

After nHz, μ Hz?

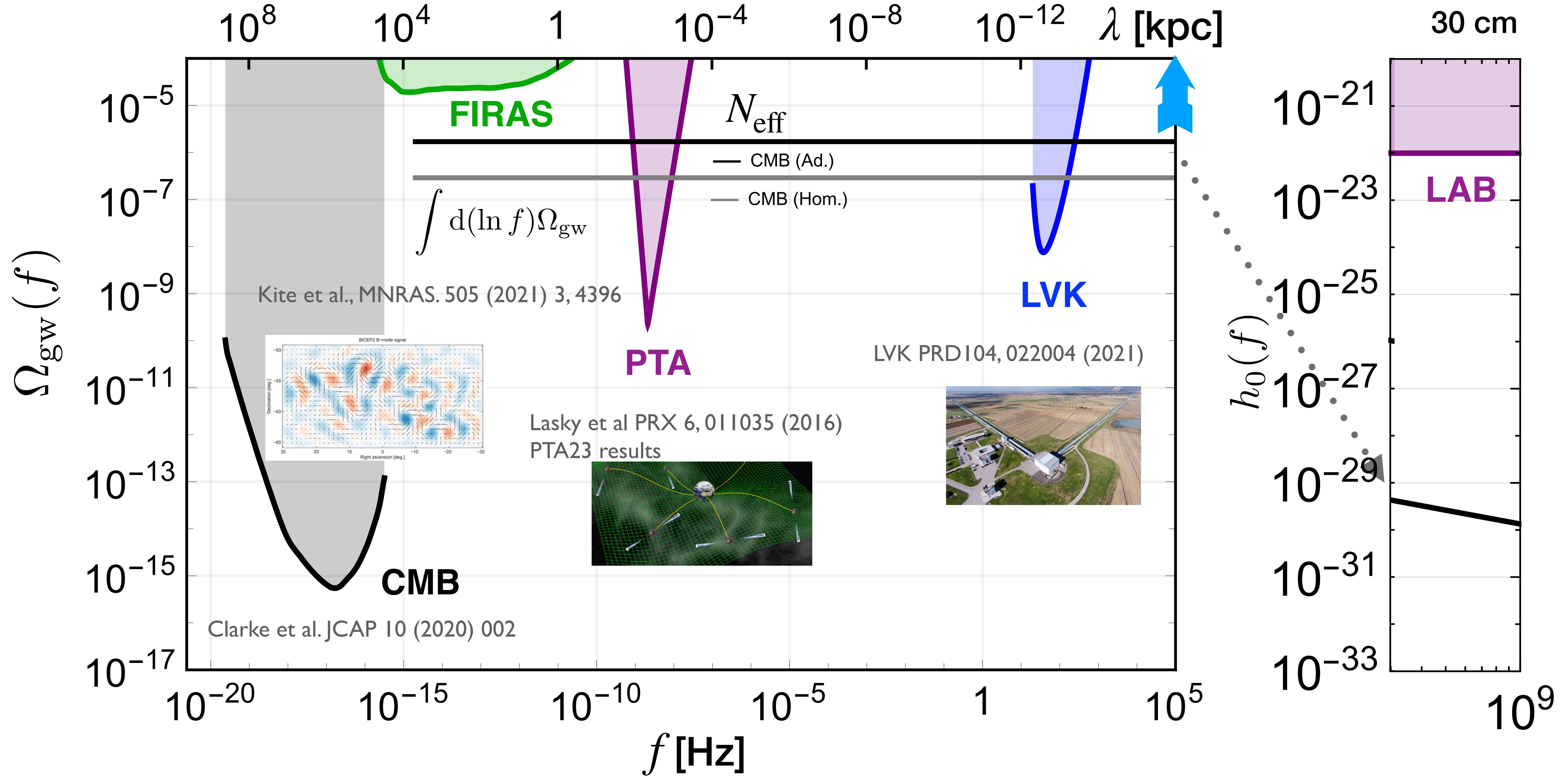
Binaries to explore the μ Hz band of GWs

Diego Blas (UAB/IFAE)
w/ A. Jenkins (UCL)

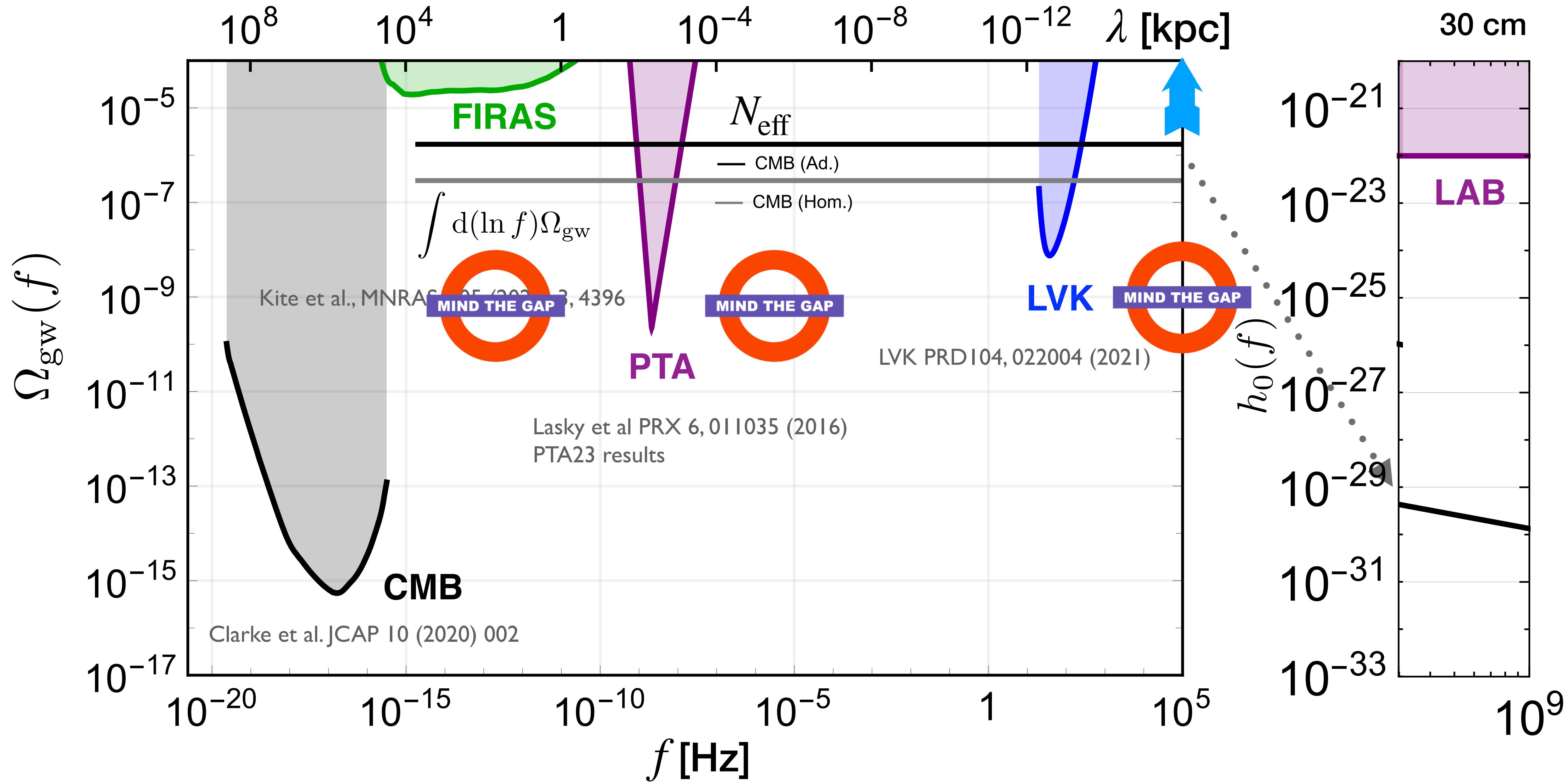
based on 2107.04063 (PRL)/2107.04601 (PRD)



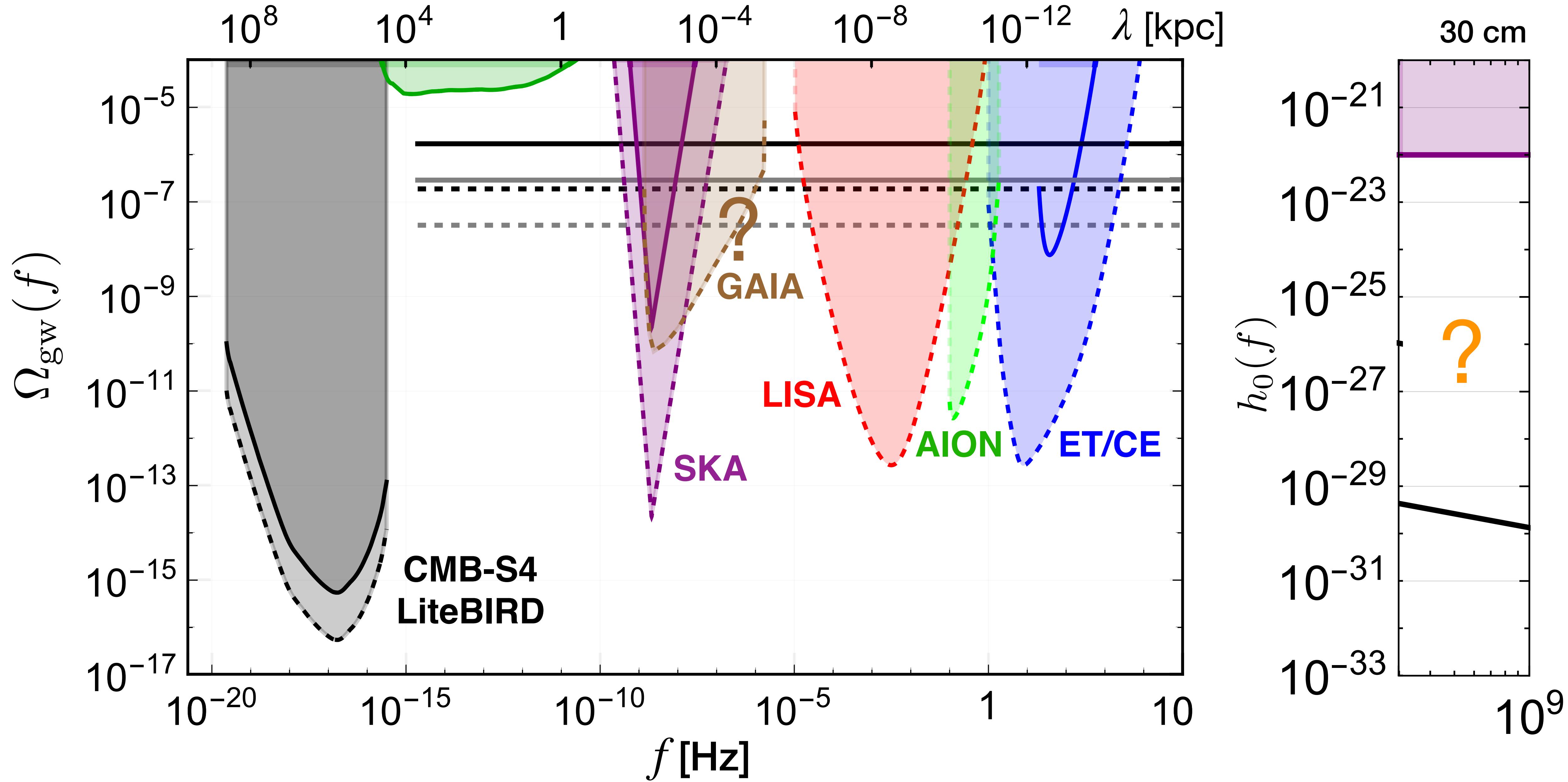
Soundscape today



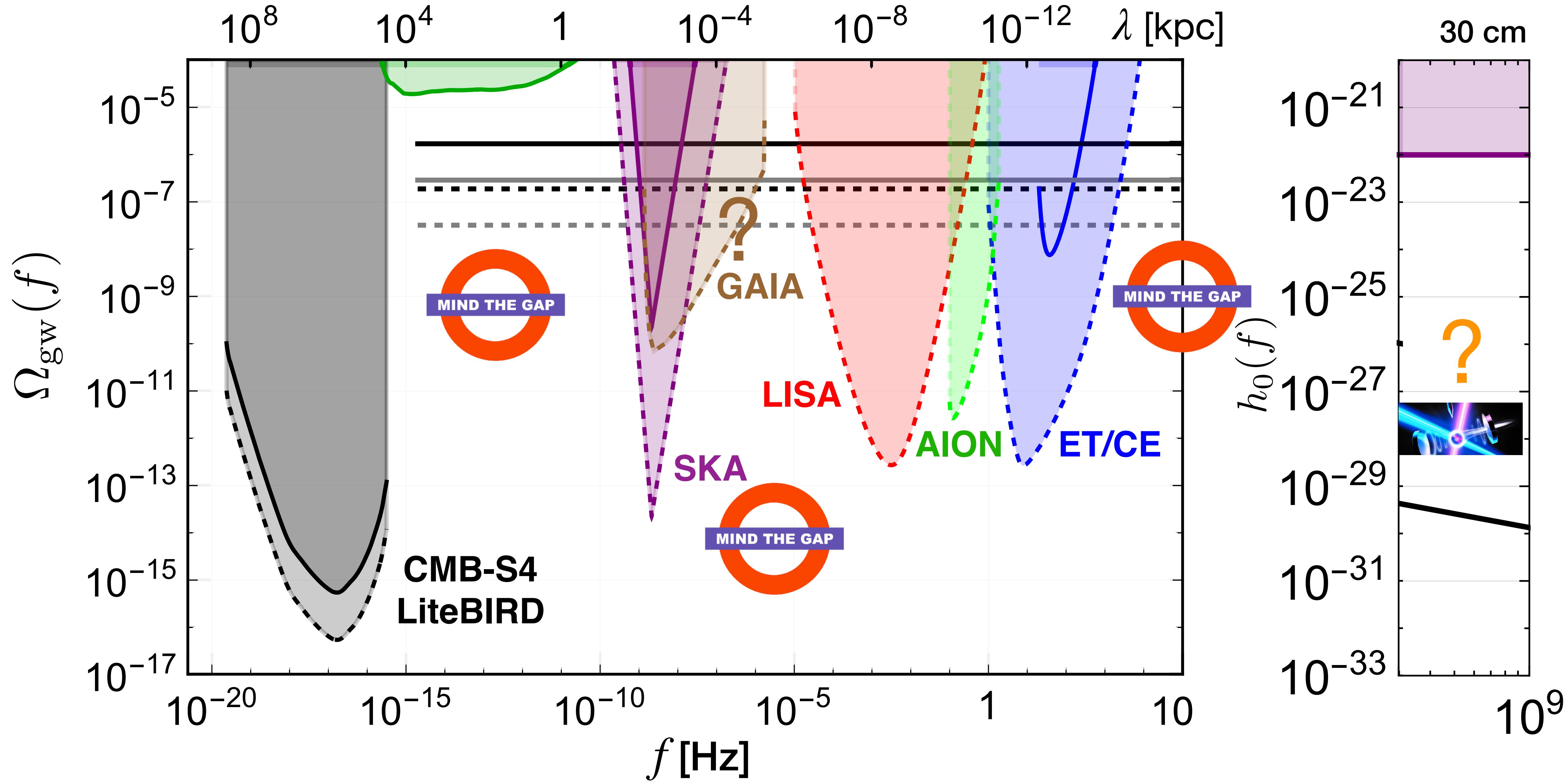
Soundscape today



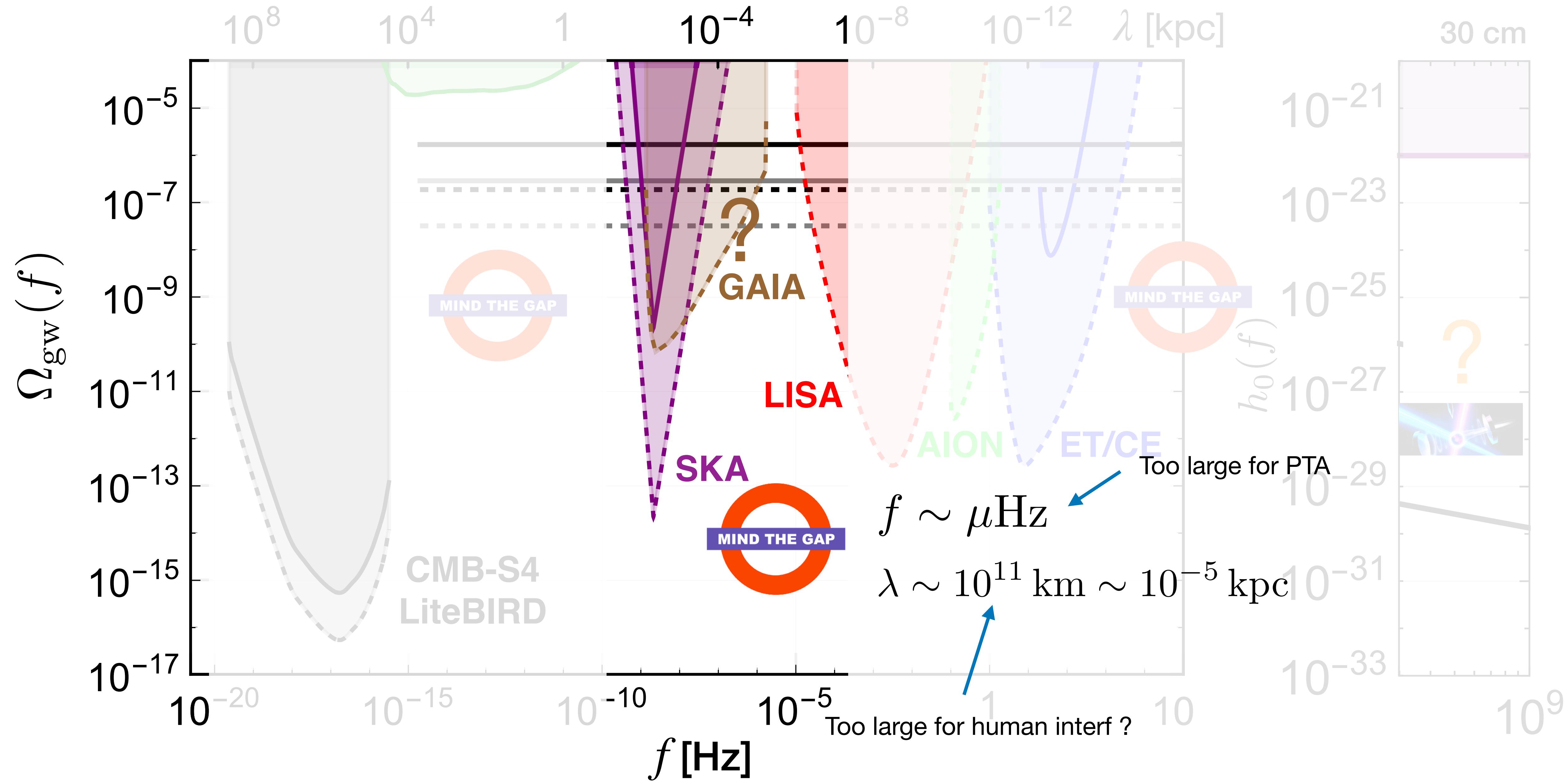
Future soundscape (maybe 2040)?



