

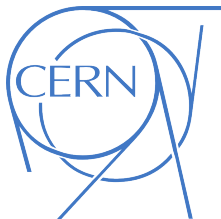
PBH bounds on scalar-induced GWs in the PTA band



Valerie Domcke
CERN

@ MITP workshop on PTAs
14.-18.08.2023

based on
2302.07901
with Virgile Dandoy and
Fabrizio Rompineve



probes of scalar power spectrum

CMB: $P_\zeta(k) \simeq 10^{-9}$ @ $k \sim 0.05 \text{ Mpc}^{-1}$

at smaller scales much weaker constrained:

- scalar induced GWs (SIGWs) $\zeta + \zeta \rightarrow h$

$$f = k/(2\pi) \sim 10^{-9} \text{ Hz } (k \text{ pc})$$

- primordial black holes (PBHs) $\zeta \sim \delta\rho \rightarrow \text{PBH}$

$$M_H = \frac{4}{3}\pi H^{-3}\rho = 4\pi M_P^2 H^{-1} \simeq 20M_\odot (k \text{ pc})^{-2}$$

see also Fabrizio's talk yesterday



large scalar perturbations at pc scale (re-entered 15 – 20 e-fold after CMB)
→ GWs at PTA scales and PBHs around ~ solar mass

scalar induced GWs

model for scalar power spectrum

$$P_\zeta(k) = \frac{A_\zeta}{\sqrt{2\pi}\Delta} \exp\left[-\frac{\ln^2(k/k_*)}{2\Delta^2}\right]$$

k_* peak position

A_ζ amplitude

Δ width

SIGWs: Espinosa, Racco, Riotto `18, Kohri, Terada `18

$$\Omega_{\text{gw}} h^2 = 10^{-9} \left(\frac{A_\zeta}{0.01}\right)^2 \left(\frac{\Omega_r h^2}{10^{-5}}\right) S(f/f_*, \Delta)$$

