Dark energy and nonlocal gravity

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based on

Jaccard, MM, Mitsou,	PRD 2013, 1305.3034
MM,	PRD 2014, 1307.3898
Foffa, MM, Mitsou,	PLB 2014, 1311.3421
Foffa, MM, Mitsou,	IJMPA 2014, 1311.3435
Kehagias and MM,	JHEP 2014, 1401.8289
MM and Mancarella,	PRD 2014, 1402.0448
Dirian, Foffa, Khosravi, Kunz, MM,	JCAP 2014, 1403.6068
Dirian, Foffa, Kunz, MM, Pettorino,	JCAP 2015, 1411.7692
MM	1506.06217

the general idea: modify GR in the infrared using non-local terms

non-locality emerges from fundamental local theories in many situations

- classically, when separating long and short wavelength and integrating out the short wave-length

  (e.g cosmological perturbation theory)

- in QFT, when computing the effective action that includes the effect of radiative corrections of massless or light particles.

- a natural way of modifying GR in the IR is by introducing a mass scale (e.g. massive gravity, bigravity,...)

  we will introduce a mass scale as the coefficient of a non-local term

- phenomenological approach. Identify a non-local modification of GR that works well

- attempt at a more fundamental understanding

  IR running and dimensional transmutation in R^2 theories?

some sources of inspiration:

- massive photon $S = \int d^4x \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu\right]$

  can be described replacing

$$\partial_\mu F^{\mu\nu} = j^\nu \quad \rightarrow \quad \left(1 - \frac{m^2}{\Box}\right)\partial_\mu F^{\mu\nu} = j^\nu$$

(Dvali 2006)

- for gravity, a first guess for a massive deformation of GR could be

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \rightarrow \quad \left(1 - \frac{m^2}{\Box_g}\right)G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

(Arkani-Hamed, Dimopoulos, Dvali and Gabadadze 2002)

however this is not correct since $\nabla^\mu(\Box_g^{-1}G_{\mu\nu}) \neq 0$

we lose energy-momentum conservation.

- to preserve energy-momentum conservation:

$$G_{\mu\nu} - m^2(\Box^{-1}G_{\mu\nu})^T = 8\pi G T_{\mu\nu}$$

(Jaccard,MM, Mitsou, 2013)

however, instabilities in the cosmological evolution

(Foffa,MM, Mitsou, 2013)

- $$G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$$

(MM 2013)

stable cosmological evolution!

- a related model:

(MM and M.Mancarella, 2014)

$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

Absence of vDVZ discontinuity and of a strong coupling regime

A. Kehagias and MM 2014

- write the eqs of motion of the non-local theory in spherical symmetry:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

- for mr <<1: low-mass expansion
- for $r >> r_S$: Newtonian limit (perturbation over Minowski)
- match the solutions for $r_S << r << m^{-1}$ (this fixes all coefficients)

- result: for $r \gg r_s$

$$A(r) = 1 - \frac{r_S}{r}\left[1 + \frac{1}{3}(1 - \cos mr)\right]$$

$$B(r) = 1 + \frac{r_S}{r}\left[1 - \frac{1}{3}(1 - \cos mr - mr \sin mr)\right]$$

for $r_s \ll r \ll m^{-1}$: $A(r) \simeq 1 - \frac{r_S}{r}\left(1 + \frac{m^2 r^2}{6}\right)$

the limit $m \to 0$ is smooth !

By comparison, in massive gravity the same computation gives

$$A(r) = 1 - \frac{4}{3}\frac{r_S}{r}\left(1 - \frac{r_S}{12 m^4 r^5}\right)$$

vDVZ discontinuity

breakdown of linearity below $r_V = (r_s/m\hat{}4)^{1/5}$

Cosmological consequences.

- consider $$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - m^2 R\frac{1}{\Box^2}R\right]$$

  define $$U = -\Box^{-1}R\,,\qquad S = -\Box^{-1}U$$

  NB: auxiliary non-dynamical fields! U=0 if R=0. It is not the same as a scalar-tensor theory

- in FRW we have 3 variables: H(t), U(t), W(t)=H^2(t)S(t).

  define $x = \ln a(t)$, $h(x) = H(x)/H_0$,
  $\gamma = (m/3H_0)^2$ $\zeta(x) = h'(x)/h(x)$

$$h^2(x) = \Omega_M e^{-3x} + \Omega_R e^{-4x} + \gamma Y(U, U', W, W')$$
$$U'' + (3+\zeta)U' = 6(2+\zeta)$$
$$W'' + 3(1-\zeta)W' - 2(\zeta' + 3\zeta - \zeta^2)W = U$$

- there is an effective DE term, with

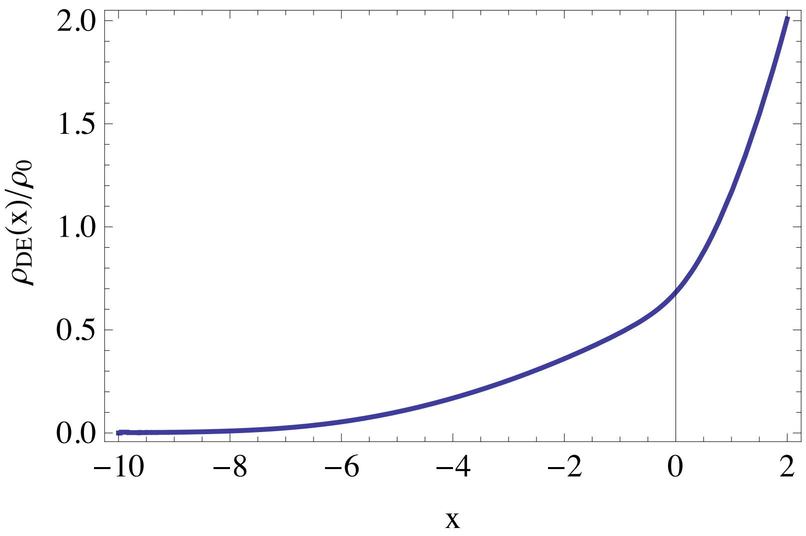
$$\rho_{\rm DE}(x) = \rho_0 \gamma Y(x) \qquad\qquad \rho_0 = 3H_0^2/(8\pi G)$$