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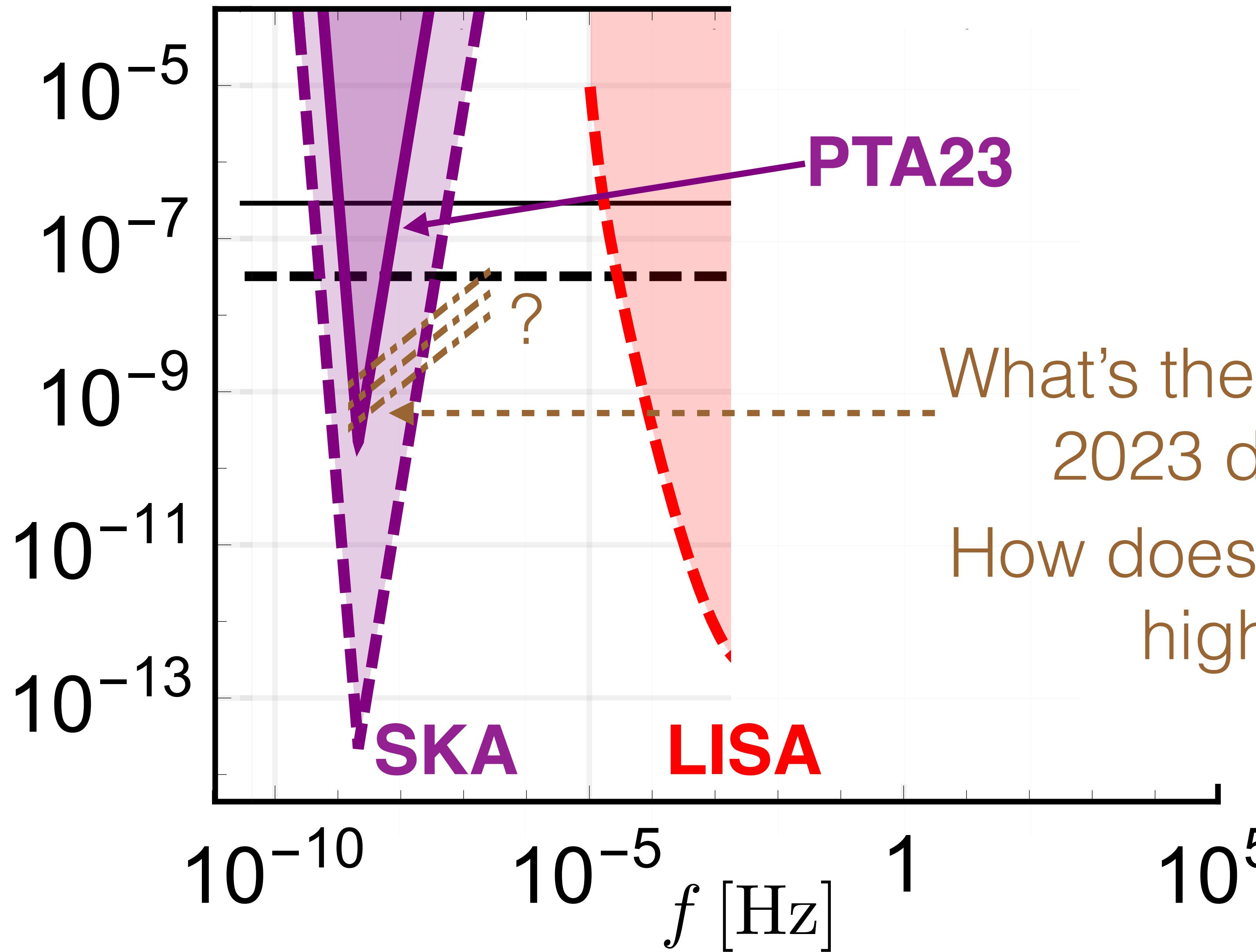


Future soundscape (maybe 2040)?



Possible backgrounds & ideas at μ Hz: a rich band

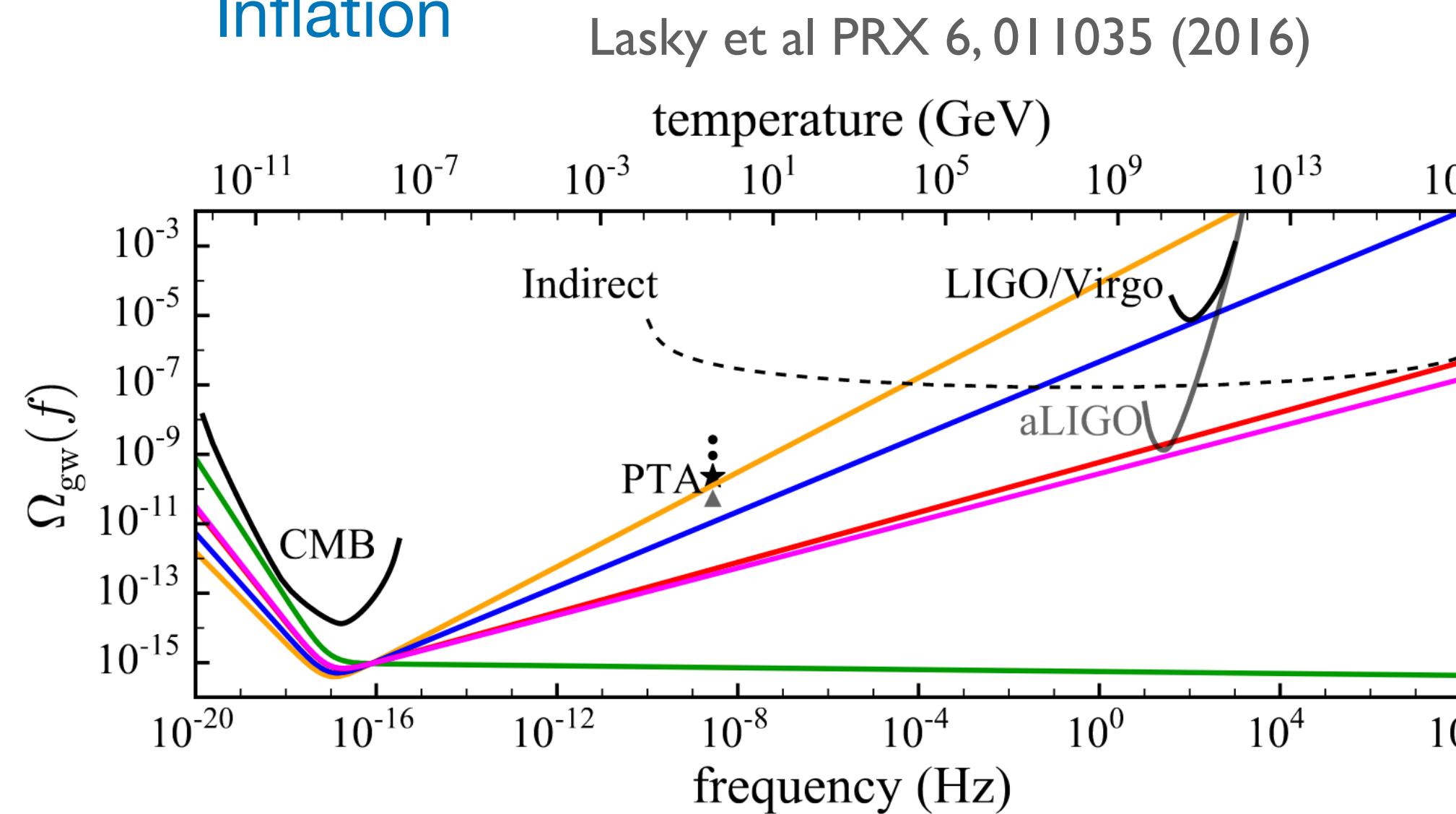
Possible backgrounds & ideas at μHz : a rich band



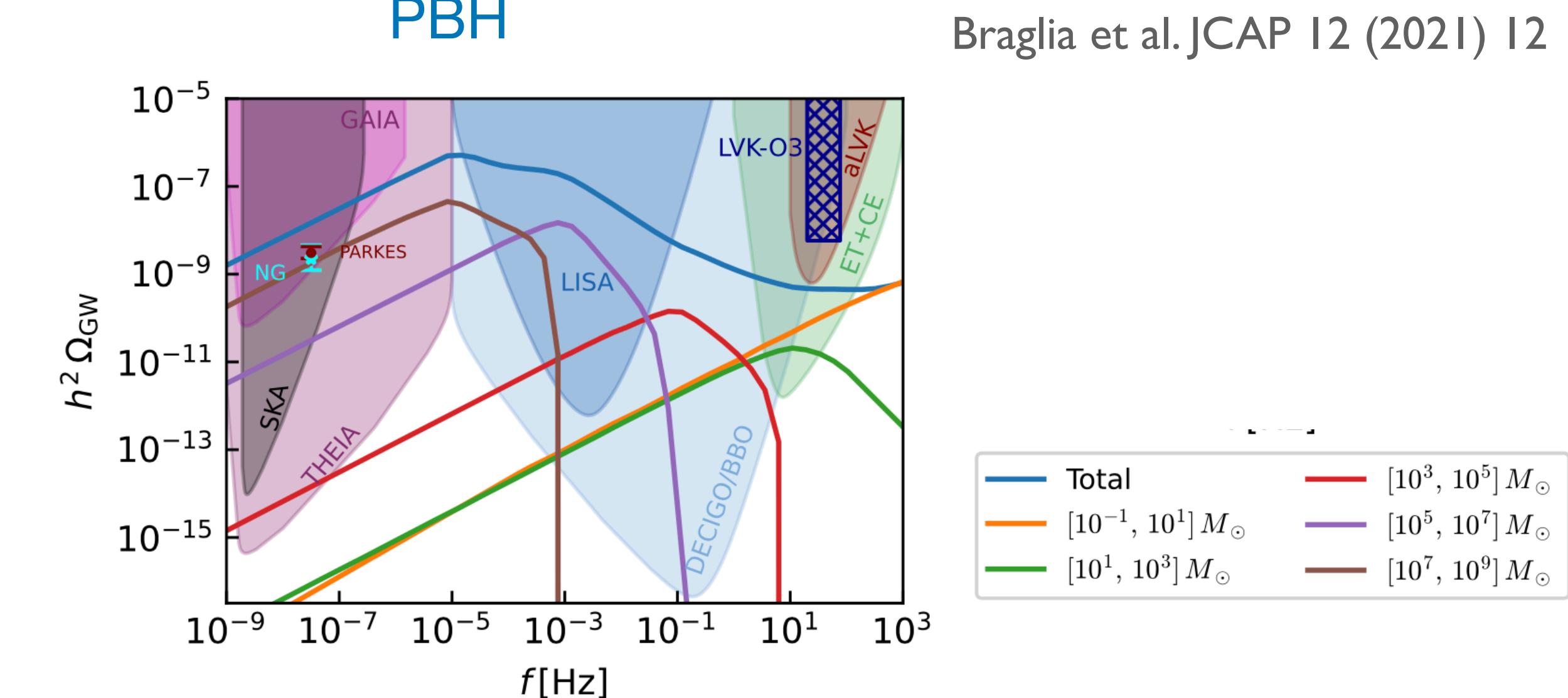
What's the origin of the
2023 detection?
How does it change at
high freq?

