Massive ghosts and stability in higher derivative gravity

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Semiclassical approach and higher derivatives.

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Three choice for Quantum Gravity (QG)

One may suppose the presence of some new fundamental physics at the Planck scale.

Possible approaches to QG can be classified into three distinct groups. Namely, we can

● Quantize both gravity and matter fields. This is the most fundamental approach and the main subject of this talk.

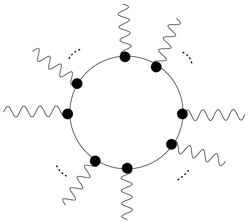
●● Quantize only matter fields on classical curved background (semiclassical approach).
QFT and Curved space-time are well-established notions, which passed many experimental/observational tests.

●●● Quantize something else. E.g., in case of (super)string theory both matter and gravity are induced.

●● **The renormalizable QFT in curved space requires introducing a generalized action of gravity (external field).**
The theory is renormalizable, but only with certain higher derivatives terms in the vacuum action.

Introduction: *Buchbinder, Odintsov & I.Sh. Effective Action in Quantum Gravity (IOPP - 1992);*
I.Sh., Class.Quant.Grav. Topical review (2008), arXiv:0801.0216.

Relevant diagrams for the vacuum sector



Possible covariant counterterms have the structure of

$$S_{vac} = S_{EH} + S_{HD}$$

The renormalizable QFT in curved space requires introducing a generalized form of the gravity (external field), "vacuum action".

$$S_{vac} \,=\, S_{EH} \,+\, S_{HD}$$

where $$S_{EH} \,=\, -\,\frac{1}{16\pi G}\,\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}\,.$$

is the Einstein-Hilbert action with the cosmological constant.

S_{HD} **includes higher derivative terms. The most useful form is**

$$S_{HD} = \int d^4x\sqrt{-g}\,\big\{a_1 C^2 + a_2 E + a_3\Box R + a_4 R^2\big\}\,,$$

where $$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2$$

is the square of the Weyl tensor in $d = 4$,

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2$$

is integrand of the Gauss-Bonnet term (topological term in d=4 **).**

General considerations about higher derivatives:

- **One should definitely quantize both matter and gravity, for otherwise the QG theory would not be complete.**

- **The diagrams with matter internal lines in a complete QG are exactly the same as in a semiclassical theory.**

- **This means one can not quantize metric without higher derivative terms in a consistent way, since these terms are produced already in the semiclassical theory.**

- **Indeed, most of the achievements in curved-space QFT are related to the renormalization of higher derivative vacuum terms, including Hawking radiation, Starobinsky inflation and others.**

Quantum Gravity (QG)

starts from some covariant action of gravity,

$$S = \int d^4x \sqrt{-g}\, \mathcal{L}(g_{\mu\nu})\,.$$

$\mathcal{L}(g_{\mu\nu})$ **can be of GR,** $\mathcal{L}(g_{\mu\nu}) = -\kappa^{-2}(R + 2\Lambda)$ **or some other.**

Gauge transformation $x'^{\mu} = x^{\mu} + \xi^{\mu}$. **The metric transforms as**

$$\delta g_{\mu\nu} = g'_{\mu\nu}(x) - g_{\mu\nu}(x) = -\nabla_{\mu}\xi_{\nu} - \nabla_{\nu}\xi_{\mu}\,.$$

In the case of $\quad g_{\mu\nu}(x) = \eta_{\mu\nu} + \kappa h_{\mu\nu}(x)\,,$

$$\delta h_{\mu\nu} = -\frac{1}{\kappa}\,(\partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}) - h_{\mu\alpha}\partial_{\nu}\xi^{\alpha} - h_{\nu\alpha}\partial_{\mu}\xi^{\alpha} - \xi^{\alpha}\partial_{\alpha}h_{\mu\nu} = R_{\mu\nu\,,\,\alpha}\xi^{\alpha}\,.$$

The gauge invariance of the action means

$$\frac{\delta S}{\delta h_{\mu\nu}}\,\cdot\, R_{\mu\nu\,,\,\alpha}\,\cdot\,\xi^{\alpha} = 0\,.$$

One can prove that the same is true for the Effective Action.

Let us use power counting.

