Loop Quantum Cosmology and the CMB

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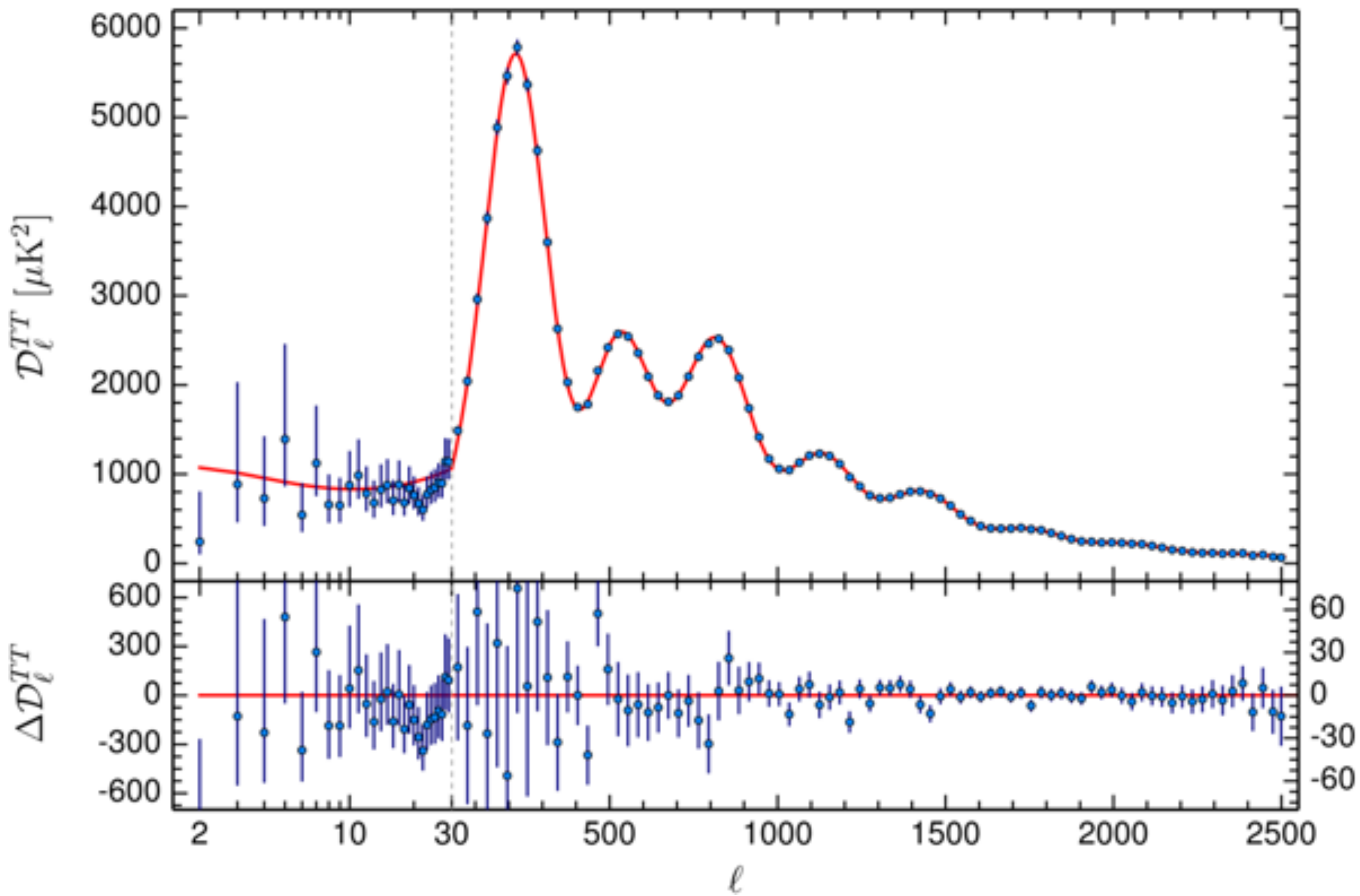
MITP, Mainz, Jun 24, 2015

Work in collaboration with: Ashtekar, Gupt, Morris, Nelson, Parker, Shandera

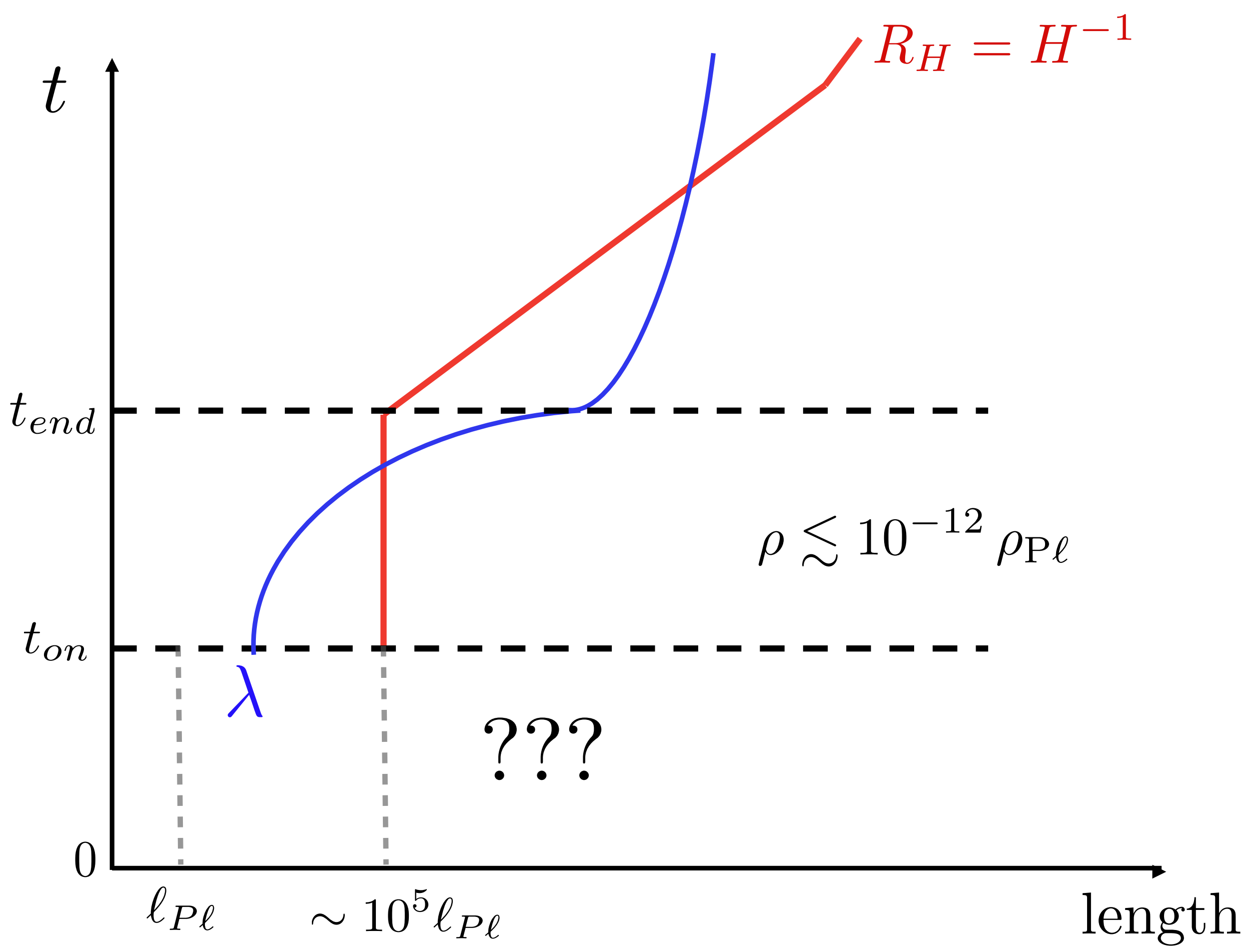
Introduction

Credits: Planck 2015



Λ CDM + simplest model of inflation work very well

Quantum gravity vs Inflation



"Safe" to describe perturbations as quantum fields propagating in a classical FRW

Quantum gravity:

- Conceptual completion of the our picture of the early universe
- Phenomenology: **initial conditions for inflation**

Inflation and the initial state of perturbations

Pre-inflationary physics: Initial state for perturbations at onset of slow-roll

For instance, a Bogoluibov transformation of the BD vacuum: $|\beta\rangle$

$$u_k(\eta) = \alpha_k \, u_k^{BD}(\eta) + \beta_k (u_k^{BD}(\eta))^\star$$

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Two relevant questions:

- Does the initial state $|\beta\rangle$ matters for the predictions?

I.A., Andersonm, Brandenberger, Danielson, Eaker, Easter, Martin, Molina-Paris, Gasperini, Greene, Kinney, Mottola, Parentani, Parker, Shiu, Veneciano, ...

QUANTUM MECHANICS

I.A. and L. Parker, 2010-11:

Stimulated particle creation (for non-conformally coupled bosons) **during inflation**

Red-shift, but no dilution $\propto a(t)^{-3}$

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- Can the effects from $|\beta\rangle$ be disentangled in observations?

If yes: Window to pre-inflationary universe!

