

What can we learn about axions with PTAs ?

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CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE



Universität Hamburg

Our Approach .

Rather than interpreting the PTA GW signal in terms of new physics, can we use this measurement to derive constraints on new physics, and in particular, on axion physics?

Which axion physics
in the early universe
produces GWs
at NanoHz frequencies today?

-1- Axionic strings (= *global strings*): **this talk**

-2- Axion fragmentation

Axions

Among the most hunted particles.

Axions = Pseudo- Nambu Goldstone bosons (PNGBs) from spontaneous breaking of global symmetry which is not exact but broken weakly.

Axion mass is proportional to this breaking.

Very general context.

Historically: QCD axion. Strong dynamics from QCD provides breaking of symmetry.

Axion-like-particles (ALPs): other axions whose mass is not affected by QCD. They get their mass from other sources.

Ubiquitous in many extensions of the Standard Model (in particular in string theory)

Axion-Like-Particles (ALPs).

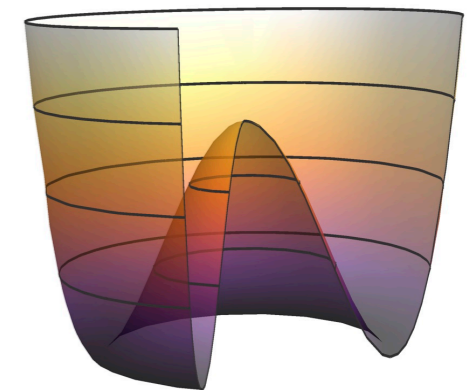
Consider complex scalar field

$$\Phi = \phi e^{i\theta}$$

charged under anomalous U(1) global symmetry (Peccei-Quinn symmetry)

Spontaneously broken at scale f_a $V(\varphi) = \lambda \left(|\varphi|^2 - \frac{f_a^2}{2} \right)^2$

$$\langle \boldsymbol{\varphi} \rangle = f_a / \sqrt{2}$$



Axion as Goldstone boson

$$\theta \rightarrow \theta + \text{const.}$$

$$\theta = a / f_a$$

ALPs.

Non-perturbative effects at energy $\Lambda_b \ll f_a$ break the shift symmetry and generate a potential/mass for the axion

$$\mathbf{V} = m_a^2(T) f_a^2 [1 - \cos(\theta)]$$

$$\mathbf{m}_a = \Lambda_b^2 / f_a$$

QCD axion

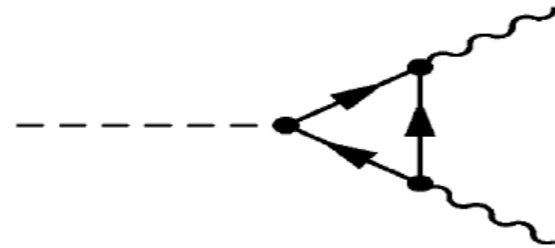
$$\mathbf{m}_a^2 f_a^2 \approx (76 \text{ MeV})^4$$

Generic ALP

m_a and f_a : free parameters

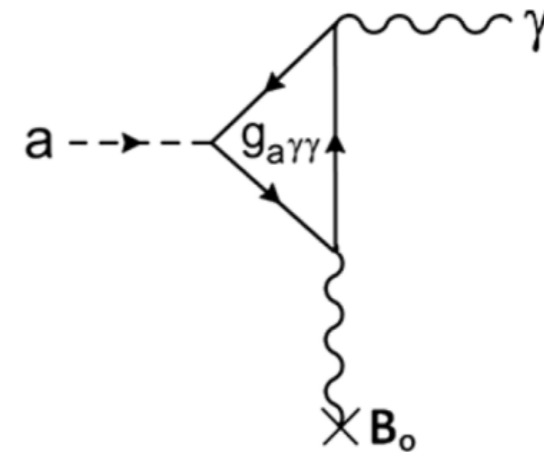
The hunt for axions.

Mainly through Axion-photon coupling



$$\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

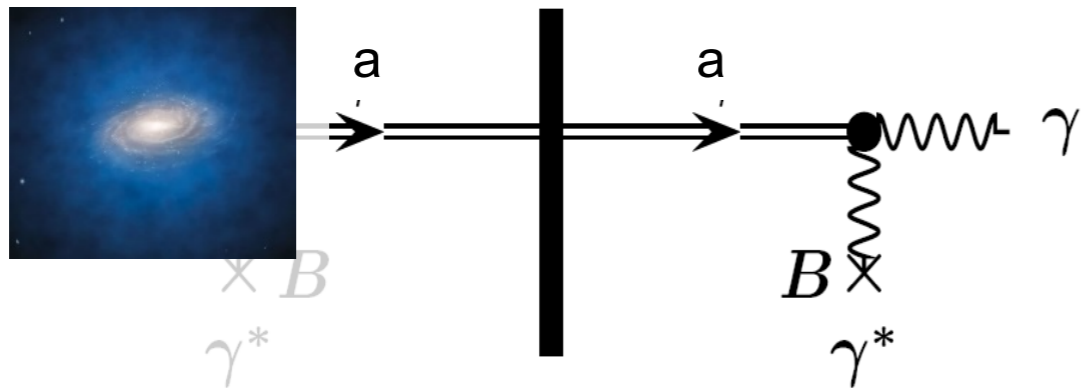
In a background magnetic field:
axion \leftrightarrow photon conversion