Backgrounds from fundamental physics

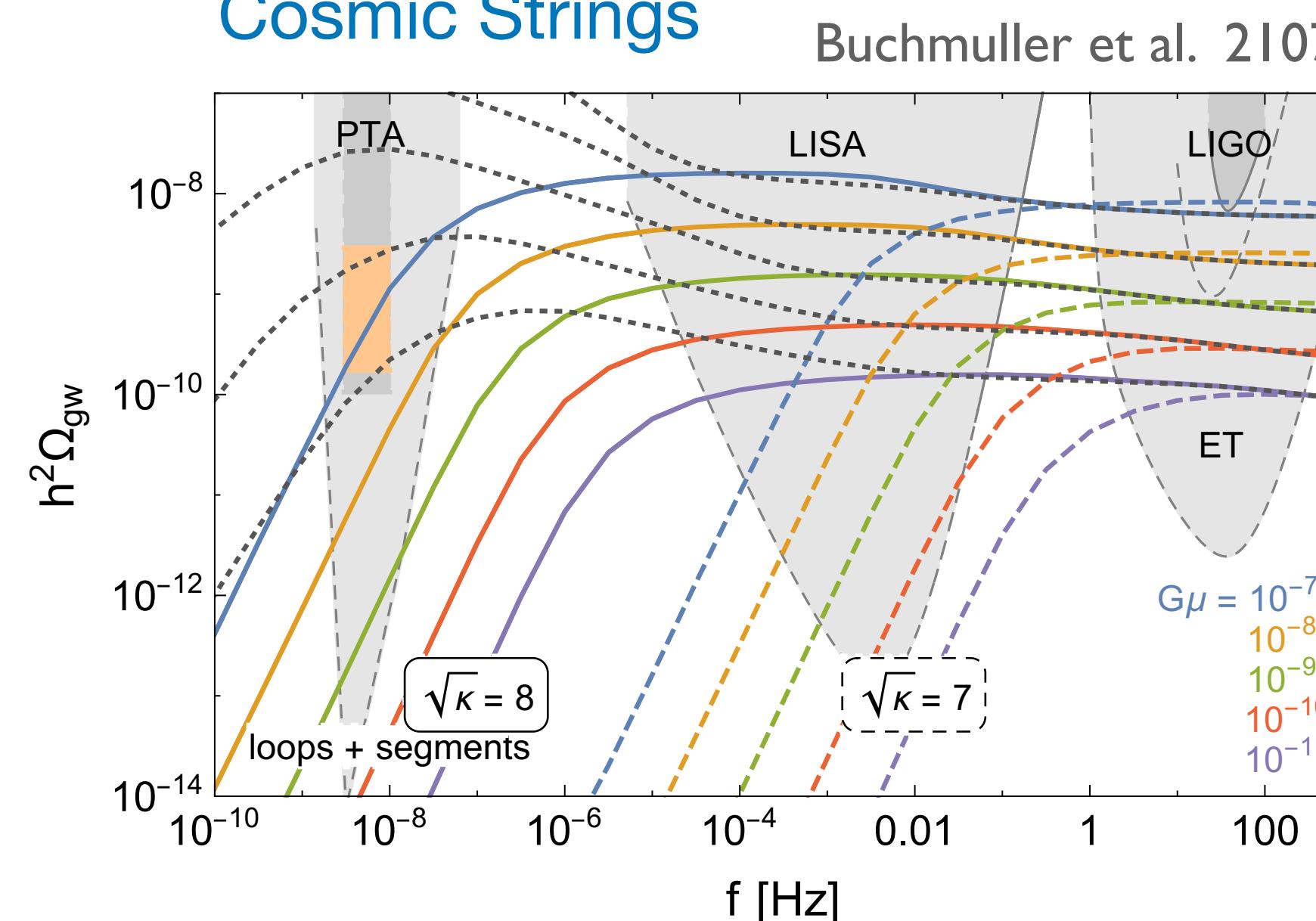
Inflation



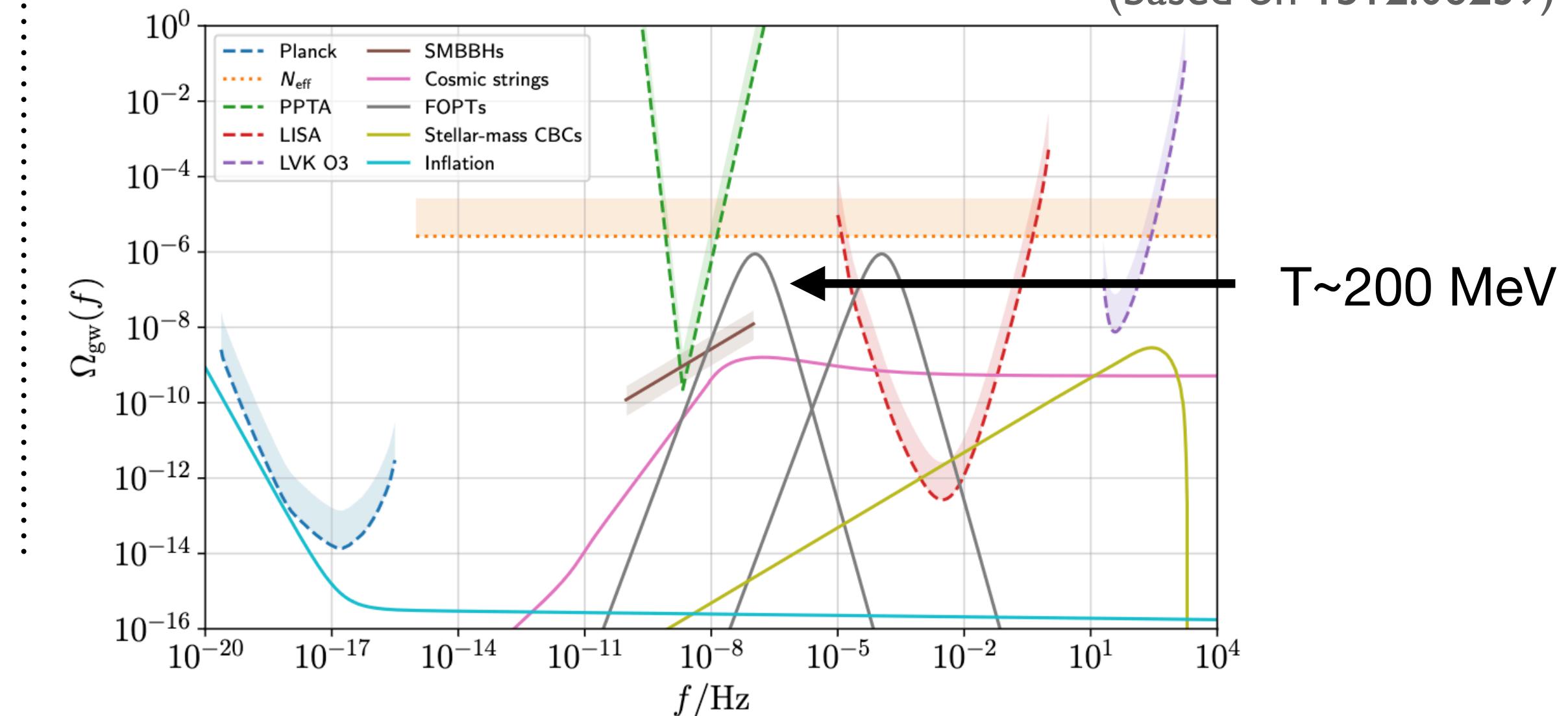
PBH



Cosmic Strings

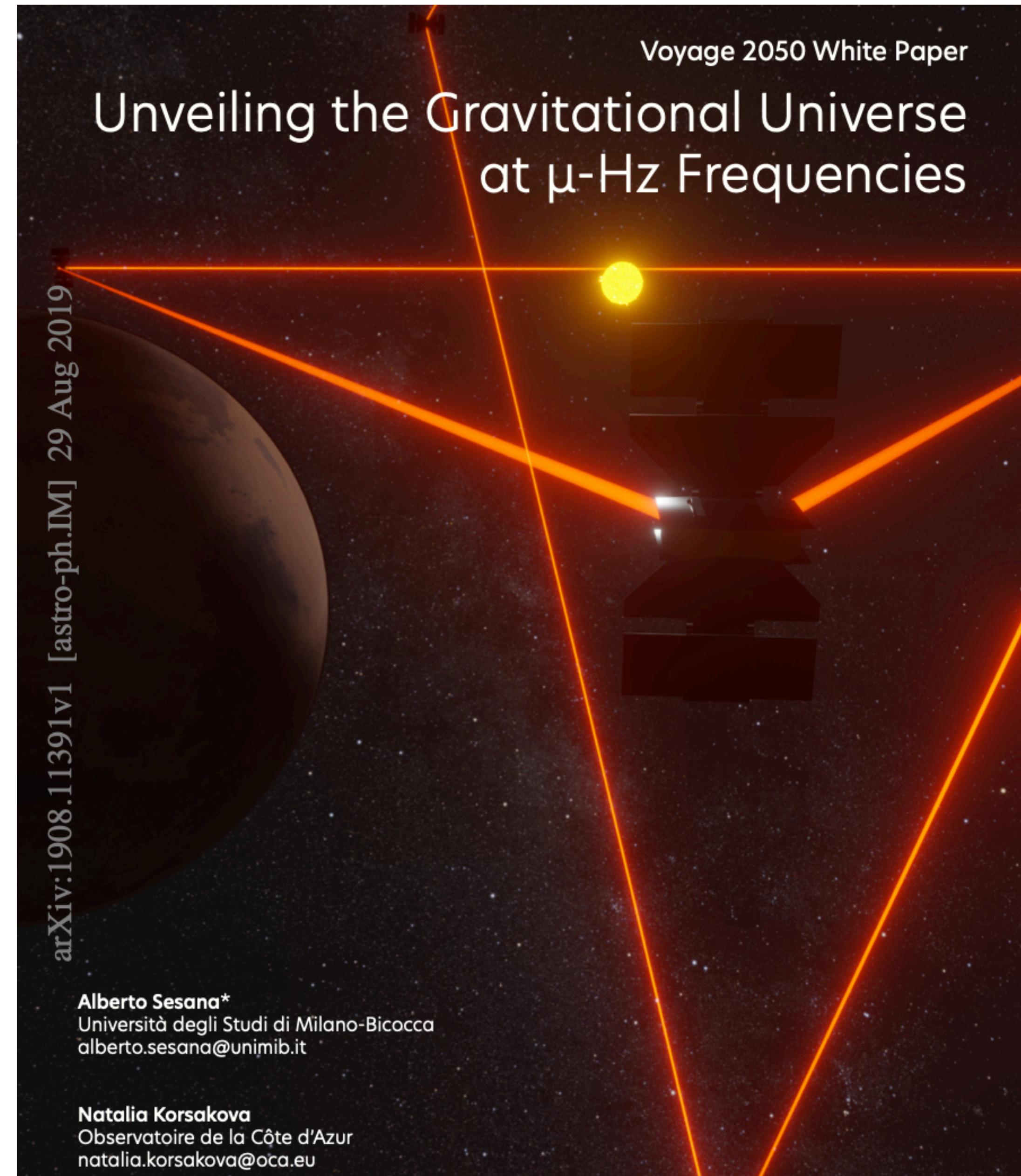


FOPT

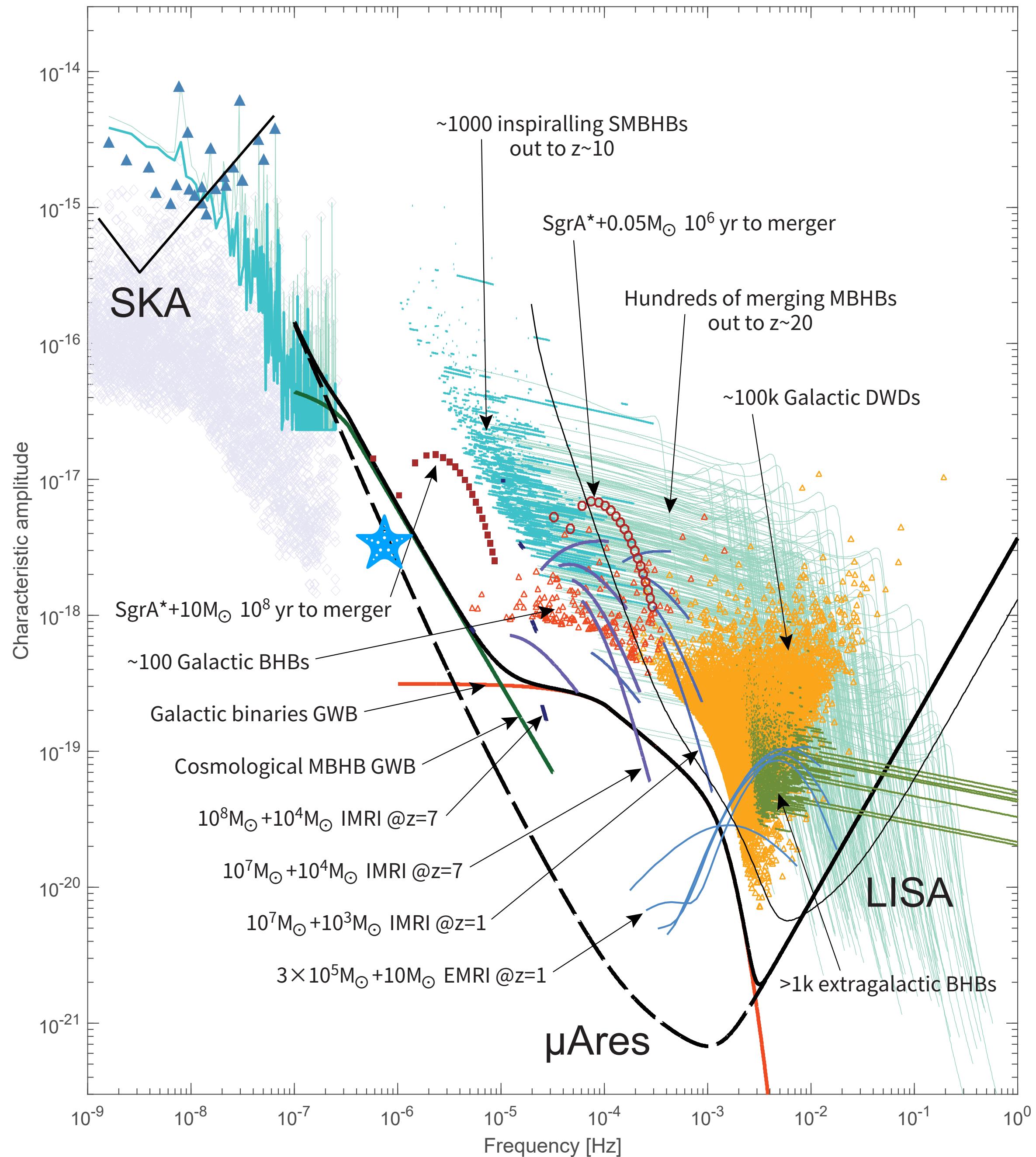
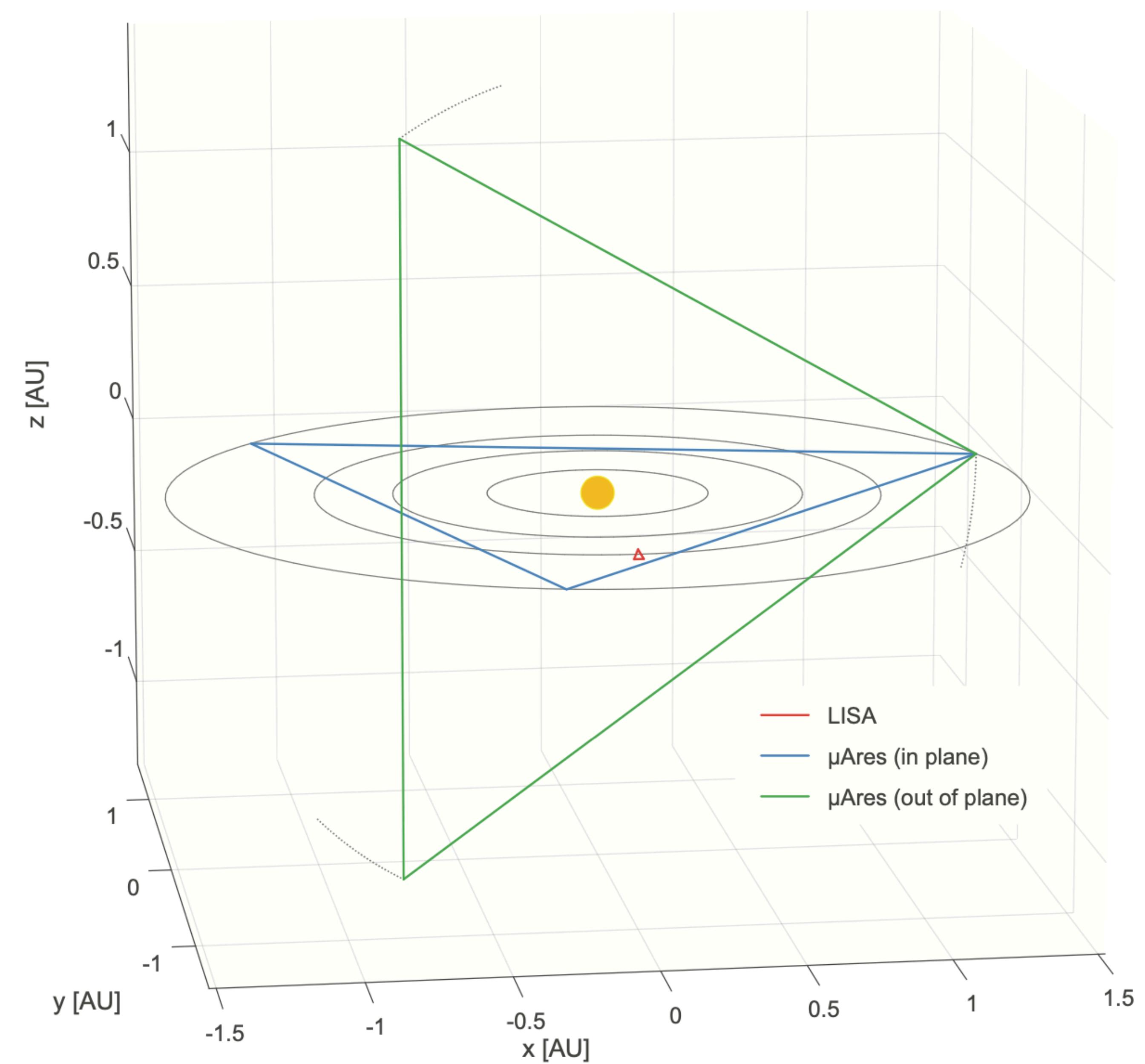


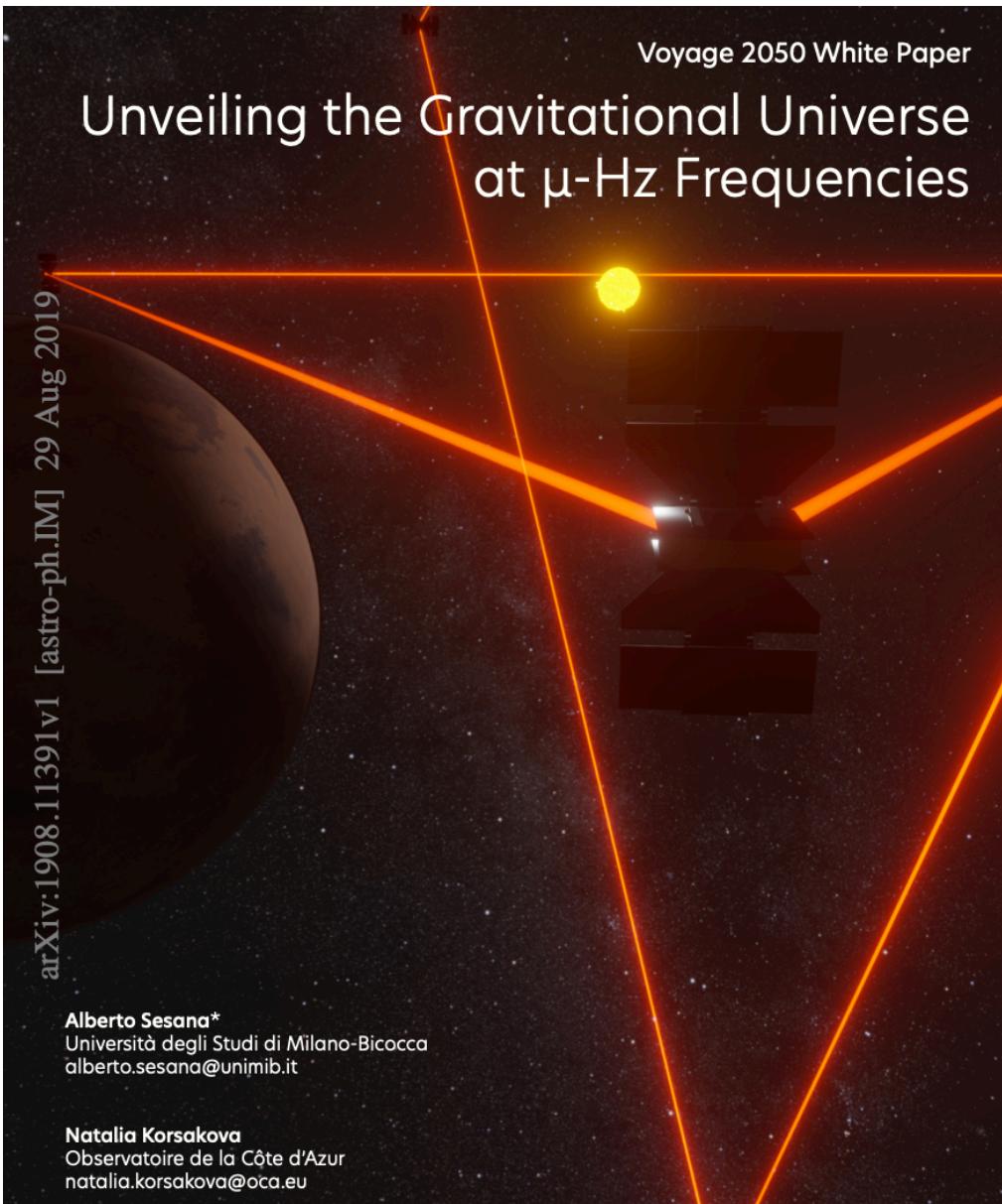
Possible backgrounds & ideas at μ Hz: a rich band

i) μ Ares: LISA-like concept



The μ Ares detection landscape





Summary of multimessenger and multiband science goals:

- identify the characteristic signatures of inspiralling MBHBs, providing the key to search for them in millions of AGN spectra and lightcurves;
- investigate the interplay between gravity, matter, and light in the dynamical spacetime of inspiralling/merging MBHBs;
- establish the connection between MBHBs and their hosts;
- provide a unique sample of standard sirens out to $z \gtrsim 5$;
- fully characterize the MBHB population at low frequency, in connection with PTAs.

Summary of Milky Way science goals:

- understand common envelope physics via detection of mixed (CO + MS star) binaries and the distribution of DWD at $f < 10^{-4}$ Hz;
- physics of contact and over-contact binaries via joint GW + EM detection;
- characterization of BHB, NSB and BH-NS population in the Galaxy, synergies with PTAs and SKA;
- unveil the dynamics of stars and COs around SgrA*.

Summary of cosmology and cosmography science goals:

- investigate a vast region of the allowed parameter space of QCD phase transitions;
- explore beyond standard physics by searching for an EW phase transition;
- explore first order phase transitions in the hidden sector;
- probe the geometry of the Universe, constraining deviations from Λ CDM out to $z \approx 10$ by means of $\mathcal{O}(10^3)$ MBHB standard sirens.

Summary of General relativity and beyond science goals:

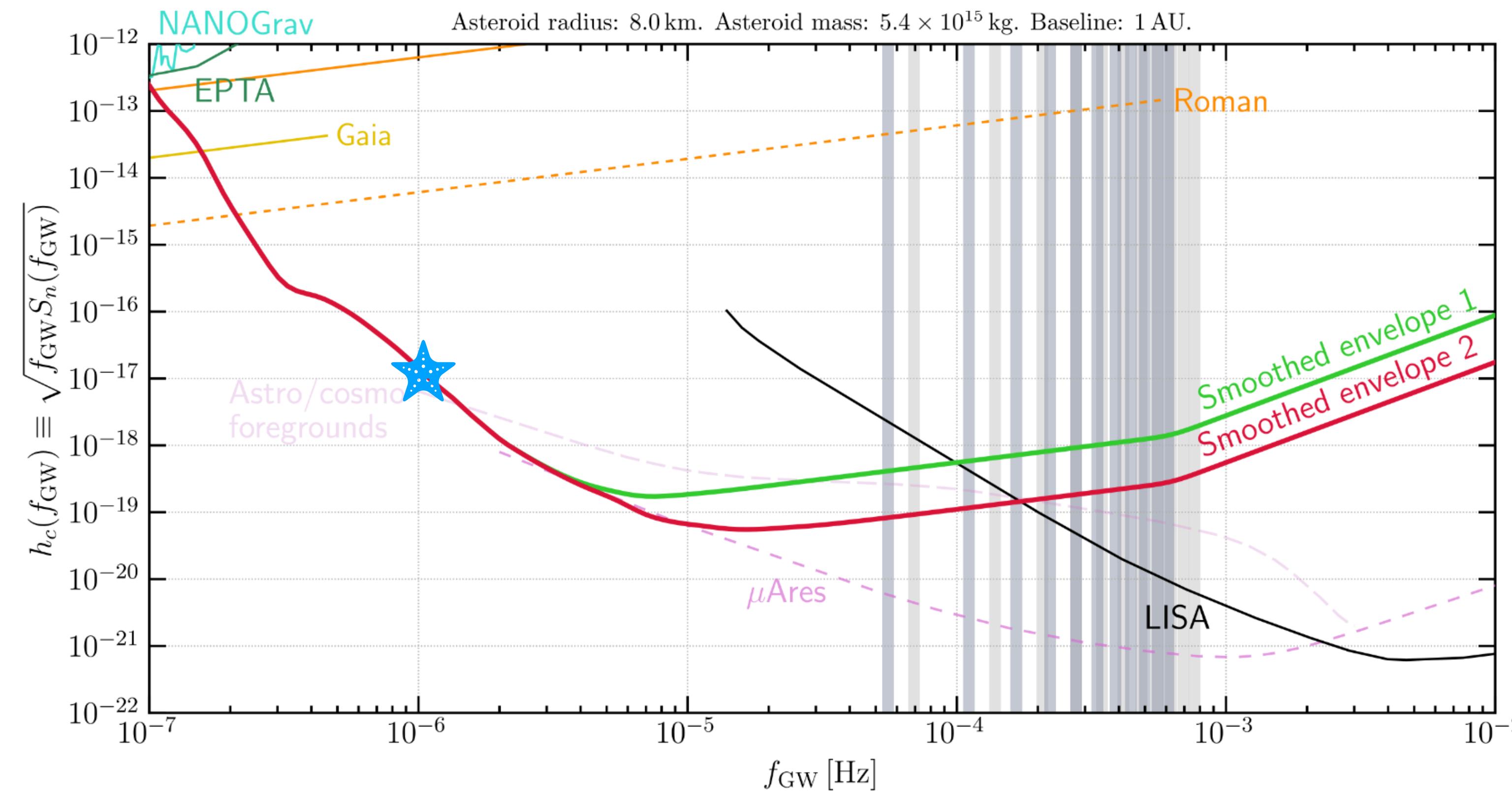
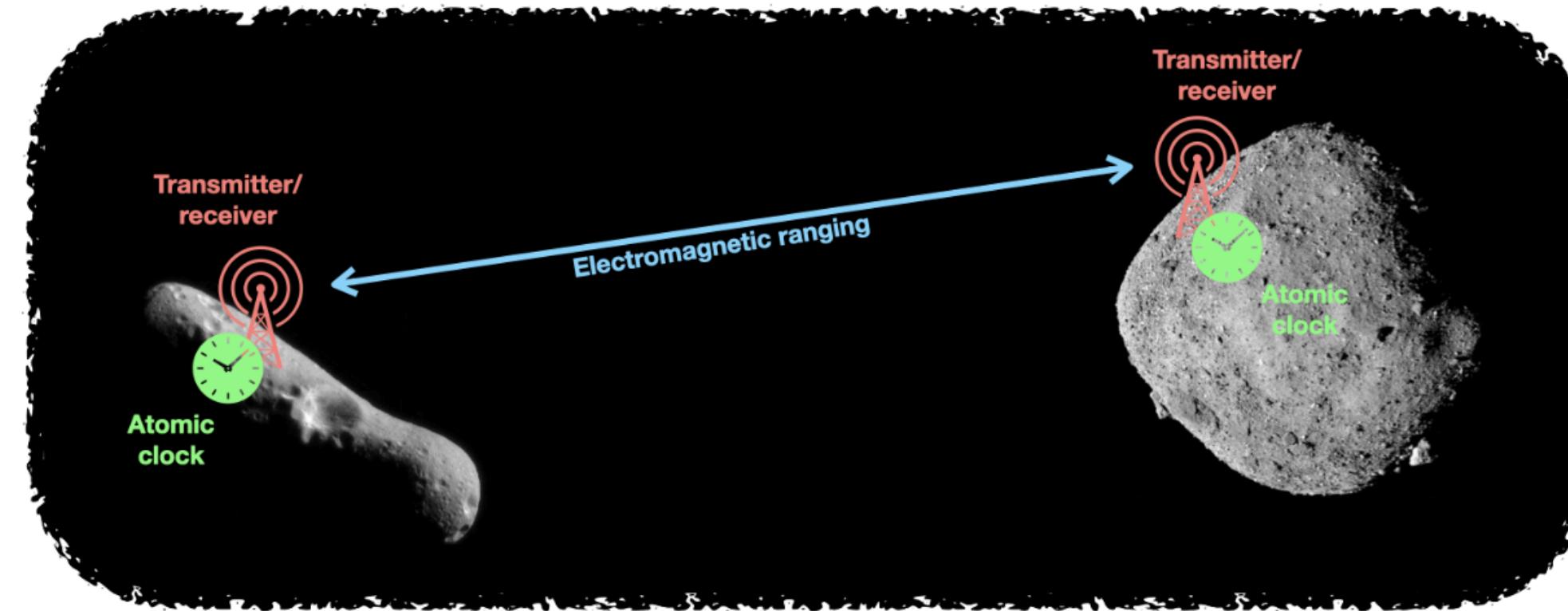
- probe DM substructures and the Universe expansion via strong GW lensing;
- detect non-linear GW memory with high SNR from merging MBHBs;
- improve sensitivity to graviton mass and other deviations from GR by more than two orders of magnitude with respect to LISA.

Summary of MBH science goals:

- probe the emergence of the high-redshift quasars;
- establish the relative importance of accretion vs mergers in growing MBHBs;
- disentangle seed formation models at the high-mass end of the seed spectrum;
- probe the population of inspiralling MBHBs in low-redshift dwarf galaxies;
- pin down the physics of MBHB dynamics, including stellar hardening, gaseous drag, triplets, and multiplets MBHB interactions;
- probe the formation and dynamics of IMBHs in galactic nuclei.

Ranging of asteroids?

Fedderke et al 2112.11431



Future astrometry?

e.g. Moore et al
Mihaylov et al.

