Dark Energy & Condensate Stars

Surface Tension, Negative Pressure & Effective Theory of Low Energy Gravity

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w. P. O. Mazur arXiv:1501.03806
Class. Quant. Grav. (2015)

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Outline

- Classical 'Black Holes' & Quantum Mechanics
  - Entropy & the Second Law of Thermodynamics
  - Temperature & the 'Trans-Planckian Problem'
  - Negative Heat Capacity & the 'Information Paradox'
- Effective Theory of Low Energy Gravity
  - Massless Scalar Poles in Flat Space Amplitudes
  - Conformal Scalar Degree of Freedom in EFT of Gravity
  - Macroscopic Effects at Event Horizons
- Gravitational Vacuum Condensate Stars
  - Already Inherent in Interior Schwarzschild Solution
  - Negative Pressure and Surface Tension
  - Quantum Final State of Gravitational Collapse
- Cosmological 'Constant' as Macroscopic Condensate

Classical Black Holes

Schwarzschild Metric (1916)

$$ds^2 = -dt^2 f(r) + \frac{dr^2}{h(r)} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right)$$

$$f(r) = 1 - \frac{2GM}{r} = h(r)$$

Classical Singularities:

- $r = 0$: Infinite Tidal Forces, Breakdown of Gen. Rel.
- $r \equiv R_S = 2GM \quad (c = 1)$: Event Horizon, Infinite Blueshift, Change of sign of f, h

Trapping of light inside the horizon is what makes a black hole
BLACK
The $r = R_S$ singularity is purely kinematic, removable by a coordinate transformation
$\boxed{\text{iff } \hbar = 0}$

And iff $T_{\mu\nu} = 0$ on the horizon

Black Holes and Entropy

- A fixed classical solution usually has no entropy :
  (What is the "entropy" of the Coulomb potential $\Phi = Q/r$?)
  … But if matter/radiation disappears into the black hole,
  what happens to its entropy? (Only M, J, Q remain)
- Horizon area A (which always increases) a kind of "entropy"?
  To get units of entropy need to divide A by $(\text{length})^2$
  … But there is no fixed length scale in classical Gen. Rel.
- Planck length $\ell_{Pl}^2 = \hbar G / c^3$ involves $\hbar$
- Bekenstein suggested $S_{BH} = \gamma\, k_B\, A/L_{Pl}^2$ with $\gamma \sim O(1)$
- Hawking (1974) argued black holes emit thermal radiation at

$$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$

Apparently then the classical Smarr relation $dE = \kappa\, dA/8\pi G$
becomes first law, $dE = T_H\, dS_{BH}$ fixes $\gamma = 1/4$
(multiply & divide by $\hbar$) *But …*

A few problems remained …

- Hawking Temperature requires trans-Planckian frequencies
- $S_{BH} \propto A$ is *non-extensive* and *HUGE*
- In the classical limit $T_H \to 0$ (cold) but $S_{BH} \to \infty$ (?)
- $E \propto T^{-1}$ implies negative heat capacity

$$\frac{dE}{dT} << 0 \quad \Rightarrow \text{highly unstable}$$

Equilibrium Thermodynamics cannot be applied

- Information Paradox: Where does the information go?

  (Pure states → Mixed States? Unitarity ?)
- What is the statistical interpretation of S_{BH} ?

  Boltzmann asks: $\mathbf{S = k_B \ln W}$??

Statistical Entropy of a Relativistic Star

- $S = k_B\, ln\, W(E)$ (microcanonical) is equivalent to

$$S = -\, k_B\, Tr\,(\rho\, ln\, \rho)$$

- Maximized by canonical thermal distribution

  Eg. Blackbody Radiation $E \sim V\,T^4$, $S \sim V\,T^3$

  $S \sim V^{1/4}\,E^{3/4} \sim R^{3/4}\,E^{3/4}$

  For a fully collapsed relativistic star $E = M$, $R \sim 2GM$,

  so $\boxed{S \sim k_B\,(M/M_{Pl})^{3/2}}$ $\leftarrow$ *note 3/2 power*

$S_{BH} \sim M^2$ is a factor $(M/M_{Pl})^{1/2}$ larger or **10^{19}** for $M = M_{\odot}$

- There is *no way* to get $S_{BH} \sim M^2$ by any standard statistical thermodynamic counting of states

Black 'Holes'… or Not

Black Holes believed 'inevitable' in General Relativity but

- Difficulties reconciling Black Holes with Quantum Mechanics
  - Hawking Temperature & the 'Trans-Planckian Problem'
  - Entropy & the Second Law of Thermodynamics
  - Negative Heat Capacity & the 'Information Paradox'
- Singularity Theorems assume Trapped Surface and Energy Conditions: Strong Energy Condition

$$\rho + \sum_{i=1}^{3} p_i \geq 0$$

Violated by Quantum Fields, e.g. by Casimir Effect & Hadron 'Bag', Cosmological Dark Energy, Inflation V(ϕ):

Negative Pressure $\boxed{p_i = -\rho < 0}$ Effective **Repulsion**

Static, Spherical Symmetry

- 2 Metric Fns.

$$f(r),\ h(r) \equiv 1 - \frac{2Gm(r)}{r}$$ ← Misner-Sharp Mass

- 3 Stress Tensor Fns.

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p_\perp & 0 \\ 0 & 0 & 0 & p_\perp \end{pmatrix}$$

- 2 Einstein Eqs.

$$\frac{dm}{dr} = 4\pi\, r^2 \rho \qquad \frac{h}{2f}\frac{df}{dr} = \frac{Gm}{r^2} + 4\pi G p r$$

- 1 Conservation Eq.

$$\nabla_\mu T^{\mu}_{\ r} = \frac{dp}{dr} + \frac{\rho + p}{2f}\,\frac{df}{dr} + \frac{2\,(p - p_\perp)}{r} = 0$$

Buchdahl Bound (1959)

Assuming classical Einstein eqs. &

- Static Killing time:

$$K^{\mu}\frac{\partial}{\partial x^{\mu}} = \frac{\partial}{\partial t}$$

- Spherical Symmetry:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{h(r)} + r^2\,d\Omega^2$$

- Isotropic Pressure: $p_i = p(r)$
- Positive Monotonically Decreasing Density: $\frac{d\rho}{dr} \leq 0$
- Metric Continuity at Surface of Star r=R
- Then $R > \frac{9}{8} R_s = \frac{9}{4} GM$

or the pressure must diverge in the Interior before horizon is reached

Schwarzschild Interior

- Constant Density $m(r) = \frac{4\pi}{3}\bar{\rho}r^3 = \frac{M}{R^3}r^3$

$$\rho' = 0$$

$$h(r) = 1 - H^2 r^2$$

Saturates

Buchdahl Bound

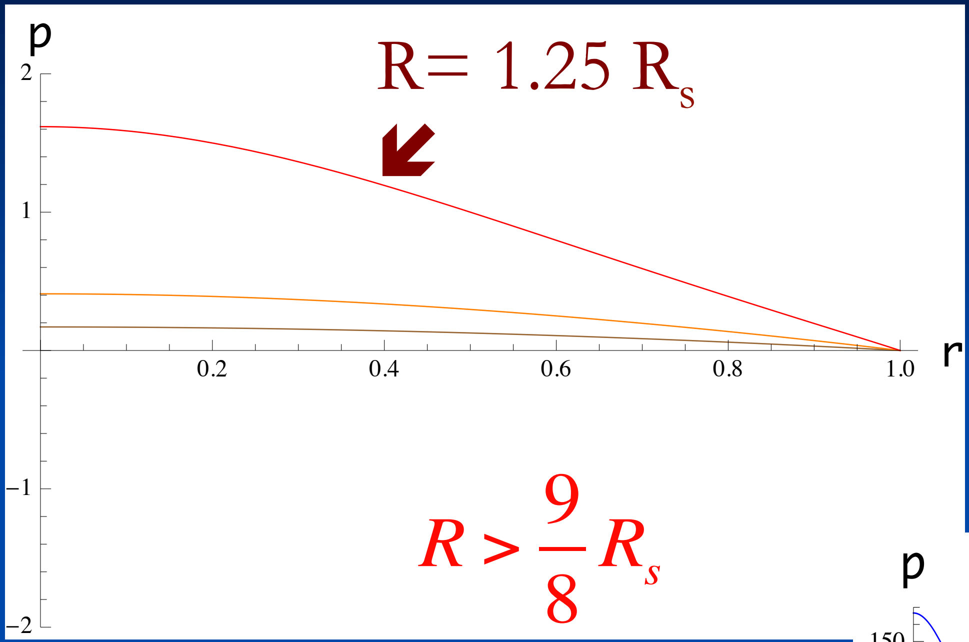
$$H^2 = \frac{8\pi G}{3}\bar{\rho} = \frac{2GM}{R^3}$$

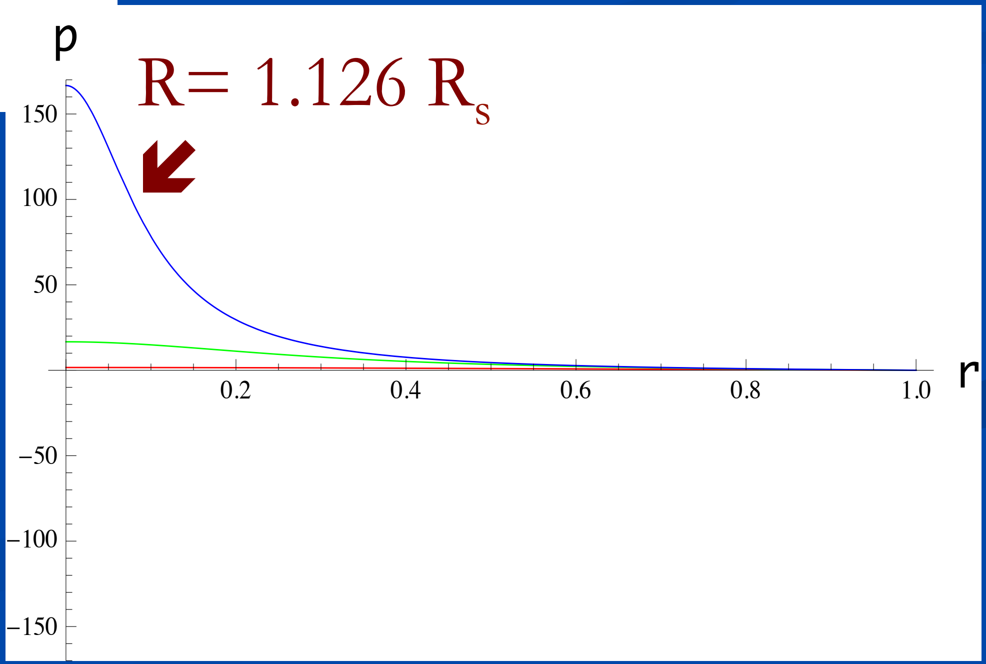
- Pressure $p(r) = \bar{\rho}\left[\frac{\sqrt{1-H^2r^2} - \sqrt{1-H^2R^2}}{3\sqrt{1-H^2R^2} - \sqrt{1-H^2r^2}}\right]$

- Diverges at $\boxed{R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}}$ iff $\boxed{R < \frac{9}{8}R_s = \frac{9}{4}GM}$