As the first example consider quantum GR.

$$S \,=\, -\,\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,.$$

For the sake of simplicity we consider only vertices with maximal K_ν. Then we have $r_l = K_\nu = 2$ and, combining

$$D+d \,=\, \sum_{l_{int}}(4-r_l) \,-\, 4n \,+\, 4 \,+\, \sum_\nu K_\nu$$

with

$$l_{int} \,= p+n-1$$

we arrive at the estimate ($D=0$ means log. divergences)

$$D+d \,=\, 2+2p\,.$$

This output means that quantum GR is not renormalizable and we can look for some other starting point.

Perhaps, the most natural is HDQG.

Reason: we need HD's anyway for quantum matter field.

Already known action: $S_{gravity} = S_{EH} + S_{HD}$

where $S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}$

and S_{HD} **include higher derivative terms**

$$S_{HD} = -\int d^4x\sqrt{-g}\left\{\frac{1}{2\lambda}C^2 + \frac{\omega}{3\lambda}R^2\right\},$$

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + 1/3\,R^2,$$

K. Stelle, Phys. Rev. D (1977).

Propagators and vertices in HDQG are not like in quantum GR.
Propagators of metric and ghosts behave like $\mathcal{O}(k^{-4})$ **and we have** $K_4,\ K_2,\ K_0$ **vertices. The superficial degree of divergence**

$$D + d = 4 - 2K_2 - 2K_0.$$

This theory is definitely renormalizable.

Including even more derivatives was initially thought to move massive pole to even higher mass scale,

$$S = S_{EH} + \int d^4x\sqrt{-g}\,\Big\{a_1 R^2_{\mu\nu\alpha\beta} + a_2 R^2_{\mu\nu} + a_3 R^2 + \,...$$

$$+ c_1 R_{\mu\nu\alpha\beta}\Box^k R^{\mu\nu\alpha\beta} + c_2 R_{\mu\nu}\Box^k R^{\mu\nu} + c_3 R\Box^k R + b_{1,2,..}R^{k+1}_{...}\Big\}\,.$$

Simple analysis shows this theory is superrenormalizable, but the massive ghosts are still here. For the case of real poles:

$$G_2(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2+m_1^2} + \frac{A_2}{k^2+m_2^2} + \cdots + \frac{A_{N+1}}{k^2+m_{N+1}^2}$$

For any sequence $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$, **the signs of the corresponding terms alternate,** $A_j \cdot A_{j+1} < 0$.

Asorey, Lopez & I. Sh., hep-th/9610006; IJMPhA (1997).

$$D + d = 4 + k(1-p)\,.$$

In what follows we deal with the four derivative theories only.

The price to pay for renormalizability: **For linearized gravity**

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,,$$

there are assive ghosts

$$G_{\rm spin-2}(k) \,\sim\, \frac{1}{m^2}\,\left(\frac{1}{k^2} - \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,.$$

The tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and huge mass.

Tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and huge mass.

- **In classical systems higher derivatives generate exploding instabilities at the non-linear level** *(M.V. Ostrogradsky, 1850).*

- **Interaction between ghost and gravitons may violate energy conservation in the massless sector** *(M.J.G. Veltman, 1963).*

- **Without ghost one violates unitarity of the S -matrix.**

There were several attempts to solve the HD ghost problem.

Stelle, Salam & Strathdee, Tomboulis,
Antonidis & Tomboulis, Johnston, Hawking, ...

In what follows we suggest a new approach which is much simpler and is probably working.

Assumption we made to declassify higher derivative gravity:

● **One can draw conclusions using linearized gravity approximation. S-matrix of gravitons is the main object.**

●● **Ostrogradsky instabilities or Veltman scattering are relevant independent on the energy scale, in all cases they produce run-away solutions and "Universe explodes".**

There is a simple way to check all these assumptions at once.

Take higher derivative theory of gravity and verify the stability with respect to the linear perturbations on some, physically interesting, dynamical background.

Stability & Gravitational Waves

As far as classical action and quantum, anomaly-induced term, both have higher derivatives, an important question is whether the stability of classical solutions in cosmology holds or not.

