asymptotic safety -
from gauge theories to quantum gravity

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workshop

Quantum Vaccuum and Gravitation

MITP, U Mainz

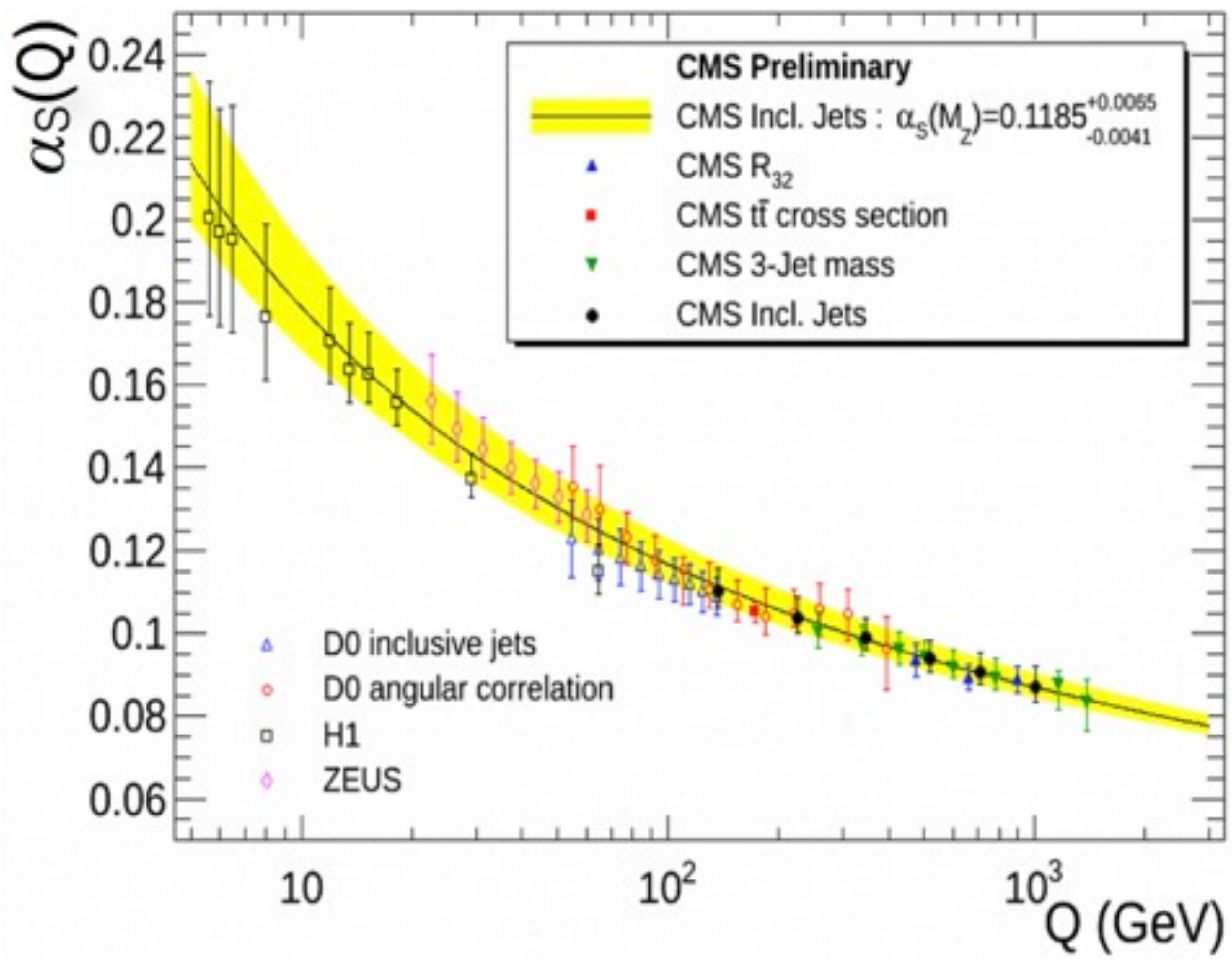
22 - 26 June 2015

Mainz Institute for Theoretical Physics

success story: standard model

running couplings

quantum fluctuations modify interactions
couplings depend on energy or distance



triumph of QFT

asymptotic freedom

't Hooft '74
Gross, Wilczek '74
Politzer '74

quantum gravity as a QFT

degrees of freedom: **spin 2**

dimensionful coupling constant: $[G_N] = 2 - D < 0$

asymptotic safety conjecture:

what, if **running couplings** reach **finite values** in the UV?

Weinberg '79

fundamental definition of QFT ⟺ UV fixed point

Wilson '71

exact asymptotic safety

	dimension	coupling	
gravitons	$D = 2 + \epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Christensen, Duff '78 Weinberg '79 Kawai et al '90

exact asymptotic safety

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exact asymptotic safety

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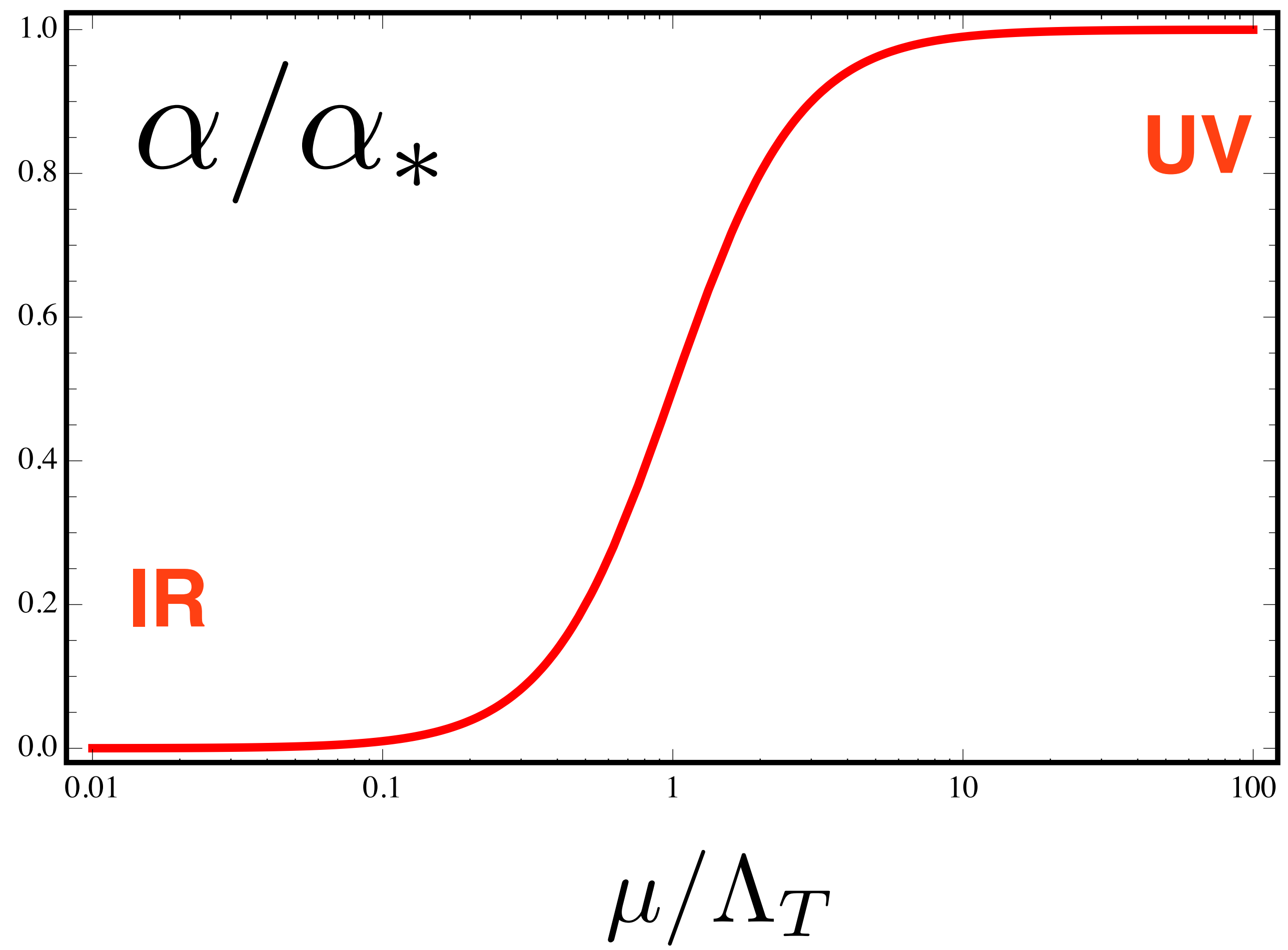


$G(\mu) \approx \frac{\alpha_*}{\mu^{D-2}}$

gravity weakens

$G(\mu) \approx G_N$

classical GR

how is this predictive?

UV: interactions are softened by fluctuations

fixed point characterised by

relevant, marginal, irrelevant invariants

predictivity ⟺ finitely many relevant invariants

exact asymptotic safety

	dimension	coupling	
gravitons	$D = 2 + \epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Christensen, Duff '78 Weinberg '79 Kawai et al '90
fermions	$D = 2 + \epsilon :$	$\alpha = g_{\rm GN}(\mu)\mu^{2-D}$	Gawedzki, Kupiainen '85 de Calan et al '91
gluons	$D = 4 + \epsilon :$	$\alpha = g^2_{\rm YM}(\mu)\mu^{4-D}$	Peskin '80 Morris '04
scalars	$D = 2 + \epsilon :$	$\alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76