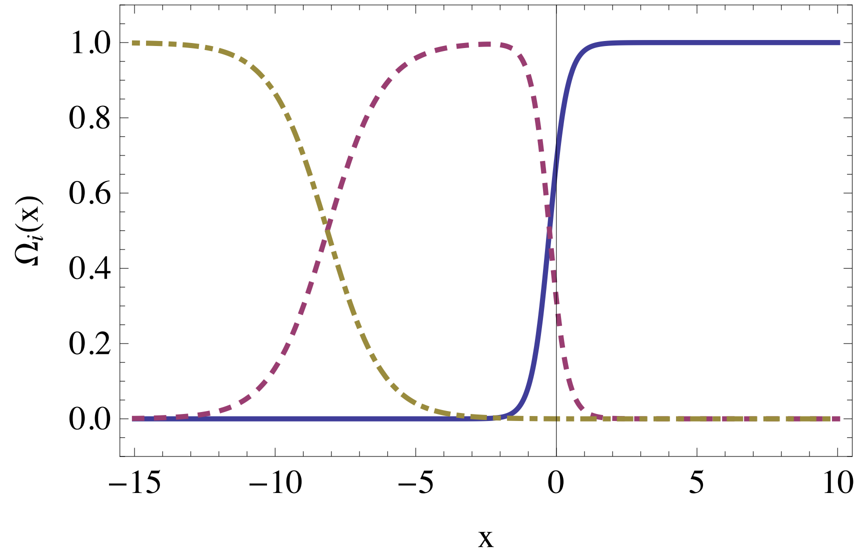
- define w_{DE} from

$$\dot\rho_{\rm DE} + 3(1 + w_{\rm DE})H\rho_{\rm DE} = 0$$

- the model has the same number of parameters as ΛCDM, with $\Omega_\Lambda \leftrightarrow \gamma$.

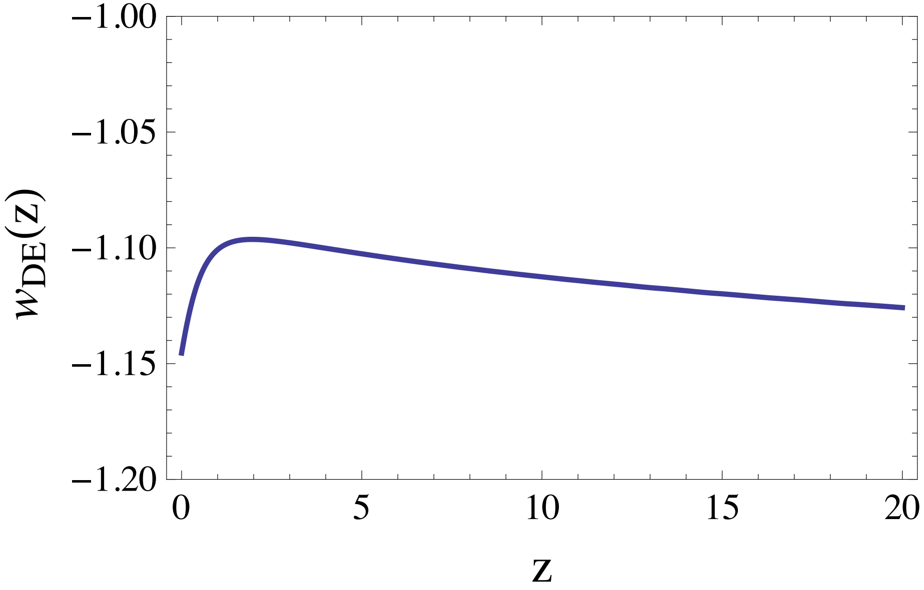
- results:





- Fixing γ = 0.0089.. (m=0.28 H_0) we reproduce Ω_{DE}=0.68

- having fixed γ we get a pure prediction for the EOS:



fit $w(a)=w_0+(1-a)\,w_a$

in the region $0< z <1.6$

w_0= -1.14, w_a=0.08

on the phantom side ! general consequence of

$$\dot{\rho}_{\rm DE} + 3(1+w_{\rm DE})H\rho_{\rm DE} = 0$$

together with ρ>0 and dρ/dt>0

The RT model $G_{\mu\nu} - m^2(g_{\mu\nu}\Box^{-1}R)^T = 8\pi G T_{\mu\nu}$

gives w_0= -1.04, w_a=-0.02

warning. This is not wCDM !!!

Cosmological perturbations

- well-behaved?

