On the partner particles for black-hole evaporation

Ralf Schützhold

Fakultät für Physik

Universität Duisburg-Essen

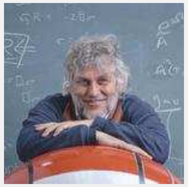
Quantum Radiation

Relativistic quantum fields ($\hbar, c$) in vacuum state $|0\rangle_{\rm in}$

- Hawking radiation
  $\rightarrow$ gravitational field

- Sauter-Schwinger effect
  $\rightarrow$ electric field

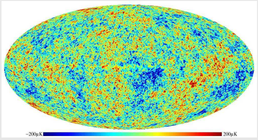
- Unruh radiation
  $\rightarrow$ acceleration



- Dynamical Casimir effect
  $\rightarrow$ mirror motion

- Cosmological particle creation
  $\rightarrow$ expansion

"Particles are created in pairs"



Squeezing

Bogoliubov transformation (linear)

$$\hat{a}_k^{\rm out} = \int dk'\, \alpha^*_{kk'} \hat{a}_{k'}^{\rm in} + \int dk'\, \beta_{kk'} \left(\hat{a}_{k'}^{\rm in}\right)^\dagger$$

Time evolution for bi-linear Hamiltonian

$$\hat{U} = \mathfrak{T}\left[\exp\left\{-i\int dt\ \hat{H}(t)\right\}\right]$$

Generalized squeezing operation

$$|0\rangle_{\rm in} = \exp\left\{\int dk\, dk'\, \xi_{kk'} \left(\hat{a}_k^{\rm out}\right)^\dagger \left(\hat{a}_{k'}^{\rm out}\right)^\dagger - {\rm h.c.}\right\}|0\rangle_{\rm out}$$

Creation of particles $\langle 0|\, \hat{n}_k^{\rm out}\, |0\rangle_{\rm in} \neq 0$ in pairs

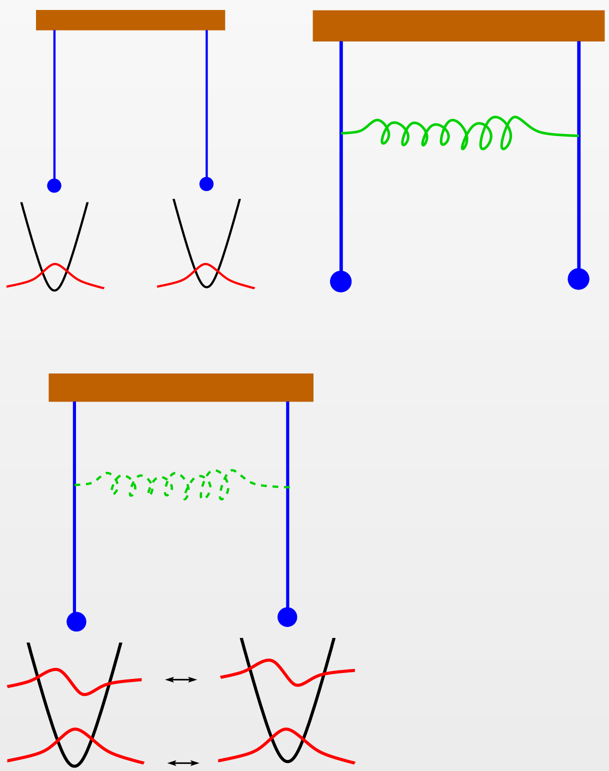
$$|0\rangle_{\rm in} = |0\rangle_{\rm out} + \int dk\, dk'\, \xi_{kk'}\, |k, k'\rangle_{\rm out} + \dots$$