- Pressure becomes negative for $0 < r < R_0$

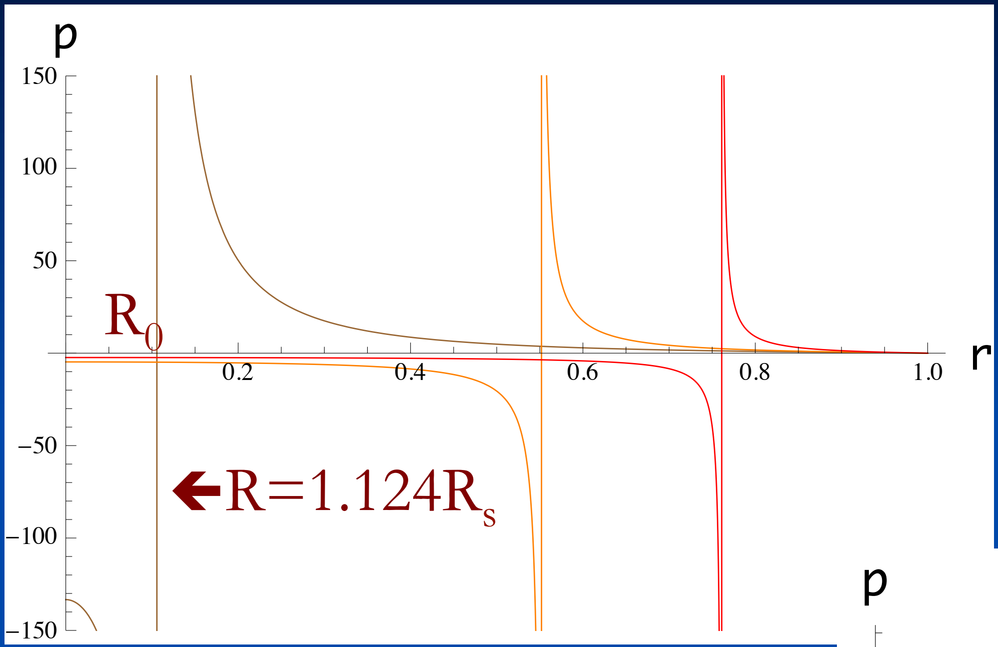
Interior Pressure





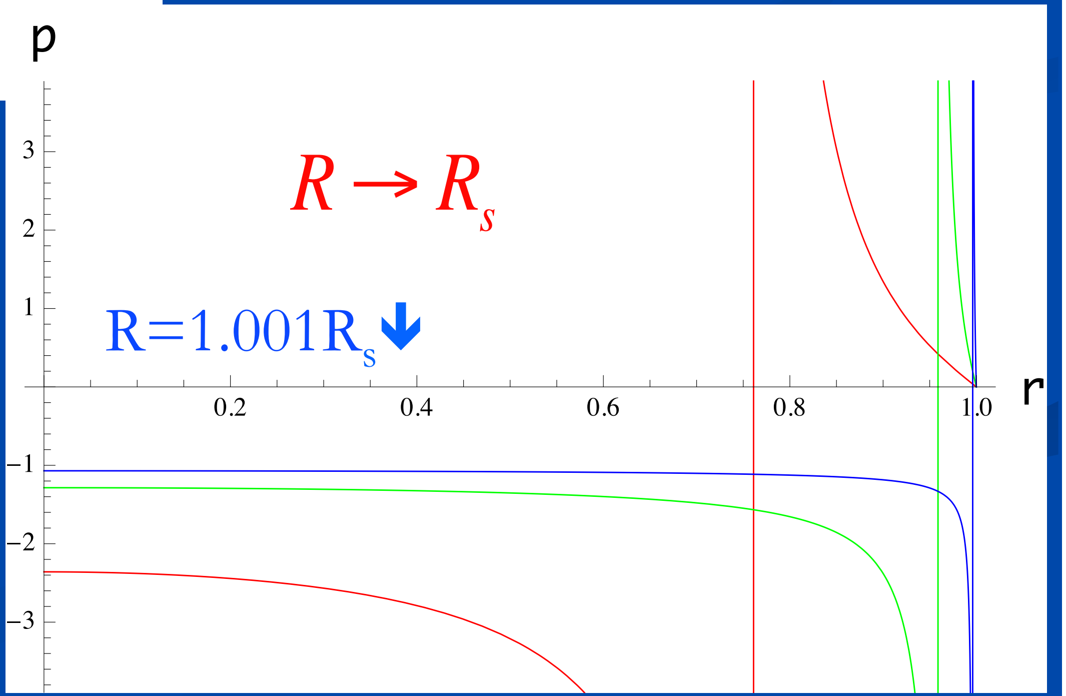
As $R \rightarrow 9/8\ R_s$ from above
central pressure diverges

Interior Pressure



$$R < \frac{9}{8} R_s$$

Negative Pressure soln. opens up for $R < R_0$



As $R \to R_s$ from above

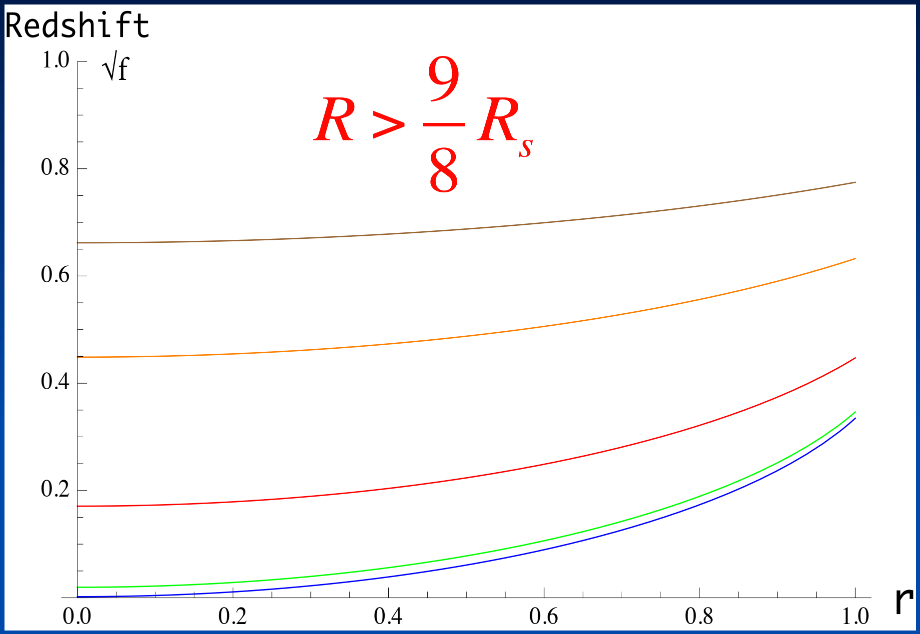
$R_0 \to R_s$ from below and

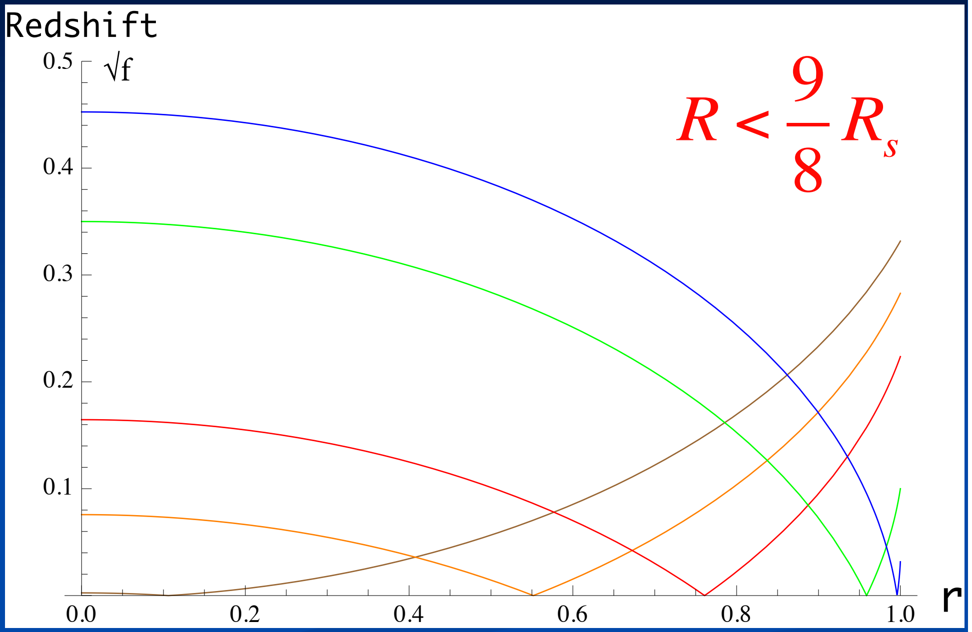
negative pressure region

fills entire interior with

$p = -\rho$, $w = -1$

Interior Redshift





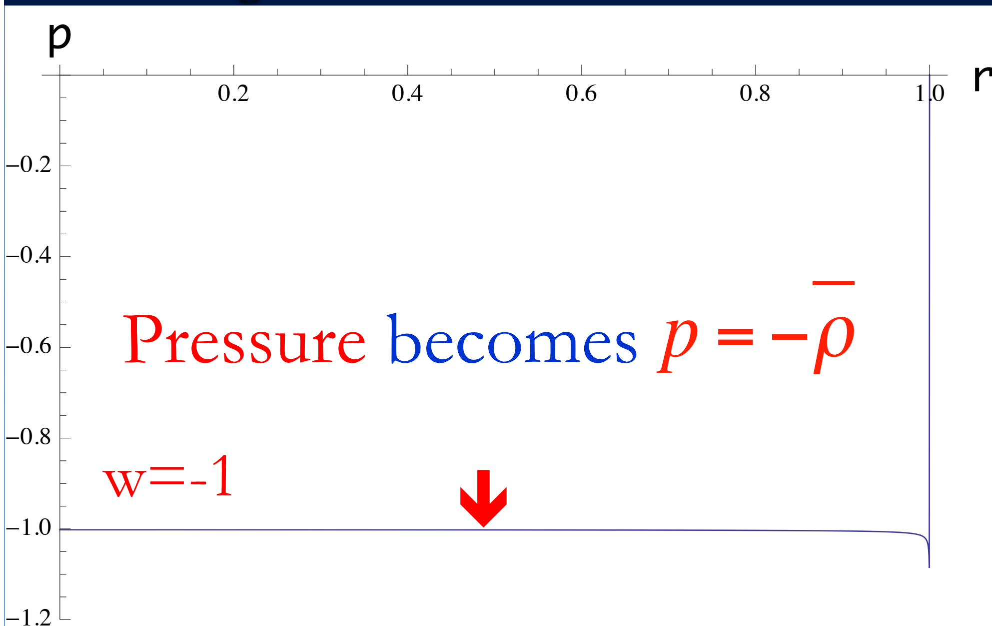
$f(r) = \frac{1}{4} D^2 \geq 0$ Non-negative (no horizon)

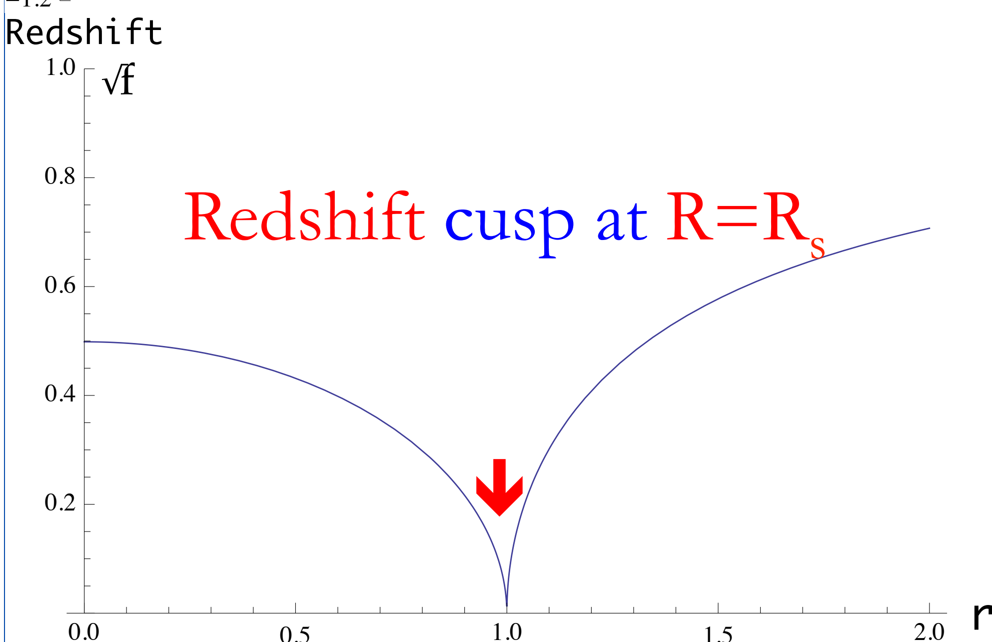
$$D \equiv 3\sqrt{1 - R_s/R} - \sqrt{1 - R_s r^2/R^3}$$

vanishes at same radius $R_0 = 3R\sqrt{1 - \frac{8}{9}\frac{R}{R_s}}$ where p diverges

