Using the cosmological constant to learn about spacetime

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Outline

- The cosmological constant problem.
- The role of the gravitational symmetries.
- Weyl transverse gravity.

The cosmological constant problem

- Failure of the decoupling principle behind effective field theory.
- Some quantities are extremely sensitive to high-energy physics.
- Relevant, but non-natural operator:

$$\Lambda\sqrt{-g}$$

- Value of the cosmological constant: both ultravioled and infrared aspects; quantum gravity.

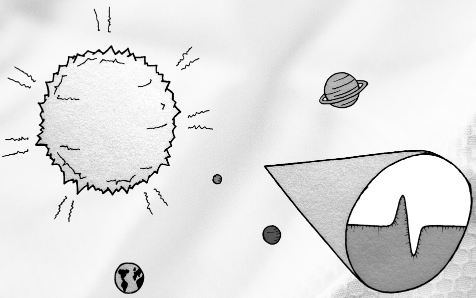
Our solar system

- Natural theoretical prediction:

$$|\Lambda_{vac}| \sim 10^8 \text{ GeV}^4$$

- Measured perihelion precession of Mercury:

$$\Delta\varphi = 574.10 \pm 0.65 \text{ arc-seconds per century}$$

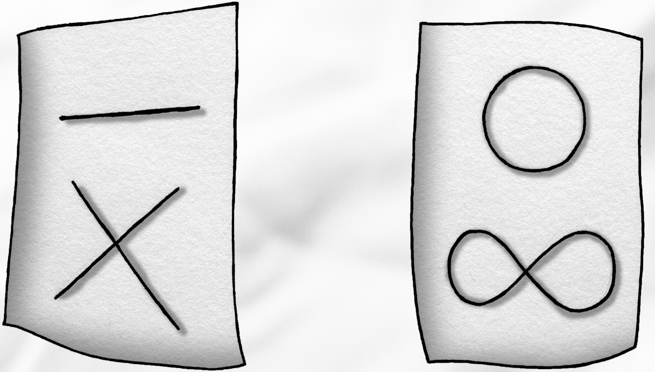


- It implies

$$|\Lambda| \leq 10^{-32} \text{ GeV}^4$$

The quantum vacuum

- Scattering amplitudes and vacuum bubbles:



- Vacuum bubbles couple to a dynamical volume form

$$\epsilon = \sqrt{|g|}\, dx^1 \wedge \ldots \wedge dx^n$$

Consistent low-energy effective description

- No modifications of the classical physics.
- Respect classical solutions of general relativity and the particle spectrum of the standard model.
- Different semiclassical and quantum physics in the presence of gravitational fields.
- Cosmological constant as mysterious as (but no more than) any other parameter: gravitational constant, electron charge, ...

First steps

- Shift symmetry

$$\mathcal{L} \longrightarrow \mathcal{L} + c_0$$

- Classically: Fixed volume element $\boldsymbol{\omega}$; corresponding contribution

$$\int \boldsymbol{\omega}\, c_0$$

- Radiative corrections: we need a symmetry that forbids the term

$$\int \boldsymbol{\epsilon}\, \Lambda = \int d^4x \sqrt{-g}\, \Lambda$$

- Scale transformations of the gravitational field:

$$g_{ab} \longrightarrow \zeta^2 g_{ab} \qquad\qquad \zeta \in \mathbb{R}$$

The gravitational action

- The Einstein-Hilbert action is not invariant under scale transformations.
- Second-order field theory: longitudinal diffeomorphisms should be broken.
- Keeping the counting of degrees of freedom: local scale transformations

$$g_{ab} \longrightarrow \zeta^2(x) g_{ab}$$

- Dynamical conformal structures, parametrized by tensor densities

$$|g|^{-1/n} g_{ab}$$

Weyl transverse gravity

$$\mathcal{A} := \frac{1}{2\kappa}\int_{\mathcal{M}} \boldsymbol{\omega}\, R[|\omega|^{1/n}|g|^{-1/n}g_{ab}]$$