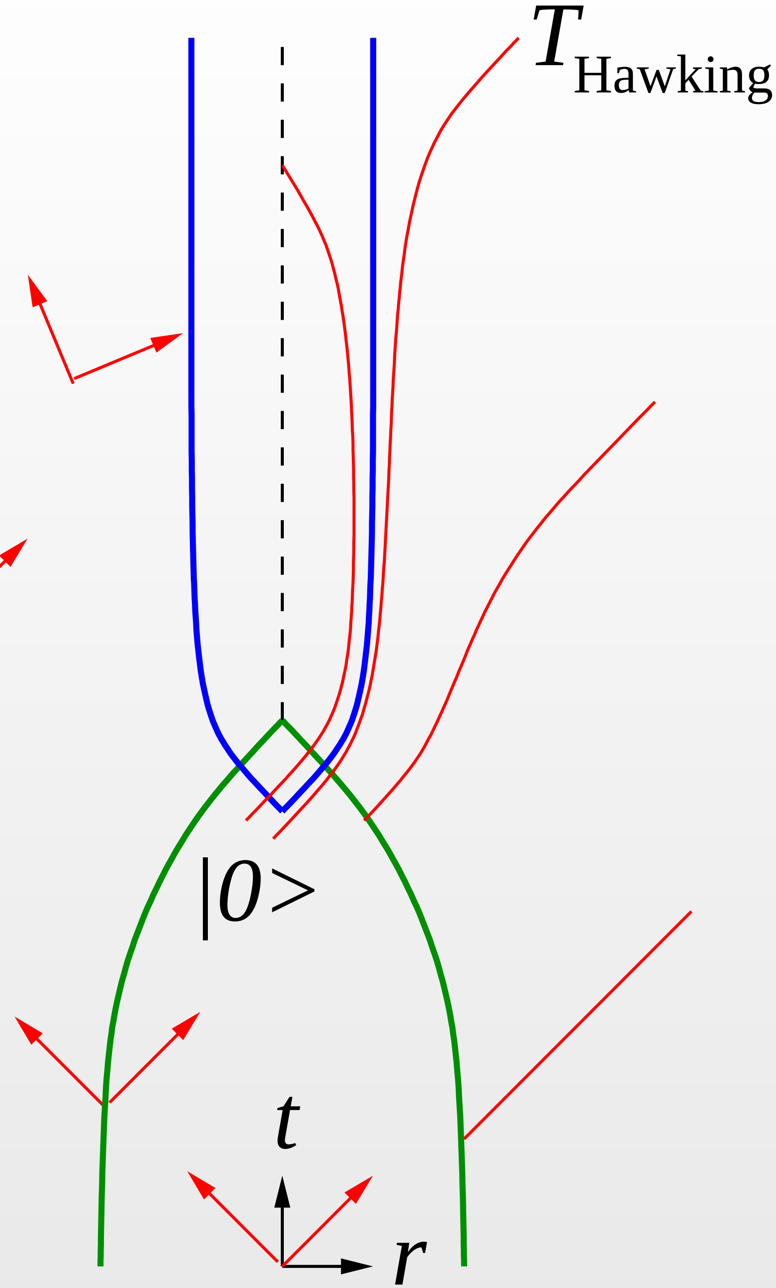
Note: asymptotics. . .

Hawking Radiation

S. W. Hawking, Nature **248**, 30 (1974);

Comm. Math. Phys. **43**, 199 (1975).





$$T_{\rm Hawking} = \frac{1}{8\pi M}\frac{\hbar c^3}{G_{\rm N} k_{\rm B}}$$

But: trans-Plankian problem, information puzzle etc.

Definition of Partner Particle

A) reduced density matrix for Hawking mode plus partner $\hat{\varrho}_{\rm HP} = {\rm Tr}_{\rm rest}\{|0\rangle_{\rm in}\langle 0|_{\rm in}\}$ is a pure state

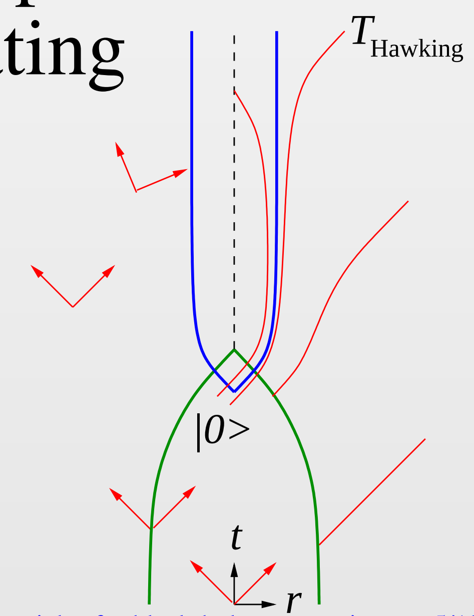
$\to$ only correlations between Hawking mode and its partner – but not with any other modes

