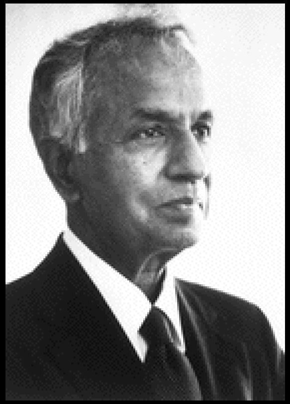
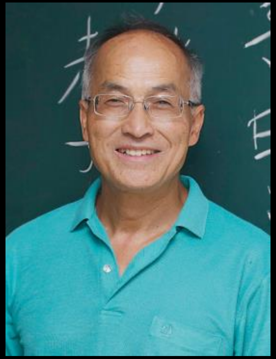
TESTING GRAVITY IN THE NANOHERTZ GW REGIME USING PTA CORRELATIONS

[ASCL:2211.001, ARXIV:2206.01056, 2208.12538, 2209.14834, 2302.11796, 2304.07040, 2304.07040, IN PREP, IN PREP, IN PREP]



Reggie Bernardo

with **Kin-Wang Ng**



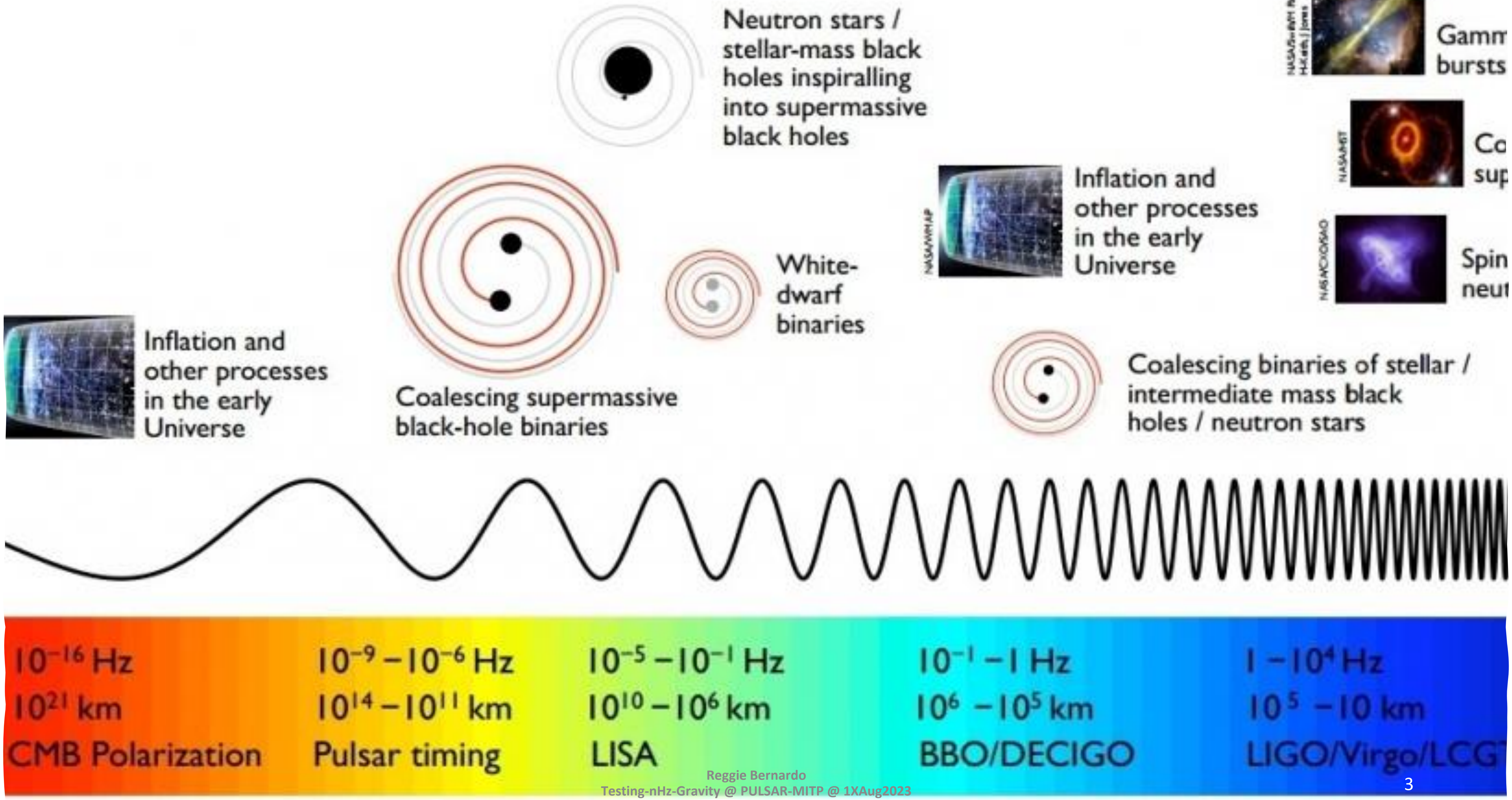
Institute of Physics, Academia Sinica

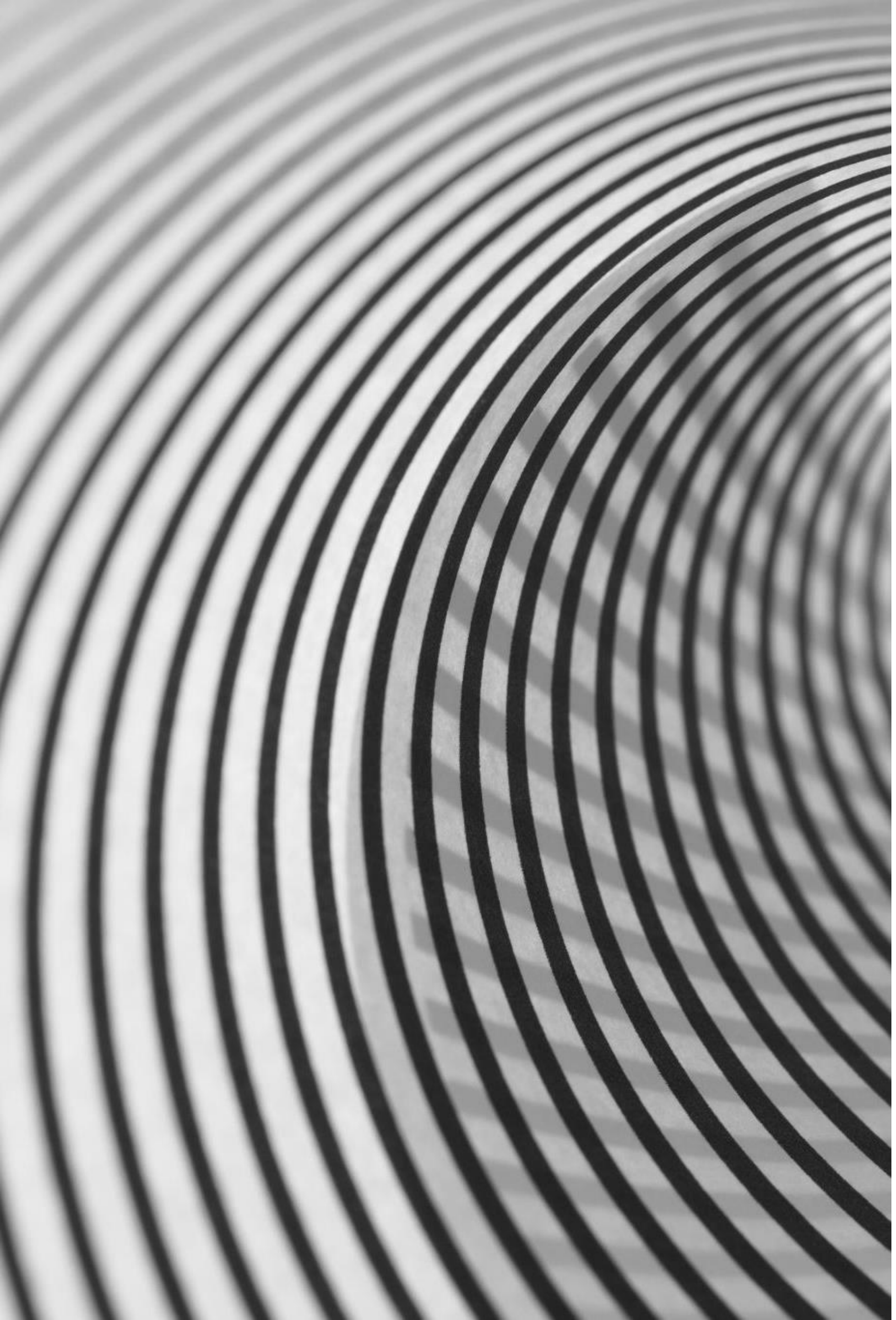
16 Aug 2023 @ PULSAR-A Star-Way To New Physics-MITP

Outline

1. **Gravitational Waves, GW Background, and Pulsar Timing Array**
2. **Stochastic GWB Phenomenology: Power Spectrum, Variance**
3. **Gravity Beyond Hellings-Downs**
4. **Testing Gravity in the Nanohertz GW Regime**
5. **Outlook**

Accessed 031022
https://www.ligo-india.in/wp-content/uploads/2016/02/gw-spectrum-1024x450-1.png





Gravitational Waves

- Spacetime distortions/perturbations

$$ds^2 = -dt^2 + (\delta_{ab} + \boldsymbol{h_{ab}})dx^a dx^b$$

- Wave properties:

  Carry energy, momentum, $v \sim 1$ @ 10^{1-3} Hz

- Tells about its sources

  e.g., BBH/BNS (LVK), IMRI/EMRI (LISA/TianQin)

- Challenge to overcome:

$$\boldsymbol{h_{ab} \sim G_{\mathrm{N}} \rightarrow \textbf{GW strains}:\ h \ll 1}$$

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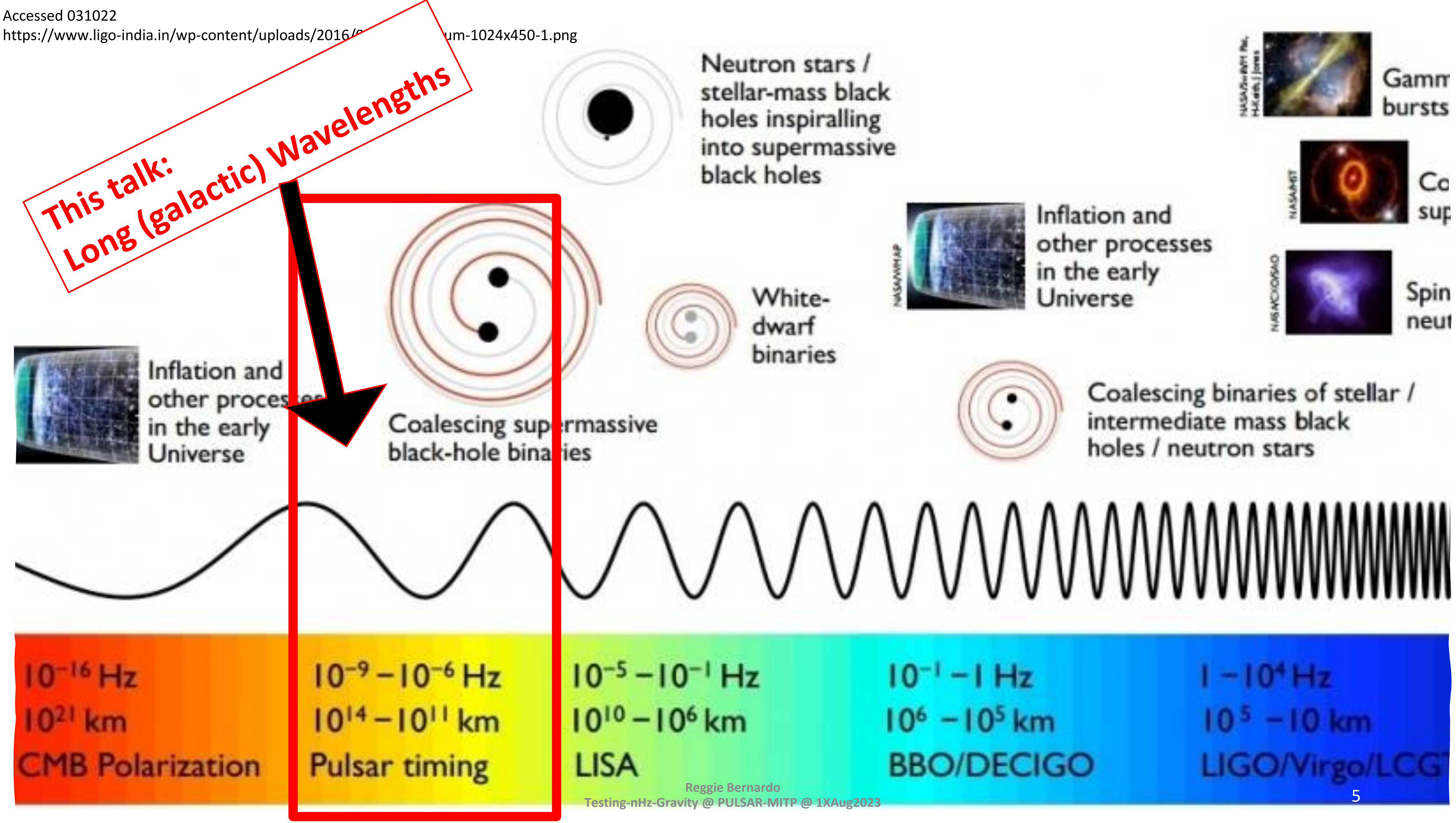
https://www.ligo-india.in/wp-content/uploads/2016/0 um-1024x450-1.png



Stochastic Gravitational Wave Background

- Results from many GWs from various sources;
- Sources tied to early cosmos.

THE ASTROPHYSICAL JOURNAL, 234:1100–1104, 1979 December 15

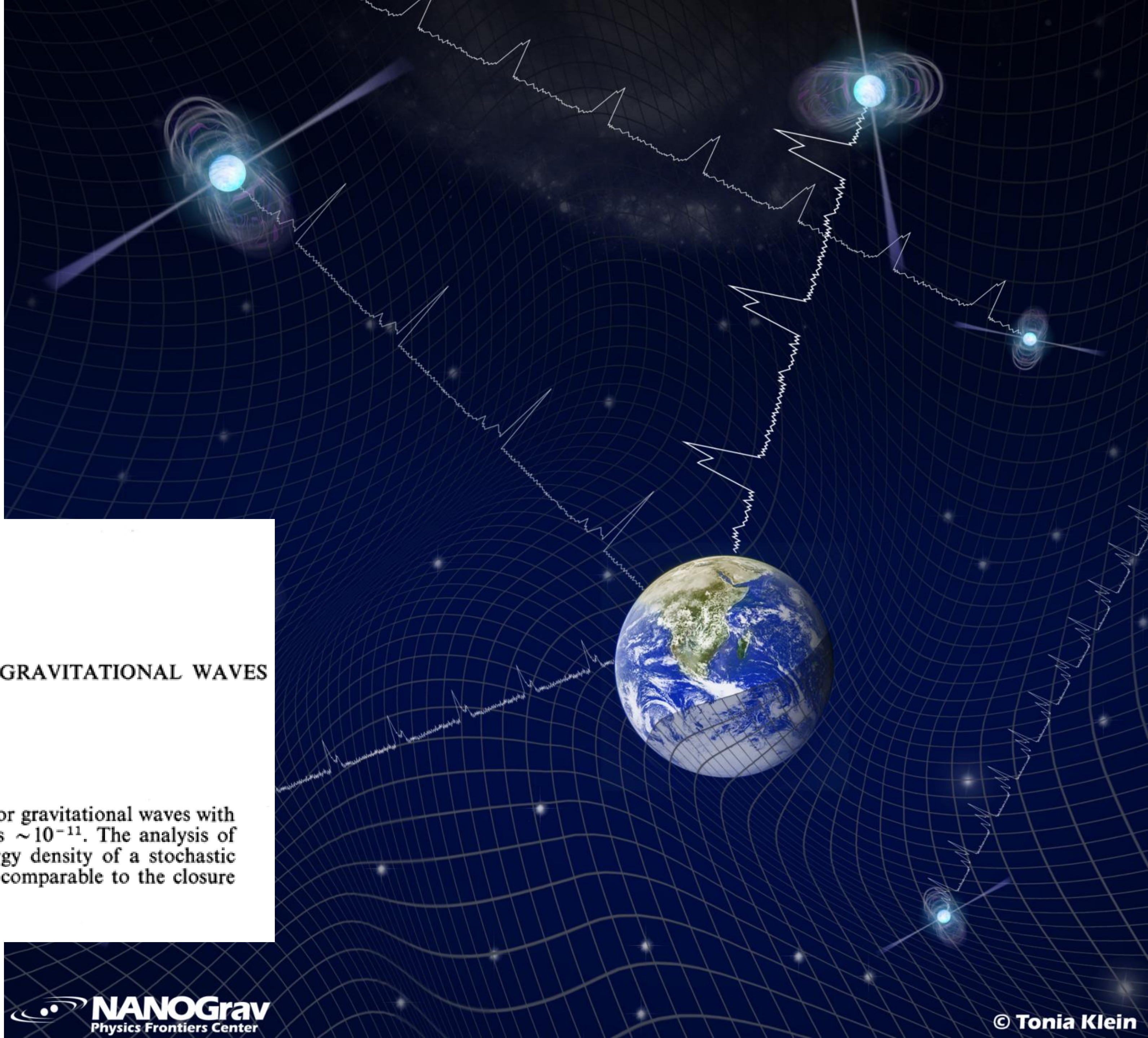
PULSAR TIMING MEASUREMENTS AND THE SEARCH FOR GRAVITATIONAL WAVES

STEVEN DETWEILER
Department of Physics, Yale University
Received 1979 June 4; accepted 1979 July 6

ABSTRACT

Pulse arrival time measurements of pulsars may be used to search for gravitational waves with periods on the order of 1 to 10 years and dimensionless amplitudes $\sim 10^{-11}$. The analysis of published data on pulsar regularity sets an upper limit to the energy density of a stochastic background of gravitational waves, with periods ~ 1 year, which is comparable to the closure density of the universe.

