Non-Singular Models of Black Holes

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Based on:

"Spherical collapse of small masses in the ghost-free gravity

V.F, A. Zelnikov, T. Netto, e-Print: arXiv:1504.00412 (2015);

(to appear in JHEP)

"Mass-gap for black hole formation in higher derivative and

ghost free gravity", V. F. ,arXiv:1505.00492 (2015);

"Information loss problem and a 'black hole` model with a

closed apparent horizon", V.F., JHEP 1405 (2014) 049,

arXiv:1402.5446

Outline of the talk:

1. Brief Introduction
2. Higher Derivative (HD) and Ghost Free (GH) Gravity
3. Weak Gravity: Gravitational Field of a Point Mass
4. HD and GF Gyratons
5. Null Shell Collapse in HD and GH Gravity
6. Mass Gap for Mini-BH formation
7. Strong Gravity: Models with Closed Apparent Horizon
8. New Universe Formation inside a Black Hole?
9. Summary and Discussions

(Remark: Everything in four dimensions, however 3 and 4 have been done for arbitrary D)

1.Brief Introduction

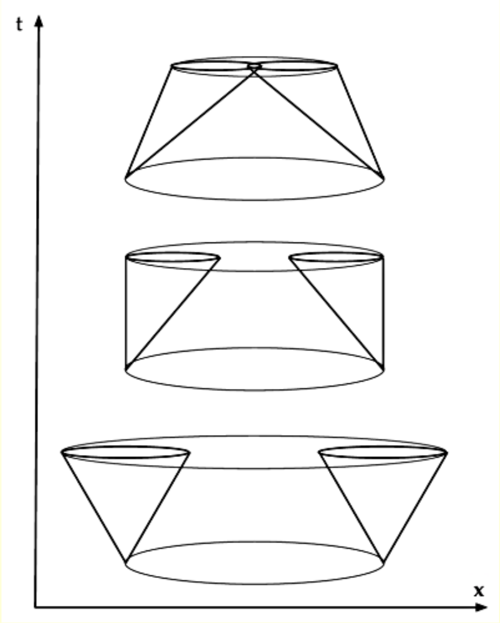
Black hole is a spacetime domain from where no information carrying signals can escape to infinity. The black hole boundary is an event horizon.

Can we prove that an object in the center of our Galaxy is a black hole (according to this definition) ? Yes, only if you expect to live forever.

This definition is very useful for proof of theorems, but certainly is not very practical.

Event horizon vs. apparent horizon

`Quasi-local definition' of BH: Apparent horizon



A compact smooth surface *B* is called a trapped surface if both, in- and out-going null surfaces, orthogonal to *B*, are non-expanding .

A trapped region is a region inside *B*.

A boundary of all trapped regions is called an apparent horizon.

Null energy condition: $T_{\mu\nu} l^{\mu} l^{\mu} \geq 0$

Trapped surface + NEC

=Event horizon existence

According to GR: Singularity exists inside a black hole.

Theorems on singularities: Penrose and Hawking.

Penrose theorem: Assume

1. The null energy condition holds $T_{\mu\nu} l^{\mu} l^{\nu} \geq 0$;
2. There exists a noncompact connected Cauchy surface.
3. There exist a closed trapped null surface .

Then, we either have null geodesic incompleteness, or closed timelike curves.

Schwarzschild ST has a spacelike singularity.

RN and Kerr ST have a timelike singularity.

In both cases this is a curvature singularity.

Expectation 1: When curvature becomes high (e.g. reaches the Planckian value) the classical GR must be modified (quantum corrections, it is an emergent theory, etc.).

Expectation 2: Singularities of GR would be resolved.

Regularity at r=0 and AH

$$ds^2 = -F(t,r)\,dt^2 + \frac{dr^2}{g(t,r)} + r^2 d\omega^2,$$

$$F(t,r) \sim F_0(t) + F_1(t)\,r^2,\ g(t,r) \sim g_0(t) + g_1(t)\,r^2,$$

$$\Re^2 \sim \frac{4(g_0 - 1)^2}{r^4} = 4\left(\frac{[(\nabla r)^2 - 1]}{r^2}\right)^2.$$

Apparent horizon: g=$(\nabla r)^2 = 0$. If an AH crosses $r = 0,$ then before this the curvature singularity is developed at $r = 0$.

Schwarzschild metric: $\varphi = -GM/r.$

$$ds^2 = -Fdt^2 + \frac{dr^2}{F} + r^2 d\omega^2,\ F = 1 + 2\varphi.$$

Apparent (event) horizon at $F = 0,\ r = 2GM.$

Kretschmann scalar $\mathbb{R}^2 = \dfrac{48\,(GM)^2}{r^6}.$

Linearized version

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\varphi)(dr^2 + r^2 d\omega^2).$$

Three connected problems:

1. Regularity of potential φ at $r=0$;

2. Finiteness of the self-energy of a point charge;

3. Existence of AH: $|\varphi| \leq CM$. For $M < C/2,\ F > 0$.

Regularization:

$$\Delta\varphi = 4\pi GM\delta(\vec{r}) \rightarrow \varphi = \frac{GM}{r},$$

$$(\Delta+\mu^2)\tilde{\varphi} = 4\pi GM\delta(\vec{r}) \rightarrow \tilde{\varphi} = \frac{GMe^{-\mu r}}{r},$$

$$\varphi_{reg}(r) = \varphi(r) - \tilde{\varphi}(r) = \frac{GM(1-e^{-\mu r})}{r},$$

$\varphi_{reg}(0) = GM\mu \rightarrow$ Pauli-Villars regularization

$$\Delta G = -I,\ (\Delta + \mu^2)\tilde{G} = -I,$$

$$G_{reg} = G - \tilde{G} = \frac{1}{\Delta + \mu^2} - \frac{1}{\Delta} = -\frac{1}{\Delta(1 + \Delta / \mu^2)};$$

$$\Delta(1 + \Delta / \mu^2)\varphi_{reg} = 4\pi GM\delta(\vec{r}) \to$$

Higher-derivative theory.

Source-smearing vs non-locality:

$$\Delta\varphi_{reg} = 4\pi G\tilde{\rho},\ \tilde{\rho} = (1 + \Delta / \mu^2)^{-1}\rho = Me^{-\mu r} / r.$$

2.Higher Derivative and Ghost Free Gravity

Quadratic in Curvature Action

$$S = \int dx \sqrt{-g}\left[R/2 + R_{\bullet} \hat{O}^{\bullet\bullet} R_{\bullet} \right],$$

$\hat{O}^{\bullet\bullet}$ is an operator constructed from ∇ and g.

Biswas, Gerwick, Koivisto, Mazumdar (2012):

The number of arbitrary functions of $\square$ operator (after using the Bianchi identities) is 6.

For metric perturbations over the flat background only 2 arbitrary functions survive.

$$S=-\int d^4x\left[\frac{1}{2}h_{\mu\nu}a(\Box)h^{\mu\nu}+h_\mu^{\ \sigma}b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu}\right.$$

$$+hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu}+\frac{1}{2}hd(\Box)h$$

$$\left.+h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\right].$$

$$a+b=0,c+d=0,b+c+f=0.$$

IR GR limit: $a(0)=c(0)=-b(0)=-d(0)=1$

1. General Relativity (GR): $L = R,\ a = c = 1;$

2. $L(R)$ gravity: $L(R) = L(0) + L'(0)R + 1/2L''(0)R^2 + \ldots;$

$a = 1,\ c = 1 - L''(0);$

3. Weyl gravity: $L = R - \mu^{-2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta},\quad a = 1 - \mu^{-2}\Box,$

$c = 1 - \frac{1}{3}\mu^{-2}\Box\ ;$

4. Higher derivative (HD) gravity: $a = \prod_{i=1}^{n} (1 - \mu_i^{-2}\Box)\ ,$

$c = \prod_{k=1}^{n_c} (1 - \nu_k^{-2}\Box).$

5. Ghost free (GF) gravity: $a = c = \exp(-\Box/\mu^2).$

3.Weak Gravity: Gravitational Field of a Point Mass

Static solutions of linearized gravity equations in the Newtonian limit

Stress-energy tensor: $\tau_{\mu\nu} = \rho(\vec{r})\delta_\mu^0\delta_\nu^0$

$$ds^2 = -(1+2\varphi)dt^2 + (1-2\psi+2\varphi)d\ell^2 .$$

Biswas, Gerwick, Koivisto, Mazumdar (2012):

$$a(\Delta)\triangle\psi = 8\pi G\rho ,$$

$$(a(\Delta)-3c(\Delta))(\Delta\varphi - 2\Delta\psi) = 8\pi G\rho$$

Modesto, Netto, Shapiro (2014)

For a point mass $\rho = m\delta(\vec{r})$ the solution is spherically symmetric. We call it finite if near $r = 0$ it is of the form

$$\psi(r) \sim \psi_0 + \psi_1 r + \frac{1}{2}\psi_2 r^2 + O(r^3),$$

$$\varphi(r) \sim \varphi_0 + \varphi_1 r + \frac{1}{2}\varphi_2 r^2 + O(r^3).$$

A finite solution is not necessary regular one.

$$R^2 = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{A_2}{r^2} + \frac{A_1}{r} + O(1),$$

$$A_2 = 8(4\psi_1^2 - 5\psi_1\varphi_1 + 3\varphi_1^2),$$

$$A_1 = 16[\psi_1(5\psi_2 - 4\varphi_2) - 4\varphi_1(\psi_2 - \varphi_2)].$$

The solution is regular if $\psi_1 = \varphi_1 = 0.$

The solution is ψ-regular if $\psi_1 = 0.$

If $a = c,\ \varphi = \psi$, and ψ-regular is regular

The gravitational collapse is regular in linearized regular HD and in GF theories of gravity.

