Accurate calculation of the gravitational wave background from cosmic strings

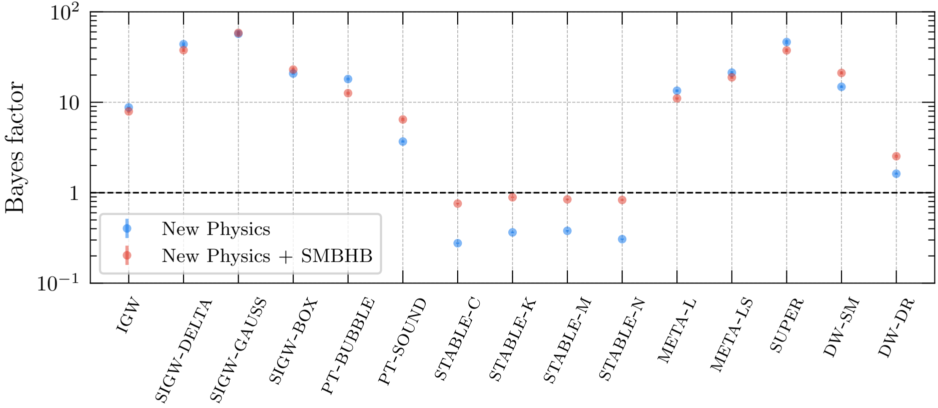
Ken D. Olum

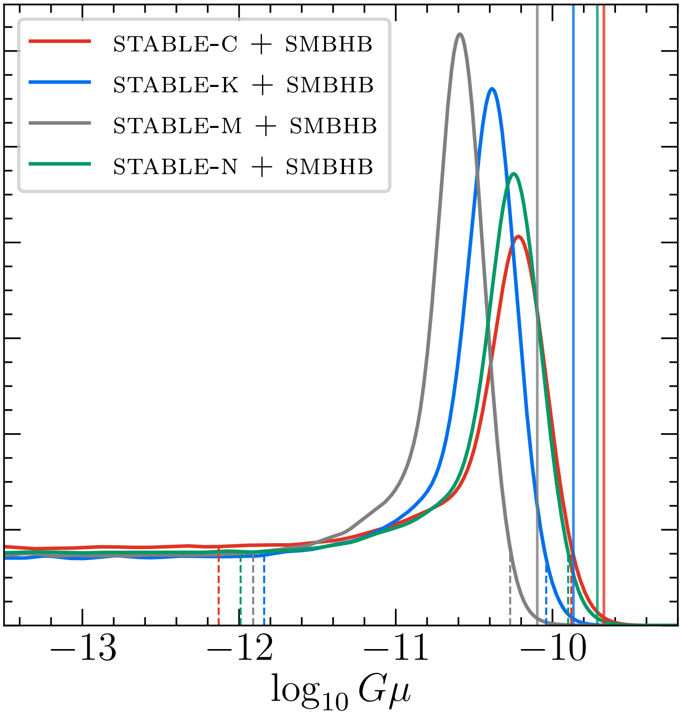
Tufts Institute of Cosmology
Department of Physics and Astronomy

Thanks to Jeremy Wachter, Jose Juan Blanco-Pillado.

Outline:
Cosmic string dynamics. Kinks and cusps.
Gravitational radiation and backreaction.
Extrapolations.
Results in progress.

Present situation





STABLE-C + SMBHB
STABLE-K + SMBHB
STABLE-M + SMBHB
STABLE-N + SMBHB
−13
−12
−11
−10
$\log_{10} G\mu$

Cosmic string introduction

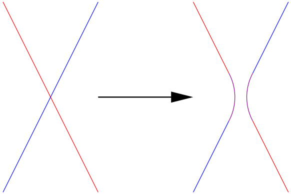
We will consider Abelian-Higgs strings and superstrings, but not hybrid models.

Strings can never have ends:
"Network" of infinite strings and loops.

μ = tension = energy density.
Relativistic motion.

