Model Independent Constraints on Correlation Functions in Invariance in CMB and on the Early Galaxy Distribution from CFT

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OUTLINE

- Physical Motivation and a Preview of Basic Results
- Hypothesis of Criticality Based on Observational Evidence
- How the 3D Conformal Group Makes its Appearance in Dark Energy Dominated Cosmology
- Implications of Conformal (Co-)In-variance
- The Form of Correlations and Graphical Representation

The presence of `dark energy' in the early universe leads to the de Sitter Phase in cosmological expansion and therefore to CMB non-Gaussianity.

This is so because the early `dark energy' driven de Sitter Phase leads to the highly correlated (entangled) initial macroscopic quantum state which necessarily leads to non-Gaussian spectrum of the primordial energy density fluctuations.

Quantum correlations in the initial macroscopic quantum state also lead to the emergent dynamical symmetry that manifests itself as conformal group of the spatially flat slices of the FLRW cosmological expansion.
This emergent dynamical symmetry constrains almost uniquely the form of the large scale two-point and three-point correlation functions of primordial energy density (temperature) fluctuations.

In addition to simple scale invariance, a universe dominated by `dark energy' naturally gives rise to correlation functions possessing full conformal invariance.
This is due to the mathematical isomorphism between the conformal group of certain three dimensional slices of de Sitter space and the de Sitter isometry group SO(4,1). In the standard homogeneous, isotropic cosmological model in which primordial density (temperature) perturbations are generated during a long vacuum energy dominated de Sitter phase, the embedding of flat spatial sections in de Sitter space induces a conformal invariant perturbation spectrum and definite prediction for the `shape' of the non-Gaussian CMB bi-spectrum. The bi-spectrum is intrinsic to the symmetries of de Sitter space.

Detection of non-Gaussian correlations in the CMB of these bi-spectral `shape' functions can both pinpoint the origins of primordial density fluctuations, and distinguish between different dynamical models of cosmological vacuum `dark energy'. The primordial power spectrum (the two-point function) and the bi-spectrum (the three-point function) is transformed by the subsequent cosmological expansion to the recombination era in the Sachs-Wolfe large angle regime. One has to convolve the primordial CMB spectrum with the photon transfer functions.
A specific kind of statistic is introduced which is well-suited to the comparison of the predictions based on conformal invariance to the experimental/ observational data.

BASIC FORMULAE FOR CMB PHYSICS

The formula for temperature fluctuations later specified for the Sachs-Wolfe regime of large angles

$$\frac{\Delta T}{T}(\mathbf{x},\tau,\mathbf{n}) = \int d^3k e^{i\mathbf{k}\cdot\mathbf{x}}\Delta(\mathbf{k},\mathbf{n},\tau)$$

where $\mathbf{n}$ is the direction of photon momentum, and we set the observation point to be at the origin $\mathbf{x} = 0$ and at the present epoch $\tau = \tau_0$

$$\frac{\Delta T}{T}(\mathbf{n}) = \int d^3k \Delta(\mathbf{k},\mathbf{n}) = \int d^3k \sum_{l=0}^{\infty}(-i)^l(2l+1)\psi(\mathbf{k})\Delta_l(k)P_l(\mathbf{k}\cdot\mathbf{n})$$

$\psi(\mathbf{k})$ is the initial gravitational-potential perturbation

$\Delta_l(k)$ is the photon transfer function; the photon evolution equation is independent of the wave vector direction

$$\langle\psi(\mathbf{k_1})\psi(\mathbf{k_2})\rangle = P_\psi^{(2)}(k)\delta_D(\mathbf{k}_1 + \mathbf{k}_2)$$

the amplitude above is the primordial power spectrum. For a scale-free primordial power spectrum, we have $P_\psi^{(2)}(k) \propto k^{n-4}$

Index n=1 corresponds to a flat scale-invariant spectrum, which is close to those favored by Harrison and Zel'dovich. The CMB temperature pattern may be written in a spherical-harmonic expansion with coefficients and the angular two-point correlation function

$$a_{lm} = \int d^2\mathbf{n} Y_{lm}(\mathbf{n}) \frac{\Delta T}{T}(\mathbf{n})$$

$$\xi(\mathbf{n_1}, \mathbf{n_2}) = \langle \frac{\Delta T}{T}(\mathbf{n_1}) \frac{\Delta T}{T}(\mathbf{n_2}) \rangle = \sum_m \frac{2l+1}{4\pi} C_l P_l(\mathbf{n_1} \cdot \mathbf{n_2})$$

where the CMB power spectrum is

$$C_l = (4\pi)^2 \int k^2 \, dk \, P_\psi^{(2)}(k) \, |\Delta_l(k)|^2$$

$$\langle a_{l_1 m_1} a^*_{l_2 m_2} \rangle = \delta_{l_1 l_2} \delta_{m_1 m_2} \, C_l,$$

Three-point correlation function and bi-spectrum

$$\langle\psi(\mathbf{k_1})\psi(\mathbf{k_2})\psi(\mathbf{k_3})\rangle = P_\psi^{(3)}(k_1,k_2,k_3)\delta_D(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

$$\begin{aligned}\xi(\mathbf{n_1},\mathbf{n_2},\mathbf{n_3}) &\equiv \langle\frac{\Delta T}{T}(\mathbf{n_1})\frac{\Delta T}{T}(\mathbf{n_2})\frac{\Delta T}{T}(\mathbf{n_3})\\ &= \sum_{l_i,m_i}\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle Y_{l_1m_1}(\mathbf{n_1})Y_{l_2m_2}(\mathbf{n_2})Y_{l_3m_3}(\mathbf{n_3})\end{aligned}$$

Two-point and three-point CFT correlation functions

$$< \mathcal{O}_\Delta(\mathbf{x_1})\mathcal{O}_\Delta(\mathbf{x_2}) >_{CFT} = A_2(\Delta)|\mathbf{x_1} - \mathbf{x_2}|^{-\Delta}$$

$$< \mathcal{O}_\Delta(\mathbf{x_1})\mathcal{O}_\Delta(\mathbf{x_2})\mathcal{O}_\Delta(\mathbf{x_3}) >_{CFT} = A_3(\Delta) \times$$
$$|\mathbf{x_1} - \mathbf{x_2}|^{-\Delta} |\mathbf{x_2} - \mathbf{x_3}|^{-\Delta} |\mathbf{x_3} - \mathbf{x_1}|^{-\Delta}$$