exact asymptotic safety

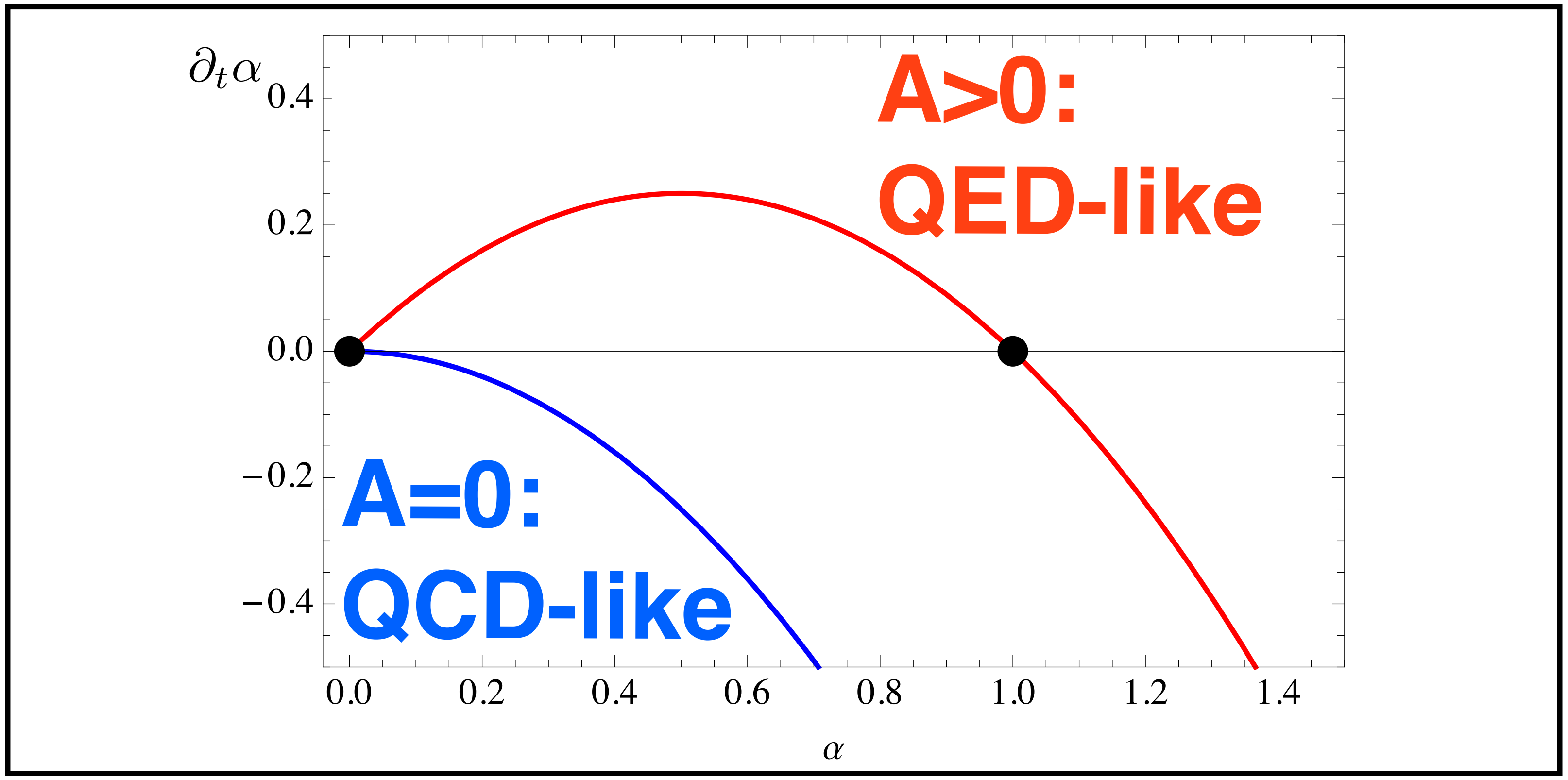
dimension coupling

gravitons

fermions

gluons

scalars



Gastmans et al ’78
Christensen, Duff ’78
Weinberg ’79
Kawai et al ’90

Gawedzki, Kupiainen ’85
de Calan et al ’91

Peskin ’80
Morris ’04

Brezin, Zinn-Justin ’76
Bardeen, Lee, Shrock ’76

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* = 0$ **IR**

$\alpha_* = A/B$ **UV**

$\alpha_* \ll 1$

exact asymptotic safety

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Q: what about D=4?

exact asymptotic safety

	dimension	coupling	
gravitons	$D = 2 + \epsilon :$	$\alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Christensen, Duff '78 Weinberg '79 Kawai et al '90
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scalars	$D = 2 + \epsilon :$	$\alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76
classes of gauge-Yukawa theories	$D = 4 :$	several α_i	Litim, Sannino 1406.2337

exact asymptotic safety of 4D gauge-Yukawa theories

gauge-Yukawa theory

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu}F_{\mu\nu}$$

$$L_F = {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right)$$

$$L_Y = y\,{\rm Tr}\left(\overline{Q}\, H\, Q\right)$$

$$L_H = {\rm Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right)$$

$$L_U = -u\,{\rm Tr}\,(H^\dagger H)^2$$

$$L_V = -v\,({\rm Tr}\, H^\dagger H)^2\,.$$

couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\qquad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}$$

$$\alpha_h = \frac{u\, N_F}{(4\pi)^2}\qquad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}\,.$$

small parameter:

$$0 < \epsilon \ll 1 \qquad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

no asymptotic freedom

gauge-Yukawa theory

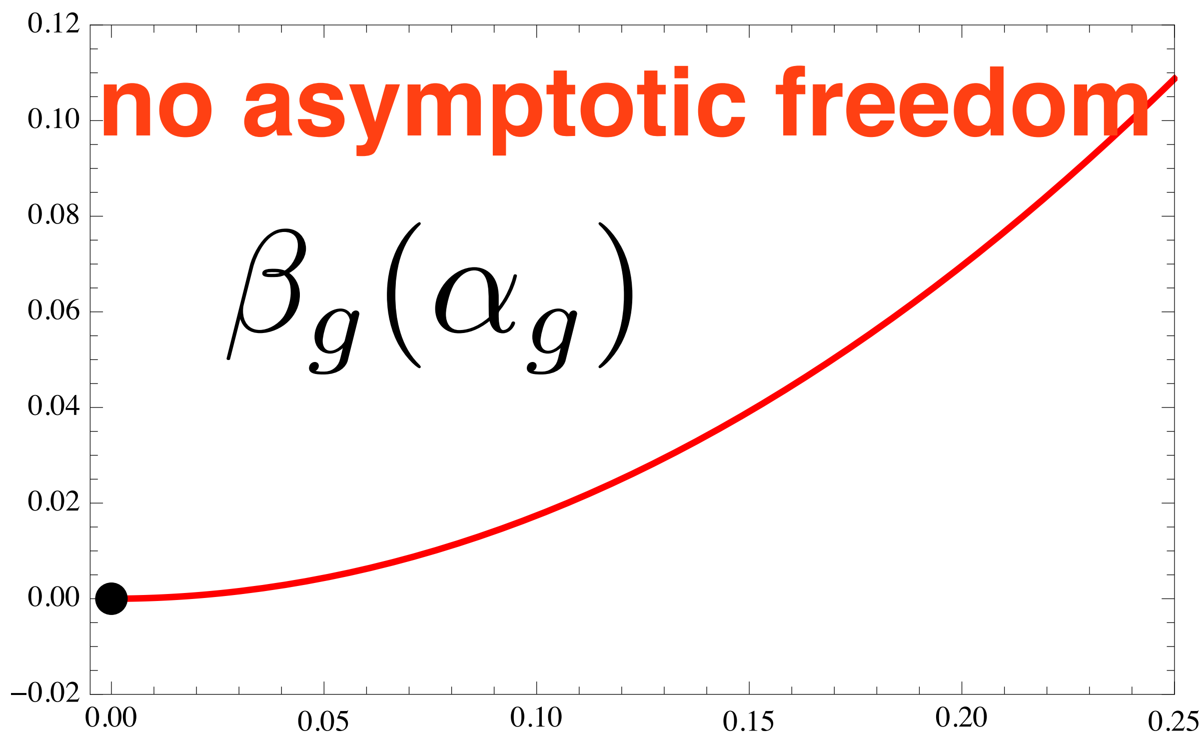
$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$ **gauge**

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} .$$ **Yukawa**

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y)\ .$$

Higgs



gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

gauge

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Yukawa

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Higgs

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gauge-Yukawa theory

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$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y)\ .$$

exact
UV fixed point

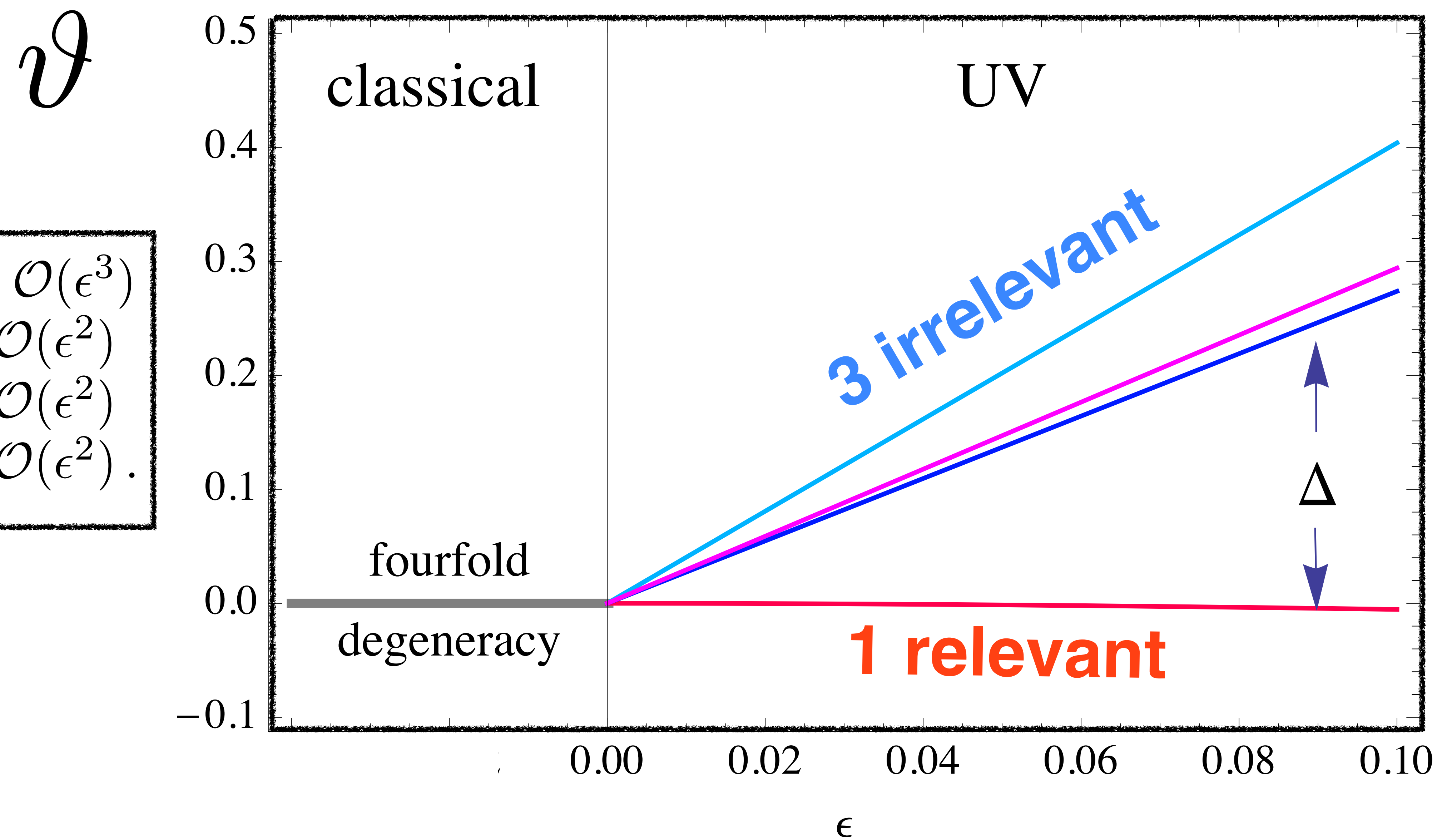
$$\begin{aligned}
\alpha_g^* &= \quad 0.4561\,\epsilon + 0.7808\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\
\alpha_y^* &= \quad 0.2105\,\epsilon + 0.5082\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\
\alpha_h^* &= \quad 0.1998\,\epsilon + 0.5042\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\
\alpha_v^* &= -0.1373\,\epsilon + \mathcal{O}(\epsilon^2)
\end{aligned}$$