$\Rightarrow$ Mass gap for mini black hole production.

$$\hat{O} = a(\Delta)\Delta,$$

$$Q(\xi) = \hat{O}^{-1}(\Delta = -\xi) = -[\xi a(-\xi)]^{-1},$$

$$Q(\xi) = \int_0^\infty ds\; f(s) e^{-s\xi},$$

$$f(s) = \frac{1}{2\pi i}\int_{\alpha - i\infty}^{\alpha + i\infty} d\xi\, Q(\xi) e^{s\xi}$$

$$\boxed{Q(\xi) \Leftrightarrow f(s)}$$

Q-image and f-image of the field equation.

Green function: $\hat{O}\,\hat{G} = -\hat{I}\,.$

$$\hat{G} = -\hat{O}^{-1} = -\int_0^\infty ds\; f(s)\, e^{s\Delta}\,,$$

Heat kernel:

$$< x' \,|\, e^{s\Delta} \,|\, x >= K(|\, x - x' \,|; s) = \frac{e^{-|x-x'|^2/(4s)}}{(4\pi s)^{3/2}}$$

$$\boxed{\psi(r) = 8\pi G m \int_0^\infty ds\; f(s)\, K(r;s),}$$

$$= \frac{Gm}{\pi i r}\int_{\alpha-i\infty}^{\alpha+i\infty} d\xi\, Q(\xi)\, e^{-\sqrt{-\xi}\,r}$$

HD gravity: $Q(\xi) = -[\xi \prod_{i=1}^{n} (1+\xi/\mu_i^2)]^{-1}$,

The Heaviside expansion theorem:

$$f(s) = -(1 - \sum_{i=1}^{n} P_i^{-1} e^{-\mu_i s}), \quad P_i = \prod_{j=1,\, j\neq i}^{n} (1 - \mu_j^2/\mu_i^2).$$

$$\boxed{\psi(r) = -2Gmr^{-1}(1 - \sum_{i=1}^{n} P_i^{-1} e^{-\mu_i r}).}$$

General Relativity: $f(s) = 1, \ \psi(r) = 2\varphi(r) = -2Gm/r$.

Solution near $r=0$: $\sum_{i=1}^{n} P_i^{-1} = 1.$

$$\psi_0 = -2GmS_1\,,\;\; \psi_1 = GmS_2\,,\;\; S_k = \sum_{i=1}^{n} \mu_i^k P_i^{-1}.$$

$\boxed{\text{The solution is } \psi\text{-regular if } S_2 = 0.}$

For the GF gravity $f(s) = -\vartheta(s-\mu^{-2})$,

$$\boxed{\psi(r) = 2\varphi(r) = -2Gm\,\mathrm{erf}\,(\mu r/2)/r.}$$

The solution is regular at $r=0$.

4.HD and GF Gyratons

We obtain now a solution for an ultra-relativistic particle.

In $4D$ Einstein gravity -- Aichelburg-Sexl solution.

Higher dimensional generalization for a particle with spin -- gyraton metric [V.F., Fursaev, PRD 71 104034 (2005); V.F., Israel, Zelnikov, PRD 72, 084031]

$$ds^2 = ds_0^2 + dh^2,$$

$$ds_0^2 = -dt^2 + d\ell^2, \quad d\ell^2 = dy^2 + d\zeta_\perp^2.$$

Boost transformation:

$$t = \lambda_- v + \lambda_+ u, \; y = \lambda_- v - \lambda_+ u,$$

$$\lambda_\pm = (1 \pm \beta)\gamma/2, \;\; \gamma = (1-\beta^2)^{-1/2}.$$

In the limit $\gamma \to \infty$ one has

$$y \sim -\gamma u,\ t \sim \gamma u,\ \ell^2 \sim \gamma^2 u^2 + \zeta_\perp^2,$$

$$ds^2 = -du dv + d\zeta_\perp^2 + dh^2,$$

$$dh^2 = \Phi du^2, \quad \Phi = -2 \lim_{\gamma\to\infty} (\gamma^2 \psi).$$

Penrose limit: $M = \gamma m = \mathrm{const};$

$$\lim_{\gamma\to\infty} \gamma \exp(-\gamma^2 u^2/(4s)) = \sqrt{4\pi s}\,\delta(u).$$

$$\psi(r) = 8\pi Gm \int_0^\infty ds\ f(s)\, K(r;s),$$

$$K(|x-x'|;s) = \frac{e^{-|x-x'|^2/(4s)}}{(4\pi s)^{3/2}}$$

$$\boxed{\begin{aligned} &\Phi = -4GMF(\zeta_\perp^2)\delta(u), \\ &F(\zeta_\perp^2) = \int_0^\infty \frac{ds}{s}\, f(s)\, e^{-\zeta_\perp^2/(4s)}. \end{aligned}}$$

For GR, as well as for GB and $L(R)$ gravity:

$F(z) = \ln(z/\eta^2)$, η is IR cut-off parameter.

For HD gravity:

$$\boxed{F(z) = \ln(z/\eta^2) + 2\sum_{i=1}^{n} P_i^{-1} K_0(\mu_i \sqrt{z}),}$$

$$F(z) \sim C - \frac{1}{4} S_2 z(\ln z - 2c) - \frac{1}{4} Sz + O(z^2),$$

$$c = 1 + \ln 2 - \gamma,\ S = \sum\nolimits_{i=1}^{n} \mu_i^2 \ln(\mu_i^2) P_i^{-1}.$$

For ψ-regular theory $S_2 = 0.$

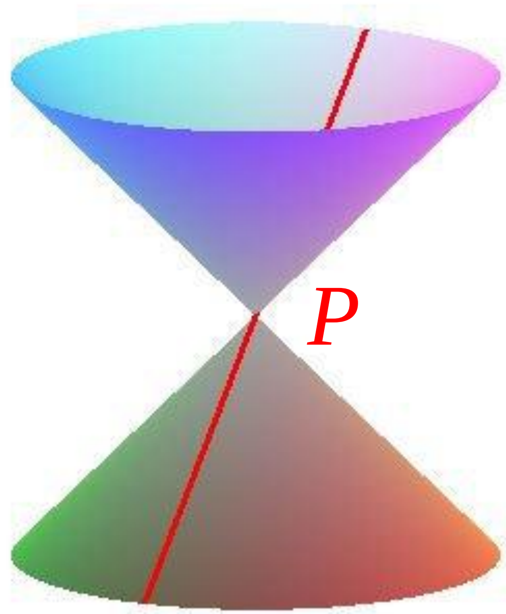
For ghost free gravity

$$\boxed{F(z) = \ln z + \gamma + \mathrm{Ei}(1, z)}$$

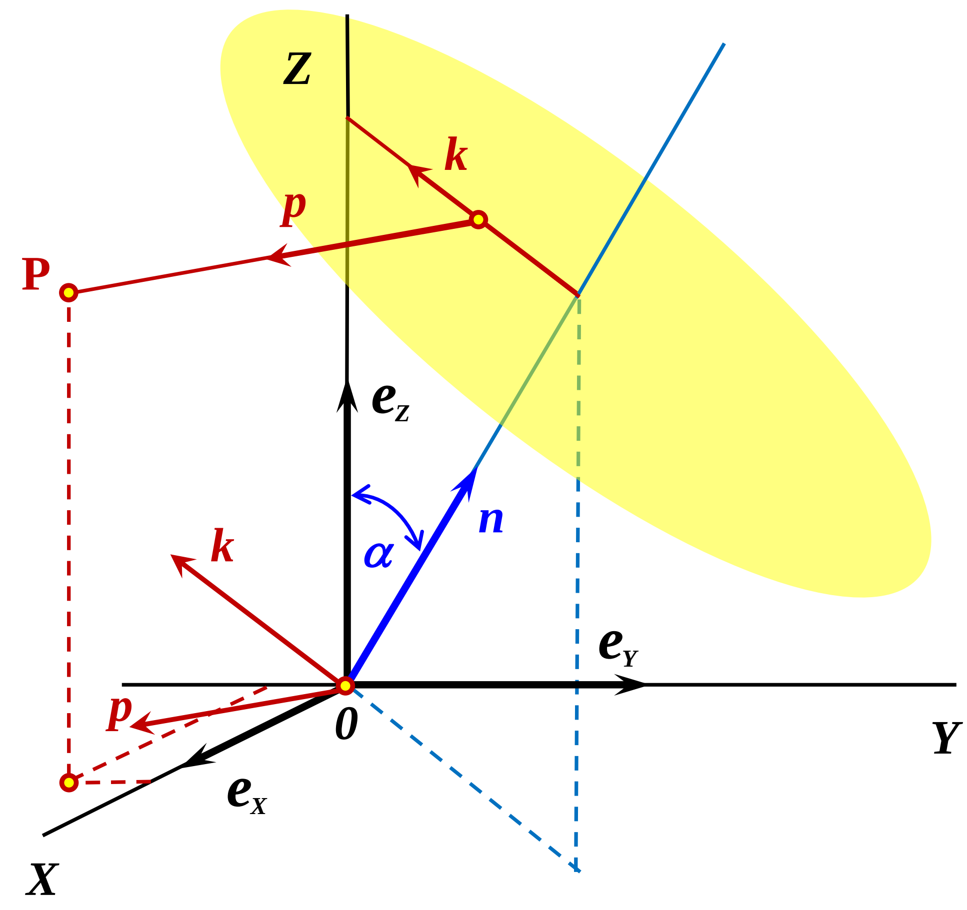
$$\sim z - \frac{1}{4} z^2 + O(z^3)$$

5.Null Shell Collapse in HD and GH Gravity

Metric for Thin Null Shell



We consider a set of gyratons, passing through a point P in Minkowski spacetime. They form a null cone with the vertex at P. A section $t = \text{const}$ of the cone is a sphere. We take a continuous limit of this destibution, assuming that the mass density per a unit solid angle is constant $M/4\pi$.



