Late-time quantum backreaction in cosmology

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in preparation

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Outline

- Physical problem & motivation
- Theoretical setting – what is quantum backreaction?
- Model & definition of quantities to calculate
- Perturbative computation
  - Calculation scheme
  - Approximations
  - Results
- Self-consistent computation
  - Stochastic approximation
  - Results
- Conclusions & outlook

Physical problem – Dark Energy

- Universe today expanding in an accelerating fashion – unknown physical origin
- Could be a cosmological constant (CC) (but we think we can calculate it?)
- New matter content? (70% of total energy density)
- Modifications of General Relativity on cosmological scales
- Backreaction from non-linear structures
- Other effects – quantum backreaction

Quantum fluctuations

- All matter is quantum and exhibits quantum fluctuations
- Quantum fluctuations carry energy
- All energy is the source for Einstein's equation
- Semiclassical gravity:

$$G_{\mu\nu} = 8\pi G_N \Big\{ T^{cl}_{\mu\nu} + \langle \hat{T}_{\mu\nu} \rangle \Big\} \tag{1}$$

- Quantum correction to the equations of motion descending from the effective action

Perturbative vs self-consistent computations

- Are there any interesting phenomena in cosmology arising from quantum corrections? – solve the full equation self consistently
- Dark Energy: looking for an effect that becomes important only at late times – it must be small during most of the history of the expansion
- Solve simpler problem first – backreaction on a fixed FLRW background

$$\left[G_{\mu\nu} = 8\pi G_N \left\{ T^{cl}_{\mu\nu} \right] + \left[\langle \hat{T}_{\mu\nu} \rangle \right\} \right] \tag{2}$$

- Backreaction initially small $\Rightarrow$ there is a regime where it can safely be treated perturbatively
- Perturbative computation will determine if there is any effect, in what regimes, and for which ranges of parameters
- After establishing that backreaction grows to be large – tackle the self-consistent problem – start numerical evolution at late matter era

FLRW space-time

- Line element ($c = \hbar = 1$):

$$ds^2 = -dt^2 + a^2(t)d\boldsymbol{x}^2 = a^2(\eta)\Big[-d\eta^2 + d\boldsymbol{x}^2\Big] \ , \quad dt = a d\eta \tag{3}$$

- Hubble rate $H = \frac{\dot{a}}{a}$ and conformal Hubble rate $\mathcal{H} = \frac{a'}{a} = aH$
- Friedmann equations:

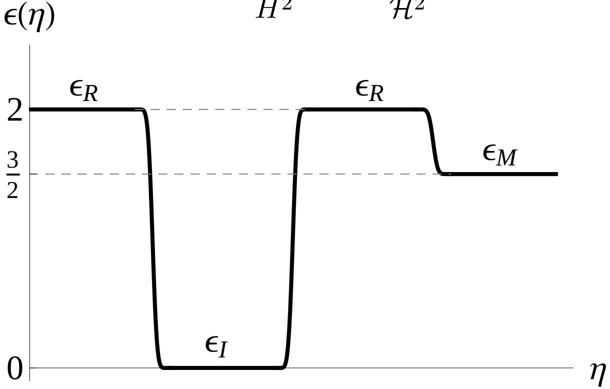
$$\left(\frac{\mathcal{H}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i \ , \qquad \frac{\mathcal{H}' - \mathcal{H}^2}{a^2} = -4\pi G \sum_i (\rho_i + p_i) \tag{4}$$

- Ideal fluids: $p_i = w_i \rho_i$, $\rho_i \sim a^{-3(1+w_i)}$
- Dominance of one fluid $\Rightarrow$ constant ϵ parameter

$$\epsilon = -\frac{\dot{H}}{H^2} = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} = \frac{3}{2}(1+w) \tag{5}$$

