Dynamical de Sitter phase and topological sectors in gauge theories (arxiv 1505.05151)

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1. Motivation. Structure of the talk.

- The main goal of this talk is to argue that QCD, being a gapped system, nevertheless exhibits the topological long range order, highly sensitive to a time-dependent and curved background.

- Furthermore, there is a novel type of energy (see definition below). This energy has "non-dispersive" nature, and can not be expressed in terms of conventional propagating degrees of freedom and Green's function (contrast with a scalar field).

- All these novel effects are due to the nontrivial topological sectors in the gauge system (no analogy with a scalar particle in a curved background).

- The effect is non-local, and can not be expressed in terms of local curvature. It is expressed in terms of a non-local characteristic, the holonomy.

2. Topological susceptibility

- A convenient way to explain the nature of new type of vacuum energy is to study the topologically susceptibility (*it is the key element in the resolution of the so-called U(1) problem in QCD, Witten, Veneziano, 1979*).

$$\chi_{YM} = \int d^4x \, \langle q(x), q(0) \rangle \neq 0 \qquad \frac{\partial^2 E_{\mathrm{vac}}(\theta)}{\partial \theta^2} = \chi_{YM}$$

To avoid confusion: This is the Wick's T-product, not Dyson's

- χ_{YM} does not vanish, though $q(x) \sim \partial_\mu K^\mu(x)$. It has ``wrong sign", see below. It can not be related to any physical propagating degrees of freedom. Furthermore, it has a pole in momentum space

$$\lim_{k \to 0} \int d^4x e^{ikx} \langle K_\mu(x), K_\nu(0) \rangle \sim \frac{k_\mu k_\nu}{k^4}$$

- There is a <u>massless</u> pole, but there are <u>no</u> any <u>physical massless</u> states in the system.

$$\chi_{dispersive} \sim \lim_{k\to 0} \sum_n \frac{\langle 0|q|n\rangle\langle n|q|0\rangle}{-k^2 - m_n^2} < 0,$$

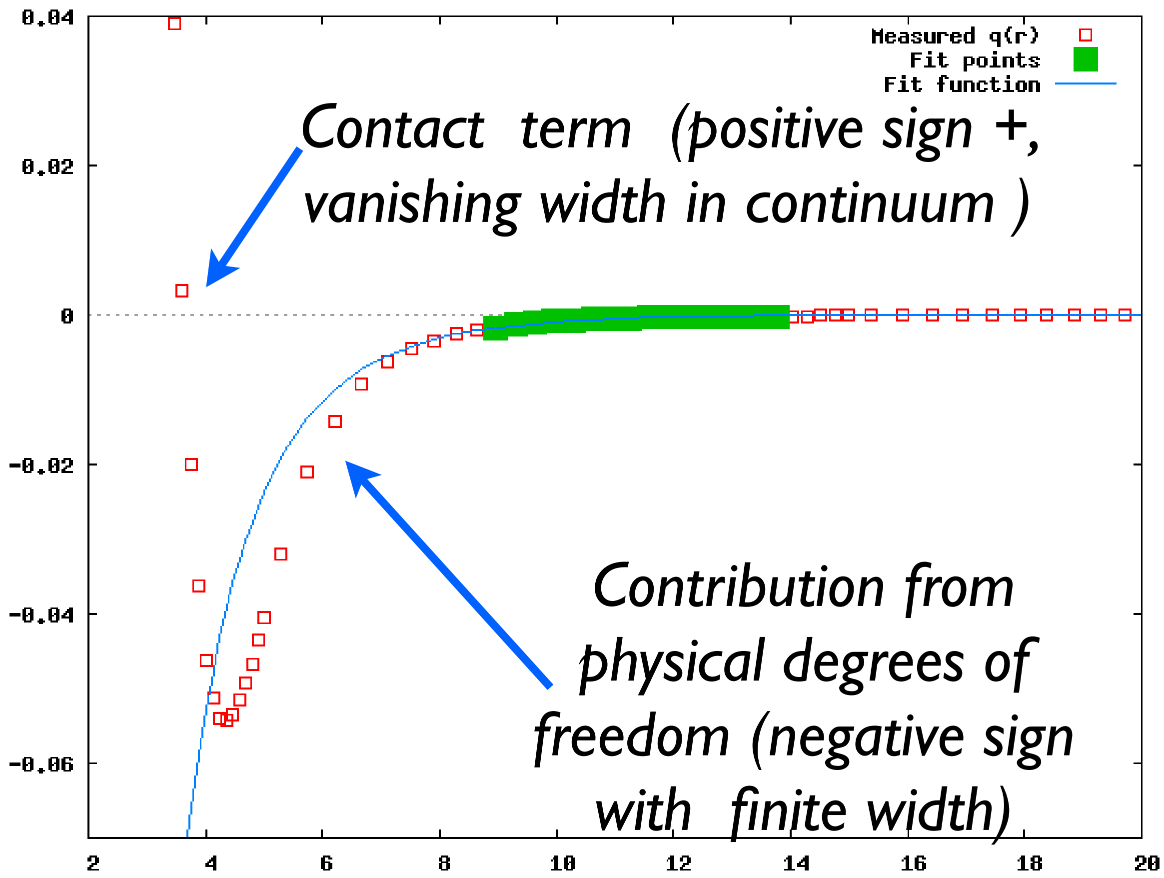
- Conventional physical degrees of freedom always contribute with sign (-) while one needs sign (+) to satisfy WI and resolve the U(1) problem