Note: $\hat{\varrho}_{\rm H} = {\rm Tr}_{\rm P}\{\hat{\varrho}_{\rm HP}\}$ is a mixed (thermal) state

B) state after annihilating one Hawking particle is the same (up to normalization) as after creating one partner particle $\hat{a}_{\rm H}\,|0\rangle_{\rm in} \propto \hat{a}^\dagger_{\rm P}\,|0\rangle_{\rm in}$

$\to$ particles come in pairs

Note: in most simple cases symmetric, i.e., equivalent to $\hat{a}_{\rm P}\,|0\rangle_{\rm in} \propto \hat{a}^\dagger_{\rm H}\,|0\rangle_{\rm in}$



Partner Particle

Hawking mode in terms of in-operators $\hat{a}_k^{\text{in}} \left|0\right\rangle_{\text{in}} = 0$

$$\hat{a}_{\text{H}} = \int dk\, \alpha_k^* \hat{a}_k^{\text{in}} + \int dk\, \beta_k \left(\hat{a}_k^{\text{in}}\right)^\dagger$$

Assume orthogonality (most simple case)

$$\int dk\, \alpha_k^* \beta_k = 0$$

Unique partner mode from conditions A and B

$$\hat{a}_{\text{P}} = \coth \xi \int dk\, \beta_k^* \hat{a}_k^{\text{in}} + \tanh \xi \int dk\, \alpha_k \left(\hat{a}_k^{\text{in}}\right)^\dagger$$

with squeezing parameter

$$\sinh^2 \xi = \int dk\, \left|\beta_k^2\right| \rightsquigarrow \left|0\right\rangle_{\text{in}} = e^{\xi \hat{a}_{\text{H}}^\dagger \hat{a}_{\text{P}}^\dagger - \text{h.c.}} \left|0\right\rangle_{\text{HP}}$$

where $\hat{a}_{\text{H}} \left|0\right\rangle_{\text{HP}} = \hat{a}_{\text{P}} \left|0\right\rangle_{\text{HP}} = 0$