Two-point function: Power Spectrum $P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|u_k|^2$

I.A., Andersonm, Brandenberger, Danielson, Eaker, Easter, Martin, Molina-Paris, Gasperini, Greene, Kinney, Mottola, Parentani, Parker, Shiu, Veneciano, ...

oscillatory term: typical from pre-inflationary physics
(difficult to observe, but not impossible)

$$P_{\mathcal{R}}(k) = \frac{1}{2\epsilon(t_k)\, M_P^2}\left(\frac{H(t_k)}{2\pi}\right)^2 \times (1 + 2\,|\beta_k|^2 + 2\,\mathrm{Re}[\alpha_k^\star \beta_k])$$

$$n_s = 6\epsilon(t_k) - 2\eta(t_k) + \frac{d\ln(1 + 2\,|\beta_k|^2 + 2\,\mathrm{Re}[\alpha_k^\star \beta_k])}{d\ln k}$$

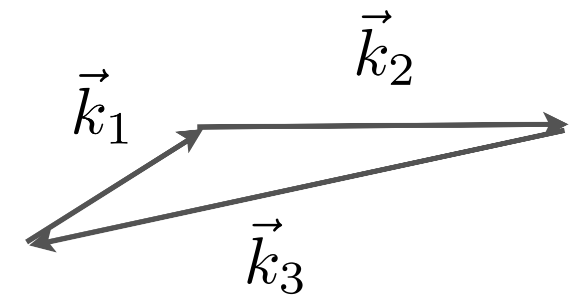
Observations may be sensitive to the k dependence of β_k (global amplitude difficult to constrain)

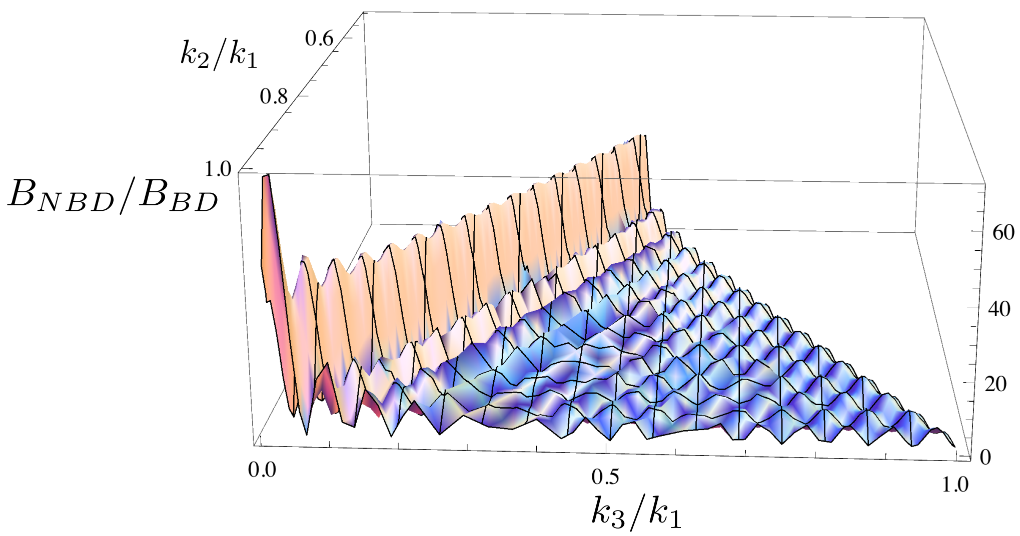
The tensor to scalar ratio r and the inflationary consistency relation $r = -8n_t$ may be modified

Three-point function: Bispectrum

Chen, Huang, Kachru, Shiu'07 , Holman, Tolley 08, I.A. Parker 10

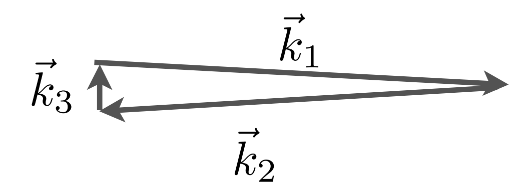
$$\langle \hat{\mathcal{R}}_{\vec{k}_1} \hat{\mathcal{R}}_{\vec{k}_2} \hat{\mathcal{R}}_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, B_{\mathcal{R}}(k_1, k_2, k_3)$$





Large amplification in squeezed and collinear configurations

I.A, Parker '10,



Planck 2015 does not find evidence for such a strong enhancement in observable modes

Exact expressions

$$B_{\rm GIS}(k_1,k_2,k_3) = P_\zeta(k_1)P_\zeta(k_2)\Big\{\frac{1}{2}\left(3\epsilon - 2\eta + \epsilon\,\frac{k_1^2+k_2^2}{k_3^2}\right) +$$

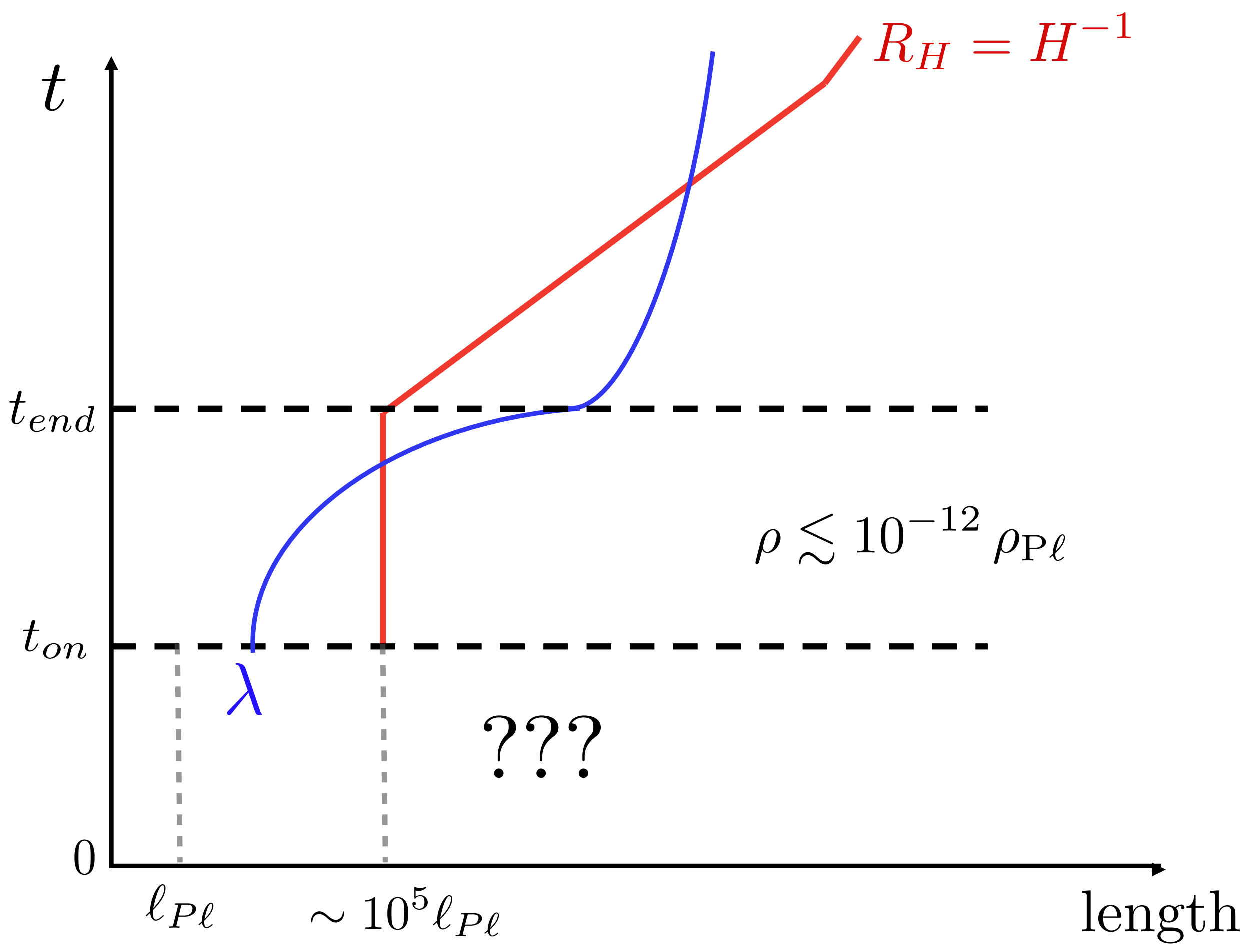
$$+\,4\epsilon\,\frac{k_1^2k_2^2}{k_3^3}\ {\rm Re}\left[f_t\frac{1-e^{ik_t/k_*}}{k_t} + f_2\frac{1-e^{i\tilde{k}_1/k_*}}{\tilde{k}_1} + f_2\frac{1-e^{i\tilde{k}_2/k_*}}{\tilde{k}_2} + f_3\frac{1-e^{i\tilde{k}_3/k_*}}{\tilde{k}_3}\right]\Big\} + 2\ {\rm perm.}$$

$$f_t = \frac{1}{\prod_{i=1}^2|\alpha_{k_i}+\beta_{k_i}|^2}\Big[\prod_{i=1}^3(\alpha_{k_i}+\beta_{k_i})(\alpha^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}) - \prod_{i=1}^3(\alpha^*_{k_i}+\beta^*_{k_i})(\beta_{k_1}\beta_{k_2}\beta_{k_3})\Big]$$

$$f_1 = \frac{1}{\prod_{i=1}^2|\alpha_{k_i}+\beta_{k_i}|^2}\Big[\prod_{i=1}^3(\alpha_{k_i}+\beta_{k_i})(\beta^*_{k_1}\alpha^*_{k_2}\alpha^*_{k_3}) - \prod_{i=1}^3(\alpha^*_{k_i}+\beta^*_{k_i})(\alpha_{k_1}\beta_{k_2}\beta_{k_3})\Big]$$

$$f_2 = \frac{1}{\prod_{i=1}^2|\alpha_{k_i}+\beta_{k_i}|^2}\Big[\prod_{i=1}^3(\alpha_{k_i}+\beta_{k_i})(\alpha^*_{k_1}\beta^*_{k_2}\alpha^*_{k_3}) - \prod_{i=1}^3(\alpha^*_{k_i}+\beta^*_{k_i})(\beta k_1\alpha_{k_2}\beta_{k_3})\Big]$$

$$f_3 = \frac{1}{\prod_{i=1}^2|\alpha_{k_i}+\beta_{k_i}|^2}\Big[\prod_{i=1}^3(\alpha_{k_i}+\beta_{k_i})(\alpha^*_{k_1}\alpha^*_{k_2}\beta^*_{k_3}) - \prod_{i=1}^3(\alpha^*_{k_i}+\beta^*_{k_i})(\beta_{k_1}\beta_{k_2}\alpha_{k_3})\Big]$$



$R_H = H^{-1}$
t
t_{end}
$\rho \lesssim 10^{-12}\,\rho_{\mathrm{P}\ell}$
t_{on}
λ
???
0
$\ell_{P\ell}$
$\sim 10^5 \ell_{P\ell}$
length

Loop Quantum Cosmology

- **Loop Quantum Gravity**: approach to QG. Not fully complete yet.

Less ambitious target: symmetry reduced scenarios: black hole space-times, FRW, etc

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The quantization program can be completed: $\Psi_{FRW}(a,\phi) \in \mathcal{H}_0$ **, dynamics, etc**

Ashtekar, Bojowald, Brizuela, Campiglia, Corichi, Garay, Lewandowsky, Martin, Martin-Benito, Mena-Marugan, Olmedo, Pawlowski, Singh, Taveras, Vandersloot, Vidoto, Wilson-Ewing,...)

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All operators representing physical quantities are **bounded above** in $\mathcal{H}_0$: **no Big Bang singularity** (Ashtekar, Corichi, Singh'06)

For example: $\langle\hat{\rho}\rangle \leq \rho_{\rm max} \propto \hbar^{-1}$ where $\rho_{max} \sim \rho_{P\ell}$

Results have been extended to more complicated space-times: Bianchi, k=1, Λ , Gowdy models.

