Secularly growing loop corrections in strong background field

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Motivation

- Secular growth of loop corrections is practically inevitable in non–stationary situations (Landau and Lifshitz, X-th volume)
- This growth is the IR effect. No modifications of UV physics.
- Quantum corrections are of the same order as classical contributions, if one weights long enough.

- de Sitter space interacting QFT (review arXiv:1309.2557).
- QED on strong electric field background beyond the background field approximation (arXiv:1405.5225).
- Loop correction to Hawking radiation (arXiv:1509. ...).

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- Suppose one would like to find:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| \overline{T}\, e^{i \int_{t_0}^{t} dt' H(t')}\, \mathcal{O}\, T\, e^{-i \int_{t_0}^{t} dt' H(t')} \right| \Psi \right\rangle, \quad (1)$$

e.g. $\langle T_{\mu\nu} \rangle$ or $\langle J_\mu \rangle$.

- Here $H(t) = H_0(t) + V(t)$.
- T — time–ordering, $\overline{T}$ — anti–time–ordering.
- t_0 — initial moment of time, $|\Psi\rangle$ — initial state, $\langle \Psi | \mathcal{O} | \Psi \rangle (t_0)$ is supposed to be given.

Adiabatic catastrophe

- Transferring to the interaction picture:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| S^+(+\infty, t_0)\, T \left[\mathcal{O}_0(t)\, S(+\infty, t_0) \right] \right| \Psi \right\rangle . \qquad (2)$$

Here $S(t_2, t_1) = T\, e^{-i \int_{t_1}^{t_2} dt' V_0(t')}$; $\mathcal{O}_0(t)$ and $V_0(t)$ are operators in the interaction picture.

- Slightly changing the problem:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| S^+_{t_0}(+\infty, -\infty)\, T \left[\mathcal{O}_0(t)\, S_{t_0}(+\infty, -\infty) \right] \right| \Psi \right\rangle . \quad (3)$$

Here t_0 is the time moment after which the interactions, $V(t)$, are adiabatically turned on.

Discussion

- When does the dependence on t_0 disappear? Otherwise we have adiabatic catastrophe and breaking of various symmetries: E.g. correlation functions stop to depend only on $|t_1 - t_2|$.

- The dependence on t_0 disappears when the situation is or becomes stationary.

Discussion

- The seminal example of the stationary situation is when the free Hamiltonian H_0 is time independent and has a spectrum bounded from below: $H_0 \left|vac\right\rangle = 0$ and $\left|\psi\right\rangle = \left|vac\right\rangle$.

- In fact, in the latter case by adiabatic turning on and then switching off $V(t)$ we do not disturb the ground state:

$$\left\langle vac \left| S^{+}(+\infty, -\infty) \right| excited\ \ state \right\rangle = 0,$$

while

$$\left| \left\langle vac \left| S^{+}(+\infty, -\infty) \right| vac \right\rangle \right| = 1.$$

It does not matter when one turns on interactions. The dependence on t_0 disappeared!

- Furthermore, in the latter case we obtain:

$$\langle \mathcal{O} \rangle (t) =$$
$$\sum_{sta} \langle vac \left| S^{+}(+\infty, -\infty) \right| sta \rangle \, \langle sta \left| T \left[\mathcal{O}_0(t) \, S(+\infty, -\infty) \right] \right| vac \rangle =$$
$$= \langle vac \left| S^{+}(+\infty, -\infty) \right| vac \rangle \, \langle vac \left| T \left[\mathcal{O}_0(t) \, S(+\infty, -\infty) \right] \right| vac \rangle =$$
$$= \frac{\langle vac \left| T \left[\mathcal{O}_0(t) \, S(+\infty, -\infty) \right] \right| vac \rangle}{\langle vac \left| S(+\infty, -\infty) \right| vac \rangle}.$$

This way we arrive at having only the T–ordered expressions and then can use Feynman technique.