Moving Mirror in 1+1 D

Toy model for black hole evaporation: accelerated mirror with

$$v = -\frac{e^{-\kappa u}}{\kappa}$$

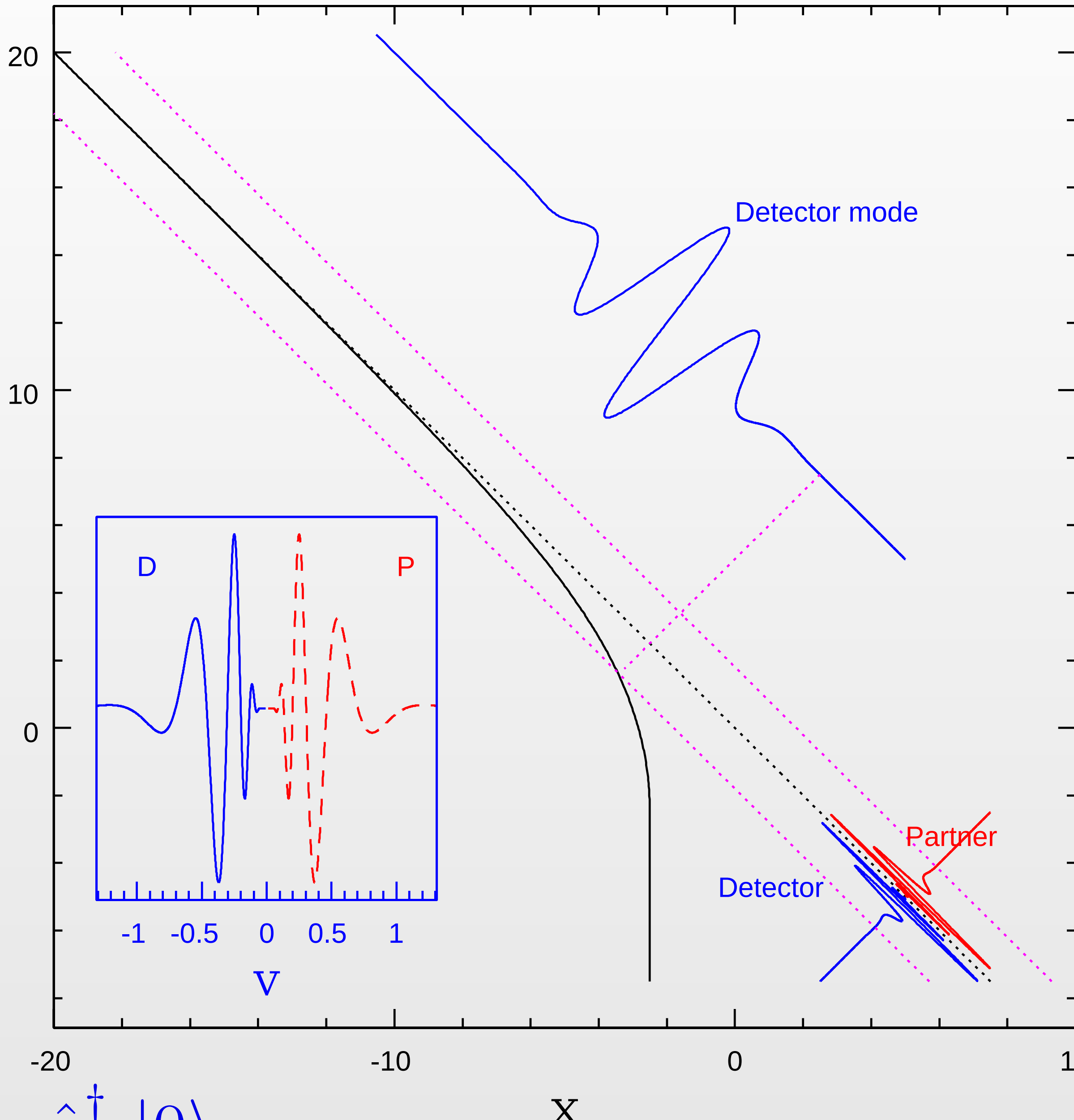
$$v = t + x$$