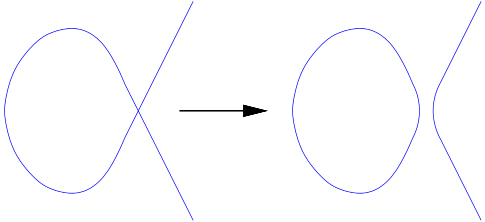
Loops oscillate.

Intercommutation:



Field theory strings: always.
Superstrings: $p = 10^{-3} \ldots 1$.

Intercommutation
$\rightarrow$ loops.



Each loop emits gravitational waves with power $P = \Gamma G\mu^2$. $\Gamma \sim 50$ for a loop without a lot of small-scale structure.

Length decreases at rate $dl/dt = \Gamma G\mu$. Loop lifetime $L/(\Gamma G\mu)$.

PTA upper limit $G\mu \lesssim 10^{-10}$. Lifetime $\gtrsim 10^8 L$.

Loops survive from the radiation era.

Scaling

Loop formation/evaporation leads to *scaling*.

A picture of the string network at time $2t$ looks (statistically) like a magnified picture of the network at t.

Fixed (average) number of long strings per horizon volume, distribution of loops in horizon units.

String density redshifts like radiation in radiation era, matter in matter era.

No "monopole problem".

In radiation era, long string density $\sim a^{-4}$
but loops are just diluted $\sim a^{-3}$
$\Rightarrow$ Most of the string is in loops.
Very many loops $\Rightarrow$ background is Gaussian, isotropic.

String dynamics

String position at time t given by $\mathbf{x}(\sigma, t)$. Choose parameter σ to parameterize energy along string, $\sigma = 0 \ldots L$.

In flat space, equations of motion $\mathbf{x}'' = \ddot{\mathbf{x}}$.

Solution $\mathbf{x} = \frac{1}{2}\left[\mathbf{a}(t-\sigma) + \mathbf{b}(t+\sigma)\right]$ with $|\mathbf{a}'| = |\mathbf{b}'| = 1$

Loops oscillate with period $L/2$.

Cusps and kinks

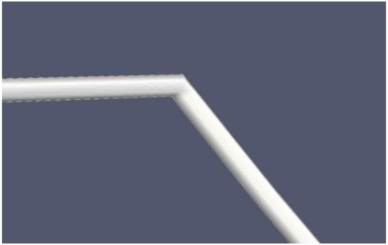
String can double back on itself to form a cusp

For a moment the cusp moves (formally) at speed of light.

Rapidly moving string beams radiation in a narrow cone.

Cusps and kinks

When strings cross and reconnect, the string develops kinks



Kinks move along the string at the speed of light.

Spectrum of GWB

Energy density in GW today:

$$\rho_{\rm gw}(t_0, f) = \int_0^{t_0} \frac{dt}{(1+z(t))^3} P_{\rm gw}\left(t, (1+z)f\right).$$

given power density emitted

$$P_{\rm gw}(t, f') = G\mu^2 \sum_{n=1}^{\infty} \frac{2n}{f'^2}\, \mathbf{n}\left(\frac{2n}{f'}, t\right) P_n\,.$$

P_n is the power in each harmonic n in units of $G\mu^2$.
n is the loop number density.

Spectrum of GWB

Loop density (radiation era)

$$\mathbf{n}(l,t) = \frac{0.2}{t^{3/2}\left(l+\Gamma G\mu t\right)^{5/2}}$$

Γ is the total power in units of $G\mu^2$, $\Gamma = \sum_n P_n$

Our goal is to compute P_n. How do we do it?

1. Simulate the string network

Piecewise-linear $\mathbf{a}$ and $\mathbf{b}$.

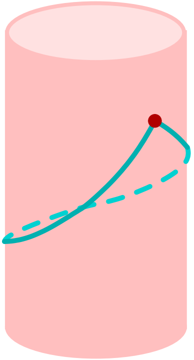
Get the loop production rate and so **n**.
Corpus of loops of various shapes.

We have loop shapes at formation, but as they evaporate their shapes will change.