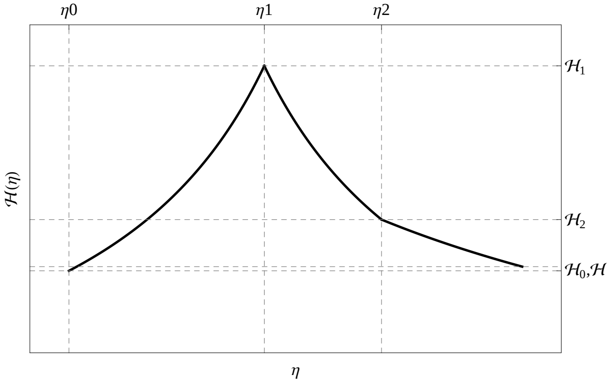
Evolution of the Universe: history

$$\epsilon = -\frac{\dot{H}}{H^2} = 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} \tag{6}$$



Transition between periods fast $\tau \ll \mathcal{H}$.

Evolution of the Universe: hierarchy of scales



Hierarchy of scales $\mathcal{H}_0, \mathcal{H} \ll \mathcal{H}_2 \ll \mathcal{H}_1$
For minimal inflation $\mathcal{H}_0 \sim \mathcal{H}$, but $\mathcal{H}_0 \ll \mathcal{H}$ not disallowed

Non-minimally coupled massive scalar: quantization

- Action

$$S = \int d^D x \, \mathcal{L}(x) = \int d^D x \sqrt{-g} \left\{ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{m^2}{2} \phi^2 - \frac{1}{2} \xi R \phi^2 \right\} \tag{7}$$

- Canonically conjugate momentum

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \phi'(x)} = a^{D-2} \phi'(x) \tag{8}$$

- Hamiltonian

$$H(\eta) = \int d^{D-1} x \, \frac{a^D}{2} \left\{ a^{2-2D} \pi^2 + a^{-2} (\nabla \phi)^2 + m^2 \phi^2 + \xi R \phi^2 \right\} \tag{9}$$

- Canonical commutation relations

$$\begin{aligned} \left[\hat{\phi}(\eta, \boldsymbol{x}), \hat{\pi}(\eta, \boldsymbol{y}) \right] &= i \delta^{D-1}(\boldsymbol{x} - \boldsymbol{y}) \ , \\ \left[\hat{\phi}(\eta, \boldsymbol{x}), \hat{\phi}(\eta, \boldsymbol{y}) \right] &= 0 = \left[\hat{\pi}(\eta, \boldsymbol{x}), \hat{\pi}(\eta, \boldsymbol{y}) \right] \end{aligned} \tag{10}$$

Non-minimally coupled massive scalar: quantization

- Heisenberg equations of motion

$$\hat{\phi}'' + (D-2)\mathcal{H}'\hat{\phi}' - \nabla^2\hat{\phi} + m^2\hat{\phi} + \xi(D-1)\Big[2\mathcal{H}' + (D-2)\mathcal{H}^2\Big]\hat{\phi} = 0 \quad (11)$$

- Expand in Fourier modes

$$\hat{\phi}(\eta, \boldsymbol{x}) = a^{\frac{2-D}{2}} \int \frac{d^{D-1}k}{(2\pi)^{\frac{D-1}{2}}} \Big\{ e^{i\boldsymbol{k}\cdot\boldsymbol{x}} U(k,\eta)\hat{b}(\boldsymbol{k}) + e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} U^*(k,\eta)\hat{b}^\dagger(\boldsymbol{k}) \Big\} \quad (12)$$

- Commutation relations

$$\begin{aligned} \Big[\hat{b}(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{q})\Big] &= \delta^{D-1}(\boldsymbol{k}-\boldsymbol{q}) \\ \Big[\hat{b}(\boldsymbol{k}), \hat{b}(\boldsymbol{q})\Big] &= 0 = \Big[\hat{b}^\dagger(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{q})\Big] \end{aligned} \quad (13)$$