Redshift $\sqrt{f(r)} = \frac{1}{2}\,|D|$ has cusp-like behavior

R=R$_s$ Limit is Grav. Condensate Star (2001-4)





No divergence in p

$$p = -\bar{\rho}$$

$$h(r) = 1 - H^2 r^2$$

$$f(r) = \frac{1}{4}\, h(r)$$

$$H = 1/R_s$$

but non-analytic cusp

Discontinuity (classically)

Interior is de Sitter space in (modified) static coordinates

(Time runs slower inside)

Komar Mass-Energy Flux (1959-62)

$$\frac{1}{G}\frac{d}{dr}\left(r^2\kappa\right) = 4\pi\sqrt{\frac{f}{h}}\, r^2\left(\rho + p + 2p_\perp\right)$$

$$\kappa(r) = \frac{1}{2}\sqrt{\frac{h}{f}}\,\frac{df}{dr} \to \frac{GM}{r^2}$$ Surface Gravity

Total Mass: Compare Gauss' Law

$$\boxed{M = 4\pi\int_0^{R_s} dr\,\sqrt{\frac{f}{h}}\, r^2\left(\rho + p + 2p_\perp\right)}$$

Transverse Pressure

Cusp in Redshift produces Transverse Pressure

$$r\frac{d}{dr}\left[(p+\bar{\rho})f^{\frac{1}{2}}\right] = 2\,(p_\perp - p)f^{\frac{1}{2}}$$

Localized at $r = R_0$

$$8\pi\sqrt{\frac{f}{h}}\,r^2\,(p_\perp - p) = \frac{8\pi}{3}\,\bar{\rho}\,{R_0}^3\,\delta(r - R_0)$$

Integrable Surface Energy

$$E_s = \frac{8\pi}{3}\,\bar{\rho}\,{R_0}^3 = 2M\left(\frac{R_0}{R}\right)^3 \to 2M$$

$$\boxed{M = E_v + E_s}$$

Surface Tension

Discontinuity in Surface Gravities

$$\kappa_\pm \equiv \lim_{r \to R_0^\pm} \kappa(r) = \pm \frac{4\pi G}{3} \bar{\rho}\, R_0$$

$$\Delta\kappa \equiv \kappa_+ - \kappa_- = \frac{R_s R_0}{R^3} \to \frac{1}{R_s}$$

is (redshifted) surface tension

$$\boxed{\tau_s = \frac{E_s}{2A} = \frac{\Delta\kappa}{8\pi G}} \to \frac{1}{8\pi G R_s} > 0$$

Interior is not analytic continuation of exterior

First Law

Classical Mechanical Conservation of Energy

$$\boxed{dM = dE_v + \tau_s\, dA}$$

Gibbs Relation

$$p + \rho = s\,T + \mu\,N = 0$$

Schw. Interior Soln. in $R \rightarrow R_s$ Limit describes a **<u>Zero Entropy/Zero Temperature Condensate</u>**

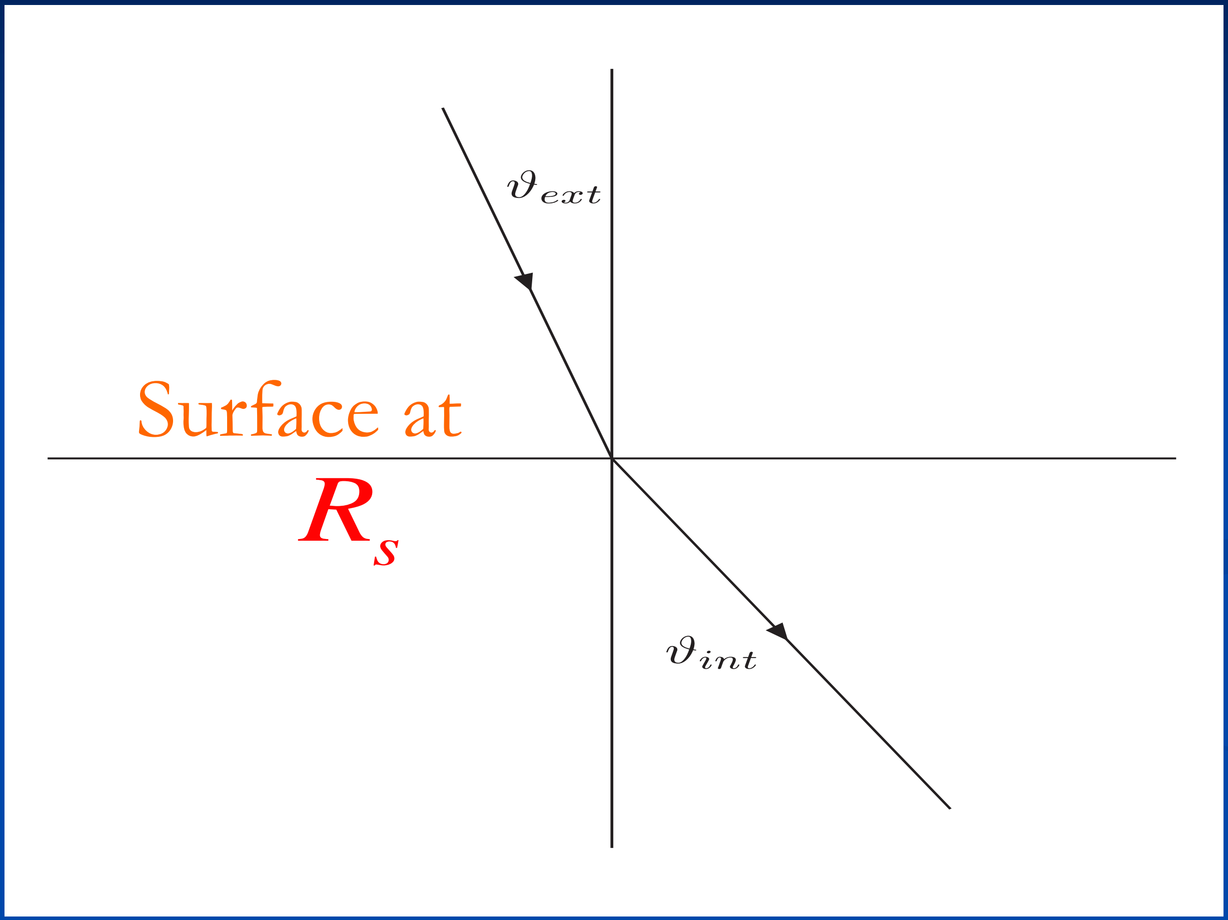
Discontinuity in κ implies <u>non-analytic</u> behavior

<u>No horizon, Truly Static,</u> t is a <u>Global Time</u>

Surface Area is Surface Area not Entropy

Surface Gravity is Surface Tension not Temperature

Refraction of Null Rays at Surface



Solve Geodesic Eq.

➔ Snell's Law

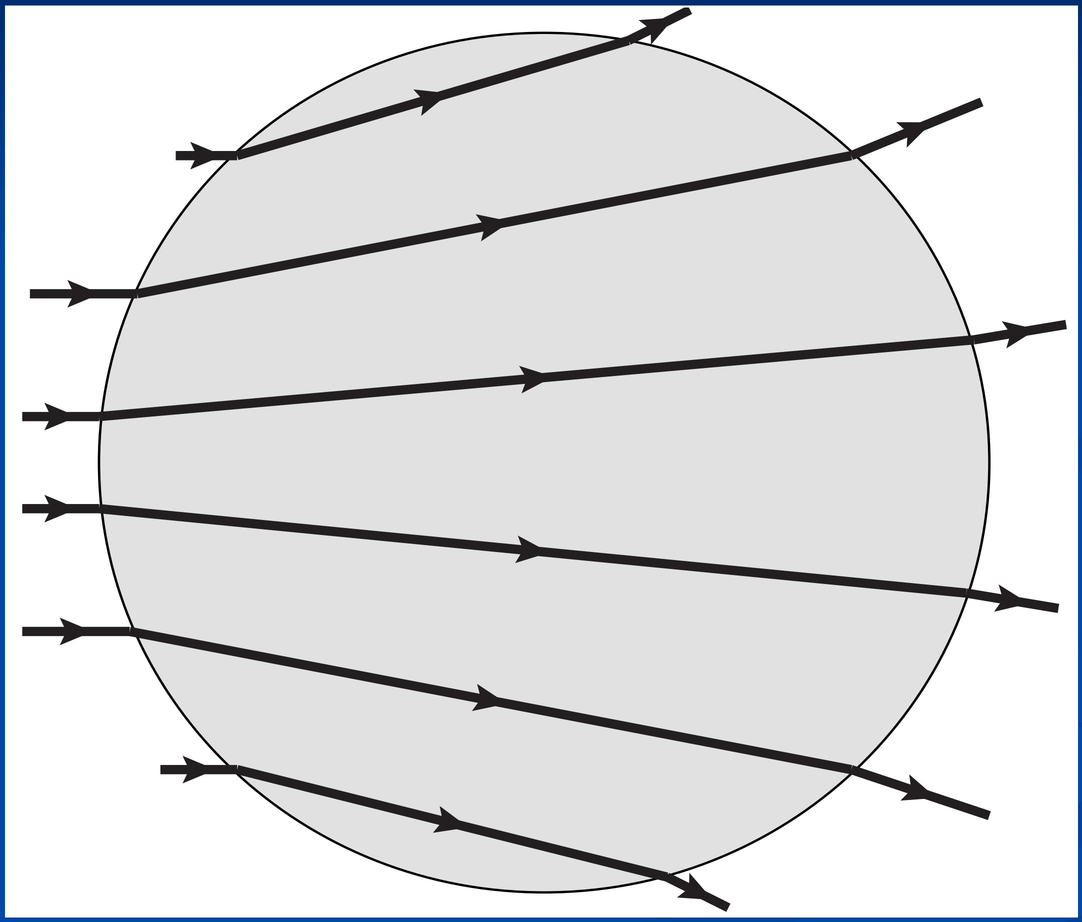
n < 1

Impact Parameter b

$$\sin\vartheta_{ext}\,\sqrt{1+\frac{4R_s^2}{b^2}} \;=\; \sin\vartheta_{int}\,\sqrt{1+\frac{R_s^2}{b^2}}$$

Defocusing of Null Rays

No Horizon ➔ Light Rays Penetrate Interior



Completely Different Imaging from a Black Hole

Summary so far

- Buchdahl Bound → Interior Pressure Divergence Develops **before** Event Horizon Forms for $R > \frac{9}{8} R_s = \frac{9}{4} GM$
- Constant Density Interior Schwarzschild Solution **Saturates Bound** & illustrates the generic behavior
- Infinite Redshift at the Central Pressure Divergence
- Pressure Singularity is **Integrable**
- Implies Formation of a **δ-fn.** Surface & **Surface Tension**
- & a Non-Singular (de Sitter $p = -\rho$) 'BH' Interior
- Area term is **Mechanical Surface Energy not Entropy**
- QM, Unitarity ✓ **No 'Information Paradox'**
- **Condensate Star negative pressure** $p = -\rho$

  already realized/inherent in Classical General Relativity

Effective Field Theory & Quantum Anomalies

- Expansion of Effective Action in Local Invariants assumes Decoupling of UV from Long Distance Modes
- But Massless Modes do not decouple
- Chiral, Conformal Symmetries are Anomalous
- Special Non-local Additions to Local EFT
- **IR** Sensitivity to **UV** degrees of freedom
- Conformal Symmetry & its Breaking controlled by the Conformal Trace Anomaly
- Macroscopic Effects in Black Hole Physics, Cosmology

2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\,\sigma$$

gives a total derivative in S_{cl}

Quantum Trace or Conformal Anomaly

$$\langle T^a_{\ a}\rangle = -\frac{c_m}{24\pi}R$$

$c_m = N_S + N_F$ for massless scalars or fermions

Linearity in σ in the variational eq.