Spectrum of GWB

2. Simulate gravitational back reaction.

We use a Green's function approach.

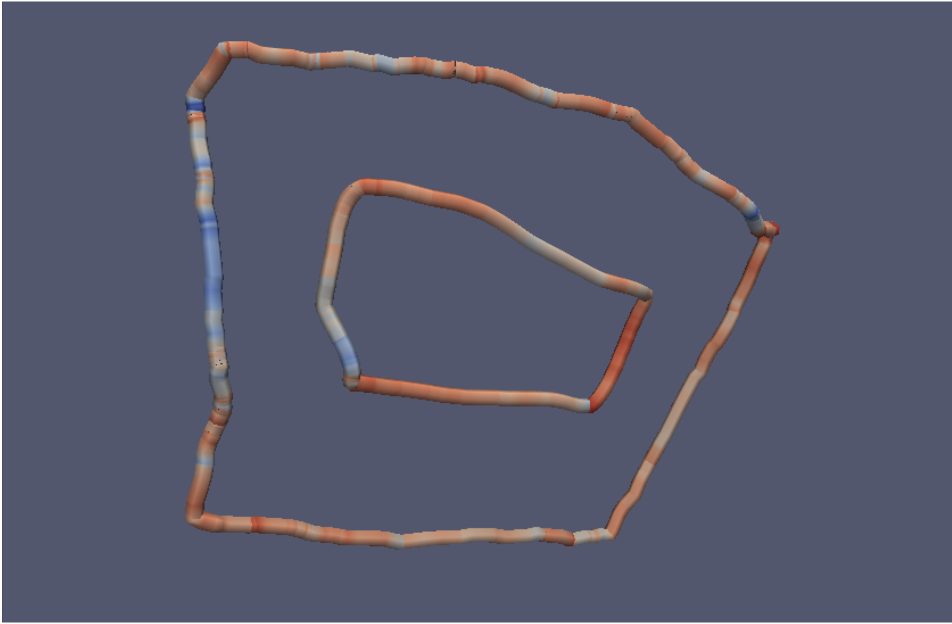


• Find the gravitational effect of the entire loop on a particular worldsheet point. Time $O(N)$.
• Integrate to find how one segment of $\mathbf{a}'$ or $\mathbf{b}'$ of the string is affected by gravitation. Time $O(N^2)$.

Repeat for all $\mathbf{a}'$ and $\mathbf{b}'$. Time $O(N^3)$. Get "gravitational acceleration" that acts on string segments.

Write this as an ODE (with 100's or 1000's of variables) and solve with ODE solver (DOP853).

