Nonperturbative dynamics of scalar fields in de Sitter space

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Motivations

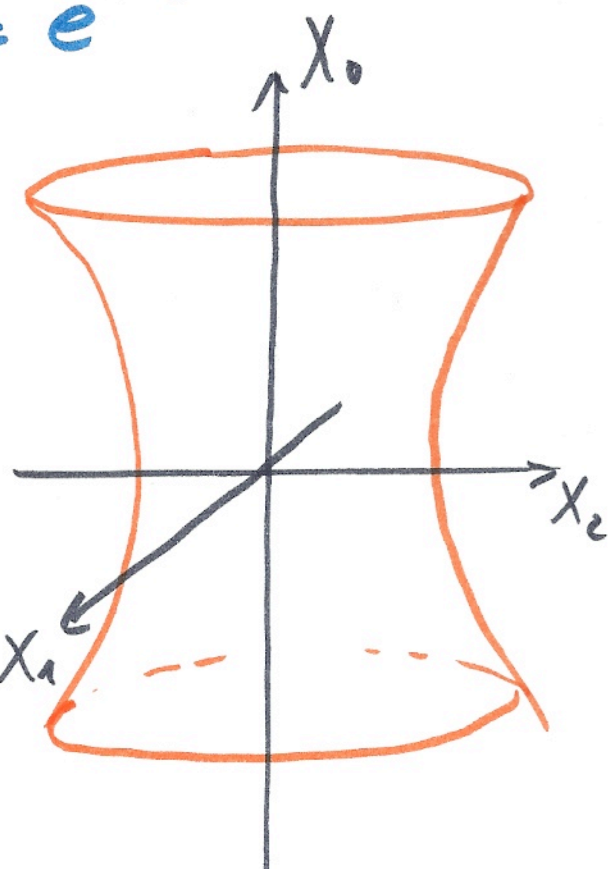
- Radiative corrections to inflationary dynamics
- (Analog) black hole radiation
- Curvature-induced phase transitions
- Foundations of QFT in curved space-times

⋮

Scalar fields in de Sitter space (I)

$$ds^2 = -dt^2 + \bar{a}^2(t)\, d\vec{X}^2$$

$$\bar{a}(t) = e^{Ht}$$



$$\boxed{d\eta = dt/\bar{a}(t)}$$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{X}^2\right)$$

spatially homogeneous
but nonstationary

$$\boxed{\vec{x} = a(t)\,\vec{X}}$$

$$ds^2 = -(1-x^2)\,dt^2 - 2\,\vec{x}\cdot d\vec{x}\,dt + d\vec{x}^2$$

stationary but inhomogeneous

Scalar fields in dS space (II)

$$S = \int d^D x \sqrt{-g(x)} \left(\frac{1}{2} \phi \Box \phi - \frac{m^2}{2} \phi^2 \right)$$

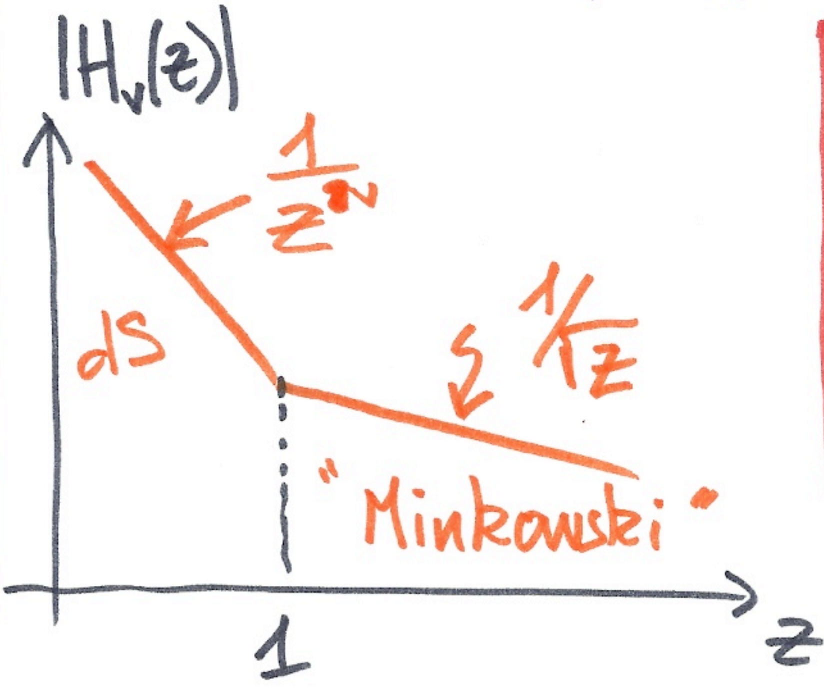
$$D = d + 1 \; ; \; \Box = \frac{1}{a^2(\eta)} \left(-\partial_\eta^2 + \frac{d-1}{\eta} \partial_\eta + \vec{\nabla}_x^2 \right)$$

$$\boxed{(-\Box + m^2)\phi = 0}$$

$$\phi(\eta, \vec{x}) \sim \int \frac{d^d k}{(2\pi)^d} \left(e^{i \vec{k} \cdot \vec{x}} H_\nu\left(\frac{k}{a(\eta)} \right) a_k + h.c. \right)$$