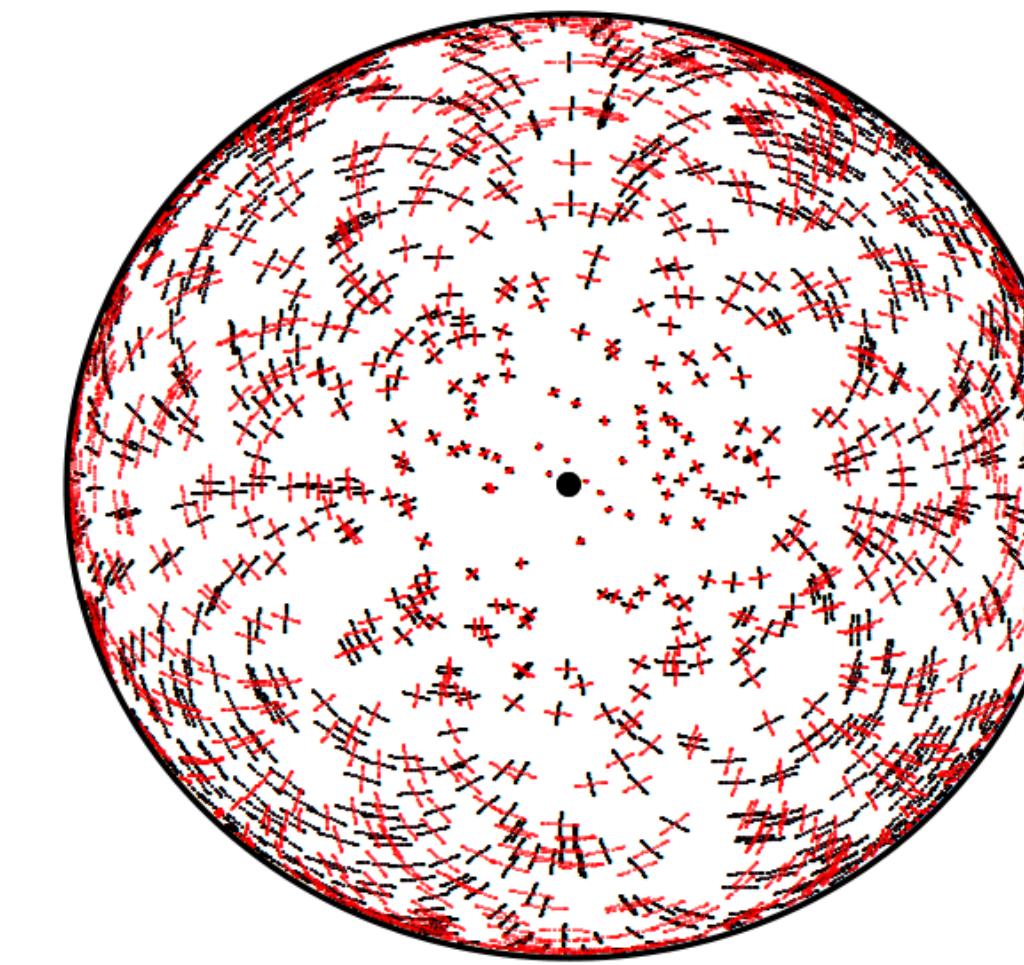
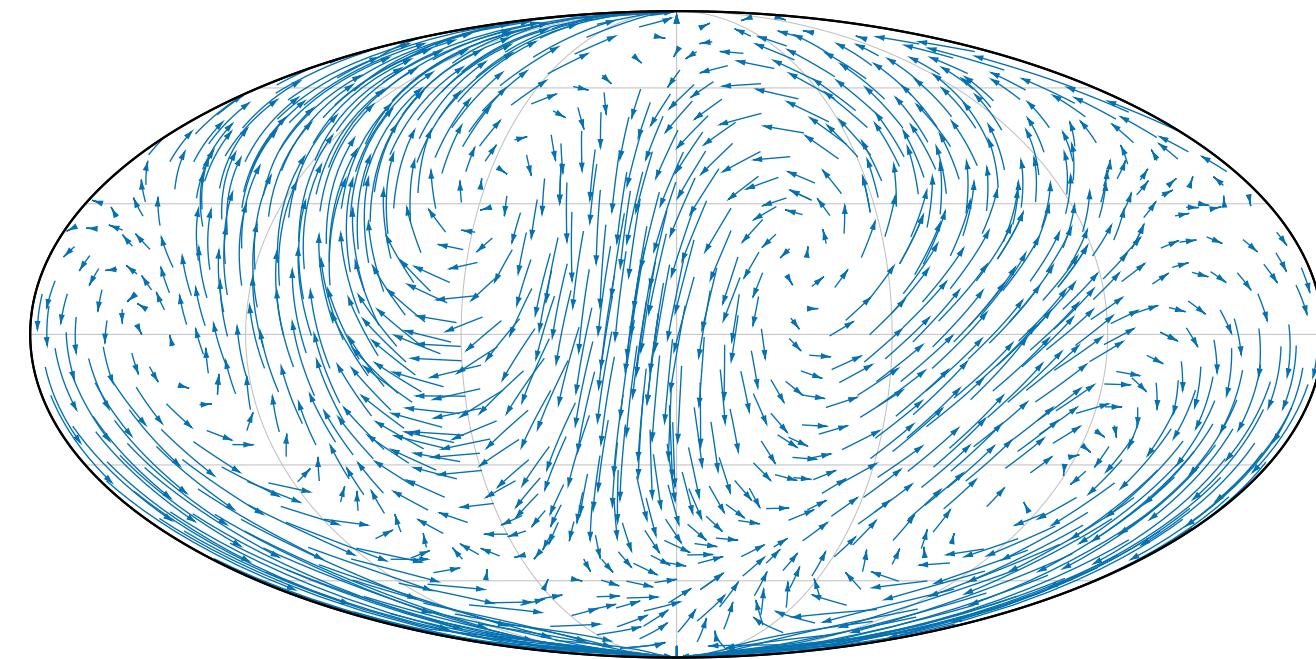
1707.06239
1804.00660

Klioner
1710.11474

Garcia-Bellido et al. 2104.04778

Fedderke et al 2204.07677

Monitoring many stars (GAIA or better)



Stellar interferometry

We evaluate the potential for gravitational-wave (GW) detection in the frequency band from 10 nHz to 1 μ Hz using extremely high-precision astrometry of a small number of stars

at characteristic strains around $h_c \sim 10^{-17} \times (\mu\text{Hz}/f_{\text{GW}})$. The astrometric angular precision required to see these sources is $\Delta\theta \sim h_c$ after integrating for a time $T \sim 1/f_{\text{GW}}$. We show that jitter in the photometric center of WD of this type due to starspots is bounded to be small enough to permit this high-precision, small- N approach. We discuss possible noise arising from stellar reflex motion induced by orbiting objects and show how it can be mitigated. The only plausible technology able to achieve the requisite astrometric precision is a space-based stellar interferometer. Such a future mission with few-meter-scale collecting dishes and baselines of $\mathcal{O}(100 \text{ km})$ is sufficient to achieve the target precision. This collector size is broadly in line with the collectors proposed for

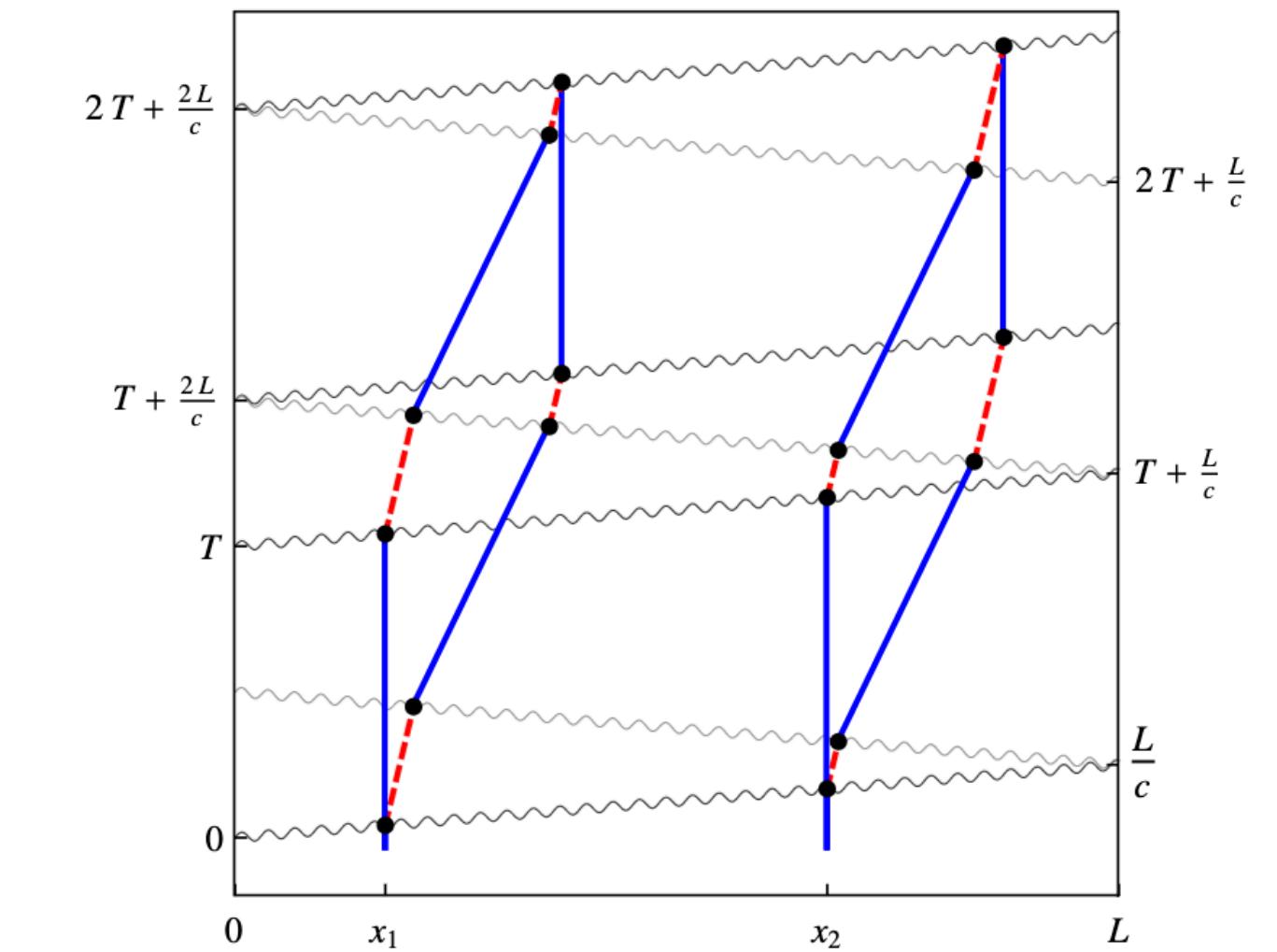
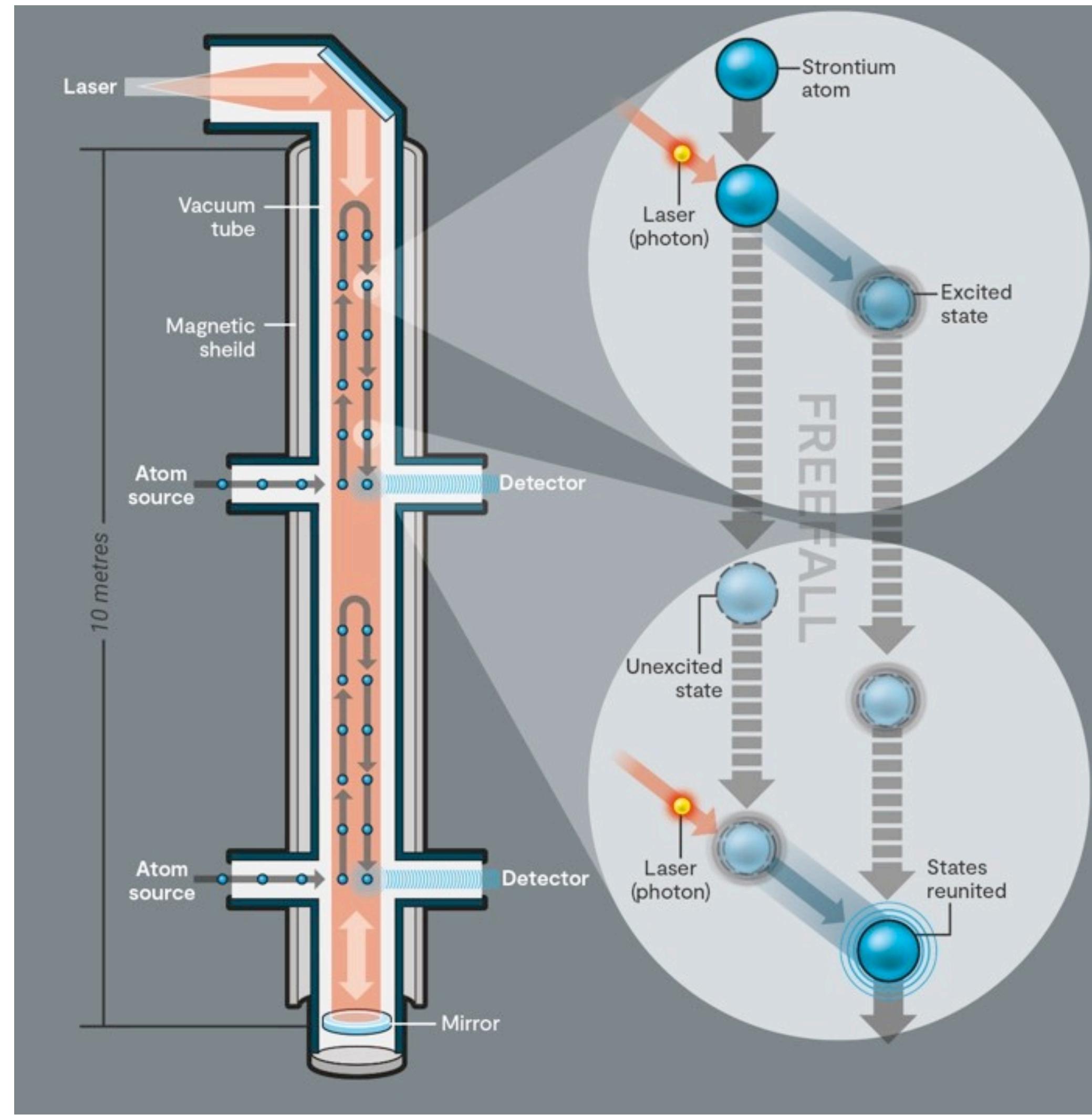
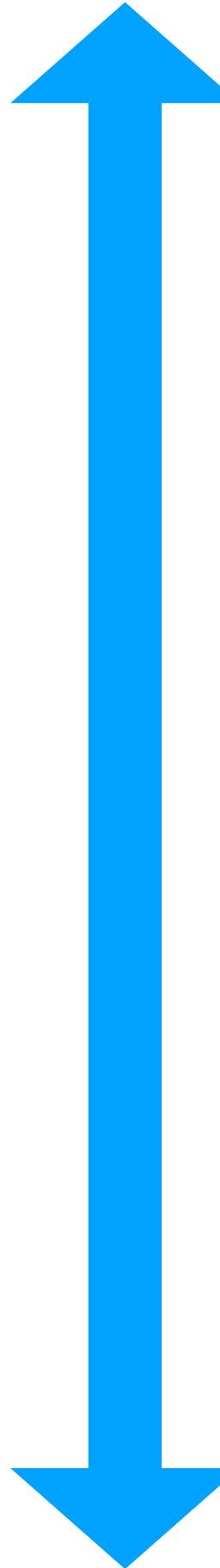
Atomic interferometry in space: AEDGE

Abou El-Neaj et al 1908.00802

Graham et al 1206.0818 (MAGIS)

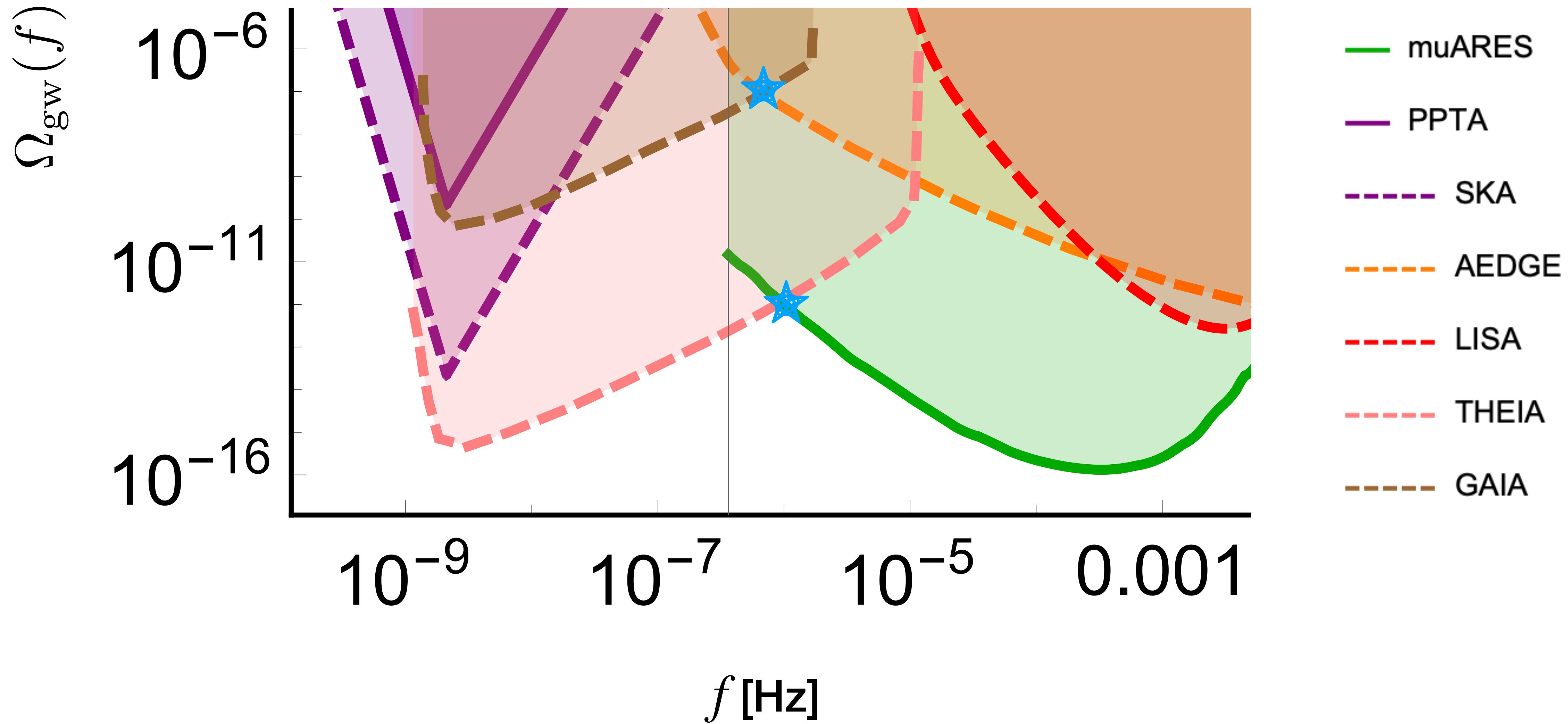
Badurina et al 2108.02468 (AION)