Non-minimally coupled scalar: mode function

- Wronskian normalization for mode function

$$U(k,\eta)U'^{*}(k,\eta) - U'(k,\eta)U^{*}(k,\eta) = i \qquad (14)$$

Equation of motion for modes – HO with time-dependent frequency

$$\boxed{U''(k,\eta) + \Big[k^2 + \mathcal{M}^2(\eta)\Big]U(k,\eta) = 0} \qquad (15)$$

$$\mathcal{M}^2(\eta) = m^2a^2 - \frac{1}{4}\Big[D-2-4\xi(D-1)\Big]\Big[2\mathcal{H}'+\mathcal{H}^2\Big] \qquad (16)$$

- Construction of Fock space: $\hat{b}(\boldsymbol{k})|\Omega\rangle = 0$, and creation operators generate the rest; mode function determines the properties of $|\Omega\rangle$
- State with no condensate: $\boxed{\langle\Omega|\hat{\phi}|\Omega\rangle = 0}$
- Choice of $U(k,\eta)$ not unique! Basic requirements: IR finiteness and reduces to positive-frequency Bunch-Davies (adiabatic) in the UV

$$U(k,\eta) \xrightarrow{k\to\infty} \frac{e^{-ik\eta}}{\sqrt{2k}}\Big[1+\mathcal{O}(k^{-1})\Big] \qquad (17)$$

Non-minimally coupled massive scalar: Energy-momentum tensor

- Energy-momentum tensor operator

$$\hat{T}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}\bigg|_{\hat{\phi}} = \partial_\mu \hat{\phi}\, \partial_\nu \hat{\phi} - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \hat{\phi}\, \partial_\beta \hat{\phi} + g_{\mu\nu} m^2 \hat{\phi}^2 + \xi \Big(G_{\mu\nu} - \nabla_\mu \nabla_\nu + g_{\mu\nu} \Box \Big) \hat{\phi}^2 \tag{18}$$

- Expectation value in state $|\Omega\rangle$ diagonal

$$\rho_Q = \frac{a^{-D}}{(4\pi)^{\frac{D-1}{2}} \Gamma(\frac{D-1}{2})} \int_0^\infty dk\, k^{D-2} \bigg[2k^2 |U|^2 - \frac{1}{2} \Big[D-2 - 4\xi(D-1) \Big] \mathcal{H}' |U|^2 + 2m^2 a^2 |U|^2 - \frac{1}{2} \Big[D-2 - 4\xi(D-1) \Big] \mathcal{H}^2 \frac{\partial}{\partial \eta} |U|^2 + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} |U|^2 \bigg] \tag{19}$$

Non-minimally coupled massive scalar: Energy-momentum tensor

$$p_Q = \frac{a^{2-D}}{(4\pi)^{\frac{D-1}{2}}\Gamma(\frac{D-1}{2})}\int_0^\infty dk\, k^{D-2}\Bigg[\frac{2k^2}{D-1}|U|^2 - \frac{1}{2}\Big[D-2-4\xi(D-1)\Big]\mathcal{H}'|U|^2$$

$$-\frac{1}{2}\Big[D-2-4\xi(D-1)\Big]\mathcal{H}^2\frac{\partial}{\partial\eta}|U|^2 + \frac{1}{2}(1-4\xi)\frac{\partial^2}{\partial\eta^2}|U|^2\Bigg] \qquad (20)$$

- Goal of perturbative calculation: find if and when $\rho_Q/\rho_B \sim 1$
- ρ_Q and p_Q exhibit standard quartic, quadratic and logarithmic divergences. Dimensional regularization automatically subtracts power-law divergences and logarithmic one has to be absorbed into CC and mass counterterms, and higher-derivative counterterms $(R^2, (R_{\mu\nu})^2, (R_{\mu\nu\alpha\beta})^2)$