- Theory of dynamical conformal structures on n−dimensional manifold $\mathcal{M}$.
- Invariant under transverse diffeomorphisms and Weyl transformations:

$$\delta_{\xi,\varphi}g_{ab} = \mathcal{L}_\xi g_{ab} + \varphi g_{ab} \qquad \nabla_a\xi^a = 0$$

- Dynamical volume element $\sqrt{-g}$ forbidden by symmetries.
- Matter couples to the composite field

$$|\omega|^{1/n}|g|^{-1/n}g_{ab}$$

From gravity to gravitons

- Expand around flat spacetime η_{ab}:

$$g_{ab} = \eta_{ab} + \lambda h_{ab}$$

- At the lowest order: invariant under

$$h'^{ab} = h^{ab} + \eta^{ac}\partial_c\xi^b + \eta^{bc}\partial_c\xi^a + \phi\eta^{ab} \qquad \partial_a\xi^a = 0$$

- On-shell equivalence to Fierz-Pauli theory (Izawa1995,Alvarez2005).
- Higher orders describe the interaction vertices of gravitons.
- Nonlinear theory of a spin-2 particle.

Classical theory

- In the gauge $g = \omega$, one recovers the traceless Einstein equations

$$R_{ab} - \frac{1}{4} R g_{ab} = \kappa \left(T_{ab} - \frac{1}{4} T g_{ab} \right)$$

- These equations are equivalent to Einstein field equations in the same gauge (Ellis2010).
- The cosmological constant Λ is a constant of integration.
- Shift symmetry $\mathcal{L} \to \mathcal{L} + c_0$:

$$T_{ab} \longrightarrow T_{ab} + g_{ab} c_0, \qquad \Lambda \longrightarrow \Lambda - \frac{\kappa}{4} c_0$$

Anomalies

- Generic result: not all symmetries can be preserved in the quantization. Path integral:

$$\int [\mathcal{D}\Psi] \exp(iS[\Psi])$$

- A symmetry is not anomalous per se, but with respect to other symmetries.
- A necessary condition is that different symmetries act on the same degrees of freedom.

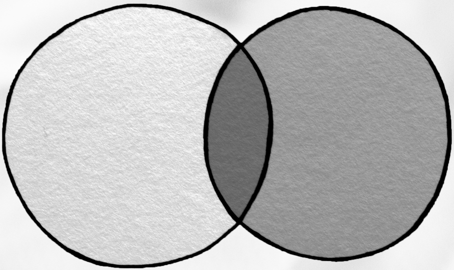
Conformal anomaly

- Diffeomorphisms:

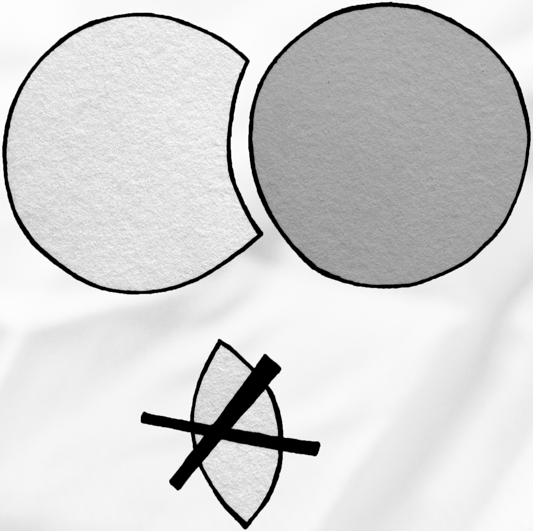
$$\delta\sqrt{|g|} \propto \nabla_a \xi^a$$