Consider small perturbation

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu}\,, \quad h_{\mu\nu} = \delta\, g_{\mu\nu}\,,$$

where $g^0_{\mu\nu} = \{1,\ -\delta_{ij}\, a^2(t)\}$, $\mu = 0,1,2,3$ **and** $i = 1,2,3$.

$$h_{\mu\nu}(t,\vec{r}) = \int \frac{d^3k}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{k}}\, h_{\mu\nu}(t,\vec{k})\,.$$

Using the conditions $\partial_i h^{ij} = 0$ **and** $h_{ii} = 0$, **together with the synchronous coordinate condition** $h_{\mu 0} = 0$, **we arrive at the equation for the tensor mode**

Fabris, Pelinson and I.Sh., (2001);
Fabris, Pelinson, Salles and I.Sh., (2011).

$$\frac{1}{3}\stackrel{....}{h} + 2H\stackrel{...}{h} + \left(H^2 + \frac{{M_P}^2}{32\pi a_1}\right)\ddot{h} + \frac{2}{3}\left(\frac{1}{4}\frac{\nabla^4 h}{a^4} - \frac{\nabla^2 \ddot{h}}{a^2} - H\frac{\nabla^2 \dot{h}}{a^2}\right)$$

$$-\left[H\dot{H} + \ddot{H} + 6H^3 - \frac{3{M_P}^2 H}{32\pi a_1}\right]\dot{h} - \left[\frac{{M_P}^2}{32\pi a_1} - \frac{4}{3}\left(\dot{H} + 2H^2\right)\right]\frac{\nabla^2 h}{a^2}$$

$$-\left[\left(24\dot{H}H^2 + 12\dot{H}^2 + 16H\ddot{H} + \frac{8}{3}\stackrel{...}{H}\right) - \frac{{M_P}^2}{16\pi a_1}\left(2\dot{H} + 3H^2\right)\right]h = 0\,.$$

It looks much simpler than Eqs. with semiclassical corrections:

Fabris, Pelinson and I.Sh., NPB, hep-th/0009197;
Fabris, Pelinson, Salles and I.Sh., JCAP, arXiv:1112.5202;
F. Salles and I.Sh., PRD, arXiv:1401.4583.

Net Result: The stability does not actually depend on quantum corrections. It is completely defined by the sign of the classical coefficient a_1 **of the Weyl-squared term. The sign of this term defines whether graviton or ghost has positive kinetic energy!**

We can distinguish the **three** cases. First two are:

- **The coefficient of the Weyl-squared term is** $a_1 < 0$ **Then**

$$G_{\rm spin-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,,$$

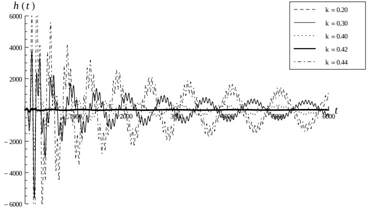
there are no growing modes up to the Planck scale, $\vec{k}^2 \approx M_P^2$**.**

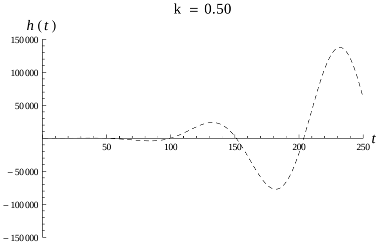
For the dS background this is in a perfect agreement with
Starobinsky, Let.Astr.Journ. (in Russian) (1983);
Hawking, Hertog and Real, PRD (2001).

- **The coefficient** $a_1 > 0$ **or** $a_1 > 0\,,\; G \to -G$**.**

$$G_{\rm spin-2}(k) \sim \frac{1}{m^2}\left(-\frac{1}{k^2} + \frac{1}{m^2+k^2}\right), \quad m \propto M_P\,.$$

and there are rapidly growing modes at any scale.





$a_1 < 0$ **Radiation-dominated Universe. There are no growing modes until the frequency** k **achieves the value** ≈ 0.5 **in Planck units. Starting from this value, we observe instability as an effect of massive ghost.**

The anomaly-induced quantum correction is $\mathcal{O}(R^3_{....})$. **Until the energy is not of the Planck order of magnitude, these corrections can not compete with classical** $\mathcal{O}(R^2_{....})$ **- terms.**

Massive ghosts are present only in the vacuum state. We just do not observe them “alive” until the energy scale M_P

CONSIDER THE THIRD CASE.

Giulia Cusin, Filipe de O. Salles, I.Sh., arXive:1503.08059.