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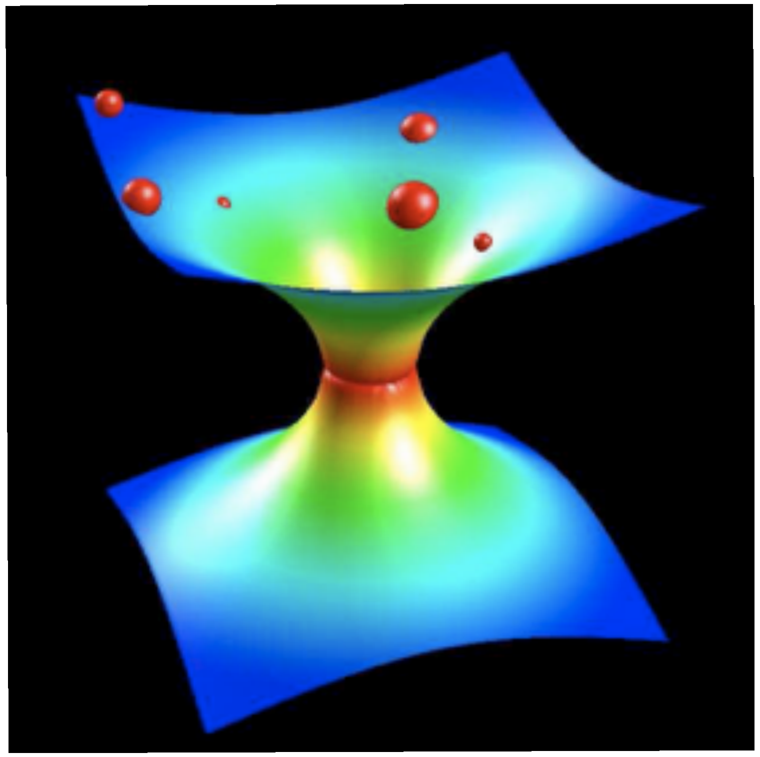
Simplifications also imply limitations. Checks are needed

A couple of relevant results:



- **Bounce replacing the Big Bang singularity**

For semiclassical states $\Psi_{\text{FRW}}(a, \phi)$, effective equations k=0 FRW

$$H^2 = \frac{8\pi G}{3}\,\rho\left(1 - \frac{\rho}{\rho_{\text{max}}}\right)$$

- If $(\phi, V(\phi))$ in the matter sector: **Inflation, almost unavoidably, appears at some time after the bounce** Ashtekar, Sloan (see also Corichi, Karami)

Arena to construct a Quantum Gravity extension of the inflationary scenario

In this talk:

LQC extension of the inflationary scenario and its observable consequences

I.A., Ashtekar, Nelson: PRL 109 251301 (2012); PRD 87 043507 (2013); CQG 30 085014 (2013)

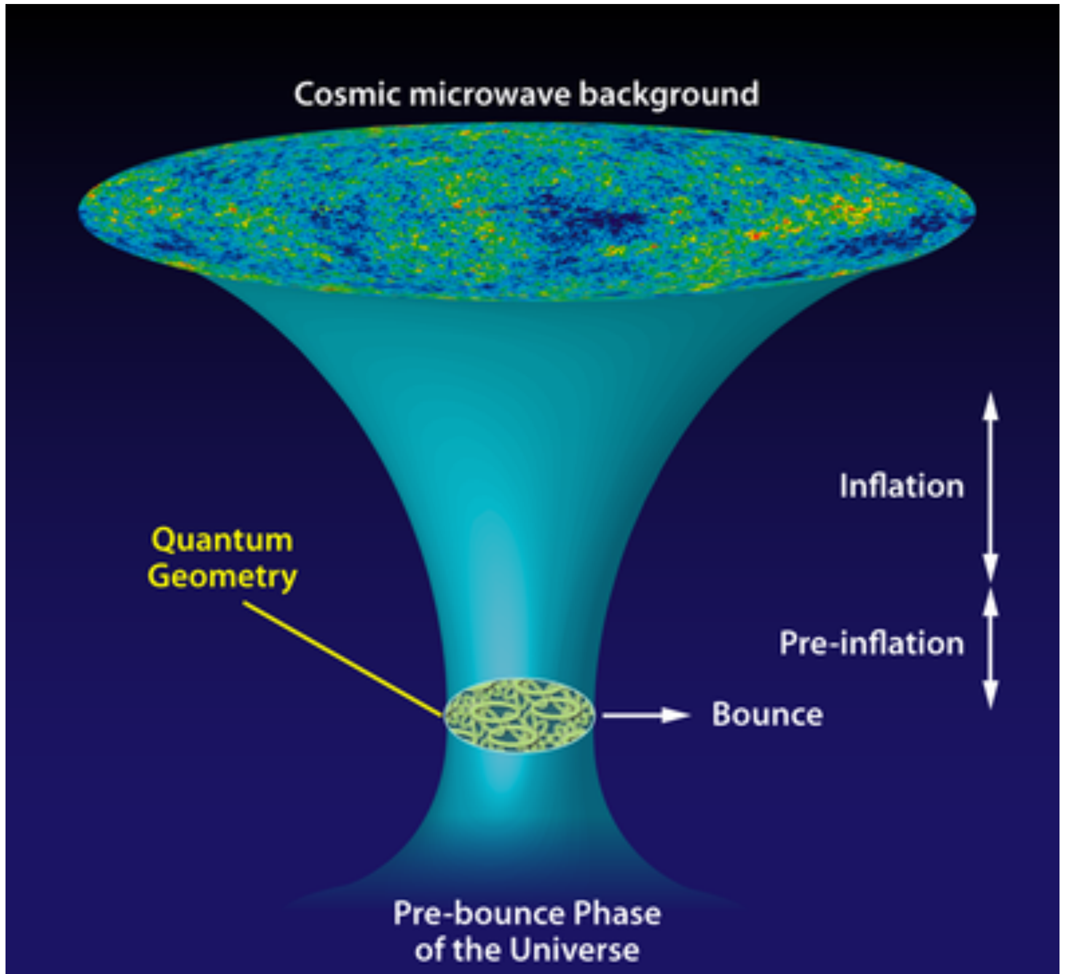


Fig. Credits: A. Stonebraker

Summary of strategy:

- Perturbations start in the vacuum at early times
- Evolution across the bounce amplifies curvature perturbations for long wavelengths (compared to the space-time curvature scale)
- Then standard slow-roll inflation begins, but perturbations reach the onset of inflation in an **excited state,** rather than the Bunch-Davies vacuum

Observational consequences

The Power Spectrum

We chose $V(\phi) = \frac{1}{2}m^2\phi^2$ (of course, other choices possible)

For $\phi_B = 1$ and $m = 1.3 \times 10^{-6}$ (Planck units) and vacuum initial conditions in the past