$$\nu = \sqrt{\frac{d^2}{4} - \frac{m^2}{H^2}}$$

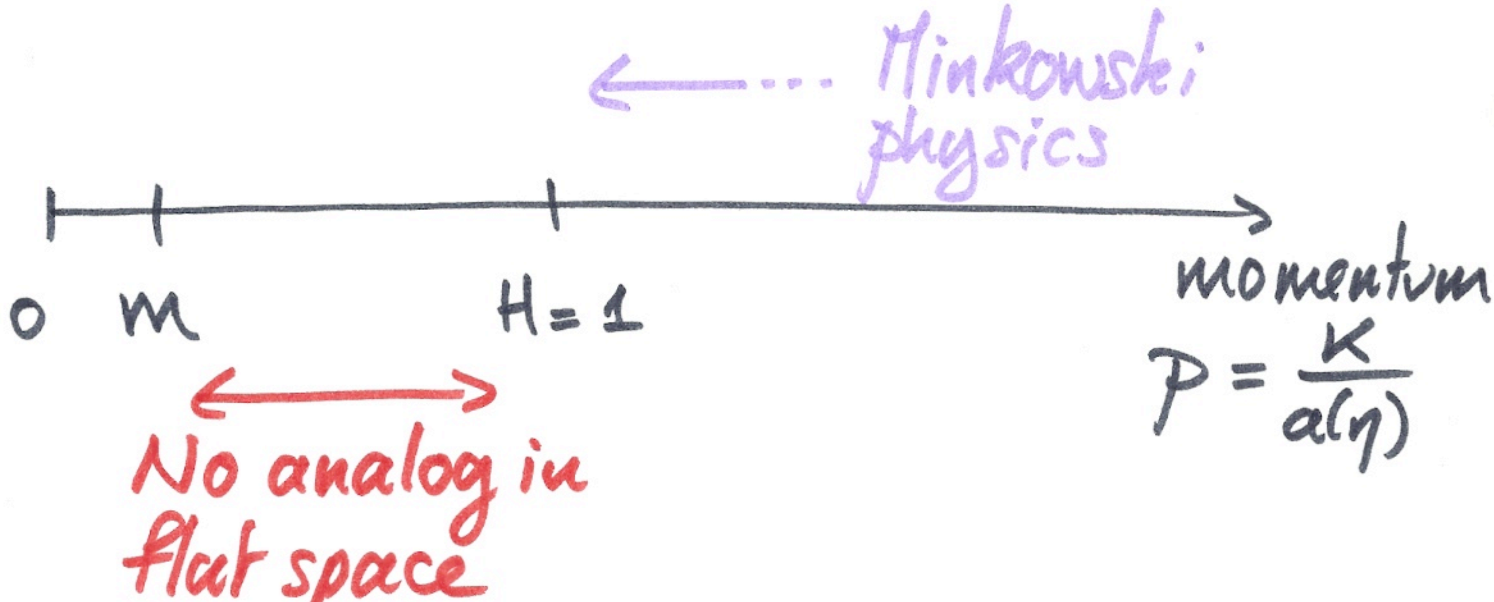
Redshift.



Stationary gravitational redshift leads to strong infrared (IR) fluctuations

Scalar fields in dS (III)

The case of light fields $m \ll H = 1$



Loop corrections :

 $\sim \frac{H^4}{m^2}$: IR divergencies

 $\sim H^2 \ln(P/H)$: large logs (secular divergencies)

$\Rightarrow$ $\boxed{\text{NEED FOR RESUMMATION}}$

Resummation / nonperturbative methods in de Sitter.

difficulty : nonequilibrium system

- Stochastic approach
  [Starobinsky, Yokoyama ('94)]
- Euclidean de Sitter
  [Rajaraman ('10); Beneke, Moch ('13)]
- Dynamical R.G.
  [Burgess et al. ('10)]
- Wigner-Weisskopf method
  [Boyanovsky ('12)]

- Large-N
  [Riotto, Sloth ('08); Mazzitelli, Paz ('89); J.S. ('11)]
- Dyson-Schwinger equations
  [Garbrecht, Rigopoulos ('11); Akhmedov, Burda ('12); Parentani, J.S. ('12) ; Gautier, J.S. ('13)]
- Nonperturbative / Functional R.G.
  [Kaya ('13) ; Serreau ('14); Guilleux; JS ('15)]

Large N

[Mazzitelli, Paz (89), Riotto, Sloth (08), Serreau (11)]

Local IR terms :

$\sim N$

Effective mass resumation

$$\sim \lambda \int \frac{d^d p}{(2\pi)^d} \left|H_\nu(p)\right|^2 \sim \frac{\lambda H^4}{M^2}$$

GAP EQUATION

$$M^2(\phi) = \underbrace{m^2 + \lambda\phi^2}_{m_{cl}^2(\phi)} + c\,\frac{\lambda H^4}{M^2(\phi)}$$

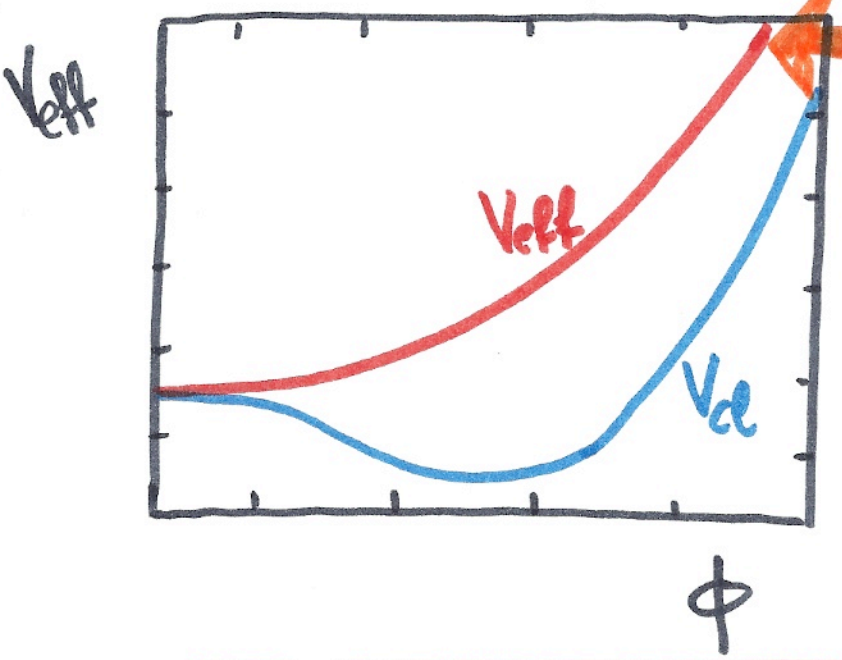
$$\boxed{M^2(\phi) = \frac{m_{cl}^2(\phi)}{2} + \sqrt{\frac{m_{cl}^4(\phi)}{4} + c\lambda H^4}}$$