- **The coefficient of the Weyl-squared term is** $a_1 > 0$**. Then**

$$G_{\rm spin-2}(k) \sim -\frac{1}{m^2}\left(\frac{1}{k^2 - m^2} - \frac{1}{k^2}\right), \quad m \propto M_P\,,$$

a graviton plus a ghost-tachyon with the Planck-scale mass.

In this case there are growing modes with all frequencies!

Why is that? What is the notion of tachyon?

Consider general second-order action of a free field $h(x) = h(t, \vec{r})$

$$S(h) = \frac{s_1}{2}\int d^4x\,\{\dot{h}^2 - (\nabla h)^2 - s_2 m^2 h^2\}\,.$$

$s_{1,2} = \pm 1$ **for different types of fields.**

Perform the Fourier transform in the space variables,

$$h(t,\vec{r}) \,=\, \frac{1}{(2\pi)^3}\int d^3k\, e^{i\vec{k}\cdot\vec{r}}\, h(t,\vec{k})\,,$$

and consider the dynamics of each $h \equiv h(t,\vec{k})$ **separately.**

$$S_{\vec{k}}(h) \,=\, \frac{s_1}{2}\int dt\,\big\{\dot{h}^2 - k^2h - s_2m^2h^2\big\} \,=\, \frac{s_1}{2}\int dt\,\big\{\dot{h}^2 - m_k^2h^2\big\}\,,$$

where

$$k^2 = \vec{k}\cdot\vec{k}\,, \qquad m_k^2 = s_2m^2 + k^2\,.$$

- **Normal healthy field corresponds to** $s_1 = s_2 = 1.$

Kinetic energy is positive. The minimal action can be achieved for a static configuration. The equation of motion is of the oscillatory type,

$$\ddot{h} + m_k^2 h \,=\, 0\,.$$

with the usual periodic solution.

- **Massive ghost has** $s_1 = -1, \quad s_2 = 1.$

It is not a tachyon, because $m_k^2 = s_2 m^2 + k^2 \geq 0.$

Kinetic energy is negative, but one can postulate zero variation of the action and arrive at the normal oscillatory equation.

A particle with negative kinetic energy has the tendency to achieve a maximal speed, but a free particle can not accelerate, for this would violate energy conservation.

A free ghost does not produce any harm to the environment, being isolated from it.

However, if we admit an interaction with healthy fields, the tendency of a ghost is to accelerate and transmit a positive energy difference to these healthy fields.

Tachyon has $s_2 = -1$. For relatively small momenta $m_k^2 < 0$ in Eq. the equation of motion is

$$\ddot{h} - \omega^2 h \,=\, 0\,, \qquad \omega^2 = \left|m_k^2\right|\,, \qquad m_k^2 = s_2 m^2 + k^2\,.$$

If the particle moves faster than light the solution is of the oscillatory kind, indicating that such a motion is "natural" for this kind of particle (Sudarshan et al, 1962 - ...).

But for a smaller velocities the equation is anti-oscillatory, with exponential-type unstable solutions,

$$h \,=\, h_1 e^{\omega t} \,+\, h_2 e^{-\omega t}\,. \qquad (*)$$

For a field interacting with an external gravitational background, it is possible that the same wave changes from being a normal healthy state to a tachyonic one.

In principle, this situation may produce very strong effects at both quantum and classical levels.

D. Vanzella et all, 2010-2015.

In the fourth-order gravity the equations for the metric perturbations in flat space are

$$\ddot{\ddot{h}} + 2k^2\ddot{h} + k^4 h \,-\, \frac{1}{32\pi G a_1}\,\big(\ddot{h} + k^2 h\big) \,=\, 0\,.$$

It proves useful to introduce a new notation

$$\frac{1}{32\pi G a_1} \,=\, -\,s_2\,m^2\,, \quad \textbf{where} \quad s_2 = -\mathrm{sign}\,a_1 \quad \text{and} \quad m^2 > 0\,.$$

Then

$$\Big(\frac{\partial^2}{\partial t^2} + k^2\Big)\,\Big(\frac{\partial^2}{\partial t^2} + m_k^2\Big)h \,=\, 0\,, \quad \textbf{where} \quad m_k^2 \,=\, k^2 + s_2 m^2\,.$$

The general formula for the frequencies is

$$\omega_{1,2} \,\approx\, \pm\, i\big(k^2\big)^{1/2} \quad \textbf{and} \quad \omega_{3,4} \,\approx\, \pm\big(-\,m_k^2\big)^{-1/2}\,,$$

For a negative a_1 there are only imaginary frequencies and hence oscillator-type solutions (ghost case).