results

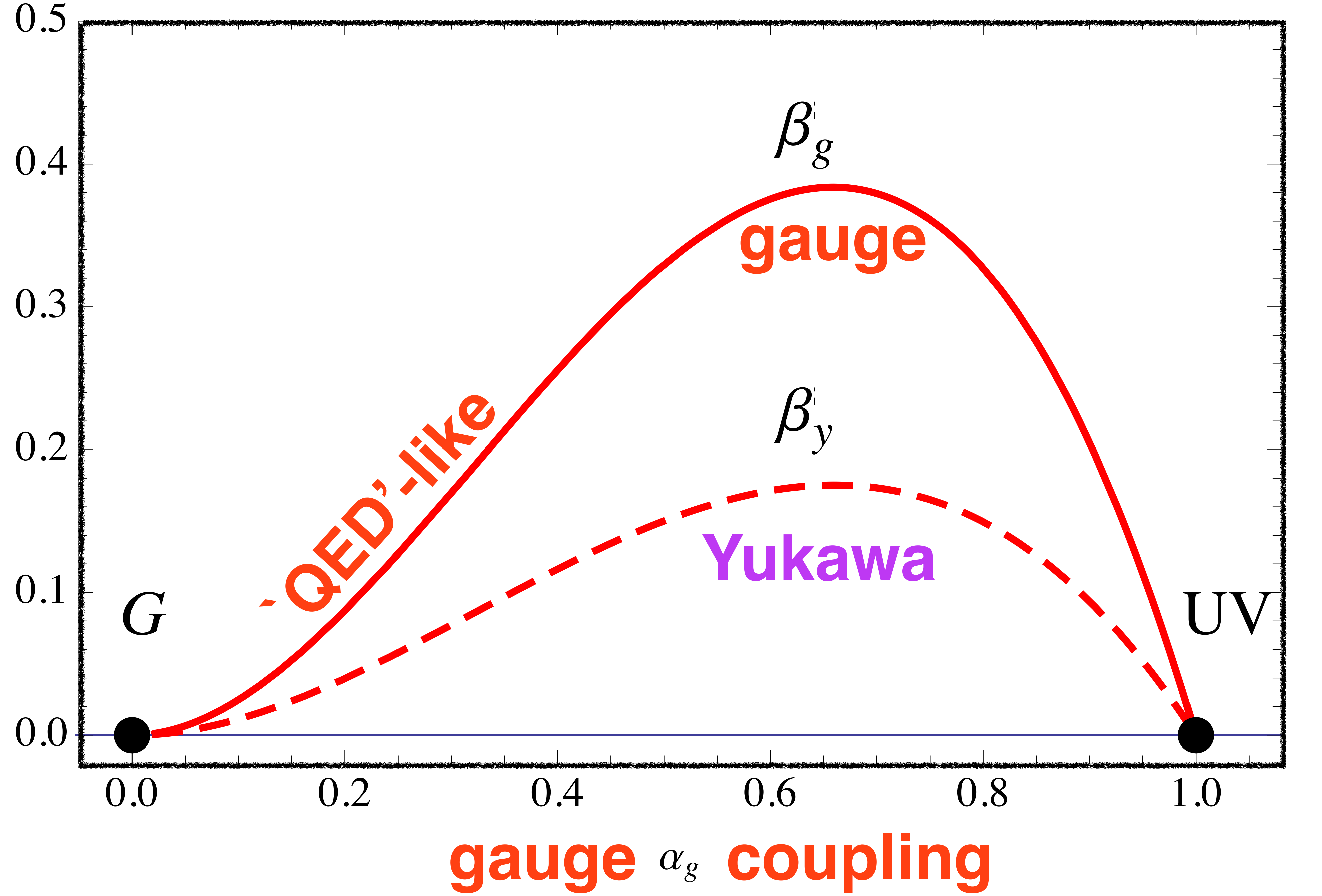
UV scaling exponents

$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$

$$\begin{aligned}
\vartheta_1 &= -0.608\,\epsilon^2 + \mathcal{O}(\epsilon^3)\\
\vartheta_2 &= 2.737\,\epsilon + \mathcal{O}(\epsilon^2)\\
\vartheta_3 &= 4.039\,\epsilon + \mathcal{O}(\epsilon^2)\\
\vartheta_4 &= 2.941\,\epsilon + \mathcal{O}(\epsilon^2)\,.
\end{aligned}$$



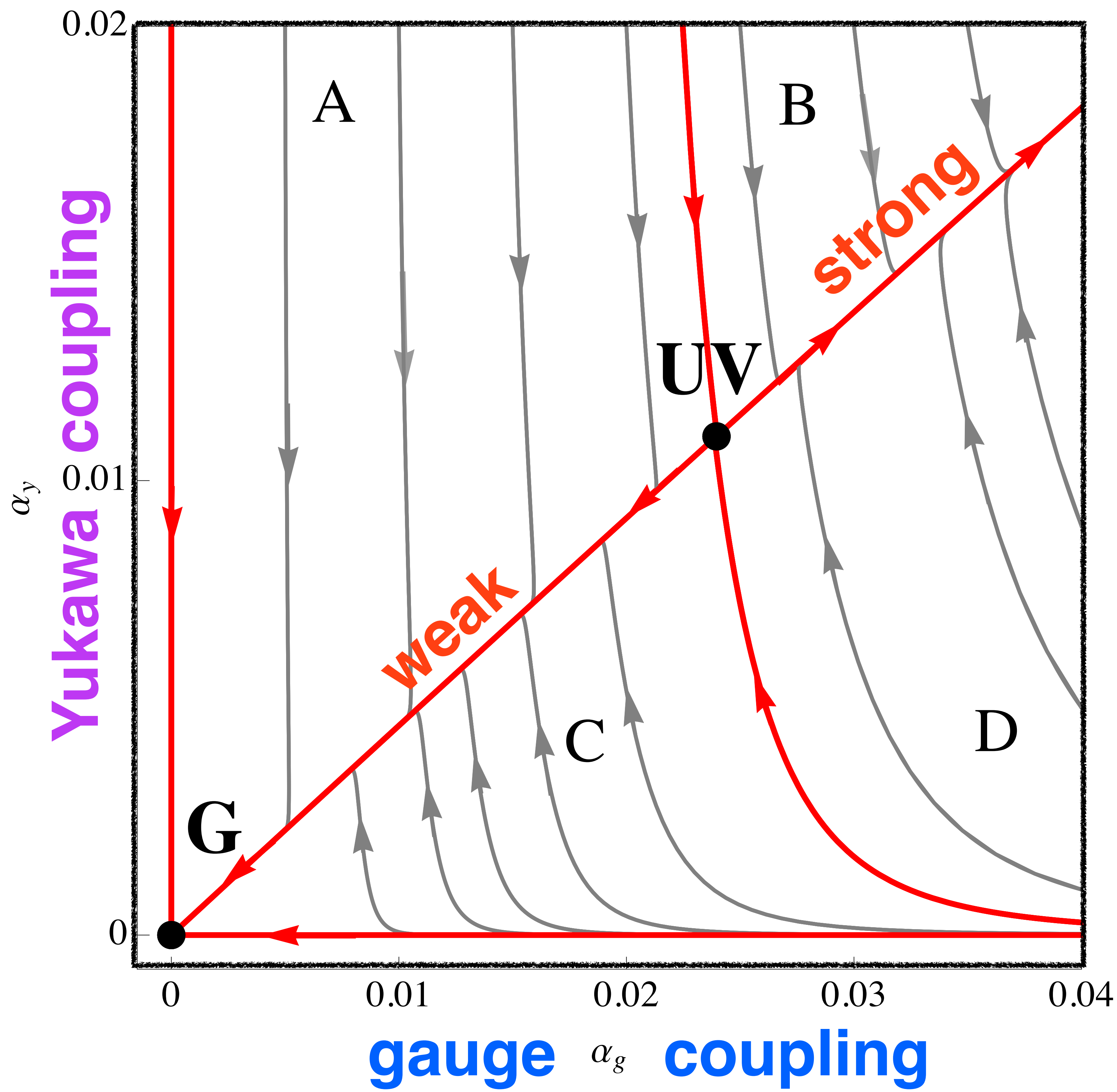
results



interacting UV fixed point
entirely due to `fluctuations'