Large N : Effective potential

[Serreau, PRL (11)]

$$V_{eff}(\phi) = \int_0^{\phi^2} dx \, \frac{M^2(x)}{2}$$

$$= \frac{3}{2\lambda}\left(M^4(\phi) - M^4(0)\right) + \frac{3H^4}{16\pi^2} \ln \frac{M^2(\phi)}{M^2(0)}$$



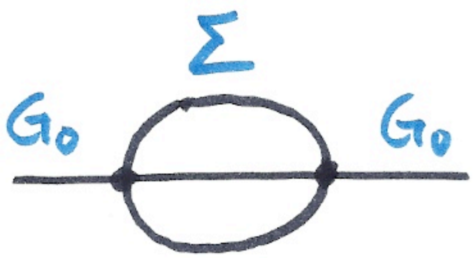
$$\boxed{M^2(\phi) > 0}$$

$\Rightarrow$ **Radiative symmetry restoration $\forall d$, $\forall H$**

[See also: Ratra ('85); Mazzitelli et al. (89), Lazzari et al. (13)]

Dyson-Schwinger Eqs. (DSE)

Nonlocal IR logarithms



$\sim \quad \lambda^2 H^2 \ln(P/H)$

→ need to resume the series

DSE : $\boxed{G^{-1} = G_0^{-1} - \Sigma}$

$$(\Box + m^2) G(x,y) - \int_z \Sigma(x,z) G(z,y) = -i\delta(x-y)$$

Integro-diff. eqn. with nontrivial metric

[Akhmedov et al. (12), (14); Gautier, J.S (13)
Garbrecht, Rigopoulos ('11)]

DSE : p-representation

$$G(x,x') = \int \frac{d^d K}{(2\pi)^d} e^{i\vec{K}.(\vec{x}-\vec{x}')} \tilde{G}(t,t',K)$$

$$G(t,t',K) = \frac{\boxed{\hat{G}(p,p')}}{K\,[a(t)a(t')]^{\frac{D-2}{2}}}$$

$$p = \frac{K}{a(t)} \quad , \quad p' = \frac{K}{a(t')}$$

Similarly : $\tilde{\Sigma}(t,t';K) = K^3 \dfrac{\hat{\Sigma}(p,p')}{[a(t)a(t')]^{\frac{D+2}{2}}}$

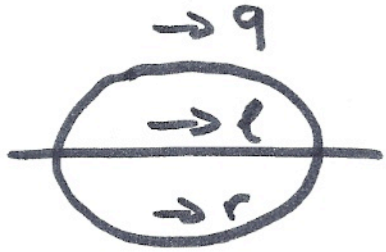
$$\left(\partial_p^2 + 1 - \frac{\nu^2 - 1/4}{p^2}\right)\hat{G}(p,p') + \int dp''\,\hat{\Sigma}(p,p'')\,\hat{G}(p'',p') = i\,\delta(p-p')$$

$$\nu = \sqrt{\frac{d^2}{4} - \frac{M^2}{H^2}}$$

0+1 dimensional system

DSE : two-loop

[Gautier, J.S., PLB ('13)]



$$\underline{\hat{\Sigma}(p,p')} \propto \lambda^2 \int_{\vec{q},\vec{\ell}} \frac{\hat{G}_0(qp,qp')\,\hat{G}_0(\ell p,\ell p')\,\hat{G}_0(rp,rp')}{q\,\ell\,r}$$

Exact analytical solution of DSE for $p,p' \ll 1$

$$\hat{G}(p,p') = c_+\, G^{free}_{M_+}(p,p') + c_-\, G^{free}_{M_-}(p,p')$$

$$c_+ + c_- = 1$$

IR logs. resum to modified power laws + "splitting"

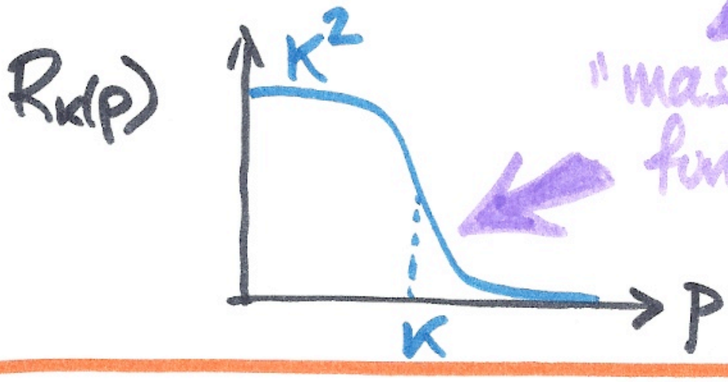
Nonperturbative Renormalization Group (NPRG)

[Kaya ('13); Serreau ('14)]

- **Infrared regulator**:

$$S[\varphi] \longrightarrow S[\varphi] + \frac{1}{2}\int_{x,y} \varphi(x) R_K(x,y) \varphi(y)$$