Gyraton frame vs Minkowski frame

The result of averaging:

$$< dh^2 >= \frac{-2MF(r^2-t^2)}{r}\left[\left(dt-\frac{t}{r}dr\right)^2+\frac{r^2-t^2}{2}d\omega^2\right].$$

$F=0$ inside the null cones.

For $F=$ const the metric $ds^2 = ds_0^2 + < dh^2 >$ is flat.

$< dh^2 >$ can be "gauged away" .

For $F(z)=\ln z$, $ds^2 = ds_0^2 + \frac{2M}{r}(dt^2+dr^2)$

6.Mass Gap for Mini BH formation

Apparent Horizon

$$g = (\nabla\rho)^2, \rho^2 \equiv g_{\theta\theta} = r^2 - \frac{GM}{r} zF(z),$$

$$\boxed{g = 1 - 2GMr^{-1}q(z), \quad q(z) = zF'(z).}$$

For GR (as well as for GB and $L(R)$-gravity) $q(z) = 1$, so that an apparent horizon exists for any value of M.

For HD gravity $q(z) = 1 - \sqrt{z}\sum_{i=1}^{n} \mu_i P_i^{-1} K_1(\mu_i \sqrt{z}),$

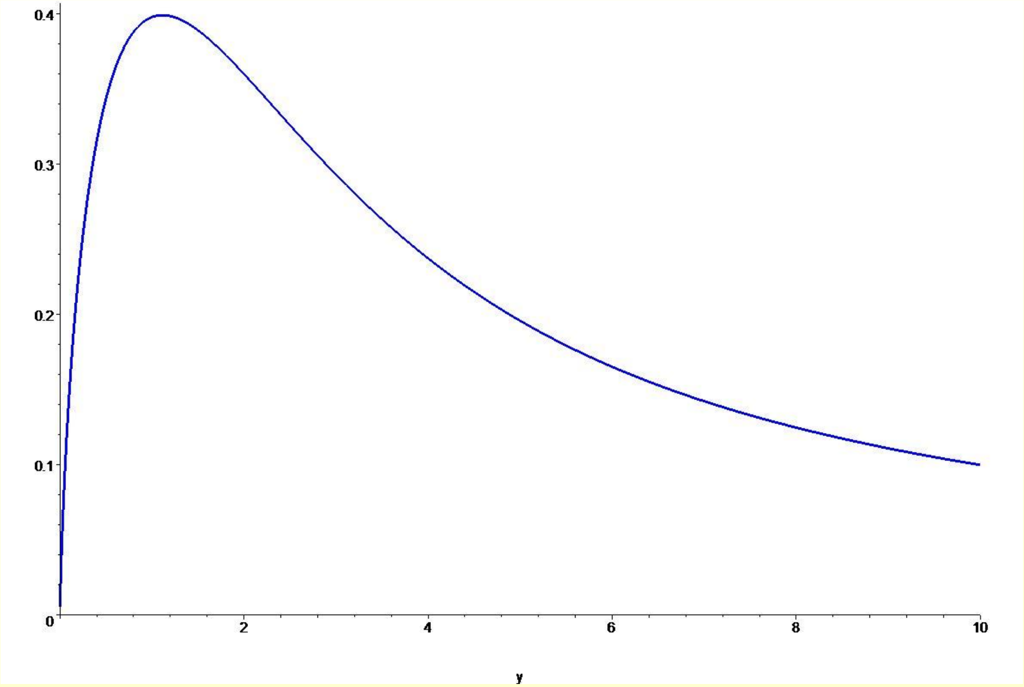
$$q(z) = -\frac{1}{4} S_2 z(\ln z - 2c + 1) - \frac{1}{4} Sz + O(z^2)$$

$$S = \sum_{i=1}^{n} \mu_i^2 \ln(\mu_i^2) P_i^{-1}.$$

Outside the null shell $|t|/r \le 1, t = \pm\sqrt{1-\beta^2}\,r,$

$$q(z)/r = \beta \sum_{i=1}^{n} \mu_i P_i^{-1} Z(y_i), \quad y_i = \beta \mu_i r,$$

$$|q(z)|/r < 0.4 \sum_{i=1}^{n} \mu_i |P_i|^{-1}$$



$$Z(y) = \frac{1}{y} - K_1(y).$$

$Z(y)$ is positive,

$Z_{\max} = 0.399$ at $y = 1.114$

In the "regular" HD theory and GF gravity,

if $GM\mu < 1$, then there is no apparent horizon.

Curvature Invariant

$$R^2 = \frac{48G^2M^2}{r^6} F\,, F = 2z^2 q'^2 - 2zqq' + q^2,$$

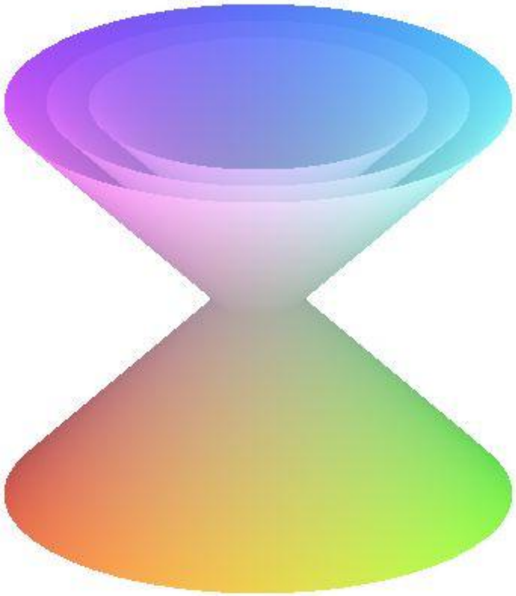
$$F \sim \frac{1}{16} z^2 [(w^2 + 4w + 5)S_2^2 + 2(w+2)SS_2 + S^2],$$

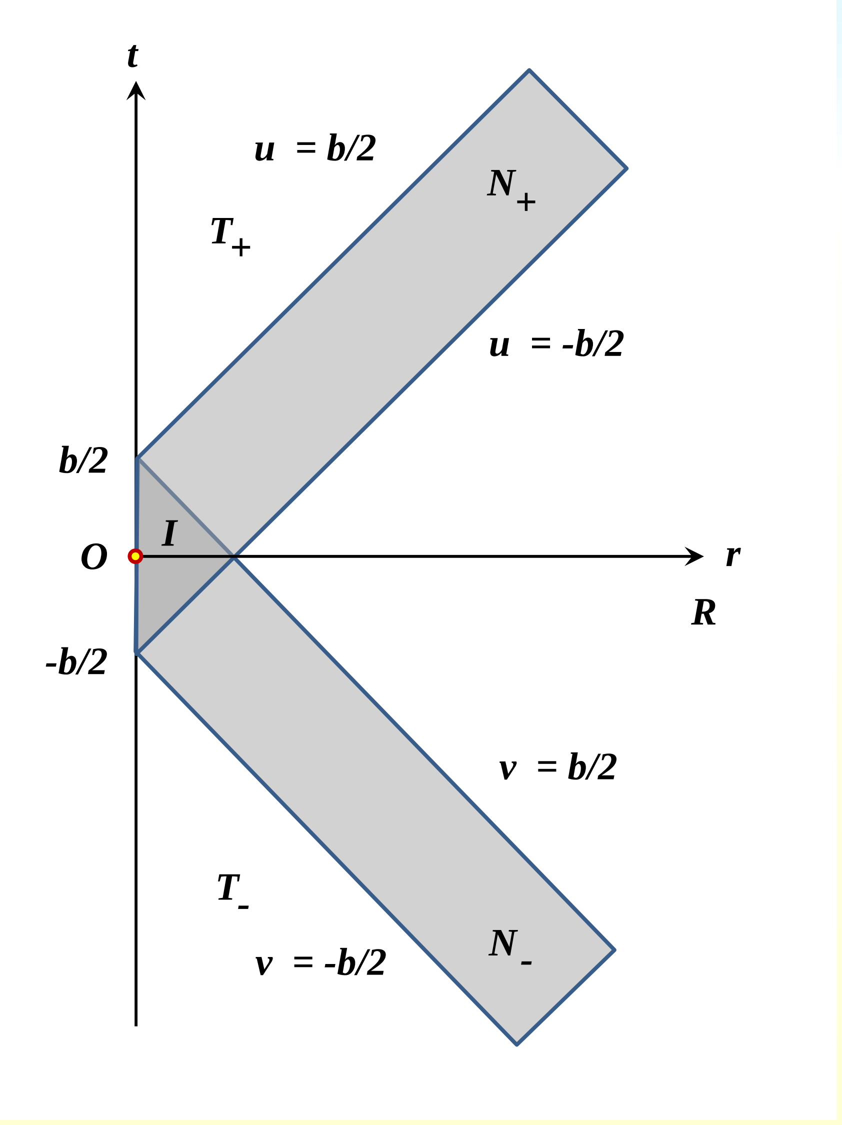
$$w = \ln z - 2c.$$

The Kretschmann curvature vanishes on the null shells.
However, in a general case it is divergent at $r = 0$.