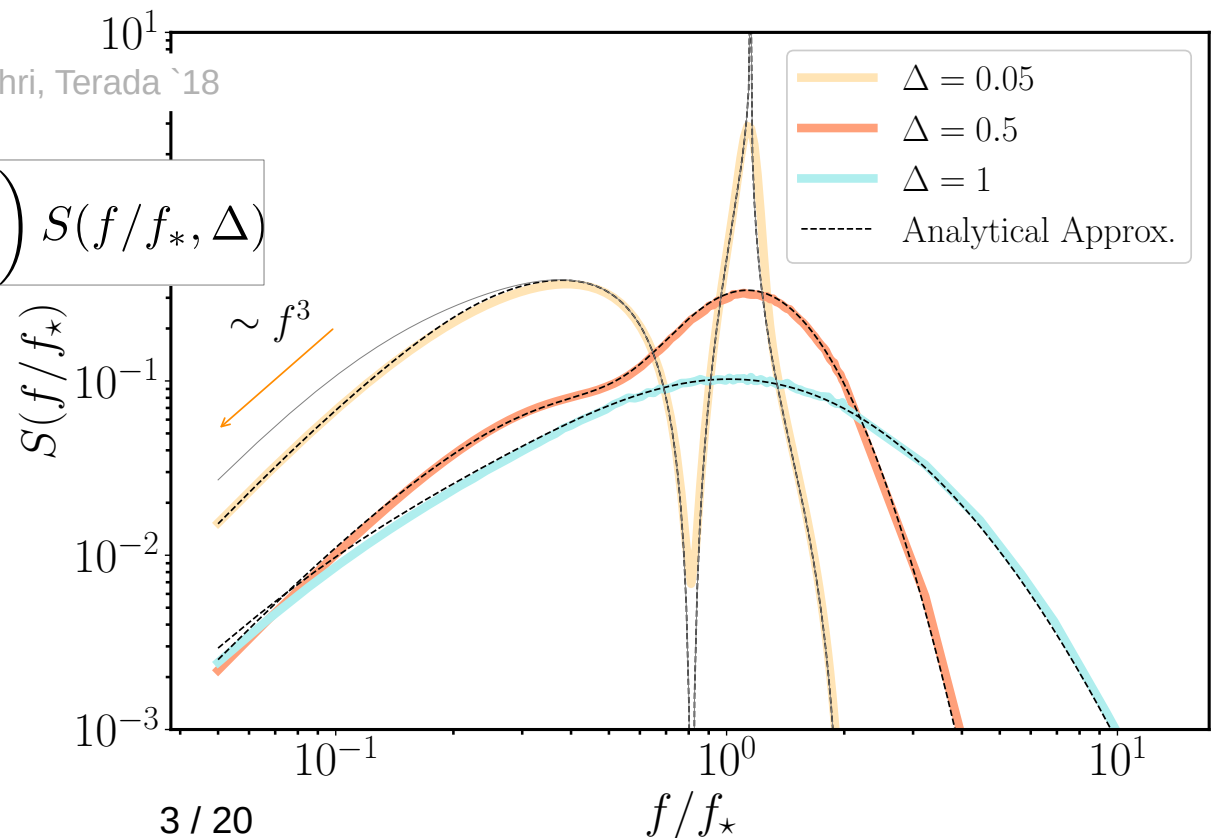
resonance for narrow peak

Ananda, Clarkson, Wands `06

analytical approximation

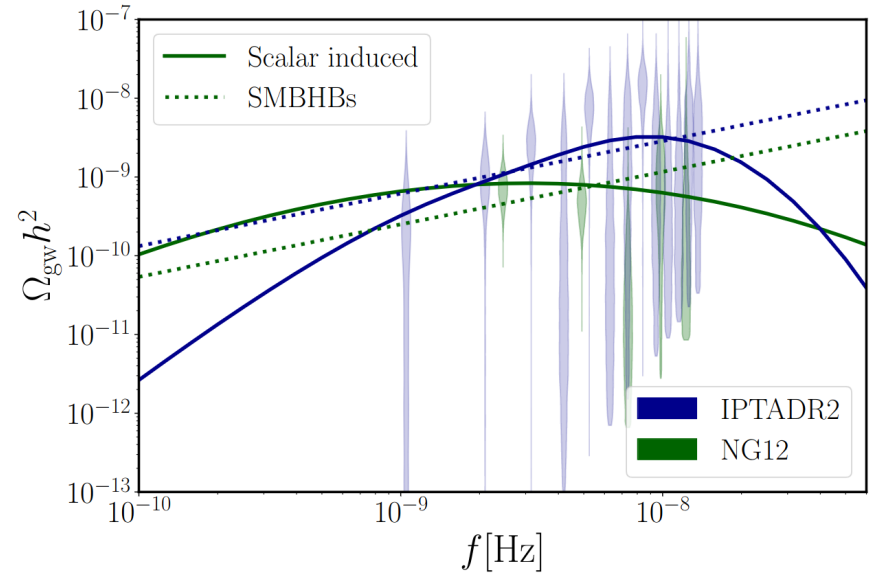
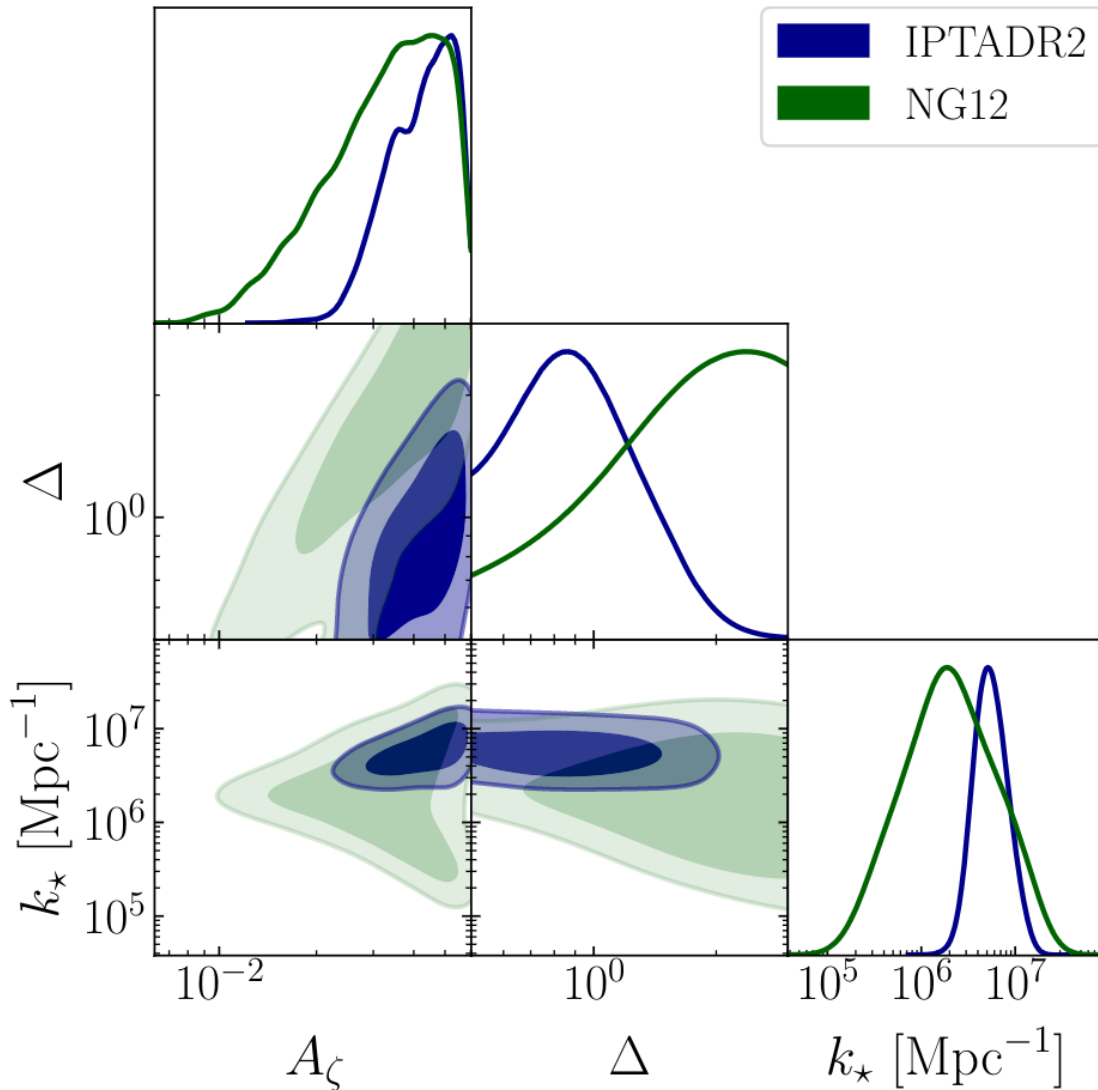
Pi, Sasaki `20,

Dandoy, VD, Rompineve `23



Bayesian search in PTA data

Dandoy, VD, Rompineve '23

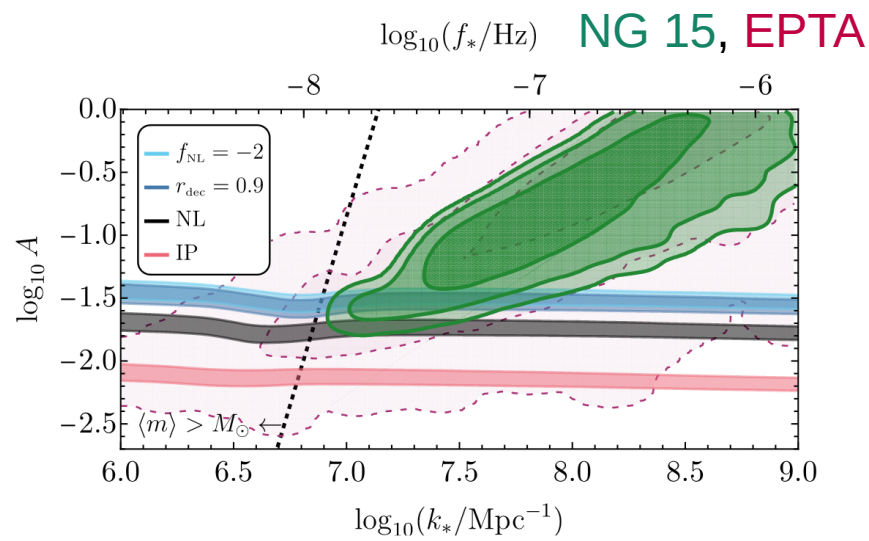
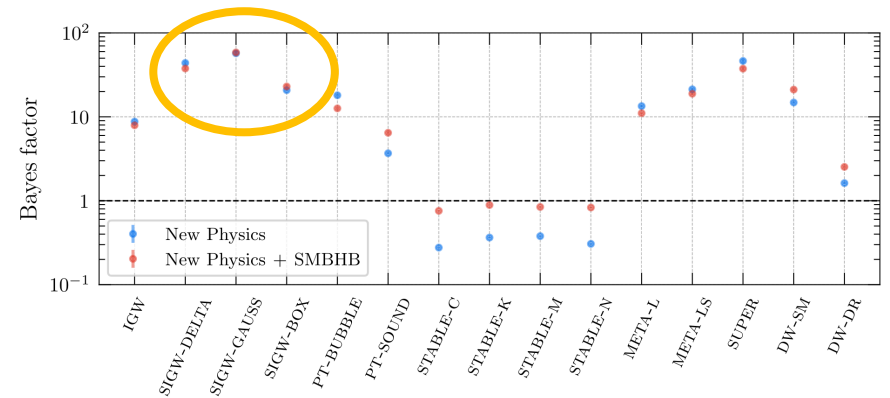
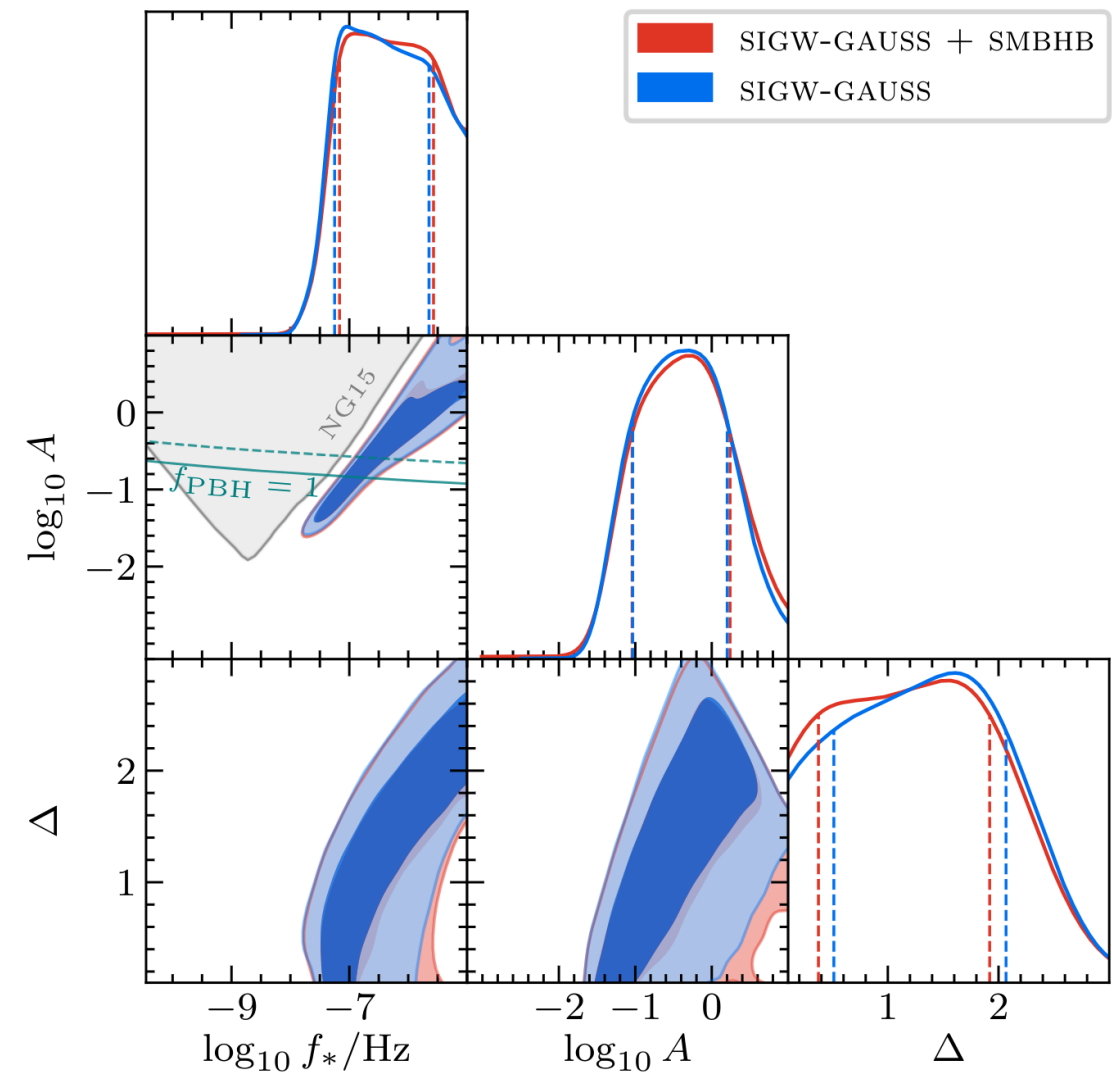