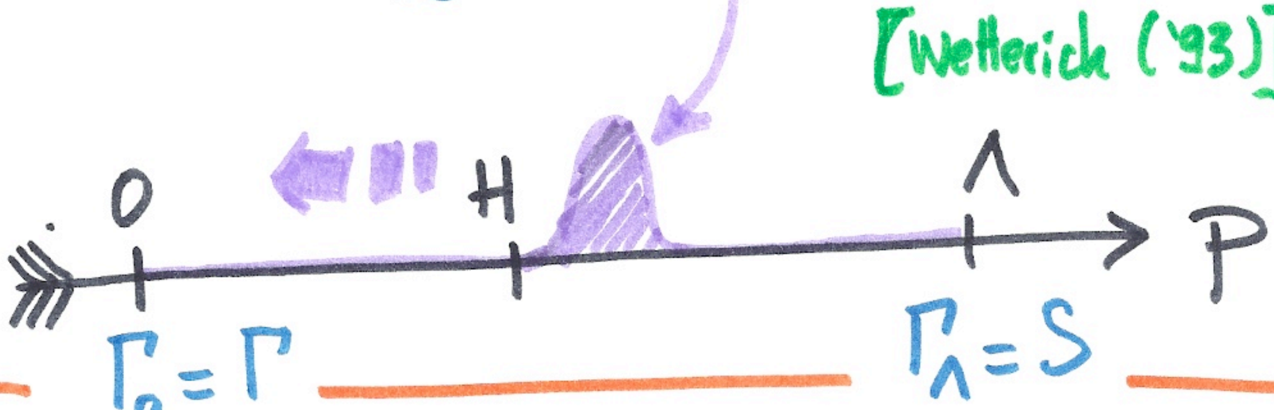
"mass function"



- Regulated effective action $\boxed{\Gamma_K[\phi]}$

$$\partial_K \Gamma_K[\phi] = \frac{1}{2} \mathrm{Tr}\left\{ \boxed{\partial_K R_K} \cdot \left(\Gamma_K^{(2)} + R_K\right)^{-1} \right\}$$

[Wetterich ('93)]



NPRG : Local Potential Approx. (LPA)

$$\Gamma_\kappa[\phi] = -\int_x \left\{ \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi + \boxed{V_\kappa(\phi)} \right\}$$

Full field dependence

+ choose $\quad R_\kappa(t,t',p) = \delta(t-t')(\kappa^2-p^2)\theta(\kappa^2-p^2)$

$$\boxed{\kappa\partial_\kappa V_\kappa = \frac{C_d\,\kappa^{d+2}}{\kappa^2+V_\kappa''}\,\underline{B_d}(\nu_\kappa,\kappa)}$$

$$C_d = \frac{\pi}{16d}\frac{\Omega_d}{(2\pi)^d}\ , \quad \nu_\kappa = \sqrt{\frac{d^2}{4}-V_\kappa''}$$

$$\underline{B_d}(\nu,\kappa) = e^{-\pi\,\mathrm{Im}(\nu)}\Big\{(d^2-2\nu^2+2\kappa^2)\,|H_\nu(\kappa)|^2 + 2\kappa^2|H_\nu'(\kappa)|^2 - 2d\kappa\,\mathrm{Re}\left[H_\nu^*(\kappa)H_\nu'(\kappa)\right]\Big\}$$

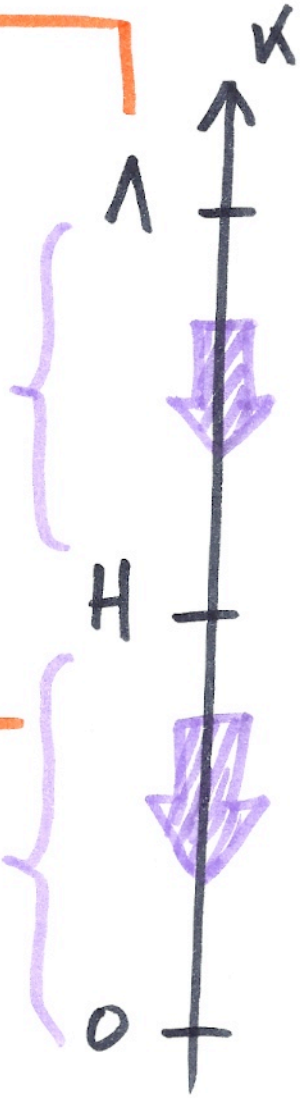
[Guilleux, Serreau ('15)]

From UV to IR : onset of gravitational effects

$\Rightarrow$ $\kappa \gtrsim H$

$$\kappa \partial_\kappa V_\kappa \approx \frac{8C_d}{\pi} \frac{\kappa^{d+2}}{\sqrt{\kappa^2 + V_\kappa''}}$$

Minkowski Regime



$\Rightarrow$ $\kappa \lesssim H$ and $|V_\kappa''| \ll H^2$

$$\kappa \partial_\kappa V_\kappa \approx \frac{1}{\Omega_{D+1}} \frac{\kappa^2}{\kappa^2 + V_\kappa''}$$

[Compare to $\frac{\kappa^{D+2}}{\kappa^2 + V_\kappa''}$ in flat Euclidean space]

$\Rightarrow$ **Effective dimensional reduction**

$$D_{\text{eff}} = 0$$

Zero-dimensional field theory

$$\boxed{e^{-\Omega_{D+1} W_\kappa(J)} = \int d\varphi \, e^{-\Omega_{D+1}\left(V_{\text{eff}}(\varphi) + J\varphi + \frac{\kappa^2}{2}\varphi^2\right)}}$$

$$V_\kappa(\phi) = W_\kappa(J) - J\phi - \frac{\kappa^2}{2}\phi^2$$

with $W'_\kappa(J) \equiv \phi$

$$\Rightarrow \boxed{\kappa \partial_\kappa V_\kappa(\phi) = \frac{1}{\Omega_{D+1}} \frac{\kappa^2}{\kappa^2 + V''_\kappa(\phi)}}$$