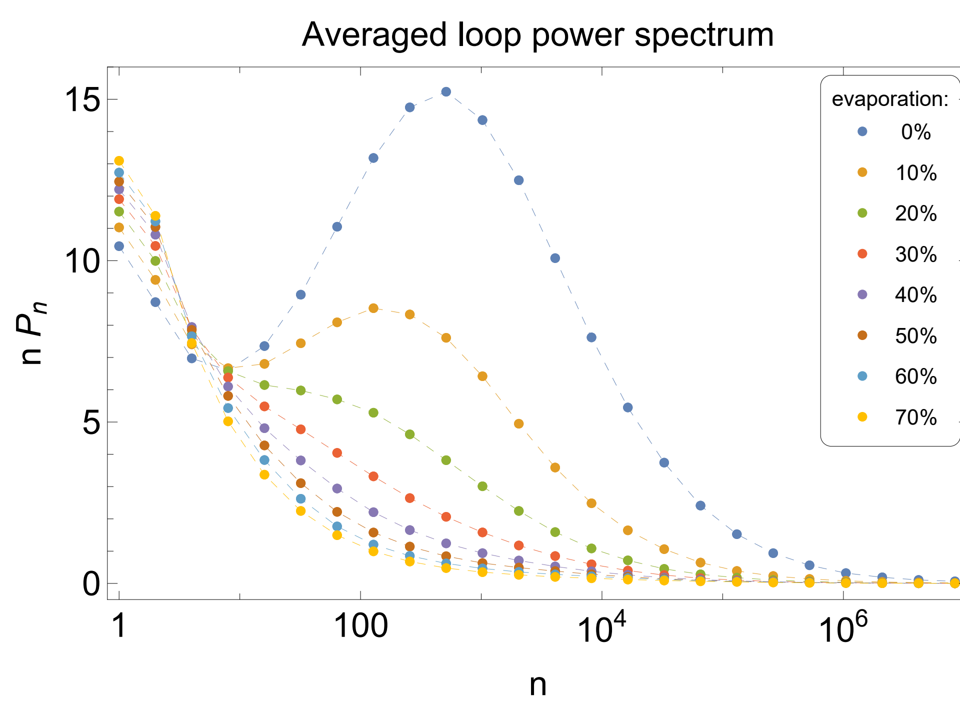
Example of 50% evaporation



Spectrum of GWB

Loops we computed:

Target evaporation fraction	Count	Range of # segments
0.7	105	$[187, 582]$
0.5	44	$[499, 956]$
0.1	51	$[852, 3304]$



Averaged loop power spectrum
15
10
5
0
n P_n
1
100
10^4
10^6
n
evaporation:
0%
10%
20%
30%
40%
50%
60%
70%

Extrapolations. 1. Small-scale structure.

Our loops come from a radiation-era run started at conformal time 6 and run until 500.

Small-scale structure = initial conditions + intersections that happened later.

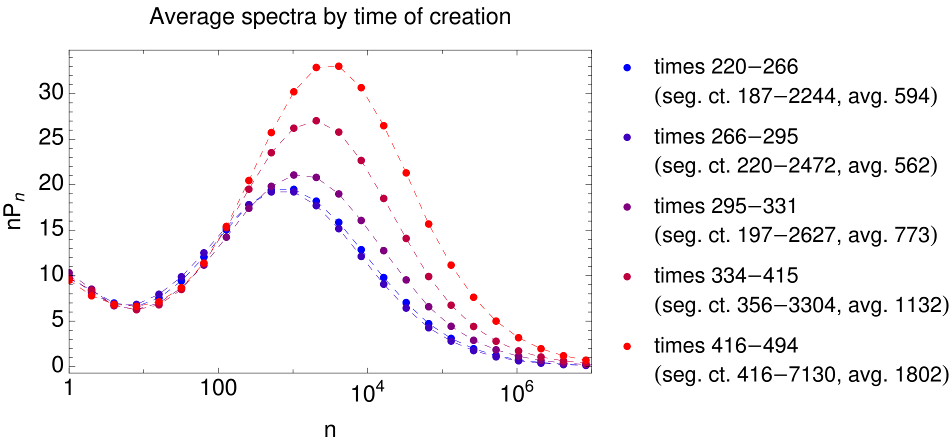
A real loop has a much longer history so more time to develop structure. Structures grow until cut off by gravitational backreaction on long strings

Number of kinks that haven't been significantly smoothed $\sim (\Gamma_\infty G\mu)^{-0.9} \sim 10^6$.

Real loops have much more structure $\Rightarrow$ larger Γ.

But it is quickly smoothed out.

Extrapolations. 1. Small-scale structure.