If long-lived: Dark Matter candidate

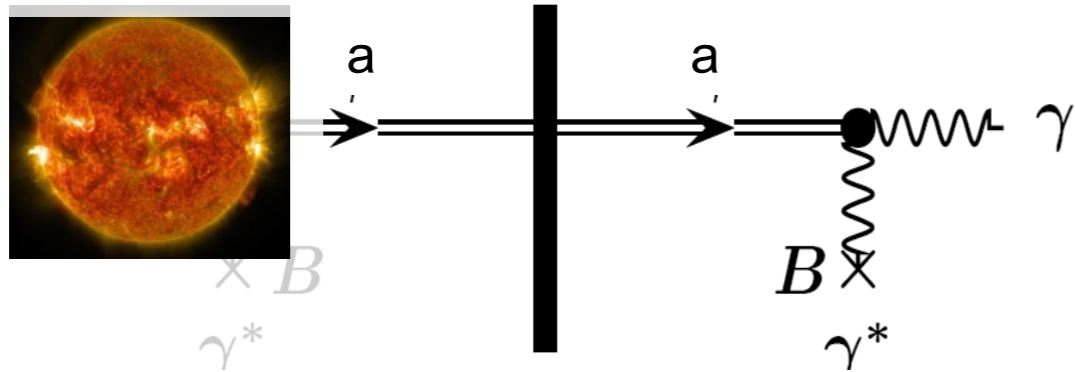
Three main ways to search for ALPs.