Subject headings: cosmology — gravitation — pulsars — relativity



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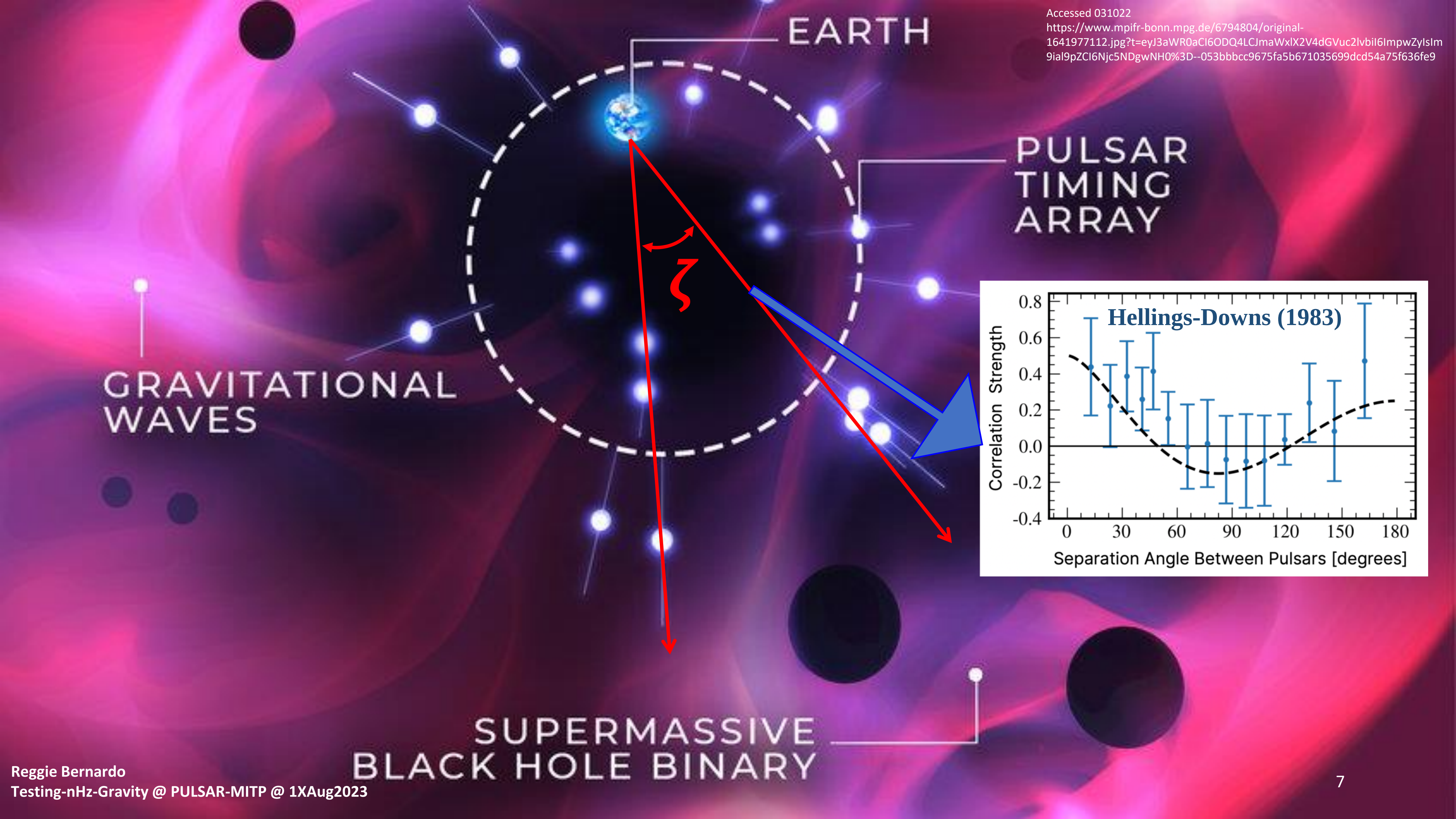


Pulsar Timing

- The **timing residual** = observation - timing model

$$r(t) = \int dt'\, z(t')$$

- Redshift fluctuation from **GW $\boldsymbol{h_{ij}(t)}$**:

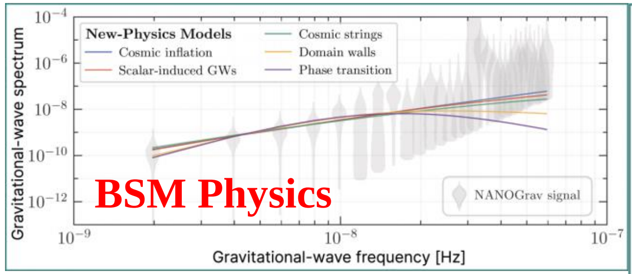
$$z(t) = -\frac{1}{2}\int d\eta\, \hat{e}^i \otimes \hat{e}^j\, \partial_\eta \boldsymbol{h_{ij}(\eta)}$$

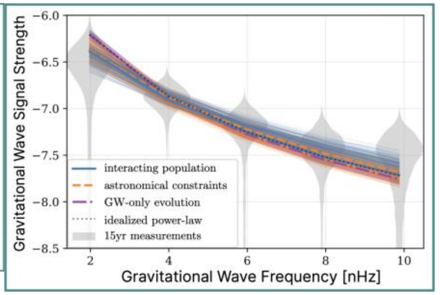
- **Two-point function**

Frequency Spectrum

$$\langle r_a(t) r_b(t)\rangle = \sum a_{lm} Y_{lm}(\hat{e}_a \cdot \hat{e}_b)$$

$$\sim \sum_A \int df\, \left(1 - e^{-2\pi i f t}\right)\left(1 - e^{2\pi i f t}\right) \times \boxed{\frac{P_{AA}(f)}{f^2}} \times \boldsymbol{\gamma^A_{ab}(\hat{e}_a \cdot \hat{e}_b)}$$





Pulsar Timing

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GW Correlation/
Overlap Reduction Function

$$\langle r_a(t) r_b(t) \rangle = \sum a_{lm} Y_{lm}(\hat{e}_a \cdot \hat{e}_b)$$

$$\sim \sum_A \int df \left(1 - e^{-2\pi i f t}\right)\left(1 - e^{2\pi i f t}\right) \times \frac{P_{AA}(f)}{f^2} \times \boxed{\boldsymbol{\gamma^A_{ab}(\hat{e}_a \cdot \hat{e}_b)}}$$

- **Quantifies SGWB induced correlation between pulsars *a* and *b*;**
- **A function of the frequency and the angle between pulsars across PTA.**

(Some) Correlations Theory

~2000s
NANOGrav
EPTA, PPTA,
InPTA, CPTA

~1-2 decades
<100 MSP

No GWB detection yet

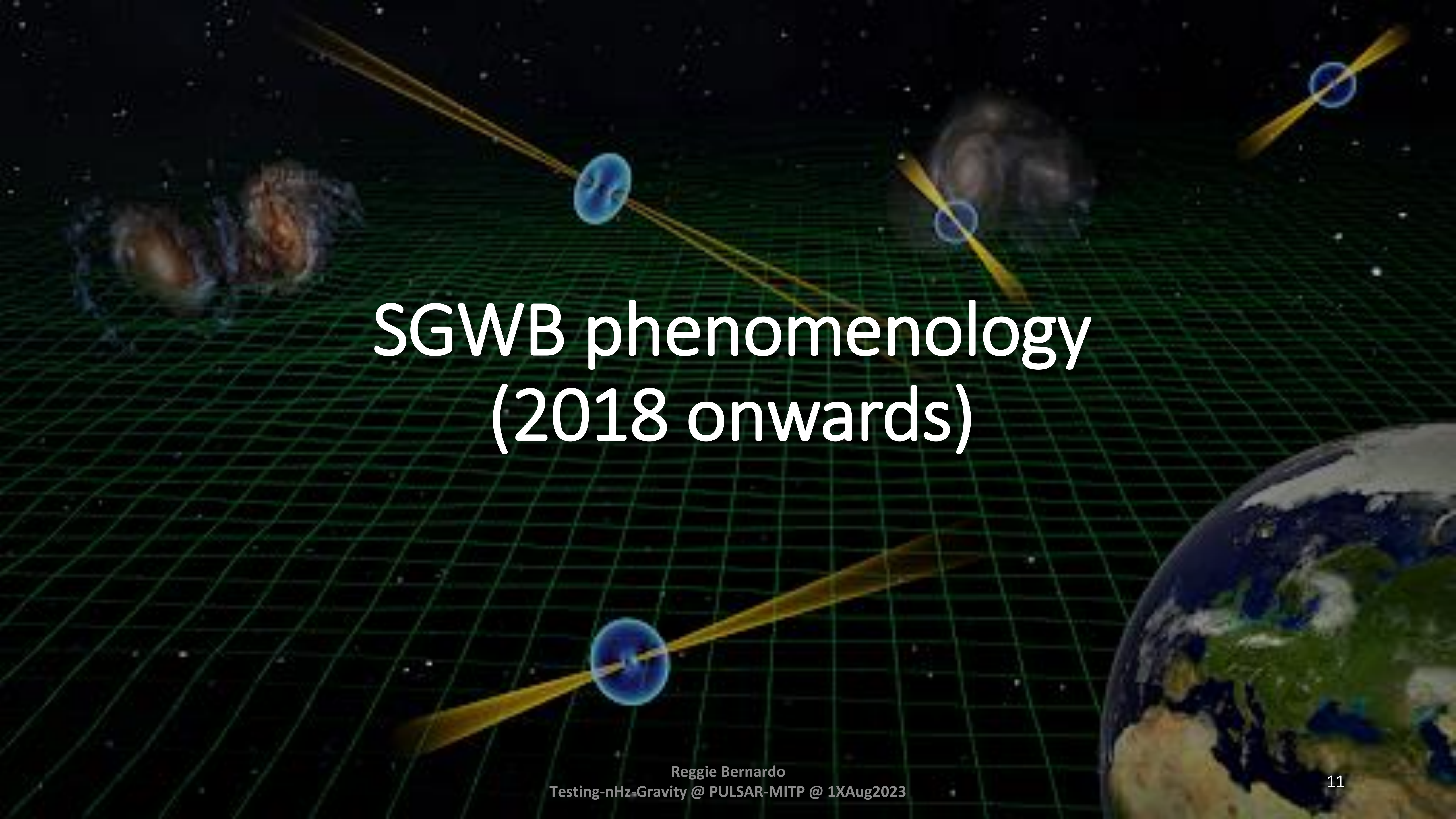
- ✓ 1979-Pulsar timing was proposed for the detection of nanohertz GWs (Detweiler)
- ✓ 1983-The SGWB (HD) correlation was derived (Hellings & Downs)
- ✓ 2001-The spectral profiles were derived for SGWB given their sources (Phinney)
- ✓ 2011-SGWB correlations were derived for non-Einsteinian GW polarizations propagating at the speed of light (Chamberlin)
- ✓ 2014-Power spectrum form of the HD was derived (Gair et al.)
- ✓ 2018-A power spectrum approach (PSA) was advanced for the calculation of SGWB correlations for non-Einsteinian GW modes (Qin, Boddy, Kamionkowski)
- ✓ 2020-PSA was generalized for subluminal SGWB correlations (Qin, Boddy, Kamionkowski)
- ✓ 2021-SGWB correlations were calculated for the massive gravity degrees of freedom (Liang & Trodden)
- ✓ 2021-PSA for luminal tensor GW modes was revisited, and generalized to finite pulsar distances (KWN)
- ✓ 2022-The variance of HD was calculated (Allen)
- ✓ 2022-HD variance was generalized to consider arbitrary pulsar sky distributions (Allen & Romano)
- ✓ 2022-The PSA was generalized for subluminal SGWB correlations by non-Einsteinian modes and finite pulsar distances (RCB & KWN)
- ✓ 2022-Variance of non-Einsteinian subluminal SGWB correlations (RCB & KWN)
- ✓ 2023-…

This talk!

Precision PTA era

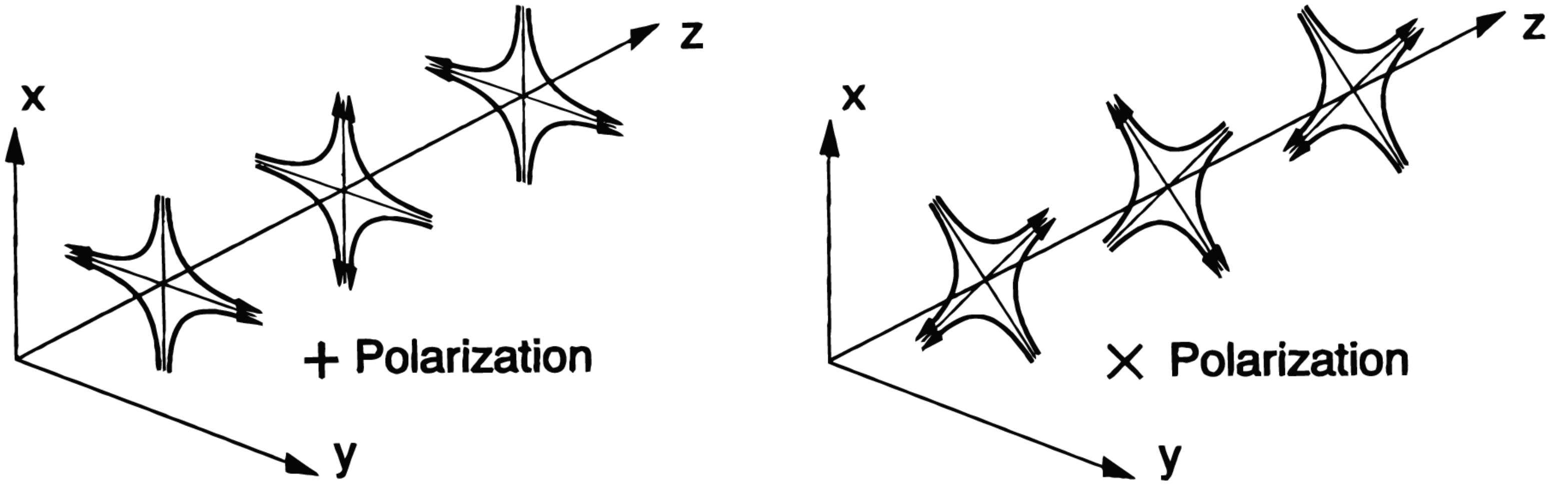
- GWs beyond (+, x)
- SMBBH
- Non-Gaussianity
- Cosmic strings, PTs, etc…

GWB Detection! ☺

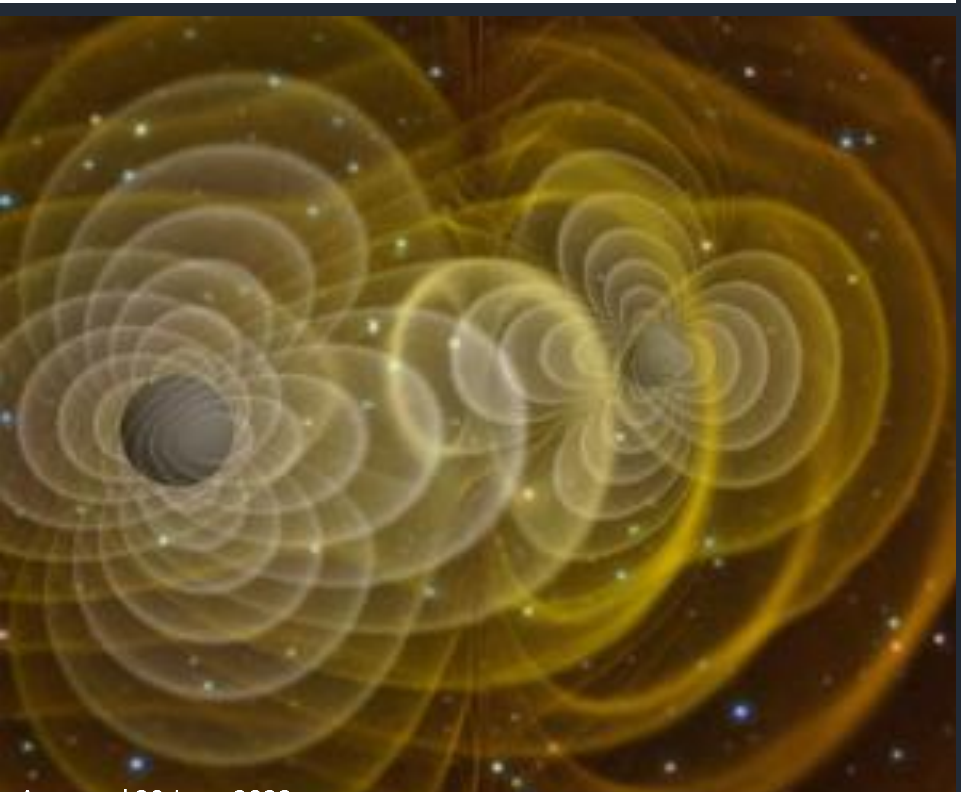


SGWB phenomenology (2018 onwards)

Gravitational Wave Polarizations



Accessed 30 June 2022: https://i.stack.imgur.com/IW50W.png



Accessed 30 June 2022: https://www.ligo.org/science/GW-Overview/images/GWvisual_tn.jpg

$$h_{ij}(\eta, \vec{x}) = \sum_A \int df \int d\hat{k}\ \tilde{h}_A\left(f, \hat{k}\right) \boldsymbol{\varepsilon}^{\boldsymbol{A}}_{\boldsymbol{ij}}\, e^{-2\pi i f\left(\eta - v\hat{k}\cdot\vec{x}\right)}$$

GW amplitude → $\tilde{h}_A$

polarization basis tensor → $\boldsymbol{\varepsilon}^{\boldsymbol{A}}_{\boldsymbol{ij}}$