On the contrary, for a positive a_1 the roots $\omega_{3,4}$ are real, since in this case $-m_k^2 > 0$ for sufficiently small k^2 (tachyonic ghost).

The main difference between ghosts and tachyons is that a ghost may cause instabilities only when it couples to some healthy fields or to the background, while with tachyons there is no such a protection.

The situation with ghosts can be kept under control in the effective field theory framework, and in general when the intensity of the background fields is low and involved energies insufficient to generate a ghost from vacuum.

On the contrary, no low-energy protection can be expected in the theory with tachyons, because they produce instabilities independently on their interaction to normal particles or on the intensity of the background.

In other words, for tachyons the exponential behavior occurs at all frequencies, and not only above the Planck threshold.

The bad news for those expecting an eternal ΛCDM universe is that massive ghost will eventually become tachyon.

At the end of the ΛCDM universe the energy of the background is very law. At low energies all massive fields decouple and only the quantum effects of virtual photons are relevant.

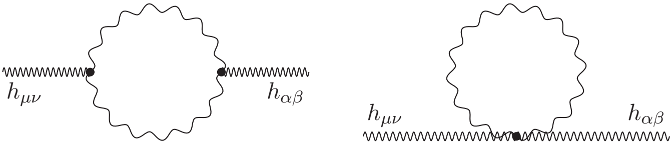


Figure: **Photon loops with two external gravitational lines.**

These quantum effects are pretty well-known,

$$\Gamma_{Weyl-squared} = -\frac{1}{320\pi^2}\int\sqrt{-g}\,C_{\alpha\beta\lambda\tau}\,\log\left(\frac{\Box}{\mu^2}\right)C^{\alpha\beta\lambda\tau}\,.$$

$$\Gamma_{Weyl-squared} = -\frac{1}{320\pi^2}\int \sqrt{-g}\, C_{\alpha\beta\lambda\tau}\, \log\left(\frac{\Box}{\mu^2}\right) C^{\alpha\beta\lambda\tau}\,.$$

It is easy to see that:

- **This expression is time-dependent during the cosmological evolution.**
- **Log. function is slow, hence we can approximately treat** $\log\left(\Box/\mu^2\right)$ **as a slowly varying parameter.**
- **Namely,**

$$g_{\mu\nu} = a^2(\eta)\,\bar{g}_{\mu\nu} = e^{\sigma(\eta)}\,\bar{g}_{\mu\nu} \;\Rightarrow\; \log\left(\frac{\Box}{\mu^2}\right) \propto -2\sigma(\eta) = -2\sigma(t)\,.$$

For any initial value of a_1 (including zero!) we meet

$$a_1^{eff}(t) = a_1 + \frac{1}{160\pi^2}\,\sigma(t)\,.$$

What this means, from the side of Physics?

At the final stage of the ΛCDM universe

$$\sigma(t) = H_0 t\,, \quad H_0 \sim \sqrt{\frac{8\pi \,\times\, 0.7\,\rho_c^0}{3\,M_P^2}} \approx 10^{-42}\,GeV\,.$$

Then the effective coefficient is

$$a_1^{eff}(t) \,=\, a_1 + \frac{1}{160\pi^2}\,\sigma(t) \,=\, a_1 + \frac{1}{160\pi^2}\,H_0 t\,.$$

Earlier or later a_1^{eff} will change sign and become positive.

The moment of this occurrence will be quite remarkable, but nobody will perhaps appreciate this, because all points of the space will explode at once.

The time remaining until tachyonic modes emerge is

$$t_q \,=\, \frac{160\pi^2\,a_1}{H_0} \,\simeq\, 2.4\cdot 10^{13}\,yr \,=\, 2.4\cdot 10^{4}\,bi\,, \qquad a_1(\mathrm{initial}) = 1\,.$$

The instability corresponds to linear perturbations. Next orders in the perturbative expansion in $h_{\mu\nu}$ will restore the stability. Even this "restricted" gravitational explosion should be capable to produce relevant changes in the space-time properties.

