 + g^2 \langle \psi^2 \rangle)\phi(t) = 0$$

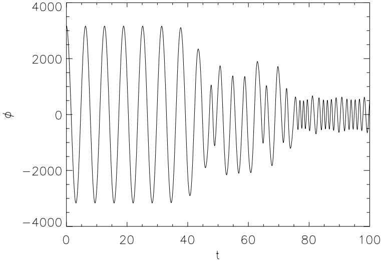
with

$$\begin{aligned}\langle \psi^2 \rangle \;=\; & \frac{1}{2\pi^2}\int_0^{\varepsilon} dk k^2 \left(|f_k(t)|^2 - \frac{1}{2k} \right) \\ & + \frac{1}{2\pi^2}\int_{\varepsilon}^{\infty} dk k^2 \left(|f_k(t)|^2 - \frac{1}{2k} + \frac{g^2\phi^2}{4k^3} \right) \\ & - \frac{g^2\phi^2}{8\pi^2}\left[1 - \log\left(\frac{2\varepsilon}{M} \right) \right]\end{aligned}$$

$$\ddot{f}_k + [k^2 + g^2\phi^2(t)] f_k = 0$$

Two Results for $g = 10^{-3}$: Anderson, Molina-París, Evanich, Cook





- Plot on left is for $\phi(0) = 10^3$. Plot on right is for $\phi(0) = \sqrt{10} \times 10^3$
- Rapid damping occurs for $g^2\phi^2(0) \gtrsim 2$

General form of linear response equation for preheating

- Perturb the semiclassical equations and find

$$(\Box - m^2 - g^2\langle\psi^2\rangle)\delta\phi - g^2\delta\langle\psi^2\rangle\phi = 0$$

$$\delta\langle\psi^2\rangle = \delta\langle\psi^2\rangle_{\mathrm{SI}} + \delta\langle\psi^2\rangle_{\mathrm{SD}}$$

$$\delta\langle\psi^2\rangle_{\mathrm{SI}} = -ig^2\int d^4x'\phi(x')\delta\phi(x')\theta(t-t')\langle[\psi^2(x),\psi^2(x')]\rangle$$

- Note that this is an integro-differential equation

Specific form of linear response equation for preheating

- For our model $\phi = \phi(t)$ so

$$\delta\ddot{\phi} + (1 + g^2\langle\psi^2\rangle)\delta\phi + g^2\delta\langle\psi^2\rangle\phi = 0$$

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$$\delta\ddot{\phi}_e + (1 + g^2\langle\psi^2\rangle_1)\delta\phi_e + g^2(\langle\psi^2\rangle_2 - \langle\psi^2\rangle_1)(\phi_1 + \delta\phi_e)$$

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- $\delta\phi_e \approx \delta\phi$ if the amplitude of ϕ_1 is large compared to $\delta\phi_e$ and

$$\delta\langle\psi^2\rangle \approx \langle\psi^2\rangle_2 - \langle\psi^2\rangle_1$$

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- Recall $\delta\phi_e = \phi_2 - \phi_1$
- Define

$$\delta\phi_c \equiv \delta\phi_e - [(\phi_2(0) - \phi_1(0))/\phi_1(0)]\phi_1$$

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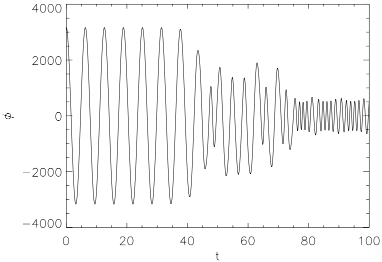
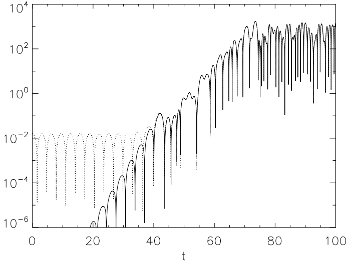
$$\delta\phi_c \equiv \delta\phi_e - [(\phi_2(0) - \phi_1(0))/\phi_1(0)]\phi_1$$

- Then $\delta\phi_c(0) = \delta\dot{\phi}_c(0) = 0$ and at early times

$$\delta\ddot{\phi}_c \approx -g^2\phi_1\delta\langle\psi^2\rangle\Big|_{\substack{\phi=\phi_1 \\ \delta\phi=c\phi_1}}$$