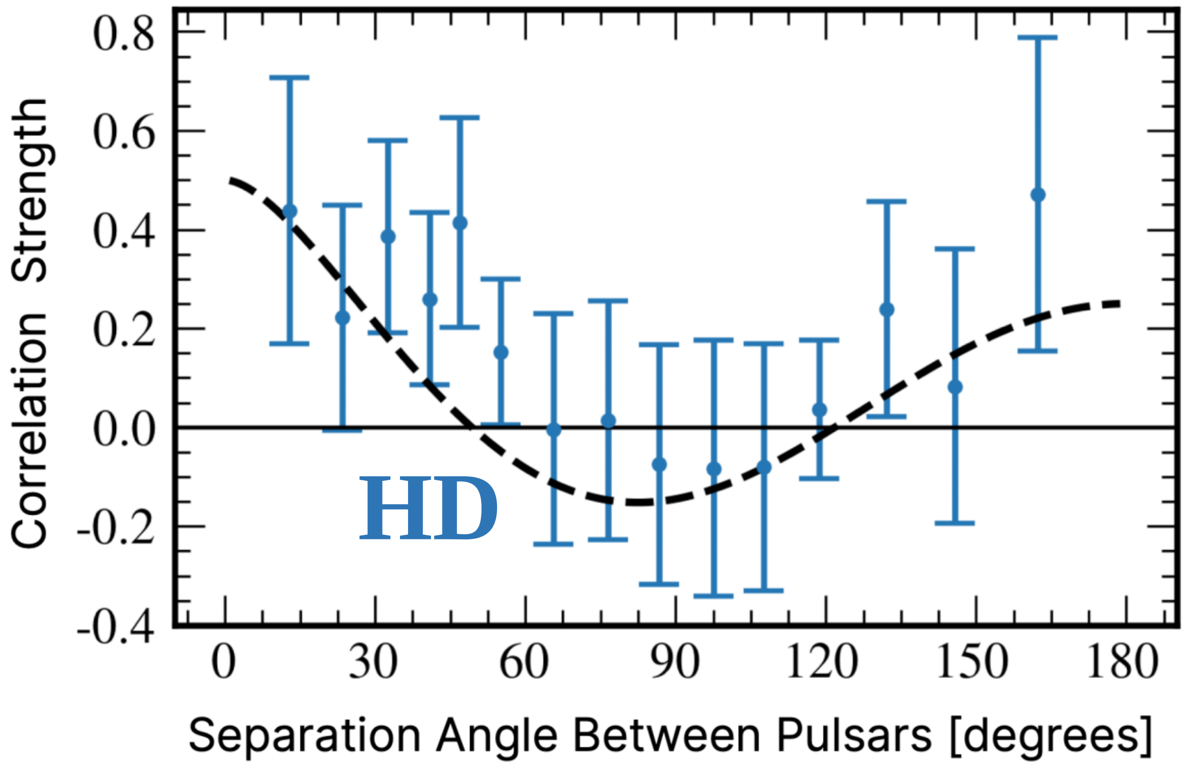
velocity → v

Hellings-Downs Curve

- ORF limit:

  TT tensor w/ $\underline{v = 1}$ and $\underline{D \to \infty}$;
- Mainly the **quadrupole**;
- Analytic formula by HD;
- In PSF,

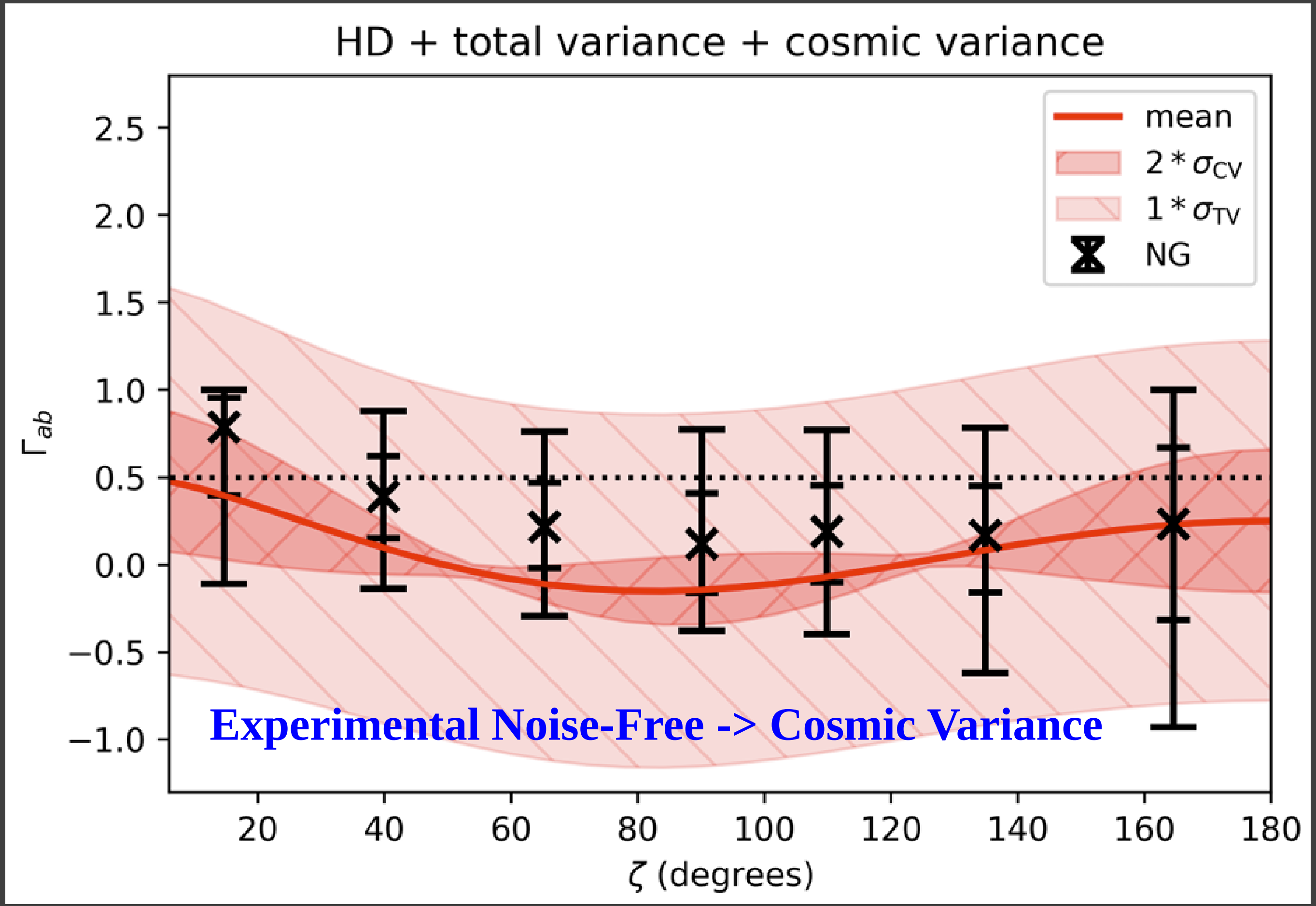
$$C_l \sim \frac{2\pi}{(l-1)l(l+1)(l+2)}$$

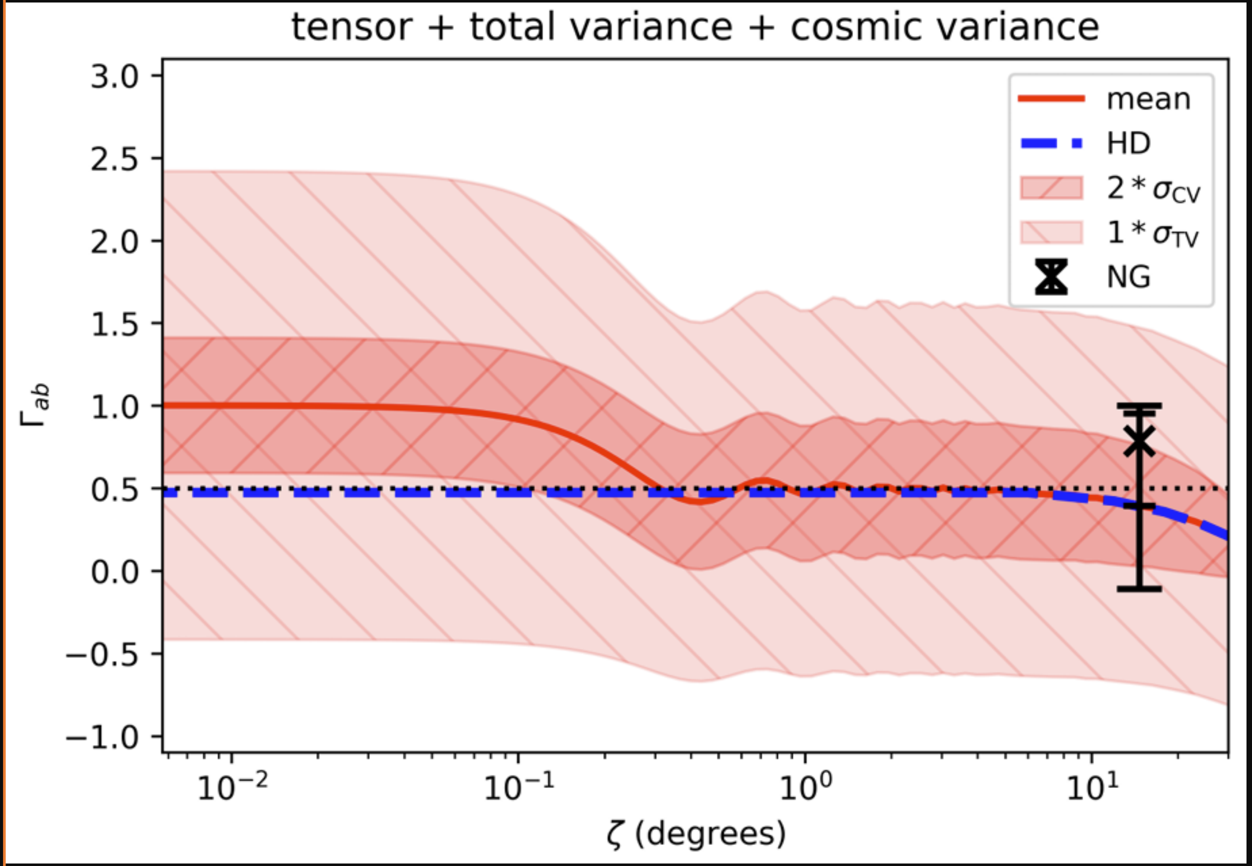


Theoretical Uncertainty

2205.05637 (Allen)
- Theory uncertainty of the HD

2209.14834 (RCB & KWN)
- General GW polarizations
- On/off light cone
- Finite pulsar distances

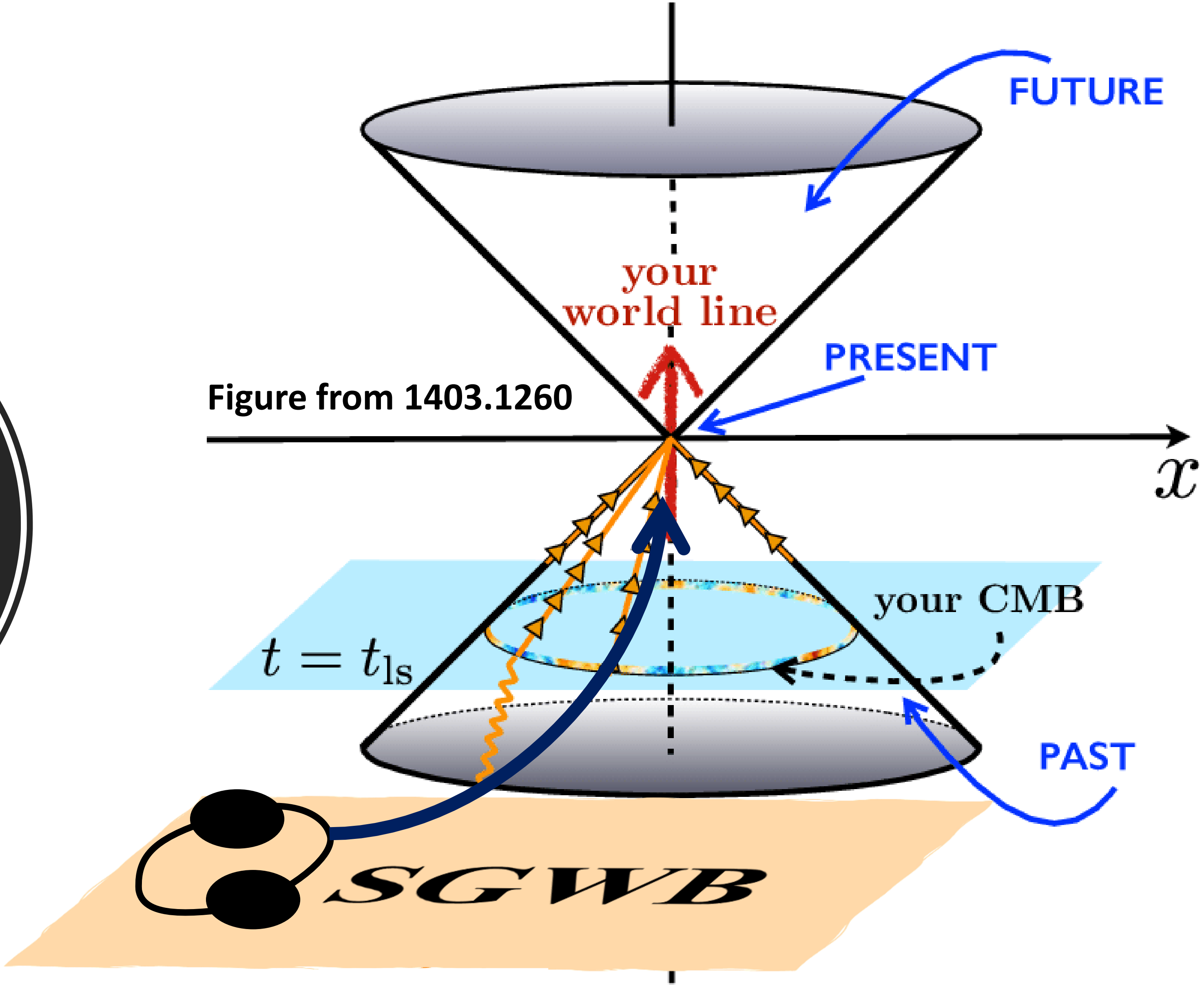


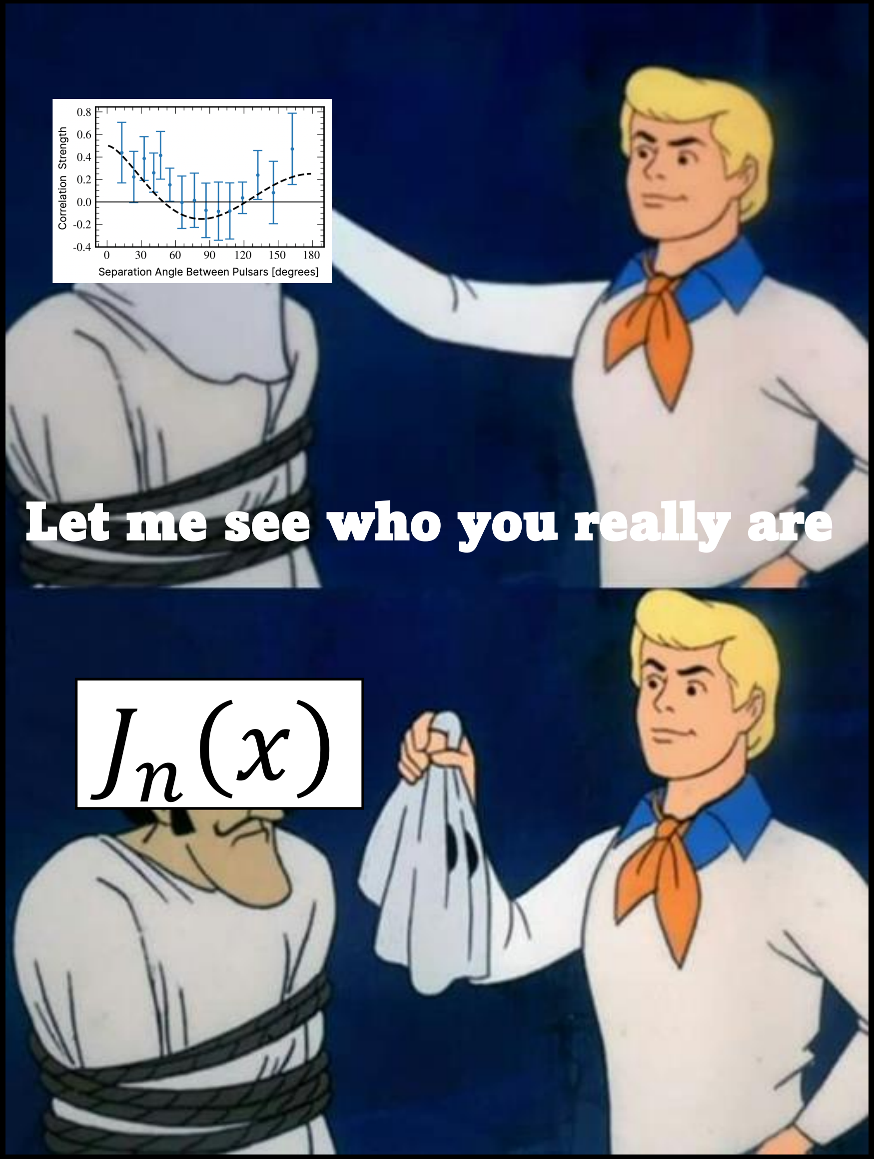


Finite pulsar distance

- $D \sim \mathrm{O}\left(10^{2-3}\right)$ pc, HD out of $2\sigma_{\mathrm{CV}}$;
- Accounts for power at small scales;
- Easy to accommodate using PSF
  - ❑ Angular resolution $l \leq l_{\max}$