Dirian, Foffa, Khosravi, Kunz, MM
JCAP 2014

this step is already non-trivial: see eg DGP, (massive gravity), bigravity

- consistent with data?

this step rules out eg the Deser-Woodard non-local model

- comparison with ΛCDM

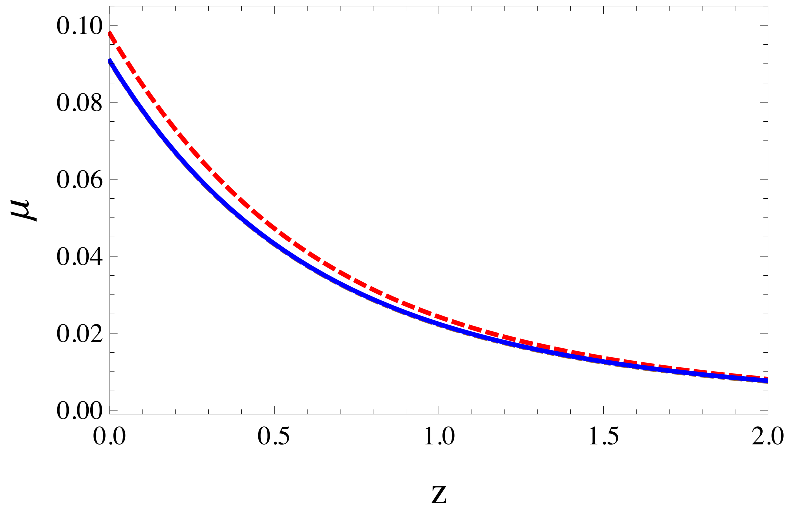
Dirian, Foffa, Kunz, MM, Pettorino,
JCAP 2015

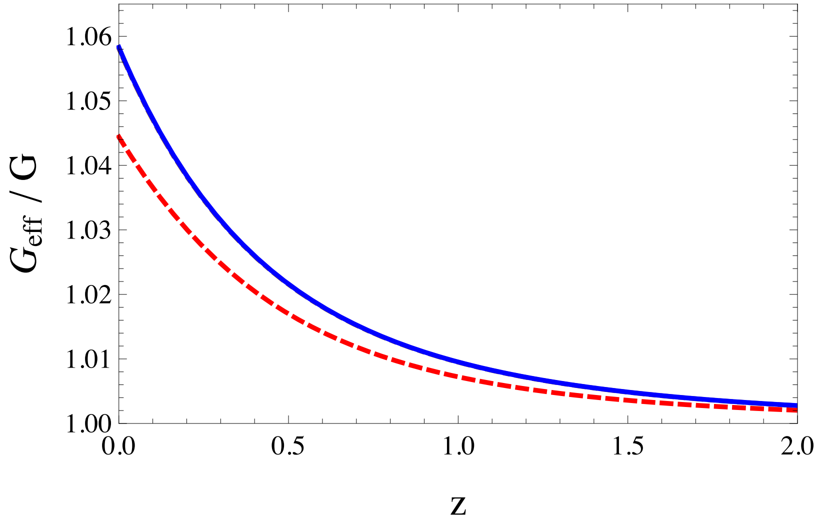
implement the perturbations in a Boltzmann code

compute likelihood, χ^2, perform parameter estimation

- the perturbations are well-behaved and differ from ΛCDM at a few percent level

$$\Psi = [1 + \mu(a;k)]\Psi_{\rm GR}$$
$$\Psi - \Phi = [1 + \Sigma(a;k)](\Psi - \Phi)_{\rm GR}$$



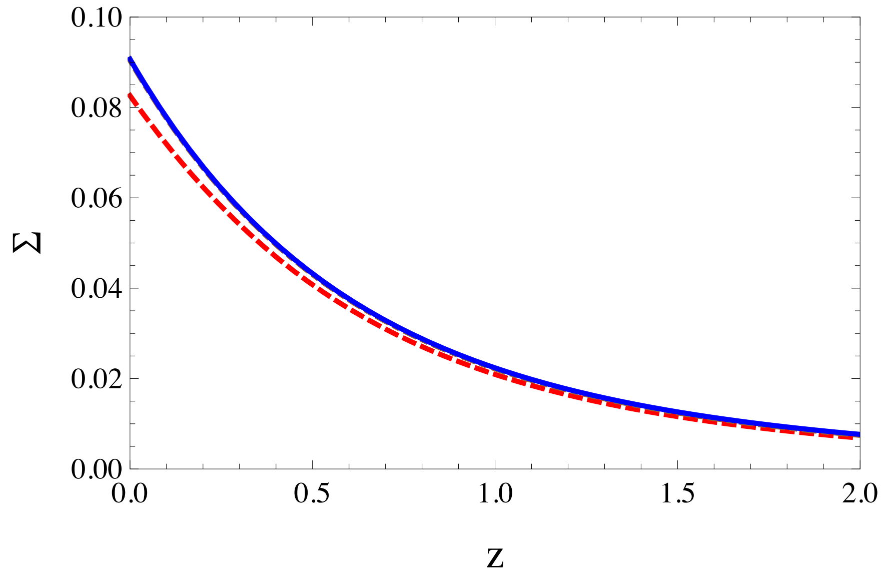


- deviations at z=0.5 of order 4%

- consistent with data: CFHTLenS gives ΔΨ/Ψ=0.05±0.25

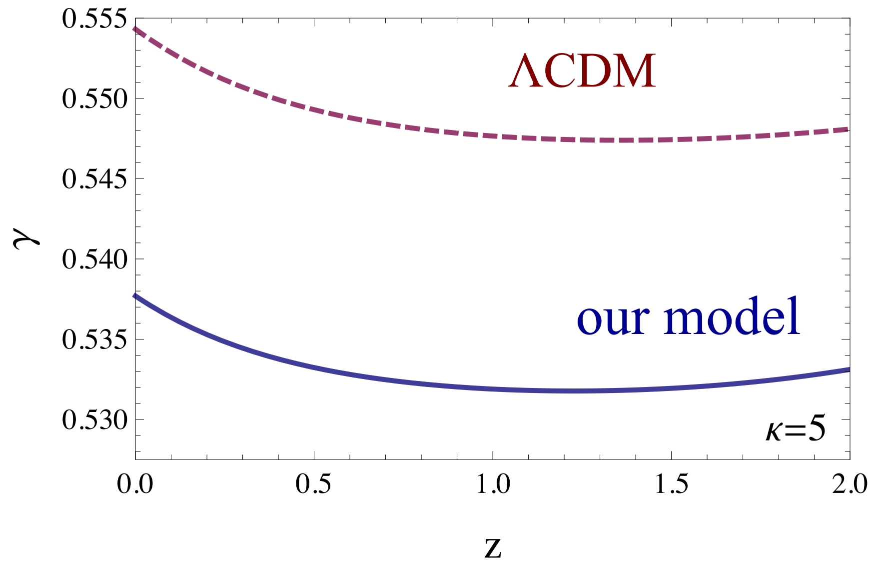
(Simpson et al 1212.3339)