$$\chi_{non-dispersive} = \int d^4x \, \langle q(x), q(0)\rangle = \frac{1}{N^2} |E_{vac}| > 0$$

- Conventional terms (related to propagating degrees of freedom) always produce $\exp\left(-\Lambda_{QCD} L\right)$ behaviour at large distances.

- Witten simply postulated this term, while Veneziano assumed the unphysical field, the so-called the "Veneziano ghost" to saturate "wrong" sign in χ.

- In "deformed QCD" this contact non-dispersive term with "wrong" sign (+) can be explicitly computed. It is originated from the tunnelings between the degenerate topological sectors of the theory.



The topological susceptibility $\chi(r)$ as a function of r. Wrong sign for χ is well established phenomenon; it has been tested on the lattice (plot above is from C. Bernard et al, LATTICE 2007). This $\chi(r=0)$ contribution is not related to any physical degrees of freedom, and can be interpreted as a contact term.

3. Some important features of "non-dispersive" contributions to the energy

- These contributions can not be described in terms of conventional degrees of freedom (wrong sign);

- They are inherently non-local in nature as they are related to the tunnelling processes which are formulated in terms of the non-local large gauge transformation operator and holonomy;

- These terms may exhibit the long range features even through QCD has a gap (similar to the CM topologically ordered systems);

- The θ -dependent portion of energy $E_{\rm vac}(\theta)$ (which is generated due to the tunnelling transitions) has all these unusual features as $\frac{\partial^2 E_{\rm vac}(\theta)}{\partial\theta^2} = \chi_{YM}$

4. Applications to Cosmology

- We assume (see next few slides) that the non-dispersive θ- dependent portion of the vacuum energy $E_{\rm vac}(\theta)$ shows the linear correction with respect to Hubble "H" in the background, i.e.

$$E_{\rm FLRW} = c_0\Lambda^4_{\rm QCD} + c_1 H\Lambda^3_{\rm QCD} + \mathcal{O}(H^2\Lambda^2_{\rm QCD}) + ...$$

- We also assume that the relevant (gravitating) energy which enters the Friedman's equation is the difference $\Delta E = (E_{\rm FLRW}(H) - E_{\rm Mink})$ similar to computations of the Casimir energy, when the difference ΔE is observed. This assumption was, in fact, originally formulated by *Zeldovich in 1967*.