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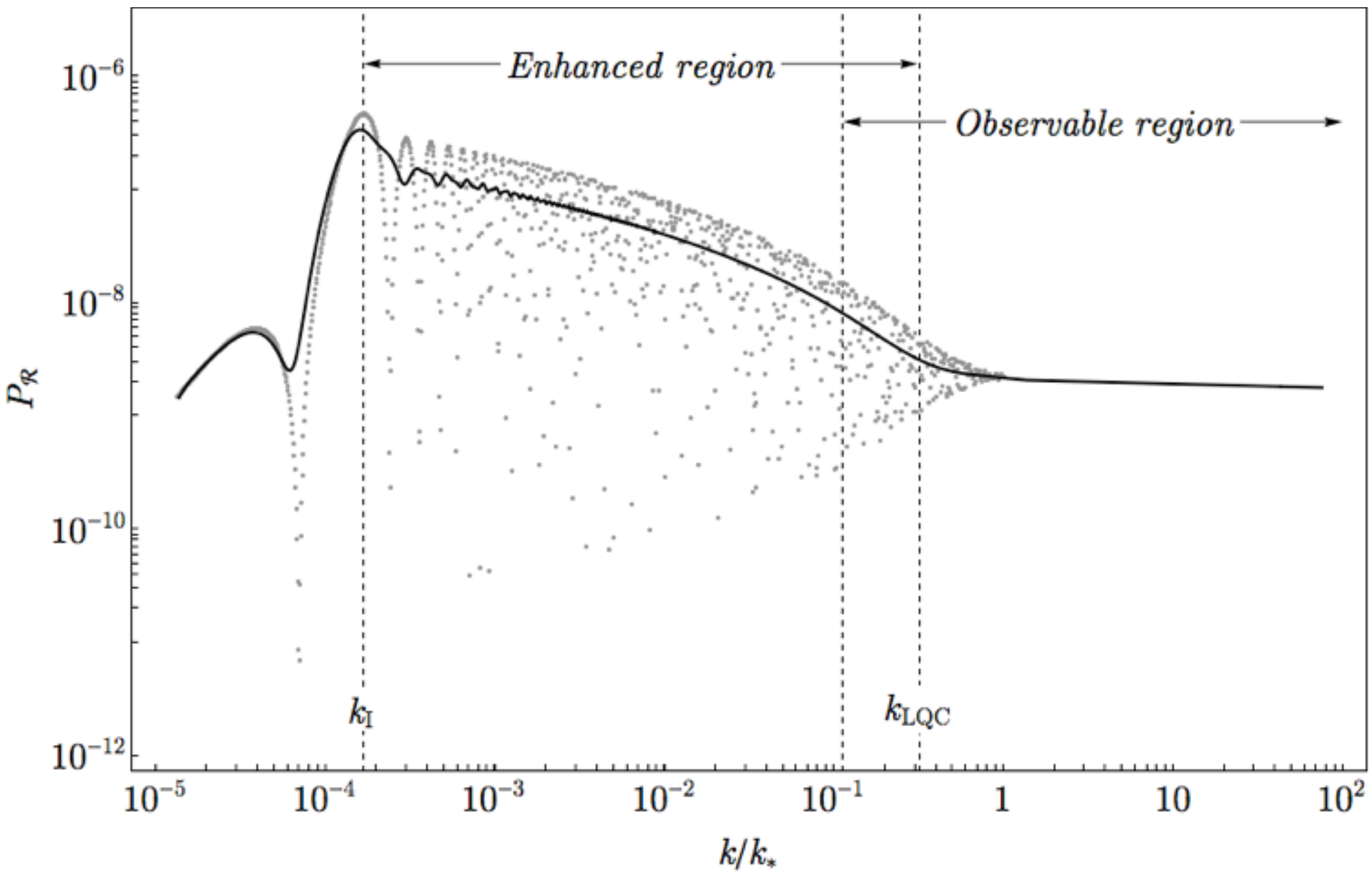
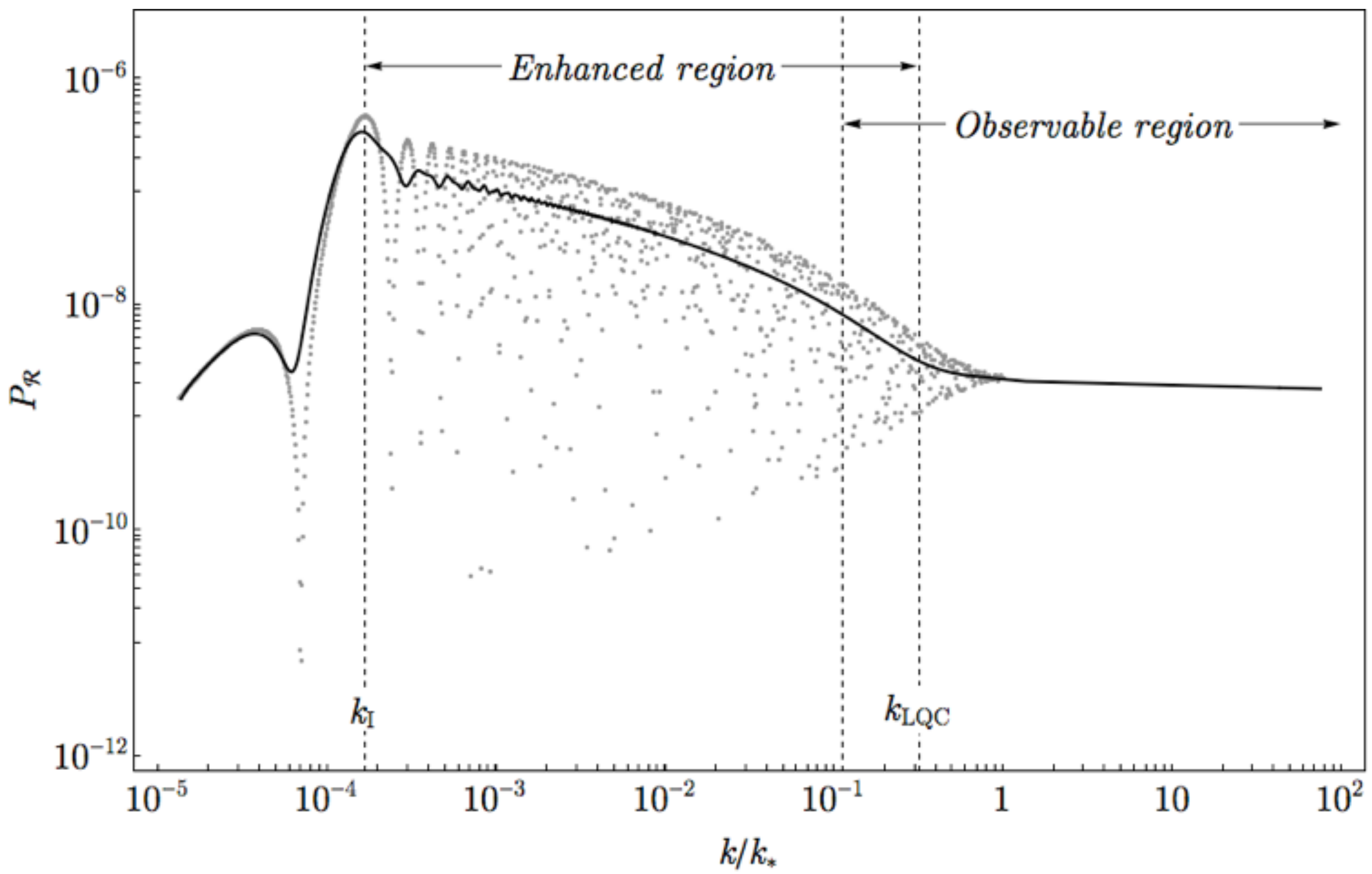
Scalar Power spectrum:

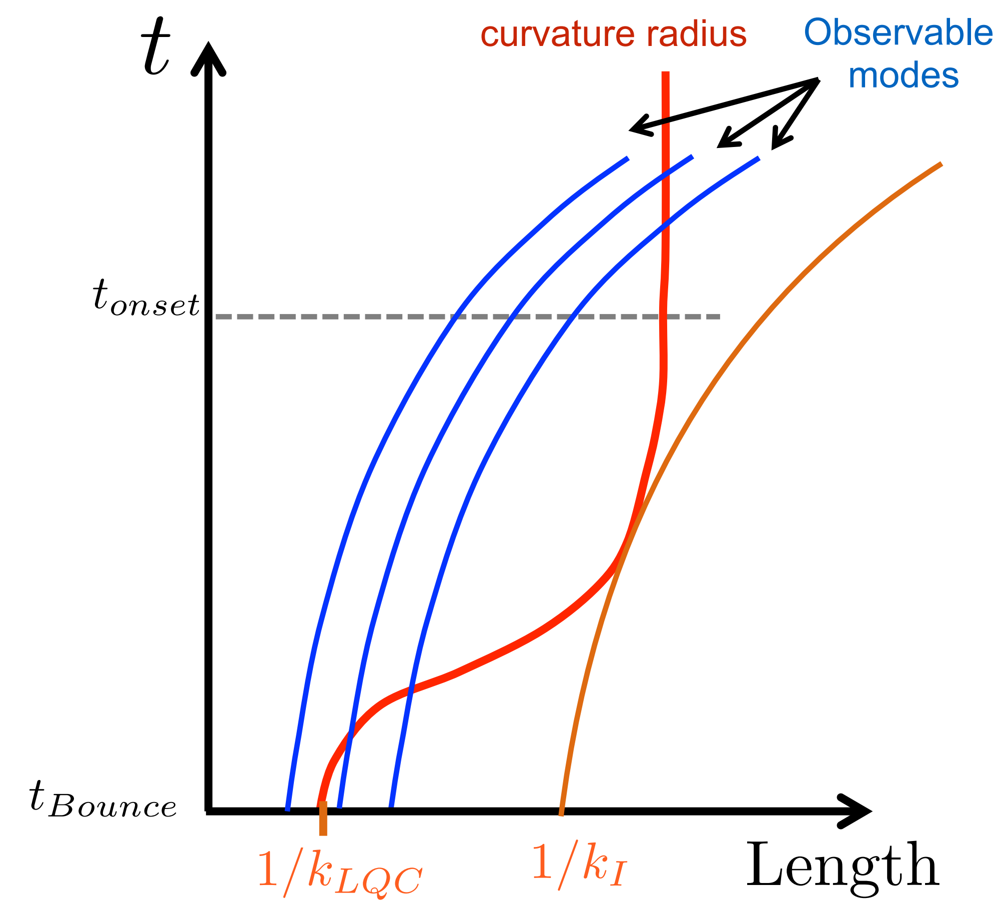


Gray points: numerical result for individual k's
Black line: average



$k_\star = 0.002\,\mathrm{Mpc}^{-1}$

Loop Quantum Cosmology



Two relevant scales:

$$\frac{k_{LQC}}{a(t_B)} = \sqrt{R_B/6}$$ Where $R_B = 48\pi G\,\rho_{\rm max} \approx 62$ scalar curvature at the bounce

$$\frac{k_I}{a(t_I)} = \sqrt{R_I/6}$$ Where $R_I \sim 10^{-10}$ scalar curvature at the onset of accelerated expansion

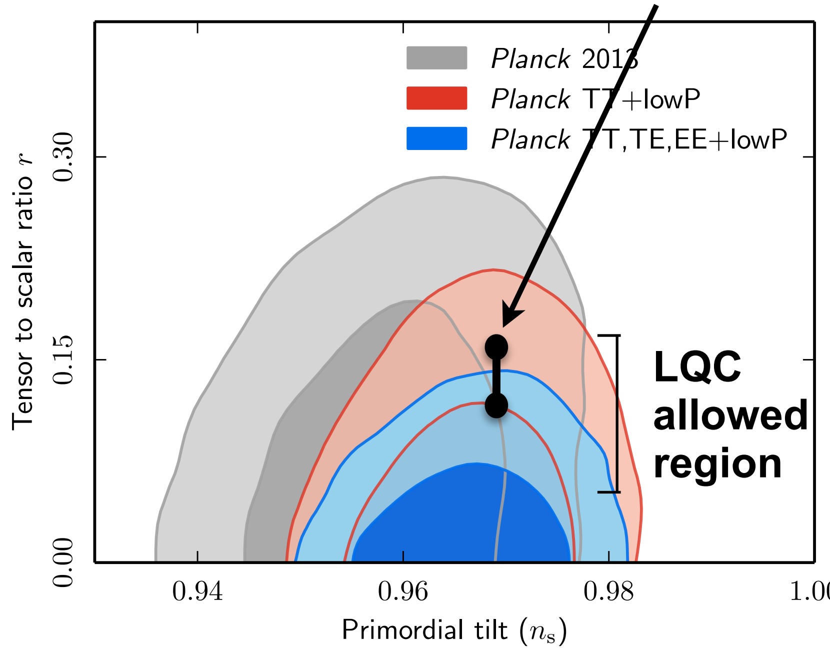
We have explored the full parameter space and evaluated LQC correction to the scalar and tensor power spectra I.A., Morris 15

Main features in the results:

- LQC contributions at low k's:
  - Oscillations
  - Power amplification for the initial conditions we have considered

- Also:
  - r can decrease (w.r.t. inflationary predictions)
  - n_t more negative
  - consistency relation may get modified $r \leq -8n_t$
  - positive running