GWs On & Off Lightcone

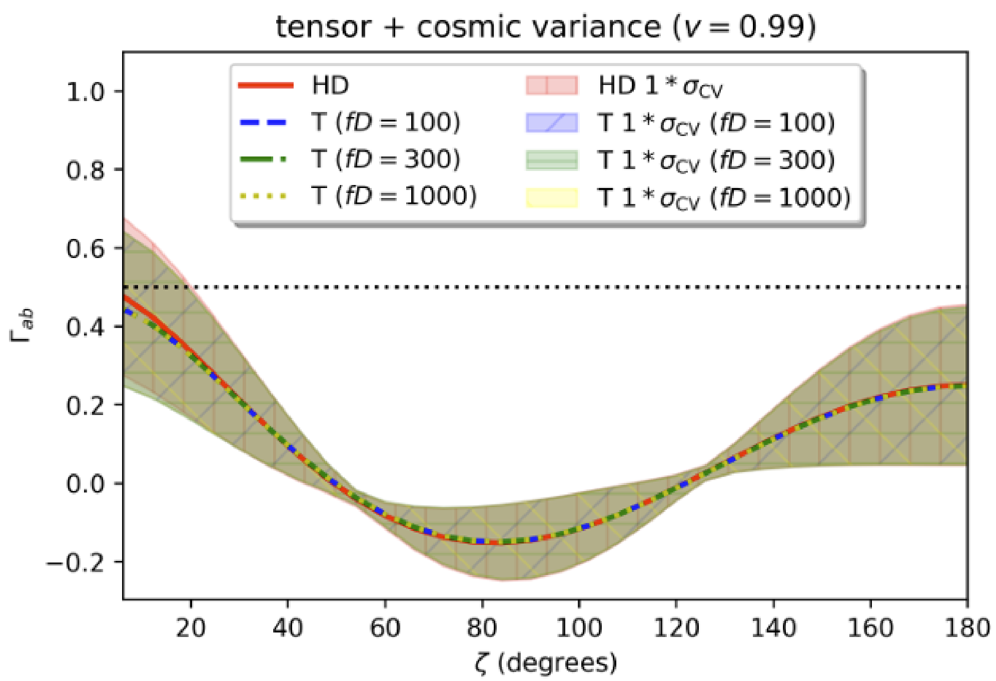


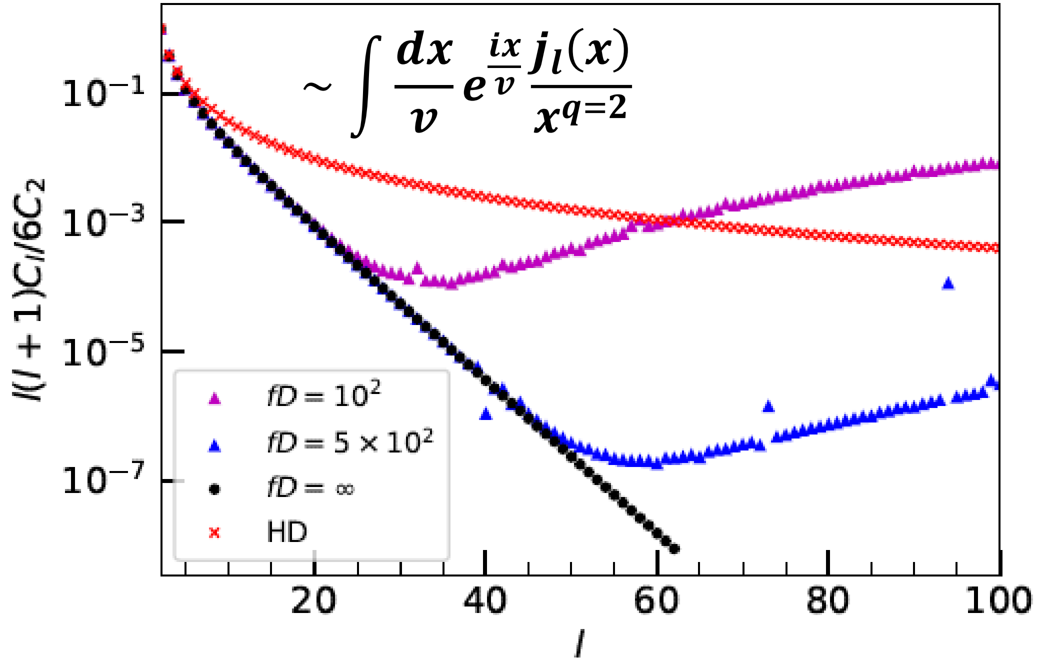


GWB correlation

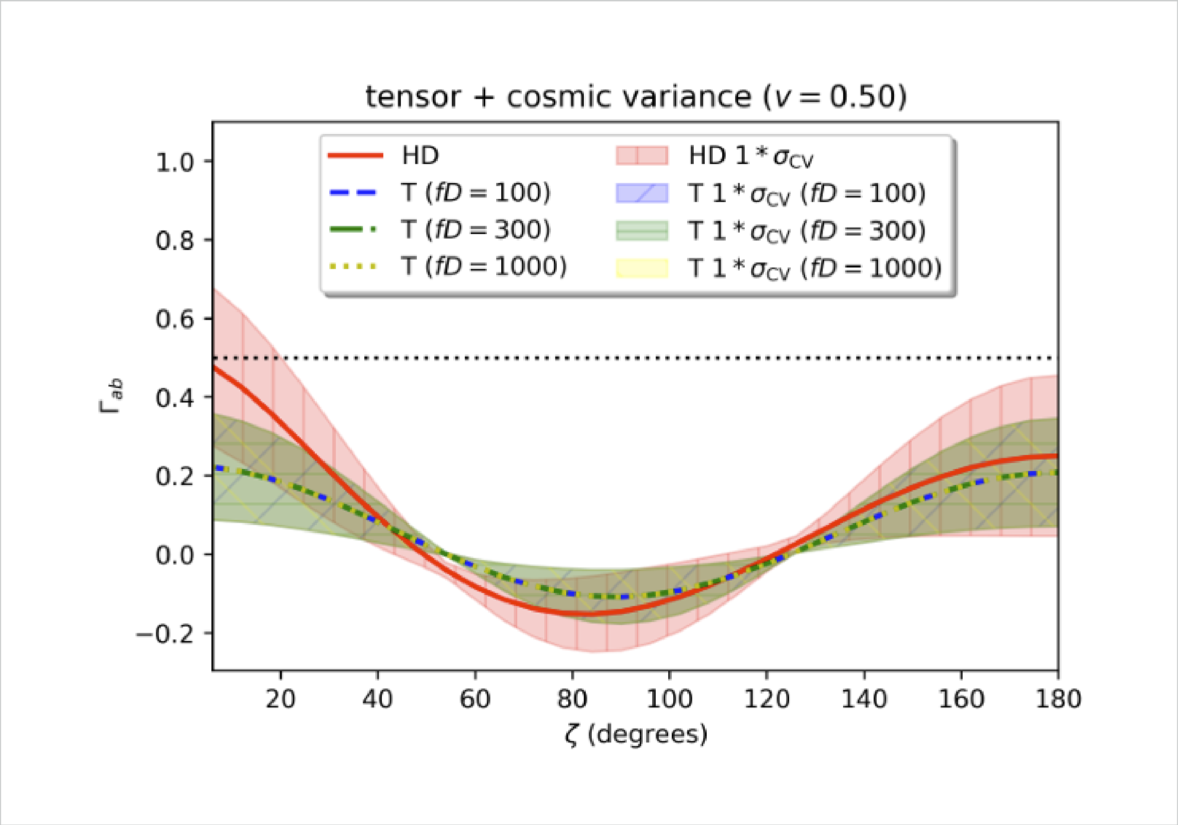
$$\langle r_a r_b \rangle \sim (2l+1)\left(\int \frac{dx}{v}\, e^{\frac{ix}{v}} \frac{j_l(x)}{x^q}\right) P_l(\hat{e}_a \cdot \hat{e}_b)$$

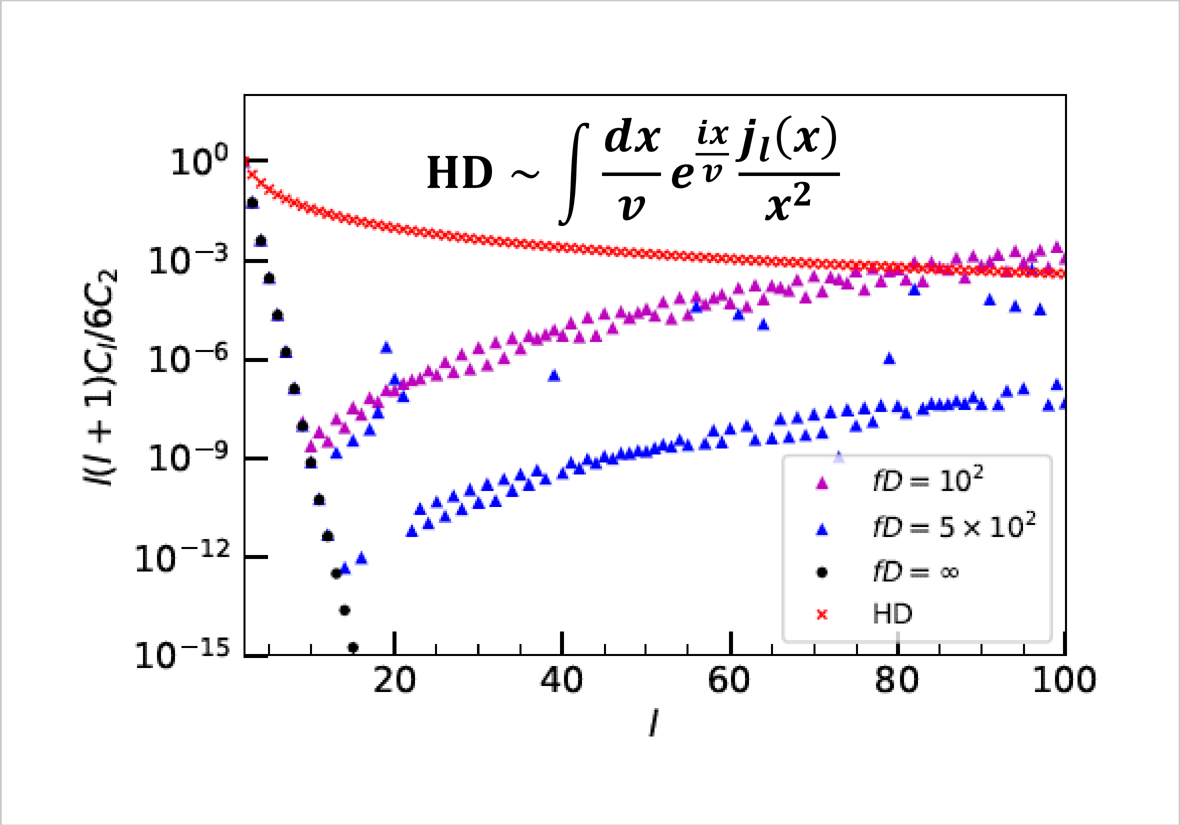
Tensor PS and ORF ($v \sim 1$, near luminal)



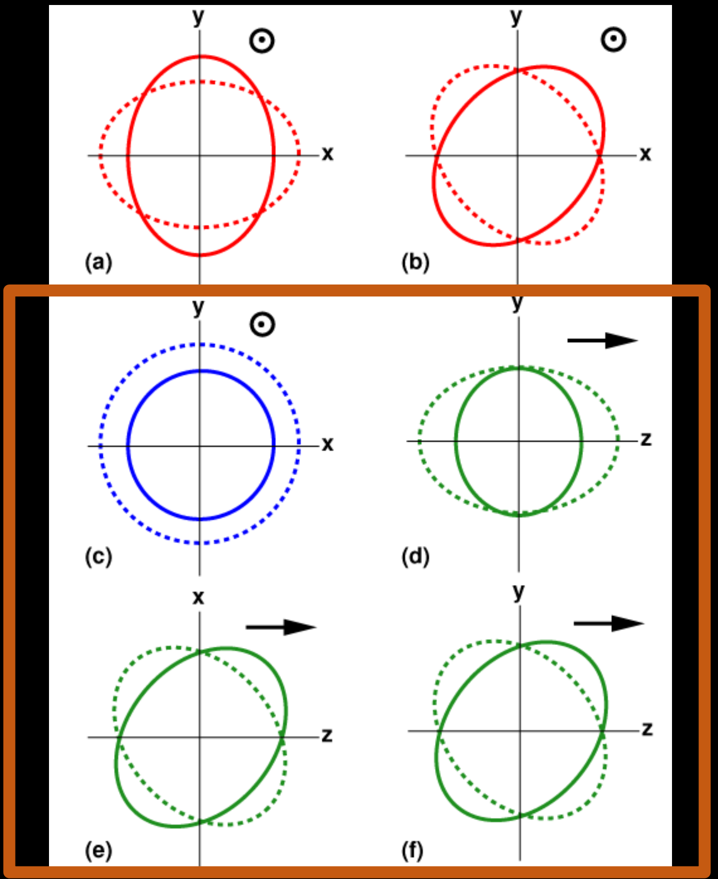


Tensor PS and ORF ($v \sim 1/2$, half luminal)

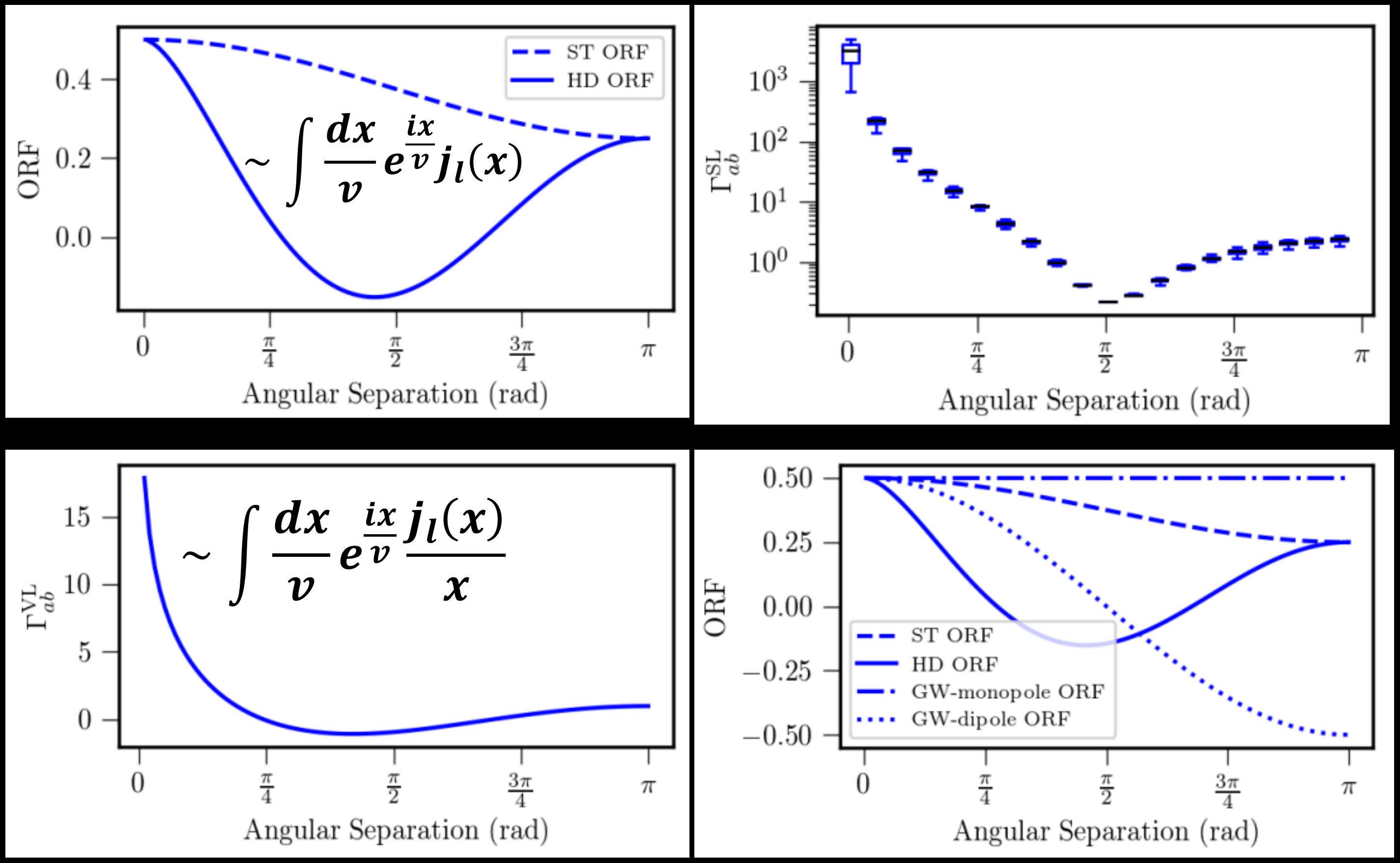




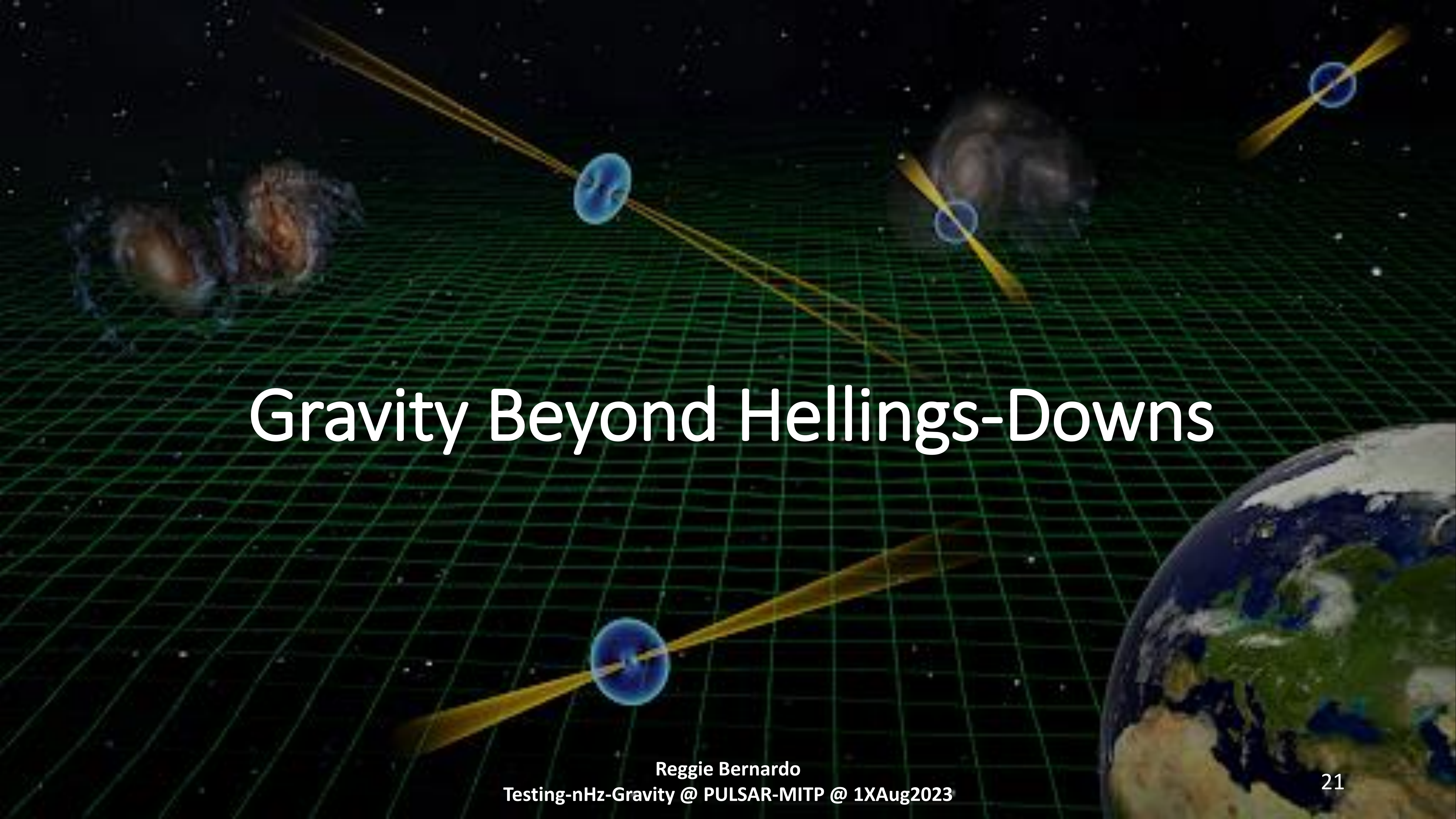
GW Polarizations: Beyond Einstein



Accessed 30 June 2022:
https://www.ligo.org/science/Publication-GW170814/images/figure5.png