- Conformal transformations:

$$\delta\sqrt{|g|} \propto \Omega^4$$



Avoiding the anomalies



Path integral for a scalar field

- Inner product:

$$\langle \phi, \phi' \rangle := \int_{\mathcal{M}} d^D x \sqrt{|\hat{g}|}\, \phi(x) \phi'(x) = \int_{\mathcal{M}} \boldsymbol{\omega}\, \phi\, \phi'$$

- Decomposition coefficients:

$$c_n := \langle \phi_n, \phi \rangle = \int_{\mathcal{M}} \boldsymbol{\omega}\, \phi_n\, \phi$$

- Path integral measure:

$$\prod_{n=0}^{\infty} \frac{dc_n}{\sqrt{2\pi}}$$

- Absence of anomalies:

$$\delta c_n = \int_{\mathcal{M}} d^D x\, \phi_n(x) \phi(x)\, \delta \sqrt{|\hat{g}|} = 0$$

Semiclassical theory

- Classical gravitational fields, quantum matter fields: effective action
$$\mathcal{S}_{g_{ab}} = \frac{1}{2} \ln \det(\mathcal{O}_{g_{ab}})$$

- Heat kernel expansion of the effective action:
$$\mathcal{S}_g = \mathcal{S}_{g_0} - \frac{1}{32\pi^2} \int_{\mathcal{M}} \boldsymbol{\omega} \left\{ \mu^2 [a_1(\hat{g}_{ab}) - a_1(\hat{g}^0_{ab})] + \right. \\ \left. + \ln(\mu^2/m^2)[a_2(\hat{g}_{ab}) - a_2(\hat{g}^0_{ab})] \right\}$$

- No term corresponding to a_0, which in general relativity leads to the renormalization of the cosmological constant.

- Due to the non-dynamical volume form $\boldsymbol{\omega}$.

Renormalization group

- Renormalization of gravitational couplings; e.g., gravitational constant
$$\frac{1}{\kappa} = \frac{1}{\kappa_0} + c_1\mu^2 + c_2 \log\left(\frac{\mu}{c_3}\right)$$
- There is NO renormalization equation for the cosmological constant.
- Cosmological constant protected by gravitational scale transformations:
$$g_{ab} \to \zeta^2 g_{ab}$$
- Shift symmetry on the Lagrangian $\mathcal{L} \to \mathcal{L} + c_0$; c_0 drops off from field equations.

Quantum theory

- Quantum gravitational field.
- Formal definition of the path integral.
- Correlation functions:

$$\langle \Phi(x_1)\Phi(x_2)...\Phi(x_n)\rangle = \frac{\int [\mathcal{D}g][\mathcal{D}\psi] \exp(iS)\Phi(x_1)\Phi(x_2)...\Phi(x_n)}{\int [\mathcal{D}g][\mathcal{D}\psi] \exp(iS)}$$

- Contributions that in general relativity would renormalize the cosmological constant are cancelled out:

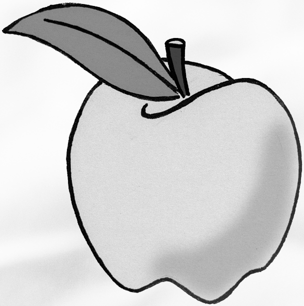
$$S_0 := c_0 \int_{\mathcal{M}} \omega$$

Discussion

- Metric volume form $\boldsymbol{\epsilon}$ versus non-dynamical volume form $\boldsymbol{\omega}$.
- Essentially equivalent classical field equations. What does it tell us?
- But different semiclassical properties.
- A non-dynamical volume form $\boldsymbol{\omega}$ seems less elegant, but provides a better fit to the properties we observe.

Conclusions

- Theory of dynamical causal structures that uses an auxiliary, non-dynamical volume form.
- Standard classical solutions for the gravitational field equations.
- Effective description that avoids the (ultraviolet) cosmological constant problem: non-anomalous gravitational scale invariance.
- Being an effective description, it invites looking for completions. New suggestions for quantum gravity?



Thank you for your attention.