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\,\langle T^a_{\ a}\rangle$$

determines the Wess-Zumino Action by *inspection*

2D Anomaly Action

- Integrating the anomaly linear in σ gives

$$\Gamma_{WZ}[\bar{g},\sigma] = \frac{c_m}{24\pi}\int d^2x\sqrt{\bar{g}}\,\left(-\sigma\,\bar{\Box}\,\sigma + \bar{R}\,\sigma\right)$$

- This is local but **non-covariant**. Note kinetic term for σ
- By solving for σ the WZ action can be also written

$$\Gamma_{WZ}[\bar{g},\sigma] = S_{anom}[g = e^{2\sigma}\,\bar{g}] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but **non-local**

$$S_{anom}[g] = -\frac{c}{96\pi}\int d^2x\sqrt{g}\int d^2x'\sqrt{g'}\;R_x\left(\Box^{-1}\right)_{x,x'}R_{x'}$$

- A covariant local form implies a **dynamical scalar** field

$$S_{anom}[g;\varphi] = \frac{c}{96\pi}\int d^2x\sqrt{g}\left[g^{ab}(\nabla_a\varphi)(\nabla_b\varphi) + 2R\varphi\right]$$

$$-\Box\varphi = R \qquad\qquad \varphi \leftrightarrow 2\sigma$$

Ward Identity and Massless Poles

Effects of Anomaly may be seen in flat space amplitudes

$$\Pi_{abcd}(x,x') = \langle T_{ab}(x) T_{cd}(x') \rangle$$

Conservation of T_{ab} Ward Identity in 2D implies

$$\boxed{\Pi_{abcd}(k) = (\eta_{ab}k^2 - k_a k_b)(\eta_{cd}k^2 - k_c k_d)\,\Pi(k^2)}$$

Anomalous Trace Ward Identity in 2D implies

$\boxed{k^2\,\Pi(k^2) \neq 0}$ at $k^2 = 0$ **massless pole**

IR Relevant Degree of Freedom

2D Anomaly Stress Tensor

- The stress-energy tensor of the 2D anomaly action is

$$T_{ab}^{(anom)}[g;\varphi] \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_{anom}[g;\varphi]}{\delta g^{ab}} = \frac{c}{24\pi}\left[\nabla_a\nabla_b\varphi - g_{ab}\Box\varphi + \frac{1}{2}\nabla_a\varphi\nabla_b\varphi - \frac{g_{ab}}{4}\nabla_c\varphi\nabla^c\varphi\right]$$

- **General soln. to $\Box\varphi = -R = f''$ with $\varphi(r^*)$ easily found in 2D Schwarzschild or de Sitter $f = 1\text{-}2M/r$, $f = 1\text{-}H^2r^2$**

$$ds^2 = f(r^*)(-dt^2 + dr^{*2}) \qquad \varphi = \ln f + (qr^* + pt)/M$$

$$T_t^{\ t} = \frac{cH^2}{24\pi}\left\{-\frac{1}{f}\left(p^2+q^2-1\right)+1\right\}$$

$$T_{r^*}^{\ r^*} = \frac{cH^2}{24\pi}\left\{\frac{1}{f}\left(p^2+q^2-1\right)+1\right\}$$

$$T_t^{\ r^*} = \frac{cH^2}{12\pi}\,\frac{pq}{f}$$

- **Quantum stress tensor fully determined from the anomaly**
- **Generally divergent at $f=0$ on horizon**
- **Finite if $p = 0$, $q = \pm 1$ (or visa versa)**

Quantum Effects of 2D Anomaly Action

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes dynamical & itself fluctuates freely
- Gravitational 'Dressing' of critical exponents: long distance/IR macroscopic physics
- Topological Properties, Large Effects near Horizons
- Additional non-local Infrared Relevant Operator in S_{EFT}

New Massless Scalar Degree of Freedom at low energy

Constructing the EFT of Gravity

- Assume *Equivalence Principle* (Symmetry)
- Metric Order Parameter Field g_{ab}
- Only two strictly *relevant* operators (R, Λ)
- Einstein's General Relativity is an EFT
- But EFT = General Relativity + Quantum Corrections
- Semi-classical Einstein Eqs. $(k << M_{pl})$:

$$G_{ab} + \Lambda\, g_{ab} = 8\pi\, G\, \langle T_{ab} \rangle$$

- But there is also a quantum conformal anomaly:

$$\langle T_a{}^a \rangle = b\, C^2 + b' \left(E - \tfrac{2}{3} \Box R\right) + b'' \Box R$$

- **New** (marginally) relevant operator(s)appear in EFT

4D Anomalous Effective Action

Conformal Parametization

$$\rightarrow \qquad g_{ab} = \exp(2\sigma)\,\bar{g}_{ab}$$

Since $\sqrt{g}\,F = \sqrt{\bar{g}}\,\bar{F}$

is independent of σ, and

$$\sqrt{g}\,(E - \tfrac{2}{3}\Box R) = \sqrt{\bar{g}}\,(\bar{E} - \tfrac{2}{3}\bar{\Box}\bar{R}) + 4\sqrt{\bar{g}}\bar{\Delta}_4\sigma$$

is linear in σ, the variational eq.,

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\langle T_a{}^a\rangle = b\,\sqrt{g}F + b'\sqrt{g}\,(E - \tfrac{2}{3}\Box R)$$

determines the Wess-Zumino Action by inspection:

$$\Gamma_{WZ} = 2b'\int d^4x\sqrt{\bar{g}}\,\sigma\bar{\Delta}_4\sigma$$

$$+\int d^4x\sqrt{\bar{g}}\left[b\bar{F} + b'\left(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\ ,$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b - \tfrac{2}{3}R\Box + \tfrac{1}{3}(\nabla^a R)\nabla_a$$

$$\mathbf{F = C_{abcd}C^{abcd}}$$

$$\mathbf{E = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2}$$

Effective Action for the Trace Anomaly

- **Non-Local Covariant Form**

$$S_{anom}[g] = \frac{1}{2}\int_x\int_{x'}\left(\frac{E}{2}-\frac{\Box R}{3}\right)_x \left(\Delta_4^{-1}\right)_{x,x'}\left[b\,C^2 + b'\left(\frac{E}{2}-\frac{\Box R}{3}\right)\right]_{x'}$$

- **Local Covariant Form**

$$S_{anom}[g;\varphi] = \frac{b'}{2}\int d^4x\sqrt{g}\,\left[-\varphi\,\Delta_4\,\varphi + \left(E - \tfrac{2}{3}\,\Box\,R + \tfrac{b}{b'}C^2\right)\varphi\right]$$

$$\Delta_4\varphi = \tfrac{1}{2}\left(E - \tfrac{2}{3}\,\Box\,R + \tfrac{b}{b'}C^2\right)$$

- **Dynamical Scalar in Conformal Sector**

$$\Delta_4 = \Box^2 + 2R^{ab}\nabla_a\nabla_b - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^a R)\nabla_a$$

- **Expectation Value/Classical Field is Scalar Condensate**

IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

Not given purely in terms of Local Curvature

$$\boxed{S_{EFT}[g,\varphi] = S_{cl}[g] + S_{anom}[g,\varphi]}$$

This is a modification of classical General Relativity with macroscopic quantum effects

Fluctuations of conformal scalar degree of freedom can generate a Quantum Phase Transition in which Λ changes

Why is this Important for Black Holes? Stress Tensor of the Anomaly

Variation of the Effective Action with respect to the metric gives stress-energy tensor

$$T_{\mu\nu}^{anom}[g,\varphi] = -\frac{2}{\sqrt{-g}}\,\frac{\delta S_{anom}}{\delta g^{\mu\nu}}$$

- Quantum Vacuum Polarization in Terms of (Semi-) Classical Scalar 'Potential'
- φ is a scalar degree of freedom in low energy gravity which depend upon the global topology ofspacetimes and its boundaries, horizons

Anomaly Scalars in Schwarzschild Space

- **General solution of φ equation as function of r are easily found in Schwarzschild case**

$$\left.\frac{d\varphi}{dr}\right|_S = -\frac{1}{3M} - \frac{1}{r} + \frac{2Mc_H}{r(r-2M)} + \frac{c_\infty}{2M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right) + \frac{q-2}{6M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right)\ln\left(1 - \frac{2M}{r}\right) - \frac{q}{6r}\left[\frac{4M}{r-2M}\ln\left(\frac{r}{2M}\right) + \frac{r}{2M} + 3\right]$$

- q, c_H, c_∞ are integration constants, q topological charge
- Linear time dependence pt can be added
- Only way to have vanishing φ as $r \to \infty$ is $c_\infty = q = 0$
- But only way to have finiteness on the horizon is

$$\boxed{c_H = 0,\ q = 2}$$

- Topological obstruction to finiteness vs. falloff of stress tensor
- 2 conditions on 3 integration constants for horizon finiteness
- Set of Measure Zero

Schwarzschild Spacetime (again)

$$ds^2 = -(1 - \tfrac{2M}{r})dt^2 + \frac{dr^2}{(1 - \frac{2M}{r})} + r^2 d\Omega^2$$

$$\varphi = \sigma = \ln\sqrt{f} = \frac{1}{2}\ln\left(1 - \frac{2M}{r}\right) \to \infty$$

solves homogeneous $\Delta_4\varphi = 0$

Timelike Killing field (Non-local Invariant)

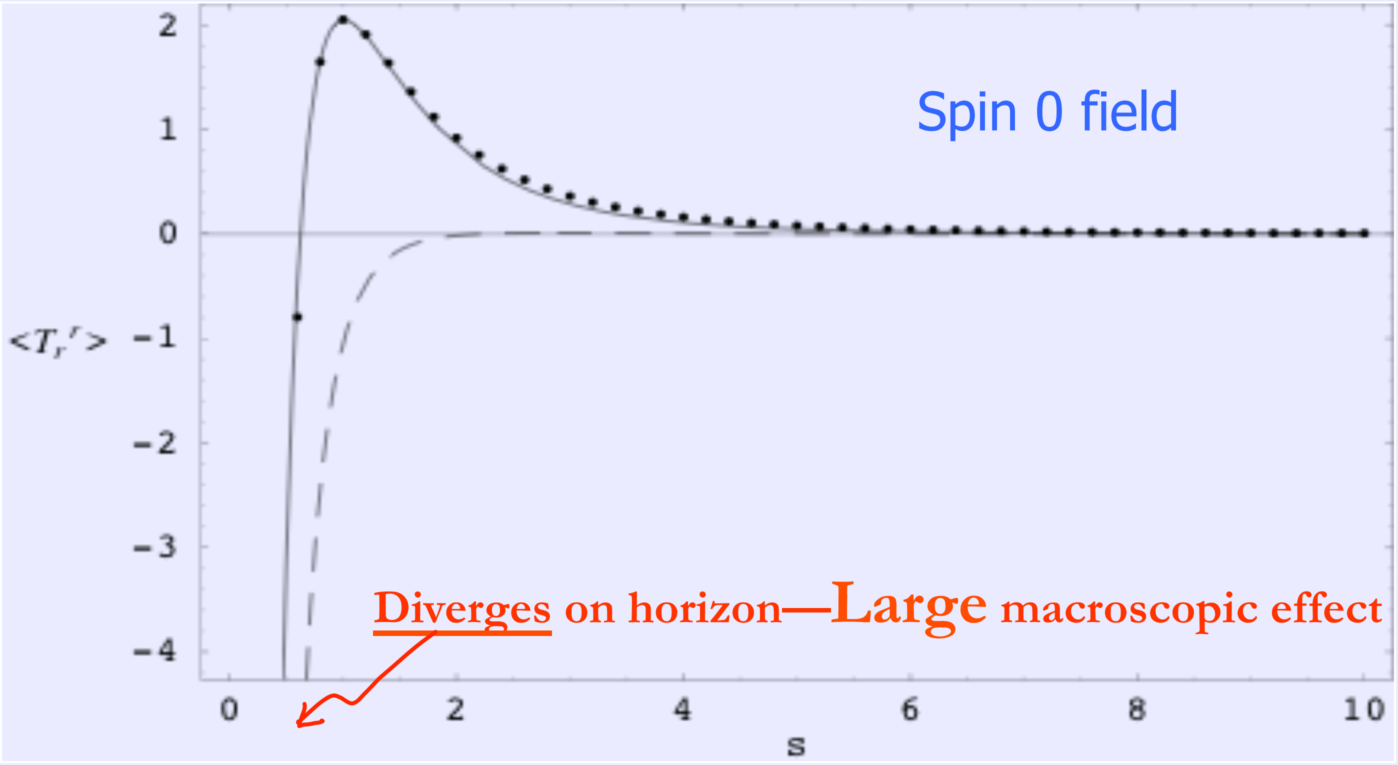
$$\xi^a = (1, 0, 0, 0) \quad e^{\sigma} = (-\xi_a\xi^a)^{\frac{1}{2}} = f^{\frac{1}{2}}$$