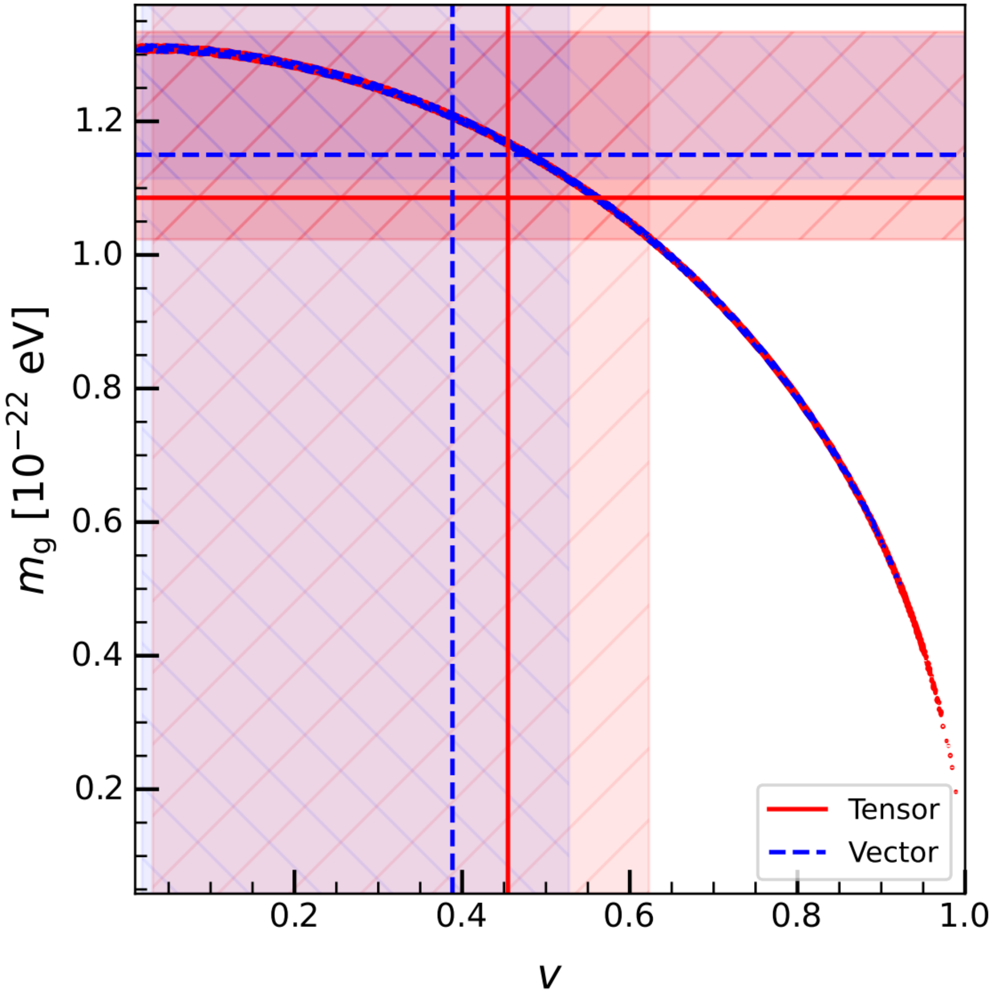
NANOGrav: arXiv:2109.14706



Gravity Beyond Hellings-Downs

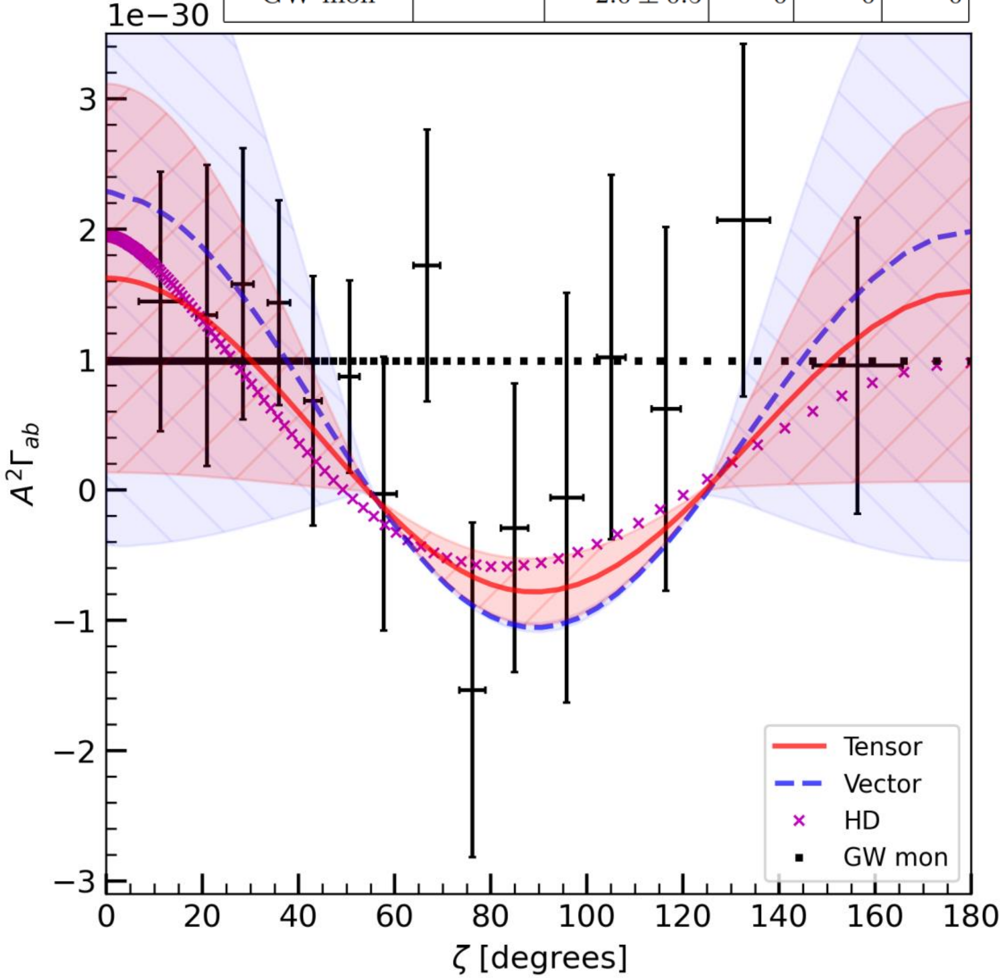
GW propagation : 2302.11796

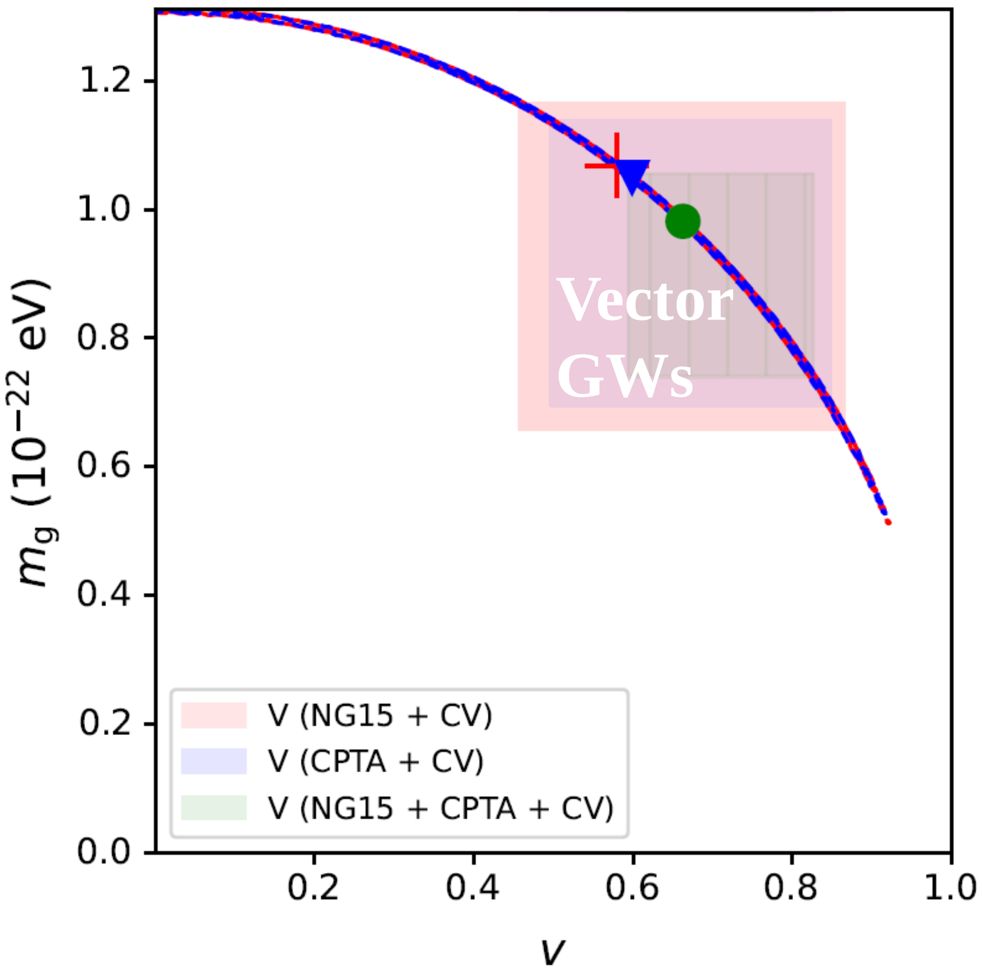
Tensor + Vector + NANOGrav

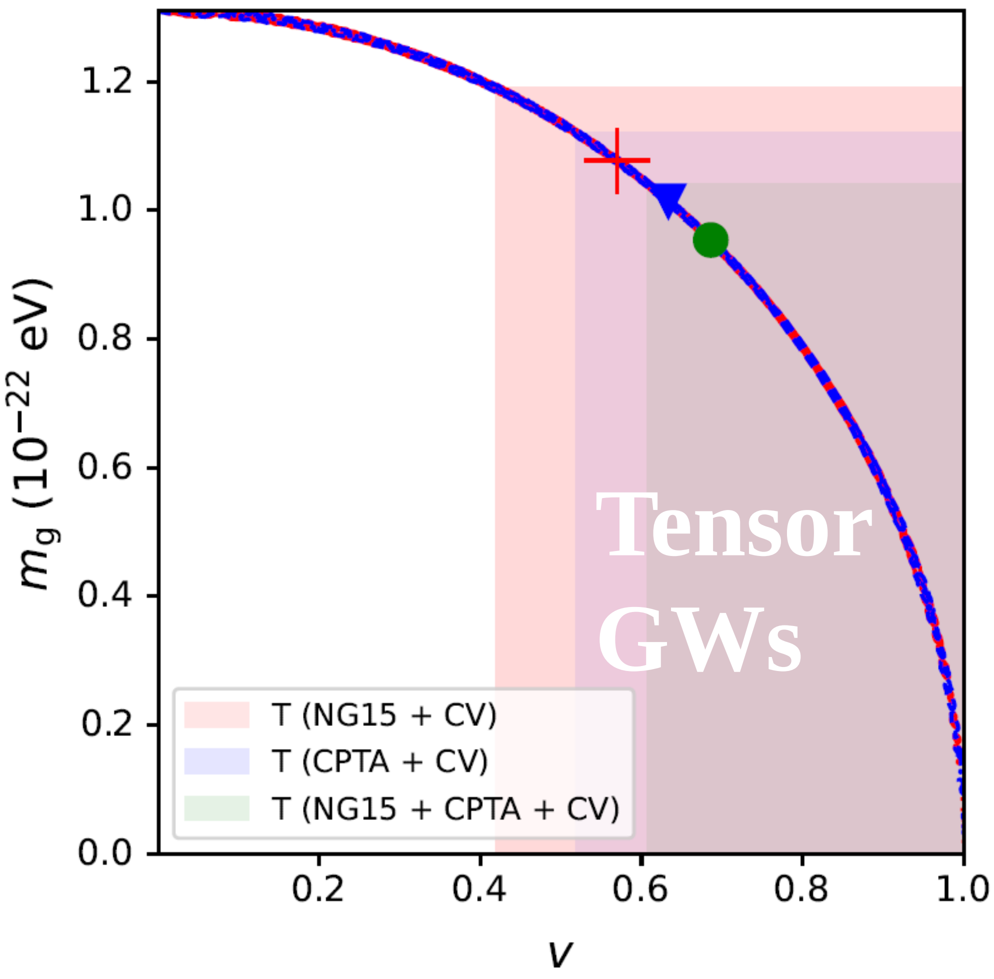


(+) is good | (-) not so

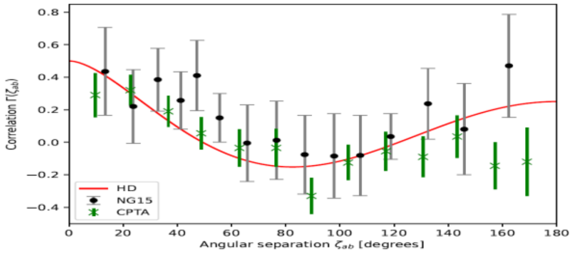
mode	v	A^2 [$\times 10^{-30}$]	$\Delta\bar{\chi}^2$	ΔAIC	ΔBIC
Tensor	$0.45^{+0.17}_{-0.42}$	$7.4^{+2.1}_{-2.4}$	0.06	−1.01	−1.79
Vector	$0.39^{+0.14}_{-0.37}$	$9.2^{+2.4}_{-6.0}$	−0.11	−3.76	−4.53
HD	$v = 1$	3.9 ± 1.1	−0.10	−1.66	−1.66
GW mon	——	2.0 ± 0.5	0	0	0







In preparation (NG15 + CPTA)



Scalar/Galileon GWs

- **Tensor** perturbations -> **+, x polarizations**
- **Scalar** perturbations satisfy the **massive KG eq**.

$$D^2\psi - \ddot{\psi} - \underbrace{m_{\text{eff}}^2(\mu, \alpha, \lambda)}\psi = 0$$

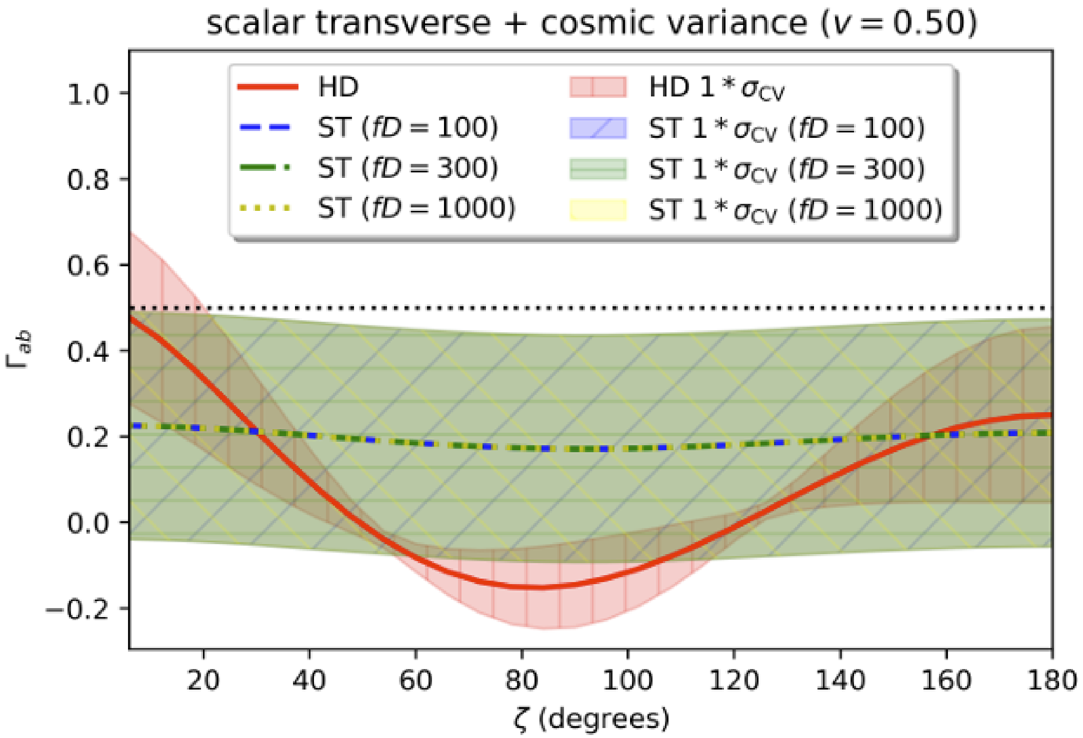
- Brings in scalar transverse (**ST**) and longitudinal (**SL**) **pols**:

$$m_{\text{eff}}^2 = \mu^2 \left(\frac{1 - \frac{\alpha\lambda^3}{M_{\text{P}}\mu^2}}{1 + \frac{3\alpha^2}{2} - \frac{\alpha\lambda^3}{M_{\text{P}}\mu^2}} \right)$$