Inflation
(without LQC corrections)
Out of the 95% CL region



Testable predictions if primordial grav. waves are observed

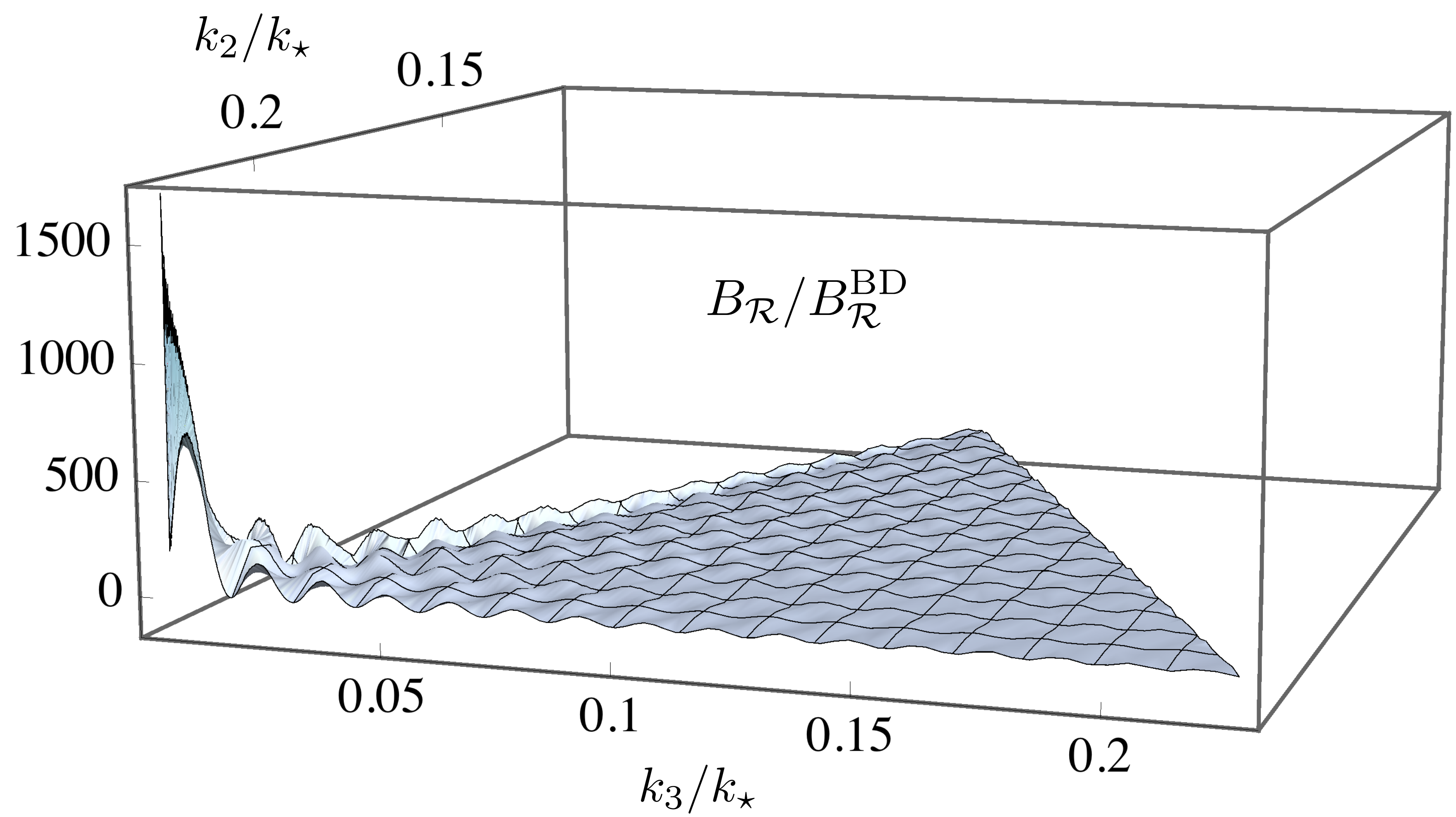
$$V(\phi) = \frac{1}{2}m^2\phi^2$$

Non-Gaussianity

Inflationary non-Gaussianity from LQC (I.A. 2015)

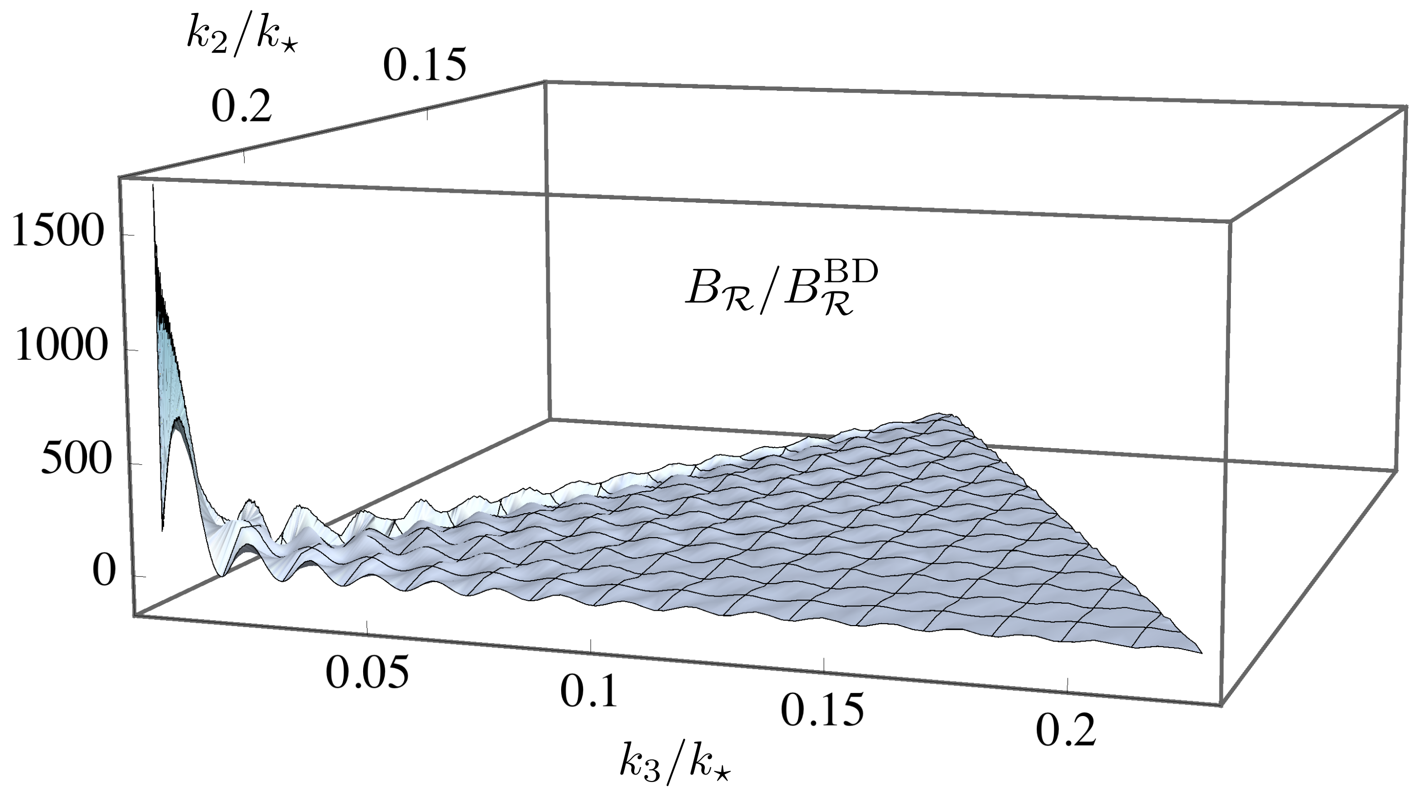
$$\langle \mathcal{R}_{\vec{k}_1}\mathcal{R}_{\vec{k}_2}\mathcal{R}_{\vec{k}_3}\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, B_{\mathcal{R}}(k_1,k_2,k_3)$$

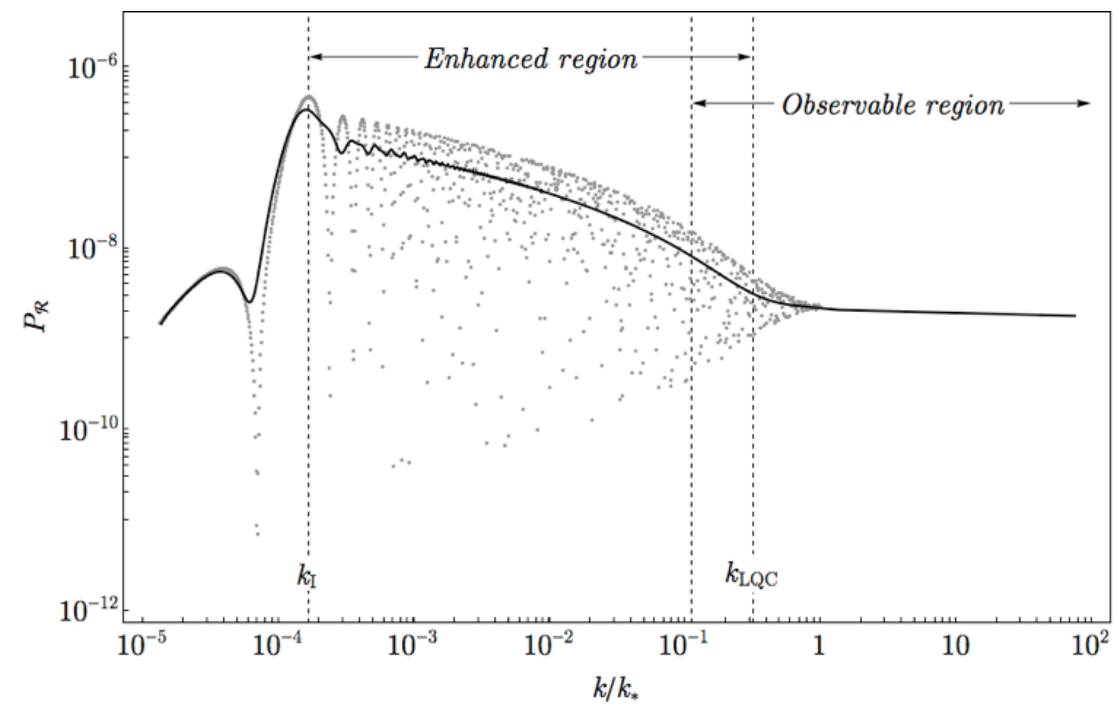
I.A. & Parker: expected enhancement in squeezed configurations $k_1 \approx k_2 \gg k_3$



$k_1 = 0.22 k_\star$
$k_\star/a_0 = 0.002\,\mathrm{Mc}^{-1}$

Origin: quantum interaction between "particles" during inflation
Single-field consistency reaction relies on classical arguments. Therefore its conclusion are neatly bypassed





Take home points:

1) Compatible with observations: very Gaussian for observable scales

2) Large correlations between longest wavelengths in the CMB and super-Horizon modes

These correlations may be relevant for the CMB: the may modify the statistics of observable modes

(Bartolo, Bramante, Byrnes, Carrol, Dimastrogiovanni, Erickcen, Hui, Jeong, Kamionkowski, Liddle, Lyth, LoVerde, Matarrese, Mota, Nelson, Nurmi, Peloso, Ricciardone, Shandera, Schmidt, Tasinato, Thorsrud, Urban,...)