40000 km

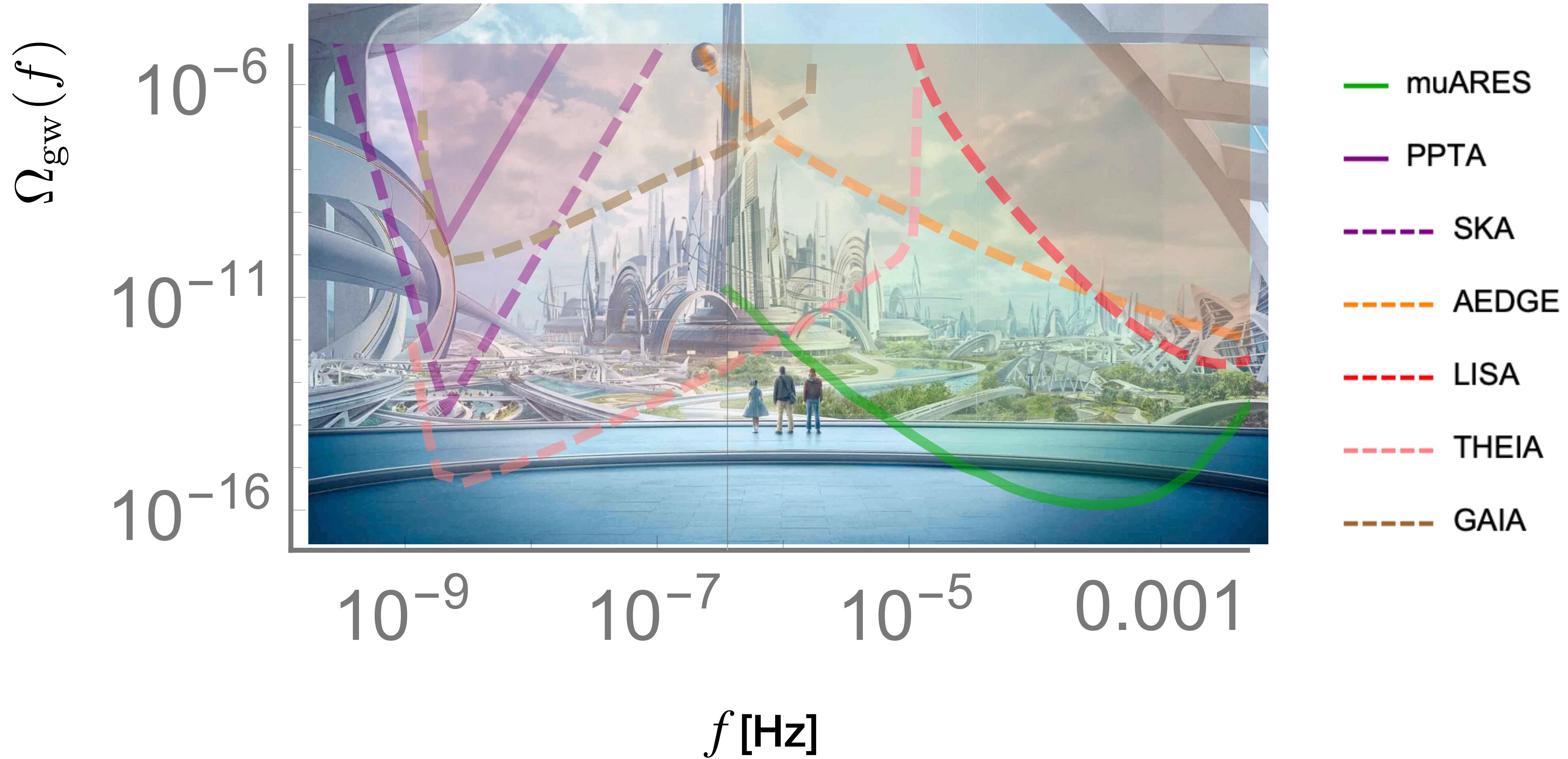


$$\Delta\phi \sim \omega L h$$

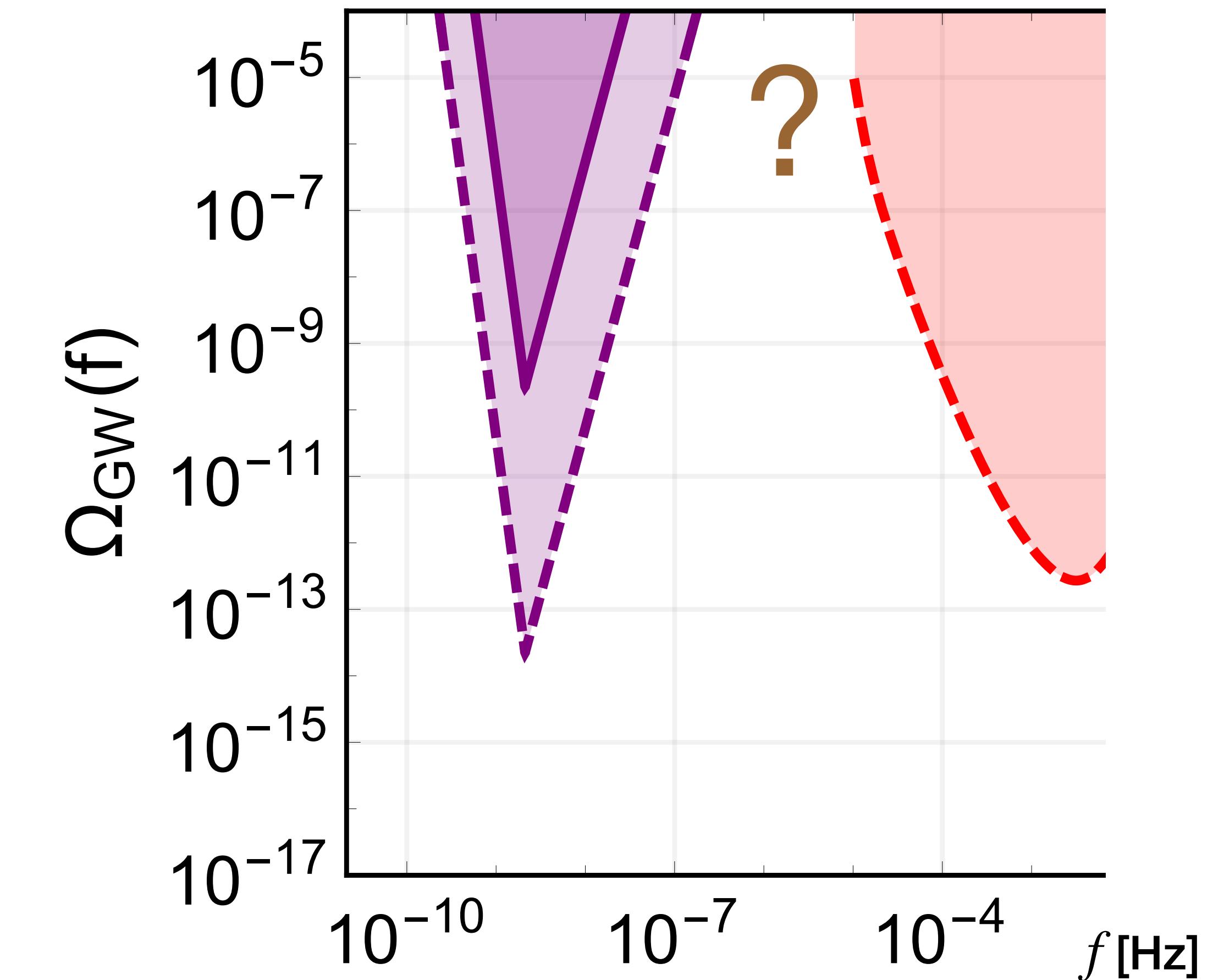
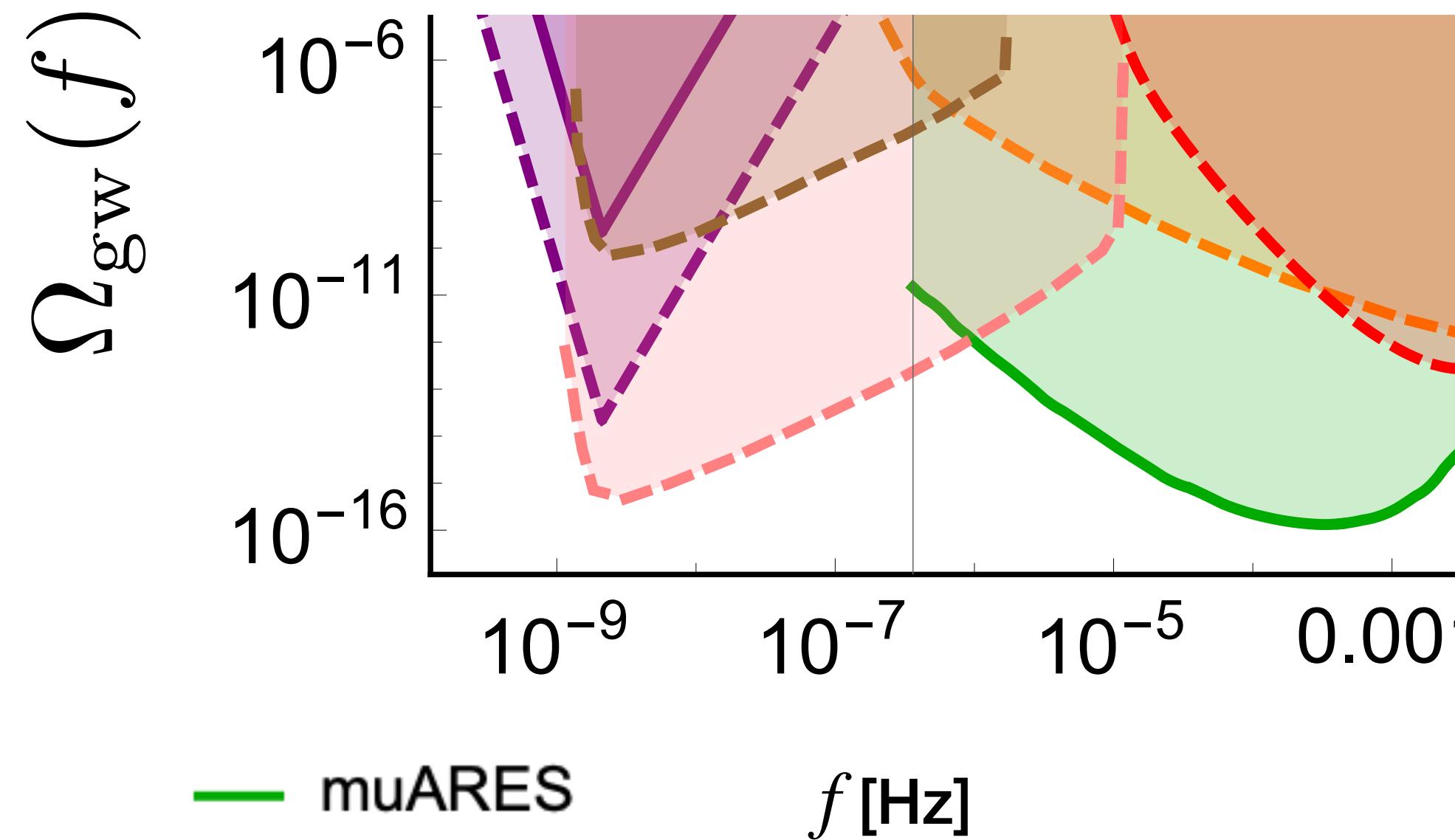
The most optimistic future...



The most optimistic future...



The most optimistic future... vs 2038



GAIA DR3 Jaraba et al 2304.06350

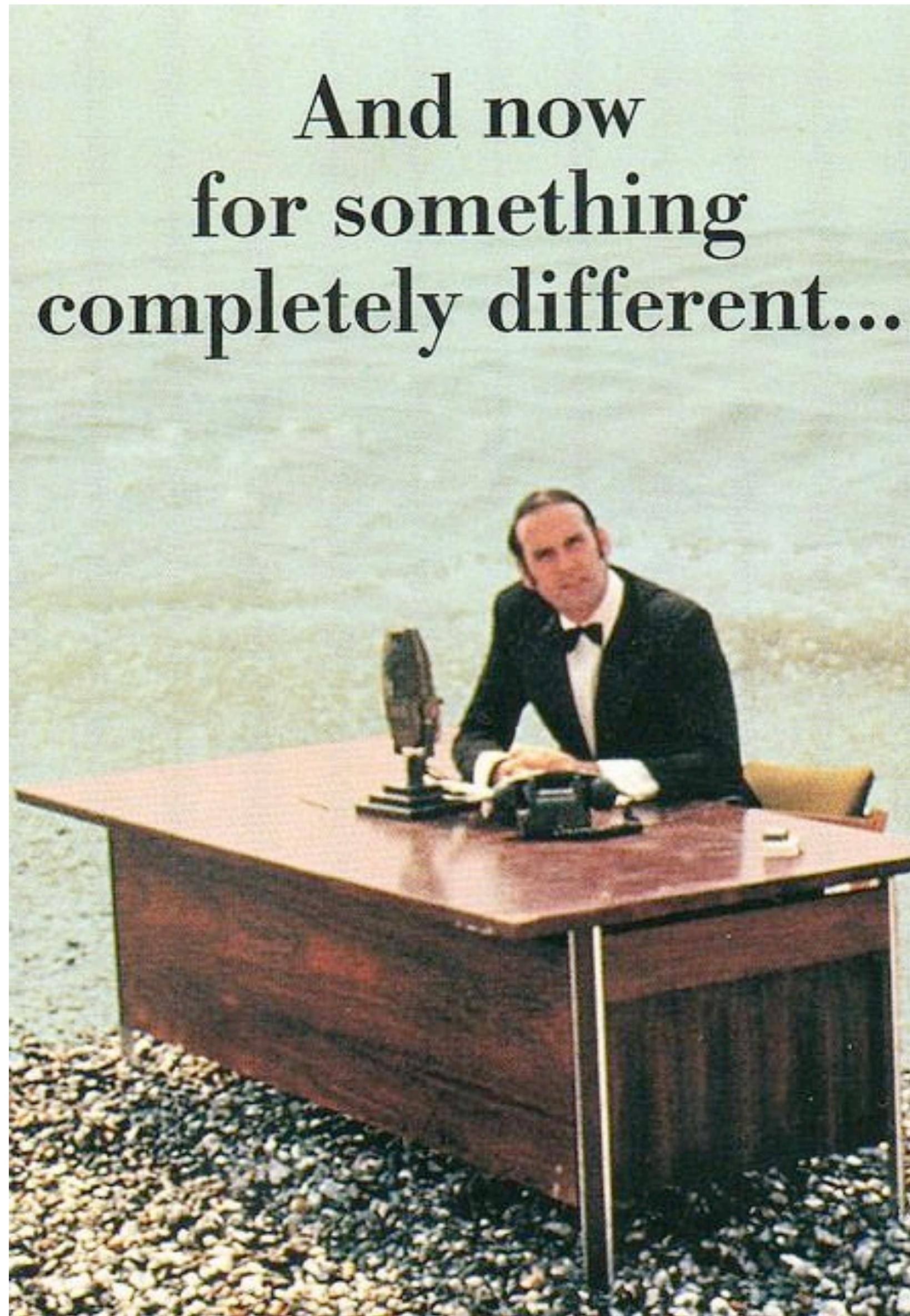
$h_{70}^2 \Omega_{\text{gw}} \lesssim 0.087$ for $4.2 \times 10^{-18} \text{ Hz} \lesssim f \lesssim 1.1 \times 10^{-8} \text{ Hz}$

ROMAN? Wang et al 2205.07962

AION 10m/MAGIS 100m in 2025? (small interferometers)

Is this all we can do in this band?

Is this all we can do in this band? No!



And now
for something
completely different...

$f \sim \mu\text{Hz}$
few days

Absorption of GWs by binaries

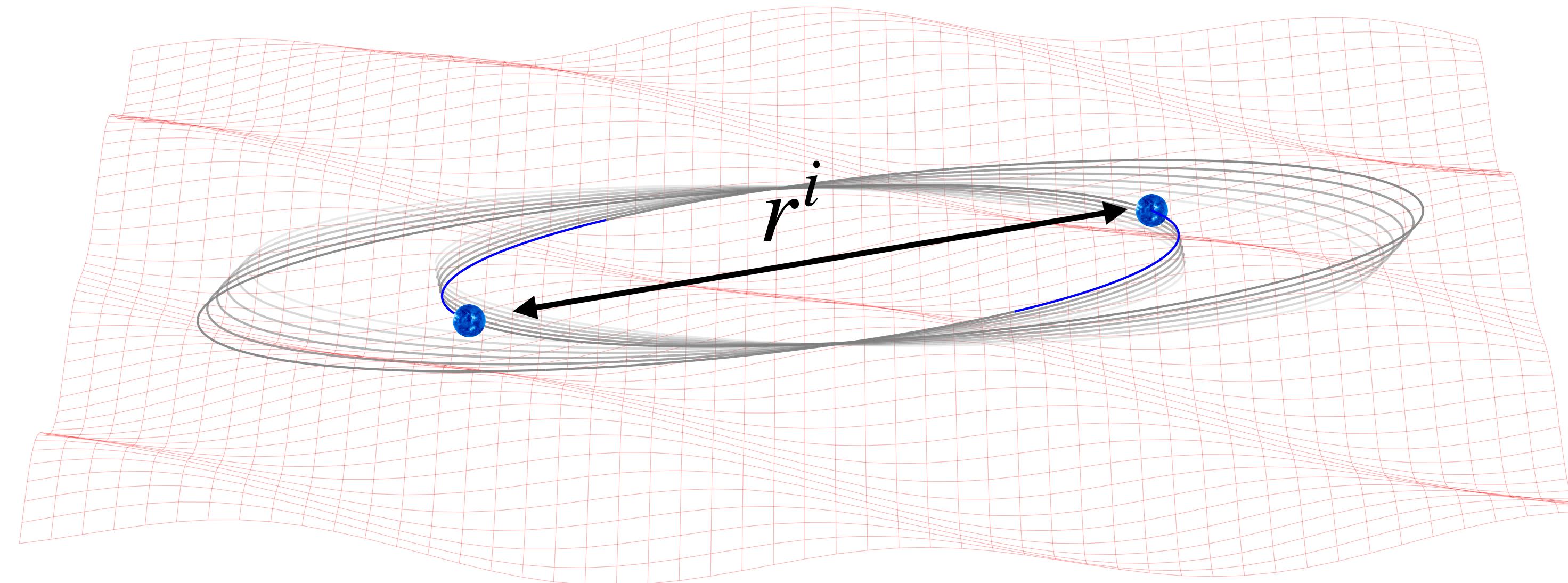
$f \sim \mu\text{Hz}$
few days

Intuitive idea (from '60s)

Influence of a GW on a binary system (e.g. non-relativistic)

$$\ddot{r}^i + \frac{GM}{r^3}r^i = \delta^{ik}\frac{1}{2}\ddot{h}_{kj}r^j$$

Newtonian potential ... GW

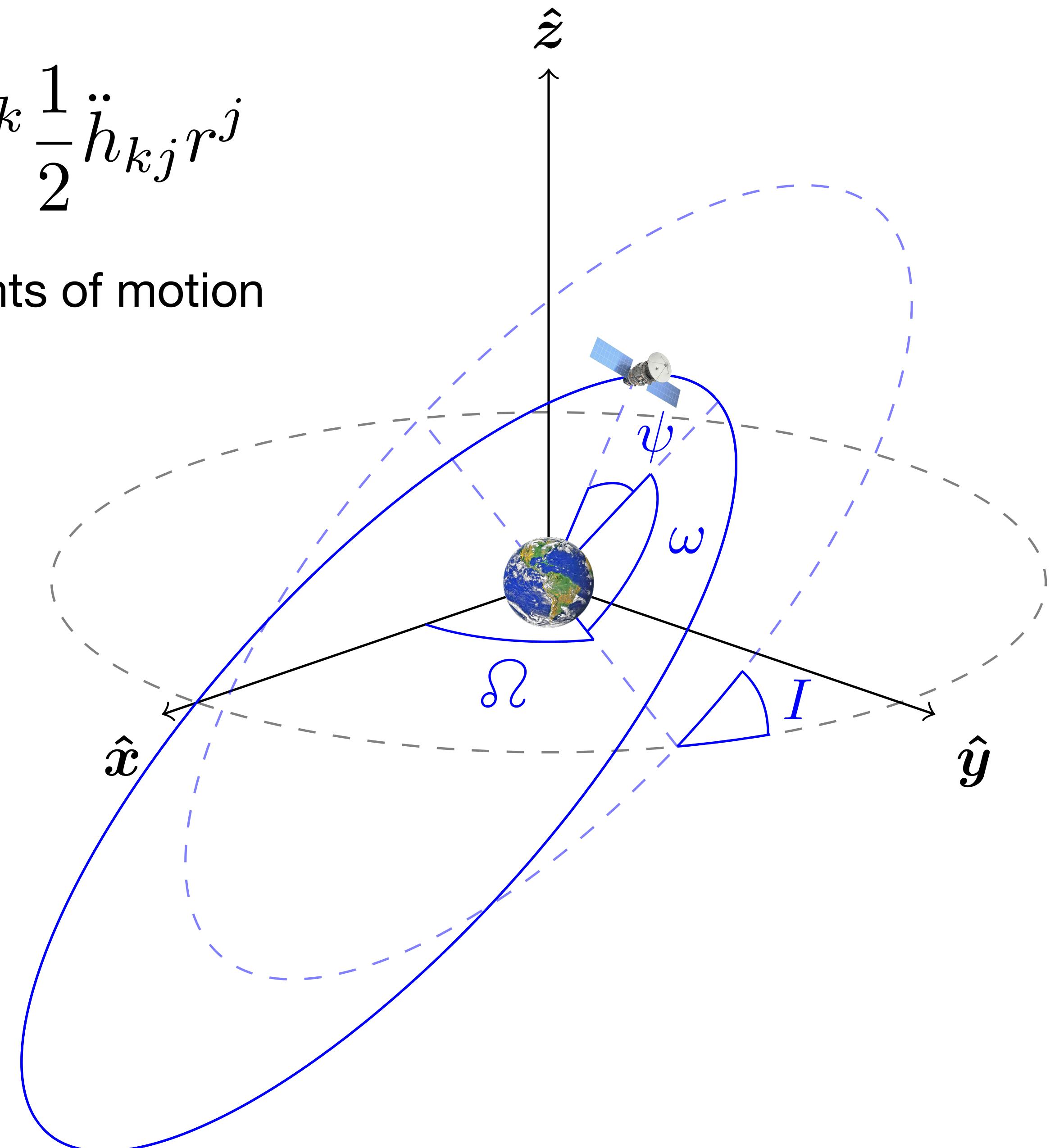


Absorption of GWs by binaries

$$\ddot{r}^i + \frac{GM}{r^3}r^i = \delta^{ik}\frac{1}{2}\ddot{h}_{kj}r^j$$

Better characterised for its 6 Newtonian constants of motion

- **period P , eccentricity e :**
size and shape of orbit
- **inclination I , ascending node Ω :**
orientation in space
- **pericentre ω ,**
mean anomaly at epoch ε :
radial and angular phases



Absorption of GWs by binaries

$$\ddot{\mathbf{r}} + \frac{GM}{r^2} \hat{\mathbf{r}} = \delta \ddot{\mathbf{r}}.$$

■ for generic perturbation:

$$\delta \ddot{\mathbf{r}} = r(\mathcal{F}_r \hat{\mathbf{r}} + \mathcal{F}_\theta \hat{\boldsymbol{\theta}} + \mathcal{F}_\ell \hat{\boldsymbol{\ell}}),$$