All rely on ALP-photon mixing in magnetic field



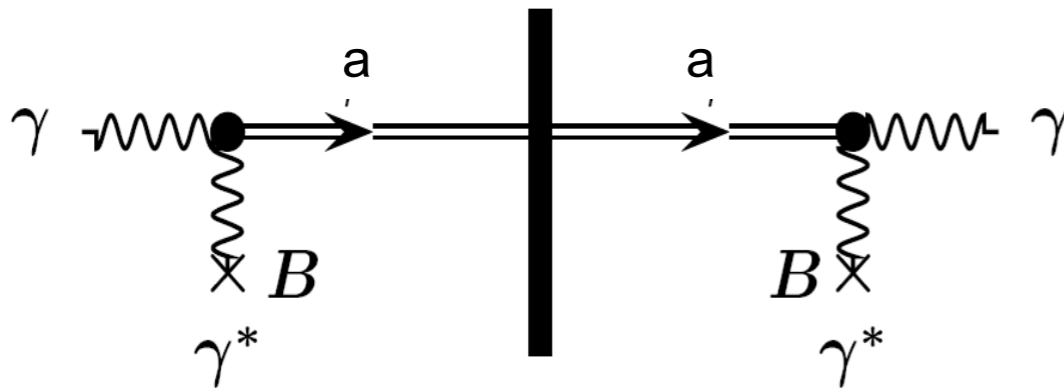
Haloscopes

looking for dark matter constituents, microwaves



Helioscopes

Axions emitted by the sun, X-rays

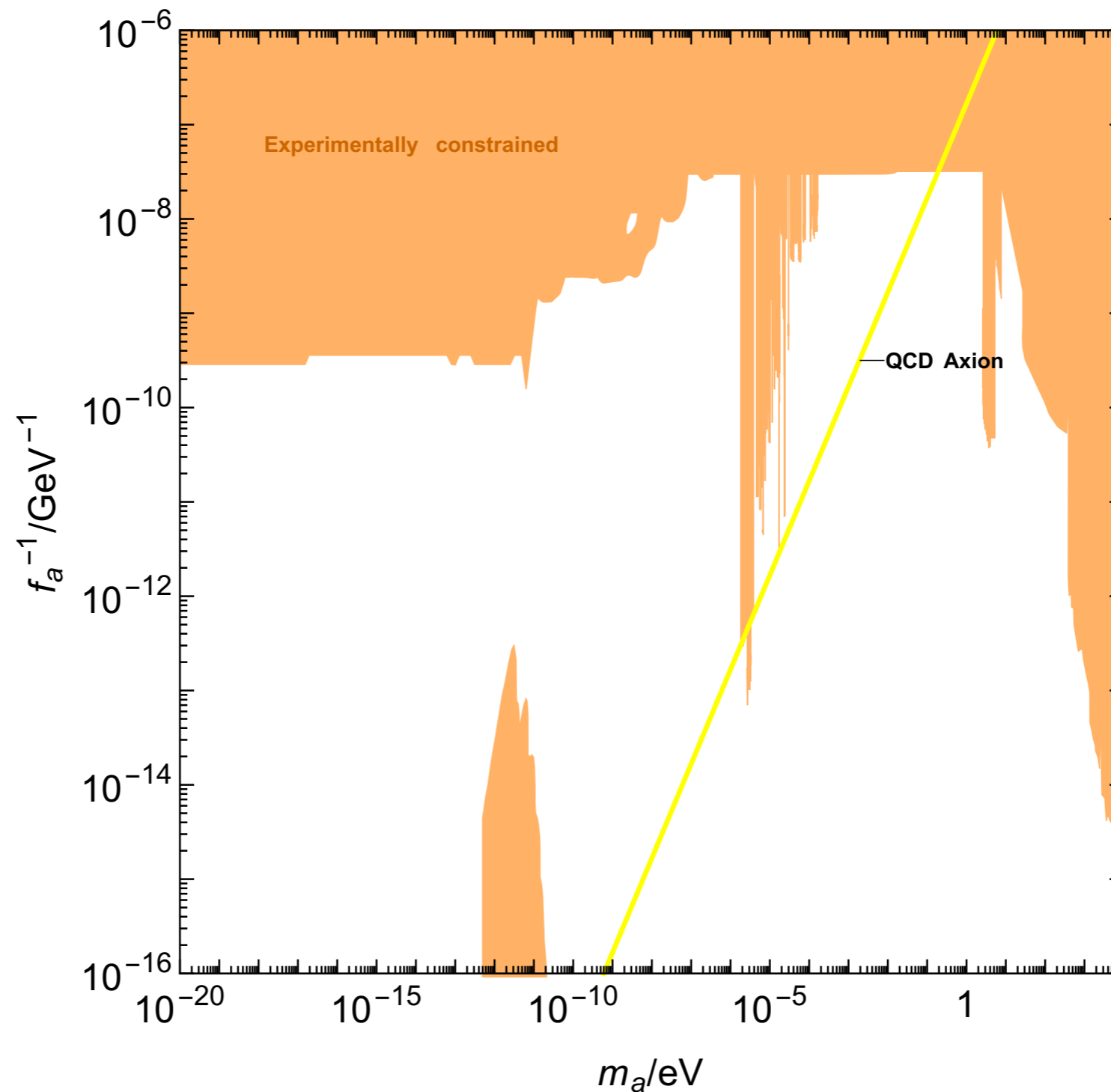


Purely laboratory experiments

“light-shining-through-walls”,
microwaves, optical photons

The Axion-Like-Particle (ALP) parameter space.

If axions are given an interaction to photons then a long list of constraints from ALP searches apply