Two-point function for GWs in CMB following from 3D CFT

$$\langle h_{\mu\nu}(x_1)h_{\rho\sigma}(x_2)\rangle = P_{\mu\nu\rho\sigma}(x_1,x_2)(x^2)^{-\eta}$$

Where $x = x_1 - x_2$ scaling dimension η

$$P_{\mu\nu\rho\sigma}(\lambda x_1,\lambda x_2) = P_{\mu\nu\rho\sigma}(x_1,x_2)$$

$$P_{\mu\mu\rho\sigma}(x_1, x_2) = P_{\mu\nu\rho\rho}(x_1, x_2) = 0$$

$$P_{\mu\nu\rho\sigma}(x_1, x_2) = P_{\rho\sigma\mu\nu}(x_2, x_1)$$

$$P_{\mu\nu\rho\sigma}(x_1, x_2) = P_{\mu\nu\rho\sigma}(n)$$

$$P_{\mu\nu\rho\sigma}(n) = aH^1_{\mu\nu}(n)H^1_{\rho\sigma} + bH^2_{\mu\nu\rho\sigma}(n) + cH^3_{\mu\nu\rho\sigma}$$

$$H^1_{\mu\nu}(n) = n_\mu n_\nu - \frac{1}{d}\delta_{\mu\nu}$$

$$H^2_{\mu\nu\rho\sigma}(n) = n_\mu n_\rho \delta_{\nu\sigma} + n_\nu n_\rho \delta_{\mu\sigma} + n_\mu n_\sigma \delta_{\nu\rho} + n_\nu n_\sigma \delta_{\mu\rho} - \frac{4}{d} n_\mu n_\nu \delta_{\rho\sigma} - \frac{4}{d} n_\rho n_\sigma \delta_{\mu\nu} + \frac{4}{d^2}\delta_{\mu\nu}\delta_{\rho\sigma}$$

$$H^3_{\mu\nu\rho\sigma} = \delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \frac{2}{d}\delta_{\mu\nu}\delta_{\rho\sigma}$$

Imposing the condition of transversality on the two-point function of traceless tensors we obtain three (3) algebraic eqs. for the amplitudes a, b, and c. However, one can compute this two-point function in much easier way in momentum space

$$P_{\mu\nu\rho\sigma}(n)(x^2)^{-\eta} = C\int (dk)e^{ikx}(k^2)^{-\eta'}\left(\Pi_{\mu\rho}(k)\Pi_{\nu\sigma}(k) + \Pi_{\mu\sigma}(k)\Pi_{\nu\rho}(k) - \frac{2}{d-1}\Pi_{\mu\nu}(k)\Pi_{\rho\sigma}(k)\right)$$

$$\Pi_{\mu\nu}(k) = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$\eta' = \frac{d}{2} - \eta$$

$$C = a\frac{(d-1)\eta'(\eta'+1)A(\eta)}{2(d-2)\eta(\eta+1)}$$

$$(x^2)^{-\eta} = A(\eta)\int (dk)e^{ikx}(k^2)^{-\eta'}$$

$$A(\eta) = \frac{\Gamma(\eta')}{2^{2\eta}\pi^{\frac{d}{2}}\Gamma(\eta)}$$

The spin-two CFT correlation function can then be used to compute the effect of relic gravitons on the CMB polarization in the so-called B-modes. This is being done (I. Antoniadis, POM and E. Mottola)

Coming back to the issue of non-Gaussianity of CMB we have previously predicted (the non-vanishing three-point functions in 3D CFT) we shall present the computation of the bi-spectrum in the large angle Sachs-Wolfe regime which is also the regime where Conformal Invariance holds (the case of intermediate asymptotic). The computation must involve the transfer functions in the S-W regime. They are well known (it is a textbook material now)

At the large angular scales relevant for CFT behavior the Sachs-Wolfe
effect dominates, and we simply have

$$\frac{\Delta T}{T}(\mathbf{n}) = A_{\mathrm{sw}} \int d^3k \psi(\mathbf{k}) e^{i\Delta\eta \mathbf{k}\cdot\mathbf{n}}$$

Where $\Delta\eta = (\eta_0 - \eta_*)$

is the conformal time between now and the surface of last scatter, and A=1/3 for a critical-density model with primordial adiabatic perturbations.

Using this formula one can immediately compute the `processed' scalar three-point correlation function. The `processing' is done by the transfer function in the Sachs-Wolfe regime. The final result is given by a very simple formula

$$\xi(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3) = C\left|\mathbf{n}_1 - \mathbf{n}_2\right|^{-\Delta}\left|\mathbf{n}_2 - \mathbf{n}_3\right|^{-\Delta}\left|\mathbf{n}_3 - \mathbf{n}_1\right|^{-\Delta}$$

For temperature fluctuations the scaling exponent is close to zero. One can directly expand this expression in terms of spherical harmonics.

$$C = A_3 \Delta\eta^{-3\Delta}$$

The conformal time difference between the time of last scattering and the present epoch enters this formula. One then computes the bi-spectrum in the l-space because this is what is measured in CMB

$$B_{l_1 l_2 l_3}$$

The Connection Between QFTs on de Sitter and the Euclidean CFTs Was Established Before the AdS/CFT Conjecture

Ignatios Antoniadis, Pawel O. Mazur, and Emil Mottola:
Relevant papers and applications to physics and cosmology:

Phys. Rev. Lett. 79, 14 (1997);   October 1996 at Ecole Polytechnique

E-print arXiv: astro-ph/9705200, (unpublished);
May 1997, University of South Carolina

"Conformal Invariance, Dark Energy, and CMB Non-Gaussianity";
E-print arXiv: 1103.4164 (March 21st, 2011);
From Dec. 2009 at CERN to October 2010 at JPL

In order to see this relationship [AMM 1997; Mazur & Mottola, 1998, unpublished; and MM in PRD, 2001; this is the so-called dS/CFT correspondence] consider for illustrative purposes a free massive scalar field on the (d+1)-dimensional de Sitter space with the metric given in spatially flat coordinates

$$ds^2 = d\tau^2 - e^{2H\tau} d\vec{x}^2$$

One can easily see that the rescaling of a spatial slice coordinates can be compensated by a shift in the cosmic time. This symmetry of the vacuum energy dominated Universe, and an approximate symmetry of the `slow roll inflation', is responsible for the Zel'dovich-Harrison power spectrum of the primordial energy density fluctuations (modulo the assumption of `steady pumping') .