Adjust V_{eff} by matching at the scale $\kappa \approx H$

$$\boxed{V_{\text{eff}}(\varphi) \approx V_H(\varphi)}$$

Relation to the stochastic approach

[Starobinsky, Yokoyama ('94)]

Effective Langevin eqn. for IR modes

$$\dot{\varphi} + \frac{1}{d} V_{soft}'(\varphi) = \xi \quad ; \quad \langle \xi(t)\xi(t') \rangle = \frac{2\,\delta(t-t')}{d\,\Omega_{D+1}}$$

Focker-Planck equation

$$\partial_t P(\varphi,t) = \frac{1}{d}\frac{\partial}{\partial\varphi}\left\{ V_{soft}' P + \frac{1}{\Omega_{D+1}} P' \right\}$$

$$\boxed{P(\varphi, t\to\infty) \propto \exp\left\{-\Omega_{D+1} V_{soft}\right\}}$$

$$\langle \varphi^2 \rangle = \int d\varphi\, P(\varphi)\,\varphi^2 = \frac{1}{\Omega_{D+1}} \underline{\underline{W_{k=0}''(J=0)}}$$

provided $V_{soft} = V_{eff} \approx V_H$

Relation to Euclidean de Sitter

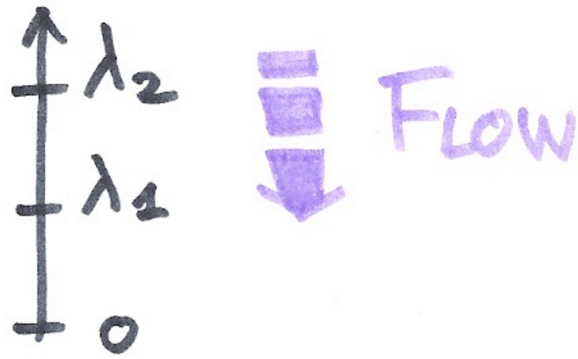
[Rajaraman ('10); Beneke, Moch ('13); Benedetti ('15)]

NPRG on S_D : $S \longrightarrow S + \frac{1}{2}\int_{xy} \varphi R_\kappa \varphi$

Compact space $\Rightarrow$ discrete spectrum

$\Box Y_{\vec{L}} = -\lambda_L Y_{\vec{L}}$

$\lambda_L = L(L+D-1)H^2$



For $\boxed{\kappa^2 < \lambda_1}$, heavy modes decouple

$\Rightarrow$ zero mode only $\boxed{\bar{\varphi}}$

$$e^{-\Omega_{D+1}\bar{W}_\kappa(J)} = \int d\bar{\varphi}\, e^{-\Omega_{D+1}\left(\bar{V}_{eff}(\bar{\varphi}) + J\bar{\varphi} + \frac{\kappa^2}{2}\bar{\varphi}^2\right)}$$

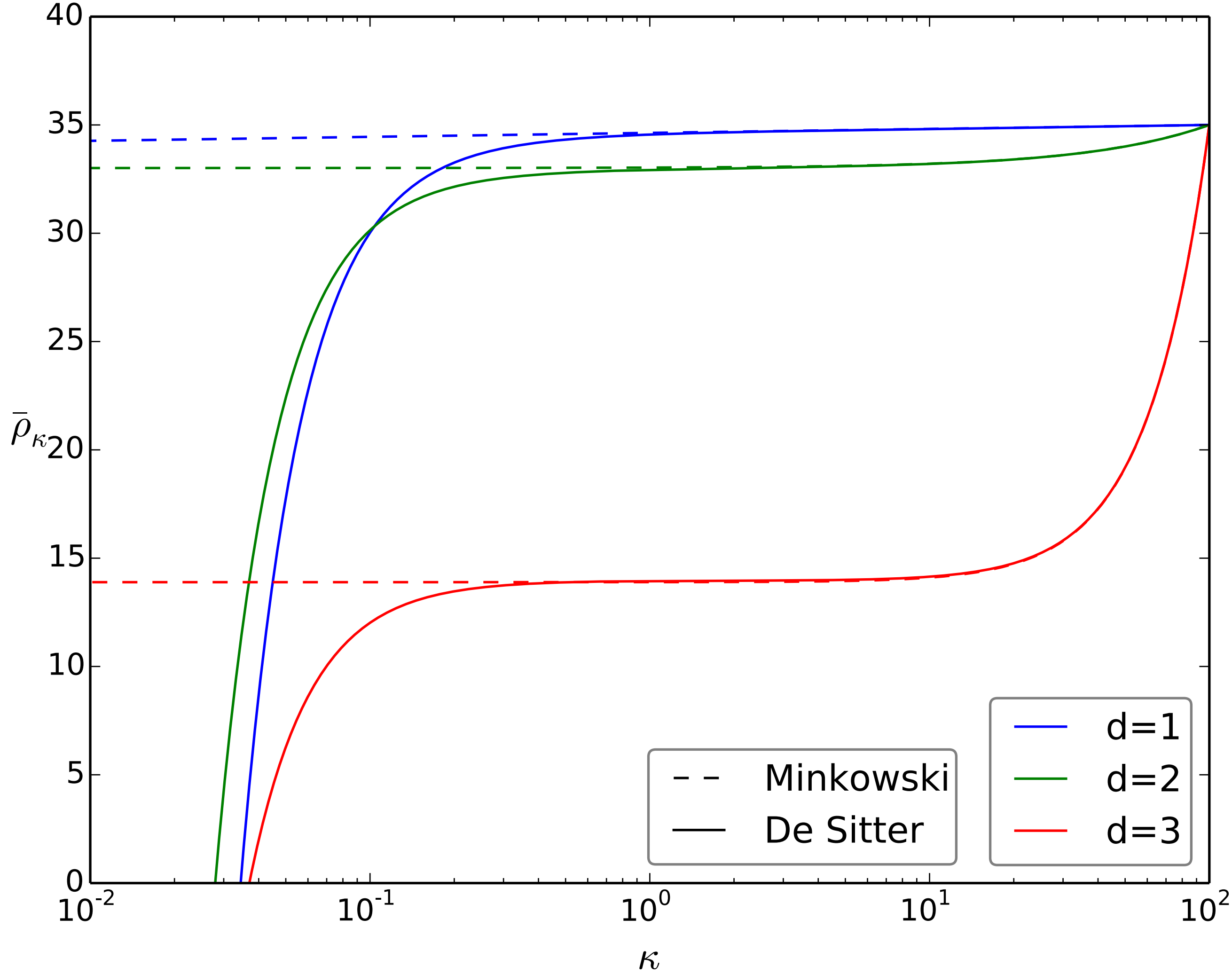
with $$e^{-\Omega_{D+1}\bar{V}_{eff}(\bar{\varphi})} = \int \mathcal{D}\hat{\varphi}\, e^{-\bar{S}[\bar{\varphi},\hat{\varphi}]}$$