HD and GF theories have the mass scale parameter μ. "Physical" null shell should be constructed from flields. One can expect that the thickness of the shell should be larger than $\lambda = \mu^{-1}$. Let us show that this makes curvature finite.

Metric for Thick Null Shell





t
u = b/2
N+
T+
u = -b/2
b/2
I
O
r
R
-b/2
v = b/2
T-
N-
v = -b/2

Additional averaging results in the metric in I-domain:

$$<< dh^2 >> = -\frac{2GM}{br}[c_0 dt^2 + c_2 \frac{dr^2}{r^2} + \frac{1}{2}(c_0 r^2 - c_2) d\omega^2],$$

$$c_k = \int_{-r}^{r} dx x^k F(r^2 - x^2),$$

$$c_0 = -\frac{r^3}{9}[(6u - 5)S_2 + 3S], \; u = \ln r - c - \ln 2,$$

$$c_2 = -\frac{r^5}{225}[(30u - 31)S_2 + 15S].$$

$$R^2 \sim \frac{32}{27} G^2 \dot{M}^2 [(36u^2 + 5)S_2^2 + 36uS_2 S + 9S^2].$$

For HD gravity R^2 is finite for a ψ-regular theory, that is when $S_2 = 0$.

For ghost free gravity: $R^2 \sim \frac{32}{3} G^2 \dot{M}^2 \mu^4$.

In both cases for small G$M\mu$ the linear perturbarion is uniformly small and higher order corrections can be neglected. This means that a no-apparent-horizon result is robust.

Denote $\mu^{-1} = \lambda$. Then we have

$$R \sim \frac{GM}{T\lambda^2}.\ \text{For } T \geq \lambda \ \Rightarrow R \sim \frac{M}{\mu}\frac{1}{\lambda^2}.$$

If $M \ll \mu \ \Rightarrow R \ll R_{cr}$. Then one can neglect all higher in curvature corrections (weak field regime!)

For regular HD and GF theories there exists the mass gap for mini black hole formation. For small mass the (time-dependent) gravitational field of the collapsing body is regular and no apparent horizon.

7. Strong Gravity: Models with Closed Apparent Horizon

Limiting curvature conjecture: $\| \mathrm{R} \| \leq \frac{1}{\lambda^2}$

Markov, JETP Letters, 36 (1982) 266;
Ann.Phys., 155 (1984) 333;
Polchinski, Nucl.Phys. B325 (1989) 619.

Spherically symmetric ST:
An apparent horizon does not cross r=0. It is either closed or unlimited.

Main assumptions of the model

ST with a geometry $g_{\mu\nu}$ satisfying modified gravitational equations;

Limiting curvature conjecture ;

Geometry differes from the classical one in the domains, where

$r < r_0 = r_S (l_{Pl} / r_S)^{2/3}$;

Hawking radiation to the infinity is accompanied by negative energy flux through the horizon, which slowly reduces the black hole's mass;

Null fluid approximation for incoming and outgoing energy fluxes;

This massive shell approximation for the region near the horizon, where massless quanta are created.

SPHERICALLY SYMMETRIC COLLAPSE IN QUANTUM GRAVITY

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State Committee of Standards, Moscow, USSR

Received 27 July 1981

The problem of classical singularities is revised on the basis of the quantum-gravity effective equations. We find a simple rule for establishing the Birkhoff theorem in spherically symmetric problems. All exact solutions of the lagrangian with $C^2_{\alpha\beta\gamma\sigma}$ are obtained. Spherically symmetric collapse of the thin null shell of mass M is considered in the framework of a local theory describing vacuum polarization effects. The boundary-value problem is set and the asymptotic solution is obtained. It is found that the shell collapses to $r = 0$ without the rise of a singularity, and begins expanding. The global behaviour of the solution is obtained for small M. For large M it is conjectured that the event horizon does not form, and the apparent horizon is closed. An object forms, possessing the observable properties of a black hole, but living a finite time.

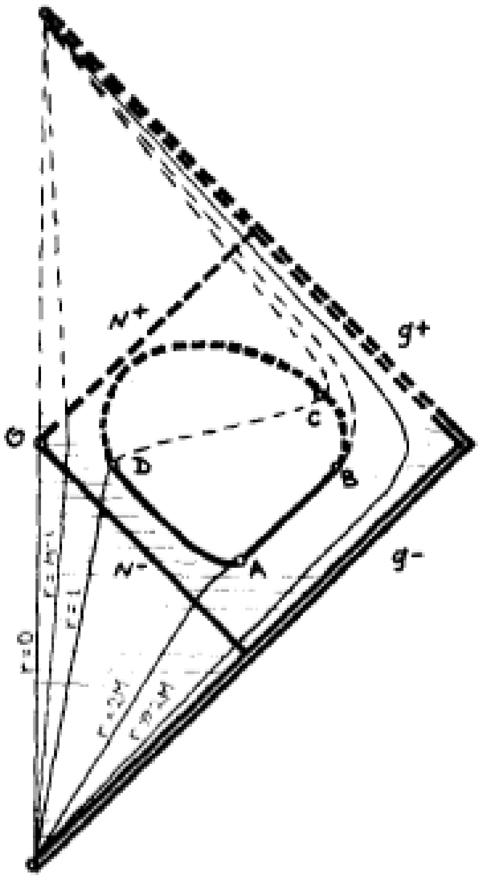


Fig. 1. Penrose diagram for the collapse of the null shell ($M \gg 1$). Solid (dashed) lines are used for the known (hypothetical) details of the picture. The shaded region is the region of validity of the obtained asymptotic solution. The line $N^- \cup N^+$ is the world line of the null shell. The closed and dashed bold line $ABCD$ is the apparent horizon. The light lines are the level lines r = const.

Other publications on regular BH models with closed apparent horizons

T. A. Roman and P.G. Bergmann (1983);
P. Bolashenko and V.F. (1986)
S. N. Solodukhin, (1999);
S. A. Hayward (2006);
S. Ansoldi (2008);
C. Bambi, D. Malafarina, L. Modesto (2013);
V. N. Lukas and V. N. Strokov (2013);
V. Frolov (2014);
J.M. Bardeen (2014);
C. Rovelli and F. Vidotto (2014);
T. De Lorenzo, C. Pacilio , C. Rovelli, S. Speziale (2015);

D.I. Kazakov, S.N. Solodukhin (1993)
P. Hajicek (2002);
D. Grumiller (2003, 2004;
J. Ziprick and G. Kunstatter (2010)

Model for Particle Creation Domain

Hawking radiation at far distance is effectively described by a properly chosen null fluid flux. The conservation law requires that this radiation is accompanied by the negative energy flux through the horizon, which we also approximate by the null fluid. In order to make a model consistent one needs to assume that between the two regions with pure outgoing and pure incoming fluxes there exists a transition region, corresponding to the domain where the particle are created.

We assume that this region is narrow and approximate it by a massive thin shell. Main conclusion: For slow change of the black hole the back-reaction of the shell is negligible.

Modified Vaidya model

[Hayward '06, Frolov '14]

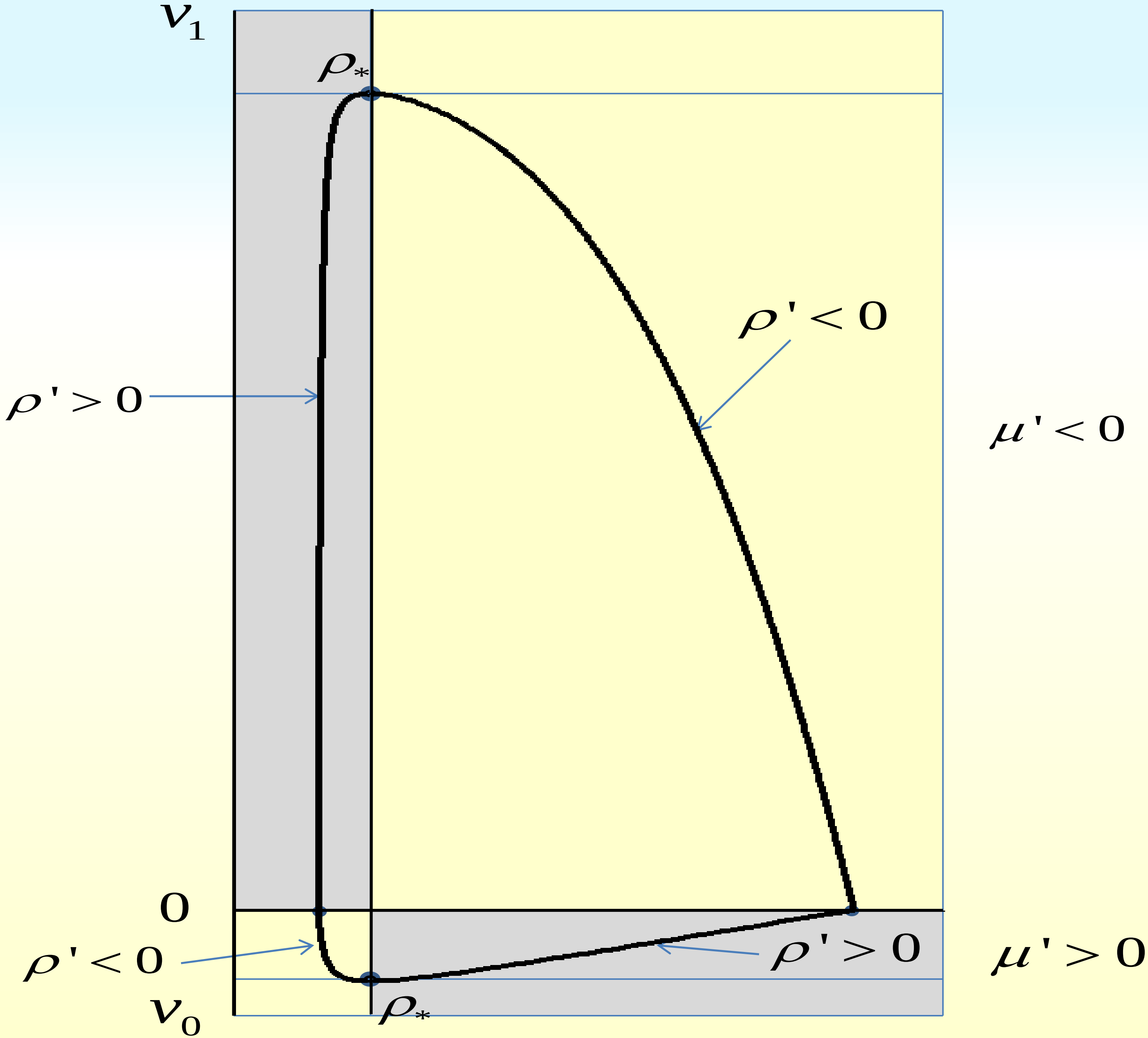
Use `Plankian scale parameter' b to transform the metric into its dimensionless form