The massive scalar is described by the action:

$$I = \frac{1}{2} \int d\tau (dx)_d \sqrt{-g} \left(g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - m^2 \Phi^2 \right)$$

The Wightman two-point correlation function, for equal cosmic times, has the following asymptotic, with D(x,x’) an invariant distance in de Sitter

$$< \Phi(x)\Phi(x') >\sim A\, D(x,x')^{-2\Delta_+} + B\, D(x,x')^{-2\Delta_-}$$

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - m^2\, R^2}$$

is the highest weight for SO(1,d+1), or the scaling dimension in CFT, and

$$C_2 = m^2\, R^2$$

is an eigen-value of the first quadratic Casimir operator for SO(1,d+1) acting on scalars and R is the radius of curvature of de Sitter space.

The fundamental formula for the scaling dimensions given above describes the highest weight for the UIRRs of the Lorentz Group SO(1,d+1), and at the same time the Isometry Group of the D=d+1 dimensional de Sitter space. The Dirac's (1935) group isomorphism (Ann. Math.) relating the Conformal Group C(d) of the d-dimensional Euclidean space and SO(1,d+1) = de Sitter(d+1) Group was noticed many times before. For d=2, it is the well known SO(1,3)=SL(2,C) (modulo the center) isomorphism standing behind Cartan's theory of spinors. The geometry of orbits of SO(1,3), UIRRs of SL(2,C), Integral Geometry, the Gelfand-Graev integral transform are the irreducible components of theoretical physicist's tools.
It easily generalizes to the case of an arbitrary dimension of `space' d.

It has been known now for 40 years [A.M. Polyakov, 1970]
 that conformal invariance leads to the universal form of the two- and three-point correlation functions.

One may reason, in analogy to the theory of the continuous second order phase transitions where the scaling hypothesis was enunciated [A.Z. Patashinskii & V.L. Pokrovskii, 1966; L. Kadanoff, 1966], explained by QFT [A.M. Polyakov, 1968; A.A. Migdal 1968; K. Wilson, 1969], and then generalized to the full conformal invariance hypothesis [A.M. Polyakov, 1970], that in the other physical situations where the similar scaling properties have been uncovered experimentally/observationally the full conformal invariance should demonstrate itself as an emergent dynamical symmetry of the general critical points (manifolds, generally). We used to call this the Conformal Invariance Hypothesis. One can show more

Our Universe seems to be an example of the dynamical system near (quantum) criticality. This is the QC Conjecture. This conjecture seems to be supported by the observational/experimental data.
The evidence comes from the behavior of the mass spectrum of the Standard Model, the existence of a non-vanishing, positive and non-negative Lorentz (de Sitter) invariant vacuum energy density, and the following cosmological data:

★ The COBE, WMAP CMB data indicate that the primordial energy density fluctuations are in the scaling regime

★ The Galaxy Counting, the LSS, again, indicates that the distribution of central Black Hole Candidates (BHCs) displays scaling properties

At large distances, or large angles, in the so-called Sachs-Wolfe (S-W) regime the primordial density fluctuations seen in the CMB are characterized by the power spectrum

$$< \delta\epsilon(\vec{x})\delta\epsilon(\vec{y}) > \sim |\vec{x} - \vec{y}|^{-2\Delta_s}$$

where $\Delta_s \approx 2.0$

The spectral index in the momentum space is:

$$n_s = 2\Delta_s - 3$$

$$n_s \approx 1.0$$

The spatial distribution of galaxies is characterized by the connected two-point function for the number density

$$< n(x)n(y) >_{conn} \sim |x - y|^{-2\delta}$$

where

$$\delta \approx 0.9$$

Now, it has been proposed that the final state of gravitational collapse is the gravitational Bose-Einstein Condensate vacuum energy `bubble', a.k.a. `gravastar' [Mazur and Mottola 2001,2004]

It appears that the formation process of vacuum energy `bubbles' is an example of the quantum critical phenomenon of the same kind as the formation process of our Universe (Quantum Criticality Hypothesis).

It must be then characterized by critical exponents and, therefore, the scaling laws emerge. Now, the `vacuum energy bubble' formation at criticality is an example of critical percolation.

It was observed around 1990/91 that the critical 3D percolation of (Kelvin) vortices can explain the celebrated K41 Kolmogorov-Obukhov 5/3 law

$$E(k) \sim k^{-\frac{5}{3}}$$

Bershadskii (1990/91) has derived the most remarkable formula for the fractal dimensions in the fully developed isotropic, small scale turbulence

$$D_{\pm} = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{2}{\nu}}$$

Here the correlation length critical exponent ν characterizing critical 3D percolation makes its appearance. It is defined in the following way

$$L \sim |p - p_c|^{-\nu}$$

where L is the length of the percolating cluster, and p is the probability to find a `bubble’ in a given spatial `cell’.

It goes without saying that once I have seen this formula I immediately realized that its similarity to the formulae one encounters in QFT on de Sitter cannot be an accident at all. It appears that the 3D critical percolation and QFTs on de Sitter space are related in the sense that they are different dynamical realizations, in different dimensions, of the isomorphism between the de Sitter isometry group and the conformal group of the Euclidean space of one dimension lower. This indicated to me that the fluctuation spectrum seen in CMB and galaxy formation on `seeds' which were the vacuum energy `bubbles' or BHCs are related to quantum criticality, and therefore they go the long way to support the hypothesis that the process of formation of our Universe was a Quantum Critical Phenomenon, which, as such, cannot be described adequately by Gaussian distributions or the free field QFTs.

I was then led to the conjecture that the spectrum of fluctuations `seen' in the CMB and the galaxy counting correlations must be intrinsically non-Gaussian and display the universal behavior typical of critical phenomena.
In addition I find that the critical exponents for the 3D critical percolation and the scaling dimension for galaxy distribution seem to coincide

$$\nu \approx \delta \approx 0.9$$

It is because the scaling dimensions must be real that the nice inequality derives

$$\nu \approx \delta \geq \frac{8}{9} = 0.888...$$

Most recently [arXiv:1103.4164, Conformal Invariance, Dark Energy, and CMB Non-Gaussianity, March 21, 2011, Ignatios Antoniadis, Pawel O. Mazur, Emil Mottola]

we have presented the detailed analytic (and graphical) form of the universal non-Gaussianity in the CMB, the so-called Bispectrum.