LQC and Power asymmetry

WMAP: anisotropies in the CMB: Power Asymmetry

Confirmed by Planck at same level of significance $\sim 3\sigma$

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$$\Delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

angular Power spectrum

Wigner 3j-symbol

$$\langle a_{\ell m} a^{\star}_{\ell' m'} \rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell + \underbrace{\sum_{LM} A_{LM}\, \mathcal{G}^{\ell\ell' L}_{-mm' M}\, (C_\ell + C_{\ell'})}_{\text{off-diagonal = anisotropies}}$$

Planck 2013

$A_{L=1} = 0.07 \pm 0.02$ for $\ell < 64$ and compatible with zero for $\ell > 64$

$A_L = 0 \quad L > 1$

Scale dependent dipole modulation

Anisotropies found significantly larger than expected for a typical realization of an isotropic Gaussian distribution

(Bartolo, Bramante, Byrnes, Carrol, Dimastrogiovanni, Erickcen, Hui, Jeong, Kamionkowski, LoVerde, Matarrese, Mota, Nelson, Nurmi, Peloso, Ricciardone, Shandera, Schmidt, Tasinato, Thorsrud, Urban,...)

Two (at least) possible solutions:

1) Break isotropy at the fundamental level (e.g. Bianchi phase of early universe)

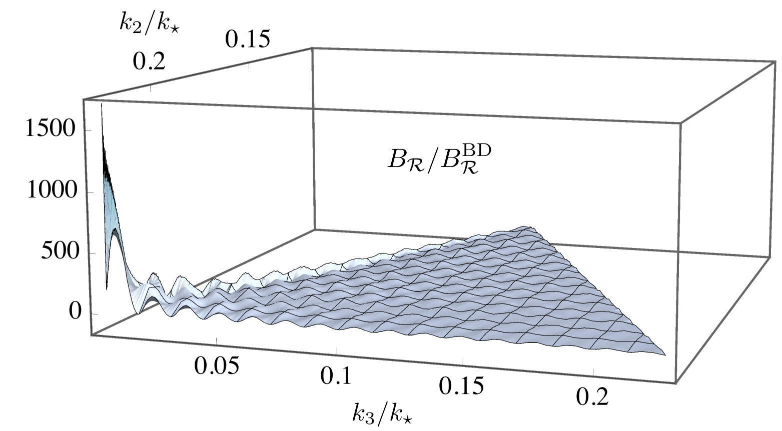
2) Keep isotropy, but break Gaussianity: a typical realization may look significantly more anisotropic if the spectrum is not Gaussian

But careful: our CMB is very Gaussian (Planck)

Large correlations between observable modes and super-horizon modes to be observed can do the job (Schmitd & Hui 2013)

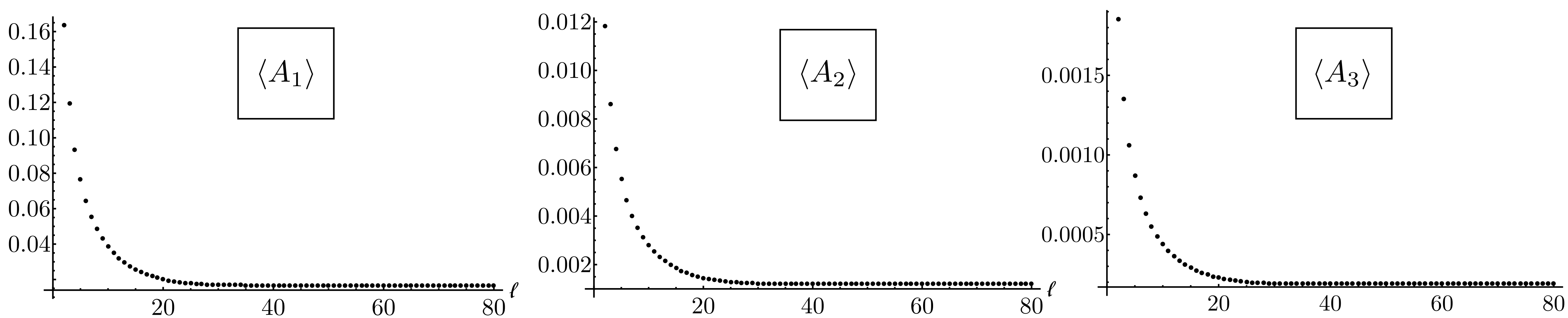
Challenge: build a model with such non-Gaussianity but with negligible correlations among observable modes

This is precisely what we find form the bounce



I've computed the expected value of $A_L = \frac{1}{(2L+1)}\sum_M A_{LM}$ **for different value of our parameters. For some cases I find results in agreement with observations**

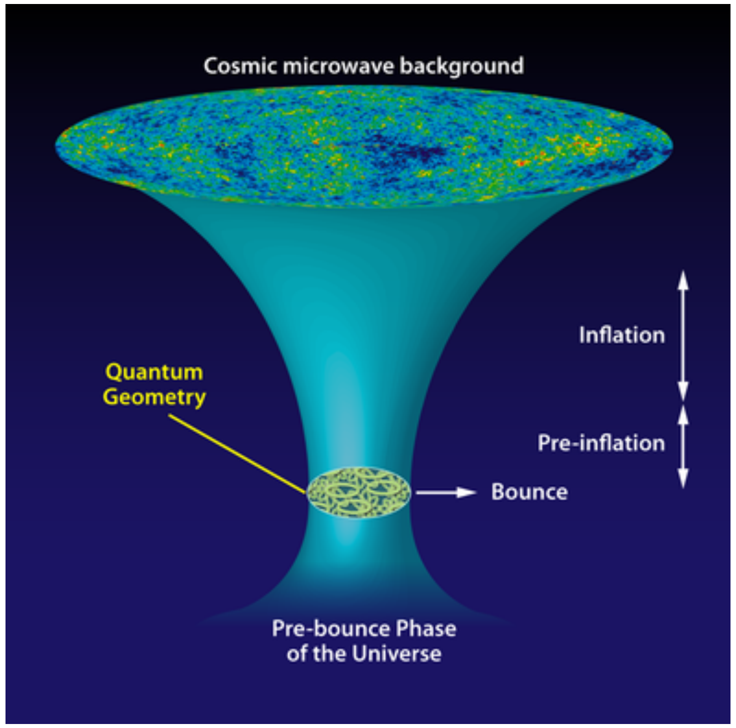
E.g. $\phi_B = 1.22$ $m = 1.1\times 10^{-6}$ and vacuum initial condition before the bounce



Scale dependent dipole modulation

Claim: a bounce preceding inflation can originate the "anomalous" observed power asymmetry

LQC and FRW space-time



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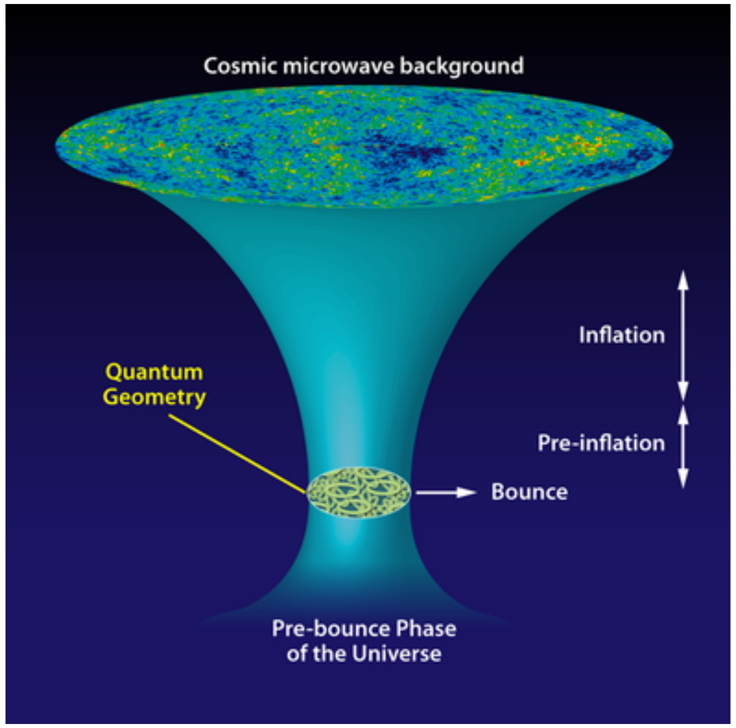
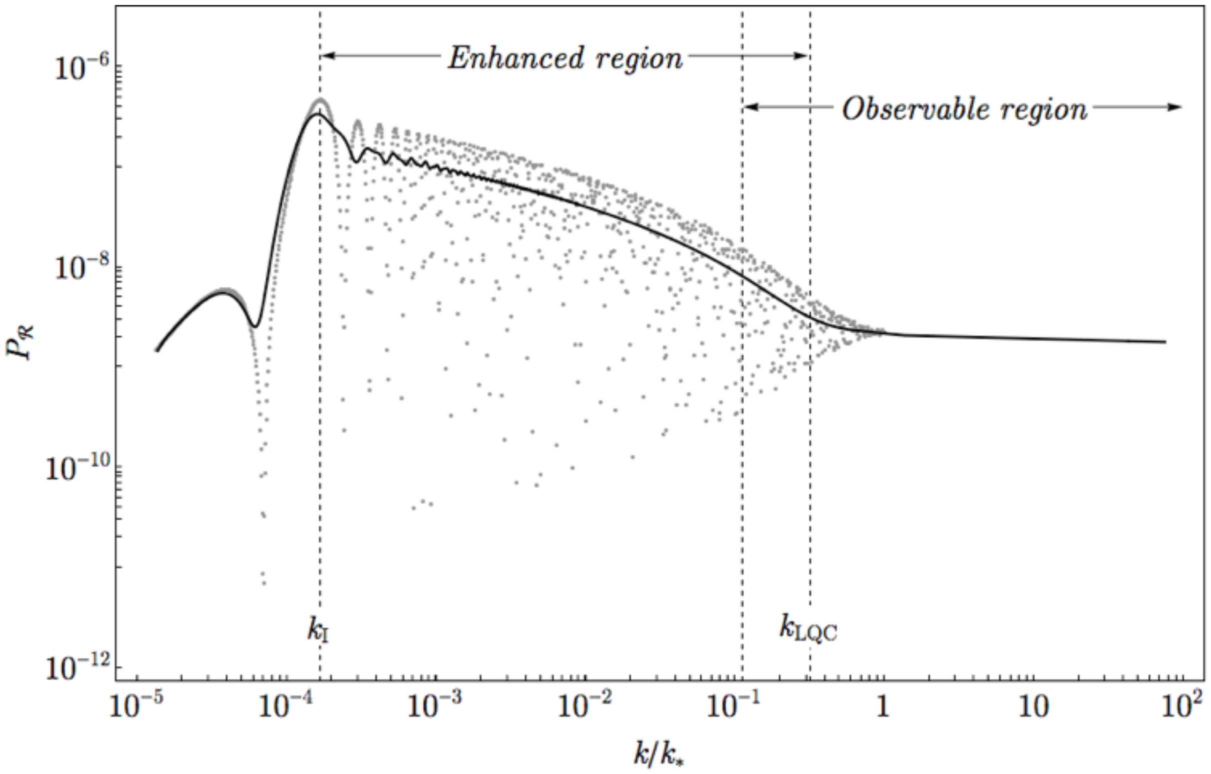