$$dS^2 = b^2 ds^2, \quad ds^2 = -f dv^2 + 2 dv\, d\rho + \rho^2 d\omega^2,$$

$$f = 1 - \frac{2\mu(v)\rho^2}{\rho^3 + 2\mu(v)}.$$

In the limit $\rho \to 0$ one has $f \sim 1 - \rho^2$ and the $(\text{curvature})^2 \sim 1$

Apparent horizon: $(\nabla\rho) = f = 0$



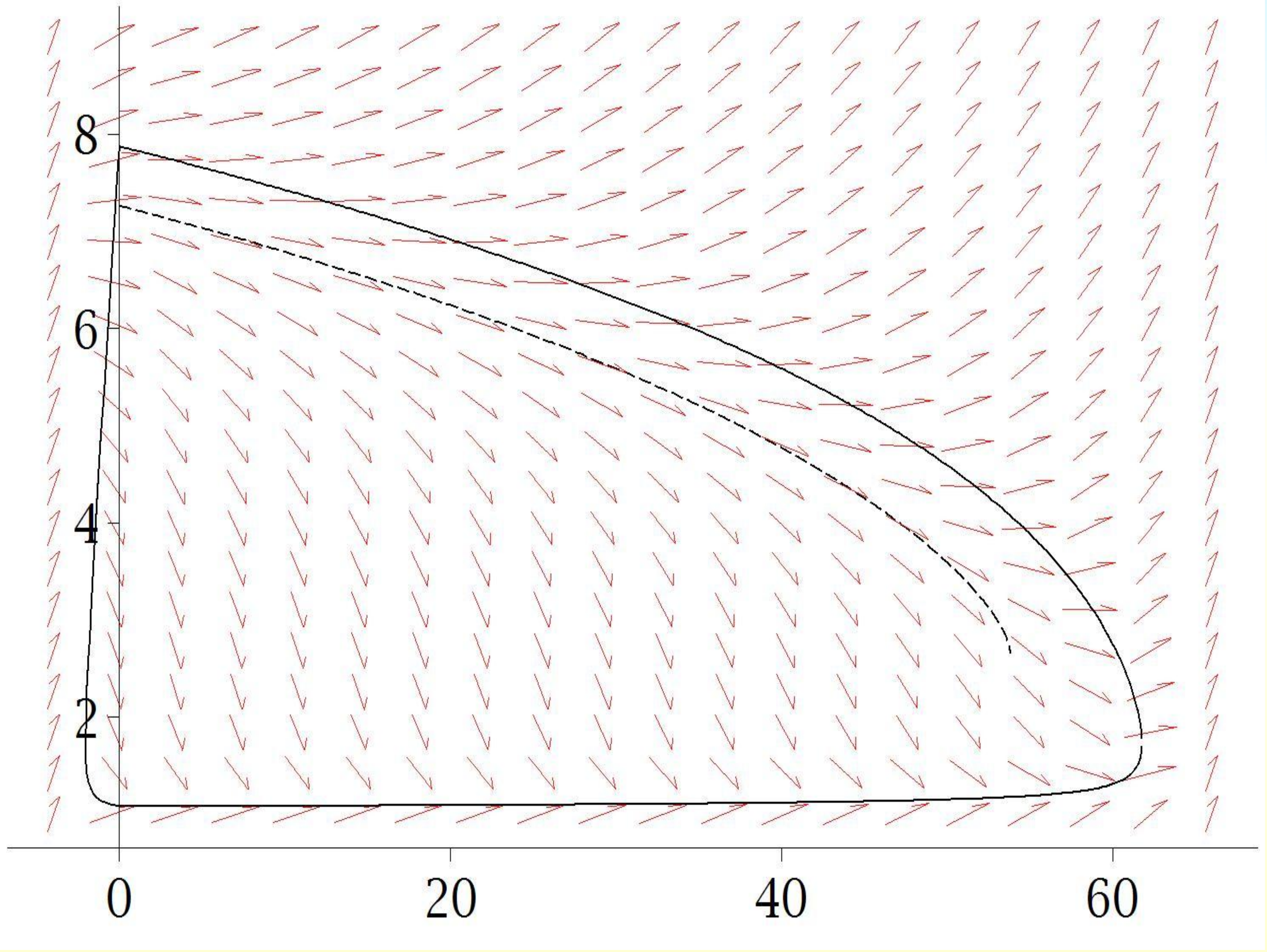
v_1
ρ_*
$\rho' < 0$
$\rho' > 0$
$\mu' < 0$
0
$\rho' < 0$
$\rho' > 0$
$\mu' > 0$
v_0
ρ_*

$\mu_* = \dfrac{3\sqrt{3}}{4}$ is the minimal mass of the black hole.

Simplest model:

$$\mu^3 = \mu_0^3 - v, \text{ for } v > 0,$$

$$\mu = \mu_0 (1 - v / v_0), \text{ for } v_0 < v < 0$$

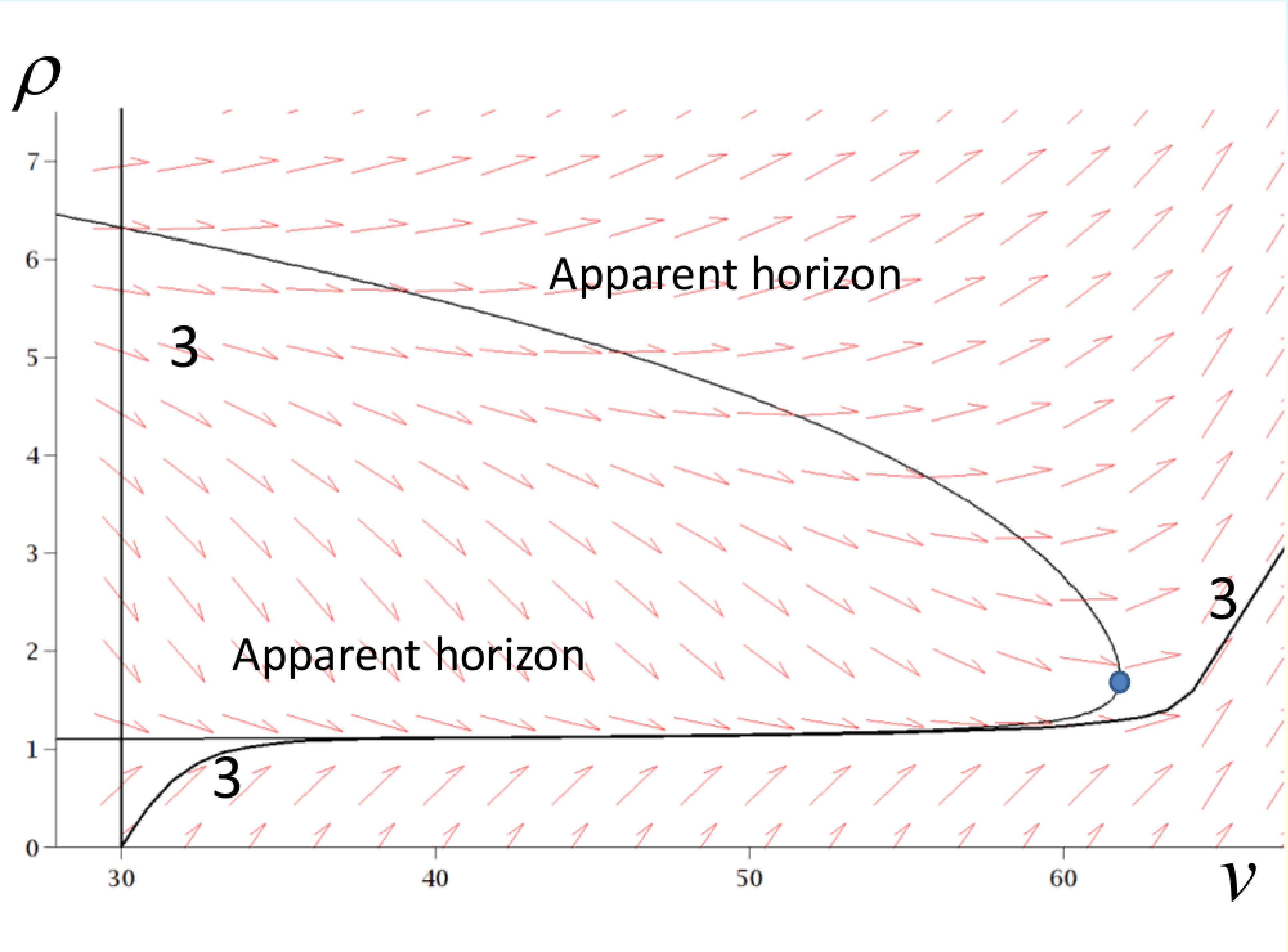

8
6
4
2
0
20
40
60

Quasi-horizon

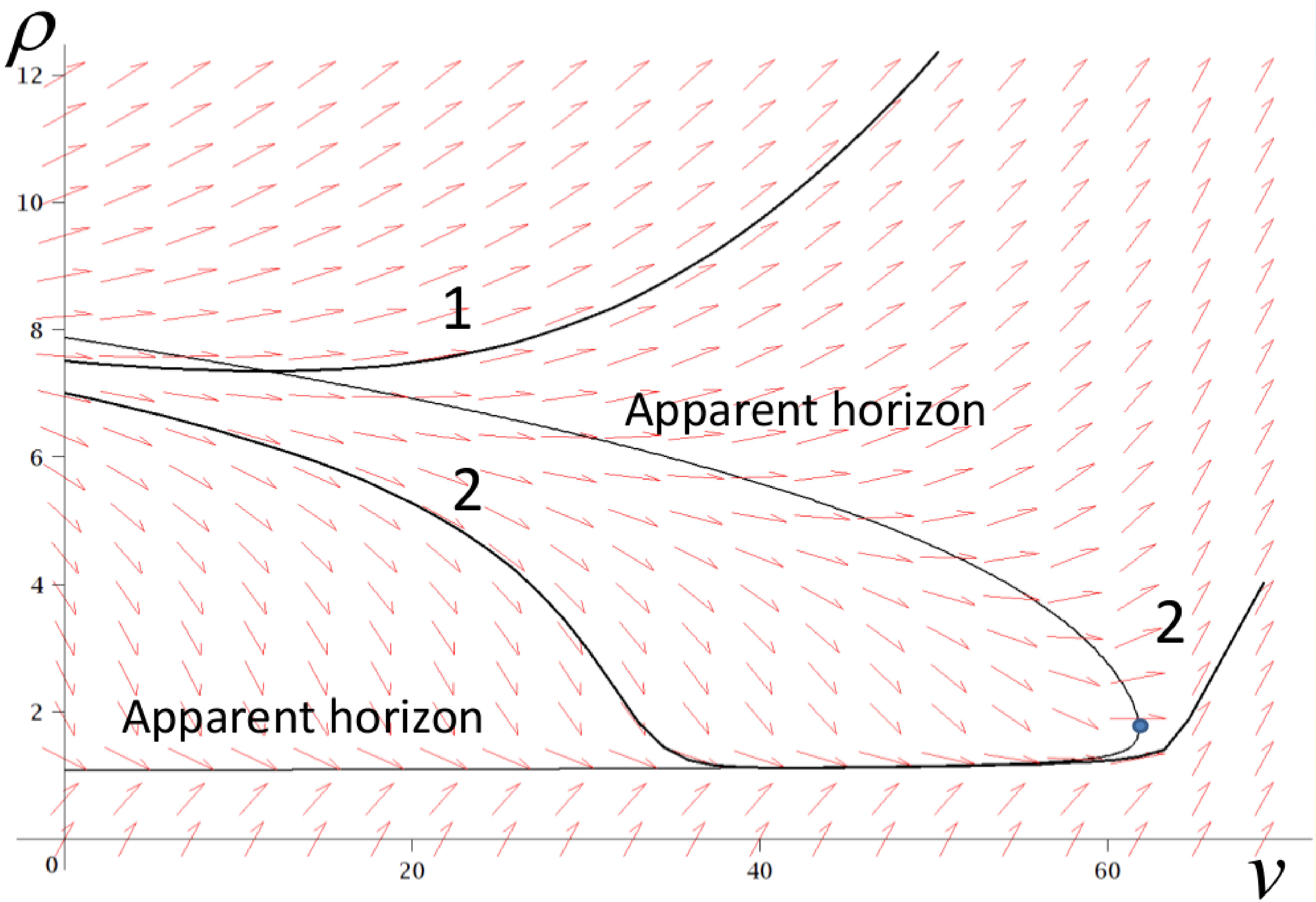
Radial out-going null rays: $\frac{d\rho}{dv} = \frac{1}{2} f$

Quasi-horizon: $\frac{d^2\rho}{dv^2} = 0 \Rightarrow 2\partial_v f + f\partial_\rho f = 0$

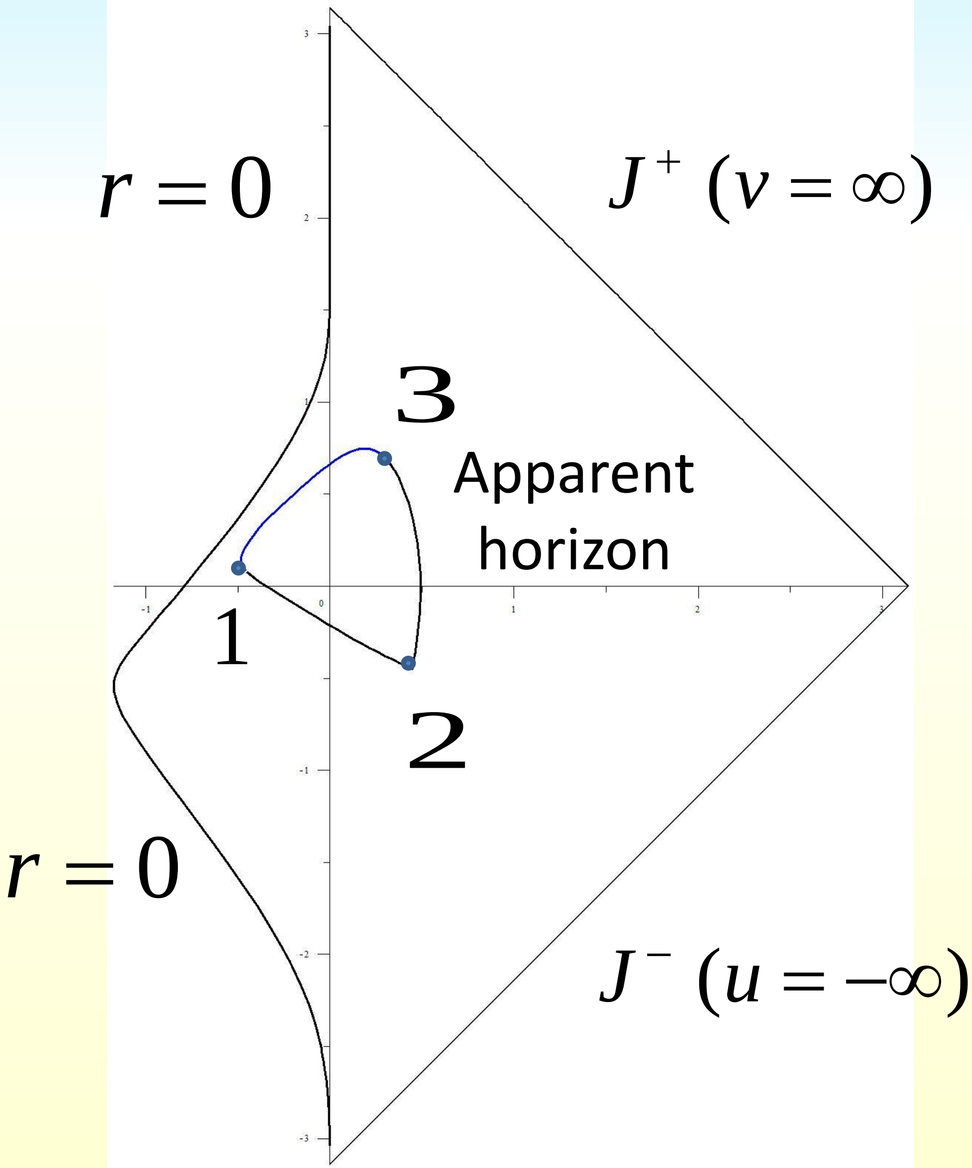
Another definition: $(\nabla f)^2 = 0$



ρ
7
6
5
4
3
2
1
0
30
40
50
60
v
Apparent horizon
3
Apparent horizon
3
3



ρ
12
10
8
6
4
2
0
20
40
60
v
1
2
2
Apparent horizon
Apparent horizon


$r = 0$
$J^+ \ (v = \infty)$
3
Apparent
horizon
1
2
$r = 0$
$J^- \ (u = -\infty)$

8.New Universe Formation inside a Black Hole?

"Through A Black Hole Into A New Universe?"
V.F., Markov, Mukhanov, **Phys.Lett. B216 (1989) 272;**
"Black Holes As Possible Sources Of Closed and
Semiclosed Worlds" ,V. F., Markov, Mukhanov,
IC/88/91. May 1988. **Phys.Rev. D41 (1990) 383;**
"How many new worlds are inside a black hole?"
Barrabes and V. F. **Phys.Rev. D53 (1996) 3215**

Buonanno, Damour, Veneziano ‘99: “Gravitational instability, leading to the possible formation of many black holes” … each of which becomes the place of “birth of a baby Friedmann universe after a period of dilaton-driven inflation”.