The 3D CFT describing the critical behavior in the primordial fluctuations seen in the CMB is closely related to the presence of the early stage in the evolution of our Universe where the vacuum energy density makes its appearance.

It does not seem likely that the non-vanishing small positive vacuum energy density made its appearance relatively recently at moderate redshifts and was not there before at the much, much larger redshifts.

In fact, there are indications both on the theoretical and the observational grounds that a small sub-leading vacuum energy was present at the much earlier times.
We know it from the observed presence of the 30 million solar masses central BHC in the protogalaxy of one billion solar masses in a galaxy protocluster observed at the redshift corresponding to some 600-700 milion years after the `BB'.

Nature, January/February 2011

The appearance of massive protoclusters with a major protogalaxy harboring a massive BHC seems to be indicative that a vacuum energy `bubbles’ (gravastars) with a mass spectrum with the mass cutoff must have percolated at very high redshifts. The mass cutoff of the order of 10000 solar masses must be present in order that the primordial vacuum energy `bubbles’ did not produce an detectable imprint on the small scale CMB structure.

If this scenario is to be consistent with the laws of QM and Gravitation, then the formation/critical percolation of the vacuum energy `bubbles’ should lead to the scaling law for their mass spectrum.
The situation here seems to be much like with the `quantum quenches’.

A FEW ELEMENTARY FACTS ABOUT 3D CFT

The two- and three-point functions are universal, i.e. in Euclidean Conformal Field Theory the two-point correlation function of two primary operators and the three-point function of primary operators is completely fixed by conformal invariance.
The correlation functions are:

$$< \mathcal{O}_\Delta(\mathbf{x_1})\mathcal{O}_\Delta(\mathbf{x_2}) >_{CFT} = A_2(\Delta)|\mathbf{x_1} - \mathbf{x_2}|^{-\Delta}$$

$$< \mathcal{O}_\Delta(\mathbf{x_1})\mathcal{O}_\Delta(\mathbf{x_2})\mathcal{O}_\Delta(\mathbf{x_3}) >_{CFT} = A_3(\Delta) \times$$
$$|\mathbf{x_1} - \mathbf{x_2}|^{-\Delta}\,|\mathbf{x_2} - \mathbf{x_3}|^{-\Delta}\,|\mathbf{x_3} - \mathbf{x_1}|^{-\Delta}$$

THE CMB BISPECTRUM FOLLOWING FROM THE 3D CFT

The three-point correlation function in 3D CFT is universal. It depends only on two parameters, the amplitude and the scaling dimension, which is the same as in the two-point function. Therefore, the power spectrum and the bi-spectrum of the CMB which follows from the critical behavior of fluctuations are characterized completely by only three parameters, two amplitudes and the scaling dimension.

The Fourier transform of the three-point function is
(for convenience of notation we change Delta to w now)

$$\tilde{G}_3(\vec{k}_1,\vec{k}_2,\vec{k}_3;w) = A_3(w)\left[\frac{2^{3-w}\pi^{\frac{3}{2}}\Gamma\left(\frac{3-w}{2}\right)}{\Gamma\left(\frac{w}{2}\right)}\right]^3 \delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\int\frac{d^3\vec{p}}{|\vec{p}-\vec{k}_1|^{3-w}|\vec{p}+\vec{k}_2|^{3-w}|\vec{p}|^{3-w}}$$

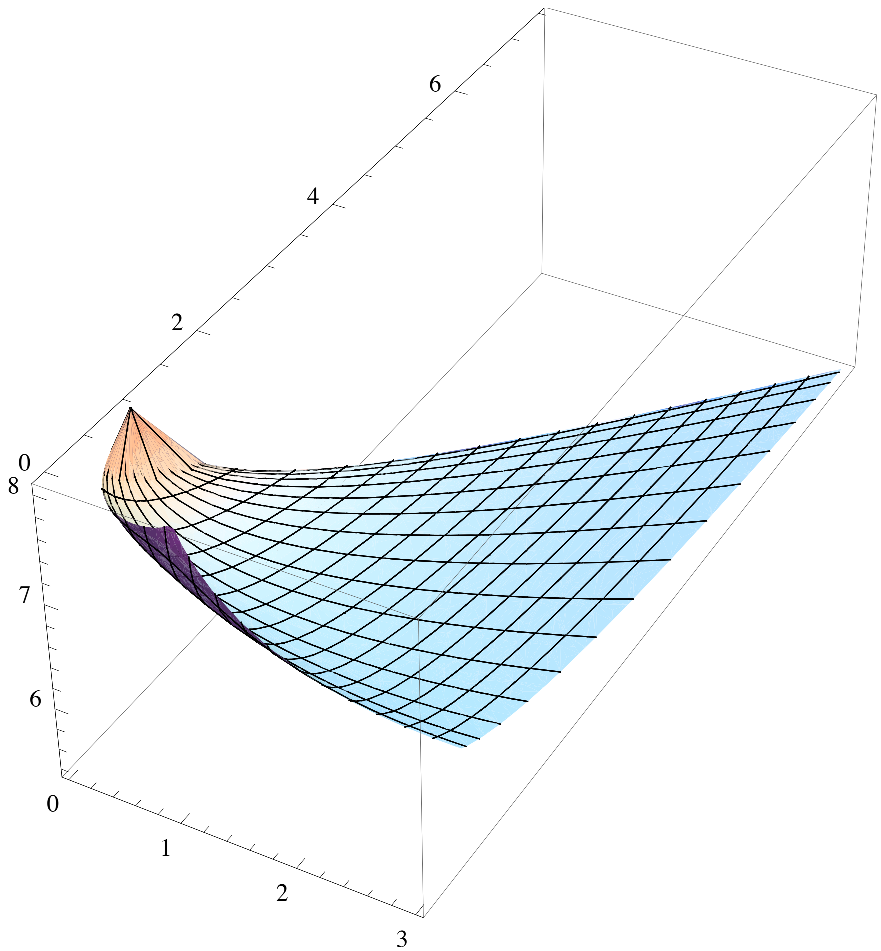
$$= A_3(w)\,\frac{2^{3-3w}}{\left[\Gamma\left(\frac{w}{2}\right)\right]^2}\,(2\pi)^6\,\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,(k_1)^{3w-6}\,S\left(X,Y;w\right)$$