Energy density scales like $e^{-4\sigma} = f^{-2}$

Conformal Scalar Potential gives Geometric (Coordinate Invariant) Meaning to Stress Tensor becoming Large on Horizon

Stress-Energy Tensor in Boulware Vacuum – Radial Component

Dots – Direct Numerical Evaluation of $<T_a^{\ b}>$ (Jensen et. al. 1992)
Solid – Stress Tensor from the Auxiliary Fields of the Anomaly (E.M & R. Vaulin 2006)
Dashed – Page, Brown and Ottewill approximation (1982-1986)



Bose-Einstein Condensation

- Bose-Einstein statistics imply any number of particles can occupy the same single particle state.
- At high enough densities and/or low enough temperatures a finite fraction of all the particles are in the lowest energy (ground) state.
- This tendency of bosons to condense takes place in the absence of interactions or even with (not too strong) repulsive interactions. Attractive interactions make it all the more favorable.
- Bose-Einstein Condensation is a generic macroscopic quantum phenomenon, observed in Superfluids, ^4He (even ^3He by fermion pairing), Superconductors, and Atomic Gases, ^{87}Rb.
- Relativistic Quantum Field Theory exhibits a similar phenomenon in Spontaneous Symmetry Breaking, in both the strong and electroweak interactions $\langle \bar{q} q \rangle \neq 0 \quad \langle \Phi \rangle \neq 0$.

A Macroscopic Quantum Effect

Gravitational Vacuum Condensates

- Gravity is a theory of spin-2 bosons
- Its interactions are attractive
- The interactions become strong near $r = R_s$
- Energy of any scalar order parameter must couple to gravity with the vacuum eq. of state,
$$p_V = -\rho_V = -V(\phi)$$
- Relativistic Entropy Density s is (for $\mu = 0$),
$$Ts = p + \rho = 0 \text{ if } p = -\rho$$
- Zero entropy density for a single macroscopic quantum state, $k_B \ln \Omega = 0$ for $\Omega = 1$
- This eq. of state violates the energy condition, $\rho + 3p \geq 0$ (if $\rho_V > 0$) needed to prove the classical singularity theorems
- Dark Energy acts as a repulsive core

A GBEC phase transition can stabilize
a high density, compact cold stellar
remnant to further gravitational collapse

Predicted by EFT of Quantum Conformal Anomaly
Scalar Mode in Gravity—Realized in Schwarzschild Soln.

Gravitational Vacuum Condensate Stars

Gravastars as Astrophysical Objects

- Cold, Dark, Compact, Arbitrary M, J
- Accrete Matter just like a black hole
- But matter does not disappear down a 'hole'
- Relativistic Surface Layer can re-emit radiation
- Can support Electric Currents, Large Magnetic Fields
- Possibly more efficient central engine for Gamma Ray Bursters, Jets, UHE Cosmic Rays
- Formation should be a violent phase transition converting gravitational energy and baryons into HE leptons and entropy
- Interior could be completely non-singular **dynamical condensate**
- Dark Energy as Condensate -- Finite Size effect of boundary conditions at the horizon ➔ Implications for Cosmology

Implication: Vacuum Energy is Dynamical

Λ as Vacuum Energy of a Gravitational Bose-Einstein Condensate

- The Conformal Factor of the metric $g_{ab} = e^{2\sigma}\bar{g}_{ab}$ is frozen by the classical Einstein's Eqs. $R = 4\Lambda$
- But the trace anomaly of massless quantum fields forces the scalar 'condensate' $\langle e^{2\sigma}\rangle$ to fluctuate Scalar: φ
- This generates a well-defined additional term in the low energy action, and
- Describes a New Conformally Invariant Phase, Infrared Fixed Point of Gravity
- The quantum phase transition to this phase is characterized by 'melting' of the scalar condensate $\langle e^{2\sigma}\rangle$ Fluctuating φ

Λ_{eff} Dynamical, generated by SSB of Global Conformal Invariance $\sigma \rightarrow \sigma + \sigma_0$

emil@lanl.gov

- Gravitational Condensate Stars resolve all 'black hole' paradoxes
- **Discrete Grav. Wave Signatures of Surface Modes**
- **Astrophysics** of gravastars & 'no-hair' **testable** also by mm **imaging** of Sgr A* w. Event Horizon Telescope)
- New Models of Dynamical Dark Energy in Cosmology

BH's and Dark Energy are related:

Both macroscopic quantum effects in gravity

Both incorporated in EFT of Anomaly

Observations are Coming

- High resolution sub-mm **Very Large Baseline Imaging** (VLBI) will zero in on event horizon of black hole (or gravastar surface) in the center of our galaxy
- Maxima of X-ray Continuum Thermal Spectra from Accretion Disk can determine the location of the **Innermost Stable Circular Orbits** (ISCO's) of candidate black holes
- X-Ray Fe Line Spectra Doppler Shifts will allow measurement of velocities and **test rotating Kerr black hole solution no-hair theorem** in external geometry
- **Gravitational Waves** expected first detection by Advanced LIGO II will observe inspiral and black hole merger events
- Millisec. Pulsar Timing Arrays may even detect **GW's** first
- Possibility of detection of **scalar 'breathing' mode** polarization from scalar φ waves

Next few years will be an exciting period

Quantum Trace Anomaly in 4D Flat Space

Massless QED in an External E&M Field

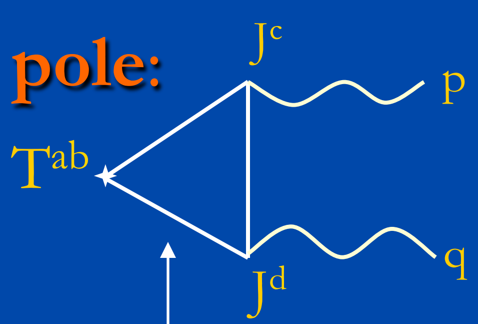
$$\boxed{\langle T_a^{\ a}\rangle = e^2 F_{\mu\nu} F^{\mu\nu}/24\pi^2}$$

Triangle Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\cdot q - q^c p^d)\, F_1(k^2) + \ldots$$

In the limit of massless fermions, $F_1(k^2)$ must have a **massless pole:**

$k = p + q$



$$F_1 = \frac{e^2}{18\pi^2 k^2}$$

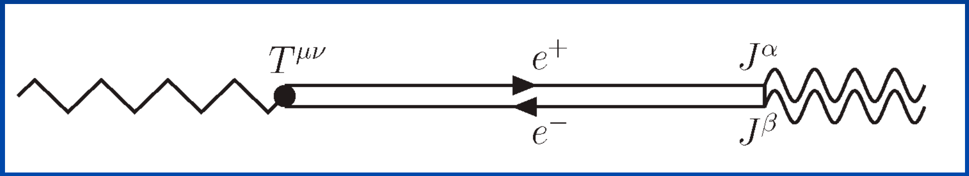
$$\rho(s) = \frac{e^2}{18\pi^2}\delta(s)$$

M. Giannotti & E. M. (2009)

Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

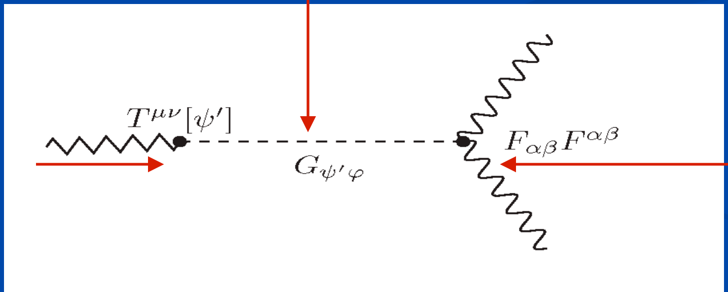
Massless Anomaly Pole

For $p^2 = q^2 = 0$ (both photons on shell) and $m_e = 0$ the pole at $k^2 = 0$ describes a *massless* $e^+ e^-$ pair moving at v=c colinearly, with opposite helicities in a total spin-0 state



a massless scalar 0^+ state ('Cooper pair') which couples to gravity

$$h_{\mu\nu}\left(\partial^\mu \partial^\nu - \eta^{\mu\nu}\,\Box\right)\Box\varphi$$



Effective vertex

$$\varphi F^{\mu\nu} F_{\mu\nu}$$

Effective Action special case of general form

$$\sim \int d^4x\sqrt{g}\left[-\varphi\,\Box^2\varphi - \frac{2}{3}\,\varphi\,\Box R - \frac{e^2}{48\pi^2}\,\varphi\, F^{\mu\nu}F_{\mu\nu}\right]$$

<TJJ> Triangle Amplitude in QED

Spectral Representation and Finite Sum Rule

$$F_1(k^2; p^2, q^2) = \frac{1}{3k^2} \int_0^\infty \frac{ds}{k^2 + s - i\epsilon} \left[(k^2 + s)\rho_T - m^2 \rho_m \right]$$

Numerator & Denominator cancel here

Im $F_1(k^2 = -s)$: Non-anomalous, vanishes when m=0

$$\rho_T(s; p^2, q^2) = \frac{e^2}{2\pi^2} \int_0^1 dx \int_0^{1-x} dy \; (1 - 4xy) \; \delta\left(s - \frac{(p^2 x + q^2 y)(1 - x - y) + m^2}{xy} \right)$$

$$\boxed{\int_0^\infty ds \, \rho_T(s; p^2, q^2) = \frac{e^2}{6\pi^2}}$$

obeys a finite sum rule independent of p^2, q^2, m^2

and as p^2, q^2, $m^2 \rightarrow 0^+$

$$\boxed{\rho_T(s) \rightarrow \frac{e^2}{6\pi^2} \delta(s)}$$

$$\boxed{F_1(k^2) \rightarrow \frac{e^2}{18\pi^2 k^2}}$$

Massless scalar intermediate two-particle state analogous to chiral limit of QCD

Trace Anomaly in Curved Space

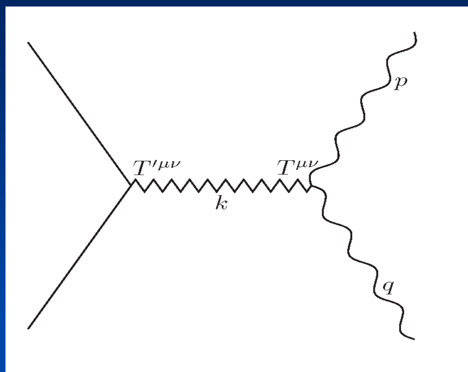
$$\boxed{\langle T_a^{\ a} \rangle = b\, C^2 + b' \left(E - \tfrac{2}{3} \Box R\right) + b'' \Box R}$$

$\langle T_{ab} \rangle$ is the Stress Tensor of Conformal Matter

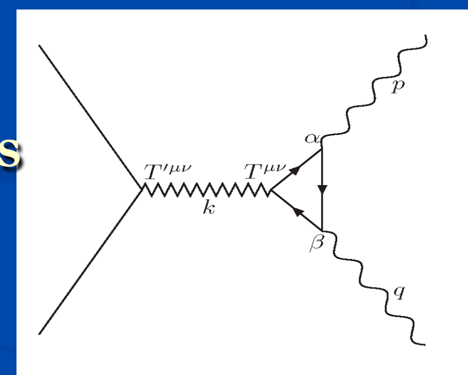
- $\langle T_a^{\ a} \rangle$ is expressed in terms of Geometric Invariants
- One-loop amplitudes similar to previous examples
- State-independent, independent of G_N
- No local effective action in terms of curvature tensor