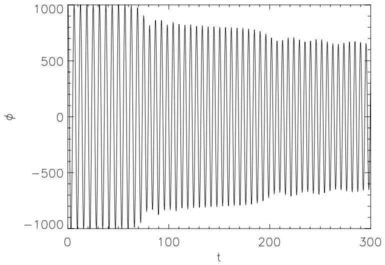
$$g^2\phi_1(0)^2 = 10$$

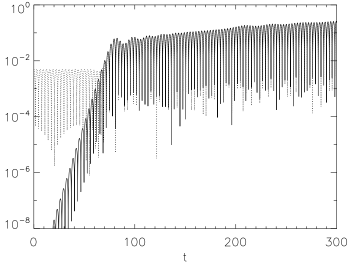




- Left plot: Solution to the semiclassical equation
- Right plot: $\phi_2(0)/\phi_1(0) = 10^{-5}$, solid line is $\delta\phi_c$, dashed is $\delta\phi_e = \phi_2 - \phi_1$
- Note that $\delta\phi_c$ first grows exponentially and then stops growing when damping of the inflaton field ceases

$$g^2 \phi_1(0)^2 = 1$$





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- Note that $\delta\phi_c$ first grows exponentially and then grows more slowly when damping of the inflaton field is slow

Comments about the results

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- Growth of $\delta\phi_c$ seems tied to particle production rate
- Early exponential growth of $\delta\phi_c$ implies quantum fluctuations are growing rapidly well before backreaction effects are large
- Thus the semiclassical approximation breaks down during the time of parametric amplification

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- After a lot of particle production, lattice simulations involving random initial conditions can be used to compute the backreaction - Khlebnikov and Tkachev, Propec and Roos, Felder and Tachev
- Qualitative agreement between our semiclassical results and lattice simulations of Propec and Roos for $g^2\phi(0)^2 = 1$

Global de Sitter space

- Metric with closed spatial sections is

$$ds^2 = H^{-2}[-du^2 + \cosh^2 u(d\chi^2 + \sin^2 \chi \, d\Omega^2)]$$

$u = Ht$

- Covers the entire manifold
- Has a contracting phase followed by an expanding one

de Sitter space is an exact solution

- to the vacuum Einstein equations with a cosmological constant

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- For the Bunch-Davies state

$$\langle T_{ab}\rangle = g_{ab}C$$

$$\Lambda_{\rm eff} = \Lambda - C$$

Bunch-Davies state as an attractor state

- Shown for the expanding part of de Sitter space for free scalar fields with $m^2 + \xi R > 0$ in homogeneous and isotropic states

$$\langle \rho \rangle \to \langle \rho \rangle_{\mathrm{BD}}$$

Anderson, Eaker, Habib, Molina-París, Mottola (2000)

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- What happens for the contracting phase of global de Sitter space?

Other homogeneous and isotropic vacuum states

- Mode functions for massive conformally coupled scalar field $u_{\mathbf{k}} = y_k(u) Y_{k\ell m_\ell}$

$$\left[\frac{d^2}{du^2} + 3\tanh u \frac{d}{du} + (k^2 - 1)\mathrm{sech}^2 u + \frac{m^2}{H^2} + 2 \right] y_k = 0$$

- If we denote the BD solution by v_k then the general solution is

$$y_k = A_k v_k + B_k v_k^*$$

with normalization

$$|A_k|^2 - |B_k|^2 = 1$$

- Energy density for an arbitrary homogeneous and isotropic vacuum state is

$$\langle \rho \rangle = \langle \rho \rangle_{\mathrm{BD}} + \frac{1}{2\pi^2} \sum_{k=1}^{\infty} k^2 \left[\mathrm{Re}(A_k B_k^* \varepsilon_1(u)) + |B_k|^2 \varepsilon_2(u) \right]$$

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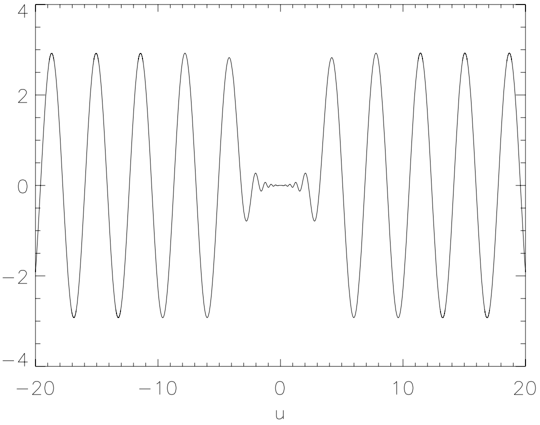
- $A_k B_k^*$ term oscillates in time and has no classical analog.
- $|B_k|^2$ term has same form as classical matter in nonrelativistic limit and classical radiation in relativistic limit
- To investigate the deviation from $\langle \rho \rangle = \langle \rho \rangle_{\mathrm{BD}}$, plot for fixed values of k the coefficients of $A_k B_k^*$ and $|B_k|^2$

$a^3 \times$ Coefficient of $\mathrm{Re} A_k B_k^*$ for $m = H$, $k = 10$



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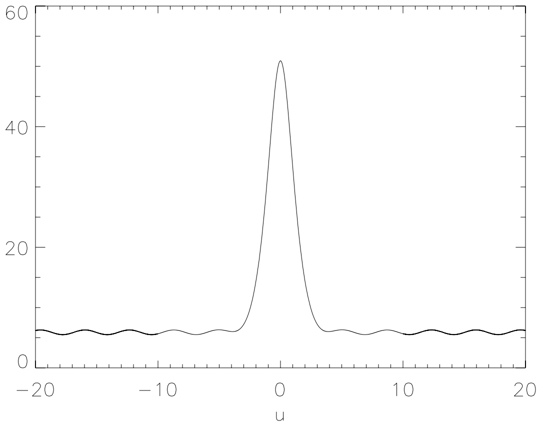