- good fit for

$$A_\zeta \sim \text{few } 10^{-2}, k \sim \text{pc}^{-1}, \Delta \sim 1$$

- ITPA DR2 similar to 2023 DR

2023 data release



primordial black holes

- fraction of PBH dark matter:

$$f_{\text{PBH}} = \frac{1}{\Omega_{\text{DM}}} \int d \ln M \int d \ln k \beta_k(M) \frac{\rho_\gamma(T_k)}{\rho_c^0} \frac{s^0}{s(T_k)}$$

radiation component

fraction of radiation collapsed
to PBH of mass M at
horizon re-entry

primordial black holes

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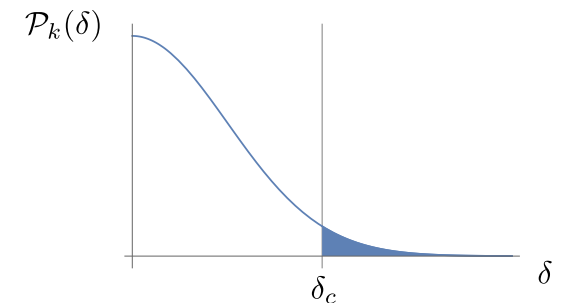
fraction of radiation collapsed to PBH of mass M at horizon re-entry

- Press-Schechter formalism for spherical collapse:

$$\beta_k(M) = \int_{\delta_c}^{\infty} d\delta \underbrace{\frac{\mathcal{P}_k(\delta)}{\frac{\exp(-\delta^2/(2\sigma_k^2))}{\sqrt{2\pi}\sigma_k}}}_{\text{probability distribution}} \underbrace{\frac{M(\delta)}{M_H(k)}}_{\simeq 1} \underbrace{\delta_D\left[\ln \frac{M}{M(\delta)}\right]}_{M(\delta)=M}$$

probability distribution

$$\delta_c \sim c_s^2 = 1/3 \quad \sigma_k^2 \sim P_\delta(k) \sim P_\zeta(k)$$



primordial black holes

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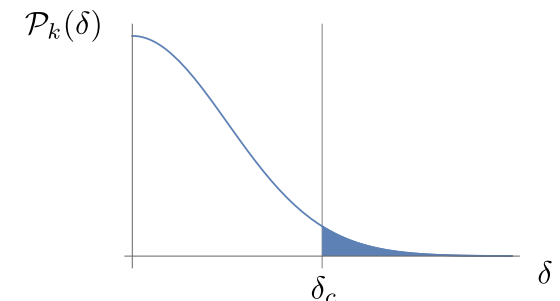
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$$\delta_c \sim c_s^2 = 1/3 \quad \sigma_k^2 \sim P_\delta(k) \sim P_\zeta(k)$$



- Bounds on PBH abundance from $f_{\text{PBH}} < 1$, microlensing, LIGO/VIRGO

→ constraints on power spectrum $P_\zeta(k)$

The devil is in the detail (1)

$$\beta_k(M) = \int_{\delta_c}^{\infty} d\delta \underbrace{\mathcal{P}_k(\delta)}_{\frac{\exp(-\delta^2/(2\sigma_k^2))}{\sqrt{2\pi}\sigma_k}} \frac{M(\delta)}{M_H(k)} \delta_D \left[\ln \frac{M}{M(\delta)} \right]$$

↑
gaussian

↖

$$M(\delta) = \kappa M_H(k) (\delta - \delta_c)^\gamma, \quad \gamma = 0.36, \quad \kappa = 1..10$$

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$$\delta \mapsto \delta_l$$

$$\delta_c \mapsto \delta_c(W)$$

↑
gaussian

$$M(\delta) = \kappa M_H(k) (\delta - \delta_c)^\gamma, \quad \gamma = 0.36, \quad \kappa = 1..10$$

smoothed density contrast: $\delta_m = \frac{1}{V} \int dR 4\pi R^2 \frac{\delta\rho}{\bar{\rho}}(R, t_H) W(R, R_m)$

non-linear relation $\delta\rho(\zeta) \mapsto \delta_m(\delta_l)$ depends on choice of window function