Lensing: again deviations at 4% level



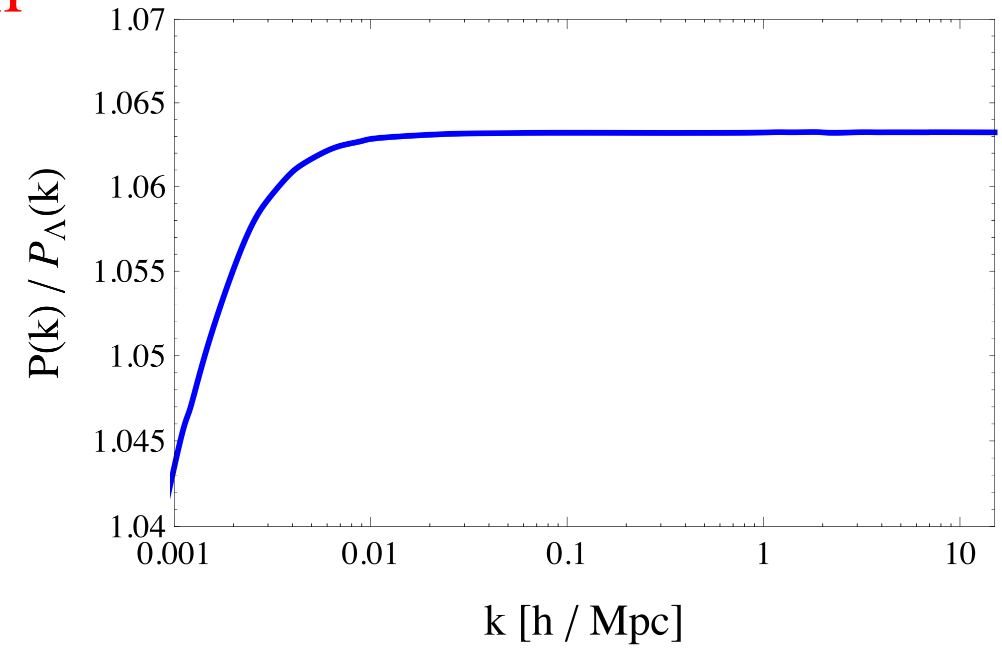
growth index:

$$\frac{d\log\delta_M(a;k)}{d\ln a} = [\Omega_M]^{\gamma(z;k)}$$

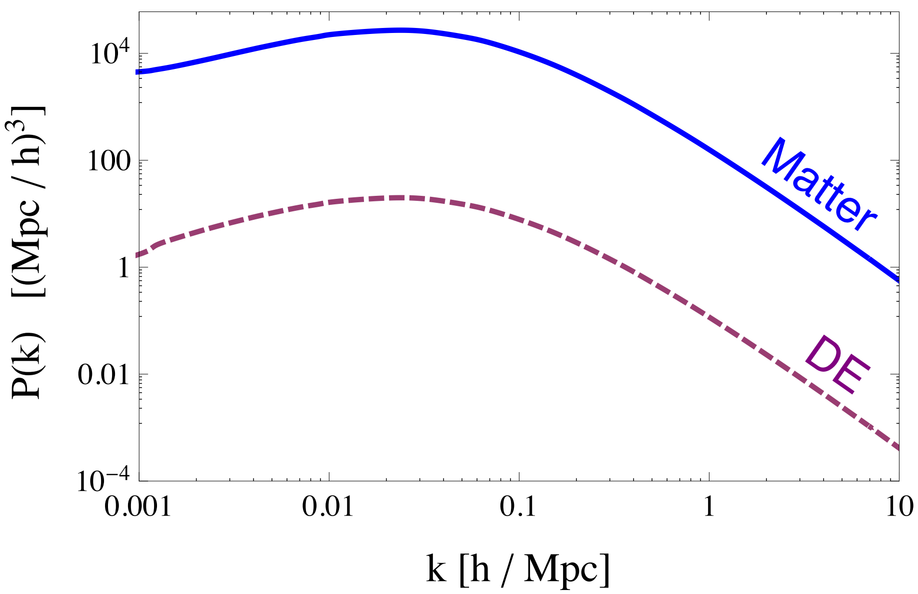


- linear power spectrum

matter power spectrum compared to ΛCDM



DE clusters but its linear power spectrum is small compared to that of matter



Boltzmann code analysis and comparison with data

Dirian, Foffa, Kunz, MM, Pettorino, JCAP 2015

- CMB data from the Planck 2013 data release, type-Ia supernovae from JLA and BAO data from BOSS

- we modified the CLASS code and use Montepython MCMC

- we vary $\omega_b = \Omega_b h_0^2,\ \omega_c = \Omega_c h_0^2,\ H_0,\ A_s,\ n_s,\ z_{\rm re}$