From UV to IR: onset of gravitational effects

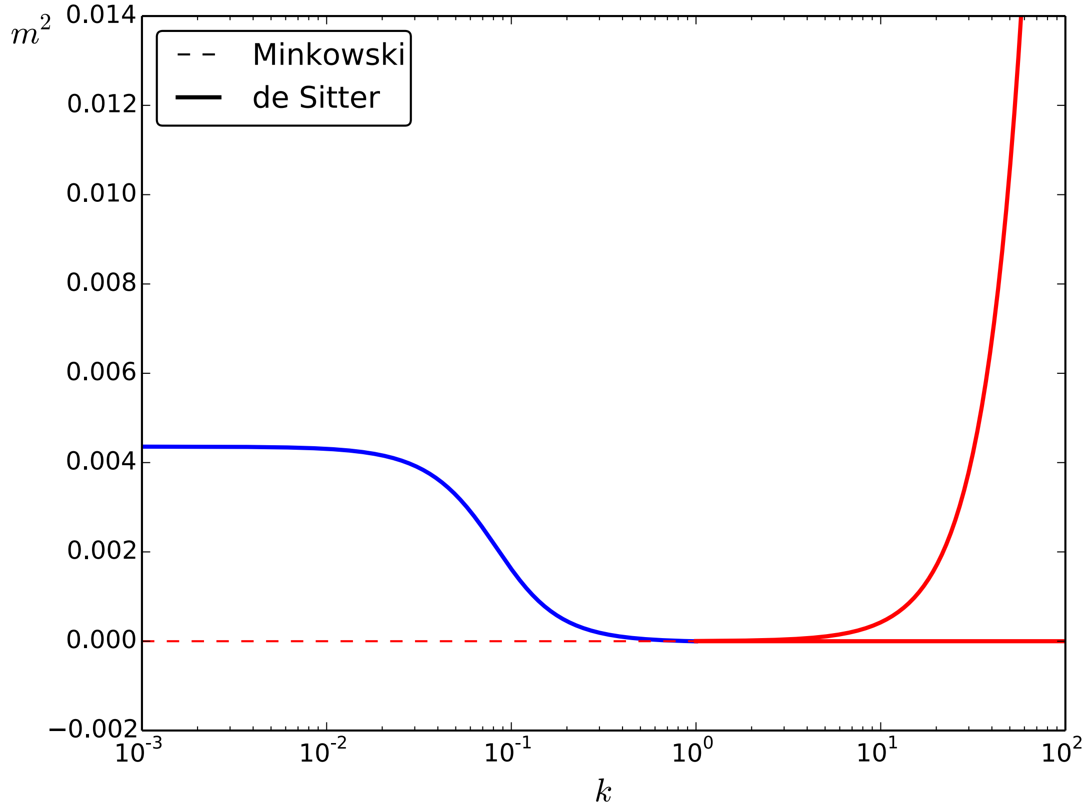
- **Massive theory in UV** ($\kappa \gtrsim H$)
(i.e. Minkowski symmetric phase)
$\Rightarrow$ $V_H'' \gtrsim H^2$: Minkowski flow

- **Massless modes / flat potential**
(e.g. Minkowski critical theory or broken phase)
$\hookrightarrow$ Regime of dimensional reduction