$\delta_l \equiv$ linearized smooth density contrast

Young `19

The devil is in the detail (1)

$$\beta_k(M) = \int_{\delta_c}^{\infty} d\delta \underbrace{\mathcal{P}_k(\delta)}_{\frac{\exp(-\delta^2/(2\sigma_k^2))}{\sqrt{2\pi}\sigma_k}} \frac{M(\delta)}{M_H(k)} \delta_D \left[\ln \frac{M}{M(\delta)} \right]$$

↑
gaussian

$$\begin{aligned} \delta &\mapsto \delta_l \\ \delta_c &\mapsto \delta_c(W) \\ \delta_c &\mapsto \delta_c(W, \Delta) \end{aligned}$$

$$M(\delta) = \kappa M_H(k) (\delta - \delta_c)^\gamma, \quad \gamma = 0.36, \quad \kappa = 1..10$$

smoothed density contrast: $\delta_m = \frac{1}{V} \int dR 4\pi R^2 \frac{\delta\rho}{\bar{\rho}}(R, t_H) W(R, R_m)$ ←

non-linear relation $\delta\rho(\zeta) \mapsto \delta_m(\delta_l)$ depends on choice of window function

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Young '19

shape parameter: $P_\zeta(k)$ broader → more modes involved in collapse → lower δ_c

The devil is in the detail (2)

$$\beta_k(M) = \int_{\delta_c}^{\infty} d\delta \underbrace{\mathcal{P}_k(\delta)}_{\frac{\exp(-\delta^2/(2\sigma_k^2))}{\sqrt{2\pi}\sigma_k}} \frac{M(\delta)}{M_H(k)} \delta_D \left[\ln \frac{M}{M(\delta)} \right]$$

see Dani's talk



↑
gaussian

Franciolini *et al* '23
Figueroa *et al* '23

$$\delta \mapsto \delta_l$$

$$\delta_c \mapsto \delta_c(W)$$

$$\delta_c \mapsto \delta_c(W, \Delta)$$

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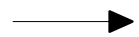
$$\delta \mapsto \delta_l$$

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$$\delta_c \mapsto \delta_c(W, \Delta)$$

$$\sigma_k \mapsto \sigma_k(W)$$

see Dani's talk



gaussian

Franciolini et al `23
Figueroa et al `23

$$\text{variance: } \sigma_k^2 = \int d \ln k' W^2(k', k) P_\delta(k') = \int d \ln k' W^2(k', k) (k'/k)^4 T^2(k', k) P_\zeta(k)$$

use same window function as in density contrast
→ most of W-dependence drops

Young `19; Musco, De Luca, Franciolini, Riotto `20

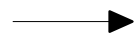
non-linear transfer function
→ density contrast at horizon crossing

Musco, De Luca, Franciolini, Riotto `20; Riotto `23

The devil is in the detail (2)

$$\beta_k(M) = \int_{\delta_c}^{\infty} d\delta \underbrace{\mathcal{P}_k(\delta)}_{\frac{\exp(-\delta^2/(2\sigma_k^2))}{\sqrt{2\pi}\sigma_k}} \frac{M(\delta)}{M_H(k)} \delta_D \left[\ln \frac{M}{M(\delta)} \right]$$

see Dani's talk



gaussian

Franciolini et al `23
Figueroa et al `23

$$\delta \mapsto \delta_l$$

$$\delta_c \mapsto \delta_c(W)$$

$$\delta_c \mapsto \delta_c(W, \Delta)$$

$$\sigma_k \mapsto \sigma_k(W)$$

$$\delta_c \mapsto \delta_c(W, \Delta, k)$$

$$\text{variance: } \sigma_k^2 = \int d \ln k' W^2(k', k) P_\delta(k') = \int d \ln k' W^2(k', k) (k'/k)^4 T^2(k', k) P_\zeta(k)$$

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Young `19; Musco, De Luca, Franciolini, Riotto `20

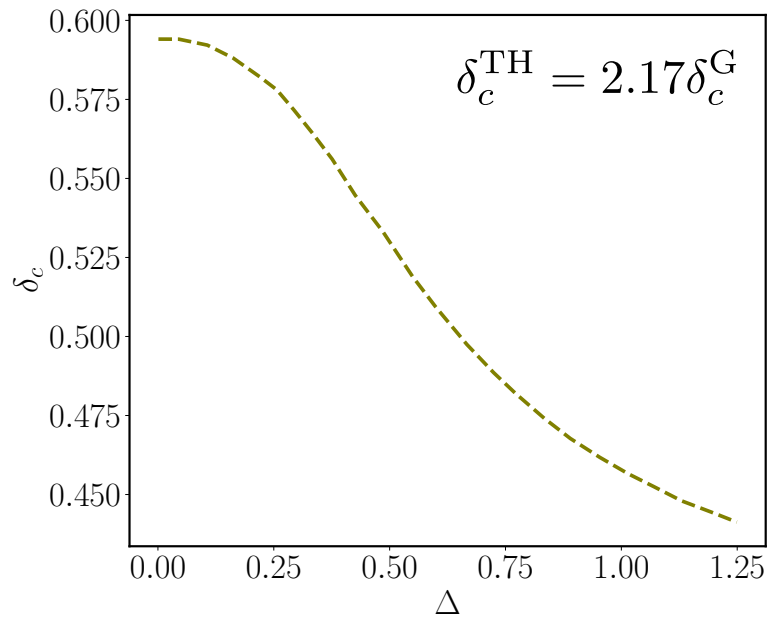
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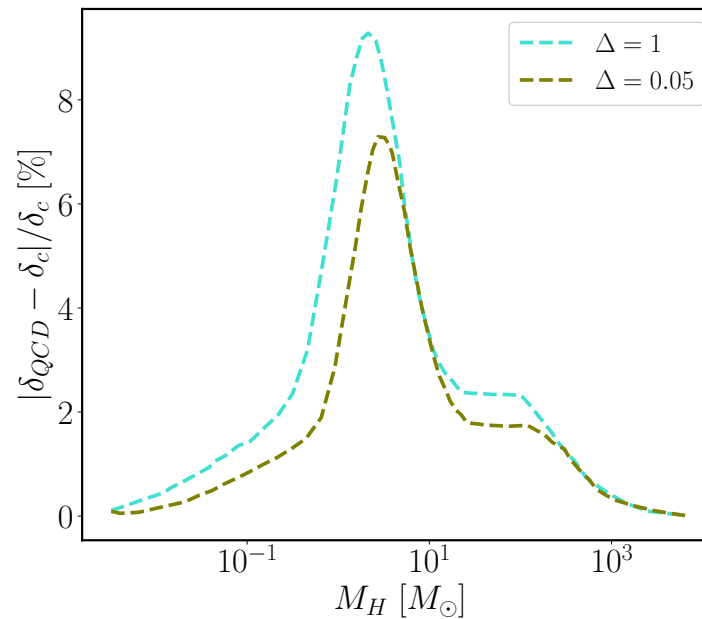
$$\text{QCD phase transition: } c_s \mapsto c_s(T) \Rightarrow \delta_c \mapsto \delta_c(k)$$