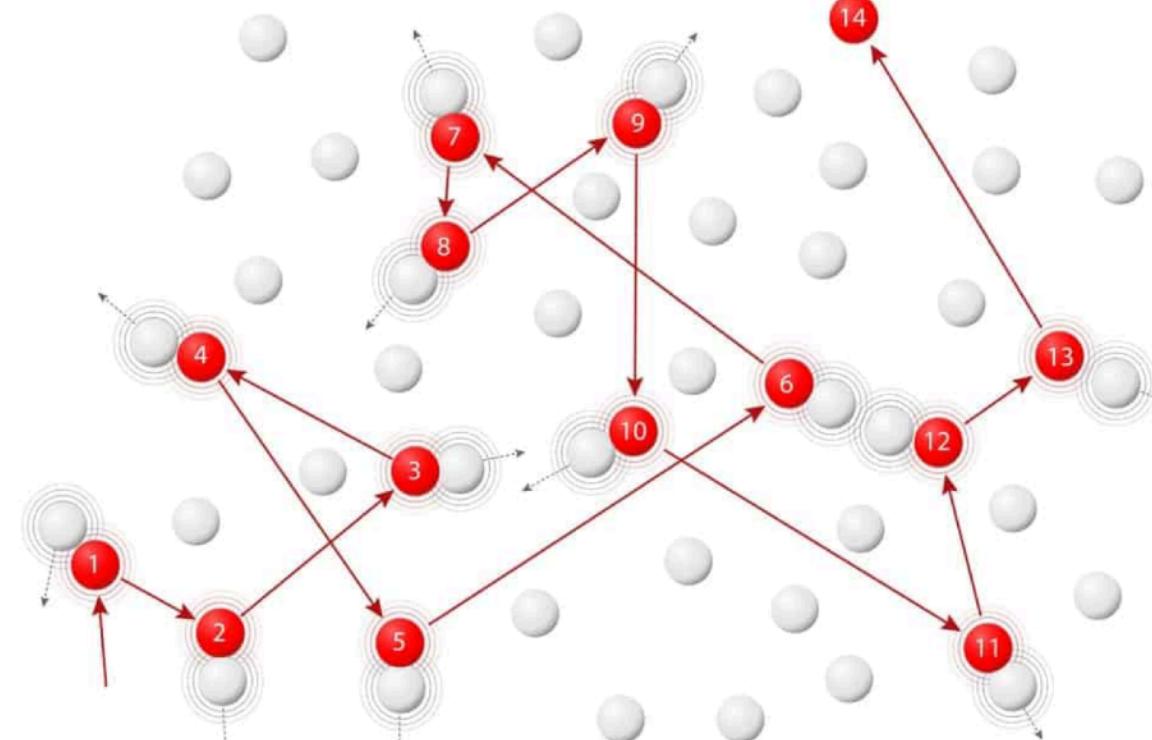
$$\begin{aligned}\dot{P} &= \frac{3P^2\gamma}{2\pi} \left[\frac{e \sin \psi \mathcal{F}_r}{1 + e \cos \psi} + \mathcal{F}_\theta \right], \\ \dot{e} &= \frac{\dot{P}\gamma^2}{3Pe} - \frac{P\gamma^5 \mathcal{F}_\theta}{2\pi e (1 + e \cos \psi)^2}, \\ \dot{I} &= \frac{P\gamma^3 \cos \theta \mathcal{F}_\ell}{2\pi (1 + e \cos \psi)^2}, \\ \dot{\Omega} &= \frac{\tan \theta}{\sin I} \dot{I}, \\ \dot{\omega} &= \frac{P\gamma^3}{2\pi e} \left[\frac{(2 + e \cos \psi) \sin \psi \mathcal{F}_\theta}{(1 + e \cos \psi)^2} - \frac{\cos \psi \mathcal{F}_r}{1 + e \cos \psi} \right] - \cos I \dot{\Omega}, \\ \dot{\varepsilon} &= -\frac{P\gamma^4 \mathcal{F}_r}{\pi (1 + e \cos \psi)^2} - \gamma (\cos I \dot{\Omega} + \dot{\omega}),\end{aligned}$$

Absorption of GWs by binaries

$$\ddot{\mathbf{r}} + \frac{GM}{r^2} \hat{\mathbf{r}} = \delta \ddot{\mathbf{r}}.$$

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For the SGWB... Fokker-Planck approach

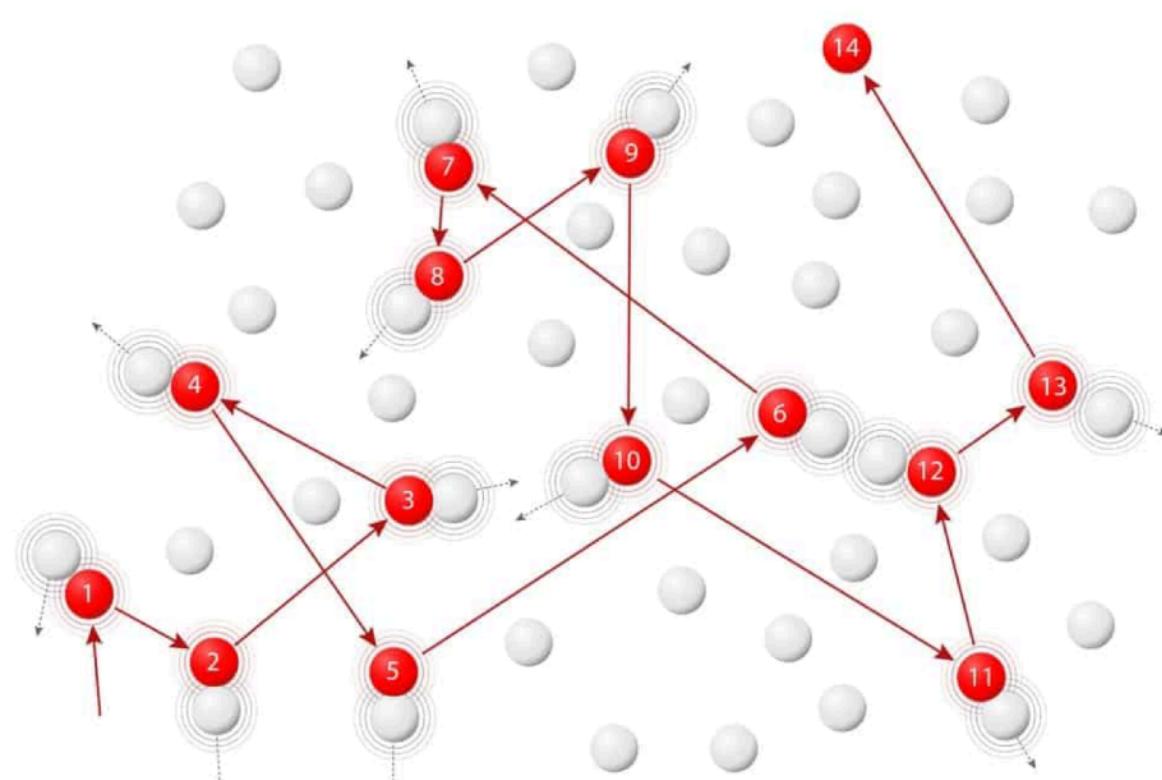
$$\ddot{r}^i + \frac{GM}{r^3} r^i = \delta^{ik} \frac{1}{2} \ddot{h}_{kj} r^j$$

deterministic

$$\dot{X}_i(\mathbf{X}, t) = V_i(\mathbf{X}) + \Gamma_i(\mathbf{X}, t)$$

stochastic

we move from dynamics of the variable to dynamics of the **distribution $W(\mathbf{X})$**



$$\frac{\partial W}{\partial t} = -\partial_i \left(D_i^{(1)} W \right) + \partial_i \partial_j \left(D_{ij}^{(2)} W \right)$$

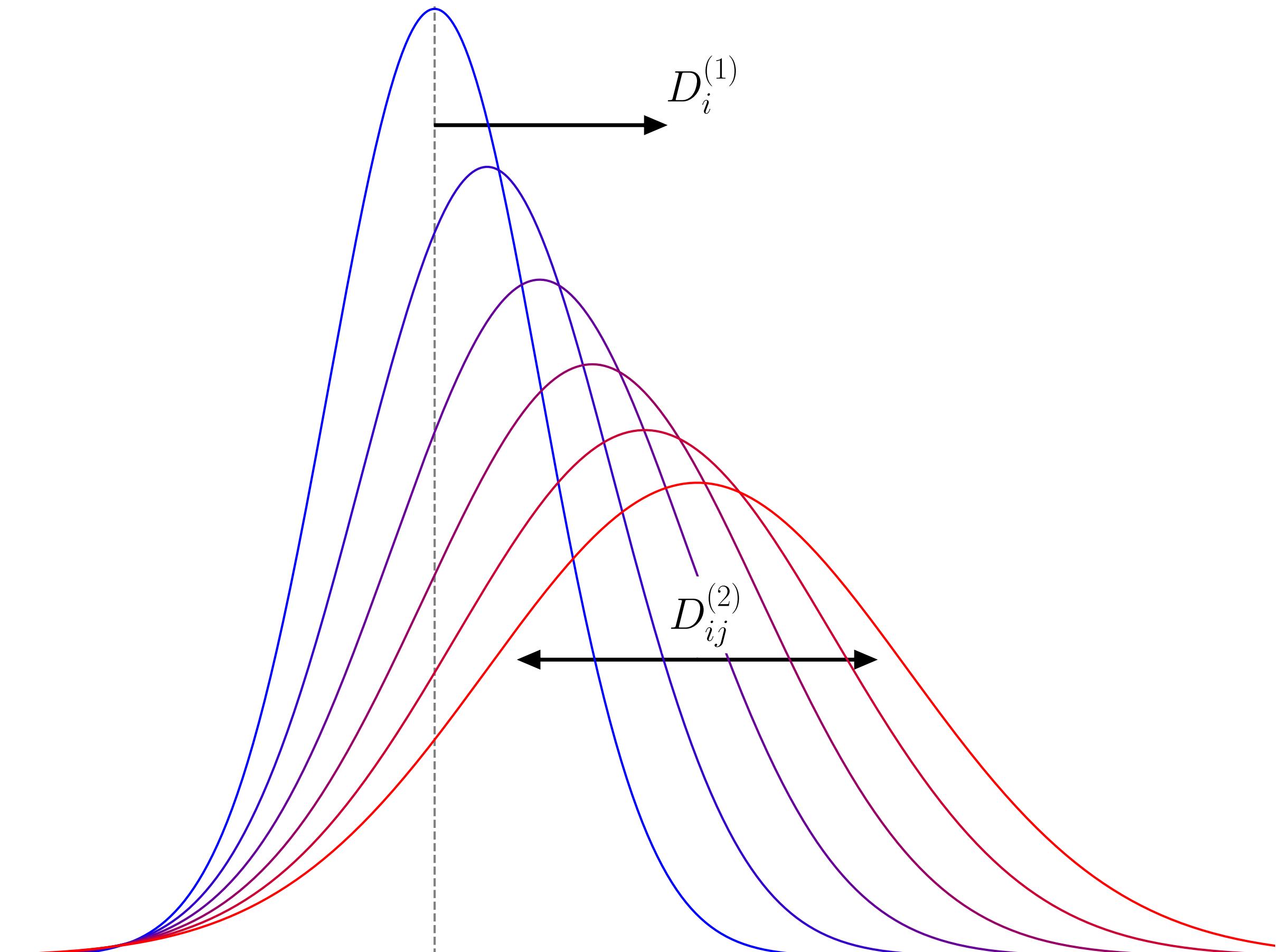
with $\partial_i \equiv \partial / \partial X_i$

$$D_i^{(1)} = V_i + \lim_{\tau \rightarrow 0} \frac{1}{\tau} \int_t^{t+\tau} dt' \int_t^{t'} dt'' \langle \Gamma_j(\mathbf{x}, t'') \partial_j \Gamma_i(\mathbf{x}, t') \rangle.$$

$$D_{ij}^{(2)} = \lim_{\tau \rightarrow 0} \frac{1}{2\tau} \int_t^{t+\tau} dt' \int_t^{t+\tau} dt'' \langle \Gamma_i(\mathbf{x}, t') \Gamma_j(\mathbf{x}, t'') \rangle.$$

Our approach to the problem

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



- track distribution function $W(\mathbf{X}, t)$ of orbital elements $\mathbf{X} = (P, e, I, \Omega, \omega, \varepsilon)$
- evolves through *Fokker-Planck eqn.*

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial \mathbf{X}_i} (D_i^{(1)} W) + \frac{\partial}{\partial \mathbf{X}_i} \frac{\partial}{\partial \mathbf{X}_j} (D_{ij}^{(2)} W)$$

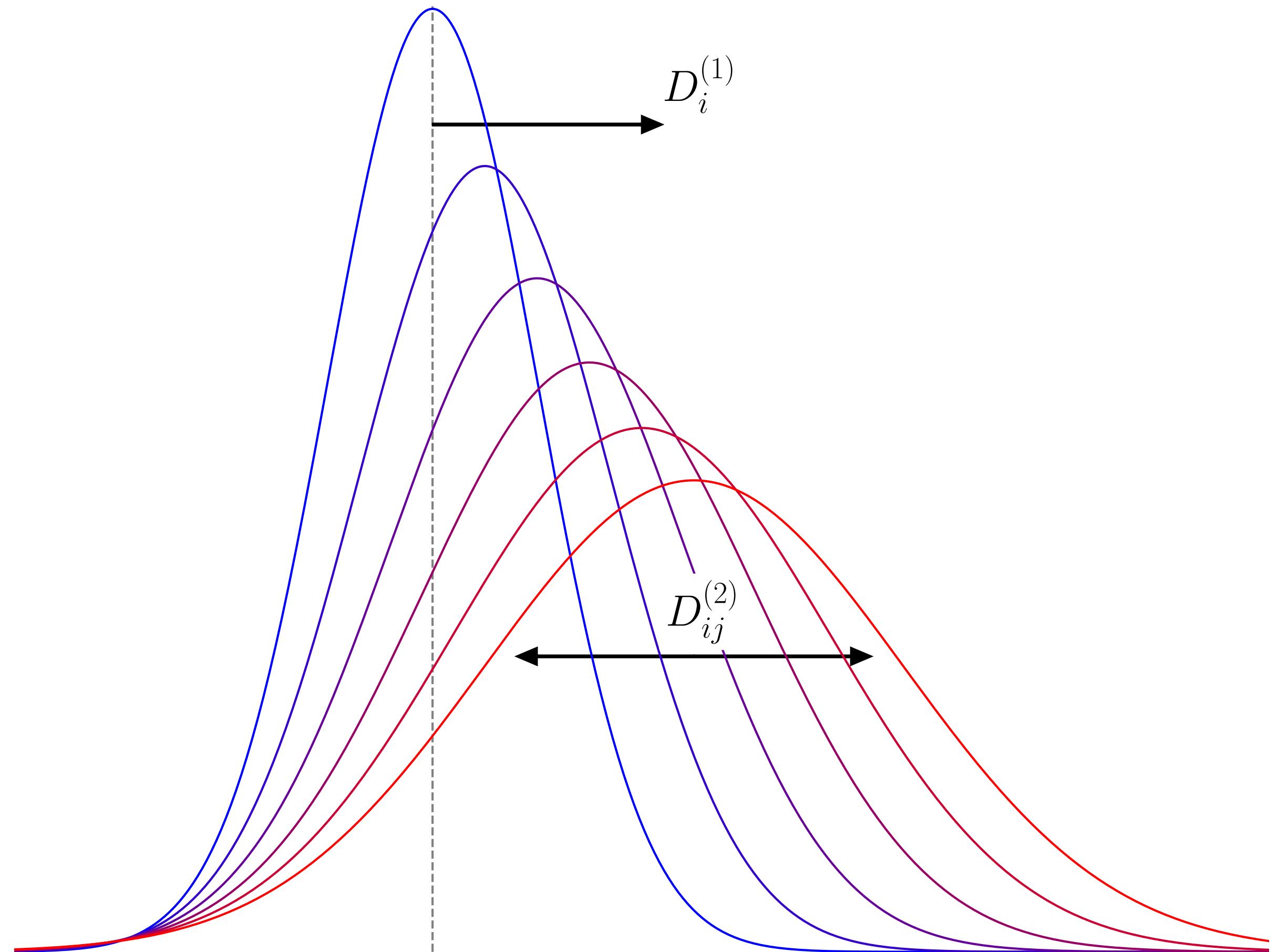
- *drift* and *diffusion* coefficients (averaged over orbits)

$$D_i^{(1)}(\mathbf{X}) = V_i(\mathbf{X}) + \sum_{n=1}^{\infty} \mathcal{A}_{n,i}(\mathbf{X}) \Omega_{\text{gw}}(n/P)$$

$$D_{ij}^{(2)}(\mathbf{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\mathbf{X}) \Omega_{\text{gw}}(n/P)$$

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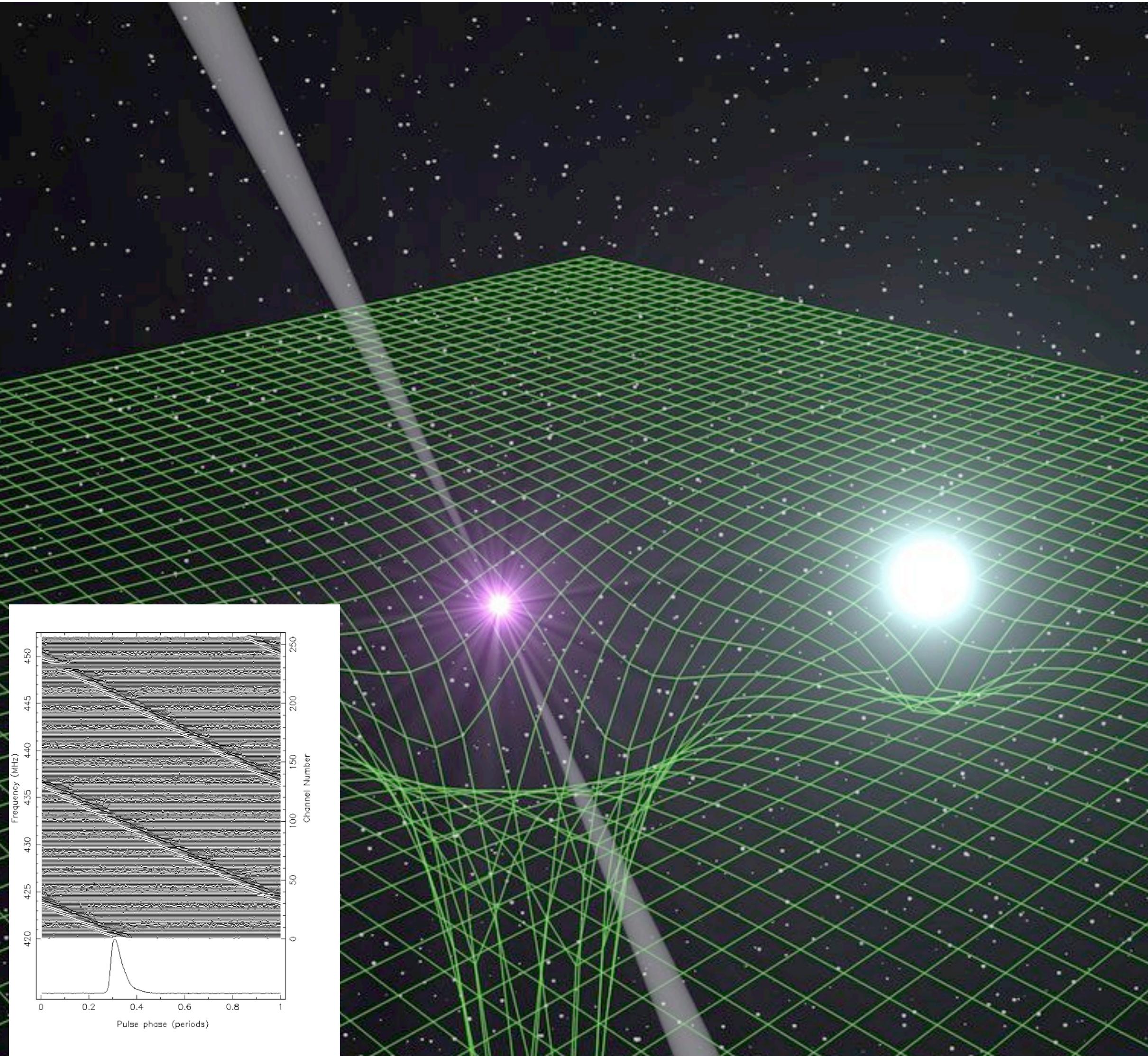
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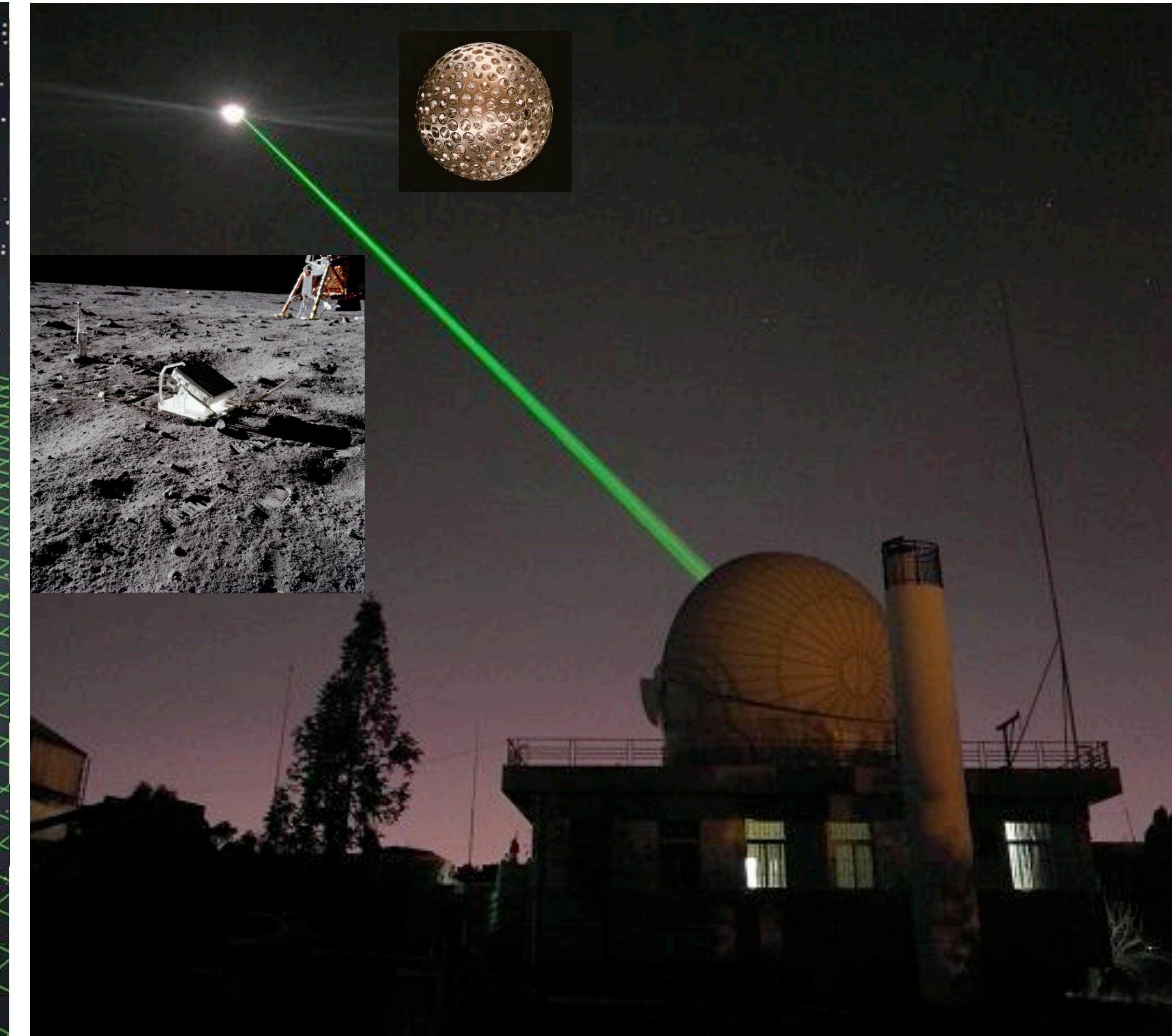
$$D_{ij}^{(2)}(\mathbf{X}) = \sum_{n=1}^{\infty} \mathcal{B}_{n,ij}(\mathbf{X}) \Omega_{\text{gw}}(n/P)$$