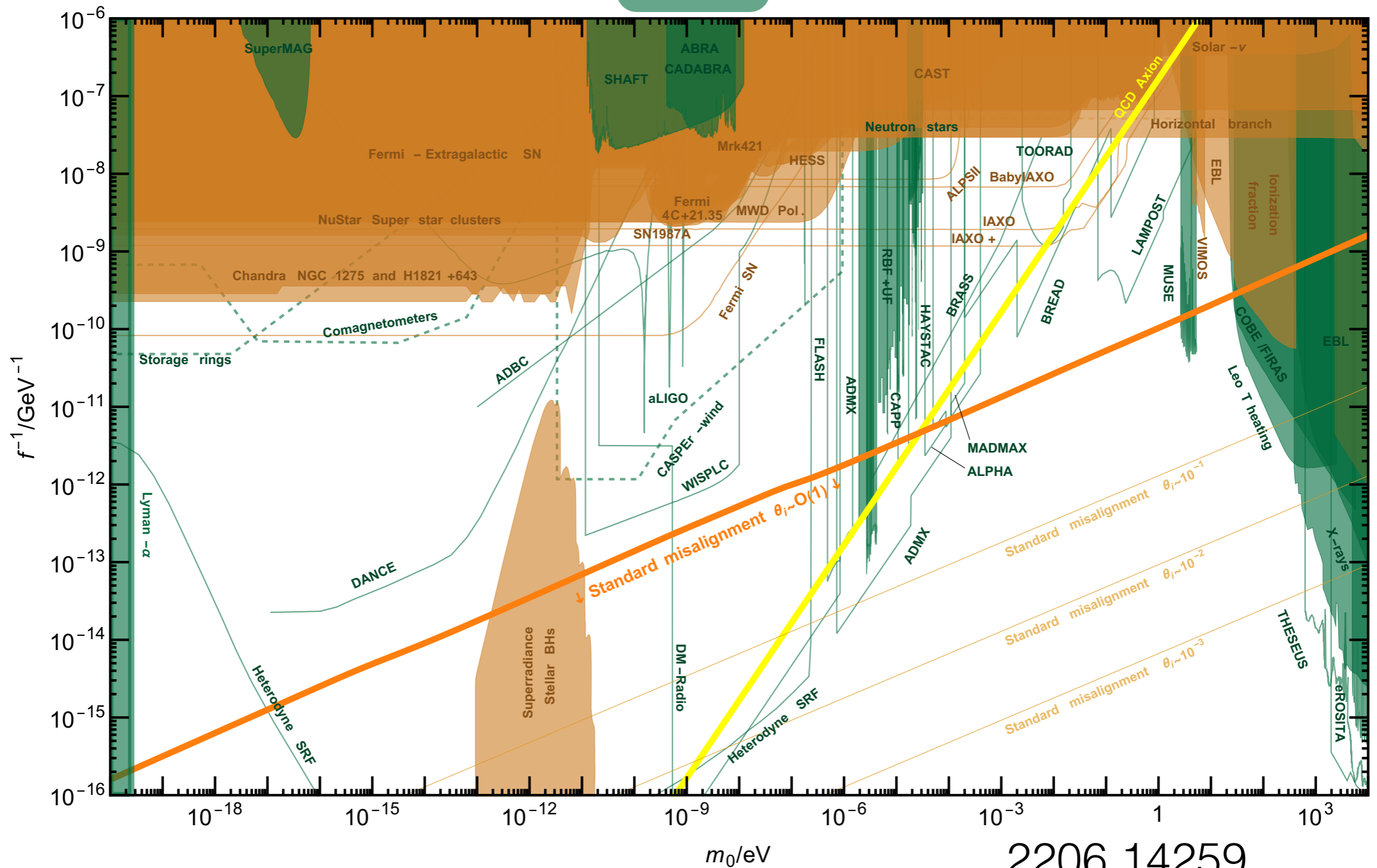
$$\frac{f_\gamma}{f_a} \approx 0.5 \times 10^3$$

assuming KSVZ-like coupling

The hunt for axions.

Any ALP

Only DM



2206.14259

A whole set of experimental constraints.

All data can be found here:

C. O'Hare, *cajohare/axionlimits: Axionlimits*, <https://cajohare.github.io/AxionLimits/> (2020) [10.5281/zenodo.3932430].

All experiments also listed in tables 1 and 2 of 2206.14259:

Experiment:	Principle	DM?	Ref.
<i>Haloscope constraints</i>			
ABRACADABRA-10cm	Haloscope	DM	[76]
ADMX	Haloscope	DM	[77–83]
BASE	Haloscope (Cryogenic Penning Trap)	DM	[84]
CAPP	Haloscope	DM	[85–87]
CAST-RADES	Haloscope	DM	[88]
DANCE	Haloscope (Optical cavity polarization)	DM	[89]
Grenoble Haloscope	Haloscope	DM	[90]
HAYSTAC	Haloscope	DM	[91, 92]
ORGAN	Haloscope	DM	[93]
QUAX	Haloscope	DM	[94, 95]
RBF	Haloscope	DM	[96]
SHAFT	Haloscope	DM	[97]
SuperMAG	Haloscope (Using terrestrial magnetic field)	DM	[98]
UF	Haloscope	DM	[99]
Upload	Haloscope	DM	[100]
<i>Haloscope projections</i>			
ABDC	Haloscope	DM	[101]
ADMX	Haloscope	DM	[102]
aLIGO	Haloscope	DM	[103]
ALPHA	Haloscope (Plasma haloscope)	DM	[104]
BRASS	Haloscope	DM	[105]
BREAD	Haloscope (Parabolic reflector)	DM	[106]
DANCE	Haloscope (Optical cavity polarization)	DM	[107]
DMRadio	Haloscope (All stages: 50L, m^3 and GUT)	DM	[108, 109]
FLASH	Haloscope (Formerly KLASH)	DM	[110, 111]
Heterodyne SRF	Haloscope (Superconduct. Resonant Freq.)	DM	[112, 113]
LAMPOST	Haloscope (Dielectric)	DM	[114]
MADMAX	Haloscope (Dielectric)	DM	[115]
ORGAN	Haloscope	DM	[93]
QUAX	Haloscope	DM	[116]
TOORAD	Haloscope (Topological anti-ferromagnets)	DM	[117, 118]
WISPLC	Haloscope (Tunable LC circuit)	DM	[119]
<i>LSW and optics</i>			
ALPS	Light-shining-through wall	Any	[120]
ALPS II	Light-shining-through wall (projection)	Any	[121]
CROWS	Light-shining-through wall (microwave)	Any	[122]
OSQAR	Light-shining-through wall	Any	[123]
PVLAS	Vacuum magnetic birefringence	Any	[124]
<i>Helioscopes</i>			
CASPEr	Helioscope	Any	[125, 126]
babyIAXO	Helioscope (projection)	Any	[1, 127, 128]
IAXO	Helioscope (projection)	Any	[1, 127, 128]
IAXO+	Helioscope (projection)	Any	[1, 127, 128]

Table 1. List of experimental searches for axions and ALPs. The table is continued in table 2. All experiments here rely on the axion-photon coupling.