The devil is in the detail (summary)

window function and shape parameter



QCD phase transition



Most important are exponential factors :

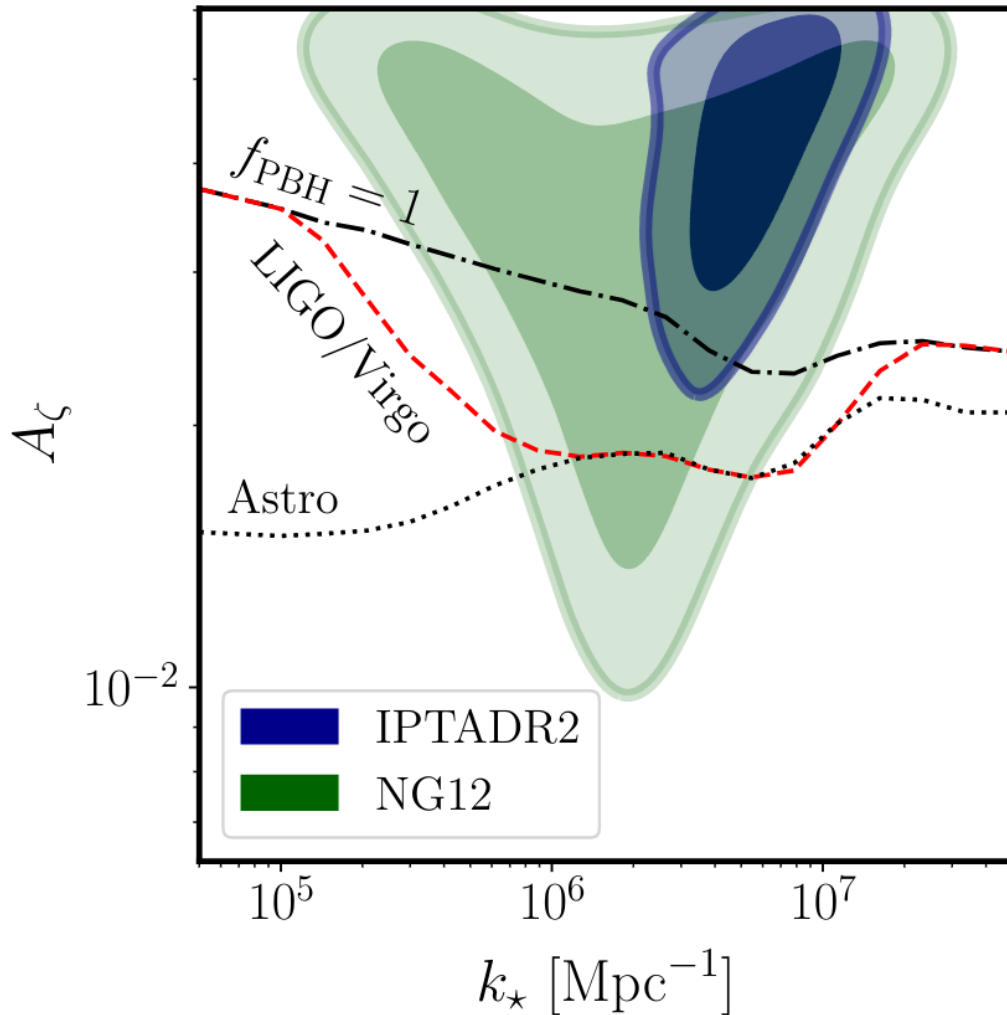
critical density δ_c → shape parameter, window function, non-linear relation

variance σ_k → window function, non-linear relation (?)

GWs vs PBHs

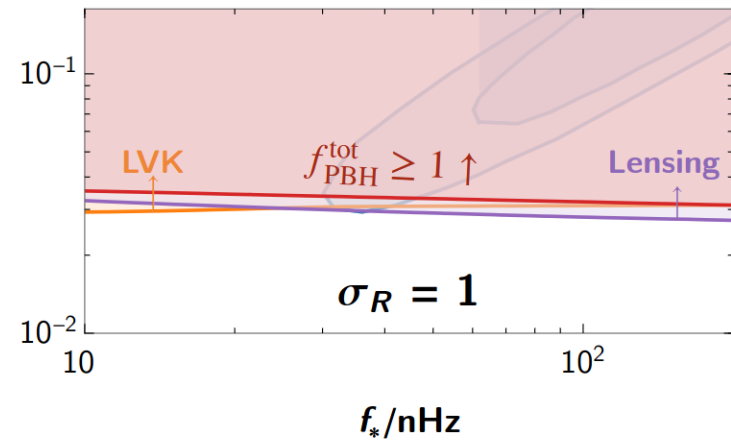
Dandoy, VD, Rompineve `23

Scalar induced GWs only



IPTA DR2 (~ DR `23) preferred region excluded by PBH constraints.

Figueroa et al `23, w NG 15 data:



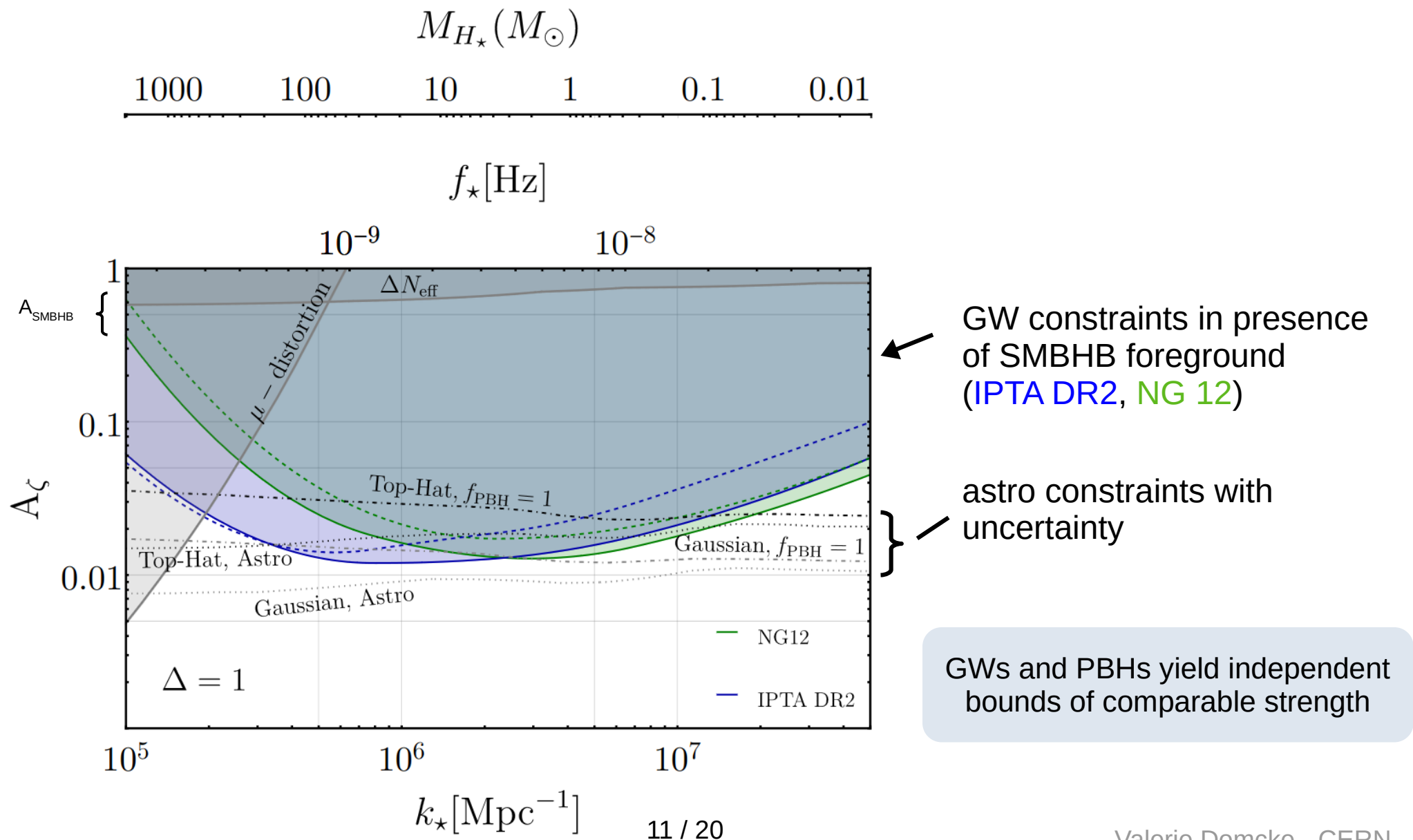
Exponential sensitivity of f_{PBH} to A_ζ :