In ΛCDM, Ω_Λ is a derived parameter, fixed by the flatness condition. Similarly, in our model the mass parameter m^2 is a derived parameter, fixed again from $\Omega_{tot}=1$

we have the same number of free parameters as in ΛCDM

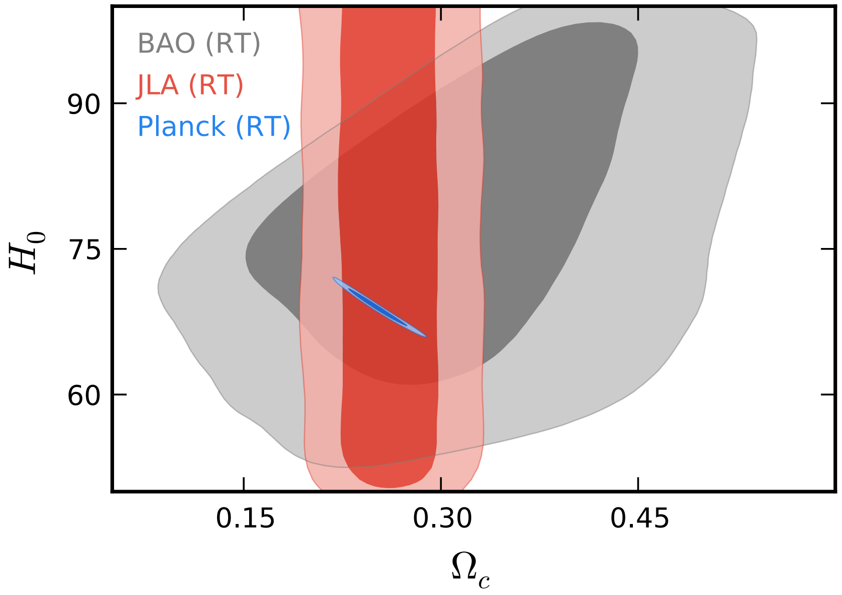
- # Results

Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
$100\ \omega_b$	$2.201^{+0.028}_{-0.029}$	$2.204^{+0.028}_{-0.03}$	$2.207^{+0.029}_{-0.029}$
ω_c	$0.1194^{+0.0027}_{-0.0026}$	$0.1195^{+0.0026}_{-0.0028}$	$0.1191^{+0.0027}_{-0.0028}$
H_0	$67.56^{+1.2}_{-1.3}$	$68.95^{+1.3}_{-1.3}$	$71.67^{+1.5}_{-1.5}$
$10^9 A_s$	$2.193^{+0.052}_{-0.06}$	$2.194^{+0.048}_{-0.062}$	$2.198^{+0.053}_{-0.059}$
n_s	$0.9625^{+0.0072}_{-0.0074}$	$0.9622^{+0.007}_{-0.0081}$	$0.9628^{+0.0074}_{-0.0073}$
z_{re}	$11.1^{+1.1}_{-1.1}$	$11.1^{+1.1}_{-1.2}$	$11.16^{+1.2}_{-1.1}$
$\chi^2_{\rm min}$	9801.7	9801.3	9800.1

Table 1: *Planck* CMB data only.

Param	ΛCDM	$g_{\mu\nu}\Box^{-1}R$	$R\Box^{-2}R$
$100\ \omega_b$	$2.215^{+0.025}_{-0.025}$	$2.207^{+0.024}_{-0.025}$	$2.197^{+0.024}_{-0.025}$
ω_c	$0.1175^{+0.0015}_{-0.0014}$	$0.1188^{+0.0014}_{-0.0014}$	$0.1204^{+0.0014}_{-0.0013}$
H_0	$68.43^{+0.61}_{-0.69}$	$69.3^{+0.68}_{-0.66}$	$70.94^{+0.74}_{-0.7}$
$10^9 A_s$	$2.199^{+0.055}_{-0.062}$	$2.196^{+0.052}_{-0.065}$	$2.192^{+0.051}_{-0.061}$
n_s	$0.9668^{+0.0055}_{-0.0054}$	$0.9636^{+0.0052}_{-0.0055}$	$0.9599^{+0.0052}_{-0.0051}$
z_{re}	$11.33^{+1.1}_{-1.1}$	$11.18^{+1.1}_{-1.2}$	$11.00^{+1.1}_{-1.2}$
$\chi^2_{\rm min}$	10485.5	10485.0	10488.7