Smolin, ***The Life of the Cosmos*** ‘97: “A collapsing black hole causes the emergence of a new universe on the "other side", whose fundamental constant parameters (speed of light, Planck length and so forth) may differ slightly from those of the universe where the black hole collapsed. Each universe therefore gives rise to as many new universes as it has black holes.”

E. Poisson, W. Israel (1990);
I.G. Dymnikova (1991);
E. Poisson (1991).
E. Elizalde, S. R. Hildebrandt (2000);
I.V. Artemova, I. D. Novikov (2002);
L. Ford (2003);
S. Conboy, K. Lake (2005);
S. Ansoldi (2008);
O. B. Zaslavskii (2009);
S. Hossenfelder, L. Smolin (2010);
S. Hossenfelder, L. Modesto, I. Premont-Schwarz (2010);
J. P.S. Lemos, V.T. Zanchin (2011);
V.N. Lukash, E.V. Mikheeva (2013)
A. Vilenkin, J. Zhang (2014)

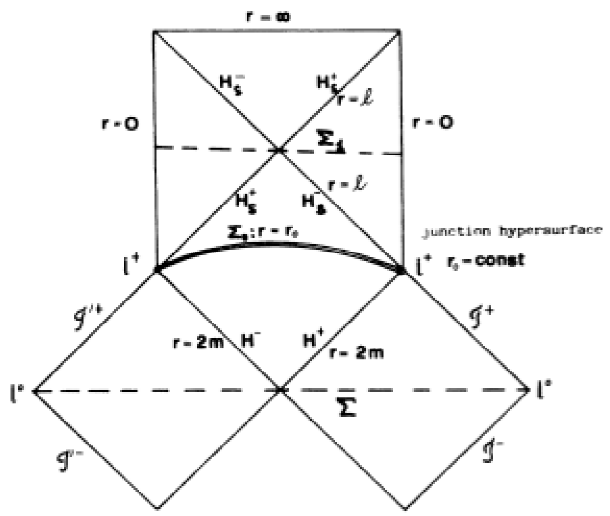


FIG. 3. Conformal diagram for an eternal black hole with the de Sitter-type world in its interior. The surface Σ_0 where $r = r_0 =$ const is a junction surface which represents a short transition layer. After passing this surface the anisotropic Kasner-type contraction of space inside the black hole is changed into the isotropic de Sitter contraction (deflation). The surface Σ_1 corresponds to the moment of "minimal size" of the de Sitter world. After passing this surface the de Sitter world begins its inflationary expansion.

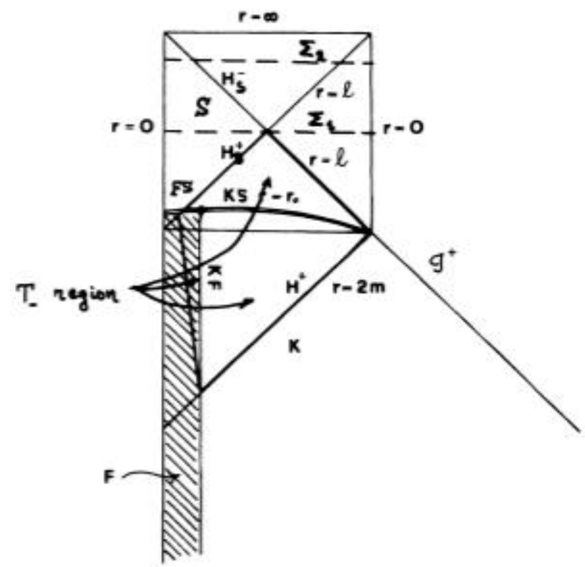
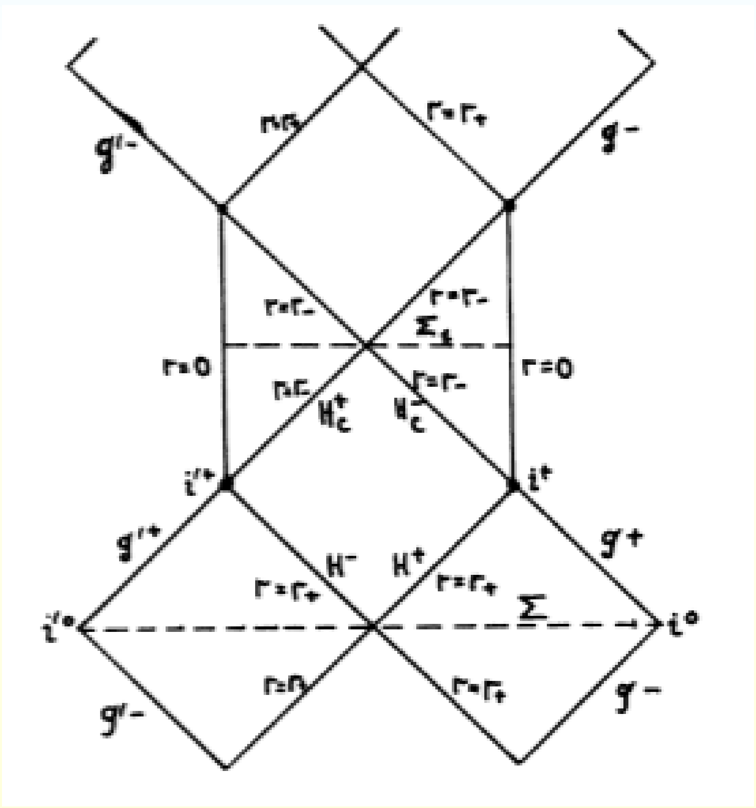


FIG. 6. Conformal diagram for the spacetime of a black hole formed by a collapsing spherically symmetric dust cloud. The domains corresponding to the Schwarzschild-Kruskal, Friedmann, and de Sitter solutions are denoted by *K*, *F*, and *S*, respectively. The boundaries *FK*, *FS*, and *KS* which separate these domains are also shown.

“Black holes in cutoff gravity”, D. Morgan, PRD 43 (1991) 3144

"Extrema of the action are either local extrema, leading to the ordinary equations of motion of general relativity, or extrema on the boundary of field space, with at least one eigenvalue of the curvature attaining its maximum $1/\lambda^2$."

"The singularities are replaced by perfectly well-behaved regions, and an infalling observer ends up in an exponentially expanding de Sitter-like core."



$$dS^2 = b^2 ds^2, \quad f = 1 - \frac{2\mu\rho^2}{\rho^3 + 2\mu},$$

$$ds^2 = -f dt^2 + \frac{d\rho^2}{f} + \rho^2 d\omega^2.$$

$$R_\mu^\nu = 3\delta_\mu^\nu + O(\rho^3).$$

Vacuum SET for 2D Black Holes

We take the 2D metric in the form

$$ds^2 = -f dv^2 + 2\,dv\,dr\,,\; f = 1 - 2M(v)/r\,.$$

To describes a formation of the 2D black hole in the collapse of the thin null shell at v=0 we shall put $M(v) = M\vartheta(v)$.

2D curvature is $R = \dfrac{4M(v)}{r^3}$.

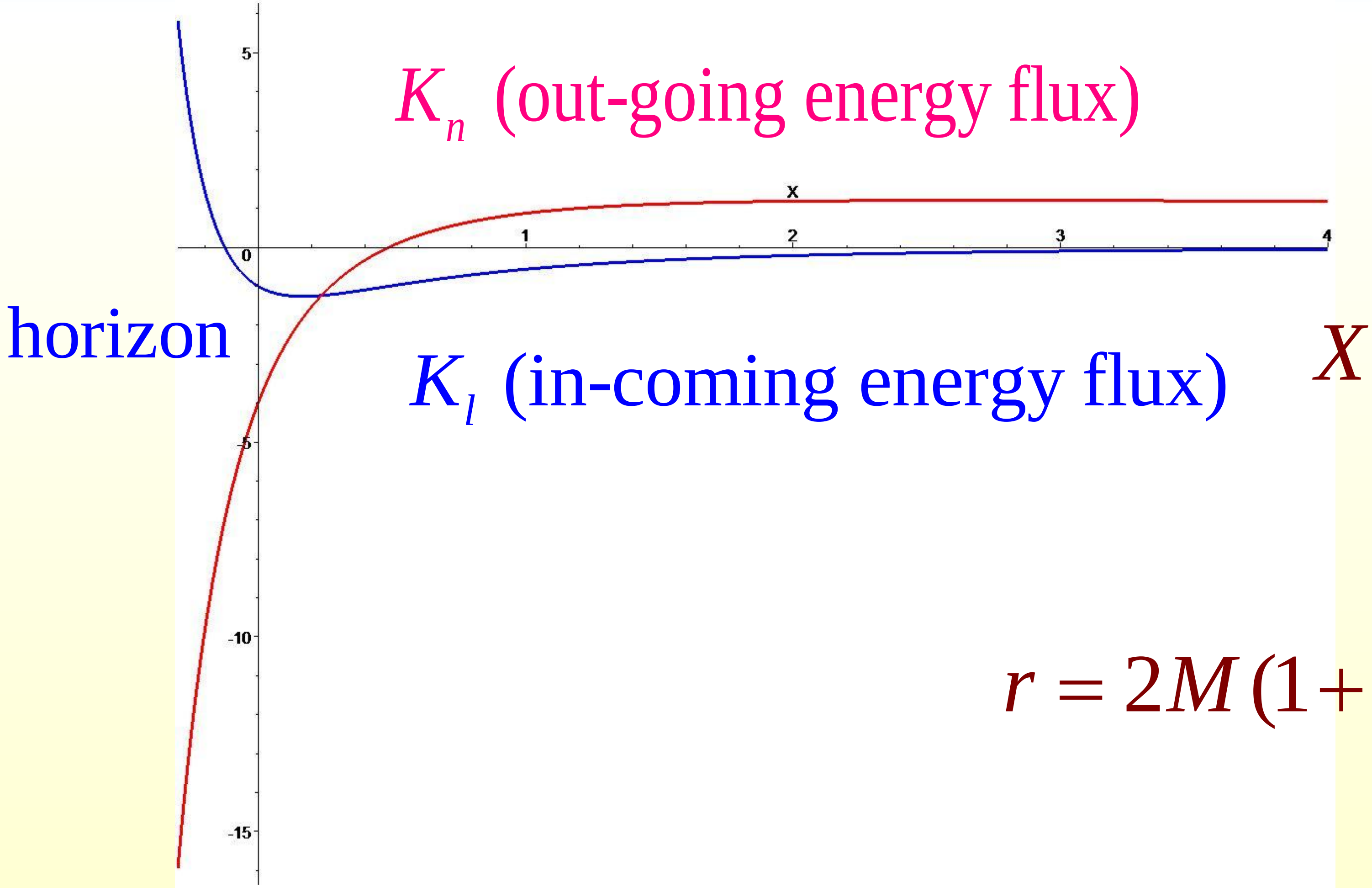
Conformal anomaly: $T^{\mu\nu}{}_{;\nu} = cR$ plus boundary conditions= no in-fluxes determines $T^{\mu\nu}$.