$$f_{\text{PBH}} \rightarrow A_\zeta \quad : \quad \checkmark$$

$$A_\zeta \rightarrow f_{\text{PBH}} \quad : \quad \times$$

constraints on scalar power spectrum

Dandoy, VD, Rompineve '23



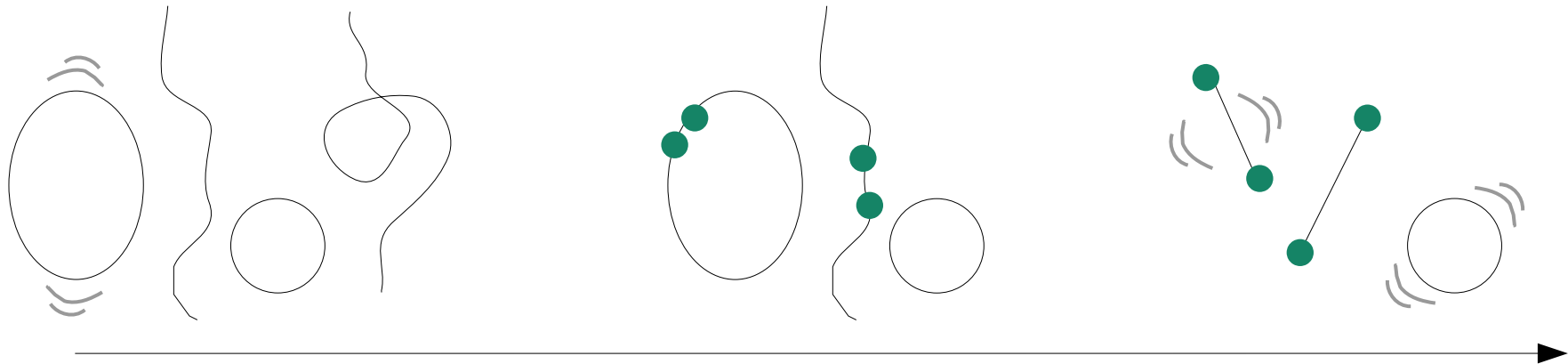
Conclusions

PBH bounds make a SIGW interpretation of PTA data difficult at best

some uncertainties and model dependence in PBH calculation remain, but a lot of progress in recent years!

PTAs are powerful probe of scalar power spectrum at \sim pc scale

One slide on metastable cosmic strings



scaling regime
(long strings & loops)

$$t_s = 1/\Gamma_d^{1/2}$$

segments & loops

[see also Leblond, Shlaer, Simons `09]

cosmic strings formed
in phase transition, eg

spontaneous creation of monopoles
due to GUT embedding

$$G_{SM} \times U(1)_{B-L} \subset SO(10)$$

$$G_{SM} \times U(1)_{B-L} \rightarrow G_{SM}$$

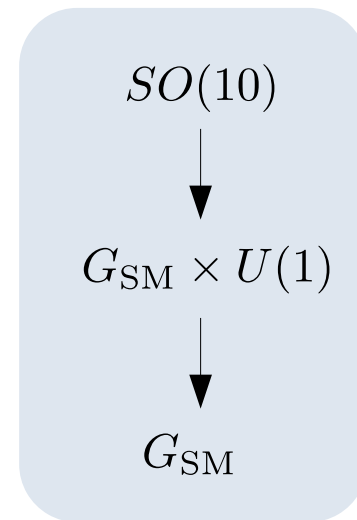
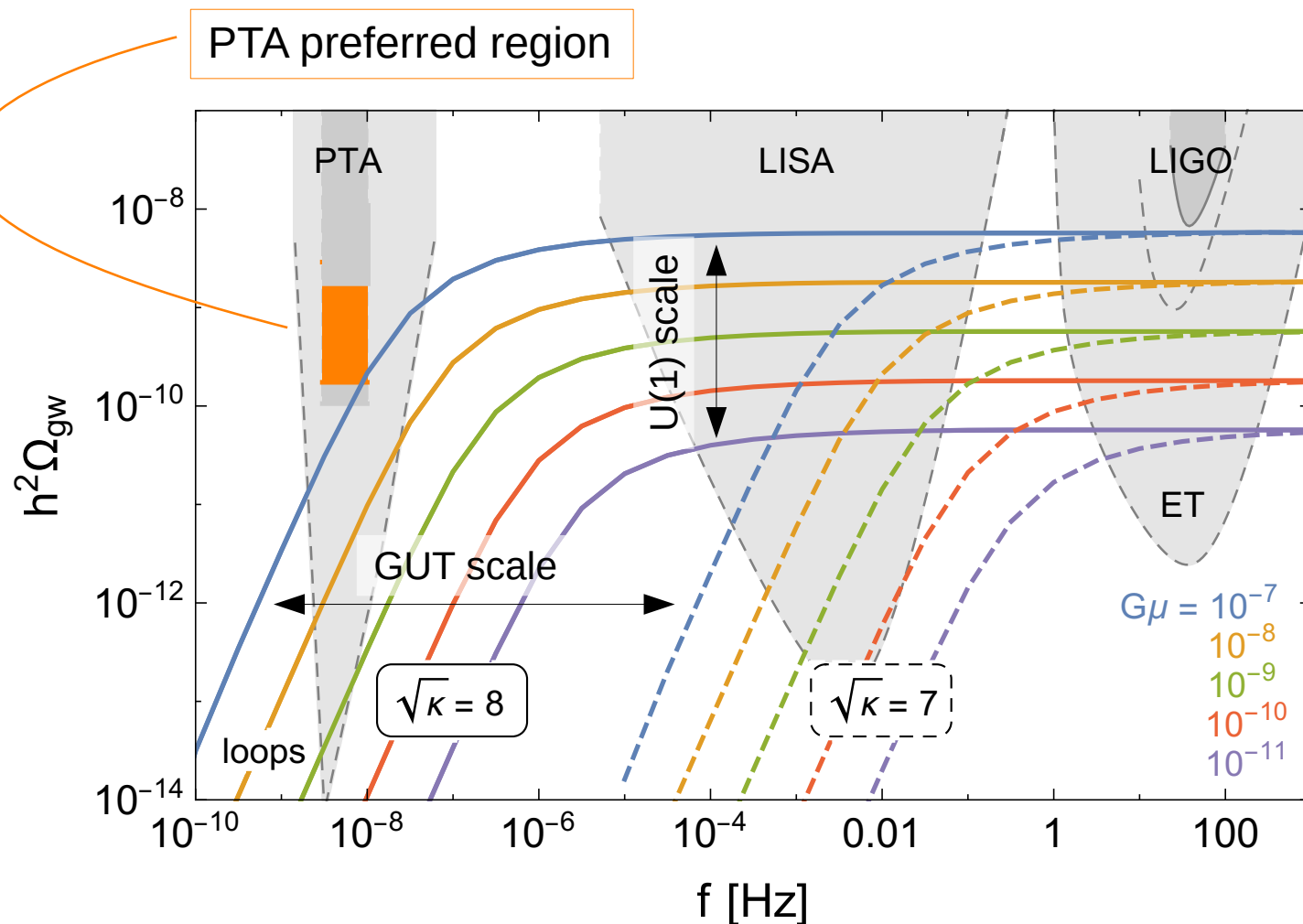
$$\Gamma_d \sim \mu \exp(-\pi\kappa^2), \quad \kappa^2 = m^2/\mu$$