Two probes

timing of binary pulsars

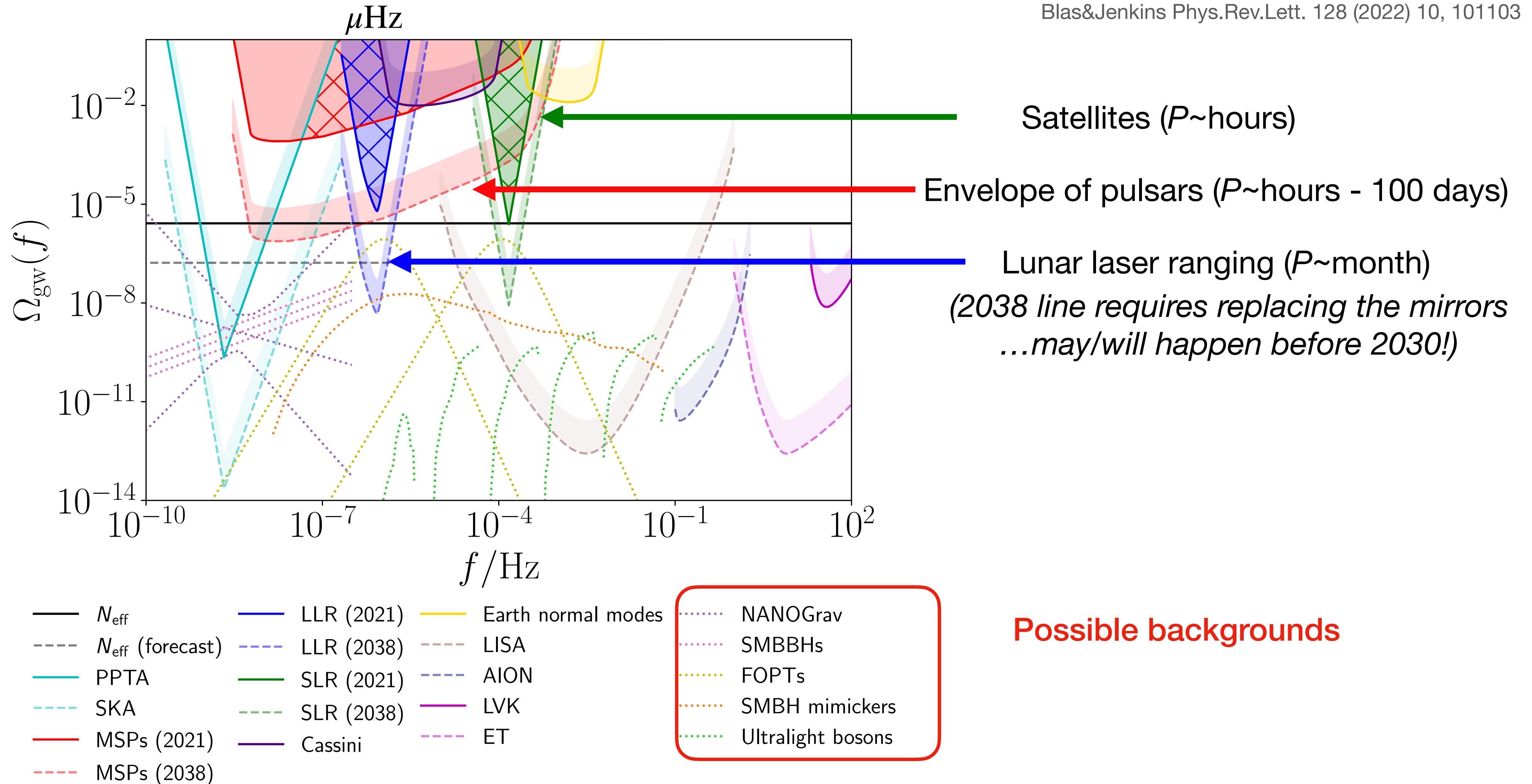


lunar and satellite laser ranging

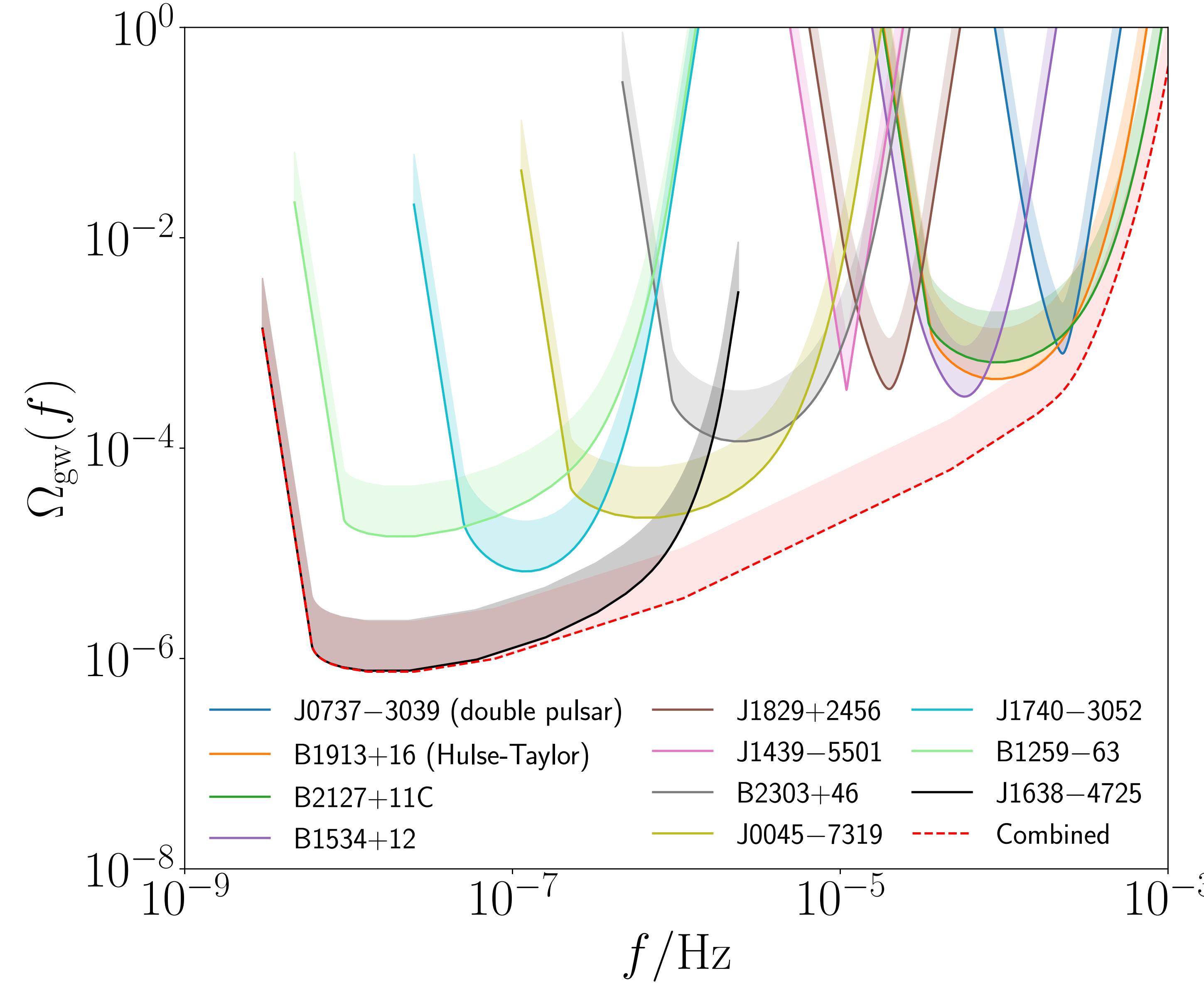


Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103

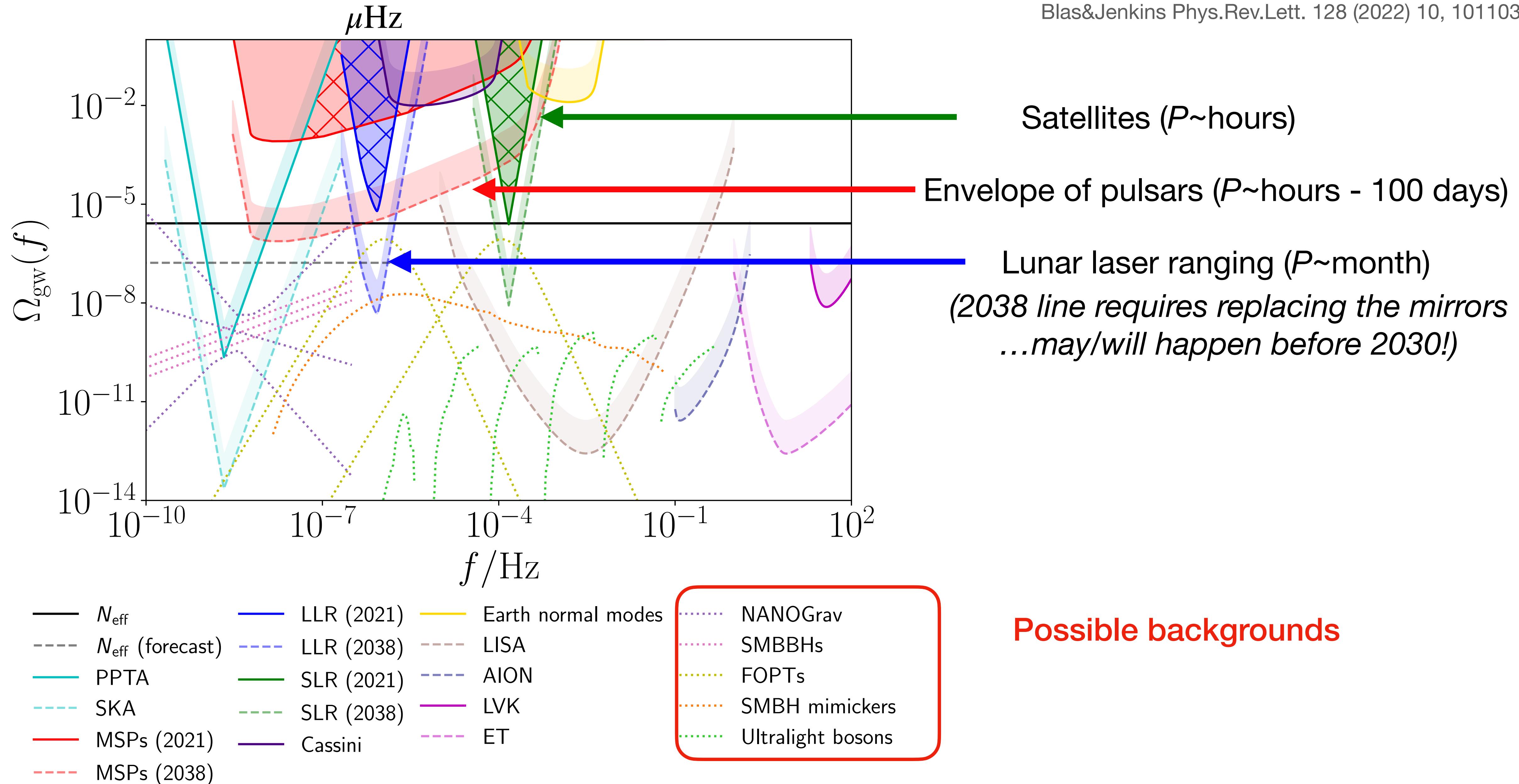


Combining binary pulsar bounds



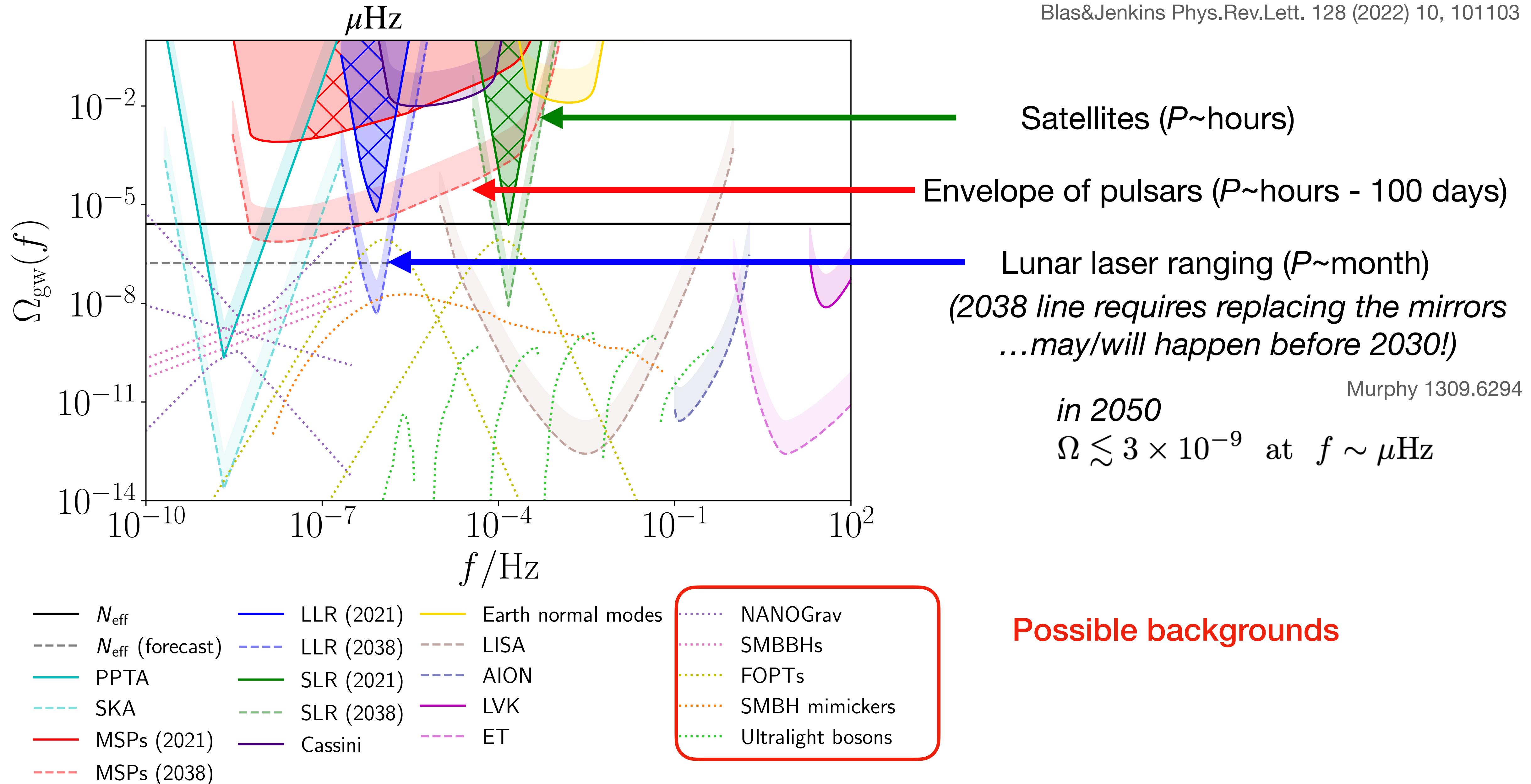
Our estimates (solid: today; dashed 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



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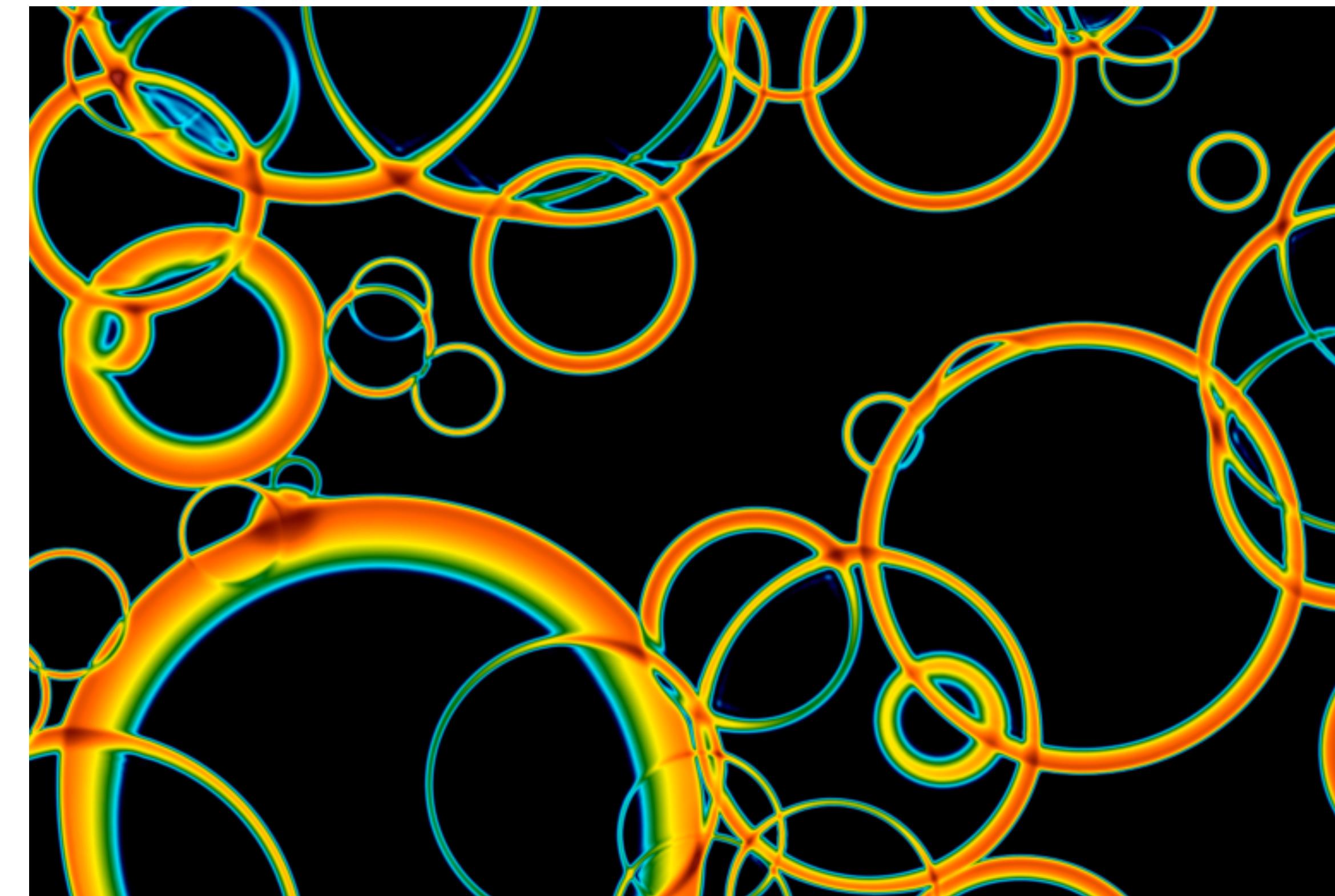
Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