LQC and the Spectrum of primordial perturbations



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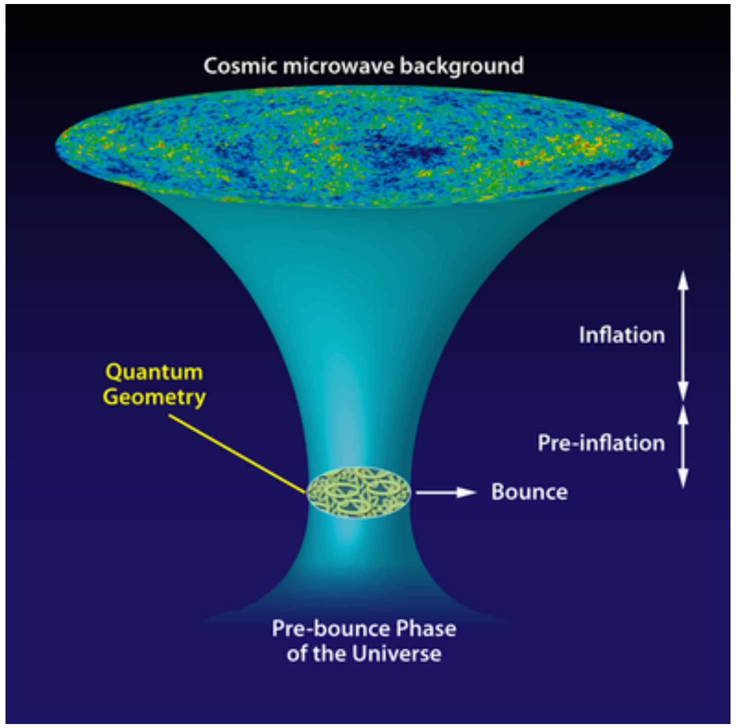

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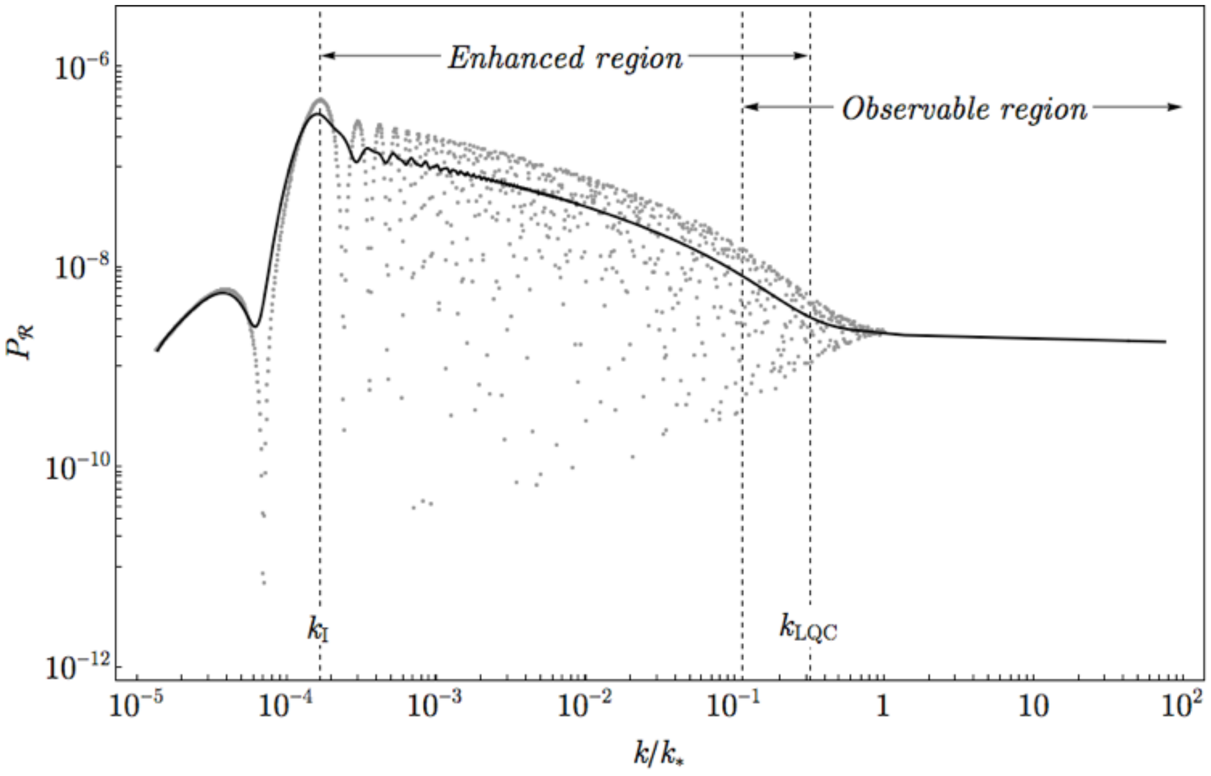
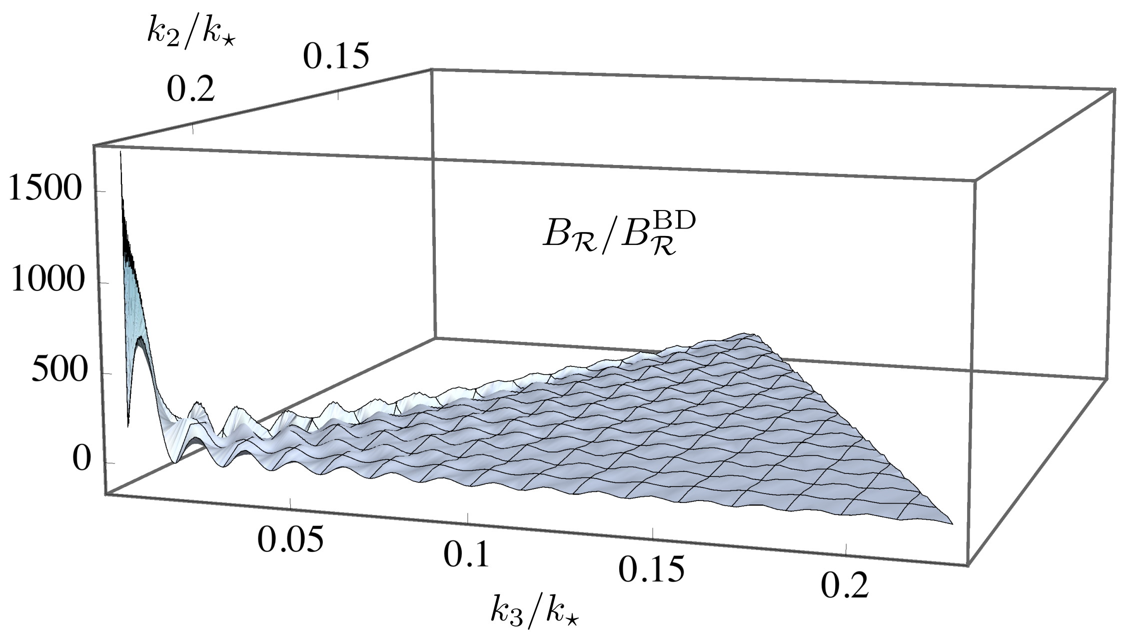


LQC and the spectrum of Non-Gaussianity



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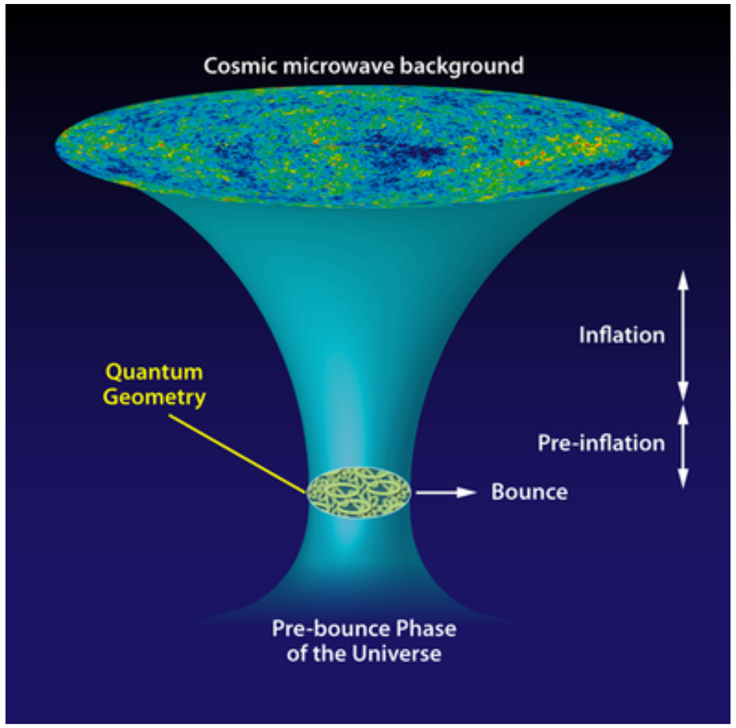