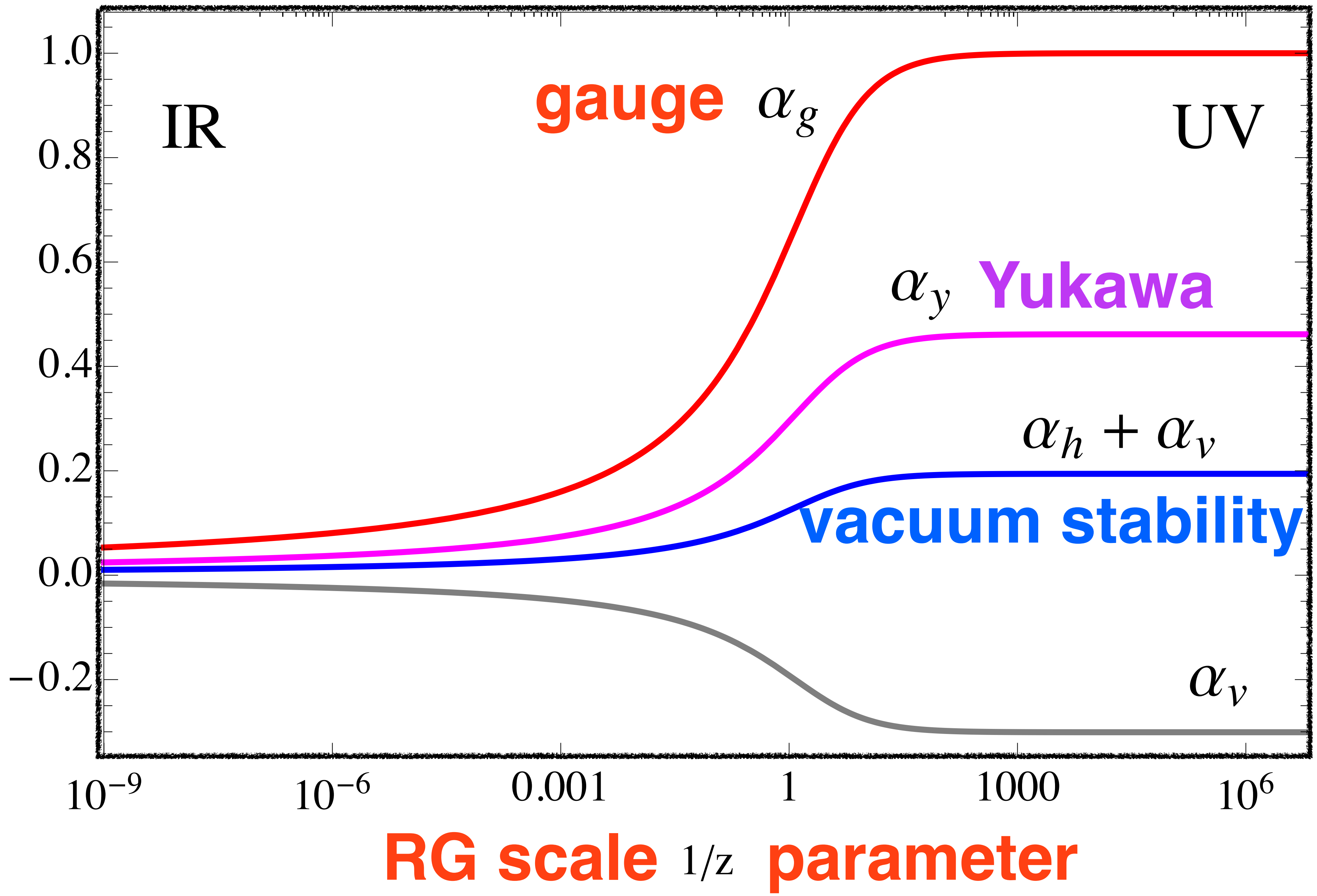
results

phase diagram



exact UV FP
strict perturbative control

results



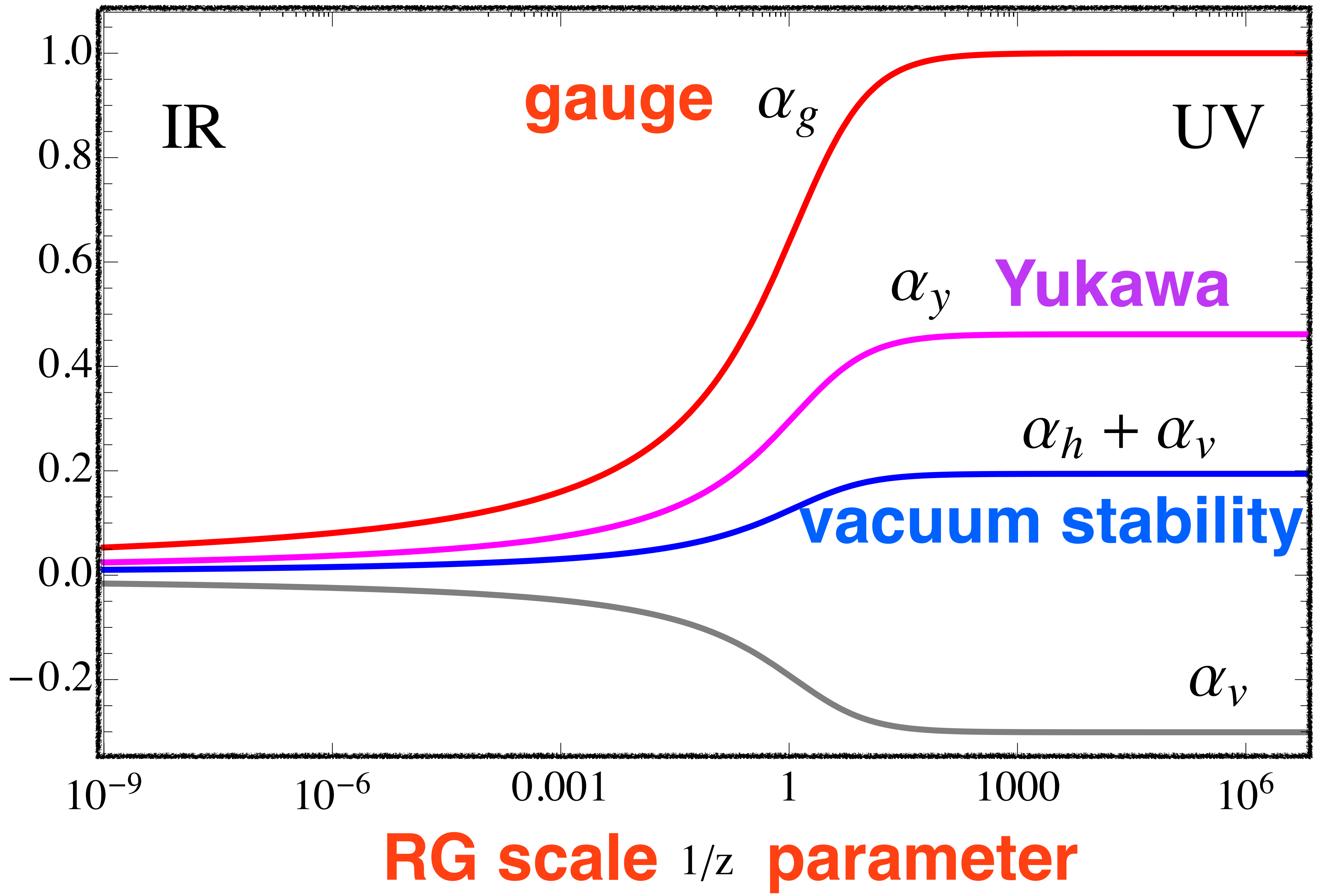
exact UV-IR cross-over
(here: eps = 0.05)

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

$$W(\mu) = W_{\text{Lambert}}[z(\mu)]$$

$$z = \left(\frac{\mu_0}{\mu}\right)^{-B\cdot\alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1\right) \exp\left(\frac{\alpha_*}{\alpha_0} - 1\right).$$

results

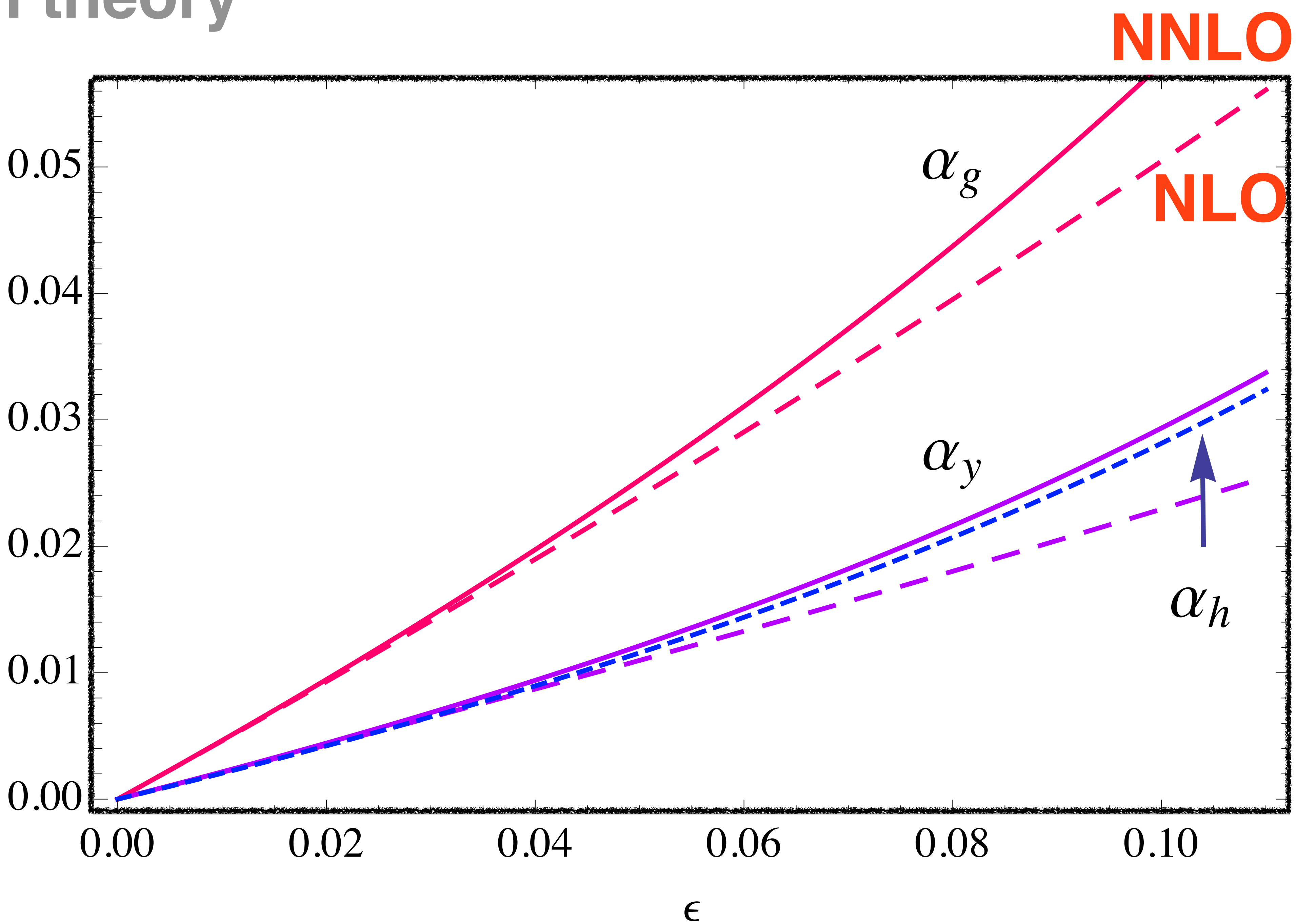


phenomenology:

- dark matter — Sannino, Shoemaker, arXiv:1412.8034
- inflation — Nielsen, Sannino, Svendsen, arXiv:1503.00702