Perturbative computation

- Compute the backreaction while it is small – neglect its influence on the background dynamics.
- Background is FLRW consisting of inflationary, radiation, and matter eras
- Check whether backreaction ever becomes important – when and for which parameters? ($\rho_Q/\rho_B \sim 1$?)
- Check the tendency of the backreaction – to accelerate or decelerate the expansion ($\rho_Q > 1$?, w_Q?)
- Interesting range of parameters:
  - $\xi < 0 \quad \Rightarrow$ IR instability for modes (inflation and matter period) ,
  - $|\xi| \ll 1 \quad \Rightarrow$ backreaction still perturbative during inflation
  - $(m/H) = (ma/\mathcal{H}) \ll 1 \quad \Rightarrow$ otherwise 'particle production' stops and backreaction behaves as a non-relativistic matter fluid

Perturbative computation: choice of initial state

- Natural choice for initial state in inflation: adiabatic vacuum - analytic extension of positive-frequency UV expansion

$$U(k,\eta) = \sqrt{\frac{\pi}{4\mathcal{H}}}\, H_\nu^{(1)}\Big(\frac{k}{\mathcal{H}}\Big) \tag{21}$$

$$\nu = \sqrt{\frac{1}{4} + 2(1-6\xi) - \frac{m^2}{H_I^2}} > \frac{3}{2} \tag{22}$$

- Problem: IR divergent state! $|U|^2 \sim k^{-2\nu}$
- IR regulator: comoving IR cutoff k_0. To be identified with the Hubble rate at the beginning of inflation $\mathcal{H}_0$ (comparison with explicit matching to pre-inflationary radiation era DG, Prokopec, van der Woude, PRD 89 (2014) 024024).

Perturbative computation: evolution of the mode function

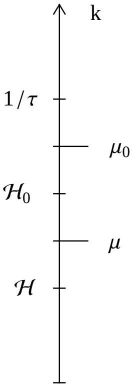
- $$U'' + \Big[k^2 + \mathcal{M}^2\Big]U = 0 \tag{23}$$
- UV modes evolve adiabatically, IR highly non-adiabatically.
- BD mode functions known for constant ϵ-periods (in massive case their expansion in small mass known)
- Represent the full mode function as expansion in terms of BD ones

$$U(k,\eta) = \alpha(k)u(k,\eta) + \beta(k)u^*(k,\eta) \tag{24}$$

- Bogolyubov coefficients suppressed in UV, $\alpha \to 1$ and $\beta \to 0$ faster than any power of k for $k \to \infty$
- For fast transitions well approximated by sudden transition approximation (limit $\tau = 0$) – calculated from continuity of U and U' at the point of transition

Perturbative computation: performing the integrals

- Integrals cannot be performed exactly
- Different approximations available on different intervals (in small $k/\mathcal{H}_i$ or $\mathcal{H}_i/k$)
- Identify intervals where dominant contribution comes from
- Example: consider one transition from accelerating to decelerating era at some $\mathcal{H}_0$
- Approximating the integrand in the end amounts to approximating in small ratios of physical scales (e.g. $\mathcal{H} \ll \mathcal{H}_0$)



Perturbative computation: Results

What (probably) does not work:

- massless, minimally coupled scalar

  DG, Prokopec, Prymidis, PRD **89** (2014) 024024
- massless, non-minimally coupled scalar ($\xi < 0$)

  DG, Prokopec, van der Woude, PRD **89** (2014) 024024

What works:

- Very light massive ($m \sim H$), minimally coupled scalar

  Ringeval, Suyama, Takahashi, Yamaguchi, Yokoyama, PRL **105** (2010) 121301

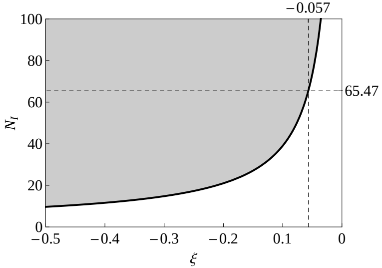
  $N_I \sim 10^{60}$ for $\hbar H_I \sim 10^4$GeV inflation

  Aoki, Iso, arXiv:1411.5129 [gr-qc]