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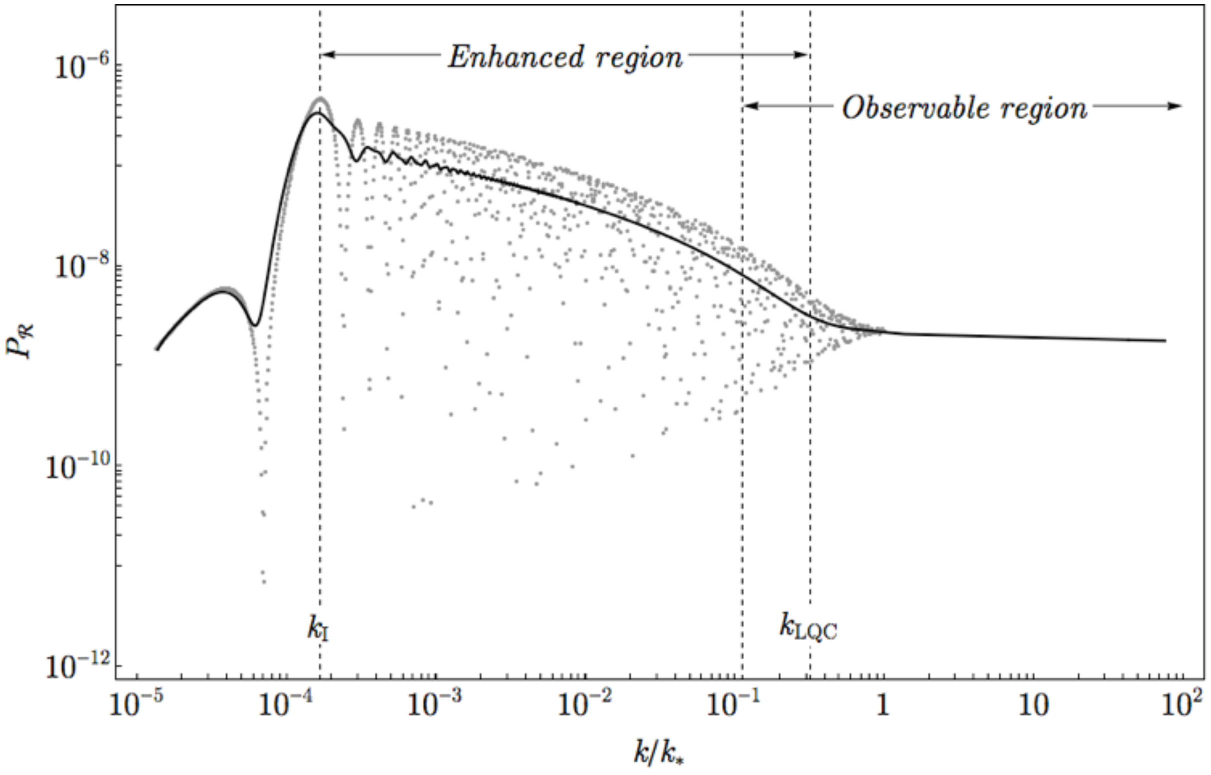

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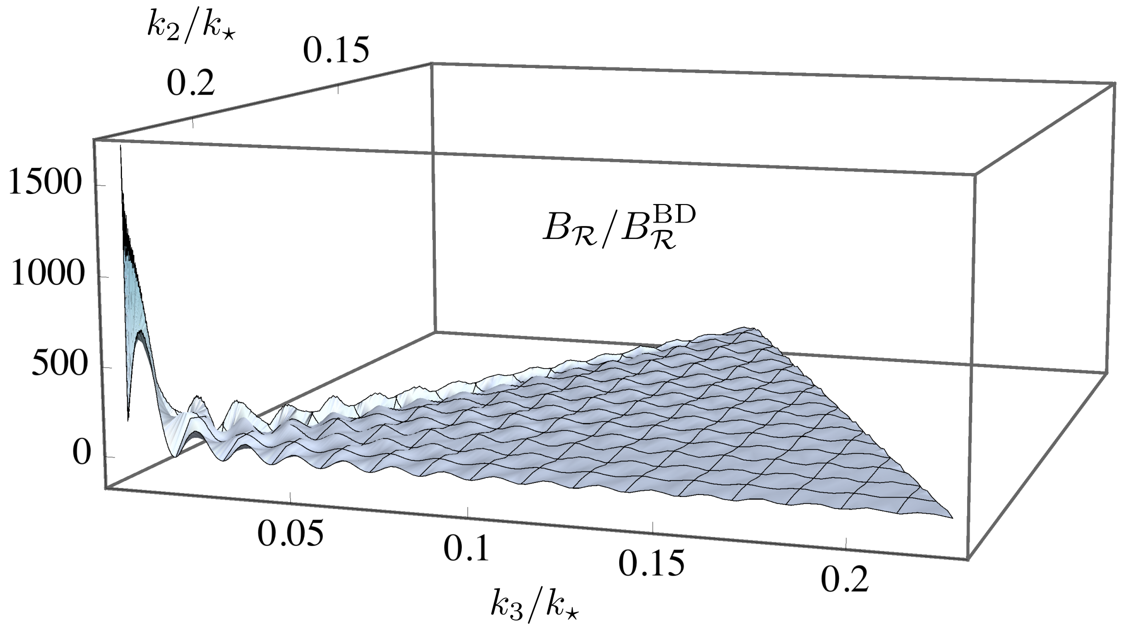
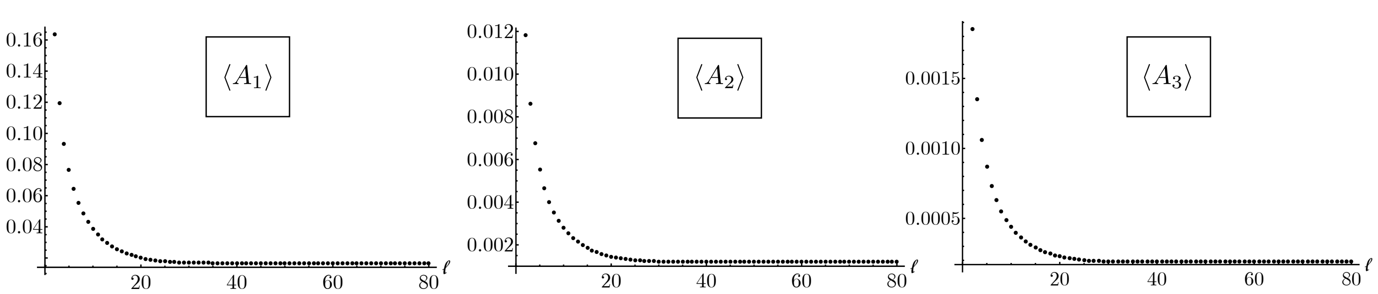


LQC and CMB anomalies



Appendix: some formulae

$$\langle A_L(\ell)\rangle = \frac{C_\ell(-\alpha_L)}{2C_\ell(0)} \left[\int \frac{dk_l\, k_l^2}{(2\pi)^3}\, |g_L(k_l)|^2 P_\phi(k_l)\right]^{1/2}$$

$$\langle \Phi_{\vec{k}} \Phi_{\vec{k}'}\rangle = P_\Phi(k)\, \left[(2\pi)^3 \delta(\vec{k}+\vec{k}') + G(\vec{k},\vec{k}_l)\, \Phi(\vec{k}_l)\right]$$

$$G(\vec{k},\vec{k}_l) = \frac{5}{3} B_{\mathcal{R}}(\vec{k}_1,\vec{k}_l)[\Delta_{\mathcal{R}}(k_1)\Delta_{\mathcal{R}}(k_l)]^{-1}$$

$$B_{\mathcal{R}} \approx 4\epsilon\, \Delta_{\mathcal{R}}(k_1)\Delta_{\mathcal{R}}(k_3)\, \times\, \mathrm{Re}\Big[f_t \frac{1-e^{ik_1\eta_0}}{1+(1+\mu)\,x} + f_1 \frac{1-e^{ik_3\eta_0}}{(1+\mu)\,x} + f_2 \frac{1-e^{ik_3\eta_0}}{(1-\mu)\,x} + f_3 \frac{1-e^{ik_1\eta_0}}{1+(-1+\mu)\,x}\Big]$$

$$x := k_3/k_1 \ll 1$$

$$\mu = \hat{k}_1 \cdot \hat{k}_3$$

$$G(\vec{k},\vec{k}_l) = \sum_L g_L(k_l)\, \left(\frac{k_\star}{k}\right)^{\alpha_L} P_L(\mu)$$

$$f_t = \big[1 + F(k_1)\,(1+F(k_2)) + \text{cyclic perm.}\big]$$

$$f_1 = F(k_1)^\star[1 + F(k_2) + F(k_3)] - F(k_2)^\star F(k_3)$$

$$F(k) = |\beta_k|^2 + \alpha_k^\star \beta_k$$

Cosmological perturbations in LQC

Difficulty: how do we study quantum fields (matter+grav. perturbations) propagating on a quantum space-time?

I.A., Ashtekar, Nelson: PRL 109 251301 (2012); PRD 87 043507 (2013); CQG 30 085014 (2013)

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Same approach as in inflation: perturbations are test field propagating in a given (now quantum) FRW background Ψ_{FRW}

$$\mathcal{H} = \mathcal{H}_{hom} \otimes \mathcal{H}_{pert} \qquad \Psi = \Psi_{FRW} \otimes \psi_{\rm pert}$$

Main idea:

One start from the LQC analog of the WdW equation for Ψ and "traces out" Ψ_{FRW}

Difficulty: how do we study quantum fields (matter+grav. perturbations) propagating on a quantum space-time?

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Reults:

Unforeseen simplification: On a given quantum space-time Ψ_{FRW} (not necessarily semiclassical), evolution of perturbations is mathematically equivalent to a propagation in a FRW smooth metric $\tilde{g}_{ab}$.

E.g., for tensor perturbations

$$\hat{\mathcal{T}}''_{\vec{k}}(\tilde{\eta}) + 2\frac{\tilde{a}'}{\tilde{a}}\hat{\mathcal{T}}'_{\vec{k}}(\tilde{\eta}) + k^2\hat{\mathcal{T}}_{\vec{k}}(\tilde{\eta}) = 0$$

$\tilde{g}_{ab}$ is constructed from expectation values of background operators. Of course, it does not satisfy Einstein eqns, and coefficients prop. to $\hbar$

$$\tilde{g}_{ab}dx^a dx^b = \tilde{a}(\tilde{\eta})[-d\tilde{\eta}^2 + d\vec{x}^2] \quad \text{with} \quad \tilde{a}^4 = \frac{\langle \hat{H}_0^{-1/2} \hat{a}^4 \hat{H}_0^{-1/2} \rangle_{\Psi_{FRW}}}{\langle H_0^{-1} \rangle_{\Psi_{FRW}}}\,, \quad d\tilde{\eta} = \tilde{a}\, \langle \hat{H}_0^{-1} \rangle_{\Psi_{FRW}}\, d\phi$$

$\tilde{g}_{ab}$ effective, dressed metric

This is a QFT in a quantum space-time

Further simplification (if desired):

If the state for the background Ψ_{FRW} is highly peaked, then $\tilde{g}_{ab}$ is simply the solution of the effective equations $H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\max}}\right)$