Hawking radiation in Lorentz-invariant theories

- Hawking radiation → physical meaning of black hole thermodynamic laws

- Lorentz-invariant theories ⇒ Killing horizon separates outgoing modes; Thermal emission spectrum\(^1\)

- Temperature of the emitted spectrum = thermodynamic temperature

Superluminal dispersion relations

- superluminal motion $\Rightarrow$ Killing horizon can be crossed in both directions

- What becomes of the laws of black hole thermodynamics? of predictability??

- Two routes to address these questions
  $\rightarrow$ analogue gravity (see the previous talks)
  $\rightarrow$ Lorentz-breaking theories of gravity (Hořava gravity, Einstein-æther theory)

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Black holes in Lorentz-violating theories

- Generally possess a universal horizon → solves the predictability issue; first law

- What about the second law?

- Does the universal horizon radiate thermally?

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Using one version of the “tunneling formalisme”\textsuperscript{4} → yes (but doubts about the validity of the procedure)

Analyzing the characteristics of the modes\textsuperscript{5} → one can define a surface gravity geometrically (but link with the emission process unclear)

Full QFT calculation → no late-time emission from the UH


Outline

1. Introduction
2. The model
3. Scattering coefficients and radiation
4. Conclusions and outlook
Section 2

The model
The collapsing shell model

- interior: Minkowski metric with non-accelerated æther
- exterior: solution of $\nabla \phi = \frac{c_{123}}{c_{123}} = 0$, $r_0 = 2M$, and $r_u = 0$
  $\rightarrow$ Killing horizon at $r = 2M$ (Schwarzschild metric)
  $\rightarrow$ Universal horizon at $r = M$

\[ u^\mu \partial_\mu = \partial_v - \frac{M}{r} \partial_r \]

- Real scalar field $\phi$ with quartic dispersion from coupling to the æther

\[ S = \int d^4x \sqrt{-g} \left( \partial_\mu \phi \partial^\mu \phi - \frac{1}{\Lambda^2} \left( \nabla_\mu h^{\mu\nu} \nabla_\nu \phi \right)^2 \right) \]

$h^{\mu\nu} \equiv g^{\mu\nu} - u^\mu u^\nu$

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Conformal diagram and characteristics
Preferred time

\[ t = \begin{cases} 
  v - r & v < 4M \\
  v - r^* & v > 4M \land r > M \\
  -(v - r^*) & v > 4M \land r < M 
\end{cases} \]

\[ X = \begin{cases} 
  r, & v < 4M \\
  r^* & v > 4M \land r > M \\
  -r^* & v > 4M \land r < M 
\end{cases} \]

\[ r^*_U = r + M \ln \left| \frac{r}{M} - 1 \right| \]
The model: summary

- Collapsing shell $\Rightarrow$ well-defined vacuum state at $t \to -\infty$

- Dispersion from coupling to the æther $\Rightarrow$ maintains causality and a standard Hamiltonian structure

- Exterior solution: exact solution of Einstein-æther for explicit calculation (but main results independent on the details)
Section 3

Scattering coefficients and radiation
WKB modes at fixed frequency

2 modes inside the shell
- $\phi_\omega$: positive energy, incoming
- $\phi_\omega^v$: positive energy, outgoing

4 WKB modes close to the universal horizon:
- $\psi_\lambda^u$: positive energy, outgoing
- $(\psi_{-\lambda}^u)^*$: negative energy, outgoing
- $\psi_\lambda^v$: positive energy, incoming
- $(\psi_{-\lambda}^v)^*$: negative energy, incoming
Modes near the universal horizon

\[ \psi_{\lambda} \]

\[ \lambda = 0.01 \]

\[ \Lambda \]

\[ \psi_{\lambda} \]

\[ \nu \]

\[ \psi_{\lambda} \]

\[ (\psi_{\lambda}^{\nu, ->})^* \]

\[ (\psi_{\lambda}^{\nu, <->})^* \]

\[ (\psi_{\lambda}^{\nu, ->})^* \]

\[ (\psi_{\lambda}^{\nu, <->})^* \]

\[ \lambda = 1. \]

\[ \Lambda \]

\[ \psi_{\lambda}^{\nu} \]

\[ \psi_{\lambda}^{\nu} \]

\[ \psi_{\lambda}^{\nu} \]

\[ \psi_{\lambda}^{\nu} \]
Mode expansion near the universal horizon

- Consider radial modes in the near-horizon approximation \((r/M) - 1 \ll 1\)

- Mode expansion:

\[
\phi^u_{\omega, \text{in}} = \int_{-\infty}^{\infty} d\lambda \left( \gamma_{\omega, \lambda} \psi^u_{\lambda} + \delta_{\omega, \lambda} (\psi^u_{-\lambda})^* + A_{\omega, \lambda} \psi^v_{\lambda} + B_{\omega, \lambda} (\psi^u_{\lambda})^* \right)
\]

- \(\delta_{\omega, \lambda}\) encodes the mixing of positive- and negative-energy modes

- Stationary, nonvanishing late-time spectrum requires

\[|\delta_{\omega, \lambda}|^2 \propto 1/\omega \text{ for } \omega \to \infty\]


Renaud Parentani, Florent Michel
Late-time scattering

- Mode matching along the matter shell trajectory $\rightarrow$ Saddle-point approximation $\rightarrow$

$$\delta_{\omega,\lambda} \approx O \left( \frac{\sqrt{M\Lambda}}{\omega} \exp \left( -2M\sqrt{\Lambda\omega} \right) \right)$$

- More general shell trajectory and dispersion relation of order $2N$ give

$$\ln \delta_{\omega,\lambda} \sim \omega^{1/N}, \quad A > 0$$
Global scattering

- Numerical integration including the propagation towards the Killing horizon and beyond $\rightarrow$ approximately thermal spectrum with temperature

$$T \approx \frac{\kappa_{\text{Killing}}}{2\pi}$$

- expected from known analogue gravity results as the universal horizon does not radiate at late times
Near-UH physics

- Contrary to what was expected using partial arguments, we found no late-time emission from the UH

\[ \ln \delta_{\omega, \lambda} \sim -A \omega^{1/N}, \ A > 0 \]

Global scattering

Approximately thermal spectrum from the Killing horizon
Outlook

- Consequences for the black hole thermodynamics laws?

- Fate of the UH? Instability from modes originating from the singularity?

- Analogue model of a UH?
Thank you for your attention!