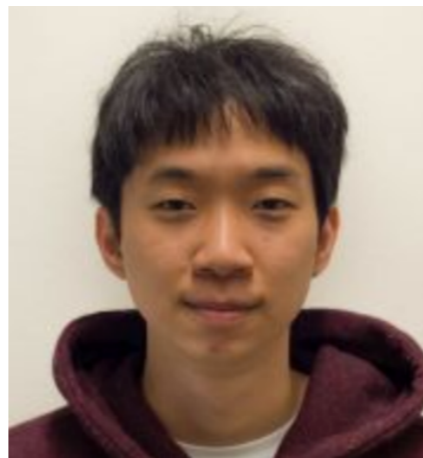
Experiment:	Principle	DM?	Reference
<i>Astrophysical constraint</i>			
4C+21.35	Photon-ALP oscillation on the γ -rays from blazars	Any	[129]
Breakthrough Listen	ALP \rightarrow radio γ in neutron star magn. fields	DM	[130]
Bullet Cluster	Radio signal from ALP DM decay	DM	[131]
Chandra	AGN X-ray prod. in cosmic magn. field	Any	[132–135]
BBN + N_{eff}	ALP thermal relic perturbing BBN and N_{eff}	Any	[136]
Chandra MWD	X-rays from Magnetic White Dwarf ALP prod.	Any	[137]
COBE/FIRAS	CMB spectral distortions from DM relic decay	DM	[138]
Distance ladder	ALP \leftrightarrow γ perturbing luminosity distances	Any	[139]
Fermi-LAT	SN ALP product. \rightarrow γ -rays in cosmic magn. field	Any	[140–142]
Fermi-LAT	AGN X-ray production \rightarrow ALP in cosmic magn. field	Any	[143]
Haystack Telescope	ALP DM decay \rightarrow microwave photons	DM	[144]
HAWC TeV Blazars	$\gamma \rightarrow$ ALP \rightarrow γ conversion reducing γ -ray attenuation	Any	[145]
H.E.S.S.	AGN X-ray production \rightarrow ALP in cosmic magn. field	Any	[146]
Horizontal branch stars	stellar metabolism and evolution	Any	[147]
LeoT dwarf galaxy	Heating of gas-rich dwarf galaxies by ALP decay	DM	[148]
Magnetic white dwarf pol.	$\gamma \rightarrow$ ALP conversion polarizing light from MWD stars	Any	[149]
MUSE	ALP DM decay \rightarrow optical photons	DM	[150]
Mrk 421	Blazar γ -ray \rightarrow ALP \rightarrow γ -ray in cosmic magn. field	Any	[151]
NuStar	Stellar ALP production \rightarrow γ in cosmic magn. fields	Any	[152, 153]
NuStar, Super star clusters	Stellar ALP production \rightarrow γ in cosmic magn. fields	Any	[153]
Solar neutrinos	ALP energy loss \rightarrow changes in neutrino production	Any	[154]
SN1987A ALP decay	SN ALP production \rightarrow γ decay	Any	[155]
SN1987A gamma rays	SN ALP production \rightarrow γ in cosmic magnetic field	Any	[156, 157]
SN1987A neutrinos	SN ALP luminosity less than neutrino flux	Any	[157, 158]
Thermal relic compilation	Decay and BBN constraints from ALP thermal relic	Any	[159]
VIMOS	Thermal relic ALP decay \rightarrow optical photons	Any	[160]
White dwarf mass relation	Stellar ALP production perturbing WD metabolism	Any	[161]
XMM-Newton	Decay of ALP relic	DM	[162]
<i>Astrophysical projections</i>			
ANOSPIN	X-ray signal from ALP DM decay	DM	[163]
Fermi-LAT	SN ALP production \rightarrow γ in cosmic magnetic field	Any	[164]
IAXO	Helioscope detection of supernova axions	Any	[165]
THESEUS	ALP DM decay \rightarrow x-ray photons	DM	[166]
<i>Neutron coupling:</i>			
CASPEr-wind	NMR from oscillating EDM (projection)	DM	[167, 168]
CASPEr-ZULF-Comag.	NMR from oscillating EDM	DM	[168, 169]
CASPEr-ZULF-Sidechain	NMR (constraint & projection)	DM	[168, 170]
NASDUCK	ALP DM perturbing atomic spins	DM	[171]
nEDM	Spin-precession in ultracold neutrons and Hg	DM	[168, 172]
K-3He	Comagnetometer	DM	[173]
Old comagnetometers	New analysis of old comagnetometers	DM	[174]
Future comagnetometers	Comagnetometers	DM	[174]
SNO	Solar ALP flux from deuterium dissociation	Any	[175]
Proton storage ring	EDM signature from ALP DM	DM	[176]
Neutron Star Cooling	ALP production modifies cooling rate	Any	[177]
SN1987 Cooling	ALP production modifies cooling rate	Any	[178]
<i>Coupling independent:</i>			
Black hole spin	Superradiance for stellar mass black holes	Any	[72–74]
Lyman- α	Modification of small-scale structure	DM	[60]