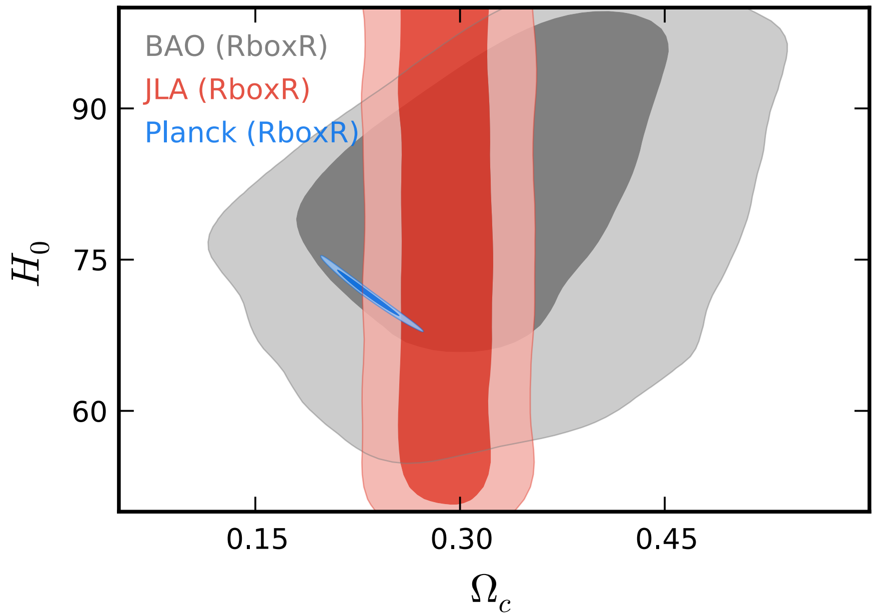
Planck+JLA+BAO

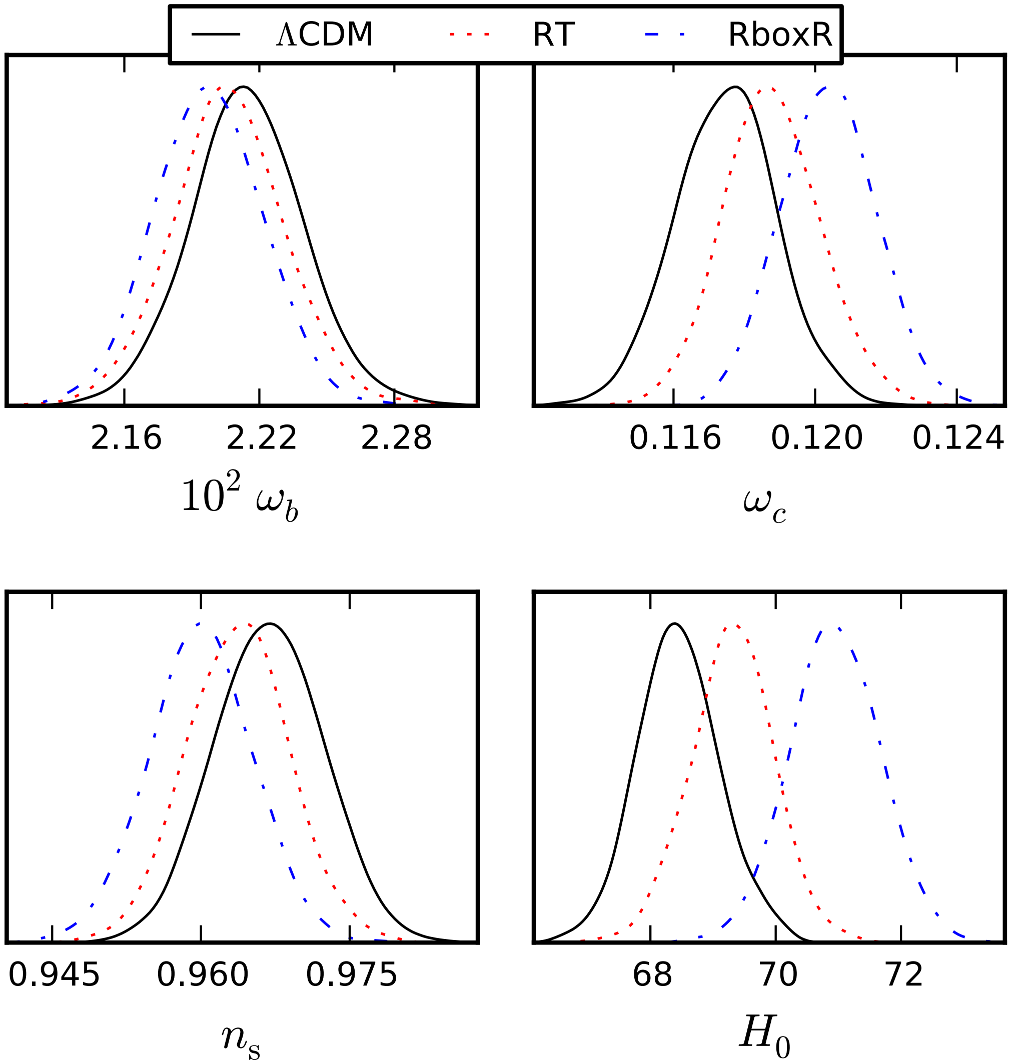


The RT model works perfectly well

(visually similar plot for ΛCDM)

The RboxR model has a slight (2σ) tension between CMB and SN





using Planck+JLA+BAO

excellent agreement with local H_0 measurements.

Latest revised value after correcting for star formation bias
$H_0 = 70.6 \pm 2.6$
(Rigault et al 1412.6501)

Conclusion: at the phenomenological level, these two non-local models work very well

- solar system tests OK
- generates dynamically a dark energy
- cosmological perturbations work well
- passes tests of structure formation
- comparison with CMB,SNe,BAO with modified Boltzmann code ok
- higher value of H0

They are the only existing models, with the same number of parameters as ΛCDM, which are competitive with ΛCDM from the point of view of fitting the data

Part 2: where such non-locality comes from?

- loop corrections involving massless or light particles give non-local terms

  e.g. in QED
$$S_{\rm eff} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}\frac{1}{e^2(\Box)}F^{\mu\nu}$$
$$\frac{1}{e^2(\Box)} = \frac{1}{e^2(\mu)} - \beta_0 \log\left(\frac{-\Box}{\mu^2}\right)$$

- in R^2 gravity
  - loops of scalar, spinor and vector field in a fixed curved background

    Barvinsky-Vilkovisky 1985,1987, [.....]
    decoupling: Gorbar-Shapiro 2003

  - graviton loops

    Fradkin-Tseytlin 1982
    Avramidi-Barvinski 1985

IR running of coupling constants and dimensional transmutation in R^2 gravity?