- With these assumptions the Friedman's equation exhibits a solution with the de-Sitter behaviour

$$H^2 = \frac{8\pi G}{3}\left(c_1 H\Lambda^3_{\rm QCD} + \rho_{\rm DM}\right) \longrightarrow H_0 \sim \frac{\Lambda^3_{\rm QCD}}{M^2_{\rm PL}}, \quad a(t) \sim \exp(H_0 t)$$

- There are few people (sitting in this room) who advocated the idea that the relevant energy entering the Einstein equation is

$$\Delta E = (E_{\rm FLRW}(H) - E_{\rm Mink})$$

- *Ralf Schuetzhold, PRL, 2002;*

  Michele Maggiore, PRD 2011

- People (not sitting in this room) from different fields who also advocated similar idea:

  James Bjorken, 2001,

  Grisha Volovik, 2008 +many more

- *I personally adopted this idea in 2009, few papers in Nucl.phys. B, Phys Lett.B*

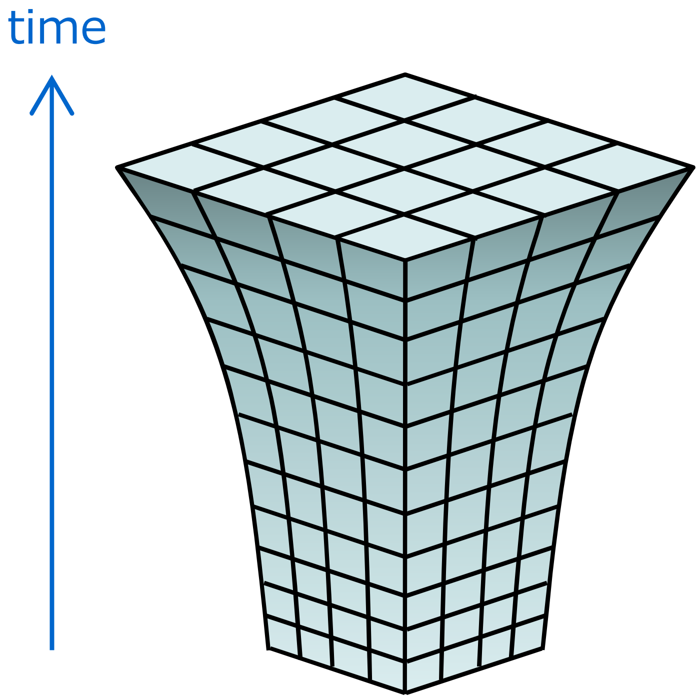
- What is the evidence for the linear dependence on cosmological scale "H" in a gapped system? (locality suggests quadratic behaviour as $R \sim H^2$)

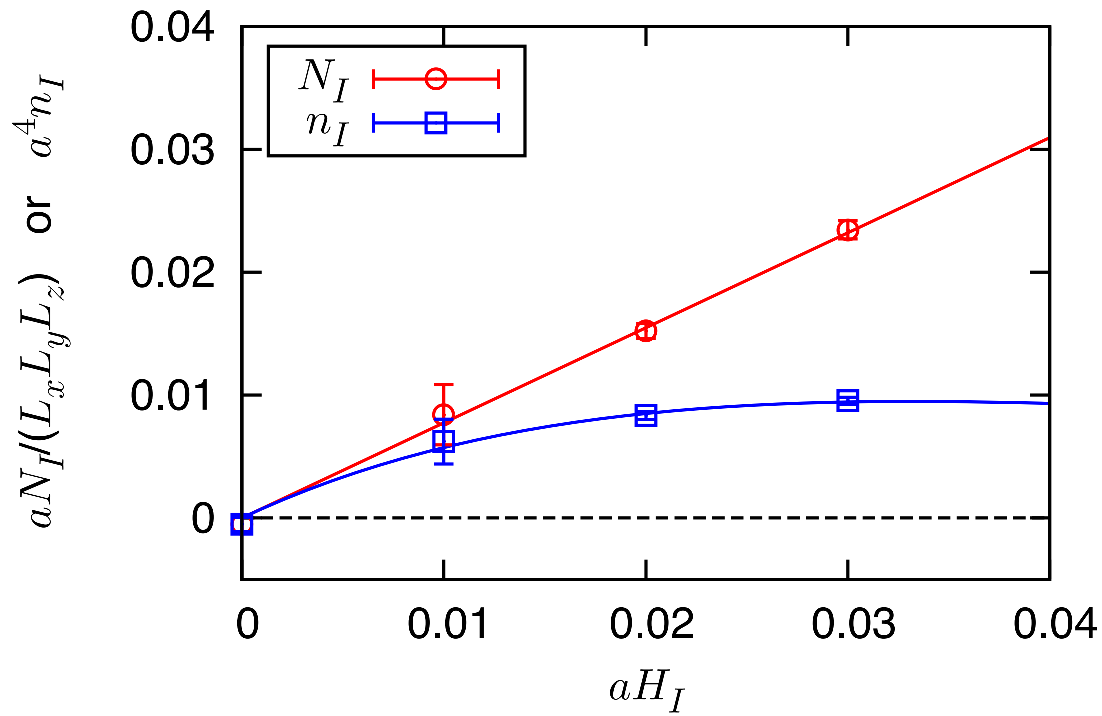
- 1. A number of analytical computations in simplified models, see few next slides below.