$$X \equiv \frac{k_2^2}{k_1^2}\,, \qquad\qquad Y \equiv \frac{k_3^2}{k_1^2}$$

$$S(X,Y;w) = \frac{\Gamma\left(3-\frac{3w}{2}\right)}{\Gamma\left(\frac{w}{2}\right)} \int_0^1 du \int_0^1 dv \, \frac{\left[u(1-u)v\right]^{\frac{1}{2}-\frac{w}{2}} (1-v)^{\frac{w}{2}-1}}{\left[u(1-u)(1-v)+(1-u)vX+uvY\right]^{3-\frac{3w}{2}}}$$

$$= \frac{2}{\sqrt{\pi}} \Gamma\left(3-\frac{3w}{2}\right) \Gamma\left(\frac{3}{2}-\frac{w}{2}\right) \int_0^1 du \, \frac{\left[u(1-u)\right]^{\frac{1}{2}-\frac{w}{2}}}{\left[(1-u)X+uY\right]^{3-\frac{3w}{2}}} F\left(3-\frac{3w}{2}, \frac{w}{2}; \frac{3}{2}; \mathcal{Z}(X,Y;u)\right)$$

$$\mathcal{Z}(X,Y;u) \equiv 1 - \frac{u(1-u)}{(1-u)X+uY}$$



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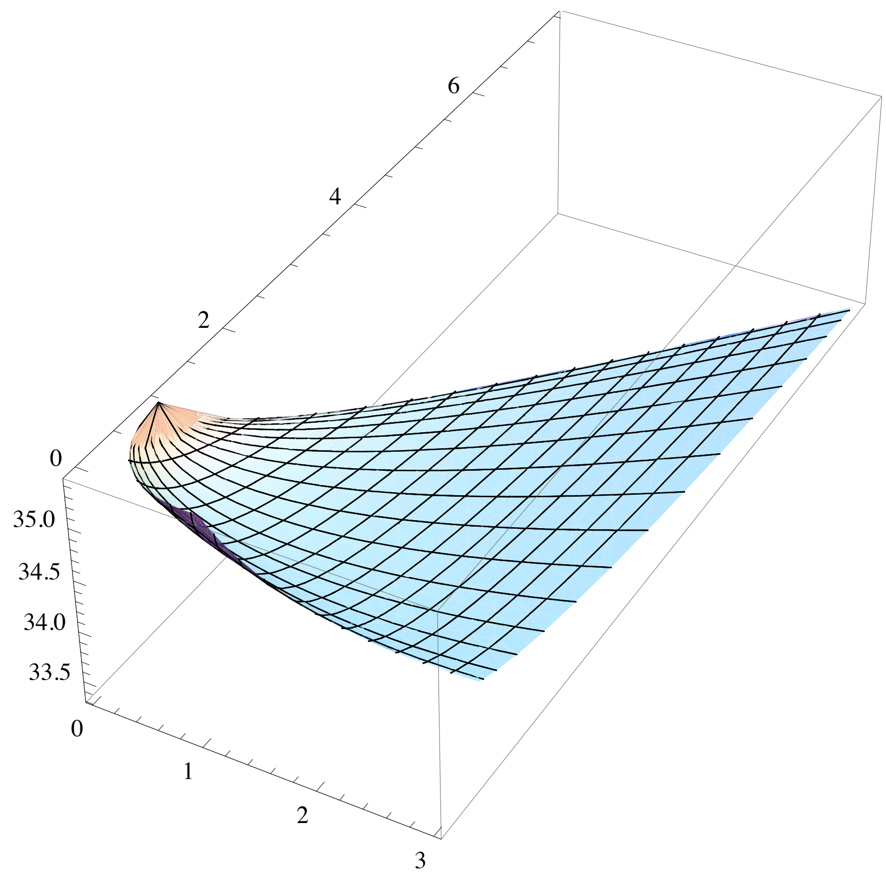
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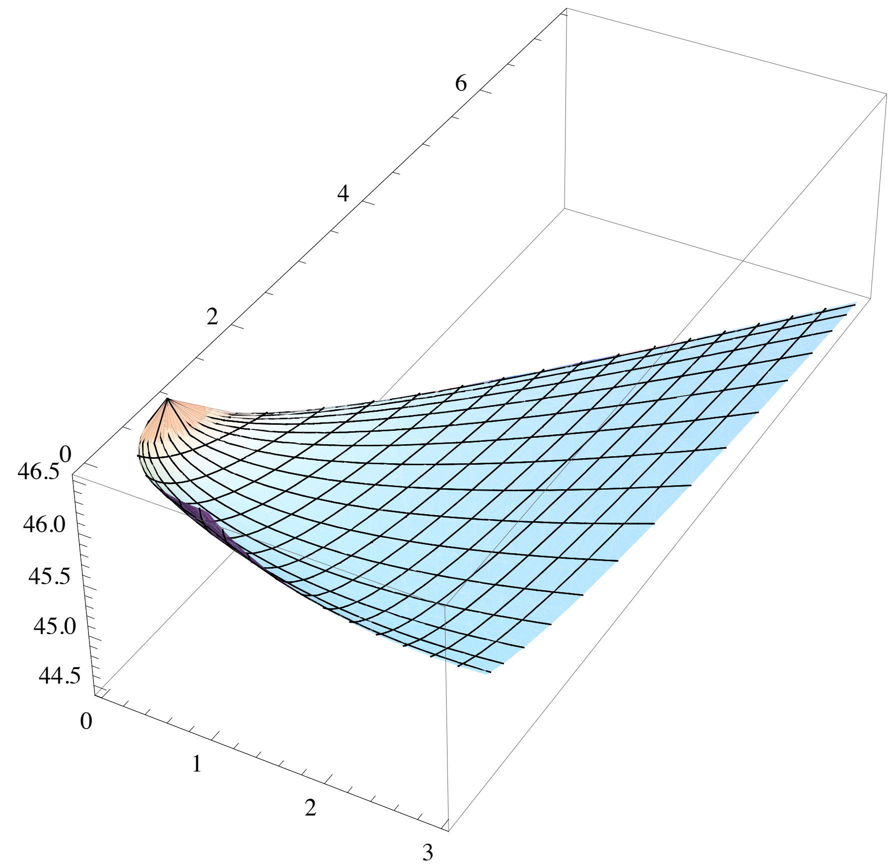