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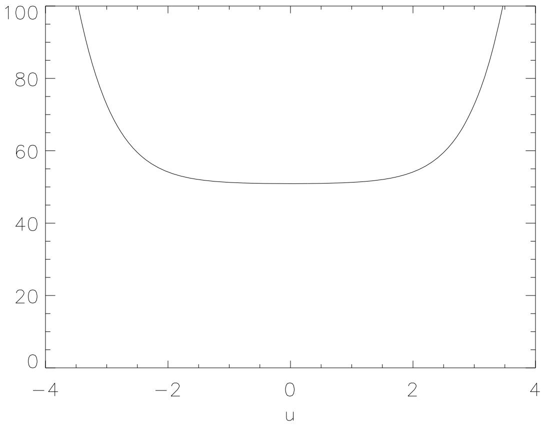

Conformally Invariant Fields

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- In contracting de Sitter for inhomogeneous states $\langle \rho \rangle \sim a^{-5}$

Backreaction for conformally invariant field

- Assume some other state than the BD state

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- Assume some other state than the BD state
- In the first approximation ignore the vacuum terms

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- Neglected vacuum effects will remain small unless the Planck scale is reached

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- Perturb by changing the state

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- Numerator grows larger in magnitude for all time
- Denominator grows smaller at early times
- Thus perturbations grow significantly at early times

Results and Conclusions for Global de Sitter space

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  - For all physically acceptable states the deviation of the energy density from the BD value is very small at early enough times

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  - For all physically acceptable states the deviation of the energy density from the BD value is very small at early enough times
  - The deviation grows exponentially with time
  - Thus the BD state is unstable to perturbations in the contracting part of de Sitter space

Details of the linear response equation for preheating

$$(\Box - m^2 - g^2\langle\psi^2\rangle)\delta\phi - g^2\delta\langle\psi^2\rangle\phi = 0$$

$$\delta\ddot{\phi} + (1 + g^2\langle\psi^2\rangle_1)\delta\phi + g^2(\delta\langle\psi^2\rangle\phi_1 = 0$$

$$\delta\langle\psi^2\rangle = \frac{1}{2\pi^2}\int_0^{\varepsilon} dkk^2\left(f_k\,\delta f_k^* + f_k^*\,\delta f_k\right) + \frac{1}{2\pi^2}\int_{\varepsilon}^{\infty} dkk^2\left(f_k\,\delta f_k^* + f_k^*\,\delta f_k + \frac{g^2\phi\,\delta\phi}{2k^3}\right)$$
$$-\frac{g^2\phi\,\delta\phi}{4\pi^2}\left[1 - \log\left(\frac{2\varepsilon}{M}\right)\right]$$

$$\delta\ddot{f}_k + (k^2 + g^2\phi^2)\delta f_k + 2g^2 f_k\phi\,\delta\phi = 0$$

$$\delta f_k = A_k f_k + B_k f_k^* + 2g^2 i\int_0^t dt'\ \phi(t')\delta\phi(t')f_k(t')\left[f_k^*(t)f_k(t') - f_k(t)f_k^*(t')\right]$$

f_k is a solution to the homogeneous equation for δf_k

A_k and B_k result in state dependent perturbations

Details relating to $\delta\phi_c$

- Choose starting values so that initially $\dot{\phi} = \delta\dot{\phi} = 0$
- Recall $\delta\phi_e = \phi_2 - \phi_1$
- Define $\delta\phi_e = c\phi_1 + \delta\phi_c$

$$c = (\phi_2(0) - \phi_1(0))/\phi_1(0)$$

$$\delta\phi_c(0) = \delta\dot{\phi}_c(0) = 0$$

- From last slide $\delta\langle\psi^2\rangle = \delta\langle\psi^2\rangle[\phi, \delta\phi, A_k, B_k]$
- If $\delta\phi_e$ is an approx. soln. to the linear response equation then $\delta\phi_c$ is an approx. soln. to

$$\delta\ddot{\phi}_c + (1 + g^2\langle\psi^2\rangle_1)\delta\phi_c + g^2\phi_1\delta\langle\psi^2\rangle[\phi = \phi_1, \delta\phi = \delta\phi_c, A_k = 0$$
$$= -g^2\phi_1\delta\langle\psi^2\rangle[\phi = \phi_1, \delta\phi = c\phi_1, A_k, B_k]$$

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- Since $\delta\phi_c = \delta\dot{\phi}_c = 0$ initially, only the $\delta\ddot{\phi}_c$ is nonzero initially on the LHS. Thus at early times $\delta\phi_c$ is driven by the RHS, i.e. by the early time values of $\delta\langle\psi^2\rangle$