results

UV fixed point from perturbation theory



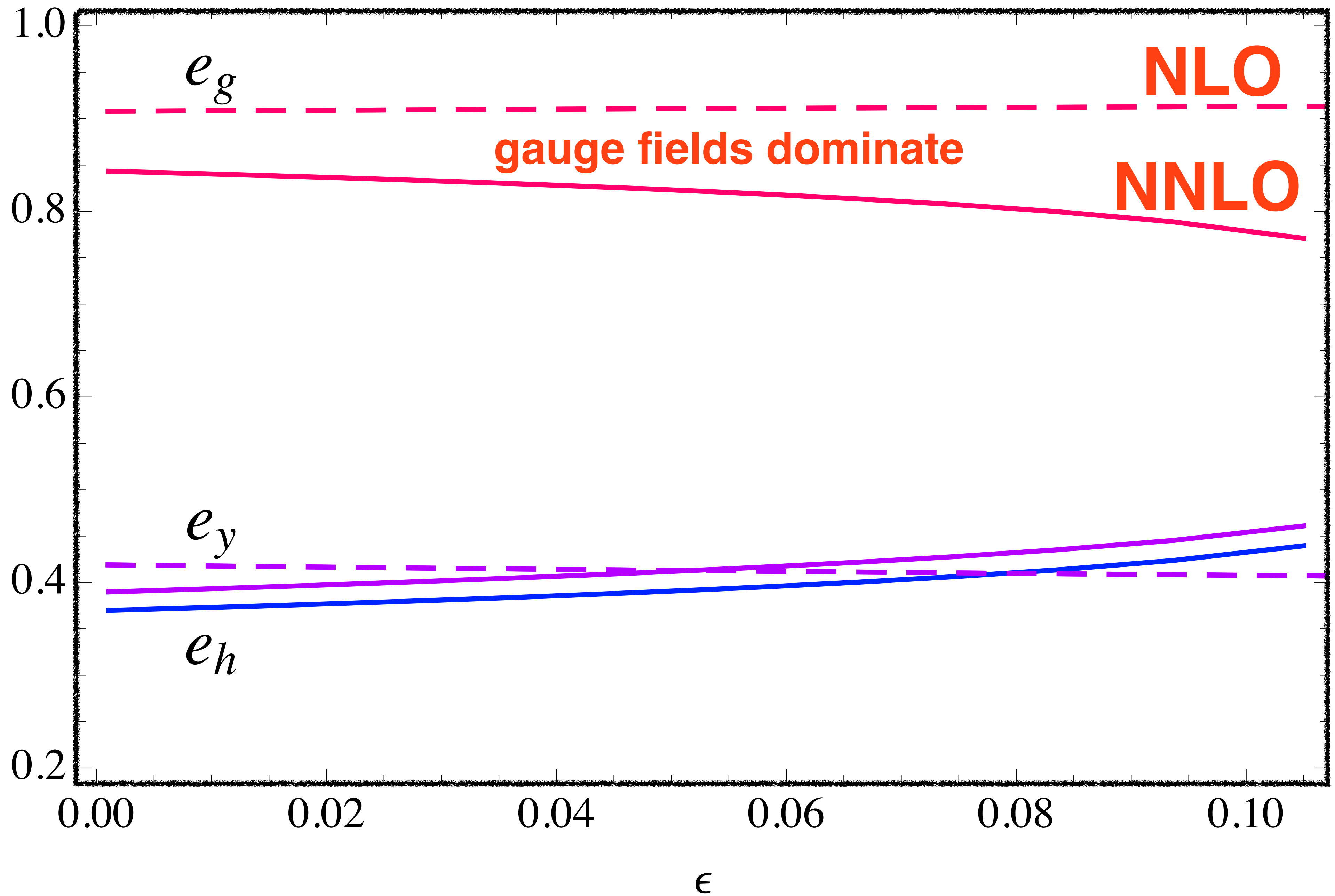
results

UV-relevant eigendirection

gauge

Yukawa

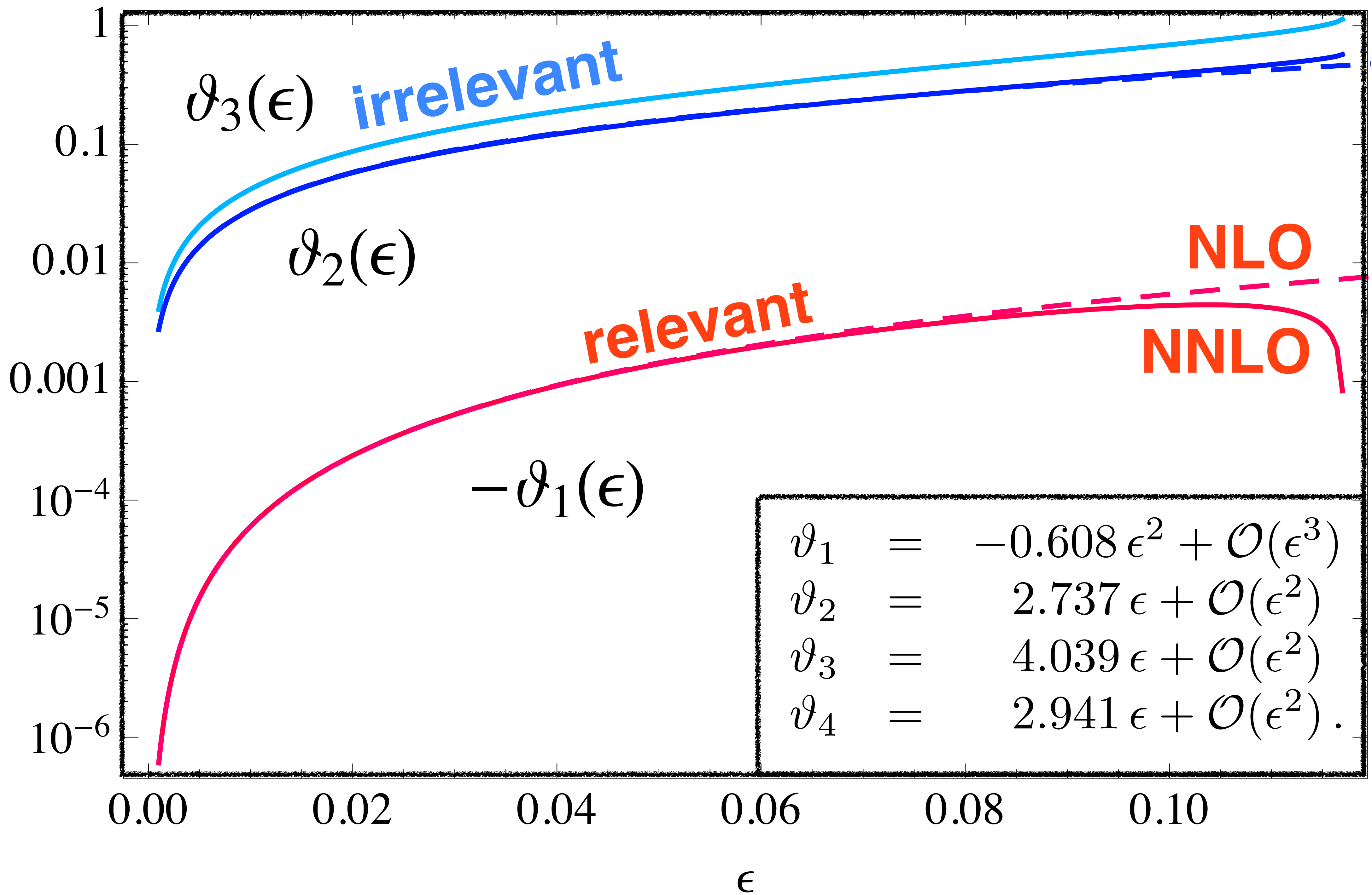
Higgs



results

UV scaling exponents

$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$



vacuum stability

vacuum must be stable classically and quantum-mechanically

$$V \propto \alpha_v \mathrm{Tr}(H^\dagger H)^2 + \alpha_h (\mathrm{Tr} H^\dagger H)^2$$

stability

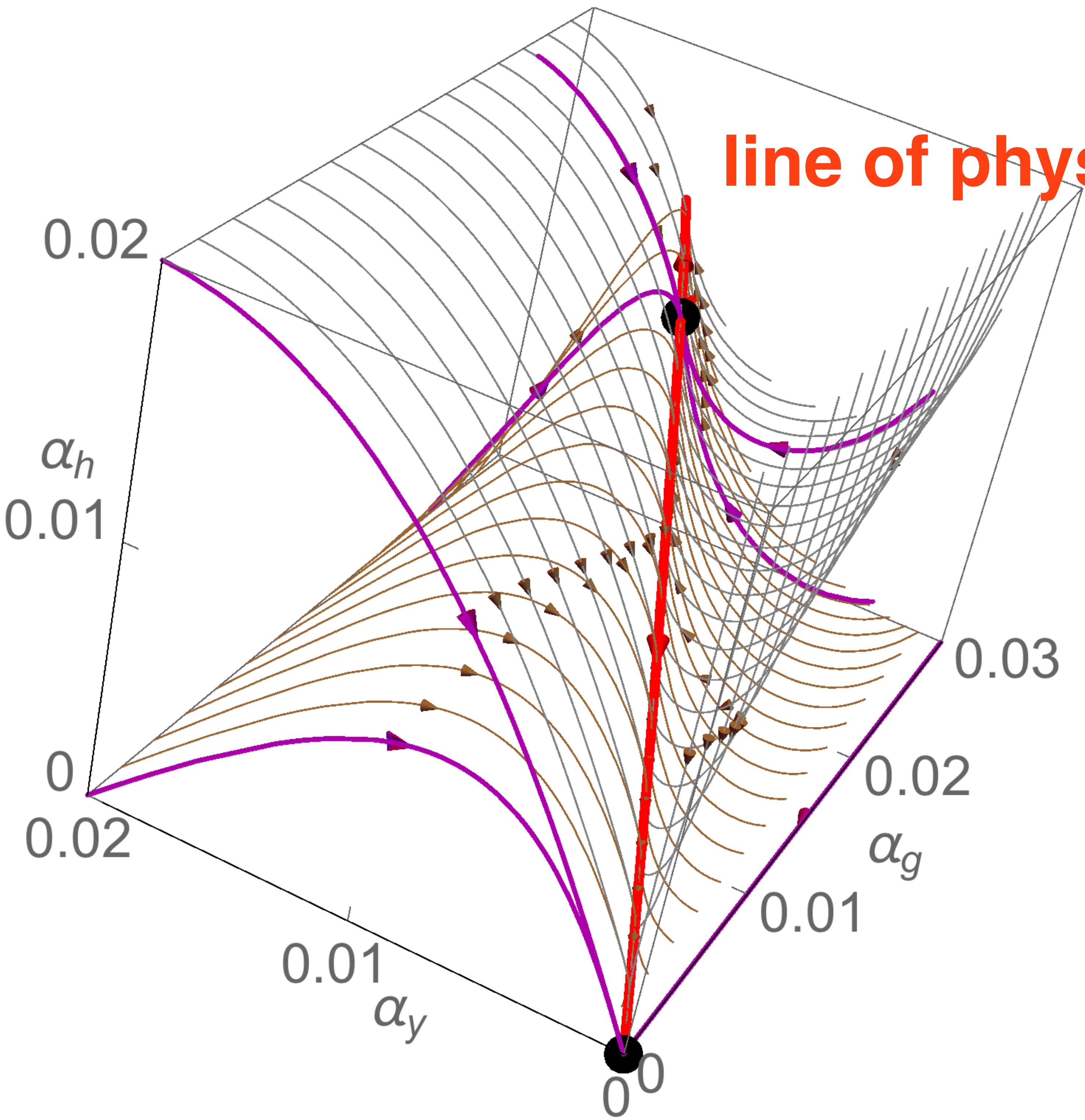
$$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0 \qquad H_c \propto \delta_{ij}$$

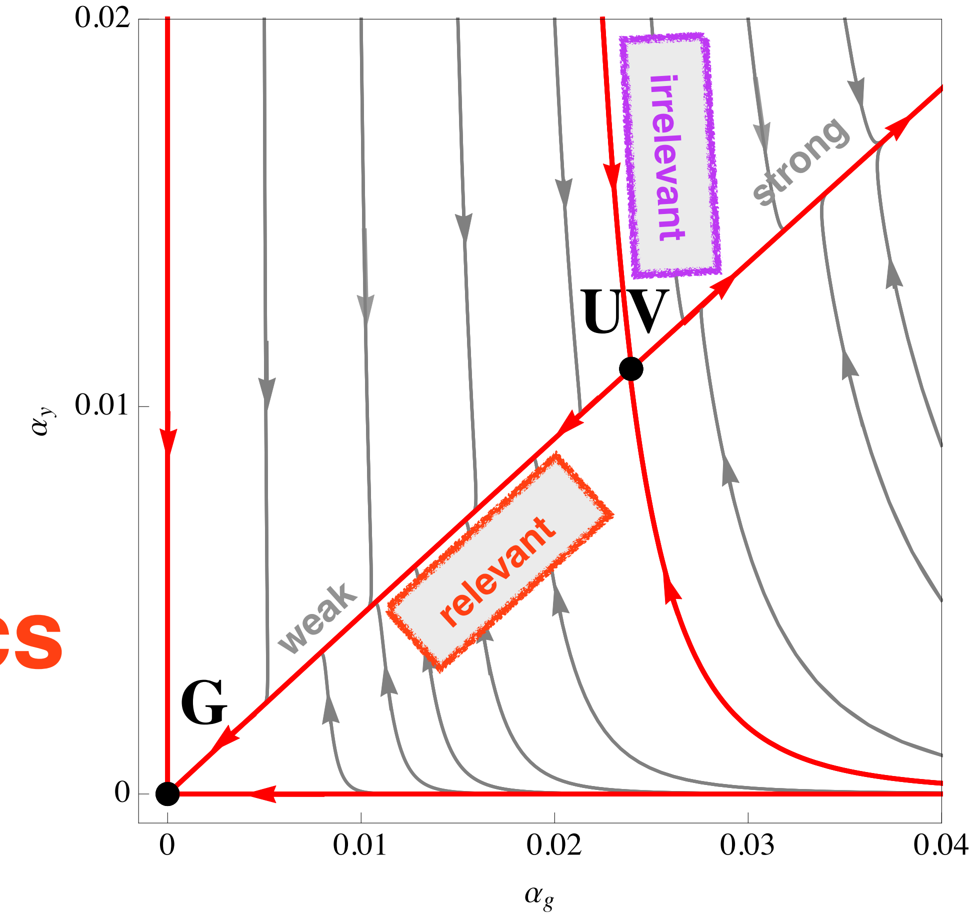
$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v / N_F \geq 0 \qquad H_c \propto \delta_{i1}$$