- **Consider the list of approximations which have been used.**

Heat-up question: "Can we trust logarithmic approximation at the one-loop level?" - Yes.

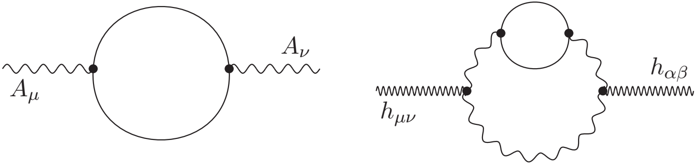
- **Can we expect qualitative change in the result by taking the higher-order loops into account?**

Formally, higher-loop contributions do not change the sign of the β_1-function (c-theorem), but this is not sufficient to draw conclusions about the (ir)relevance of the higher-loop terms.

In the UV there will be higher-log. contributions, capable to produce a strong change in the running of a_1. However, the situation at low energies (far IR) is quite different.

Let us remember that second- and higher-loop corrections to the one-photon bubble include a loop of electrons or of other massive charged fermions.

Because of the Appelquist & Carazzone decoupling theorem the contribution of the second loop is suppressed by a factor $\left(\mathcal{E}/m_e\right)^2$.



Our interest is not the dynamics of the conformal factor itself, but its interaction to gravitational waves. For instance, taking $\mathcal{E}_{GW} = 1eV$ we have "only" ten-orders decoupling.

Up to the frequencies of the order of electron mass, the one-loop approximation is completely robust. Only above this threshold there is a small chance of stabilization by higher loops.

What about quantum gravity (QG) effects, which have been neglected so far?

It is not easy to give a definite answer due to the variety of existing models of QG. Let us consider a short list of the possibilities which are better explored.

The standard effective framework for the IR effects of QG assumes that GR is a universal theory of IR quantum gravity.

J. Donoghue - 1994, PRL & PRD, gr-qc/9405057

QG based on GR is non-renormalizable, hence there is no consistent perturbative β-function for the parameter a_1.

One can easily derive the 1-loop logarithmic form factor.

G. 'tHooft and M.Veltman, (1974).

However, it is gauge-fixing dependent and vanish on-shell.

R. Kallosh, O. Tarasov and I.V. Tyutin (1978).

Therefore, no physical correction can be expected. Similar situation holds at higher loops.

- **Higher derivative QG (HDQG)**

The β-function for the parameter a_1 is well-defined, free of ambiguities. According to the well-verified calculations

E.S. Fradkin & A.A. Tseytlin (1982);
I. Avramidi & A.O. Barvinsky (1986);
I. Antoniadis and E. Mottola (1992);
G.B. Peixoto & I.Sh. (2003)

the contribution of HDQG enhance the one of the photon loop by a factor of $10-20$ **in the four-derivative QG case.**

Indeed, this statement requires great care. In fact, the IR effects of HDQG are not sufficiently well-explored.

The standard point of view is that the universal IR limit of all these theories is quantum GR. Then we come back to the irrelevant contribution of QG in the IR.

Conclusions

- In the gravity with higher derivatives the propagator includes massive nonphysical mode(s) called ghosts.

- The massive ghosts are capable to produce terrible instabilities, but ... for this end there should be at least one such ghost excitation in the initial spectrum.

- At least in the cosmological case, the ghost is not actually generated below Planck scale.

- The final conclusion is that the HDQG may be a perfect candidate to be an effective QG below the Planck scale.

- Finally, we can predict an intensive tachyonic explosion at the end of the ΛCDM universe.

- **In (super)string theory the terms providing the β-function for the parameter a_1 are removed by means of the Zwiebach transformation for the "background" metric.**

B. Zwiebach, (1985).

$$\Gamma = \int d^D x \sqrt{g}\, e^{-2\Phi} \Big\{ -R + 4\alpha' \big(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4\, R_{\alpha\beta} R^{\alpha\beta} + R^2 \big) + ... \Big\}.$$

By construction, the β-function for the parameter a_1 is zero.

However, string theory is not supposed to significantly correct QFT results at low and very low energies.

Otherwise we would observe such corrections in precision experiments, e.g., the ones that test QED and Standard Model calculations. Using string theory to evaluate the β_1-function in the far IR is not reasonable from a conceptual point of view.

After all, in known versions of QG & string theories the IR running of universe to the tachyonic end can't be cancelled.