Table 2. List of experimental searches for axions and ALPs.

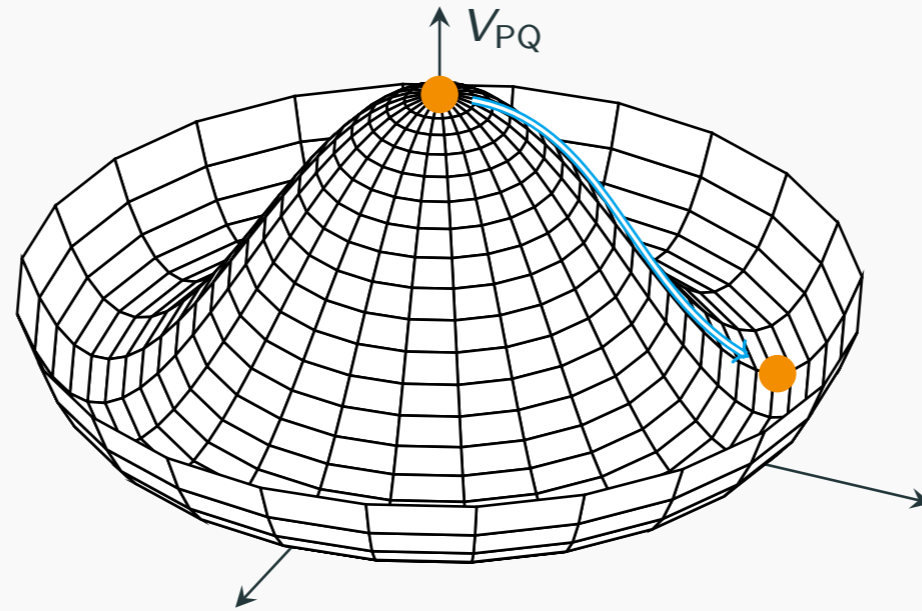
**Do PTAs have anything more
to add on this plot ?**

**Based on
arXiv 2307.03121**

with Peera Simakachorn



Pre- and post-inflationary scenarios.



Post-inflationary scenario

- **Different** initial angle in each Hubble patch.
- **Inhomogeneous** including topological defects.