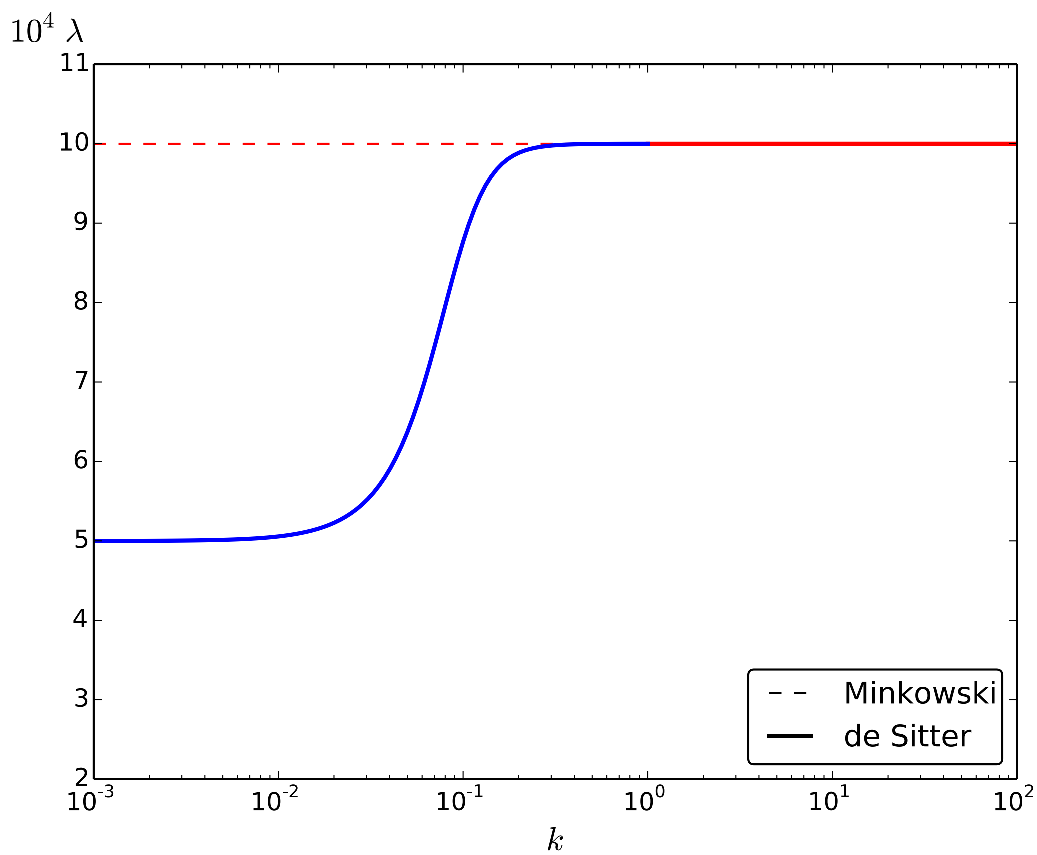
$\Rightarrow$ Symmetry restoration
Mass (re)generation

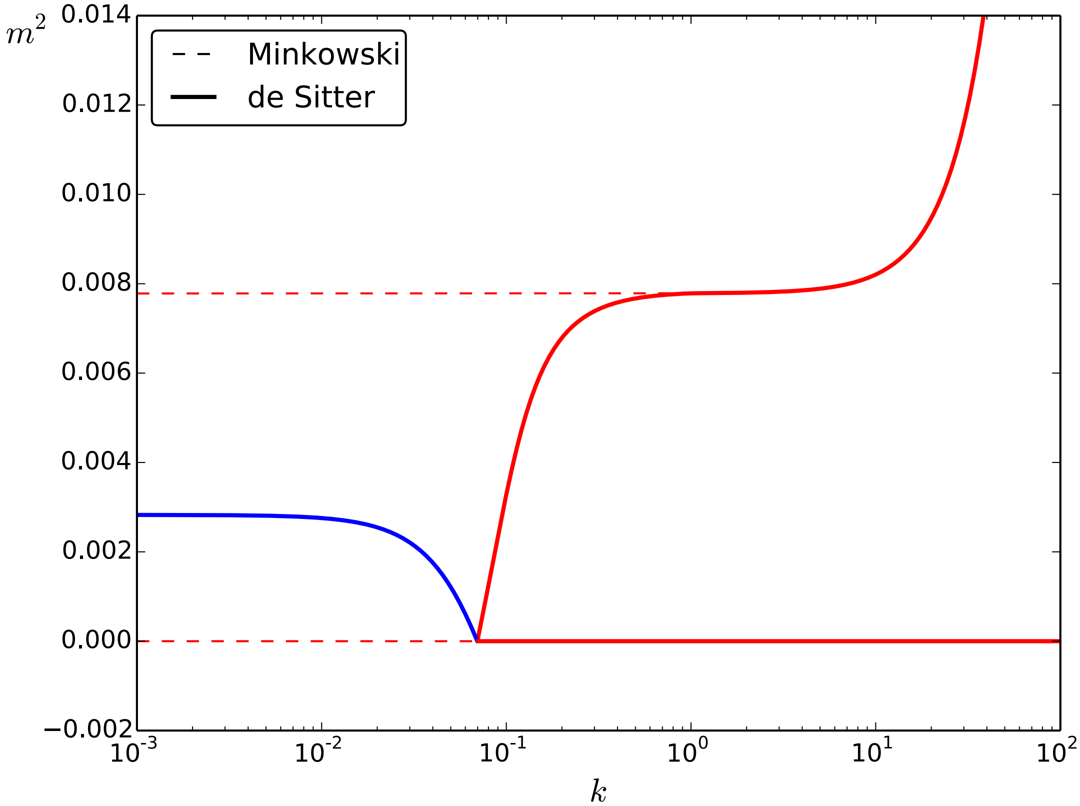


40
35
30
25
$\bar{\rho}_\kappa$
20
15
10
5
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
κ
Minkowski
De Sitter
d=1
d=2
d=3