- 2a. Lattice numerical simulations. In this case the computations of a real part of the energy - momentum tensor $Re\langle T_{\mu\nu}\rangle$ is a hard problem.

- 2b. However, the imaginary (absorptive) portion of the energy-momentum tensor $Im\langle T_{\mu\nu}\rangle$ due to particle production, can be computed, see plot below.

- 2c. Analyticity suggests that the dependence on H must be the same in $Re\langle T_{\mu\nu}\rangle$ and $Im\langle T_{\mu\nu}\rangle$





THE PLOTS FROM A. YAMAMOTO, ARXIV 1405.6665.

1. THE EXPANSION IN EUCLIDEAN SPACE-TIME WAS PARAMETRIZED BY THE “IMAGINARY” HUBBLE CONSTANT WHEN THE LATTICE ACTION IS POSITIVELY DEFINED;
2. RED CURVE DESCRIBES THE PARTICLE PRODUCTION RATE PER UNIT VOLUME PER UNIT TIME IN THE BACKGROUND H_I ;
3. THE LINEAR DEPENDENCE ON H_I HAS BEEN COMPUTED, $Im[\langle T_{\mu\nu}\rangle] \sim H_I$. IT STRONGLY SUPPORTS OUR ARGUMENTS.

- This "H"-dependent energy has the same "non-dispersive" nature, which can not be expressed in terms of any local propagating degrees of freedom (any local effective matter fields like "inflaton").

- It should not be interpreted as $\cancel{H \sim \sqrt{R}}$ as it is formulated in terms of a different characteristic, the holonomy (not expressible as local curvature)

- The de Sitter behaviour is the quantum effect describing the dynamics of the topological sectors of strongly coupled QCD in expanding background.

- With these assumptions the non-dispersive contribution to energy (at large $a(t) \sim \exp(Ht) \to \infty$) is

$$H \sim \frac{\Lambda^3_{\rm QCD}}{M^2_{\rm PL}} \sim 10^{-33}{\rm eV}, \qquad \rho_{\rm DE} \sim H\Lambda^3_{\rm QCD} \sim (10^{-3}{\rm eV})^4$$

- It is amazingly close to the observed values

5. Holonomy and the linear $\sim \kappa$ correction in $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$ hyperbolic space

- Normally it is expected that all corrections due to the time-dependent (curved) background are proportional to the local curvature $R\left[\mathbb{H}^3_\kappa\right] \sim \kappa^2$

- We want to test these ideas in gauge theories with nontrivial holonomy. In this case corrections are not reduced to the local observables. Key role of the IR regularization in all computations.

- Specifically, we compute the ratio which explicitly shows the linear correction $\sim \kappa$

$$\frac{E_{\rm vac}[\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}]}{E_{\rm vac}[\mathbb{R}^3 \times \mathbb{S}^1]} \simeq \left(1 - \frac{\nu\bar{\nu}}{2} \cdot \frac{\kappa}{\Lambda_{QCD}}\right). \quad E_{\rm vac} \equiv -\frac{1}{\beta V} \ln \mathcal{Z}$$

- The computations are based on KvBLL calorons with nontrivial holonomy (Kraan-van Baal-Lee-Lu)

$$\frac{1}{2}TrL = \frac{1}{2}Tr\mathcal{P}\exp\left(i\int_0^\beta dx_4 A_4(x_4, |\mathbf{x}| \to \infty)\right) = \cos(\pi\nu)$$

- Normally, nontrivial holonomy ($\nu \neq 0,1$) generates zero contribution to the partition function in thermodynamical limit. However, the KvBLL configurations are known to generate IR - finite contribution to the free energy (in huge contrast with instantons).