**GLOBAL (axionic)
COSMIC STRINGS**



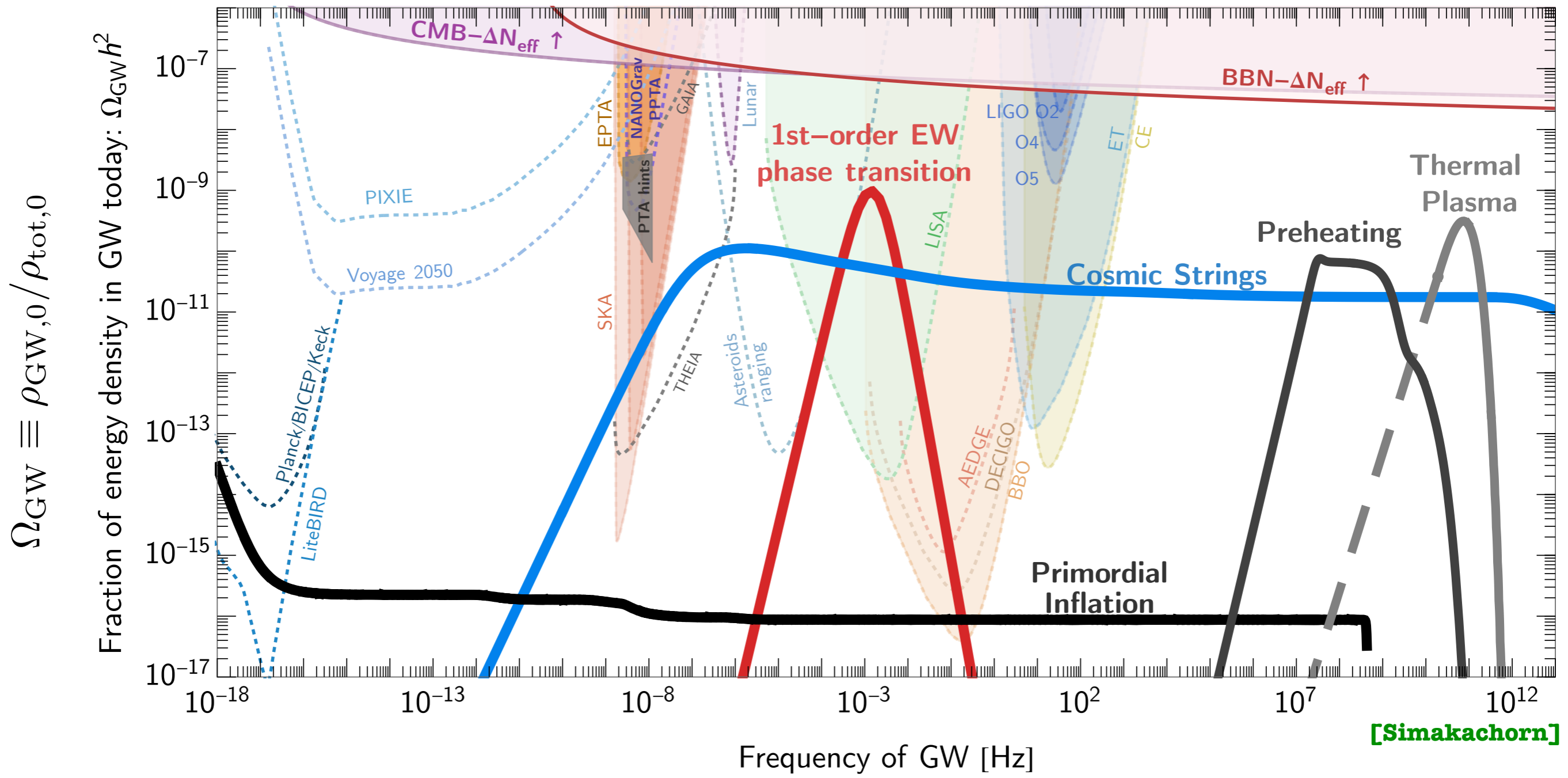
primordial GW bgd

Pre-inflationary scenario

- **Random** initial angle in the observable universe.
- Initially **homogeneous** w/o topological defects.

Primordial Gravitational Waves .

Benchmark Primordial Sources of GWs.



Reading the history of the universe.

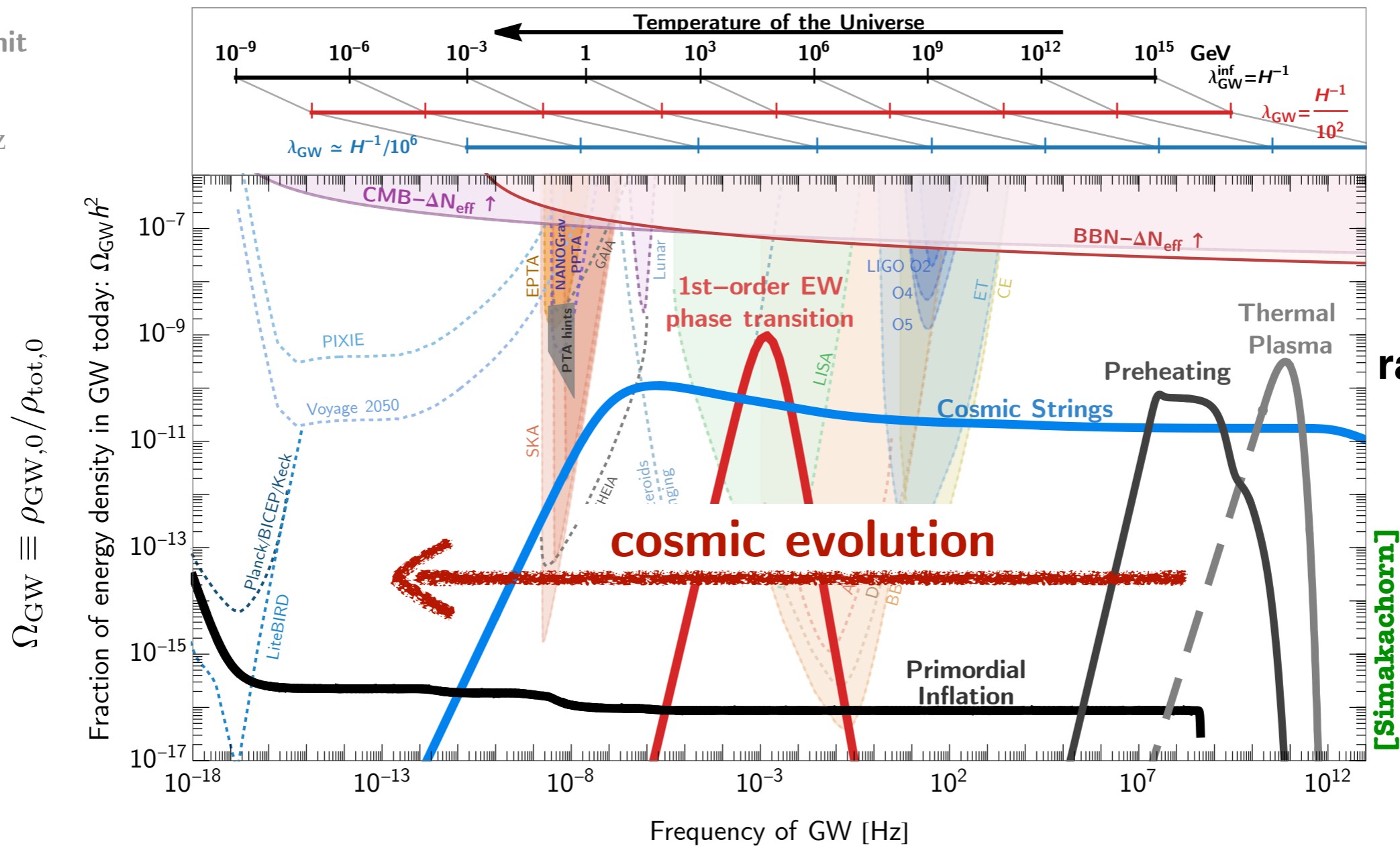
GW frequency $f_{\text{GW},0} \simeq \lambda_{\text{GW}}^{-1} \left(\frac{a_{\text{prod}}}{a_0} \right)$