  But there is a non-local effective action
  which can be rendered local in terms of
  new scalar degrees of freedom
- Quantum Modification of Classical Gravity

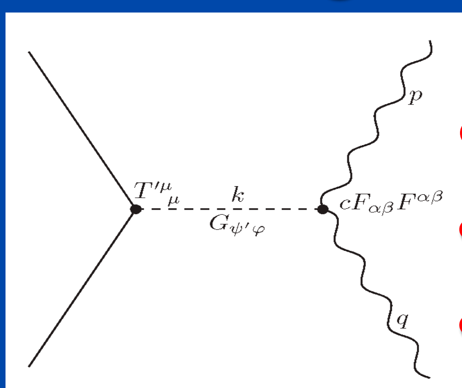
Scalar Pole in Gravitational Scattering



- In Einstein's Theory only transverse, tracefree polarized waves (<u>spin-2</u>) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)



- But for $m_e = 0$ there is a scalar pole in the <TJJ> triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$



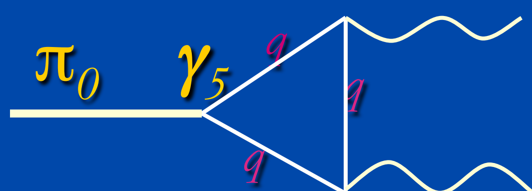
- Couples to trace $T'^{\mu}{}_{\mu}$
- <TTT> triangle of massless photons has pole
- <u>At least one</u> new scalar degree of freedom in EFT

New Horizons in Quantum 'Black Holes'

- Classical Black Holes already have some unphysical features
- The tension between General Relativity and both Quantum Mechanics and Statistical Physics in Black Holes leads to a 'Crisis in Physics'
- The most suspect assumption is the SEP which is violated by Quantum Fields in Black Hole Curved Spacetimes
- Quantum Vacuum (Casimir) Effects are computable & relevant at Macroscopic Distances & near Event Horizons
- New scalar degrees of freedom in the EFT of Gravity are required in the Standard Model by the Conformal Anomaly
- Their fluctuations induce a **Quantum Phase Transition** at the would-be 'Black Hole' horizon

Chiral Anomaly in QCD

- QCD with N_f massless quarks has an apparent
  $U(N_f) \otimes U_{ch}(N_f)$ Symmetry
- But $U_{ch}(1)$ Symmetry is Anomalous
- Effective Lagrangian in Chiral Limit has $N_f^2 - 1$
  (*not* N_f^2) massless pions at low energies
- Low Energy $\pi_0 \rightarrow 2\,\gamma$ dominated by the anomaly



$$\partial_\mu j^{\mu 5} = e^2 N_c F_{\mu\nu} \widetilde{F}^{\mu\nu} / 16\pi^2$$

- *IR Relevant Operator* that violates naïve decoupling of *UV*
- *Measured* decay rate verifies $N_c = 3$ in QCD

Anomaly Matching of *IR* ↔ *UV*

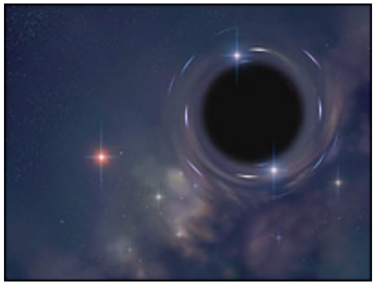
- Interior could be completely non-singular condensate
- <u>Gravitational Condensate Stars</u> resolve all 'black hole' paradoxes/ 'crisis'
- Astrophysics of gravastars & 'no-hair' testable by mm VLBI, X-rays, ISCO's, GW's in this decade (Sgr A*--gas cloud in collision this year)
- Science that 'Matters': Far-reaching implications for eventual unification of quantum matter with gravity

Gra(vitational) Va(cuum) Stars

Gravastars as Astrophysical Objects

- Cold, Dark, Compact, Arbitrary M
- Accrete Matter like a Black Hole
- But Matter does not Disappear down a Hole
- May be Re-emitted by Ultra-relativistic Shell
- Possible More Efficient Central Engine for Sporadic Gamma Ray Bursters, High-Energy Cosmic Rays, Other Sources?
- Formation could be Violent 'Bosenova'
- Should Support Angular Momentum, Magnetic Fields
- Gravitational Wave Signatures?
- Alternative to Black Holes for the Final State of Gravitational Collapse

- Cosmological Models of Dark Energy
- Much to be Done...



Main Features of New Soln.

- Vacuum Schwarzschild Exterior
- de Sitter (GBEC) Interior, No Singularity
- $\Lambda > 0$ Casimir Energy due to b.c.
- GBEC similar to Gluon Condensate in Bag Model of Hadrons
- Thin Shell of $p = \rho$, No Event Horizon
- Global Time, Unitarity, No Hawking Radiation
- Modest Entropy, No Information Paradox
- Maximizes Entropy, Completely Stable
- No Planckian Pressures or Densities
- Hydrodynamic Einstein Eqs. Valid Everywhere except at r_1, r_2 Stationary Shock Fronts
- Interior de Sitter also a Cosmological Soln.

Analog to BEC quantum transition near the classical horizon

Relation to Conformal Trace Anomaly

- Stress-Energy Tensor has Anomalous Trace

$$\langle T^a_{\ a} \rangle = bF + b' \left(E - \tfrac{2}{3}\,\Box R\right) + b''\,\Box R$$

$$E \equiv {}^*R_{abcd}\,{}^*R^{abcd} = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

$$F \equiv C_{abcd}C^{abcd} = R_{abcd}R^{abcd} - 2R_{ab}R^{ab} + \tfrac{1}{3}R^2$$

$$b' = -\frac{1}{360}\,\frac{1}{16\pi^2}\,(N_S + 11N_F + 62N_V)$$

- Non-Local Anomaly Effective Action

$$S_{anom}[g] = \frac{1}{2}\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\left(\frac{E}{2} - \frac{\Box R}{3}\right)_x \Delta_4^{-1}(x,x')\left[bF + b'\left(\frac{E}{2} - \frac{\Box R}{3}\right)\right]_{x'}$$

- Can be Made Local by Introducing a New Scalar

$$S^{(E)}_{anom}[g;\varphi] \equiv \tfrac{1}{2}\int d^4x\sqrt{-g}\left\{-\left(\Box\varphi\right)^2 + 2\left(R^{ab} - \tfrac{1}{3}Rg^{ab}\right)\nabla_a\varphi\nabla_b\varphi + \left(E - \tfrac{2}{3}\,\Box R\right)\varphi\right\}$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b - \tfrac{2}{3}R\Box + \tfrac{1}{3}(\nabla^a R)\nabla_a$$

Gravitational Vacuum Condensates

- Gravity is a theory of spin-2 bosons
- Its interactions are attractive
- The interactions become strong near $r = R_s$
- Energy of any scalar order parameter must couple to gravity with the vacuum eq. of state,
$$p_V = -\rho_V = -V(\phi)$$
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$$Ts = p + \rho = 0 \text{ if } p = -\rho$$
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- Dark Energy acts as a repulsive core

A GBEC phase transition can stabilize a high density, compact cold stellar remnant to further gravitational collapse

A New Soln. to Einstein Eqs.

$$R_a^{\ b} - \tfrac{1}{2} R\, \delta_a^{\ b} = 8\pi G\, T_a^{\ b}$$

- $1 - \frac{d(r\,h)}{dr} = 8\pi G\, \rho\, r^2$
- $\frac{r\,h}{f}\frac{df}{dr} + h - 1 = 8\pi G\, p\, r^2$
- $\frac{dp}{dr} + \frac{p+\rho}{2f}\frac{df}{dr} = 0 \qquad (\nabla_b T_r^{\ b} = 0)$

Other components follow by differentiating these

Define $\quad h \equiv 1 - \frac{2Gm(r)}{r}$

Then $\quad \frac{dm}{dr} = 4\pi\, \rho\, r^2 \quad$ and

$$\frac{dp}{dr} = -\frac{G(\rho+p)(m+4\pi p r^3)}{r\,(r-2Gm)} \qquad \text{(TOV eq.)}$$

Eqs. become closed when eq. of state is given:

$$p = \kappa\, \rho$$

with $\quad \kappa = \begin{cases} -1, & r < r_1 \\ +1, & r_1 < r < r_2 \end{cases}$

$$p = \rho = 0\,, \qquad r_2 < r$$

A Simple Model
2004

Update: Use the EFT and Stress Tensor of the Trace Anomaly to solve the matching problem in the quantum phase boundary layer

(in mean field approximation)

Conformal Symmetry Near Horizons

- An horizon is a null surface, conformal to flat space light cone & conformally invariant
- Fields become effectively massless there
- The near horizon region is conformal to $EAdS_3 \otimes$ time
- Conformal Anomaly becomes the dominant term in the effective action in the near horizon region
- Stress Tensor from S_{anom} determines $\langle T_{ab} \rangle$
- Stress Tensor is generally singular there
- Singular behavior has invariant meaning in terms of anomaly scalar degrees(s) of freedom on horizon

Dynamical Vacuum Energy

$$\boxed{S_{EFT}[g,\varphi] = S_{cl}[g] + S_{anom}[g,\varphi]}$$

- Conformal part of the metric, $g_{ab} = e^{2\sigma}\ \bar{g}_{ab}$ constrained— ---frozen– in classical GR, $R=4\Lambda$ becomes dynamical
- Λ itself is state dependent condensate determined by long distance IR breaking of Conformal Invariance
- Fluctuations of $\varphi\sim 2\sigma$ describe conformally invariant phase of gravity in 4D—likely relevant at BH horizon
- Quantum Phase Transition characterized by the 'melting' of the scalar condensate Λ

Implications for Cosmological Dark Energy and non-Gaussian statistics of CMB

I. Antoniadis, P. O. Mazur, E. M., N. Jour. Phys. 9, 11 (2007); JCAP 1209 (2012) 024

Anomaly Stress Tensor in de Sitter Space

- General soln. for φ as fn. of static r and linear in t is

$$\varphi(r,t)\Big|_{dS} = c_0 + 2Hpt + \ln\left(1 - H^2r^2\right) + \frac{q}{2}\ln\left(\frac{1-Hr}{1+Hr}\right) + \frac{2c_H - 2 - q}{2Hr}\ln\left(\frac{1-Hr}{1+Hr}\right)$$

- Bunch-Davies state has $p = 1$, $q = 0$, $c_H = 1$

$$T_{ab}\big|_{BD,dS} = 6b'H^4 g_{ab} = -\frac{H^4}{960\pi^2}\, g_{ab}\left(N_s + 11N_f + 62N_v\right)$$

- This is the soln. for conformal map to flat spacetime

$$ds^2 = e^{\varphi_{BD}}\,(ds^2)_{flat}$$

- Otherwise T_{ab} is generally divergent at the static horizon $r = H^{-1}$ behaving like $(1-H^2r^2)^{-2}$

Conformal Symmetry & de Sitter Horizon

- The de Sitter group SO(4,1) has 10 Killing vectors (isometries)

$$\nabla_a \xi_b^{(i)} + \nabla_b \xi_a^{(i)} = 0 \quad , \quad i = 1, \ldots, 10$$

- The conformal group of S^2 is the Lorentz group SO(3,1) realized projectively, includes 3 special conformal transformations

$$\boxed{\hat{n}^i = \frac{X^i}{X^0} \to \frac{L^i{}_0 + L^i{}_j \hat{n}^j}{L^0{}_0 + L^0{}_k \hat{n}^k} \,, \qquad \delta n^i = -v^i + \hat{n}^i (v \cdot \hat{n})}$$

- The 10 isometries of SO(4,1) decompose on the horizon into
    - 3 rotations
    - 3 conformal transformations (above) x 2 = 6
    - 1 time translation
- Any SO(4,1) de Sitter invariant Green's fn. becomes SO(3,1) conformally invariant on the de Sitter horizon (deS/CFT) eg.

$$\boxed{\Delta_4^{-1}(x, x') \to -\frac{1}{16\pi^2} \ln(1 - \hat{n} \cdot \hat{n}') + c_0 \propto \Delta_2^{-1}(\hat{n}, \hat{n}')}$$