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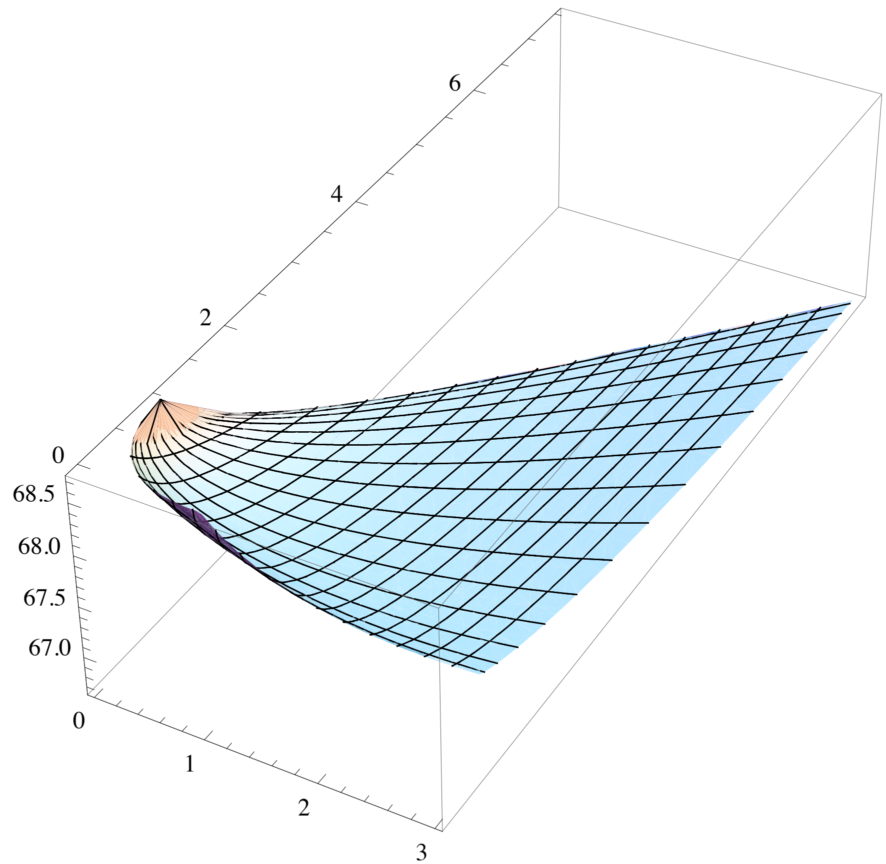
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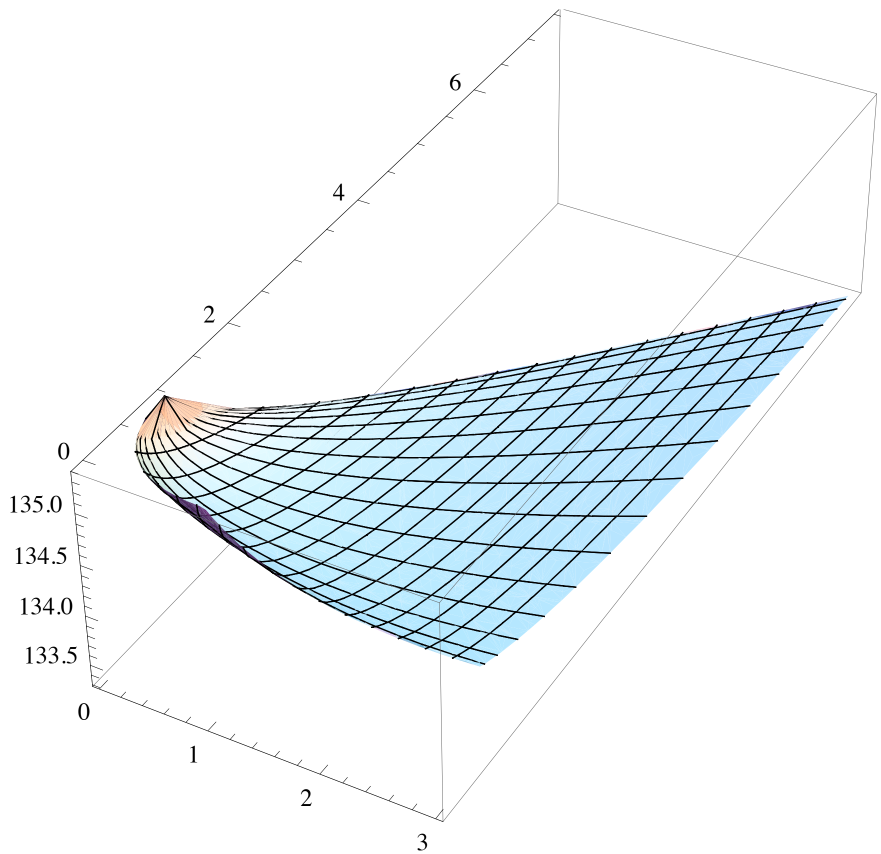
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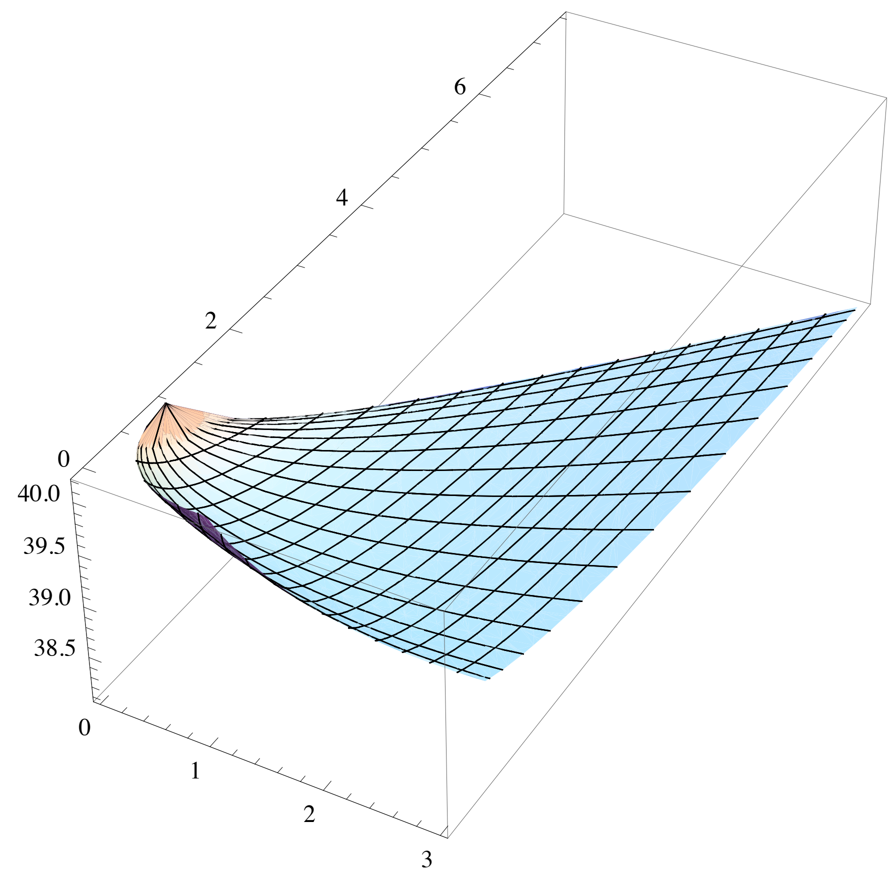