- Other situation when the dependence on t_0 disappears if there is a stationary state (e.g. thermal density matrix in flat space–time).

Discussion

- Is there a stationary state if a background field is never switched off? What is that state, if it is present? What if there is no stationary state?
- How does the dependence on t_0 reveals itself? t_0 does not appear in UV renormalization! In UV limit one always can use the Feynman technique, because high frequency modes are not sensitive to background fields.

- To answer the above questions one has to calculate directly:

$$\langle \mathcal{O} \rangle_{t_0}(t) = \left\langle \Psi \left| S^{+}_{t_0}(+\infty, -\infty)\, T\left[\mathcal{O}_0(t)\, S_{t_0}(+\infty, -\infty)\right] \right| \Psi \right\rangle \quad (4)$$

for various choices of $\mathcal{O}$.

Non–stationary case

- Schwinger notations: S — "$+$" vertexes, S^+ — "$-$" vertexes:

$$\begin{aligned}
D^{++}(1,2) &= \left\langle \Psi \left| T \left(\phi(1)\ \ \phi(2) \right) \right| \Psi \right\rangle, \\
D^{--}(1,2) &= \left\langle \Psi \left| \overline{T} \left(\phi(1)\ \ \phi(2) \right) \right| \Psi \right\rangle, \\
D^{+-}(1,2) &= \langle \Psi | \phi(1)\, \phi(2) | \Psi \rangle, \\
D^{-+}(1,2) &= \langle \Psi | \phi(2)\, \phi(1) | \Psi \rangle.
\end{aligned} \tag{5}$$

Every field is characterized by a matrix of propagators.

- After Keldysh's rotation of ϕ_+ and ϕ_-, we obtain:

$$D^{R,A}(1,2) = \theta\left(\pm\Delta t_{1,2}\right)\left(D^{+-}(1,2) \;-\; D^{-+}(1,2)\right) = \\ = \theta\left(\pm\Delta t_{1,2}\right)\left[\phi(1)\;,\;\phi(2)\right] \quad (6)$$

— state independent Retarded and Advanced propagators. They characterize only the spectrum of excitations.

- The Keldysh propagator:

$$D^{K}(1,2) = \frac{1}{2}\left(D^{+-}(1,2) \;+\; D^{-+}(1,2)\right) = \\ = \frac{1}{2}\left\langle\Psi\left|\left\{\phi(1)\;,\;\phi(2)\right\}\right|\Psi\right\rangle. \quad (7)$$

Discussion

- If we have spatially homogeneous non–stationary state: $\phi(t,\vec{x}) = \int d^{D-1}\vec{p}\,\left(a_{\vec{p}}\, e^{i\,\vec{p}\,\vec{x}}\, g_p(t) + h.c.\right)$, for the case of real scalar field, then

$$\int d^{D-1}\vec{p}\, e^{-i\,\vec{p}\,(\vec{x}_1-\vec{x}_2)}\, D^K\left(t_1, t_2, |\vec{x}_1 - \vec{x}_2|\right) \equiv D^K_p\left(t_1, t_2\right) =$$

$$= \left(\frac{1}{2} + \left\langle a^+_{\vec{p}}\, a_{\vec{p}}\right\rangle\right) g_p(t_1)\, g^*_p(t_2) + \left\langle a_{\vec{p}}\, a_{-\vec{p}}\right\rangle g_p(t_1)\, g_p(t_2) + c.c.$$

— carries information about background state!

- In QED, global de Sitter and black hole collapse case the formulas are a bit different, but the situation is conceptually the same.

Discussion

- In a free theory $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle = const, \quad \left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle = const.$ All time dependence is gone into harmonic functions — $g_p(t)$.
- If the initial state is the ground one: $|\Psi\rangle = |ground\rangle$ and $a_p\, |ground\rangle = 0$, we obviously have that $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle = \left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle = 0.$

- All quasi–classical results (non–interacting fields, background field approximation) follow from the tree–level propagator:

$$D^K_p\,(t_1\,, t_2) = \frac{1}{2}\,\Big(g_p(t_1)\, g^*_p(t_2) \;+\; g^*_p(t_1)\, g_p(t_2)\Big)\,. \qquad (8)$$

E.g. $\langle T_{\mu\nu}\rangle_0$ in de Sitter space and black hole collapse, and $\langle J_\mu\rangle_0$ in QED.