$\mu \sim v_{B-L}^2$ string tension

$m \sim v_{GUT}$ monopole mass

(One) slide on metastable cosmic strings

Buchmüller, VD, Schmitz '21, '23



$$\sqrt{\kappa} \sim v_{SO(10)} / v_{U(1)}$$

$$G\mu \sim (v_{U(1)} / M_P)^2$$

GUT-scale U(1) phase transition can be tested with GWs

... and (one) advertisement

CERN TH visitor program

<https://theory.cern/visitor-info>

short-term visits typically O(week)

long term visits (> 3 months, usually sabbaticals)

CERN fellowship program

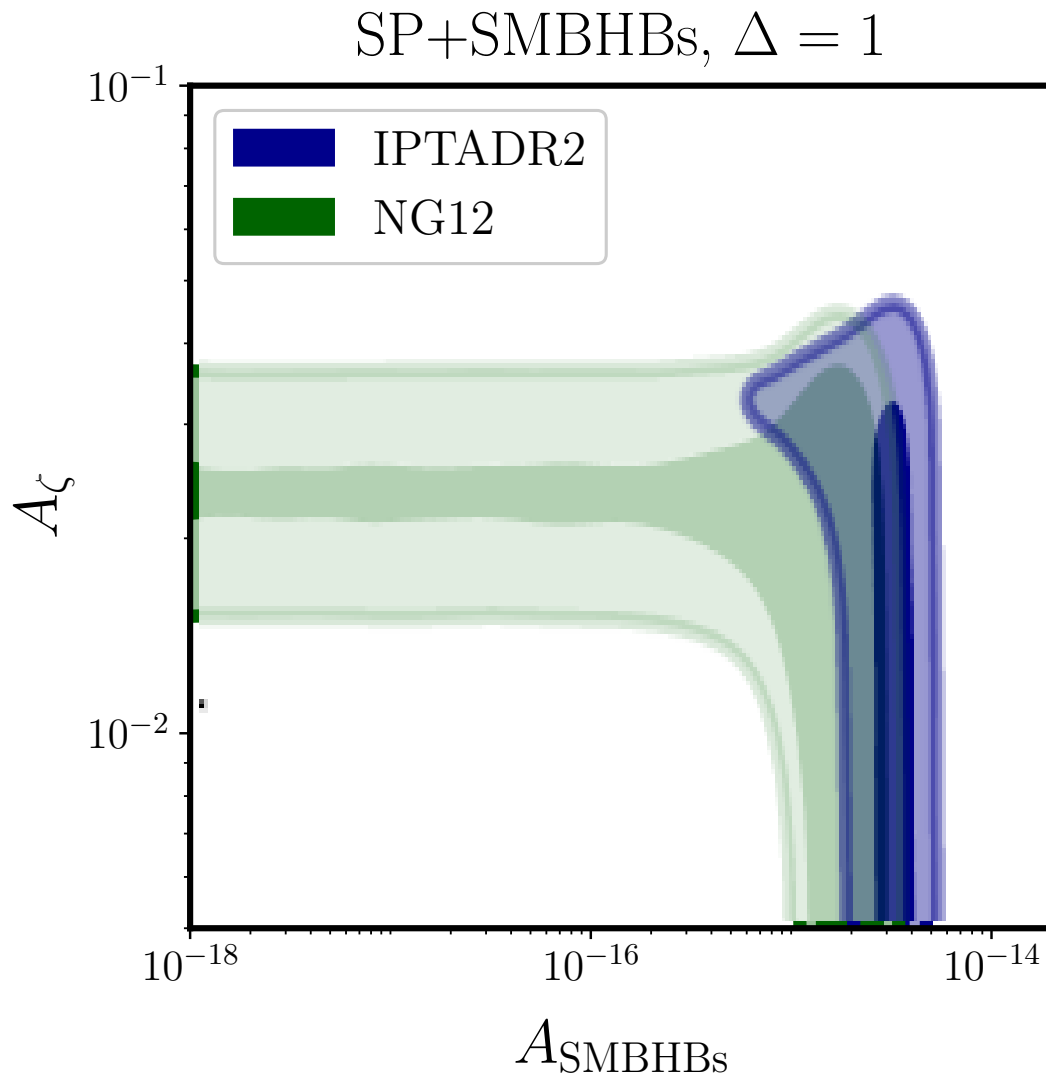
<https://theory.cern/jobs>

deadline September 3rd (!!)

consider applying!