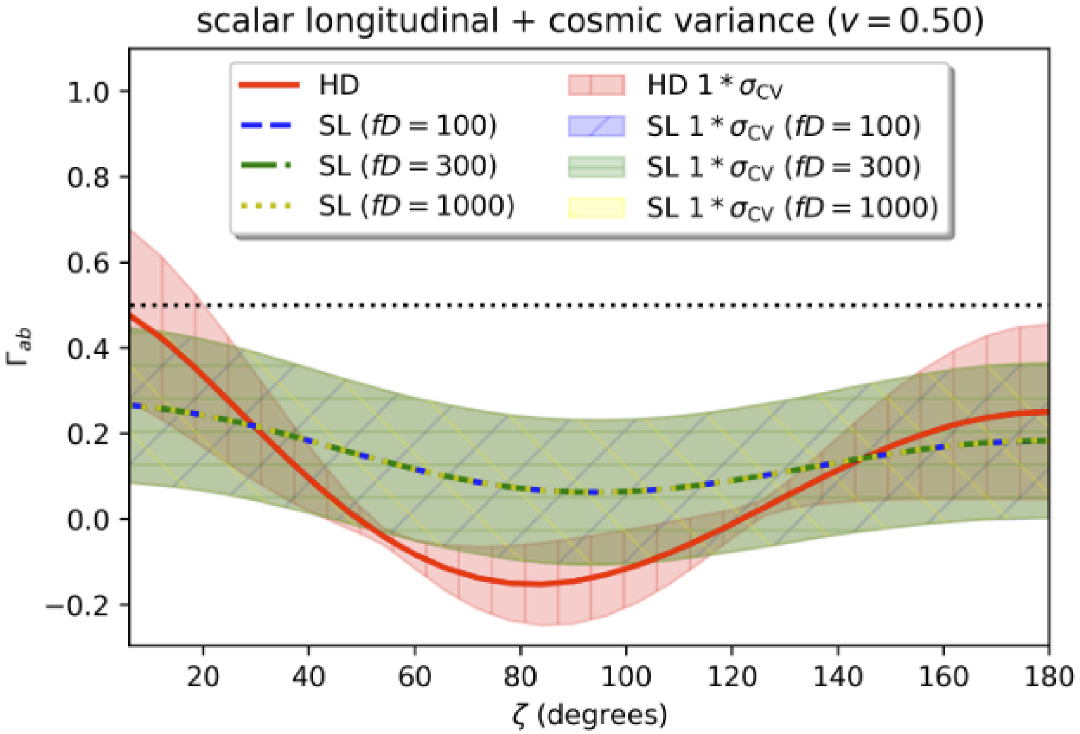
$$h_{AB} \propto \left(\boldsymbol{\varepsilon}_{AB}^{\mathbf{ST}} + \frac{1 - v(m_{\text{eff}})^2}{\sqrt{2}} \boldsymbol{\varepsilon}_{AB}^{\mathbf{SL}} \right) \times \text{plane wave}$$

Scalar ORFs ($v = 1/2$, half luminal)

ST

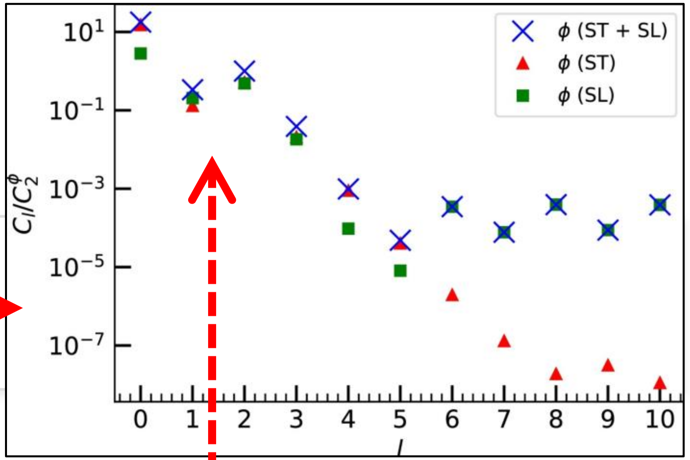


SL



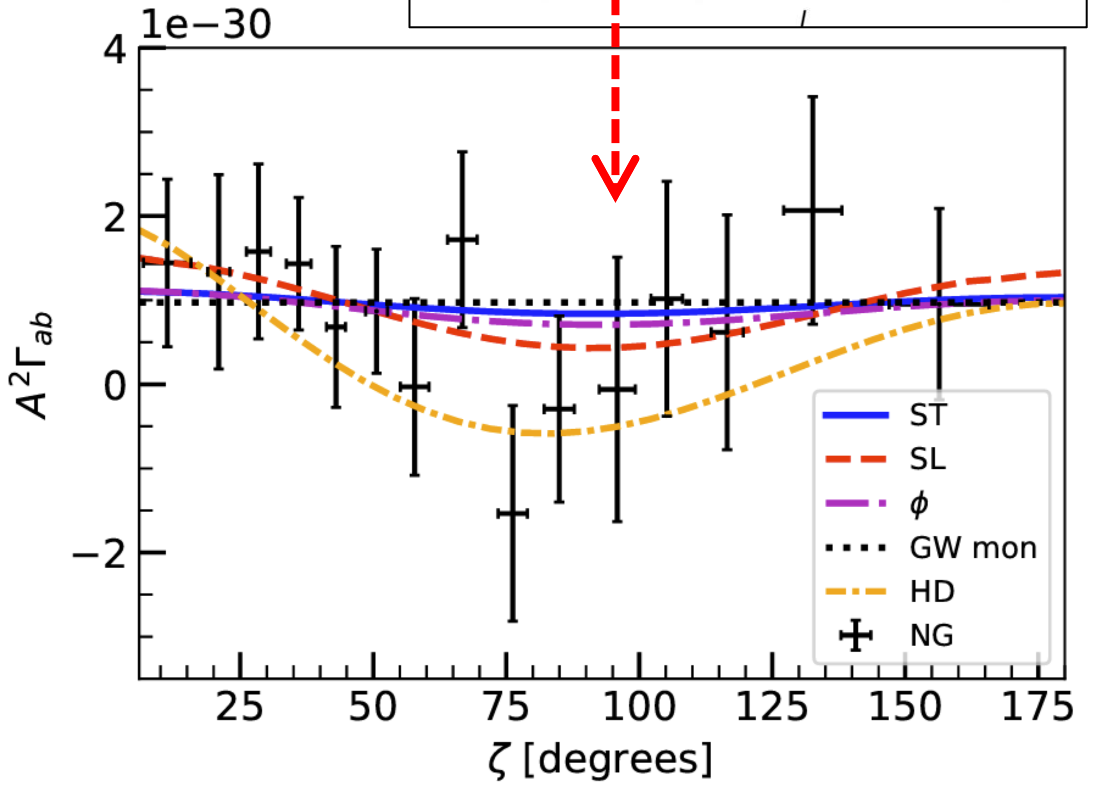
Best fit in NG12

$$v = 0.44^{+0.15}_{-0.42}\ c\ \rightarrow m_{\rm eff} \sim 10^{-22}\ \text{eV (Galileon)}$$

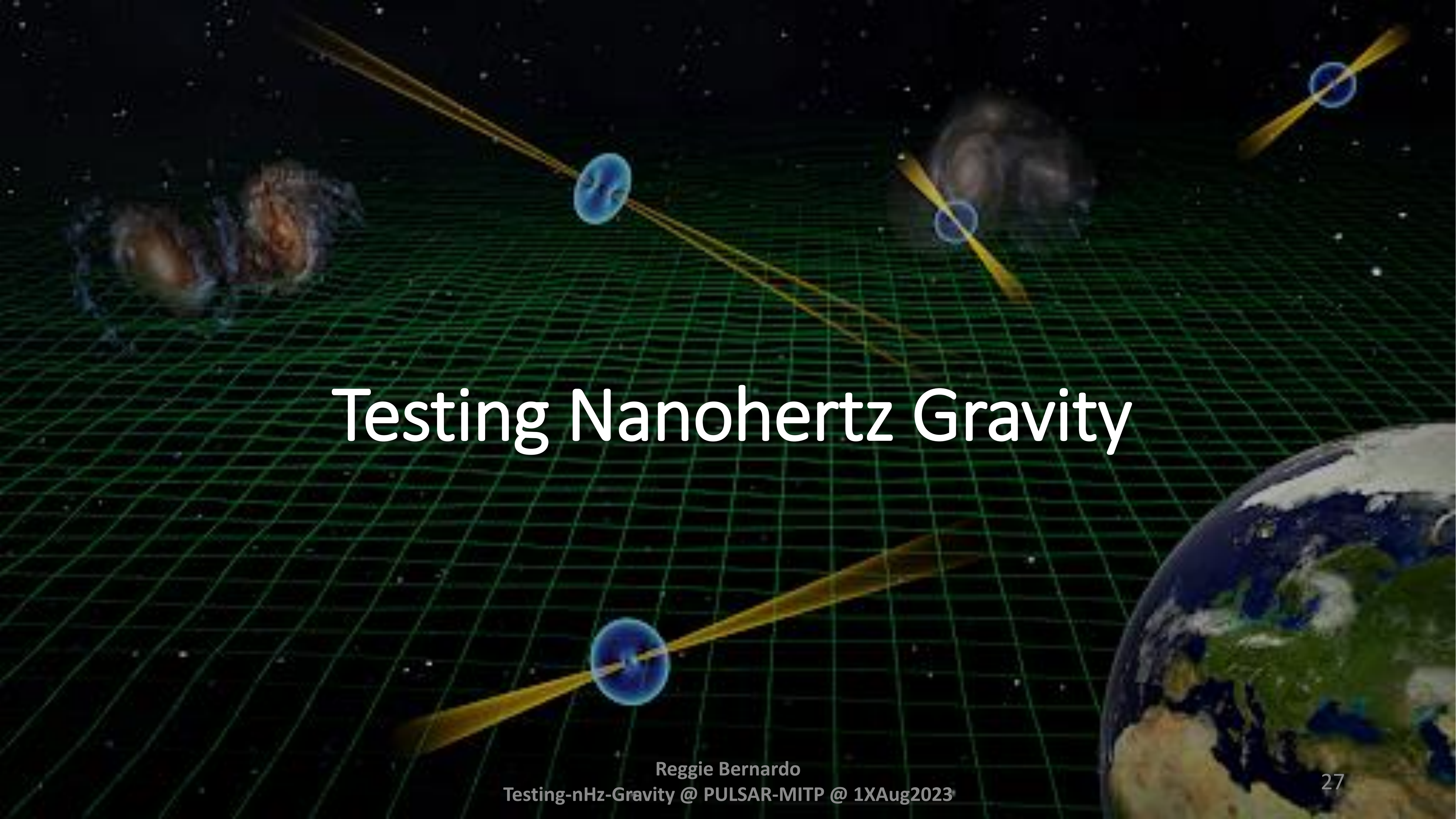


Marginalized statistics for the ST, SL, and the Galileon (ϕ) constrained by PTA observation [1]. Results for the HD correlation and the GW monopole (GW mon.) are presented for comparison. The performance statistics (chi-squared, AIC, and BIC [34,37]) are relative to the GW monopole, or that a negative value means statistical preference over the GW monopole.

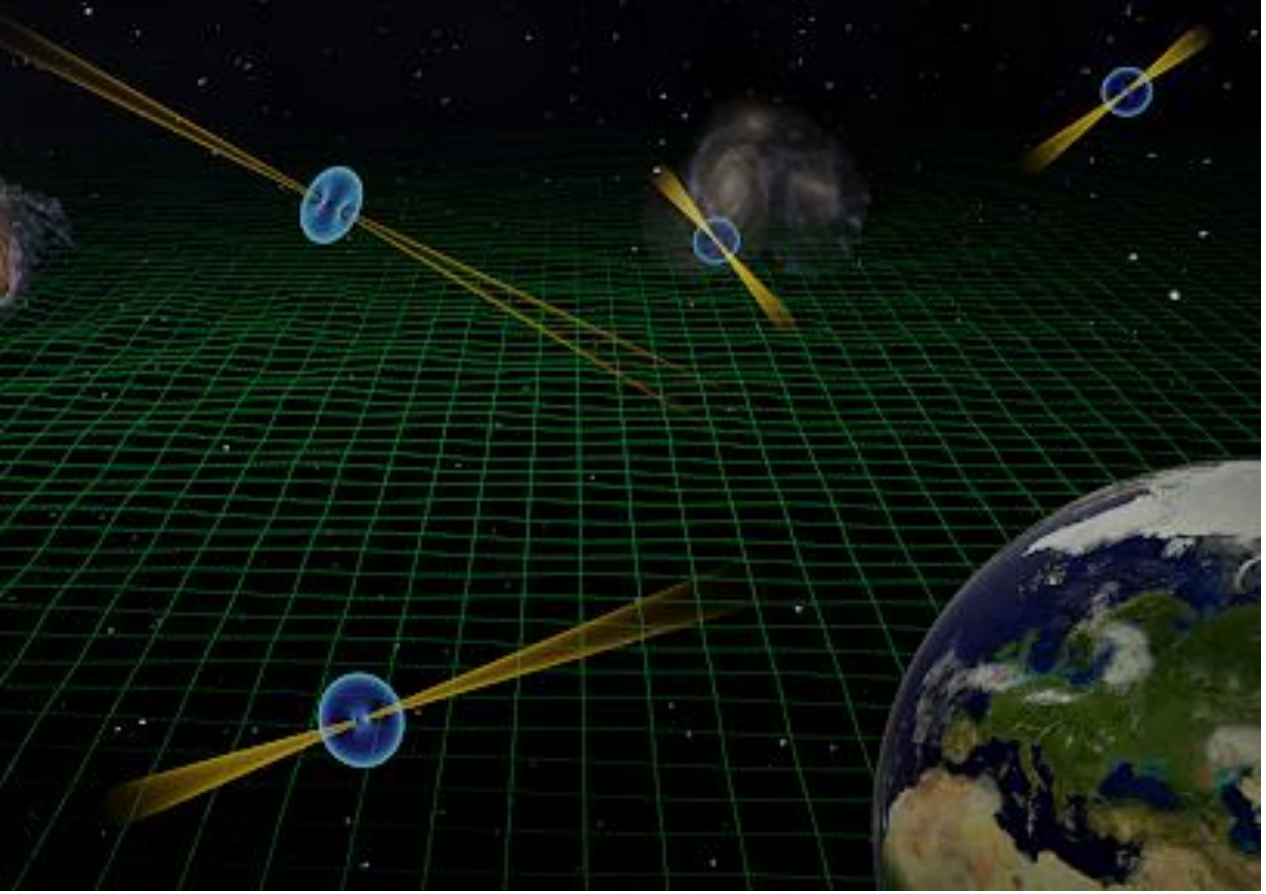
mode	v	A^2 [$\times 10^{-30}$]	$\Delta\chi^2$	ΔAIC	ΔBIC
ST	0.46 ± 0.24	$5.0^{+1.4}_{-1.6}$	-1.47	0.53	1.24
SL	< 0.44	$7.9^{+2.8}_{-3.4}$	-3.92	-1.92	-1.21
ϕ	**$0.44^{+0.15}_{-0.42}$**	**3.8 ± 1.2**	**-2.91**	**-0.91**	**-0.20**
HD	$v = 1$	3.9 ± 1.1	1.66	1.66	1.66
GW mon.	–	1.94 ± 0.48	0	0	0