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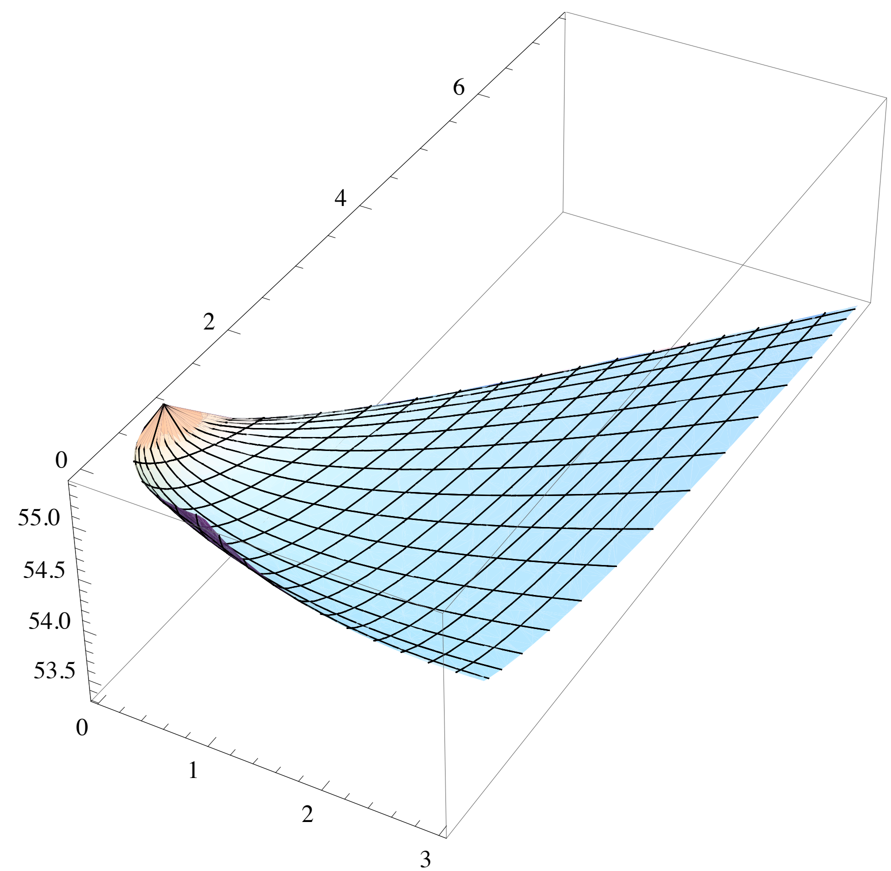
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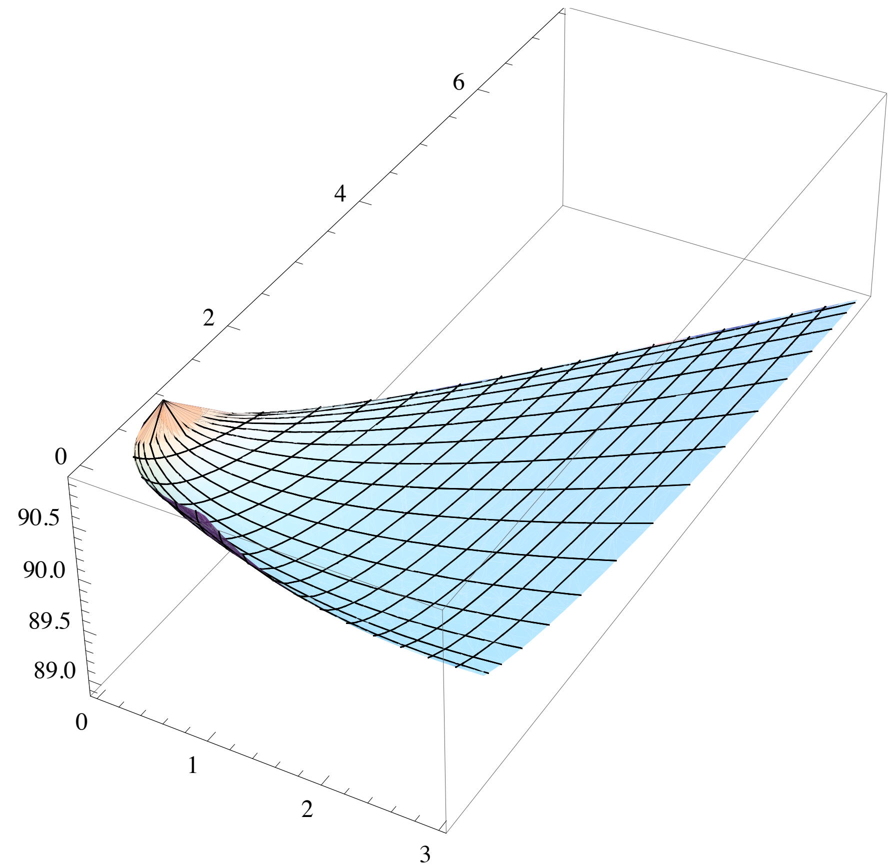
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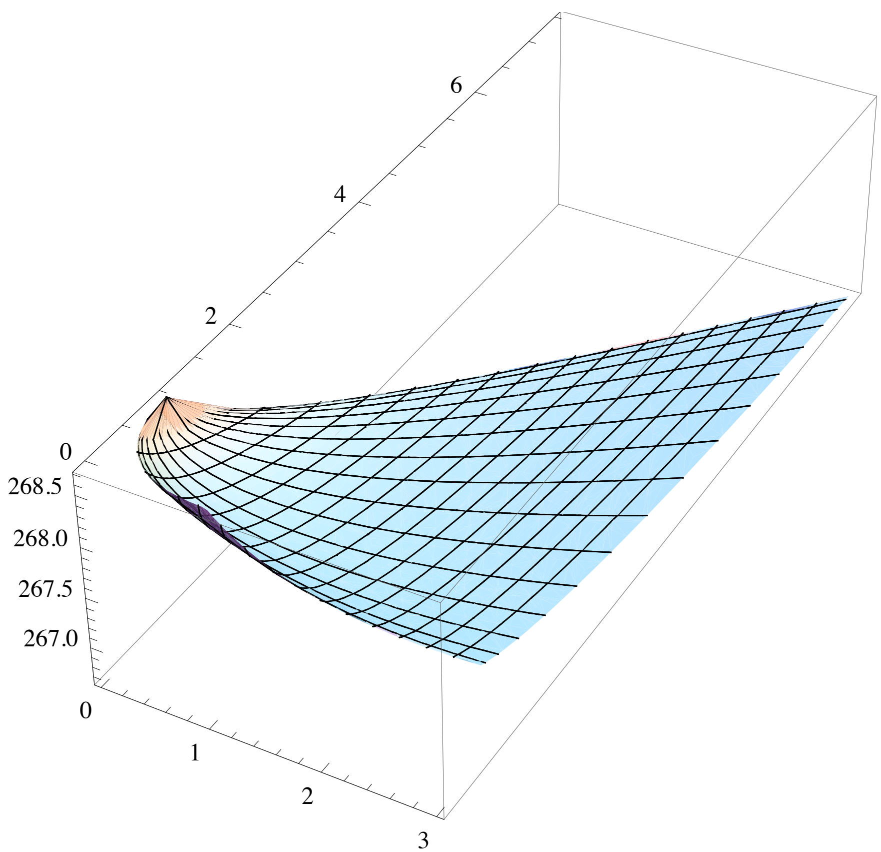
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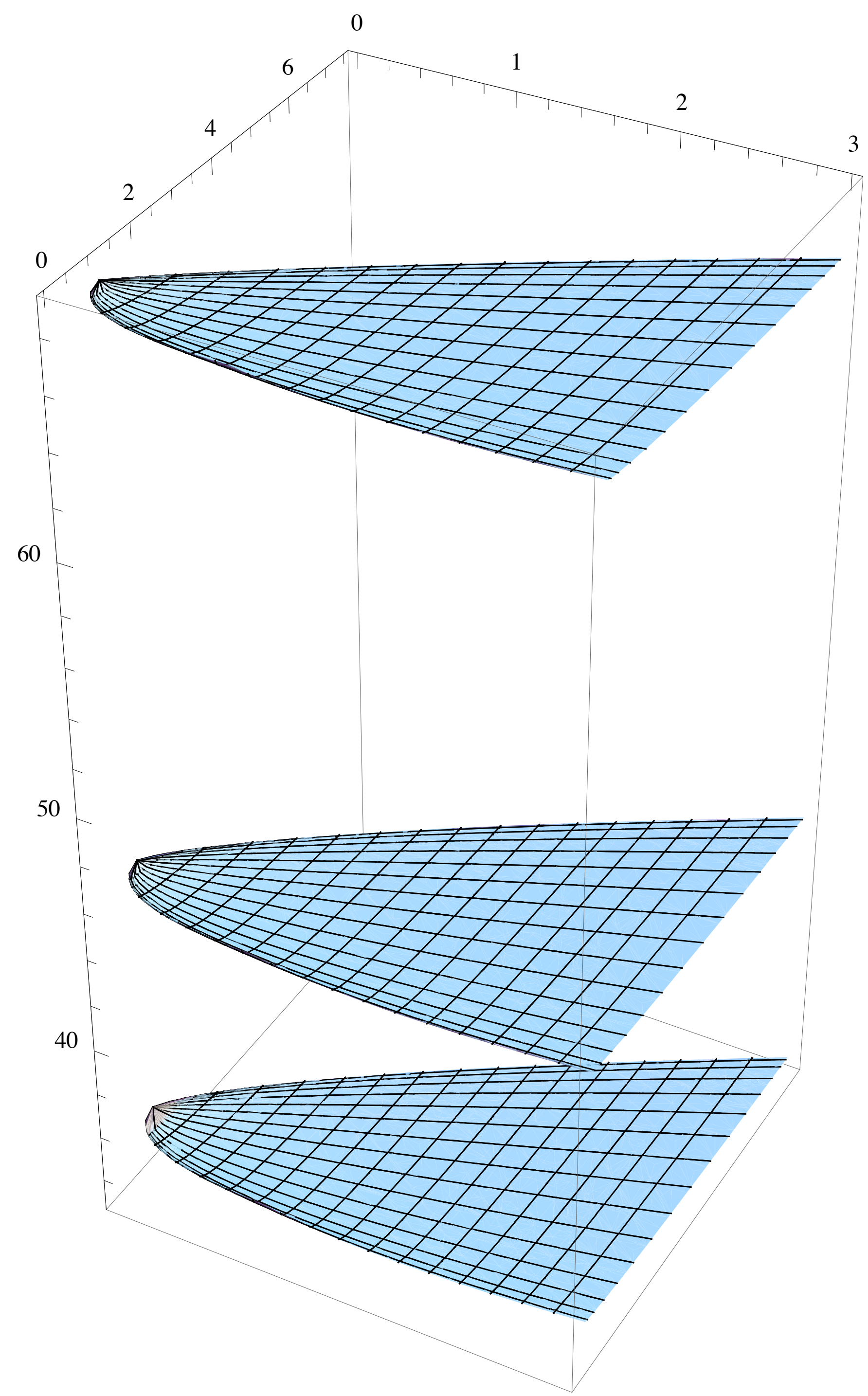
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These plots depict the shape function properly subtracted, and for different values of the spectral index n=0.75,0.96,0.97,0.98,0.99,0.965,0.975,0.985,0.995, and the stack of them.

They do seem quite `flat' except at the corners.

One then computes the FGM statistic after the transfer functions have been computed for the particular early Universe model. This has been done and the paper is in preparation. The results are ready to be compared with the data when those become available:

The PLANCK MISSION data in 2012 ?

We will have to wait then and see if Nature has chosen this scenario.

THANK YOU FOR YOUR ATTENTION