$$u = t - x$$

emits thermal radiation with

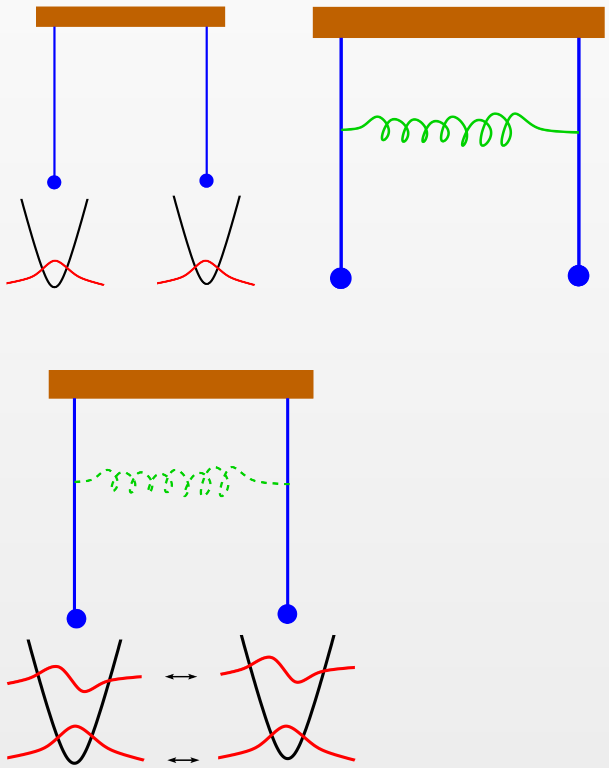
$$T = \frac{\kappa}{2\pi}$$

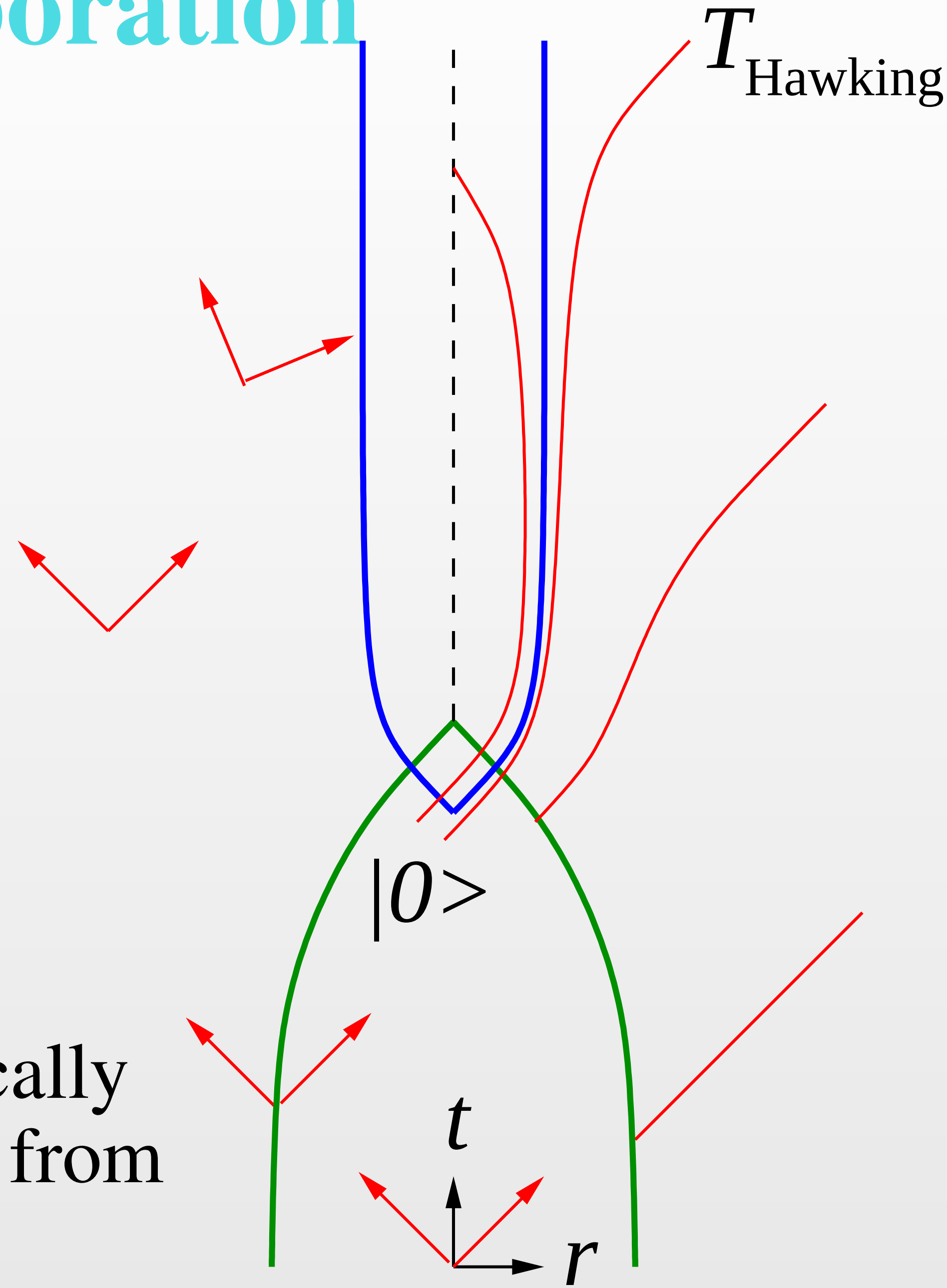
Partner mode in local vacuum!