Secular growth of loop corrections

- However, if one turns on interactions, then $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle$ and $\left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle$ start to depend on time.
- Say for $\lambda\phi^3$ (or $\lambda\phi^4$) theory at loop level, as $t = \frac{t_1+t_2}{2} \to +\infty$, we obtain that

$$D^K_p\,(t_1\,,\,t_2) = \left(\frac{1}{2} + n_p(t)\right)\, g_p(t_1)\, g^*_p(t_2) + \kappa_p(t)\, g_p(t_1)\, g_p(t_2) + c.c.$$

- At one loop level

$$n_p(t) \propto \lambda^2 \int d^{D-1}\vec{q}_1 \int d^{D-1}\vec{q}_2 \iint_{t_0}^{t} dt_3\, dt_4\, \delta\left(\vec{p} + \vec{q}_1 + \vec{q}_2\right) \times$$
$$\times g^*_p\left(t_3\right)\, g_p\left(t_4\right)\, g^*_{q_1}(t_3)\, g_{q_1}(t_4)\, g^*_{q_2}(t_3)\, g_{q_2}(t_4) + O\left(t_1 - t_2\right),$$
$$\kappa_p(t) \propto -\lambda^2 \int d^{D-1}\vec{q}_1 \int d^{D-1}\vec{q}_2 \iint_{t_0}^{t} dt_3\, dt_4\, \delta\left(\vec{p} + \vec{q}_1 + \vec{q}_2\right) \times$$
$$\times g^*_p\left(t_3\right)\, g^*_p\left(t_4\right)\, g^*_{q_1}(t_3)\, g_{q_1}(t_4)\, g^*_{q_2}(t_3)\, g_{q_2}(t_4) + O\left(t_1 - t_2\right).$$

Secular growth of loop corrections

- If there is no background field, then $g_p \propto \frac{e^{-i\,\epsilon(p)\,t}}{\sqrt{\epsilon(p)}}$ and

$$n_p(t) \propto \lambda^2\,(t - t_0) \int d^{D-1}\vec{q}_1 \int d^{D-1}\vec{q}_2 \delta\left(\vec{p} + \vec{q}_1 + \vec{q}_2\right) \times$$
$$\times\delta\left(\epsilon(p) \;+\; \epsilon(q_1) \;+\; \epsilon(q_2)\right). \quad (9)$$

Hence, $n_p(t) = 0$ due to energy conservation.

- There is no energy conservation in time–dependent background fields (or energy is not bounded from below), then we generically obtain:

$$\begin{aligned} n_p(t) &\propto \lambda^2\,(t - t_0) \times (\text{production rate}), \\ \kappa_p(t) &\propto -\lambda^2\,(t - t_0) \times (\text{backreaction rate}). \end{aligned} \quad (10)$$

The RHS is the collision integral.

Explicit examples (QED)

- In QED with $\vec{E} = const$ formulas a bit different. Harmonics are $g_p(t) = g(p + eEt)$.
- All expressions are invariant under $p \to p + a$ and $t \to t - a/eE$.

- As the result, beyond the background field approximation, for photons we obtain that:

$$n_p(t) \propto e^2 \, (t - t_0) \times (\text{production rate}), \qquad \kappa_p(t) = 0. \tag{11}$$

Because of that t_0 cannot be taken to past infinity. Hence, we have adiabatic catastrophe.

- For charged fields n_p and κ_p are time–dependent, but do not grow as $t - t_0 \to \infty$.