UV FP:

$$0 < \alpha_h^* + \alpha_v^*$$ **ok**

phase diagram





leading order

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

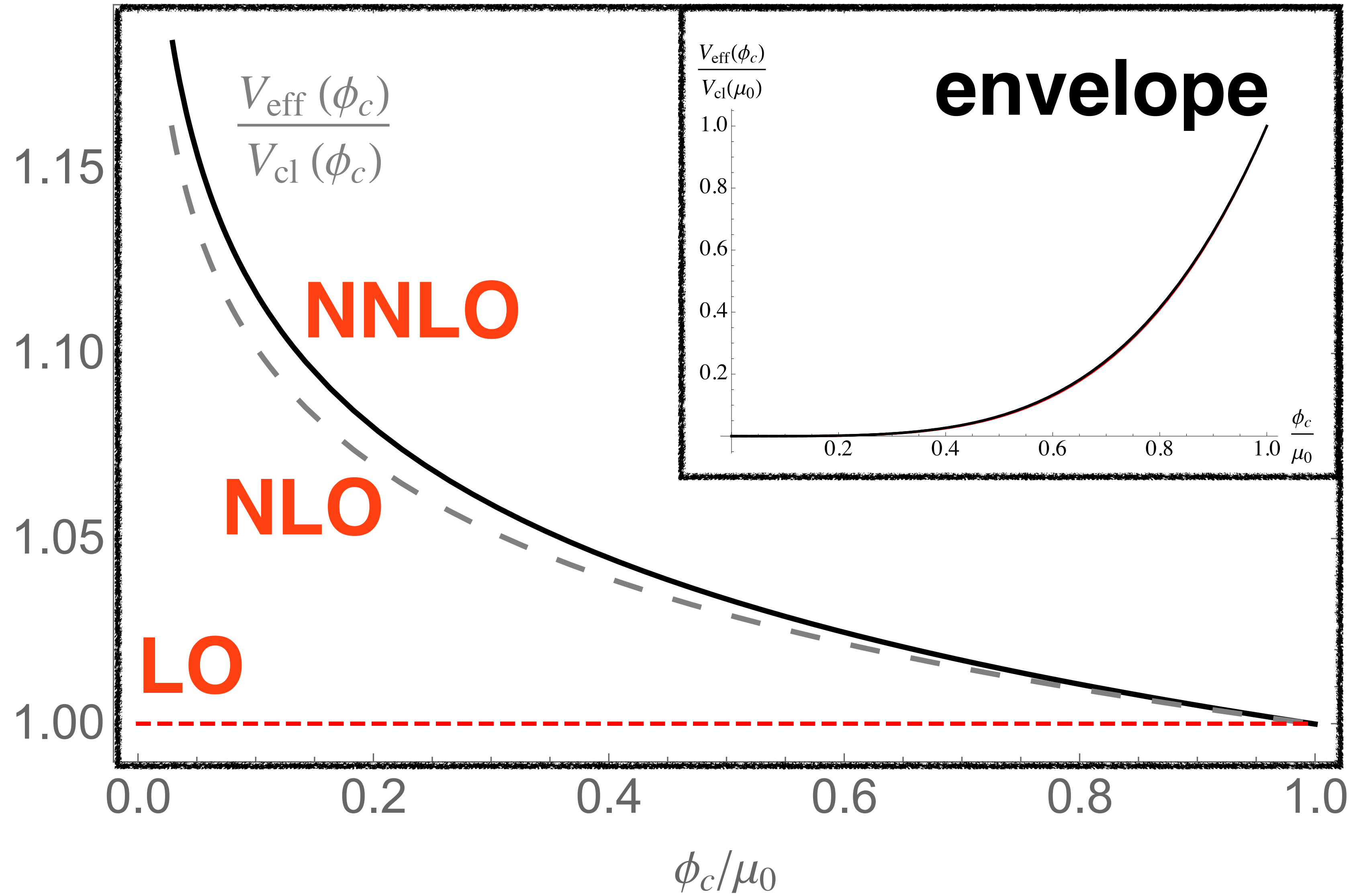
$$W(\mu) = W_{\text{Lambert}}[z(\mu)]$$

$$z = \left(\frac{\mu_0}{\mu}\right)^{-B\cdot\alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1\right) \exp\left(\frac{\alpha_*}{\alpha_0} - 1\right) .$$

vacuum stability

quantum stability: Coleman-Weinberg type resummation of logs

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j)\,\phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j)\frac{\partial}{\partial \alpha_i}\right) V_{\rm eff}(\phi_c, \mu_0, \alpha_j) = 0$$



effective potential well-defined for all scales

asymptotic safety
and quantum gravity

computational methods

4D quantum gravity:

expect large couplings
non-perturbative tools mandatory

continuum: non-perturbative renormalisation group

lattice: Monte Carlo simulations

simplicial gravity
dynamical triangulations

(AdS/CFT, holography, ...)

renormalisation group

continuum methods

functional (Wilsonian) renormalisation
`effective average action’

Polchinski ’84, Wetterich ’92
Reuter ’96, Litim ’00, ’03

vast body of results
strong evidence for interacting FP
(see e.g. 1102.4624 for an overview)

systematic search strategy (`bootstrap’)
set of relevant couplings
not known beforehand

Falls, Litim, Nikolakopoulos, Rahmede, 1301.4191

asymptotic freedom `the knowns' vs asymptotic safety `the unknowns'

asymptotic freedom	asymptotic safety
$g_* = 0$	$g_* \neq 0$
anomalous dimensions	anomalous dimensions
$\eta_A = 0$	$\eta_N \neq 0$
canonical power counting	**non-canonical** power counting
$\{\vartheta_{\mathrm{G},n}\}$ are known	$\{\vartheta_n\}$ are **not** known
F^{256} irrelevant !	R^{256} relevant / marginal / irrelevant **?**

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (no convergence) of the iteration:

hypothesis supported (refuted)

f(R)

$$\Gamma_k \propto f(R)$$

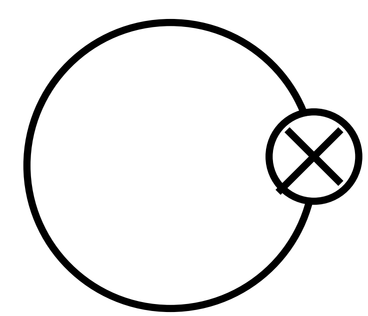
Ricci scalars

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$

effective action with invariants up to mass dimension $D = 2(N-1)$

technicalities: functional renormalisation

$$k \frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2} \mathrm{Tr} \left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi \, \delta\phi} + R_k \right)^{-1} k \frac{\mathrm{d}R_k}{\mathrm{d}k} \right] = \frac{1}{2}$$



here:

M Reuter hep-th/9605030

DL hep-th/0103195

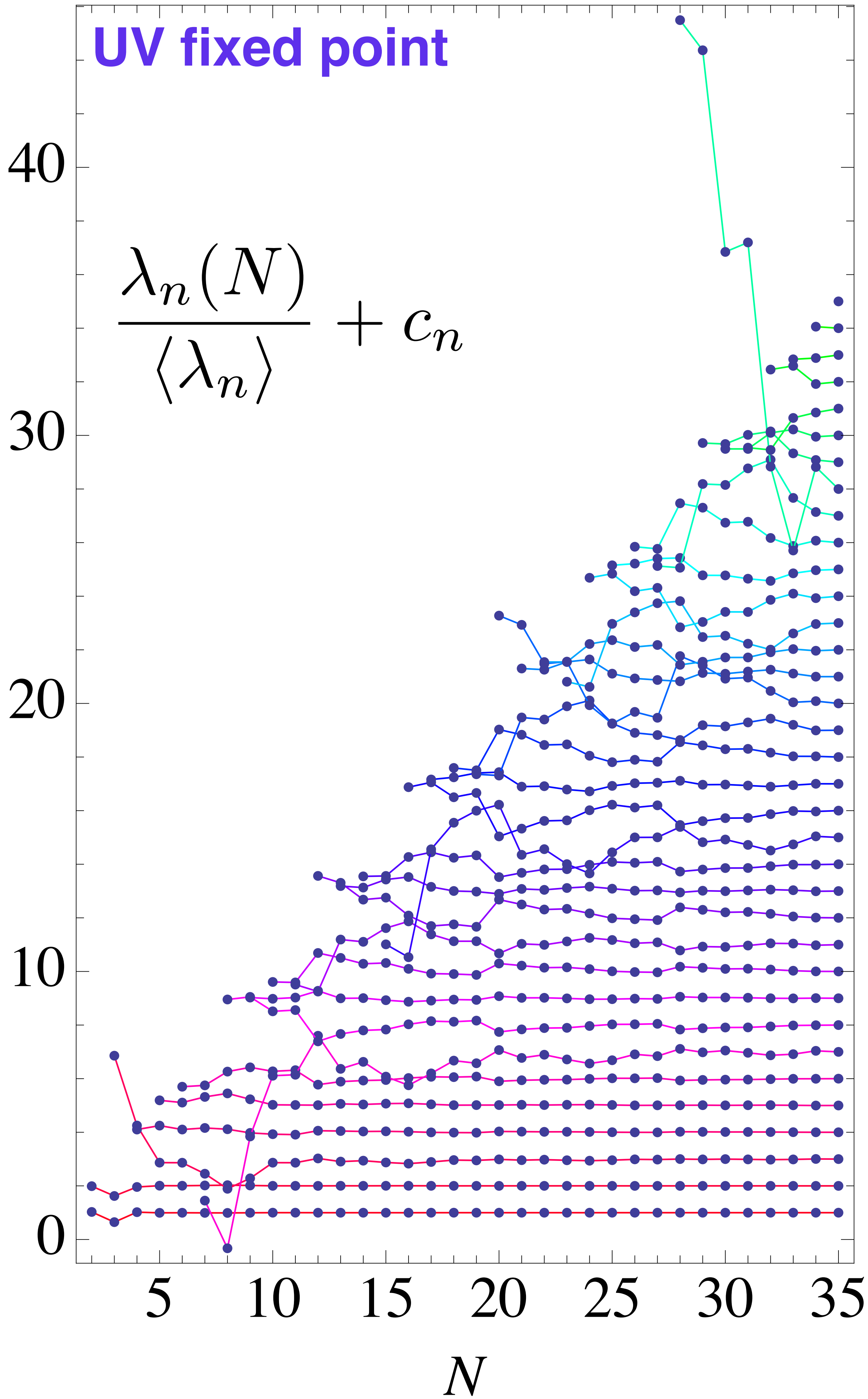
hep-th/0312114

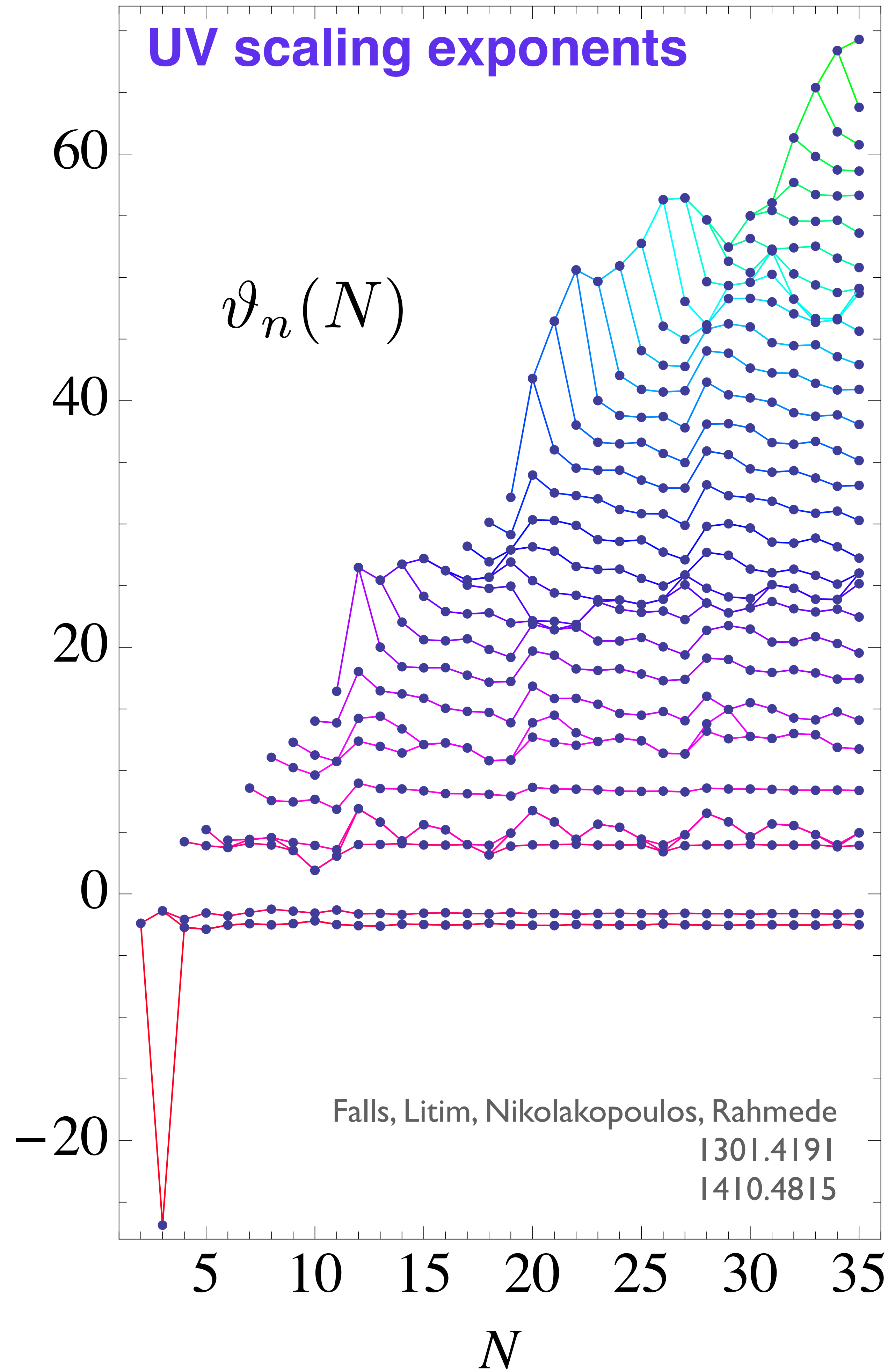
Falls, DL, Nikolakopoulos, Rahmede 1301.4191.pdf

Falls, DL, Nikolakopoulos, Rahmede 1410.4815

A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909

P Machado, F Saueressig 0712.0445

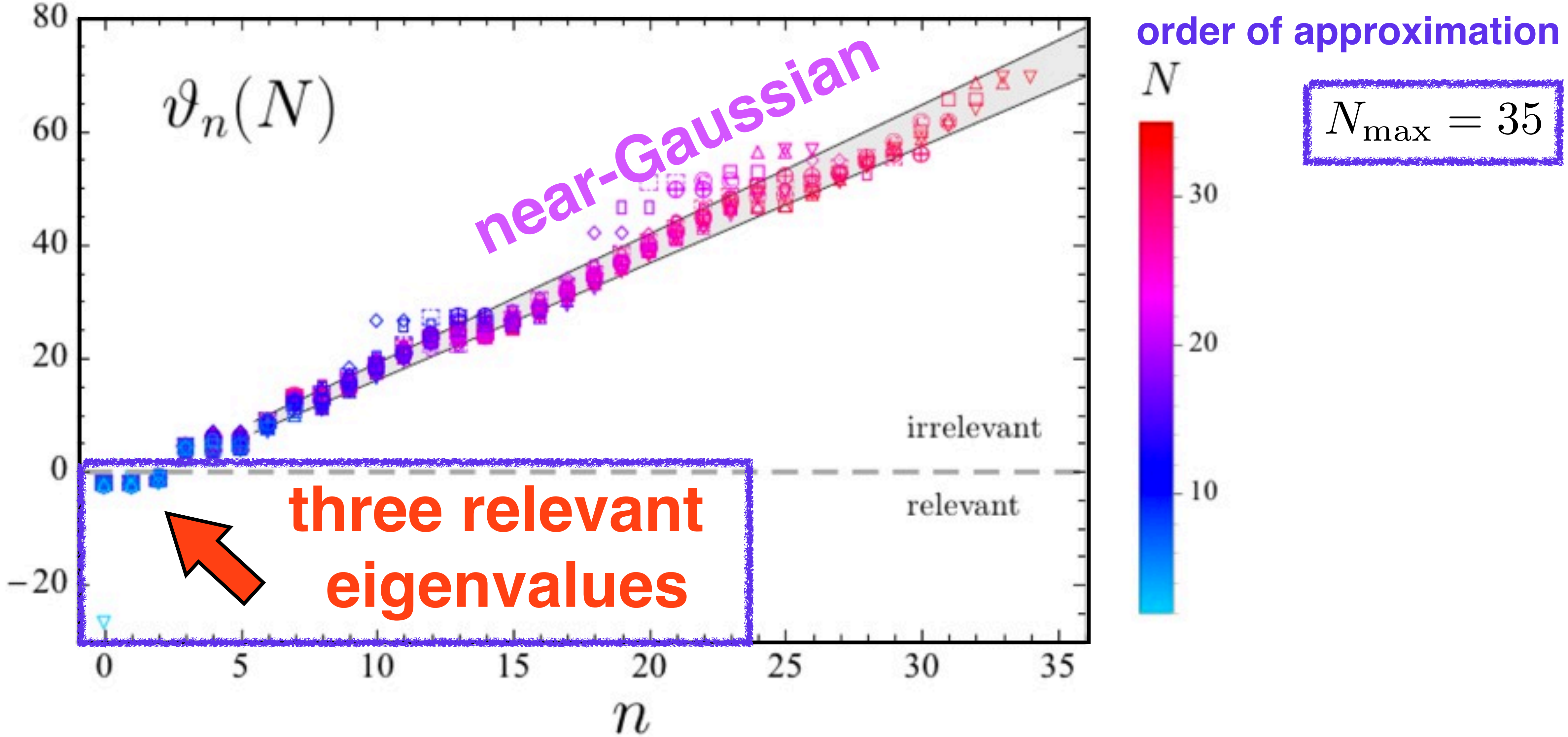

UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
40
30
20
10
0
5
10
15
20
25
30
35
N


UV scaling exponents
$\vartheta_n(N)$
60
40
20
0
−20
5
10
15
20
25
30
35
N
Falls, Litim, Nikolakopoulos, Rahmede
1301.4191
1410.4815

scaling exponents

f(R)-type gravity

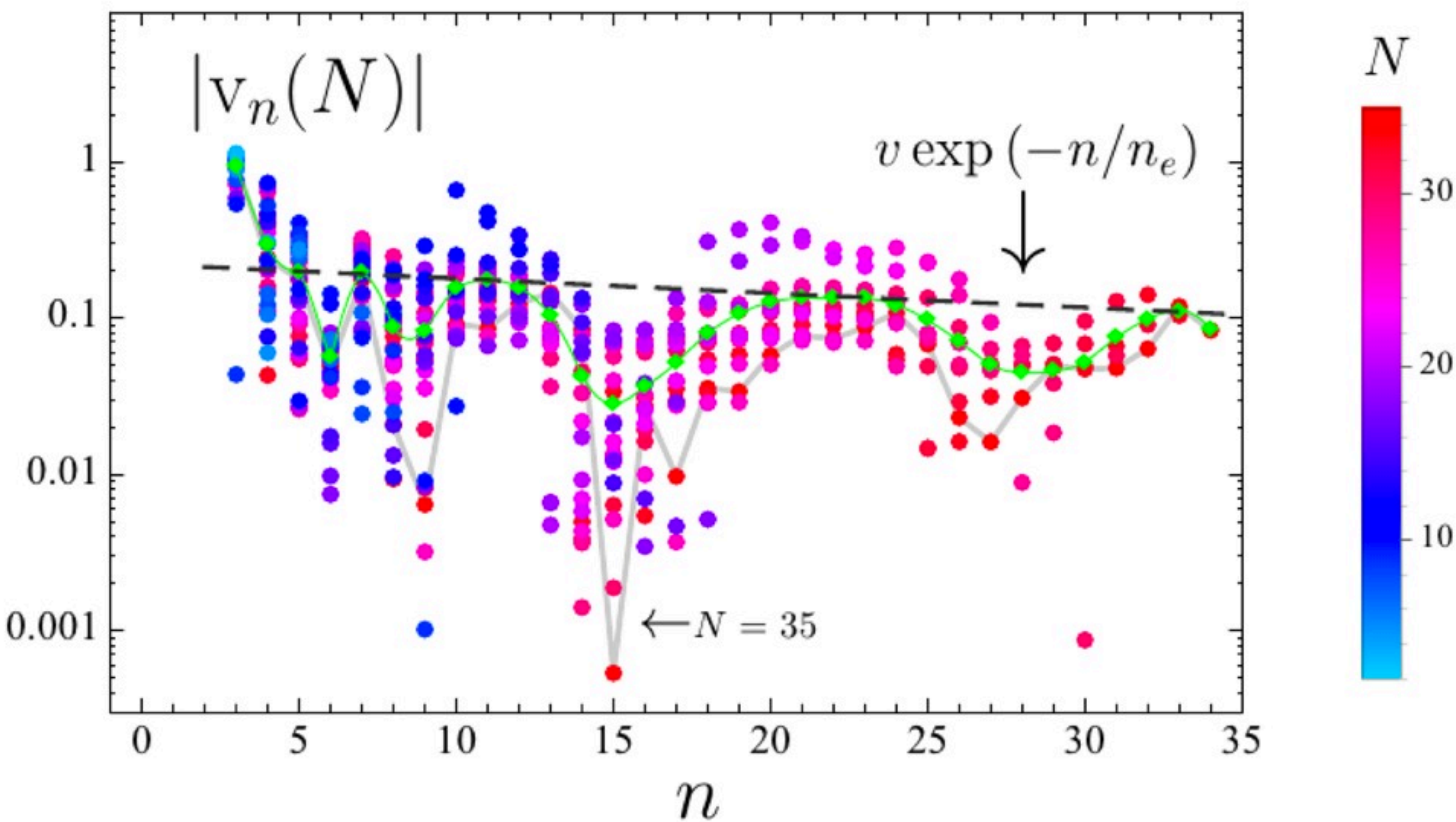
$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g}\, R^n$$