  $N_I \sim 10^{12}$ for $\hbar H_I \sim 10^{13}$GeV inflation

Perturbative computation: Results

- Negative nonminimal coupling leads to growth of backreaction during inflation
- Constraint on $(\xi - N_I)$ parameter space: $\rho_Q/\rho_B < 1$ at the end of inflation



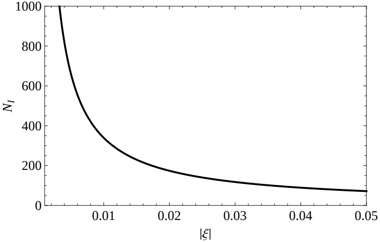
- During radiation period backreaction scales away as radiation

Perturbative computation: Results

- Backreaction in late-time matter era

$$\rho_Q = \frac{3H_I^2 H^2}{32\pi^2} \times \frac{e^{8|\xi|N_I}}{|\xi|} \times \left[-|\xi| \; + \; \frac{1}{6}\left(\frac{m}{H}\right)^2 \right] \tag{25}$$

- Inequality $6|\xi| < (m/H)^2$ for the CC term to dominate
- Backreaction becomes large in late-time matter era and has the behavior of a CC! ($\rho_Q/\rho_B \sim 1$)



Self-consistent computation

- Backreaction becomes large at late-time matter era – how exactly does it influence the dynamics of the expansion?
- Full self-consistent solution of the Friedmann equations needed – numerical problem
- Solve full integro-differential equations: mode function EOM, Friedmann equations with a sum over modes as a source

  Suen, Anderson, PRD 35 (1987) 2904
- Approximate: dominant contribution to ρ_Q comes from super-Hubble (deep IR) modes
- Gradients are negligible for super-Hubble modes and they are populated because of the (long enough) inflationary period – stochastic approximation

  Starobinsky, Lect. Notes Phys. **246** (1986) 107

Self-consistent: Stochastic formalism

- Split the fields in super-Hubble part + the rest

$$\hat{\Phi}(t,\boldsymbol{x}) = \hat{\phi}(t,\boldsymbol{x}) + \hat{\phi}_s(t,\boldsymbol{x}) \tag{26}$$

$$\hat{\phi}(t,\boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\,\theta(\mu a H - k)\Big[e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\varphi(k,t)\hat{b}(\boldsymbol{k}) + h.c.\Big] \tag{27}$$

$$\hat{\phi}_s(t,\boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}}\,\theta(k - \mu a H)\Big[e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\varphi(k,t)\hat{b}(\boldsymbol{k}) + h.c.\Big] \tag{28}$$

- $\mu \ll 1$.
- Analogously for conjugate momentum operator $\hat{\Pi}(t,\boldsymbol{x})$
- Field operators satisfy Langevin-type equations

Self-consistent: Stochastic formalism

- Easy to close the equations for correlators in linear theory (neglect gradients)

$$\mathcal{E}(t) = \langle \hat{\phi}^2(t, \boldsymbol{x}) \rangle \tag{29}$$

$$\mathcal{F}(t) = a^{-3} \langle \{ \hat{\phi}(t, \boldsymbol{x}) \hat{\pi}(t, \boldsymbol{x}) \} \rangle \tag{30}$$

$$\mathcal{G}(t) = a^{-6} \langle \hat{\pi}^2(t, \boldsymbol{x}) \rangle \, , \tag{31}$$

- Equations of motion $(M^2 = m^2 + 6\xi(2-\epsilon)H^2)$

$$\dot{\mathcal{E}} - \mathcal{F} = n_{\mathcal{E}} \tag{32}$$

$$\dot{\mathcal{F}} + 3H\mathcal{F} - 2\mathcal{G} + 2M^2\mathcal{E} = n_{\mathcal{F}} \tag{33}$$

$$\dot{\mathcal{G}} + 6H\mathcal{G} + M^2\mathcal{F} = n_{\mathcal{G}} \tag{34}$$

- Field-theoretic computations reproduced for backreaction in perturbative regime
- Noise sources mostly subdominant