Explicit examples (de Sitter, expanding patch)

- In expanding Poincare patch: $g_p(t) = \eta^{\frac{D-1}{2}} h(p\eta)$, where $\eta = e^{-t}$ and $h(p\eta)$ is a Bessel function.
- There is invariance under $p \to p\, a$ and $\eta \to \eta/a$.

- For the case of massive scalars, $m > \frac{D-1}{2}$, in the limit $p\eta \to 0$, we obtain that

$$n_p(\eta) \propto \lambda^2 \log\left(\frac{m}{p\eta}\right) \times (\text{production rate}),$$
$$\kappa_p(\eta) \propto -\lambda^2 \log\left(\frac{m}{p\eta}\right) \times (\text{backreaction rate}). \tag{12}$$

- No divergence, but there is secular growth.

Explicit examples (de Sitter, contracting patch)

- Contracting Poincare patch is just time–reversal of the expanding one.
- For the case of massive scalars, $m > \frac{D-1}{2}$, in the limit $p\eta_0 \to 0$ and $p\eta \to +\infty$, we obtain that

$$
\begin{aligned}
n_p(\eta) &\propto \lambda^2 \log\left(\frac{m}{p\eta_0}\right) \times (\text{production rate}), \\
\kappa_p(\eta) &\propto -\lambda^2 \log\left(\frac{m}{p\eta_0}\right) \times (\text{backreaction rate}).
\end{aligned}
\tag{13}
$$

  Here $\eta_0 = e^{t_0}$ is the time after which interactions are adiabatically turned on.

- In this case — IR divergence and, hence, adiabatic catastrophe.
- In global de Sitter there is also adiabatic catastrophe.

Explicit examples (black hole collapse)

- Harmonics are much more complicated, but at future infinity they depend on $\omega\, e^{-t/2r_g}$.
- Invariance under $\omega \to \omega\, a$ and $t \to t + 2r_g \log a$.

- As the result, if the collapse had started at $t = 0$, then we obtain

$$\begin{aligned} n_p(t) &\propto \lambda^2\, t \times (\text{production rate}), \\ \kappa_p(t) &\propto -\lambda^2\, t \times (\text{backreaction rate}). \end{aligned} \tag{14}$$

- Change of the Hawking's thermal spectrum? Information paradox?

Discussion

- What should one do with these growing with time quantum corrections?
- Note that if background field is on for long enough, then $\lambda^2(t - t_0) \sim 1$ and quantum corrections are of the same order as classical contributions; $n_p \sim 1$ — classical effects.

- We need to sum up leading corrections from all loops: sum $\left(\lambda^2(t - t_0)\right)^n$ and drop off e.g. $\lambda^4(t - t_0) \ll \lambda^2(t - t_0)$.
- Does the dependence on t_0 disappear after the summation?
- We did this summation in de Sitter space (expanding and contracting Poincare patches) and in QED with constant field background.

Summation of leading loop corrections

- To do the summation one has to solve the system of the Dyson–Schwinger equations for propagators and vertexes in the IR limit.
- In all the above listed cases vertexes do not receive growing with time corrections. Also retarded and advanced propagators do not secularly growing correction. Hence, to sum up leading corrections we put them to be of tree–level form.
- Ansatz for the Keldysh propagator:

$$D_p^K(t_1, t_2) = \left(\frac{1}{2} + n_p(t)\right) g_p(t_1)\, g_p^*(t_2) + \kappa_p(t)\, g_p(t_1)\, g_p(t_2) + c.c.$$

- As the result we obtain a system of Boltzmann type of equations for n_p and κ_p.
- Solution of these equations, with specified initial conditions, solves the problem of the summation of such corrections.

Summation of leading loop corrections

- Dyson–Schwinger equations are covariant under simultaneous Bogolyubov rotations of harmonics and n_p and κ_p.
- Hence, to sum up leading IR corrections we have to find harmonics for which there is such a solution that $\kappa_p = 0$. Otherwise there is no hope for stationary state!
- Inspiration from the non–stationary theory for superconductors.