Falls, Litim, Nikolakopoulos, Rahmede
1301.4191
1410.4815

near-Gaussian

$$\mathrm{v}_n(N) = 1 - \frac{\mathrm{Re}\,\vartheta_n(N)}{\vartheta_{\mathrm{G},n}}$$



K Falls, DL, K Nikolakopoulos & C Rahmede, (in prep, arXiv:1506.00xyz)

f(Ricci)

$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R\cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

f(Ricci)

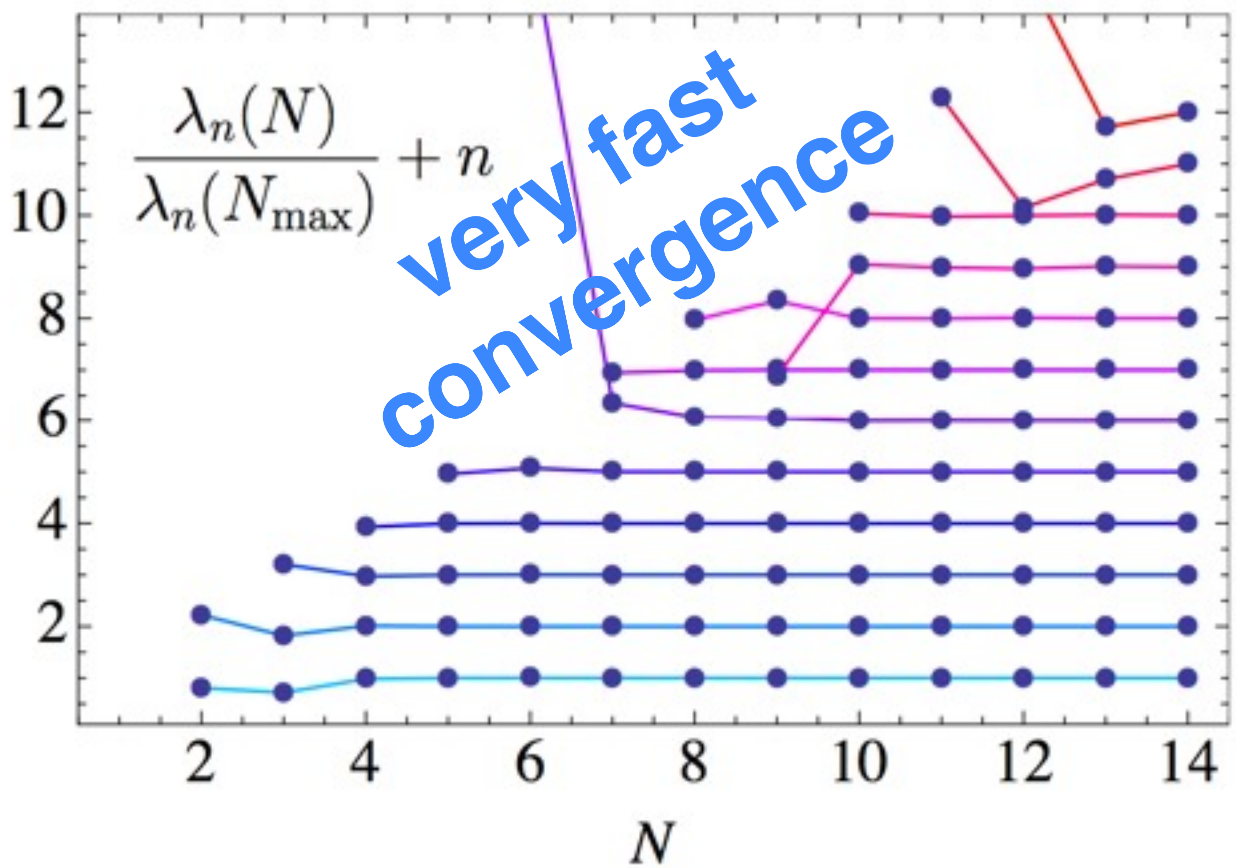
$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R\cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

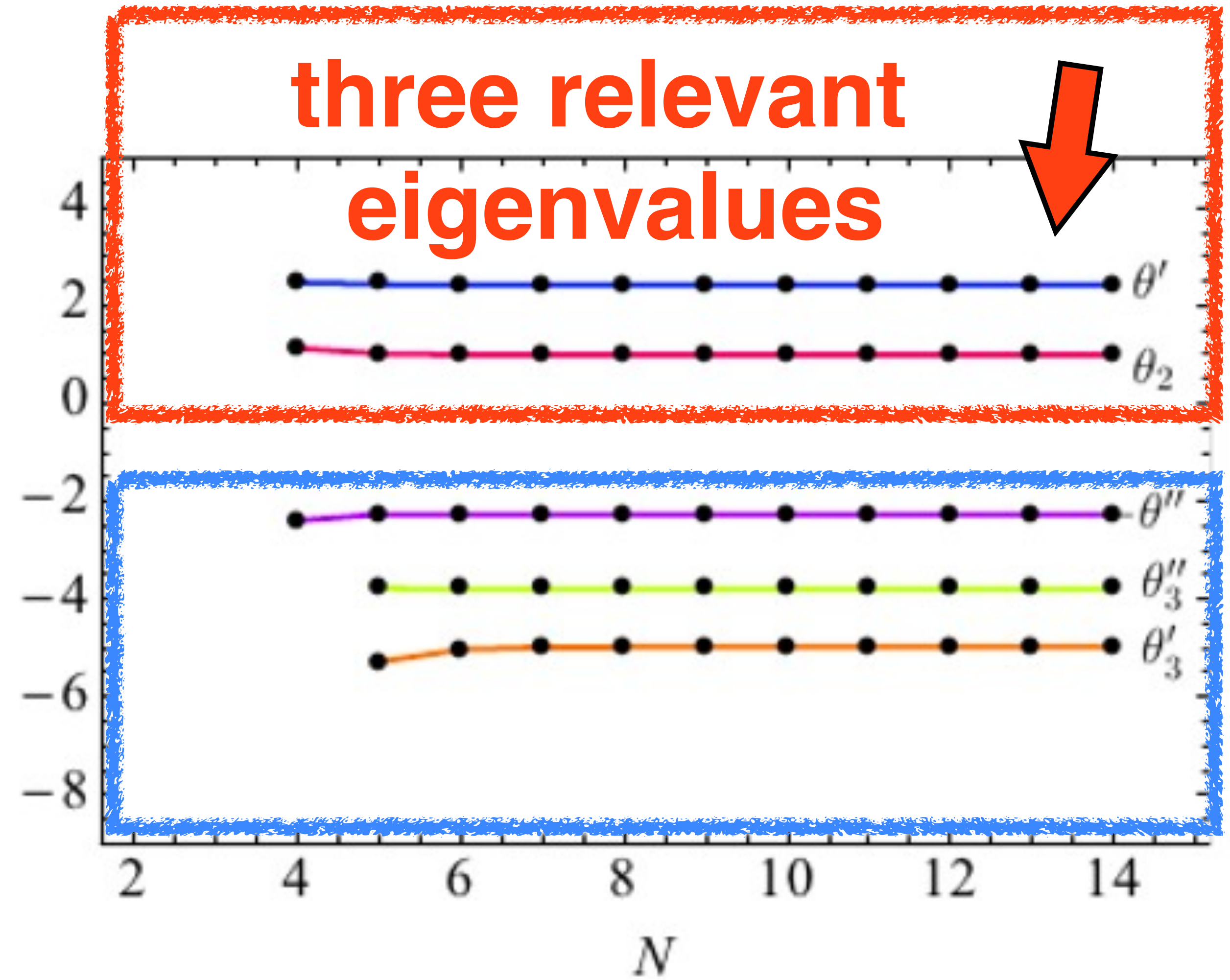
$$\begin{aligned}\partial_t\Gamma[\bar{g},\bar{g}] =& \frac{1}{2}\mathrm{Tr}_{(2T)}\left[\frac{\partial_t\mathcal{R}_k^{h^Th^T}}{\Gamma^{(2)}_{h^Th^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\xi\xi}}{\Gamma^{(2)}_{\xi\xi}}\right] + \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma\sigma}}{\Gamma^{(2)}_{\sigma\sigma}}\right] + \frac{1}{2}\mathrm{Tr}_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{hh}}{\Gamma^{(2)}_{hh}}\right]\\ &+ \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma h}}{\Gamma^{(2)}_{\sigma h}}\right] - \mathrm{Tr}_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{C}^TC^T}}{\Gamma^{(2)}_{\bar{C}^TC^T}}\right] - \mathrm{Tr}_{(0)'}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma^{(2)}_{\bar{\eta}\eta}}\right] - \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma^{(2)}_{\bar{\lambda}\lambda}}\right]\\ &+ \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\omega\omega}}{\Gamma^{(2)}_{\omega\omega}}\right] - \mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{c}^Tc^T}}{\Gamma^{(2)}_{\bar{c}^Tc^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\zeta^T\zeta^T}}{\Gamma^{(2)}_{\zeta^T\zeta^T}}\right] + \mathrm{Tr}'_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{s}s}}{\Gamma^{(2)}_{\bar{s}s}}\right]\end{aligned}$$

f(Ricci)

$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R\cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

results:





RG vs lattice

simplicial gravity

lattice fixed point in 4D

Hamber ’00, ’15

scaling exponent	lattice		RG	
ν	$0.335(9)$	Hamber ’00	0.375	Litim ’03
	$0.335(4)$	Hamber ’15 as quoted in 1503.06233	0.3333	Falls 1503.06233

dynamical triangulations (casual vs euclidean)

lattice fixed point in 4D CDT

Ambjoern, Jordan, Jurkiewicz, Loll ‘11

spectral dimension $\mathcal{D}_s$	CDT	Ambjoern, Jurkiewicz, Loll ’05
	EDT	Laiho, Coumbe ’11
	RG	Lauscher, Reuter, ’05 Reuter, Saueressig, ’11

$$\mathcal{D}_s = \frac{2D}{2+\delta}$$

testing asymptotic safety in the physical world

cosmology

early universe and inflation, late-time acceleration
asymptotically safe cosmology

★★★

particle physics

towards a Standard Model including quantum gravity
gravitational scattering: signatures at particle colliders

★★★

black holes

quantum corrections to BH space-times
quantum aspects of black hole thermodynamics

workshop announcement

Shaping UV Physics Beyond the Standard Model

12 -15 July 2015 (IPPP, U Durham)

https://conference.ippp.dur.ac.uk/event/452/

organisers:
S Abel
G Hiller
D Litim
F Sannino