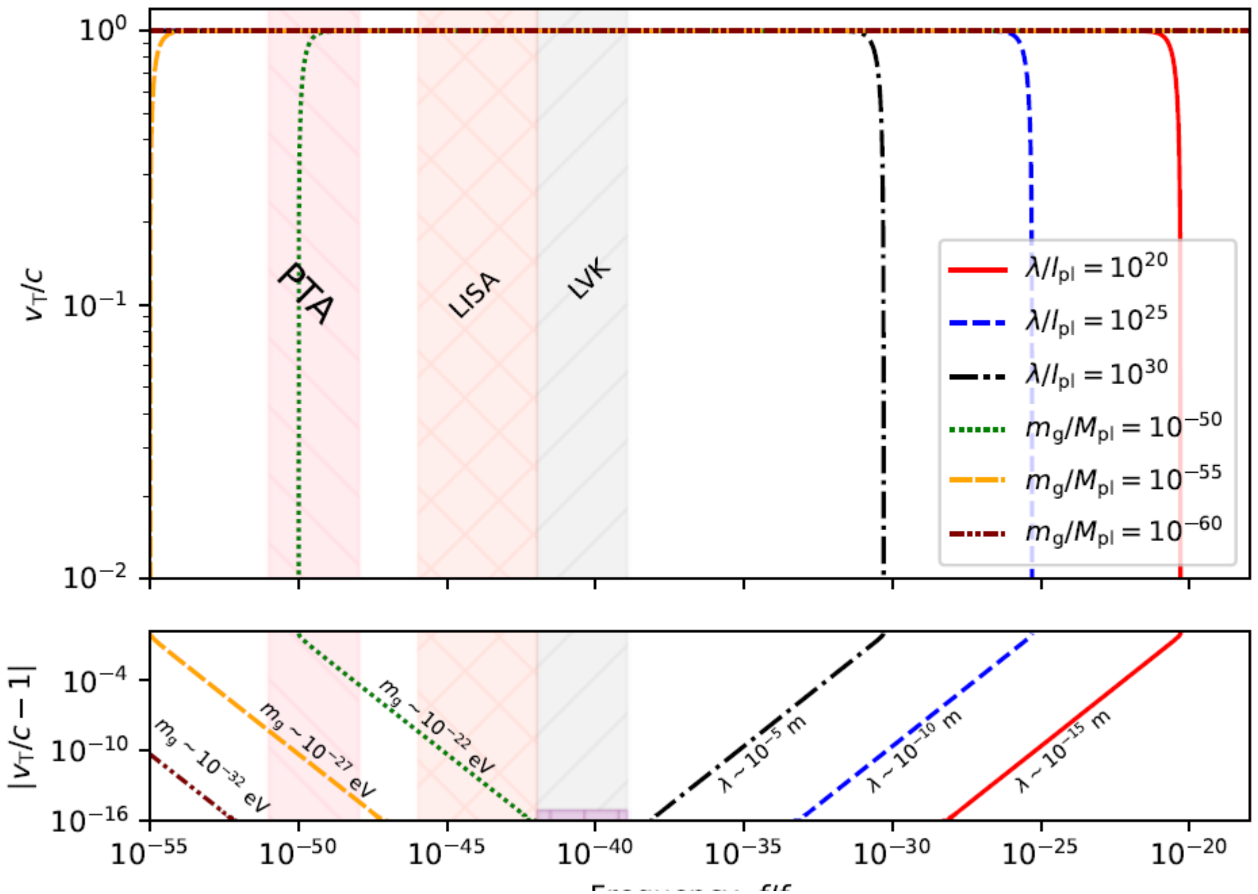


2206.01056 (RCB & KWN)
X by NG15 + PTAs
In preparation: HD + phi / T + phi



Testing Nanohertz Gravity



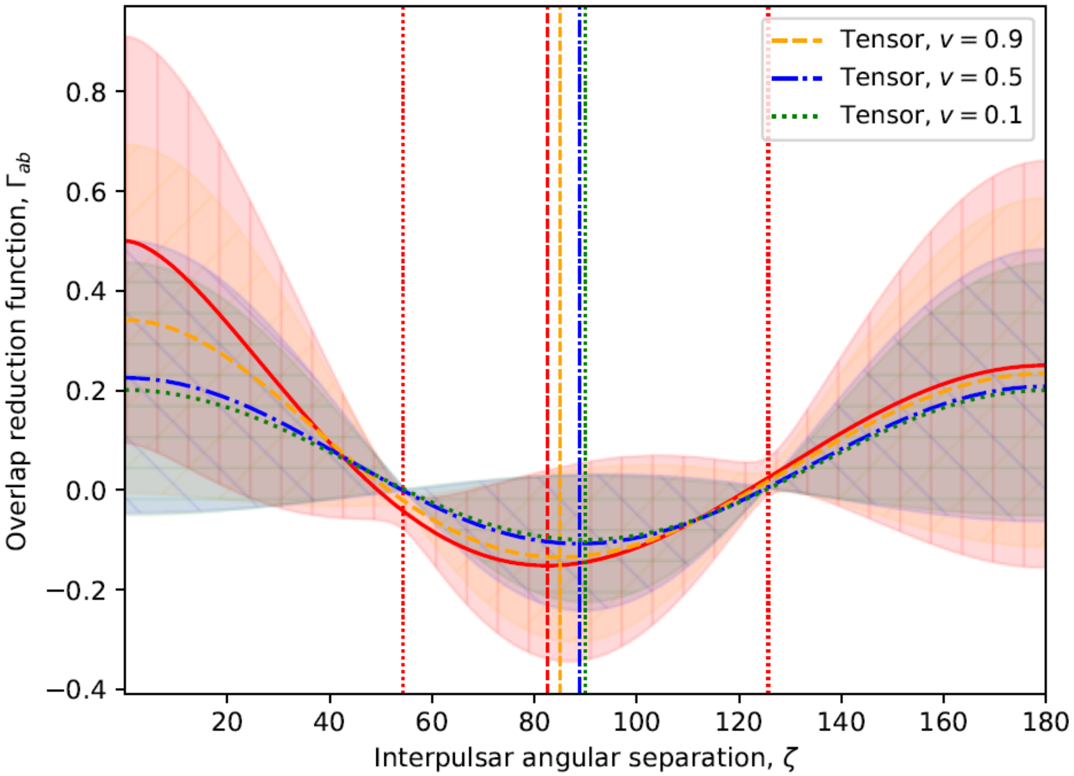


PTA Playbook: Testing Gravity

$$S[g,\phi] = \int d^4x \sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \text{Horndeski} - \text{GB terms} + \cdots\right)$$

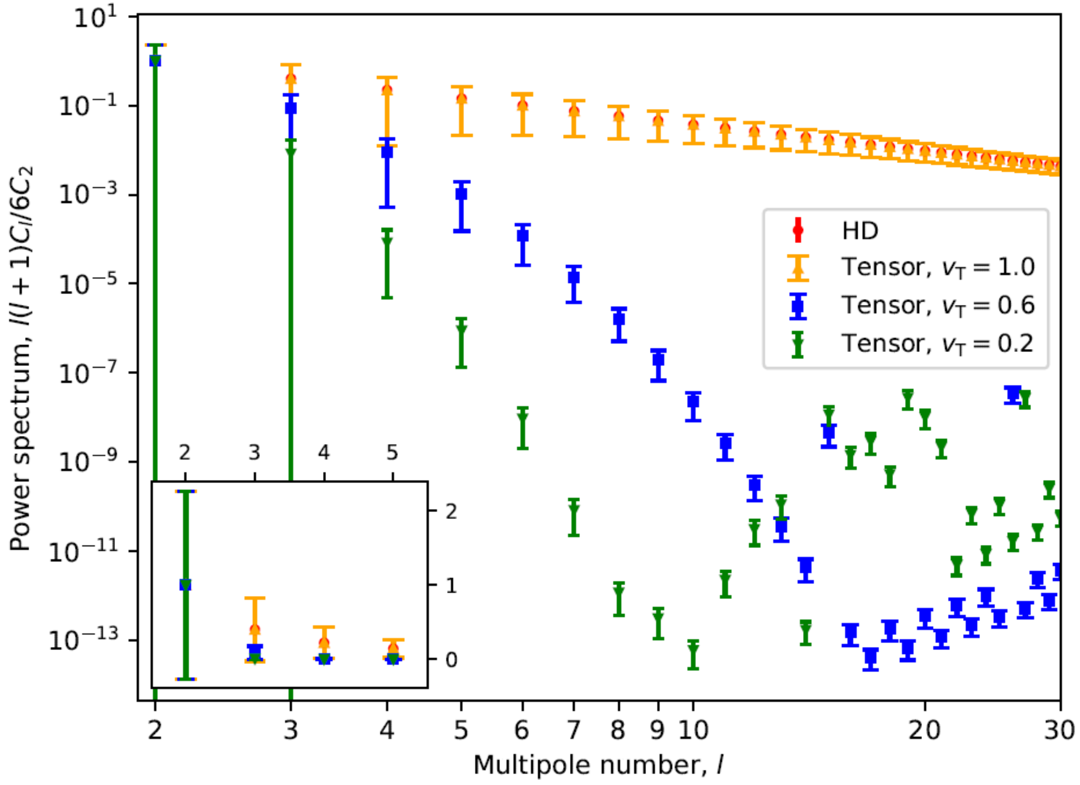
Minimal Angle

(2304.02640: Liang, Lin, Trodden)



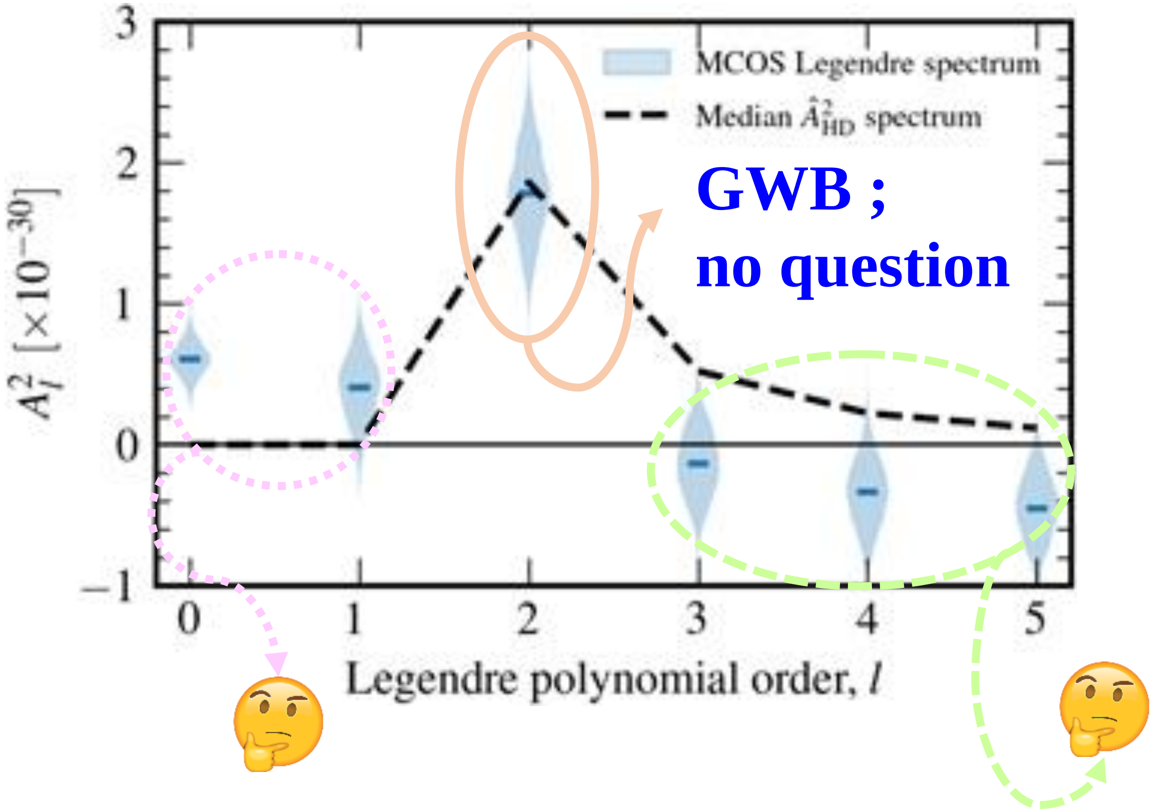
CV precise PS measurements

(2306.13593: **RCB**, Ng)

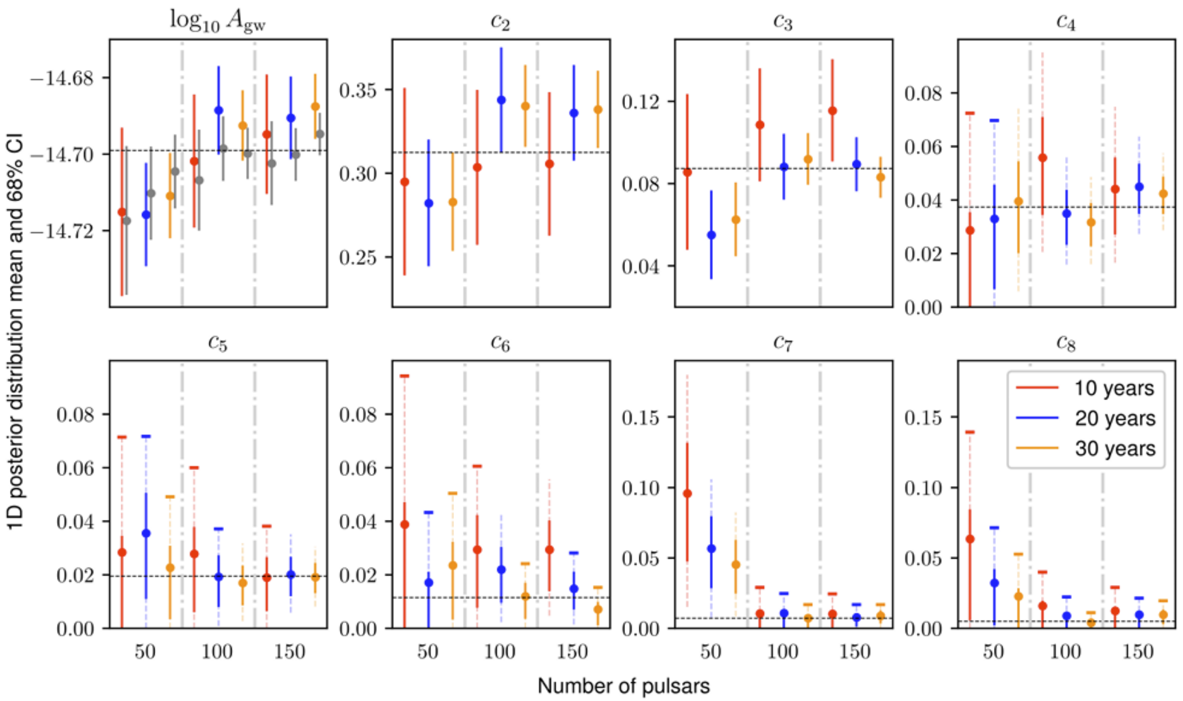


NG15 + future projections ~ CV-precise

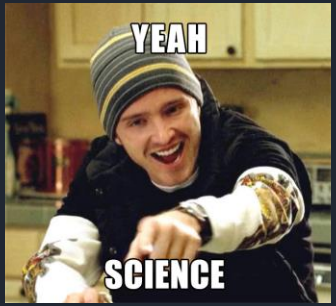
NANOGrav 15 years detection paper

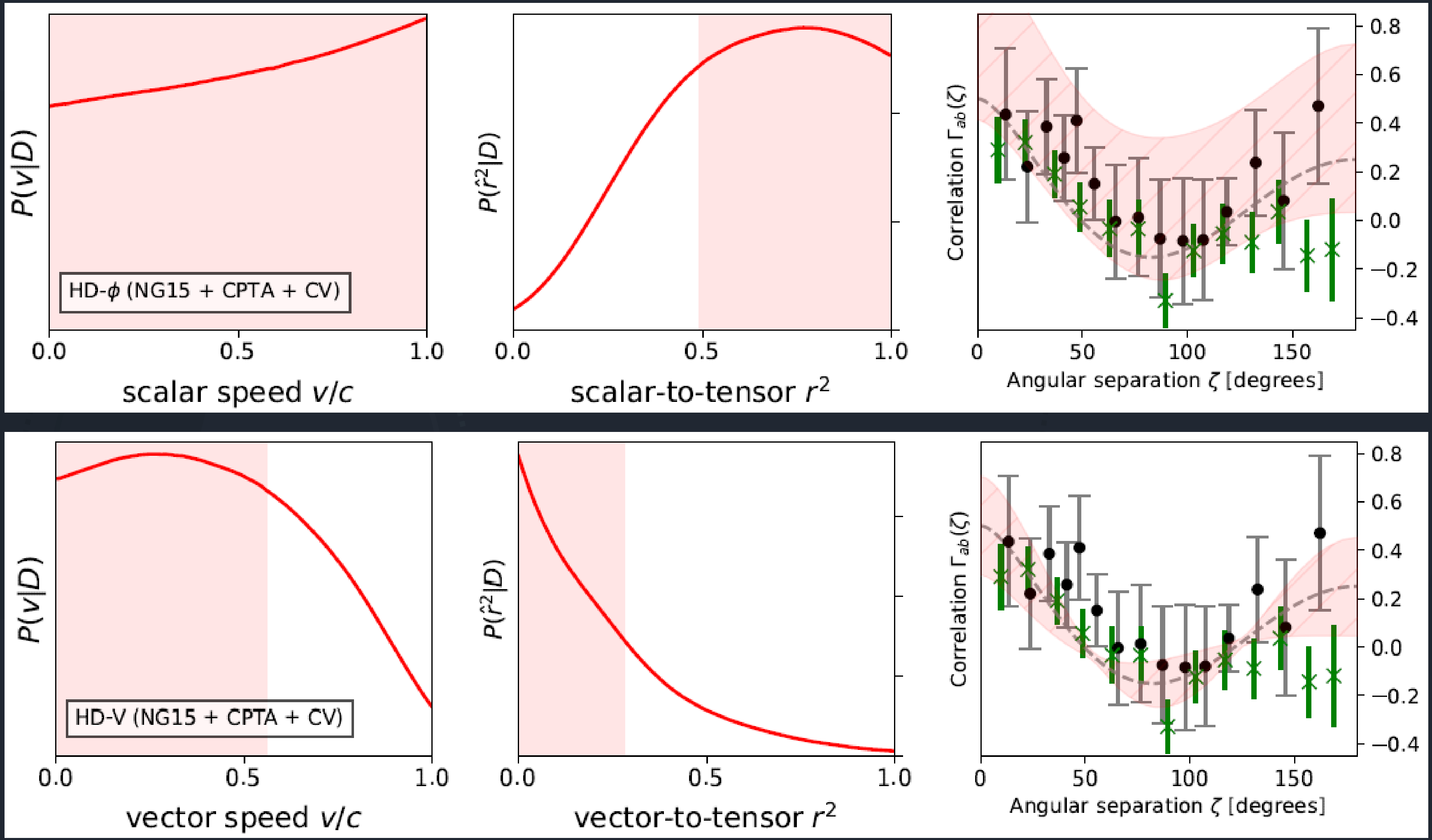


2306.06168 (Nay, Boddy, Smith, Mingarelli)



In preparation: HD + scalar/vector



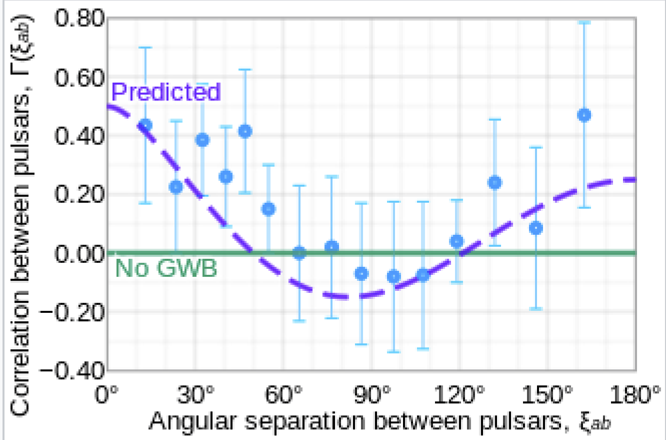


Reggie Bernardo

Retrieved 14 Aug 2023
https://media.nature.com/lw767/magazine-assets/d41586-023-02167-7/d41586-023-02167-7_25547166.jpg?as=webp
https://www.ligo.caltech.edu/system/news_items/images/213/page/NANOGrav_PTA_GWB_15yr_wide.jpg?1687824134

Amazing Time for PTA science!