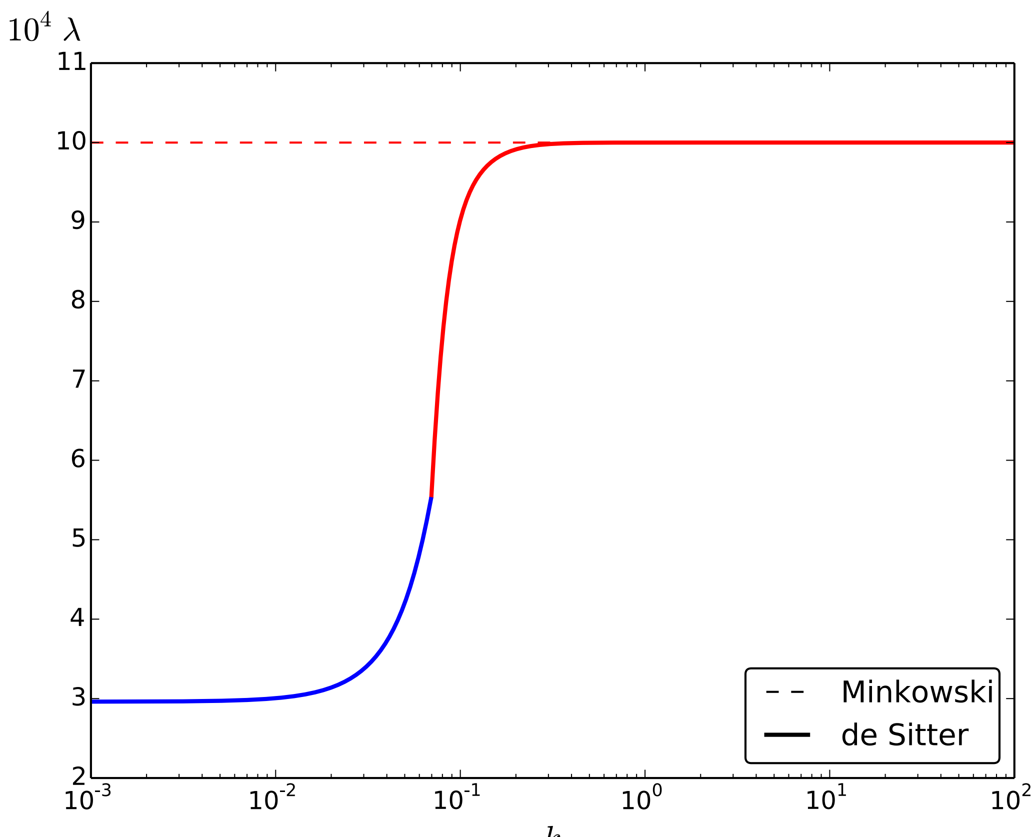


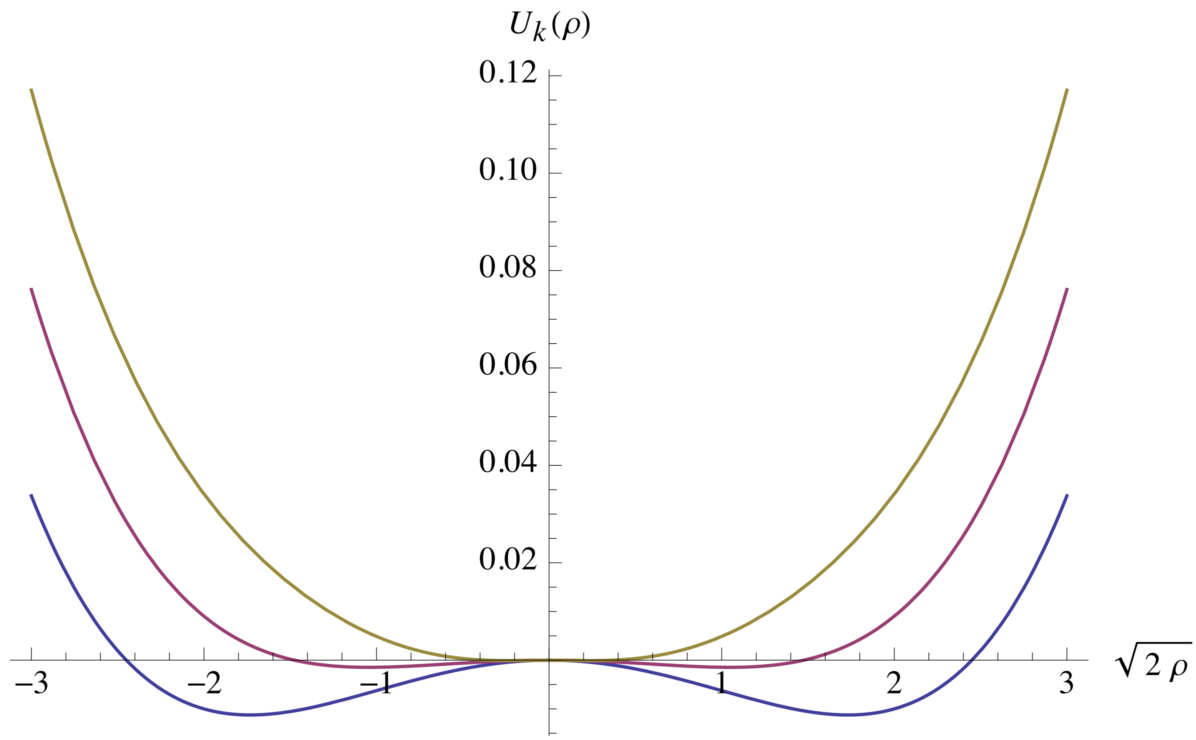
$$N \to \infty, \text{ ``critical'' case}$$





$N \to \infty$, "broken symmetry" case





$N = 1$

CONCLUSIONS

- IR physics in dS space is nontrivial and interesting

- The p-representation allows one to formulate powerful resummation or nonperturbative tools in dS.

Analytical nonperturbative results e.g. large-N ; DSE

- NPRG can be efficiently formulated on dS. space

A promising tool !

PERSPECTIVES

- **Dyson-Schwinger Equations at next-to-leading order in $1/N$**

  $\hat{\Sigma} \sim$ 

  [Gautier J.S (SOON)]

  Exact analytical result in the IR.

- **NPRG beyond the Local Potential Approximation**

  ↳ anomalous dimension

  [Guilleux, J.S. in progress]

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