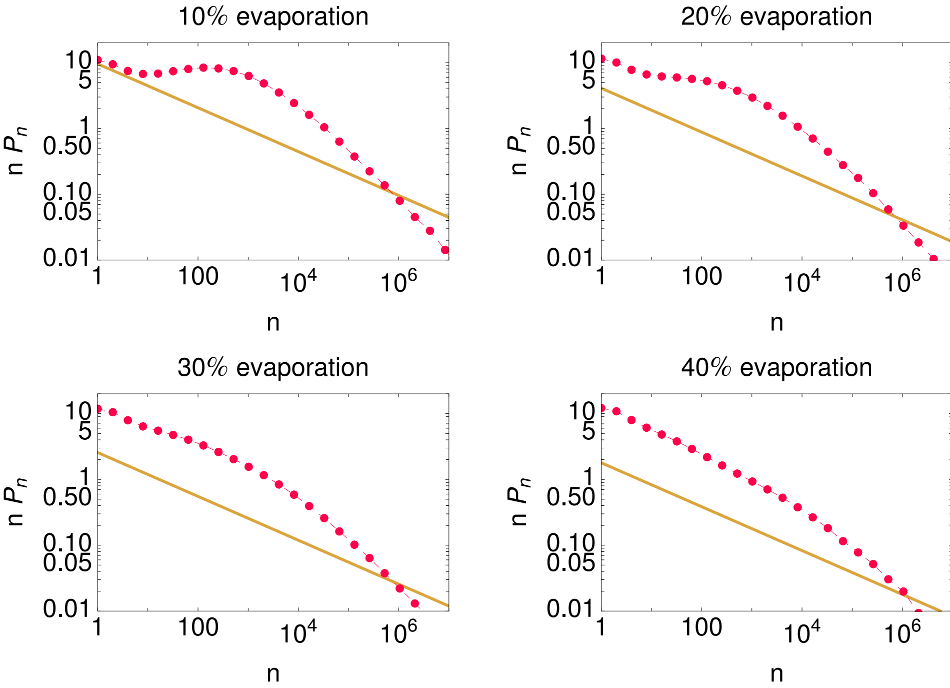


Extrapolations. 2. Cusps.

Loops as formed in simulations have no cusps (nor anything looking cusp-like).

We know that backreaction will smooth kinks, which will immediately give rise to cusps. But they are weak.

It is difficult to track this with our backreaction code, but we can give an upper limit estimate by imagining that every kink in the loop has been fully smoothed. This gives many cusps and collectively a $n^{-4/3}$ spectrum. But the amplitude is small.


10% evaporation
20% evaporation
30% evaporation
40% evaporation
n P_n
n
10
5
1
0.50
0.10
0.05
0.01
1
100
10^4
10^6

Project status

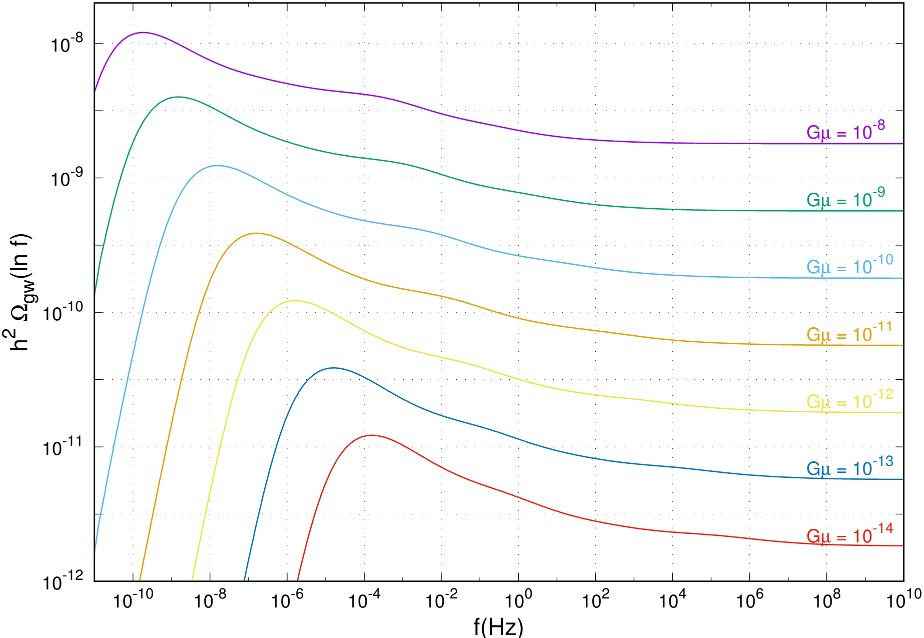
We are not done.

To do:

1. Finish small scale structure extrapolation.

2 Combine spectra for loops of different ages to give an average P_n

Using previous toy model “STABLE-N”



Thank you