Energy current: $K^{\mu} = -T^{\mu\nu}\xi_{\nu}$, $K^{\mu} = \frac{1}{\sqrt{2}}(K_l l^{\mu} + K_n n^{\mu})$,

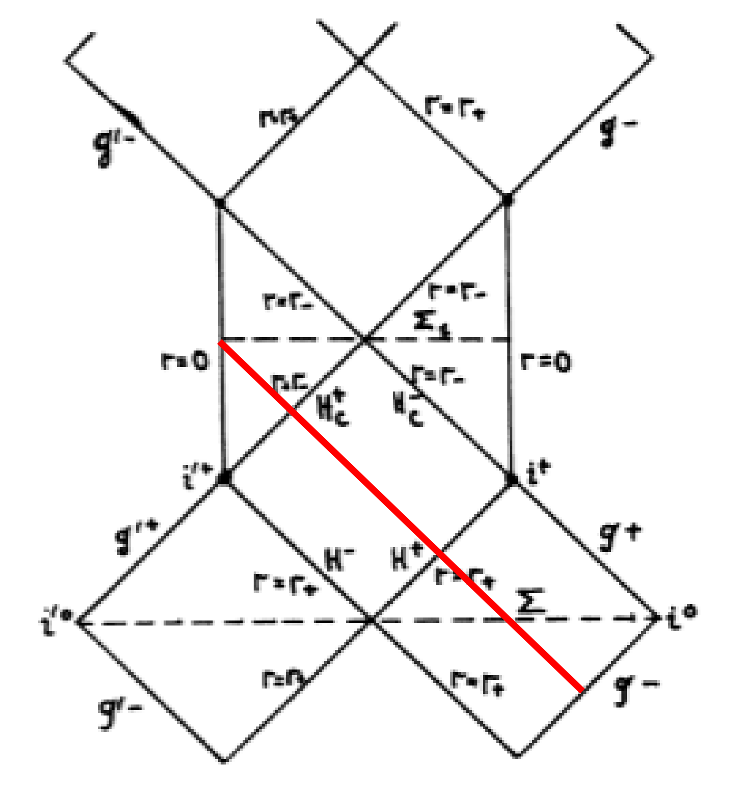
$$F = -T^{\mu\nu}\xi_{\mu} r_{\nu}\Big|_{r=\infty} = K_n - K_l\,,$$

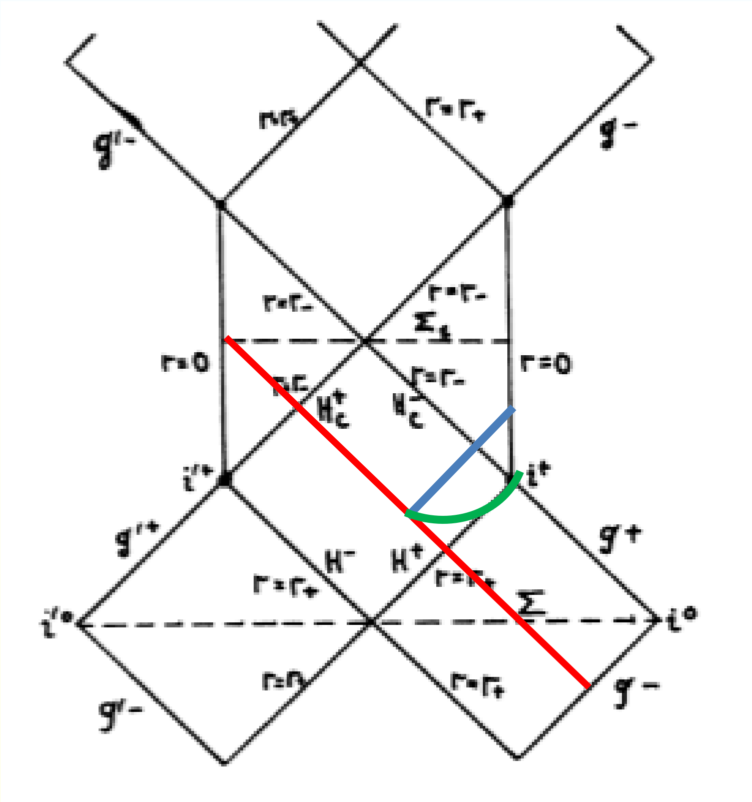
$$\xi \overset{\mathrm{H}}{=} \frac{1}{\sqrt{2}} n,\ T_{\mu\nu}\xi^{\mu}\xi^{\nu} \overset{\mathrm{H}}{=} \frac{1}{2} T_{nn} \overset{\mathrm{H}}{=} K_l\,.$$

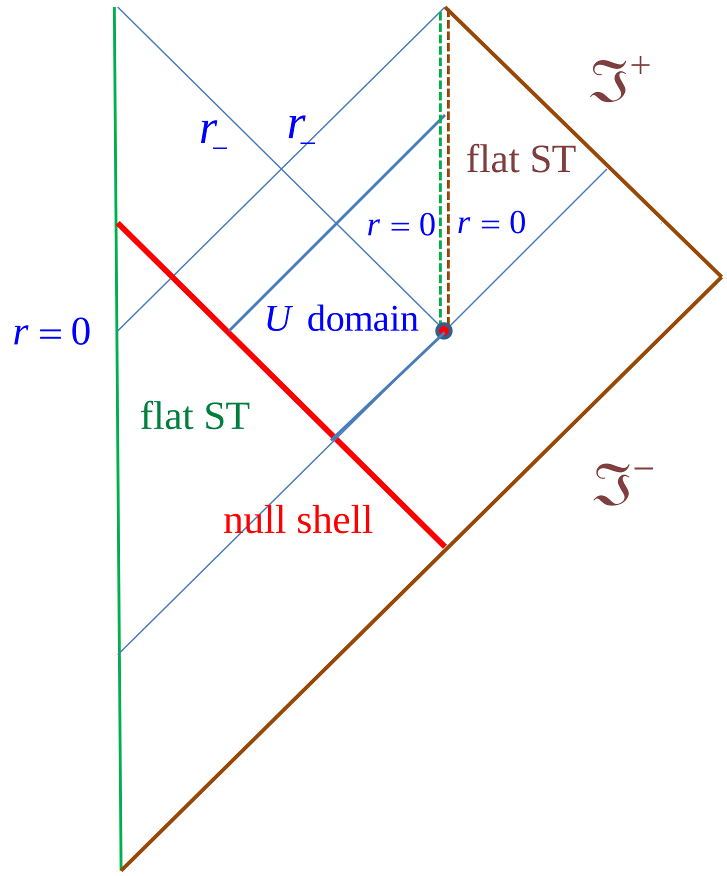
Vacuum stress-energy tensor for 2D black hole



$$r = 2M(1 + X)$$


r=r+
g''-
r=r+
g-
r=r-
r=r-
Σ_1
r=0
r=0
r=r-
r=r-
H_c^+
H_c^-
i'+
i+
g'+
g+
H-
H+
r=r+
r=r+
Σ
i'0
i0
r=r+
r=r+
g'-
g-


r=r+
g''-
r=r+
g-
r=r-
r=r-
Σ_1
r=0
r=0
r=r-
r=r-
H_c^+
H_c^-
i'+
i+
g'+
g+
H-
H+
r=r+
r=r+
Σ
i'0
i0
r=r+
r=r+
g'-
g-



$\mathfrak{I}^+$
r_-
r_-
flat ST
$r = 0$
$r = 0$
$r = 0$
U domain
flat ST
$\mathfrak{I}^-$
null shell

9.Summary and Discussions

Two type of models: With “closed” and “open” apparent horizon.

Common feature is regularity of the BH interior.

Difference: Either V or U dominated energy fluxes?

In V-model: Solution for information loss paradox;
Extended time of the final phase;
Large blue shift of out-coming particles (trans-Planckian energy);
Anti-Hawking effect, etc