MM 1506.06217

$$S = \int d^4x\sqrt{-g}\,\left[\tfrac{1}{16\pi G}\,(R-2\Lambda) - \left(a_1C^2 + a_2R^2 + a_3E\right)\right]$$

- we consider the model with
  - $a_1>0$ stability of tensor perurbations
  - $a_2>0$ for matching with the sign in

$$S_{\rm NL} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\left[R - m^2R\frac{1}{\Box^2}R\right]$$

  - define $a_1=1/f^2$, $a_2=1/g^2$

- to one-loop: (neglecting the GB term)

$$S = \int d^4x\sqrt{-g}\,\left[\frac{1}{16\pi G}\,R - C_{\mu\nu\rho\sigma}\,\frac{1}{f^2(\Box)}C^{\mu\nu\rho\sigma} - R\frac{1}{g^2(\Box)}R\right]$$

$$\frac{1}{f^2(\Box)} = \frac{1}{f^2(\mu)} + \frac{\bar{\beta}}{2(4\pi)^2}\log\left(\frac{-\Box}{\mu^2}\right)$$

$$\frac{1}{g^2(\Box)} = \frac{1}{g^2(\mu)} + \frac{\bar{\alpha}}{2(4\pi)^2}\log\left(\frac{-\Box}{\mu^2}\right)$$

$$\bar{\alpha} > 0,\ \bar{\beta} > 0$$

f^2 and g^2 are asymptotically free in the UV — Avramidi-Barvinski 1985

- effect of the log terms in cosmology only relevant near the Planck scale — Donoghue-El-Menoufi 2015
- we are rather interested in the deep IR region, for dark energy

- in the IR g^2 and f^2 grow large and the theory becomes strongly coupled!

- a mass scale is generated by dimensional transmutation, analogous to ΛQCD

$$\Lambda_{\rm RR}^2 = \mu^2 \exp\left\{-\frac{2(4\pi)^2}{\bar{\alpha} g^2(\mu)}\right\}$$

- in the IR the form factors $g^2(\Box)$ and $f^2(\Box)$ will be non-trivial functions of $\Box/\Lambda_{RR}$

  Can we get $1/g^2(\Box)=(\Lambda_{RR}/\ \Box)^2$?

- generally speaking, power-like behavior can emerge from resummation of leading logs

$$\sum_{n=0}^{\infty} \frac{(-\nu)^n}{n!} \left(\log \frac{-\Box}{\Lambda_{\rm RR}}\right)^n = \left(\frac{\Lambda_{\rm RR}}{-\Box}\right)^\nu$$

- a more stringent argument?

4D-quantum gravity in the IR is dominated by the conformal mode

$$g_{\mu\nu}(x) = e^{2\sigma(x)}\bar{g}_{\mu\nu}(x)$$

Antoniadis and Mottola 1992
Antoniadis, Mazur and Mottola 2007

in the classical theory it is a constrained variable. At the quantum level it acquires dynamics because of the conformal anomaly

– in D=2: Polyakov action (which becomes local in terms of σ)

$$S_{\rm anom} = -\frac{N}{96\pi}\int d^2x\sqrt{-g}\,R\frac{1}{\Box}R$$

$$= \frac{N}{24\pi}\int d^2x\sqrt{-\bar{g}}\,(-\sigma\bar{\Box}\sigma + \bar{R}\sigma)$$

– in D=4: covariant non-local anomaly-induced action (again local in terms of σ)

$$S_{\rm anom} = -\frac{Q^2}{16\pi^2}\int d^4x\,(\Box\sigma)^2$$

fluctuations in σ become large in the IR

$$(\Box\sigma)^2 \to G(x,x') = -\frac{1}{2Q^2}\log\left[\mu^2(x-x')^2\right]$$

1. the IR dynamics is dominated by σ
2. we expect the generation of a mass scale through dimensional transmutation

How this will be reflected on the conformal mode?

Natural expectation: dynamical generation of a mass term for σ

compare with:

- generation of a mass gap in QCD or 2-dim confining theories
- BKT transition in d=2 (which also triggered by a logarithmic growth of fluctuations)

- however, we do not want to spoil diff invariance.

  no local term starts with $m^2\sigma^2$

However, setting e.g. $g_{\mu\nu} = e^{2\sigma(x)}\eta_{\mu\nu}$

$$
\begin{aligned}
R &= -6e^{-2\sigma}\left(\Box\sigma + \partial_\mu\sigma\partial^\mu\sigma\right) \\
&= -6\Box\sigma + \mathcal{O}(\sigma^2)
\end{aligned}
$$

$$m^2 R \frac{1}{\Box^2} R = 36 m^2\sigma^2 + \mathcal{O}(\sigma^3)$$

our non-local term is just a mass-term for σ, plus non-linear completion that make it diff-invariant !

$$\rightarrow \frac{1}{g^2(\Box)} \simeq \frac{\Lambda_{\rm RR}^4}{\Box^2}$$

$$\Lambda_{\rm RR}^4 \sim m^2 M_{\rm Pl}^2$$

$$\rightarrow \Lambda_{\rm RR} \sim \rm meV$$

final comment: a new perspective on the naturalness problem for the cosmological constant:

the scale Λ_{RR} emerges from dimensional transmutation, similarly to Λ_{QCD}. There is no issue of naturalness for such a quantity, which is determined by the logarithmic running of a dimensionless coupling constant

Thank you!

a locality / gauge-invariance duality for massive gauge fields

- Proca theory for massive photons

$$S = \int d^4x \left[-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} - \tfrac{1}{2} m_\gamma^2 A_\mu A^\mu - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} - m_\gamma^2 A^\nu = j^\nu \quad \rightarrow \left\{ \begin{array}{r} m_\gamma^2 \, \partial_\nu A^\nu = 0 \\ (\Box - m_\gamma^2) A^\mu = 0 \end{array} \right.$$

- non-local formulation (Dvali 2006)

Stueckelberg trick: $A_\mu \rightarrow A_\mu + \frac{1}{m_\gamma} \partial_\mu \varphi$

we add one field and we gain a gauge symmetry

$$A_\mu \rightarrow A_\mu - \partial_\mu \theta \,, \quad \varphi \rightarrow \varphi + m_\gamma \theta$$

$$S = \int d^4x \, \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_\gamma^2 A_\mu A^\mu - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - m_\gamma A^\mu \partial_\mu \varphi - j_\mu A^\mu \right]$$

$$\partial_\mu F^{\mu\nu} = m_\gamma^2 A^\nu + m_\gamma \partial^\nu \varphi + j^\nu \, ,$$
$$\Box \varphi + m_\gamma \partial_\mu A^\mu = 0 \, .$$

If we choose the unitary gauge ϕ=0 we get back to the original formulation of Proca theory (and loose the gauge sym because of gauge fixing).