But: $\hat{a}_{\mathrm{H}} \left|0\right\rangle_{\mathrm{in}} \propto \hat{a}_{\mathrm{P}}^{\dagger} \left|0\right\rangle_{\mathrm{in}}$

Black Hole Evaporation



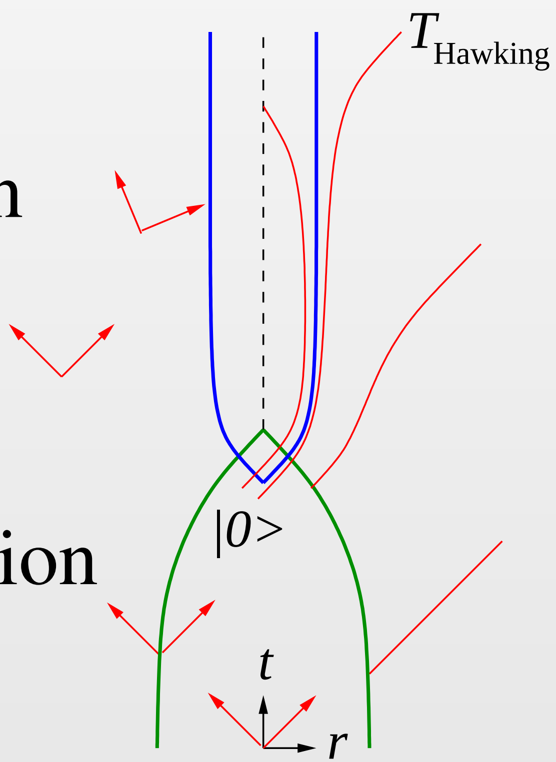


Infalling partners are locally nearly indistinguishable from vacuum fluctuations!

Black Hole Information Puzzle

Is black hole formation $\rightarrow$ evaporation unitary?

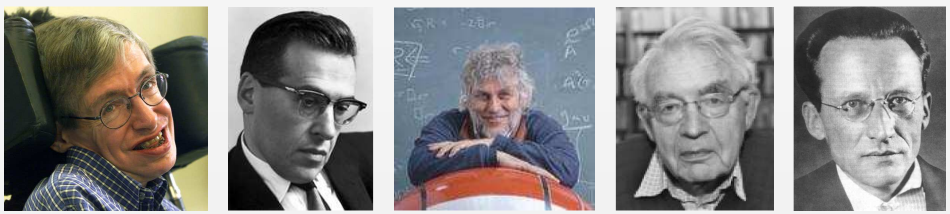
- regularity near horizon ($\leftrightarrow$ firewall etc.)
- correlations between Hawking particles and vacuum fluctuations falling towards singularity
  - a) information is lost
    $\rightarrow$ "non-unitarity"?
  - b) singularity stores information
    $\rightarrow$ black-hole entropy?
    $\rightarrow$ simple picture:
    one qubit per ℓ^3_{Planck}?
  - c) singularity re-emits information
    $\rightarrow$ causal structure?
- information $\neq$ energy

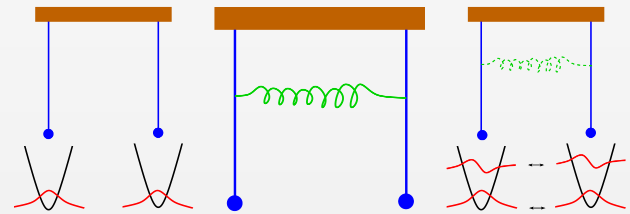


Summary

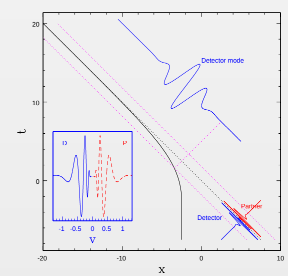
M. Hotta, R.S., W.G. Unruh, `arXiv:1503.06109`,
to appear in Phys. Rev. D (2015)