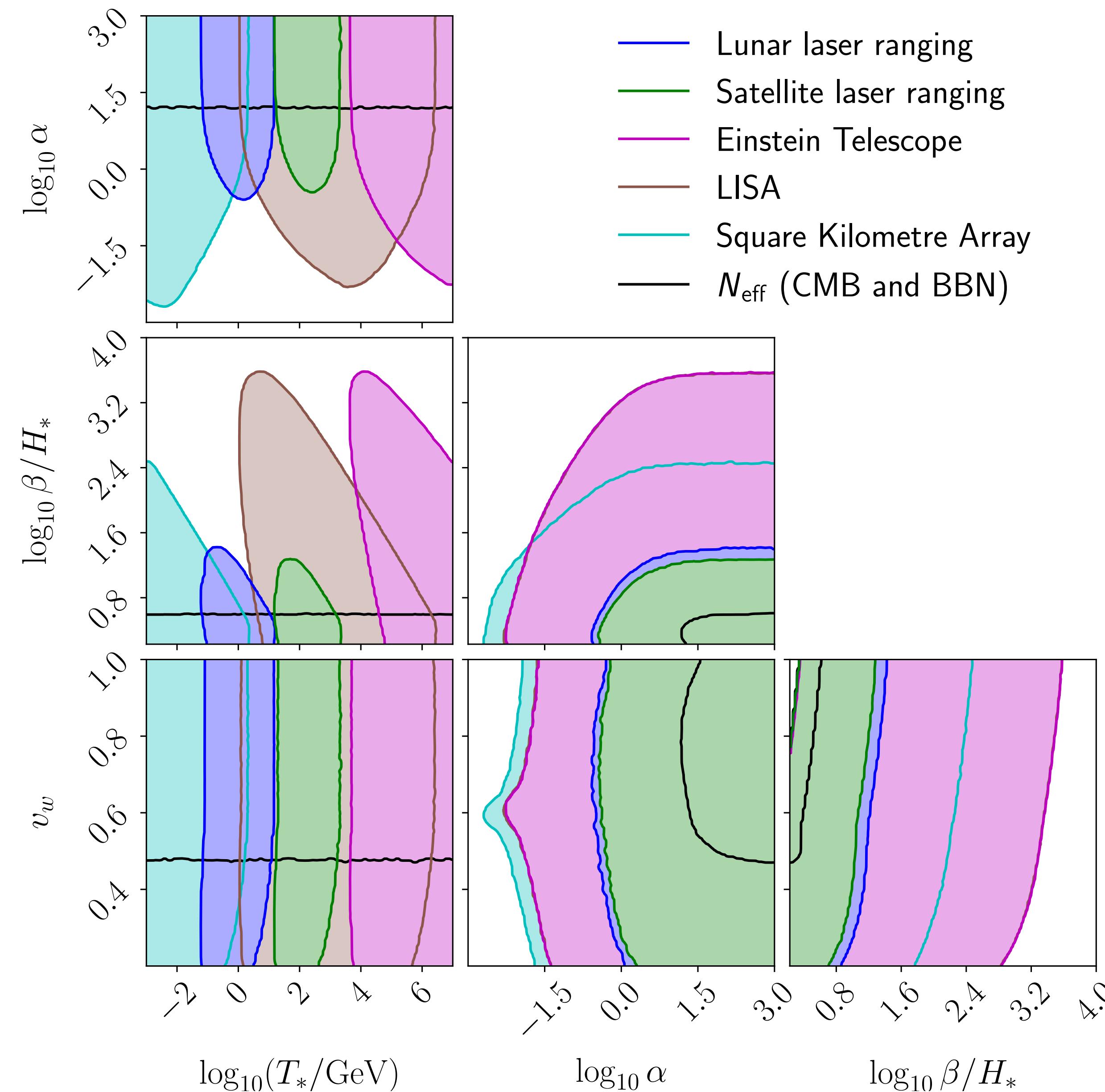
E.g. FOPT with peaks at μHz

- four parameters:
 - ▶ temperature T_*
 - ▶ strength α
 - ▶ rate β/H_*
 - ▶ bubble-wall velocity v_w
- peak frequency

$$f_* \approx 19 \mu\text{Hz} \times \frac{T_*}{100 \text{ GeV}} \frac{\beta/H_*}{v_w}$$

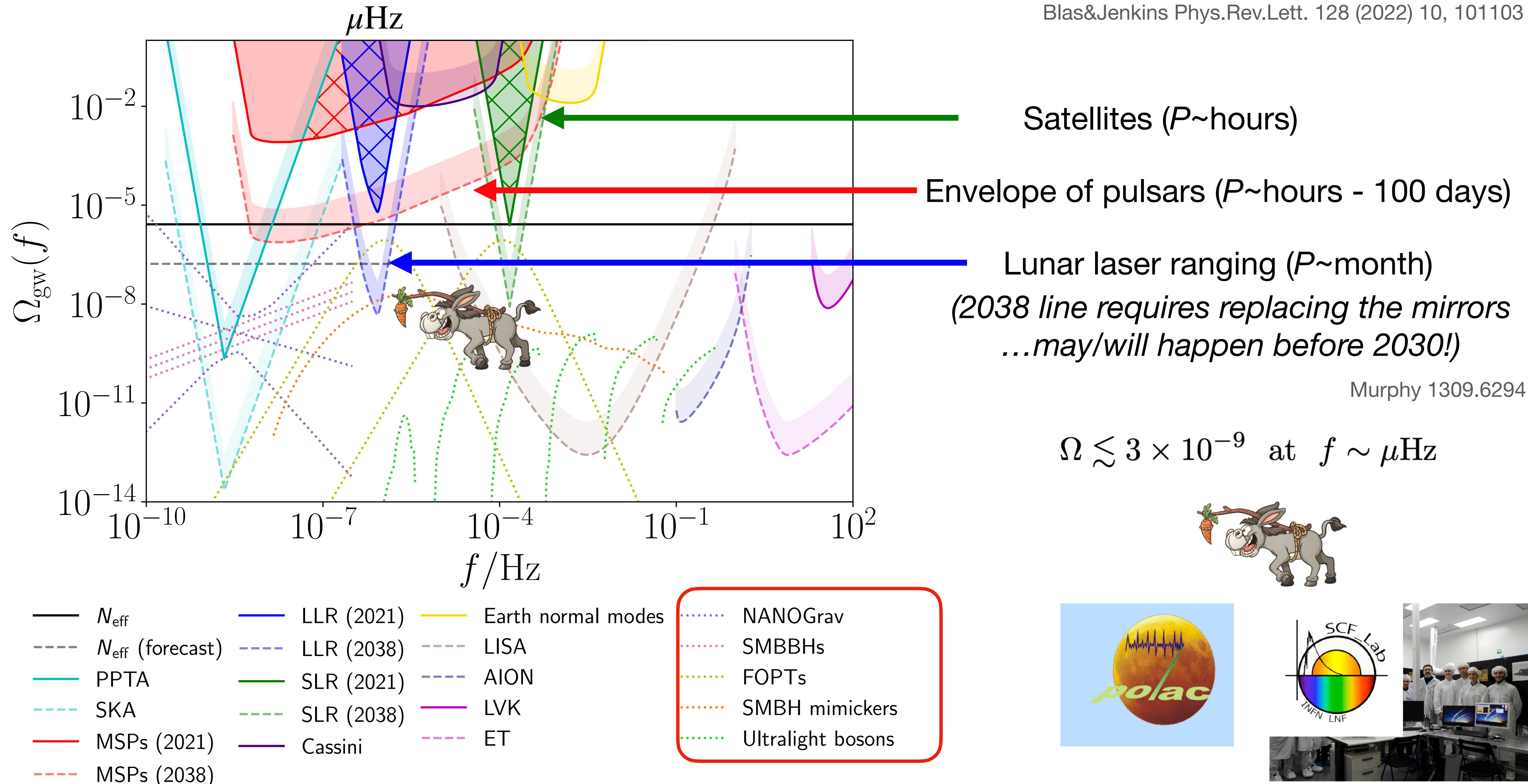


Complementarity of probes for FOPT



Our estimates (solid: today; dashed: 2038)

Blas&Jenkins Phys.Rev.Lett. 128 (2022) 10, 101103



Help to perform the
real analysis welcome!

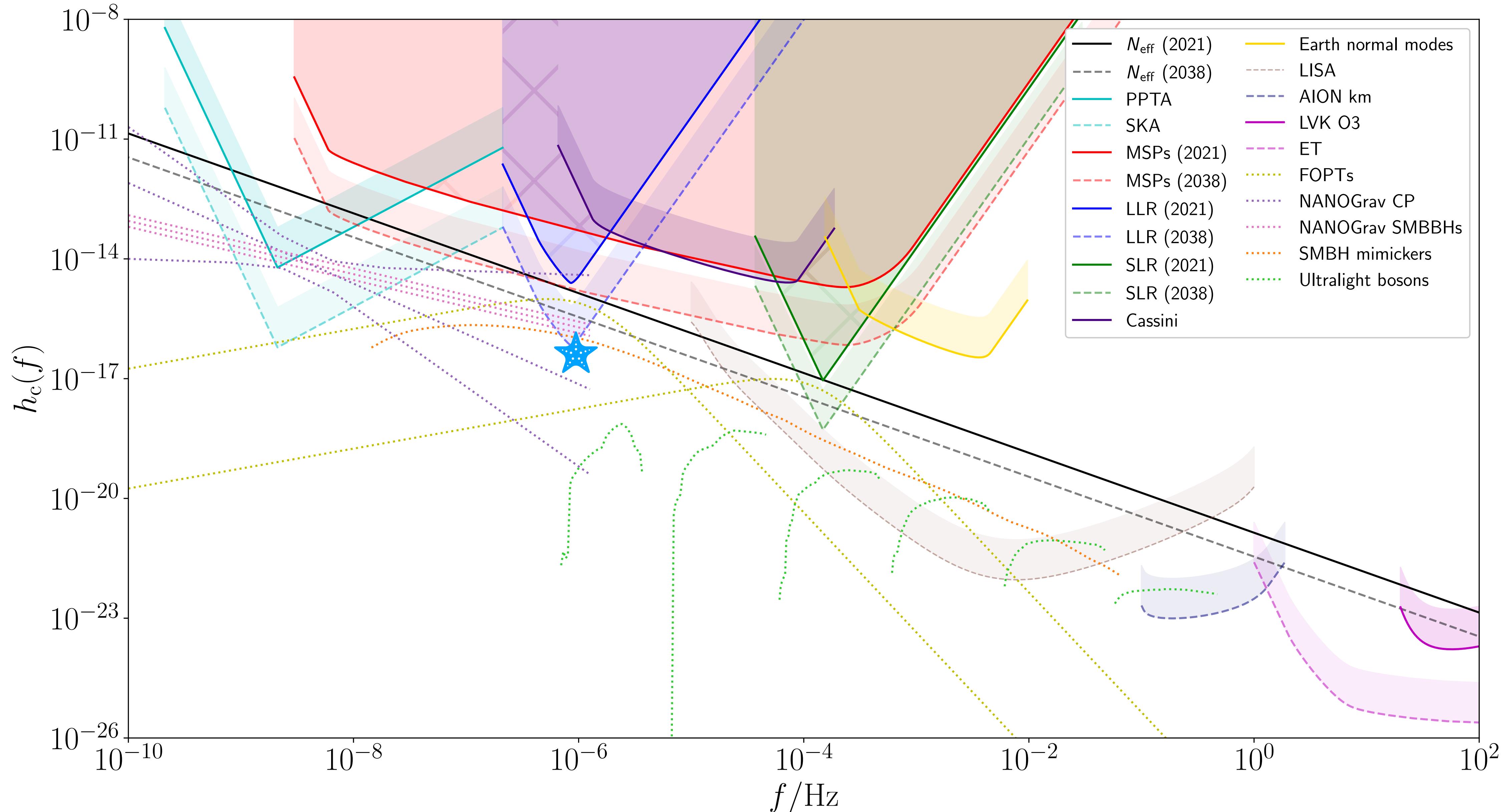


Miró, 1937.

After nHz, μ Hz?

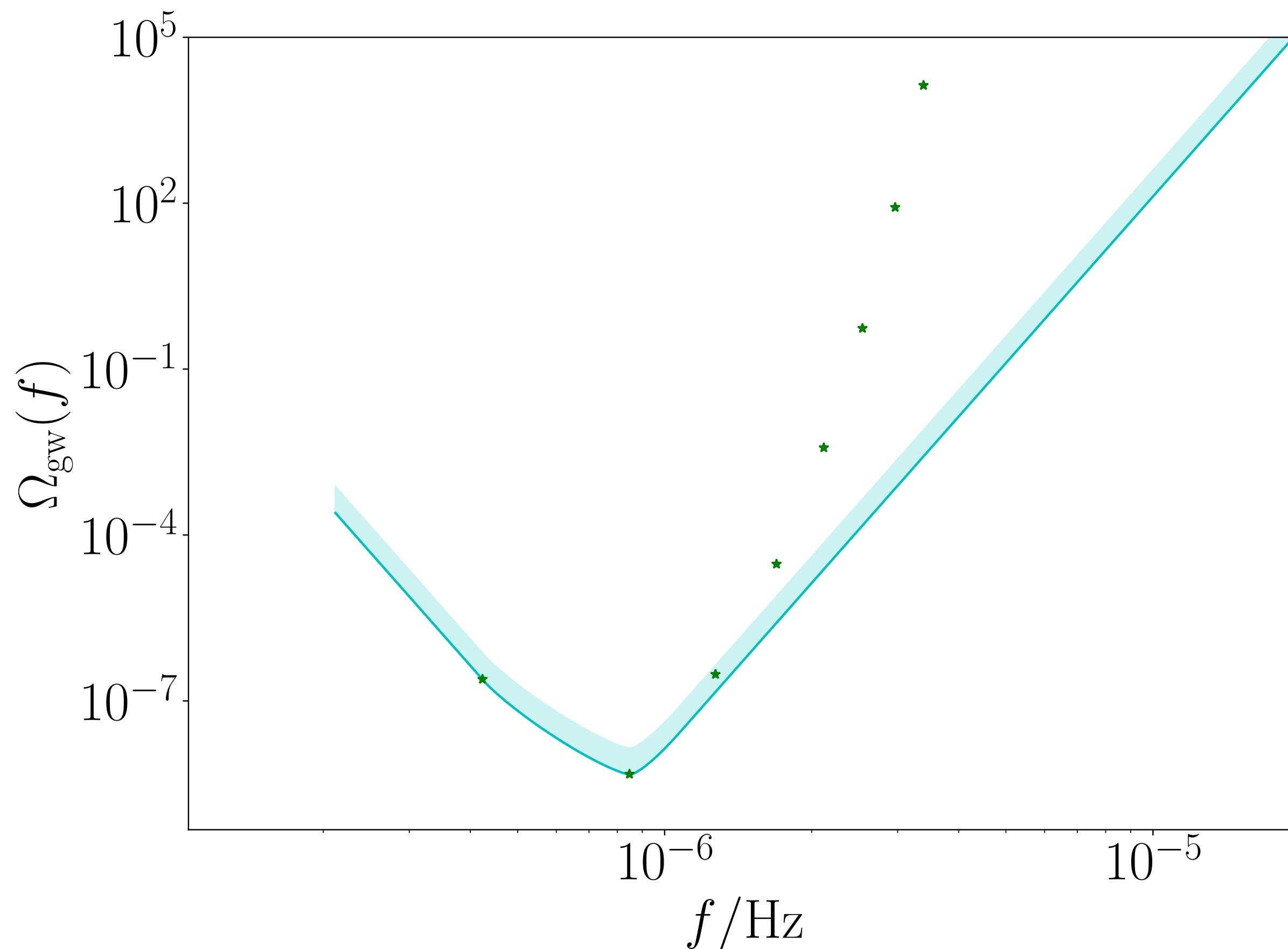
- The μ Hz band is very rich for **astrophysical** and **cosmological** sources
- There are **ideas** of how to access it, though **most** of them are **futuristic**
- The resonant **absorption of GWs by binaries** (LLR/SLR/pulsars) may give a handle at level (in 2038)
$$\Omega_{\text{gw}} \geq 4.8 \times 10^{-9} \quad f = 0.85 \mu\text{Hz}$$
$$\Omega_{\text{gw}} \geq 8.3 \times 10^{-9} \quad f = 0.15 \text{ mHz}$$
- **Future plans:** use LLR, SLR, pulsar **data** (w/ SYRTE, SCF_Lab, MPIfRA, Nanograv people...): we need/welcome new hands. Find **other resonant effects** (many possible, which frequencies?)

Characteristic strain

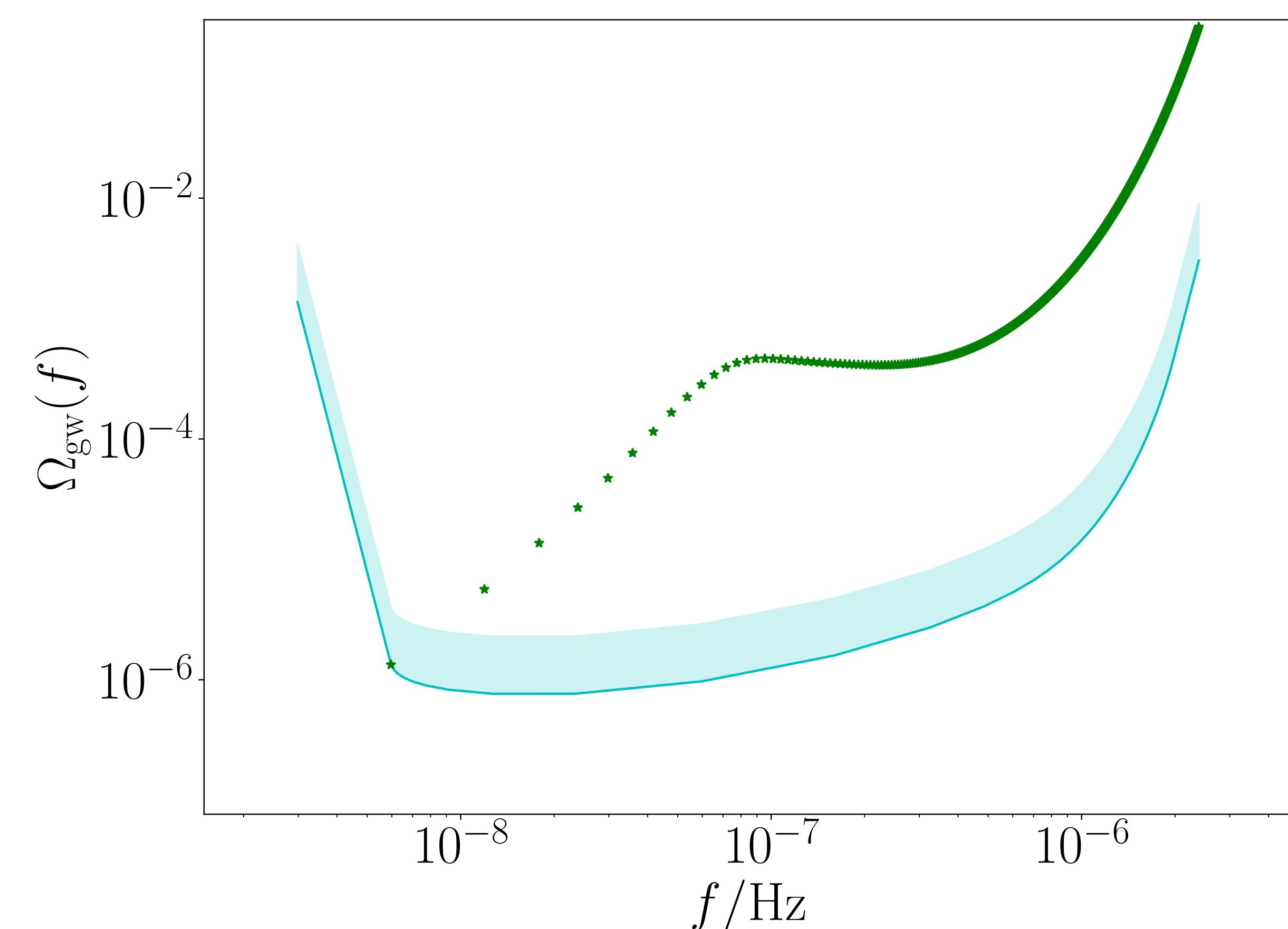


Power-law sensitivity vs. monochromatic sensitivity

lunar laser ranging, $e \approx 0.055$



pulsar timing (J1638-4725), $e \approx 0.955$



Solar system bounds