Instead, keep the gauge sym explicit and integrate out ϕ using its own equation of motion:

$$\varphi(x) = -m_\gamma \Box^{-1} (\partial_\mu A^\mu)$$

Substituting in the eq of motion for A^ν :

$$\left(1-\frac{m_\gamma^2}{\Box}\right)\partial_\mu F^{\mu\nu}=j^\nu$$

or

$$(\Box-m_\gamma^2)A^\nu=\left(1-\frac{m_\gamma^2}{\Box}\right)\partial^\nu\partial_\mu A^\mu+j^\nu$$

we have explicit gauge invariance for the massive theory,
at the price non-locality

- a sort of duality between explicit gauge-invariance and explicit locality
- we can fix the gauge $\partial_\mu A^\mu=0$ and the non-local term disappears (and we are back to Proca eqs.)
- with hindsight, the Stueckelberg trick was not needed

- sufficiently close to ΛCDM to be consistent with existing data, but distinctive prediction that can be clearly tested in the near future

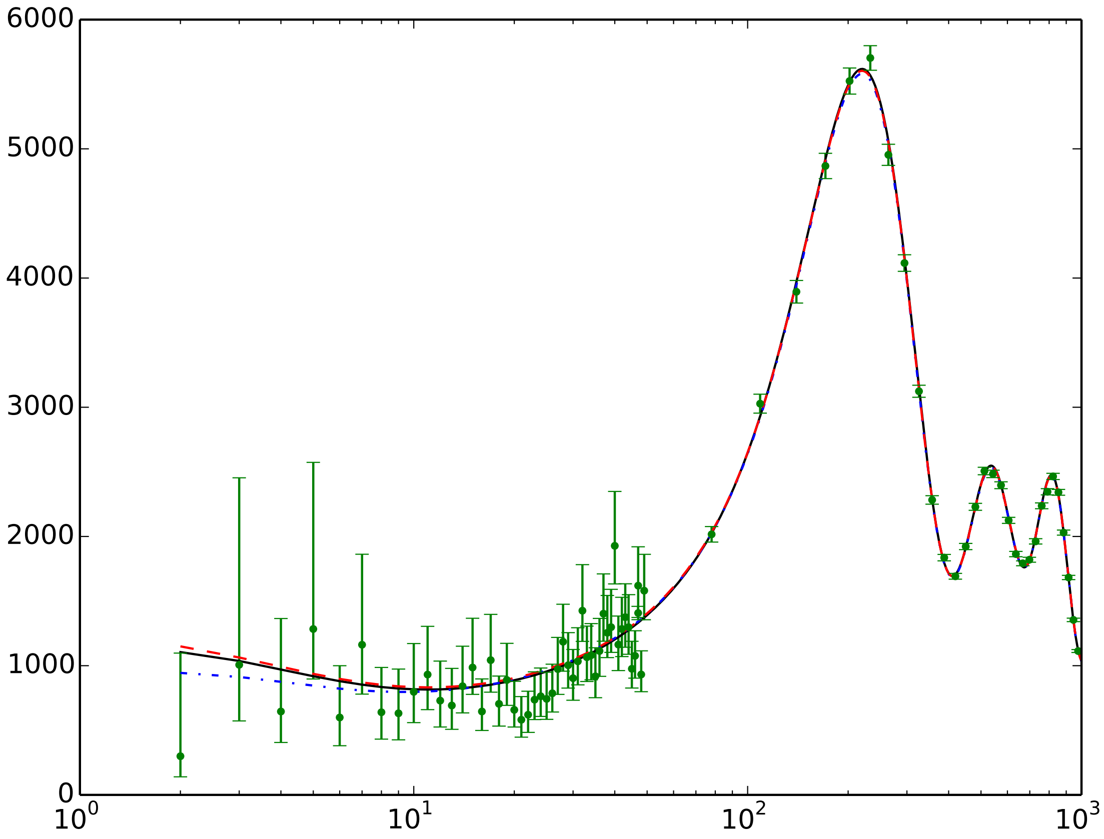
  - phantom DE eq of state: w(0)= - 1.14 (RboxR) (or -1.04 RT) + a full prediction for w(z)
    - DES Δw=0.03
    - EUCLID Δw=0.01
  - linear structure formation

    $$\mu(a) = \mu_s a^s \rightarrow \mu_{\mathbf{s}} = \mathbf{0.09}, \mathbf{s} = \mathbf{2}$$

    - Forecast for EUCLID, Δμ=0.01
  - non-linear structure formation: 10% more massive halos

Barreira, Li, Hellwing, Baugh, Pascoli 2014

  - lensing: deviations at a few %



LCDM and RT model almost indistinguishable
RboxR (blue dot-dashed) lower at low multipoles

An aside: the Deser-Woodard non-local model

with phenomenological motivations similar to ours, has been proposed a model of the form

$$S = \int d^4x \sqrt{-g}\,\left[R + Rf(\Box^{-1}R)\right]$$

Deser and Woodard 2007

much activity on ``reconstruction'' of f(R):

$$f(X) = a_1[\tanh(a_2 + a_3 X + a_4 X^2 + a_5 X^3) - 1]$$

- not predictive at the background level: chosen to mimic ΛCDM
- by comparison, our model is

$$S = \int d^4x\,\left[R - m^2 f(\Box^{-1}R)\right] \qquad f(X) = X^2$$

after fixing the background evolution in this way, one can compute cosmological perturbations in the Deser-Woodard model, and compare with data

Deser-Woodard model ruled out at the 8σ level by structure formation

Dodelson and Park 2013