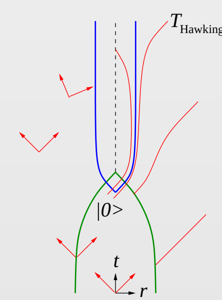
- quantum radiation: “particles in pairs”



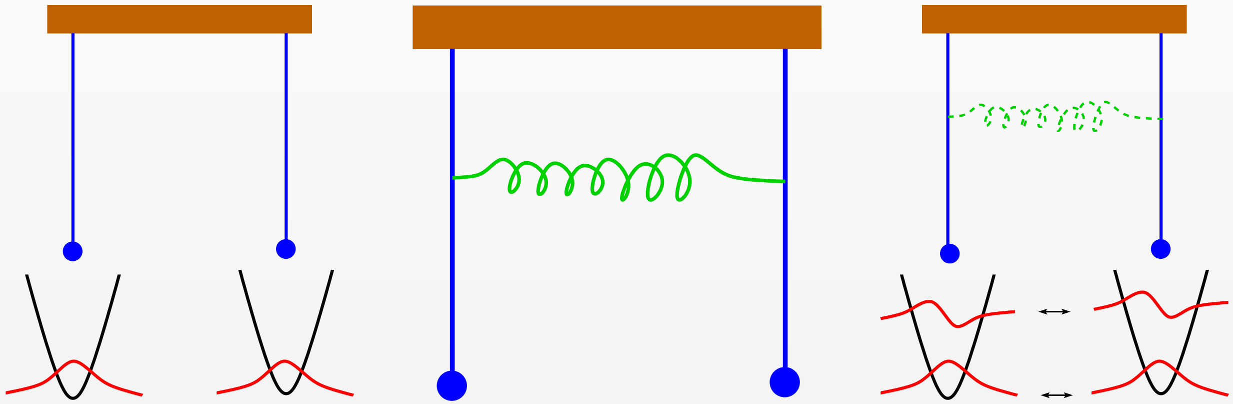


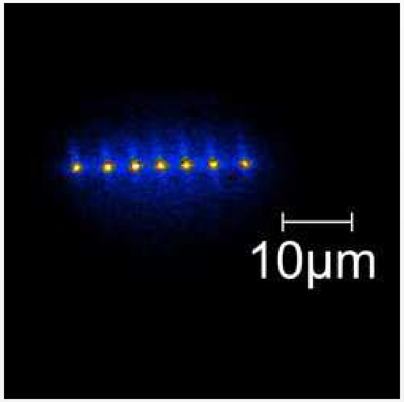
- determination of partner “particle”
- moving mirror and black hole
- partners $\approx$ vacuum fluctuations
- information $\neq$ energy
- black hole information puzzle





Ion Trap Analogue





Squeezing → creation of phonons $\langle\hat{n}\rangle_\xi \approx |\xi^2| \approx 0.1$

$$\xi \approx \exp\left\{-\pi\sqrt{\frac{8}{3}\left(\frac{\Delta x_{\text{min}}}{\Delta x_{\text{crit}}}\right)^3}\right\}$$

Creation of entanglement if

$$e^\xi > 1 + 2n_{\text{thermal}}$$

C. Fey, Ms Thesis (2014); C. Fey, T. Schätz, R.S., manuscript in preparation

See also R. S. *et al.*, Phys. Rev. Lett. **99**, 201301 (2007).

Black Hole Evaporation

Formula for Hawking temperature

$$T_{\mathrm{Hawking}} = \frac{1}{8\pi M}\frac{\hbar\, c^3}{G_{\mathrm{N}} k_{\mathrm{B}}}$$

Combines four (apparently) different areas of physics

- quantum theory $\hbar$
- relativity c
- gravity G_{N}
- thermodynamics k_{B}

Is nature trying to give us a hint?
($\rightarrow$ black hole entropy $\propto$ area etc.)

Problems: $M_{\mathrm{BH}} = 30 M_{\mathrm{sun}} \leadsto T_{\mathrm{Hawking}} \approx 2\mathrm{nK} \ldots$
+ trans-Plankian problem