- The KvBLL configurations can be thought as a superposition of "N" different monopoles which carry the fractional topological charge $Q = \pm 1/N$

- Confinement can be understood as percolation of these fractionally charged monopoles which enter the partition function in sets of "N" .

- The crucial role in generating this result is zero mode determinant. They are drastically different in hyperbolic in Euclidean spaces.

- This difference can be observed from asymptotic behaviour in these two cases

- $$A_4^M(r) = \left(v \coth(vr) - \frac{1}{r} \right) \frac{\tau^3}{2} \qquad \text{on} \qquad \mathbb{R}^3$$

$$A_\chi^M(\rho) = \Big((\nu+1) \coth\left[(\nu+1)\kappa\rho \right] - \coth \kappa\rho \Big) \frac{\kappa\tau^3}{2} \quad \text{on} \quad \mathbb{H}^3_\kappa$$

- Eventually, this difference translates into the difference in fugacities (and <u>vacuum energies</u>) as claimed above

$$f^2 = \left[\frac{4\pi\beta\Lambda_{QCD}^4}{g^4} \right]^2 \cdot \langle \frac{[1+2\pi\nu\bar{\nu}\frac{r_{12}}{\beta}]}{(\Lambda_{QCD}\ r_{12})^{2/3}} [1+2\pi\nu\frac{r_{12}}{\beta}]^{\frac{8}{3}\nu-1} [1+2\pi\bar{\nu}\frac{r_{12}}{\beta}]^{\frac{8}{3}\bar{\nu}-1} \rangle$$

- Q: *How a system with a gap could be ever sensitive to arbitrary large distances?*

- A1: The long range order in gapped QCD is similar to Aharonov -Casher effect. If one inserts an external charge into superconductor when electric field is screened $\exp(-r/\lambda)$ a neutral magnetic fluxon will be still sensitive to external charge at arbitrary large distances.

- A2: Long range order in the system emerges because the large gauge transformation operator $\mathcal{T}$ and holonomy are non-local operators sensitive to far IR-physics, similar to "modular operator" in Aharonov -Casher effect.

6. Applications to inflation

- We assume a scaled up version of QCD with the scale $M_{PL} \gg \Lambda_{QCD} \gg \sqrt[3]{M_{EW}^2 M_{PL}} \sim 10^8 \text{ GeV}$ to avoid interference with EW physics.

- The Friedman equation has a de Sitter solution after the phase transition to the confined phase when the topological susceptibility is generated

$$H^2 = \frac{8\pi G}{3}\left(\rho_{\text{Inf}} + \rho_R\right) = \frac{8\pi G}{3}\left(\overline{\alpha} H \Lambda_{QCD}^3 + \rho_R\right), \quad H_0 \sim \frac{8\pi G}{3}(\overline{\alpha}\Lambda_{QCD}^3)$$

- This non-dispersive type of energy (the contact term) is linear in "H" and drives the Universe into the de Sitter phase

- The relevant dynamics is governed by some non-propagating auxiliary topological fields without canonical kinetic term; it can not be expressed in terms of any local fields like "inflaton"

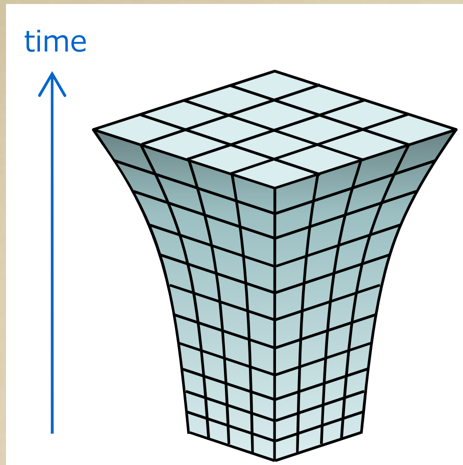
- This regime would be the final destination of our Universe if the interaction with SM fields is switched off.