Wiki: Gravitational Wave Background



Plot of correlation between pulsars observed by NANOGrav (2023) vs angular separation between pulsars, compared with a theoretical Hellings–Downs model (dashed purple) and if there were no gravitational wave background (solid green).[11][12]

Outlook

PTAfast: PTA correlations from stochastic gravitational wave background
[ascl:2211.001]

RCB & KWN:

✓ **PS formalism** for calculating the **mean** and **variance** of **SGWB correlations**;

✓ **PTA phenomenology** of **subluminal metric polarizations** for **finite pulsar distances**;

✓ analysis of **tensor & vector polarizations** off the light cone;

✓ **Scalar/Galileon** constraints;

✓ **PTA Playbook** for testing gravity.

❑ *In preparation*: Non-Einsteinian GWs; Fuzzy DM correlations; anisotropies.

Glitch in the Matrix (Accessed 16 Aug 2023):
https://preview.redd.it/660w2zbgch451.jpg?width=950&format=pjpg&auto=webp&s=4851d6685af2ab704b65d3328e61da2cdf7e453f

Extra Slides

Accessed 031022
https://scx1.b-cdn.net/csz/news/800a/2016/millisecondp.jpg

Galactic lighthouses

- Neutron Star:

  $M \sim 10^{0-1}\, M_{\odot}, D \sim 10^{0-1}$ km
- Pulsar = NS + magnetic field
- Millisecond pulsar

  - spins at ~100x per sec

PSF for variances

(2209.14834, RCB & KWN)

- Mean ORF

$$\gamma^A_{ab}(\zeta) = \sum_l \frac{2l+1}{4\pi} C_l^A P_l(\cos\zeta)$$

- Total variance [1 PP]

$$\Delta\gamma^2_{ab}(\zeta) = \left(\gamma^A_{ab}(\zeta)\right)^2 + \gamma^2_{aa}$$

- Cosmic variance [Gaussian ensemble]

$$\Delta\gamma^2_{ab}(\zeta) = \sum_l \frac{2l+1}{8\pi^2} C_l^2 P_l(\cos\zeta)$$

The International Pulsar Timing Array checklist for the detection of nanohertz gravitational waves

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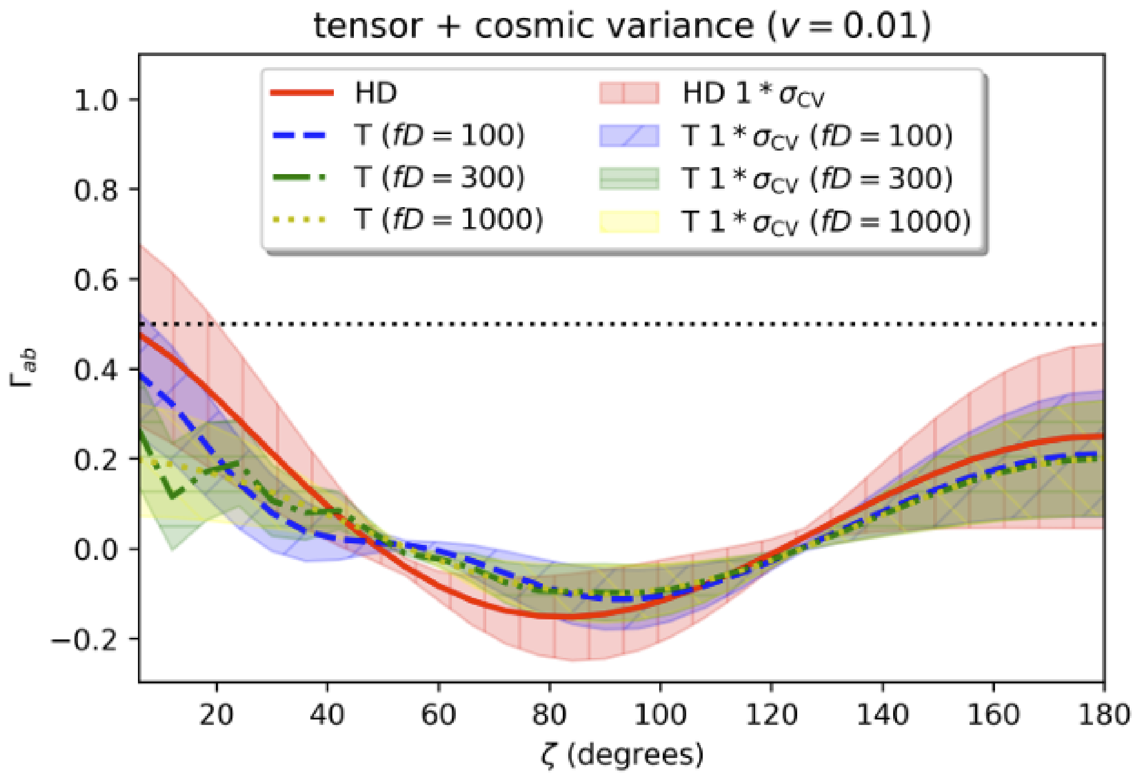
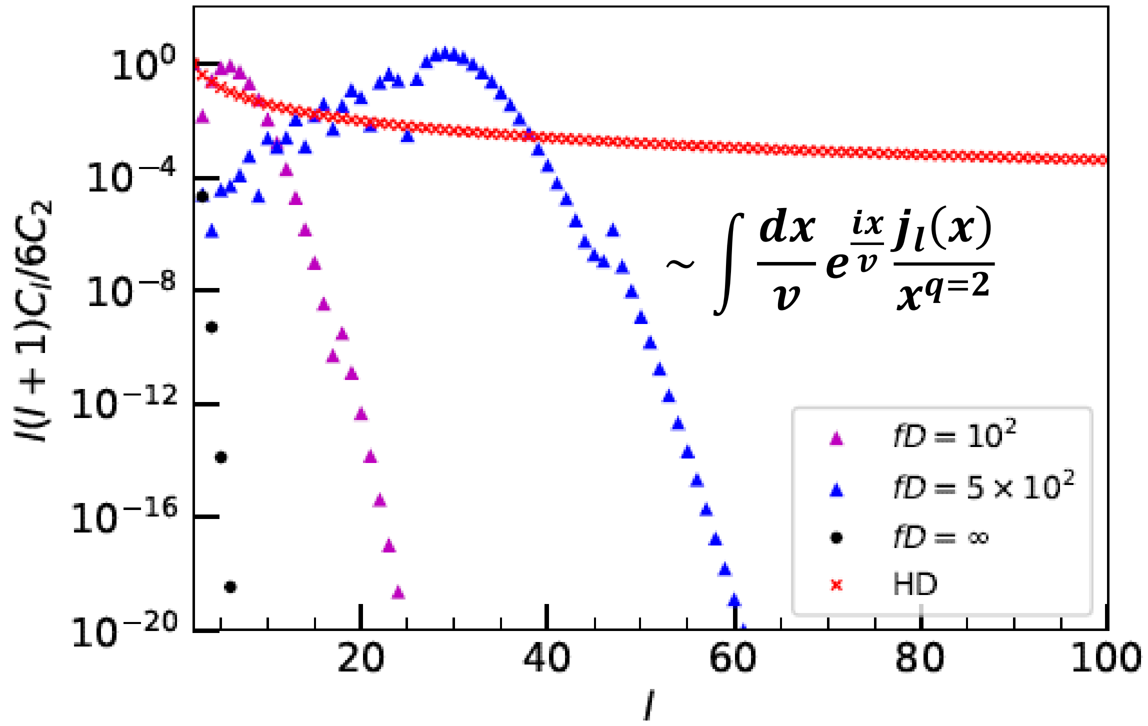
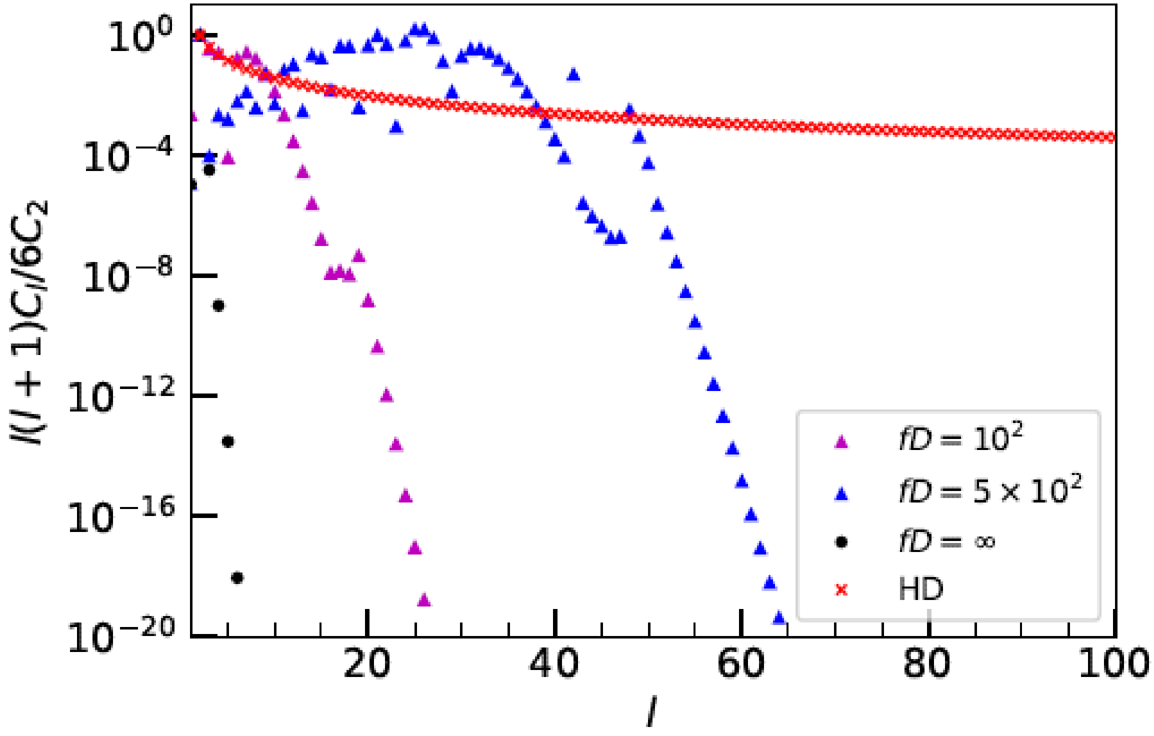
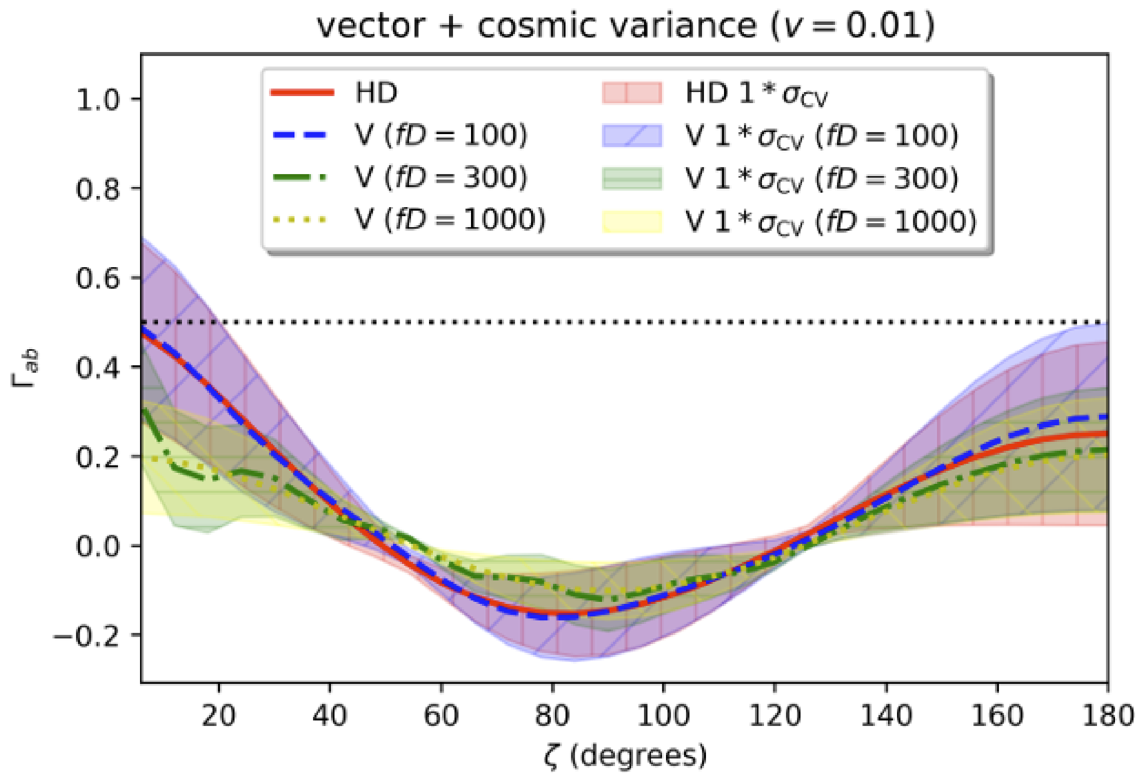
Tensor PS and ORF ($v \ll 1$, near static)

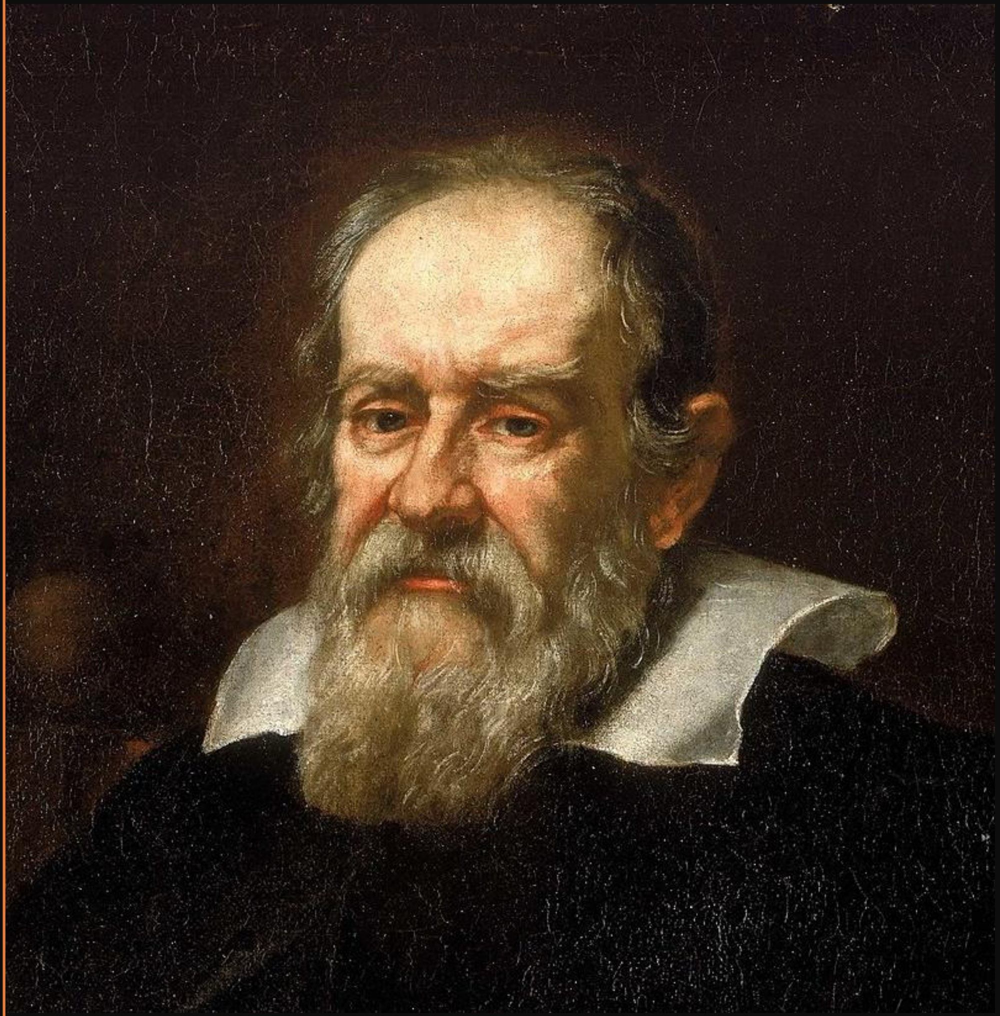








Vector PS and ORF ($v \ll 1$, near static)



Accessed 041022
https://upload.wikimedia.org/wikipedia/commons/thumb/d/d4/Justus_Sustermans_-_Portrait_of_Galileo_Galilei%2C_1636.jpg/1200px-Justus_Sustermans_-_Portrait_of_Galileo_Galilei%2C_1636.jpg

The Galileon in the nHz GW sky

$$S_G[g_{ab}, \phi] = \int d^4x\sqrt{-g}\left(\left(1 + \frac{\alpha\phi}{M_{\mathrm{P}}}\right)\mathrm{EH} - \Lambda - \lambda^3\phi + X + \frac{X}{\kappa^3}\partial^2\phi + \frac{\mu^2\phi^2}{2}\right)$$

EH = Einstein-Hilbert term

Λ = cosmological constant

κ = braiding -> Vainshtein mechanism/ϕ suppression at $R \ll L$

μ = *bare* mass -> ϕ suppression at dense environments

α = conformal coupling -> mixes the tensor and scalar modes

λ = tadpole -> self tuning mechanism (**2202.08672, Appleby, RCB**)

Metric Perturbations

Synchronous gauge:

$$ds^2 = -dt^2 + \left(\delta_{AB} - 2\psi\delta_{AB} + 2D_A D_B E + 2D_{(A}E_{B)} + 2E_{AB}\right)dx^A dx^B$$

$$\phi = \varphi + \delta\phi$$

Effective mass $\left(\omega^2 = k^2 + m_{\rm eff}^2\right)$:

$$m_{\rm eff}^2 = \mu^2 \left(\frac{1 - \dfrac{\alpha\lambda^3}{M_{\rm P}\mu^2}}{1 + \dfrac{3\alpha^2}{2} - \dfrac{\alpha\lambda^3}{M_{\rm P}\mu^2}} \right)$$

PLB 2023 (RCB & KWN)