Self-consistent: Results

- Energy density and pressure

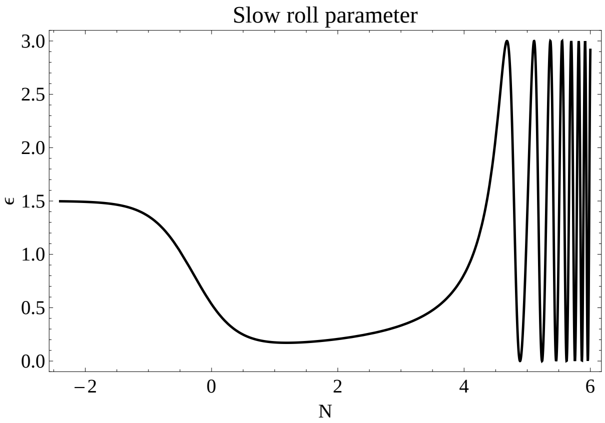
$$\rho_Q = \frac{1}{2}\mathcal{G} + 3\xi H\mathcal{F} + \frac{m^2}{2}\mathcal{E} + 3\xi H^2\mathcal{E} \tag{35}$$

$$\begin{aligned} p_Q = {} & \frac{(1-4\xi)}{2}\mathcal{G} + \xi H\mathcal{F} - \frac{m^2}{2}(1-4\xi)\mathcal{E} \\ & + 12\xi^2(2-\epsilon)H^2\mathcal{E} - \xi(3-2\epsilon)H^2\mathcal{E} \end{aligned} \tag{36}$$

- Include this as a source in Friedmann equations and solve numerically
- Impose initial condition at $z = 10$ corresponding to ones obtained by considering the Universe today with $\Omega_\Lambda \approx 0.68$, $\Omega_M \approx 0.32$

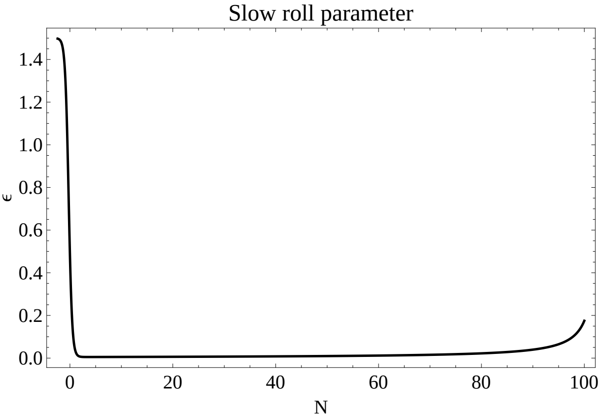
Self-consistent: Results

$$m/H = 0.5, \xi = 0$$



Self-consistent: Results

$m/H = 0.1, \xi = 0$



Conclusion & outlook

- Quantum backreaction can indeed grow large given enough time and influence the expansion dynamics
- Particular model exhibiting effects of late-time Universe acceleration – Dark Energy model
- Tiny mass parameter necessary – can it be generated dynamically?
- Clustering properties of backreaction $\langle\hat{\rho}(t,\boldsymbol{x})\hat{\rho}(t,\boldsymbol{x}')\rangle$?
- Different (interacting) models?
- (screening of the cosmological constant?)

Bonus frame 1: stochastic forces

$$n_{\mathcal{E}} = \frac{1}{2\pi^2}\mu^3 a^3 H^4 (1-\epsilon)|\varphi(k,t)|^2\Big|_{k=\mu a H} \tag{37}$$

$$n_{\mathcal{F}} = \frac{1}{2\pi^2}\mu^3 a^3 H^4 (1-\epsilon)\frac{\partial}{\partial t}|\varphi(k,t)|^2\Big|_{k=\mu a H} \tag{38}$$

$$n_{\mathcal{G}} = \frac{1}{2\pi^2}\mu^3 a^3 H^4 (1-\epsilon)|\dot{\varphi}(k,t)|^2\Big|_{k=\mu a H} \tag{39}$$

Bonus frame 2: correlators