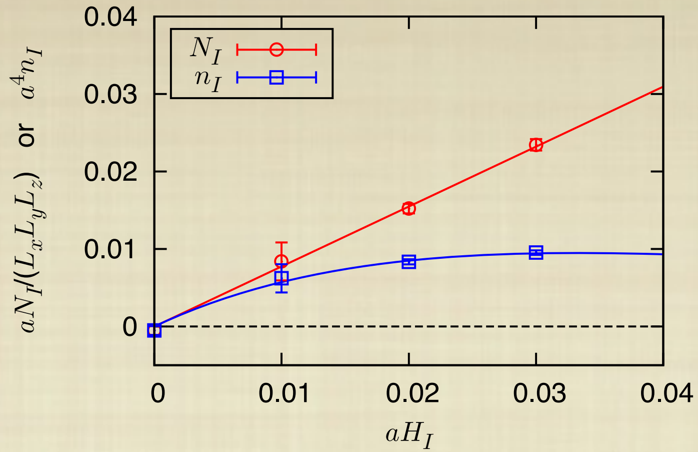
- When the coupling is switched back on, the end of inflation is triggered precisely by this interaction which itself is unambiguously fixed by triangle anomaly.

$$\mathcal{L}_{b\gamma\gamma} = \frac{\alpha(H_0)}{8\pi} NQ^2 \left[\theta - b(x)\right] \cdot F_{\mu\nu}\tilde{F}^{\mu\nu} ,$$

- where $b(x)$ is topological non-propagating field and $\alpha(H_0) \sim \alpha_{EW}(H_0) \sim \alpha_s(H_0) \sim 0.1$ is the coupling at inflationary scale

- This interaction with $\dot{b}(x) \sim \mu_5$ is known to lead to the helical instability when the vacuum releases its energy by producing the real particles





- IN QCD CONTEXT ($\mu_5 \sim H_0$) THE TYPICAL TIME SCALE FOR DEVELOPMENT OF THE HELICAL PLASMA INSTABILITY IS

$$\tau_{\text{instability}} \sim \frac{1}{\alpha_s^2 \mu_5} \quad \rightarrow \quad \tau_{\text{inflation}} \sim \frac{1}{\alpha_s^2(H_0) H_0} \quad \rightarrow \quad N_{\text{e-folds}} \sim \alpha_s^2(H_0) \sim 10^2$$

- EXACT COEFFICIENT FOR PARTICLE PRODUCTION RATE CAN BE COMPUTED ON THE LATTICE IN A TIME DEPENDENT BACKGROUND WHEN THE PRODUCTION RATE IS INDEED LINEARLY PROPORTIONAL TO “H”.

- THE NUMBER OF E-FOLDS $N_{\text{e-folds}} \sim \alpha_s^2(H_0) \sim 10^2$ IS RELATED TO THE GAUGE COUPLING CONSTANT (WE KNOW AND LOVE) AND NOT TO SOME AD HOC INFLATON POTENTIAL

Conclusion

- We speculate that a liner in "H" correction to the energy could be generated as a result of dynamics of topological configurations with nontrivial holonomy. The idea is tested in "deformed QCD" and in the system defined on hyperbolic space $\mathbb{H}^3_\kappa \times \mathbb{S}^1_{\kappa^{-1}}$

- History repeats itself as the QCD-like dynamics may be responsible for inflation in early times, while the QCD dynamics may be responsible for the dark energy now: $H \sim \frac{\Lambda^3_{\rm QCD}}{M^2_{\rm PL}} \sim 10^{-33}{\rm eV}, \qquad \rho_{\rm DE} \sim H\Lambda^3_{\rm QCD} \sim (10^{-3}{\rm eV})^4$

- History repeats itself also with technical tools: Aharonov -Bohm effect is an explicit realization of the system (including systems with a gap) when the gauge potential A_μ (rather than $F_{\mu\nu}$) is the physical observable. Effect is non-local in nature, and can not be expressed in terms of $F_{\mu\nu}$

Proposal: Instead of theoretical speculations I suggest to conduct a real tabletop experiment to study this new type of energy:

- When the Maxwell system is formulated on four-torus there will be an extra contribution to the Casimir pressure, not related to the physical propagating photons with two transverse polarizations (4-torus has nontrivial holonomy).

- This setting based on 4-torus topology should be contrasted with conventional setting when the Casimir energy is generated between two conducting plates (trivial holonomy).

- The Maxwell system on the 4-torus shows all signs (degeneracy, etc) which are normally attributed to the topologically ordered systems *(AZ, 2014, 2015)*.