- Dimension 0 field correlator = logarithm

New Cosmological Scalar Fluctuations

- Anomaly Scalar φ solutions are coherent state variations
- They are non-Planckian and can occur at any scale k
- These are new scalar degrees of freedom in cosmology
- In FRW coordinates mode with fixed k red/blueshifts
- In static de Sitter coordinates (this t is not FRW t)

$$ds^2 = -(1 - H^2r^2)\, dt^2 + (1 - H^2r^2)^{-1}\, dr^2 + r^2\, d\Omega^2$$

the modes $\sim \ln(1 - H^2r^2)$ grow large on the horizon

- Corresponding stress tensor perturbation

$$\langle T^a{}_b \rangle \sim H^4\, (1 - H^2r^2)^{-2}\, \mathrm{diag}\, (-3, 1, 1, 1)$$

diverges on the horizon, a fluctuation in the temperature away from its Hawking-deS value $H/2\pi$

Cosmological Horizon Modes

- Fluctuations of quantum fields intrinsic to de Sitter Space induce metric fluctuations on the cosmological horizon, related to the trace anomaly
- These metric fluctuations produce scalar gravitational potentials whose difference is a conformal weight $w = 0$ field on the cosmological horizon
- This is the same difference of potentials produced by adiabatic perturbations in inflation necessary for the observed CMB temperature anisotropies with scale invariant HZ Spectrum
- Arise here from conformally invariant quantum fluctuations on the de Sitter horizon without any ad hoc inflaton field or potential

Cosmological Scalar Fluctuations from the Conformal Anomaly

- These are effective quantum scalar degrees of freedom in cosmology
- No inflaton or fine tuning of its potential
- In static de Sitter coordinates

$$ds^2 = -(1 - H^2r^2)\, dt^2 + (1 - H^2r^2)^{-1}\, dr^2 + r^2\, d\Omega^2$$

the φ modes $\sim \ln(1 - H^2r^2)$ are large on the horizon

- Corresponding stress tensor perturbation

$$\langle T^a{}_b \rangle \sim H^4\, (1 - H^2r^2)^{-2}\, \mathrm{diag}\,(-3, 1, 1, 1)$$

diverges on the horizon, a coherent fluctuation in the temperature away from its Hawking-deS value $H/2\pi$

Soln. of Linear Potentials in de Sitter space

- For small fluctuations around de Sitter space $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ the linearized Einstein eqs.

$$\boxed{\delta\left(R_{\mu\nu} - \frac{R}{2}\,g_{\mu\nu} + \Lambda\, g_{\mu\nu}\right) = 8\pi G\ \delta\langle T_{\mu\nu}\rangle}$$

can be solved in terms of two scalar potentials $\Upsilon_{\mathcal{A}}, \Upsilon_{\mathcal{C}}$

$$\bar{g}_{\tau\tau} + h_{\tau\tau} = -(1 + 2\Upsilon_{\mathcal{A}})$$

$$\bar{g}_{ij} + h_{ij} = a^2(\tau)\,(1 + 2\Upsilon_{\mathcal{C}})\,\delta_{ij} + h^{\perp}_{ij}$$

the difference of potentials has a soln. in static coordinates

$$\Upsilon_{\mathcal{C}} - \Upsilon_{\mathcal{A}} = 8\pi G H^2 b' \left[\frac{c_1}{Hr}\ln\left(\frac{1-Hr}{1+Hr}\right) + \frac{c_2}{Hr}\ln f\right]$$

- Logarithmic behavior as $r \to r_H$ indicates a weight w=0 conformal field just as required for the scale invariant HZ Spectrum

Conformal Invariance & CMB Power

- Correlation Function of operators with general conformal dimension Δ in flat space is

$$\boxed{\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\rangle \sim |x_1 - x_2|^{-2\Delta}}$$

- In Fourier space this becomes

$$\boxed{G_2(k) = \langle \tilde{\mathcal{O}}_\Delta(k)\tilde{\mathcal{O}}_\Delta(-k)\rangle \sim |k|^{2\Delta-3}}$$

- Spectral index $n_s = 2\,\Delta - 3$
- Harrison-Zel'dovich argued that density perturbations obey Poisson's eq. $\boxed{-\nabla^2\delta\varphi = 4\pi G\,\delta\rho}$ so $\Delta = 2$ and $n_s = 1$ for CMB $<\delta T(k)\,\delta T(-k)>$ but in general the conformal dimension may not equal its classical dimension---anomalous dimensions are allowed in CFT

Non-Gaussianity in CMB

- Conformal Invariance also uniquely determines the form of the bi-spectrum (PRL 79, 14: 1997)

$$\langle \mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2)\mathcal{O}_\Delta(x_3)\rangle \sim |x_1 - x_2|^{-\Delta}|x_2 - x_3|^{-\Delta}|x_3 - x_1|^{-\Delta}$$

- Fourier space:

$$G_3(\vec{k}_1, \vec{k}_2) \sim \int d^3\vec{p}\ |\vec{p}|^{\Delta-3}\,|\vec{p} + \vec{k}_1|^{\Delta-3}\,|\vec{p} - \vec{k}_2|^{\Delta-3}$$

- The angular bi-spectrum is completely determined

$$C_3(\theta_{12}, \theta_{13}, \theta_{23}) \sim \int d^3\vec{k}_1 \int d^3\vec{k}_2\ \frac{G_3(\vec{k}_1, \vec{k}_2)}{\vec{k}_1^2\,\vec{k}_2^2\,(\vec{k}_1 - \vec{k}_2)^2}\ e^{i\vec{k}_1\cdot(\vec{r}_1 - \vec{r}_3)}\ e^{i\vec{k}_2\cdot(\vec{r}_2 - \vec{r}_3)}$$

The shape very different from constant f_{NL}

- Magnitude can be searched for/bounded over whole sky data
- How can it be calculated in de Sitter space?

New Horizons in Gravity

- Einstein's Theory receives Quantum Corrections relevant at **macroscopic** Distances & near Event Horizons
- These arise from **scalar degree(s) of freedom** in the extended EFT of Gravity required by the **Conformal/Trace Anomaly**
- At horizons these massless scalar degrees of freedom have **macroscopically large effects**
- Their Fluctuations can induce a **Quantum Phase Transition** at the horizon of a 'black hole'
- Λ_{eff} is a **dynamical condensate** which can change in the phase transition & remove 'black hole' interior singularity
- **Gravitational Condensate Stars** resolve all 'black hole' paradoxes ⇒ New Astrophysics of 'gravastars'
- The observed dark energy of our Universe itself may be a **macroscopic finite size effect** whose value depends not on microphysics but on the cosmological horizon scale
- Prediction for Conformal Non-Gaussian CMB Bispectral Shape

Relevance of the Trace Anomaly

- Expansion of Effective Action in *Local* Invariants assumes Decoupling of Short Distance from Long Distance Modes
- **But Relativistic Particle Creation is Non-Local**
- ***Massless* Modes do not decouple**
- **Special Non-local Additions to Local EFT**
- ***IR* Sensitivity to UV degrees of freedom**
- **QFT Conformal Behavior, Breaking & Bulk Viscosity (analog of conductivity) determined by Anomaly**
- **Blueshift on Horizons → behavior conformal there**
- **Additional Scalar Degree(s) of Freedom in EFT of Gravity allow & predict Dynamics of Λ**

Solns. of Linear Response Eqs. in deS

- Homogeneous solutions u=v=0 (variations determined solely by δg_{ab}) contain no interesting non-Planck scale solutions
- Inhomogeneous solutions have non-zero

$$H^2 u \equiv \left(\frac{d^2}{dt^2} + H\frac{d}{dt} + \frac{\vec{k}^2}{a^2}\right)\phi - 2H^2 h_{tt}$$

sa

$$H^2 v \equiv \left(\frac{d^2}{dt^2} + H\frac{d}{dt} + \frac{\vec{k}^2}{a^2}\right)\psi$$

$$\left(\frac{d^2}{dt^2} + 5H\frac{d}{dt} + 6H^2 + \frac{\vec{k}^2}{a^2}\right)\begin{pmatrix} u \\ v \end{pmatrix} = 0$$

New Cosmological Scalar Fluctuations

- Inhomogeneous solutions are coherent state variations
- They are non-Planckian and can occur at any scale k
- These are new scalar degrees of freedom in cosmology
- In FRW coordinates each mode with fixed k redshifts $\sim a^{-4}$
- In static de Sitter coordinates (this t is not FRW t)

$$ds^2 = -(1 - H^2r^2)\, dt^2 + (1 - H^2r^2)^{-1}\, dr^2 + r^2\, d\Omega^2$$

the modes $u, v \sim (1 - H^2r^2)^{-1}$ grow large on the horizon

- Corresponding stress tensor perturbation

$$\delta \langle T^a{}_b \rangle \sim H^4 (1 - H^2r^2)^{-2}\, \mathrm{diag}\, (-3, 1, 1, 1)$$

diverges on the horizon and corresponds to a fluctuation in the temperature away from its Hawking-deS value $H/2\pi$

<TJJ> Triangle Amplitude in QED

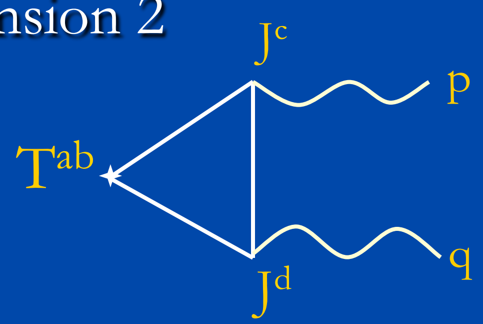
QED in an External EM Field A

M. Giannotti & E. M., *Phys. Rev. D* **79, 045014 (2009)**

$$\Gamma^{abcd}(p,q) \equiv \int d^4x \int d^4y\, e^{ip\cdot x + iq\cdot y} \frac{\delta^2 \langle T^{ab}(0)\rangle_A}{\delta A_c(x)\,\delta A_d(y)}\Big|_{A=0}$$

Mass Dimension 2

$k = p + q$



- By Lorentz invariance, can be expanded in a complete set of 13 tensors $t_i^{abcd}(p,q)$, $i = 1, \dots 13$:

$$\Gamma^{abcd}(p,q) = \Sigma_i F_i\, t_i^{abcd}(p,q)$$

- By current conservation: $p_c t_i^{abcd}(p,q) = 0 = q_d t_i^{abcd}(p,q)$
- All (but one) of these 13 tensors are dimension 4, so $\dim(F_i) = -2$ and F_i are UV Convergent

<TJJ> Triangle Amplitude in QED

Ward Identities

3. By stress tensor conservation Ward Identity: $\partial_b \langle T^{ab} \rangle_A = eF^{ab} \langle J_b \rangle_A$

$$k_b \Gamma^{abcd}(p,q) = (g^{ac} p_b - \delta^c_b p^a)\Pi^{bd}(q) + (g^{ad} q_b - \delta^d_b q^a)\Pi^{bc}(p)$$

4. Bose exchange symmetry: $\Gamma^{abcd}(p,q) = \Gamma^{abdc}(q,p)$

Finally all 13 scalar functions $F_i(k^2; p^2, q^2)$ can be found in terms of

finite (IR) Feynman parameter integrals and the polarization,

$$\Pi^{ab}(p) = (p^2 g^{ab} - p^a p^b)\,\Pi(p^2)$$

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\cdot q - q^c p^d)\, F_1(k^2; p^2, q^2) + \ldots$$

(12 other terms, 11 traceless, and 1 with zero trace when m=0)

Result:

$$F_1(k^2;p^2,q^2) = \frac{e^2}{18\pi^2 k^2}\left\{1 - 3m^2 \int_0^1 dx \int_0^{1-x} dy\, \frac{(1-4xy)}{D}\right\}$$

with $D = (p^2 x + q^2 y)(1-x-y) + xy\, k^2 + m^2$

UV Regularization Independent

Effective Action of 4D Gravity

By inverting the eq. for σ the local W-Z action may be expressed as a difference of fully covariant but non-local actions,

$$\Gamma_{WZ}[\bar{g};\sigma] = S_{anom}[g] - S_{anom}[\bar{g}] \quad \text{and}$$

$$S_{eff}[g] = \frac{1}{16\pi G}\int d^4x\sqrt{g}\,(R - 2\Lambda) + S_{anom}[g]$$

Consequences of Conformal Anomaly

- S_{anom} determined by general principles of covariance and QM, independently of any Planck scale physics.
- Additional term is relevant at large distances (scales as a marginally relevant operator under $\sigma \to \sigma + \sigma_0$).
- Conformal factor, constrained in classical Einstein theory, contains new dynamical degrees of freedom.
- New conformally invariant phase of gravity in 4D
- Running of G^{-1} and Λ to zero in this new phase.
- Possible imprint on CMBR Spectrum and Statistics.