backup

SMBHB + SIGW analysis

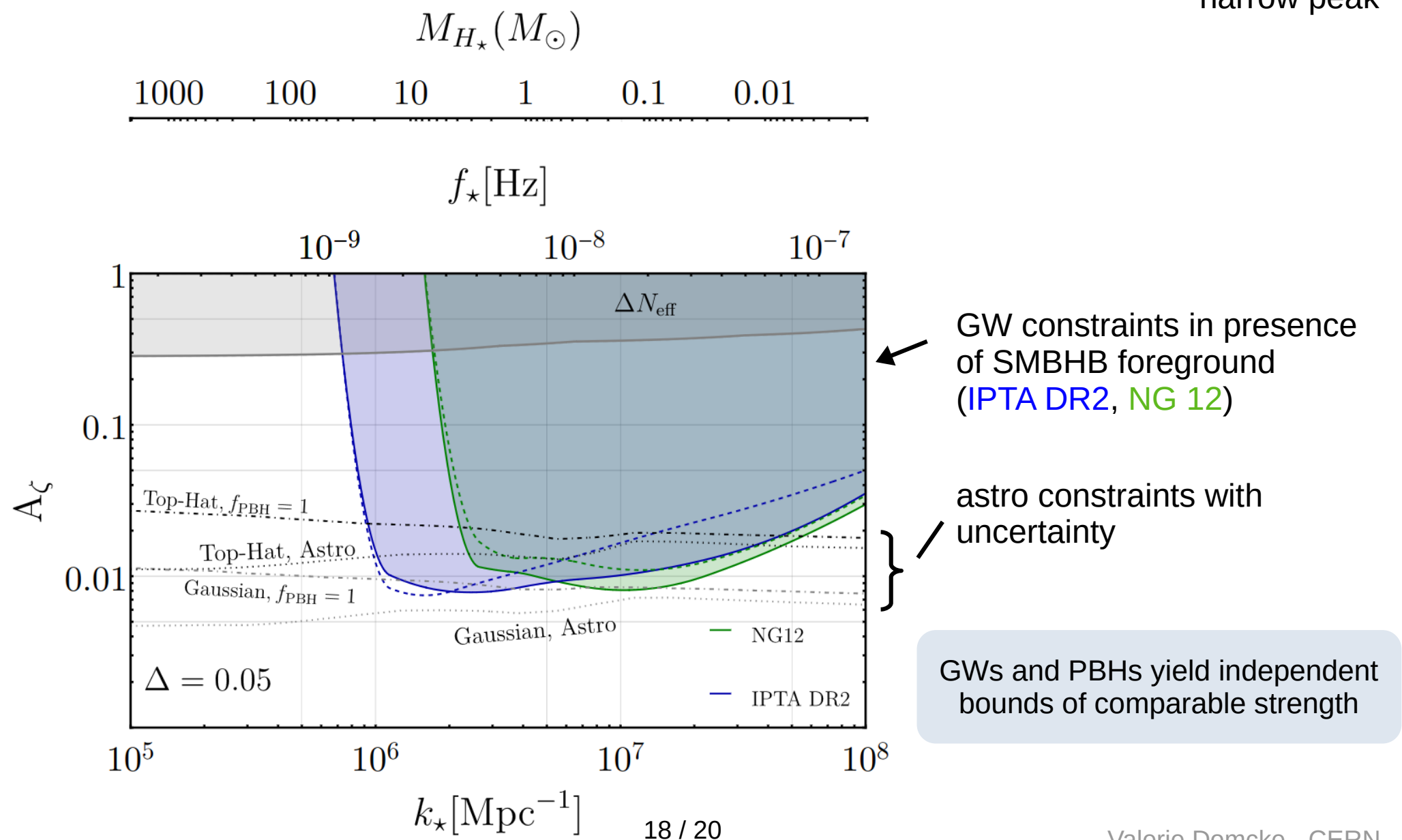


NG12: both SMBHB and SIGW
fit well
 $\ln_{10} B_{\zeta, \text{SMBHB}} = 0.05$
(inconclusive)

IPTA DR2: requires larger value
of A_ζ , above prior set by
 f_{PBH} bound.
 $\ln_{10} B_{\zeta, \text{SMBHB}} = 2.2$
(decisive)

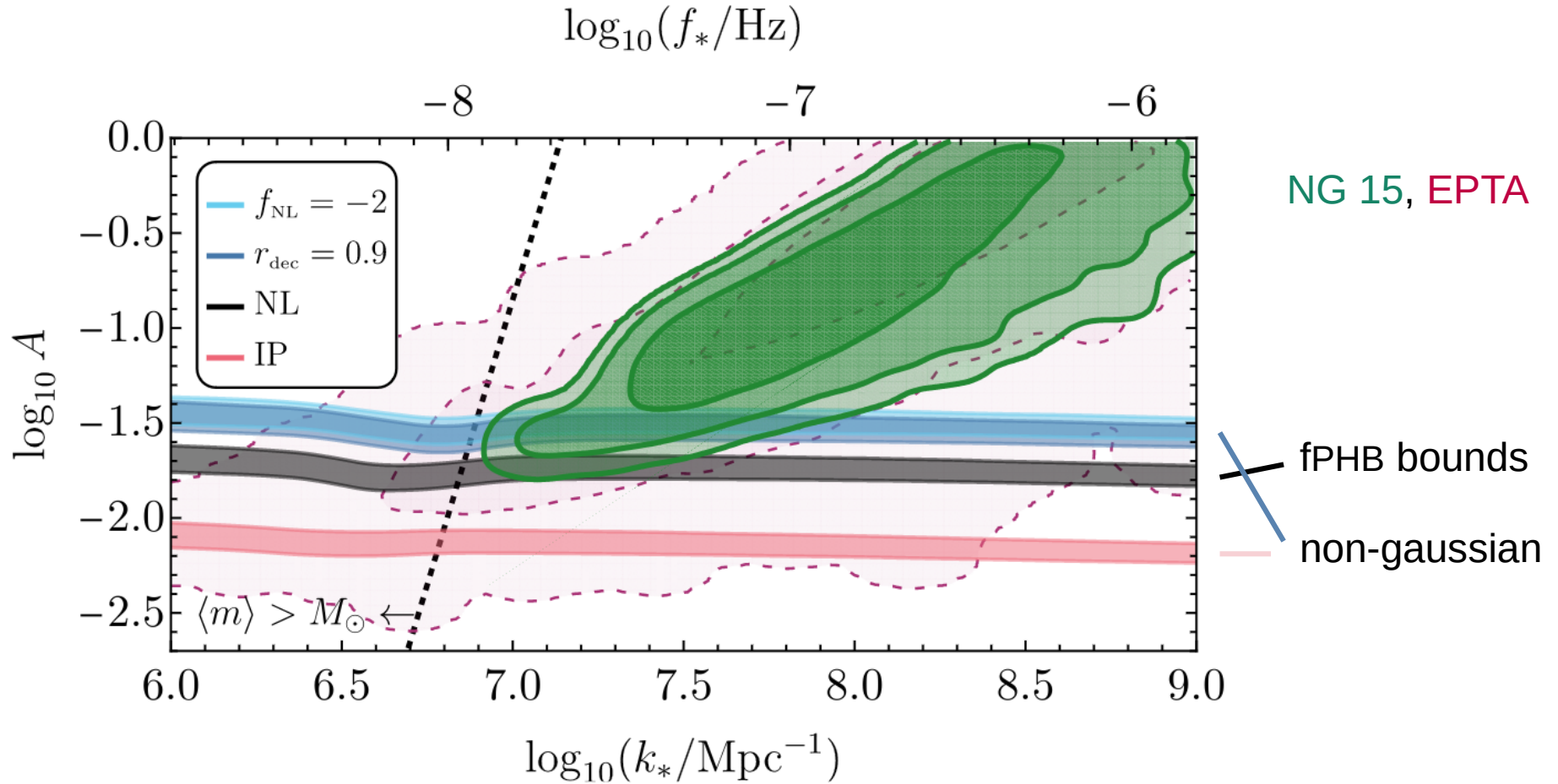
constraints on scalar power spectrum

narrow peak



GWs vs PTAs (2023 DR)

Franciolini, Iovino, Vaskonen, Veermäe '23



metastable cosmic strings : spectral tilt