I. Antoniadis, P. O. Mazur, E. M., Phys. Rev. D 55 (1997) 4756, 4770;
Phys. Rev. Lett. 79 (1997) 14

Dynamical Vacuum Energy

- Conformal part of the metric, $g_{ab} = e^{2\sigma}\, \bar{g}_{ab}$ constrained --frozen--by trace of Einstein's eq. $R=4\Lambda$ becomes dynamical and can fluctuate due to φ
- Fluctuations of $\varphi \sim 2\sigma$ describe a conformally invariant phase of gravity in 4D
- In this conformal phase G^{-1} and Λ flow to zero fixed point
- The Quantum Phase Transition to this phase characterized by the 'melting' of the scalar condensate Λ
- Λ a dynamical state dependent condensate generated by SSB of global Conformal Invariance

I. Antoniadis, E. M., Phys. Rev. D45 (1992) 2013;
I. Antoniadis, P. O. Mazur, E. M., Phys. Rev. D 55 (1997) 4756, 4770;
Phys. Rev. Lett. 79 (1997) 14; Phys. Lett. B444 (1998), 284; N. Jour. Phys. 9, 11 (2007)

Instability of de Sitter O(4,1) CTBD State

- Consider general O(4) invariant state

$$y_k(u) = A_k\, v_{k\gamma}(u) + B_k\, v^*_{k\gamma}(u)$$

- Energy Density of this state compared to CTBD

$$\varepsilon_{max} = \frac{1}{2\pi^2 a^4}\sum_{k=1}^{\infty} k^3\left[N_k + |B_k|^2(1+2N_k)\right] \simeq \frac{H^4}{8\pi^2}K^4\left[N_K + |B_K|^2(1+2N_K)\right]$$

- Starts infinitely small, blueshifted to large values for large **K**
- Becomes of order of the Background Energy Density at

$$8\pi G\varepsilon \gtrsim \Lambda = 3H^2 \Rightarrow \frac{GH^2}{3\pi}\left[N_K + |B_K|^2(1+2N_K)\right]\left(\frac{K}{\cosh u}\right)^4 \gtrsim 1$$

- O(4,1) de Sitter invariance is broken
- Quantum Backreaction must be taken into account
- Physical Momentum Scale $\boxed{\dfrac{K}{a} \sim \sqrt{HM_{Pl}} \ll M_{Pl}}$
- Interesting interplay of IR and UV physics ➔ Anomaly

Quantum Effects in de Sitter Space

- Particle Creation & Backreaction $\frac{dH}{dt} = -\frac{4\pi G}{c^2}(\rho + p)$

  Compare to 'Cosmological Electric Field Problem'

  'Shorting' the vacuum $\frac{dE}{dt} = -j$

- Hawking Temperature Instability

  Compare to Schwarzschild Black Hole $T_H = \frac{\mathrm{h}H}{2\pi k_B} \propto \left(\frac{c^5}{2GH}\right)^{-1} = E_H^{-1}$

  Negative Heat Capacity

- Graviton Propagator behaves logarithmically

  No Cluster Decomposition, S-Matrix

- Non-trivial Infrared Properties
- Infrared Relevant Operator Missing in Einstein Theory?

Cosmological Constant Electric Field Problem

- Sourcefree Maxwell's Eqs. admit a solution of a constant, uniform Electric Field
- All electric fields in Nature are associated with localized sources
- Why do we not observe some very large ***E*** in an arbitrary direction

$$\frac{\partial \vec{E}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{E} = E\,\hat{z}$$

Answer: 'Vacuum' in electric field is <u>Unstable</u> to Particle Creation

Trace Anomaly as Source of CMB

$$\langle T_a^{\ a} \rangle = b\, F_4 + b' \left(E_4 - \tfrac{2}{3}\, \Box R\right) + b''\, \Box R$$

$\langle T_{ab} \rangle$ is the Stress Tensor of Conformal Matter

- Trace $\langle T_a^{\ a} \rangle$ is a sum of Geometric Invariants

$$E_4 = R_{abcd}^{\ \ 2} - 4R_{ab}^{\ \ 2} + R^2 \qquad F_4 = R_{abcd}^{\ \ 2} - 2R_{ab}^{\ \ 2} + \tfrac{2}{3} R^2$$

- No local effective action in terms of curvature tensor

  But there exists a non-local effective action
  which can be rendered local in terms of
  new scalar degree(s) of freedom

- Macroscopic Quantum Modification of Gravity
- Anomaly Scalar Fluctuations can be source of CMB

Cosmological Scalar Fluctuations from the Conformal Anomaly

- Inhomogeneous solutions are coherent state variations
- Can occur at any scale k but couple at horizon
- These are effective quantum scalar degrees of freedom in cosmology
- No inflaton or fine tuning of its potential
- In static de Sitter coordinates

  $ds^2 = -(1 - H^2r^2)\, dt^2 + (1 - H^2r^2)^{-1}\, dr^2 + r^2\, d\Omega^2$

  the φ modes $\sim \ln(1 - H^2r^2)$ are large on the horizon
- Corresponding stress tensor perturbation

  $$\langle T^a{}_b \rangle \sim H^4\, (1 - H^2r^2)^{-2}\, \mathrm{diag}\, (-3, 1, 1, 1)$$

  diverges on the horizon, a coherent fluctuation in the temperature away from its Hawking-deS value $H/2\pi$

Linear Response in de Sitter space

- For small fluctuations around de Sitter space $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ the linearized Einstein eqs.

$$\boxed{\delta \left(R_{\mu\nu} - \frac{R}{2}\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} \right) = 8\pi G\ \delta \langle T_{\mu\nu} \rangle}$$

can be solved in terms of two scalar potentials $\Upsilon_{\mathcal{A}}, \Upsilon_{\mathcal{C}}$

$$\bar{g}_{\tau\tau} + h_{\tau\tau} = -(1 + 2\Upsilon_{\mathcal{A}})$$

$$\bar{g}_{ij} + h_{ij} = a^2(\tau)\,(1 + 2\Upsilon_{\mathcal{C}})\,\delta_{ij} + h^{\perp}_{ij}$$

the difference of potentials has a soln. in static coordinates

$$\Upsilon_{\mathcal{C}} - \Upsilon_{\mathcal{A}} = 8\pi G H^2 b' \left[\frac{c_1}{Hr} \ln \left(\frac{1 - Hr}{1 + Hr} \right) + \frac{c_2}{Hr} \ln f \right]$$

- Logarithmic behavior as $r \to r_H$ indicates a weight *w=0* conformal field just as required for the scale invariant HZ Spectrum

Latest Report from the Front of 'Black Hole Wars'

~60 'Firewall' papers in last year arguing about **mutual inconsistency** of:

- Hawking radiation is in a pure state (QM: unitarity)
- information carried by radiation in low-energy EFT
- Nothing happens at the horizon to infalling observer

"Proof" by contradiction, many assumptions but still doesn't tell you what actually happens c.f. (Aug. '13) http://online.kitp.ucsb.edu/online/fuzzorfire_m13/

NATURE | NEWS FEATURE

Astrophysics: Fire in the hole!

Will an astronaut who falls into a black hole be crushed or burned to a crisp?

Zeeya Merali

03 April 2013 Corrected: 05 April 2013

ANDY POTTS

Crisis in Foundations of Physics ?!

Desperate conditions
demand desperate
measures ?!

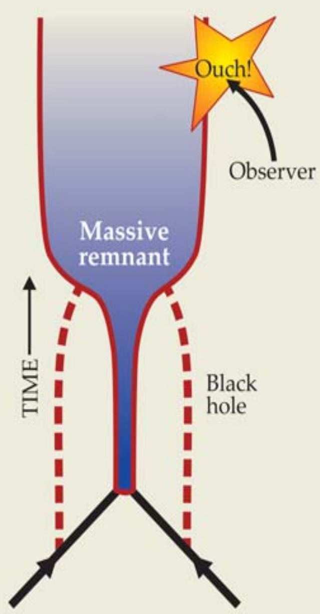
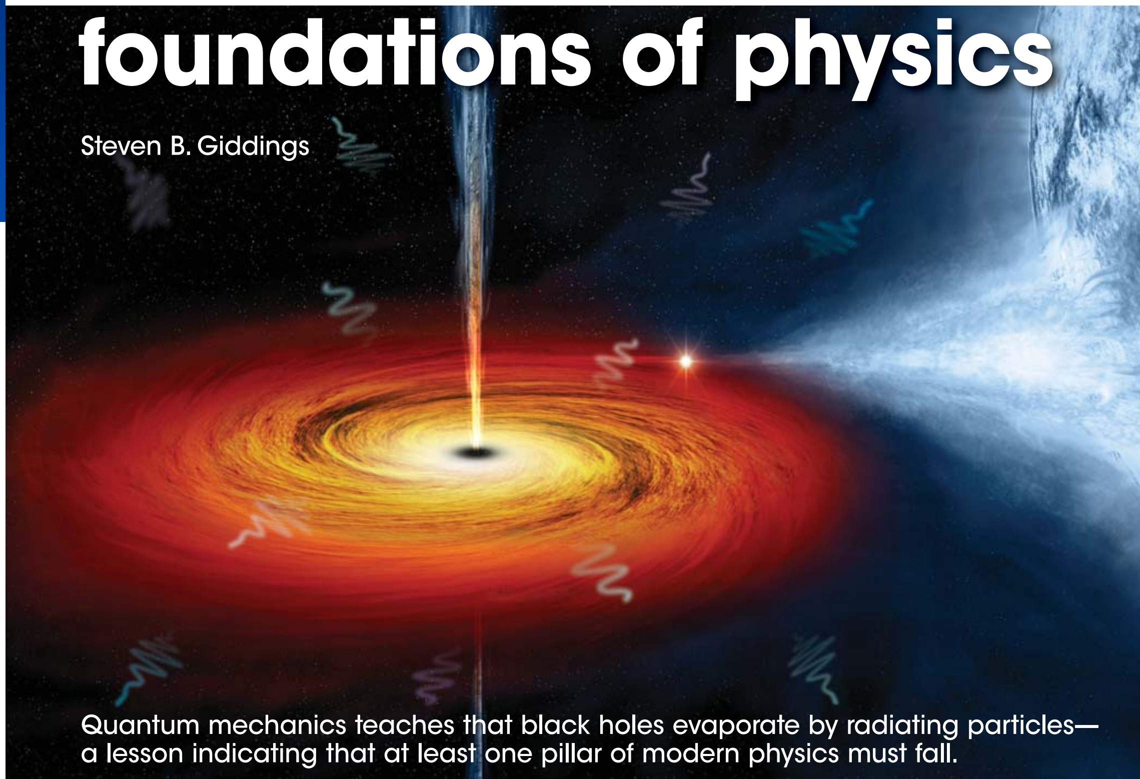


Figure 4. Massive-remnant scenarios are nonlocal. In these models, a black hole transitions to a massive object whose surface lies outside, or possibly at, the location of what would be the horizon (dashed lines on either side of the origin). In this illustration, the black hole is formed from the collision of two particles (black lines). To reach the horizon, the surface must propagate faster than the speed of light, which violates the locality of quantum field theory. An infalling observer encounters the remnant surface at a high velocity—compare falling into a neutron star—and, barring a miracle, experiences strong disruption. Variants of this general scenario include so-called fuzzballs and firewalls.

Black holes, quantum information, and the foundations of physics



Faster than Light Propagation ?