Low-freq. limit

$$f_{\text{GW}}^{\text{min}} \simeq H_0^{-1} \simeq 10^{-18} \text{ Hz}$$

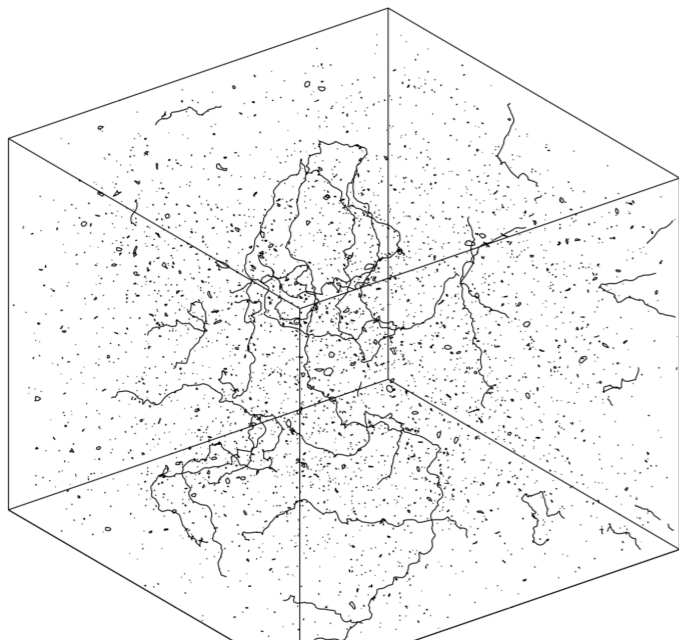
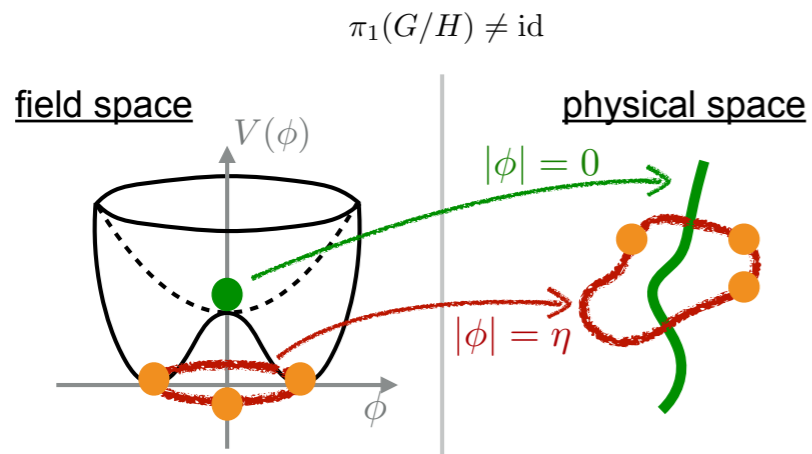
High-freq. limit

$$f_{\text{GW}}^{\text{max}} \simeq 10^{13} \text{ Hz} \quad (\lambda_{\text{GW}} \sim H^{-1} \sim M_{\text{pl}}^{-1})$$



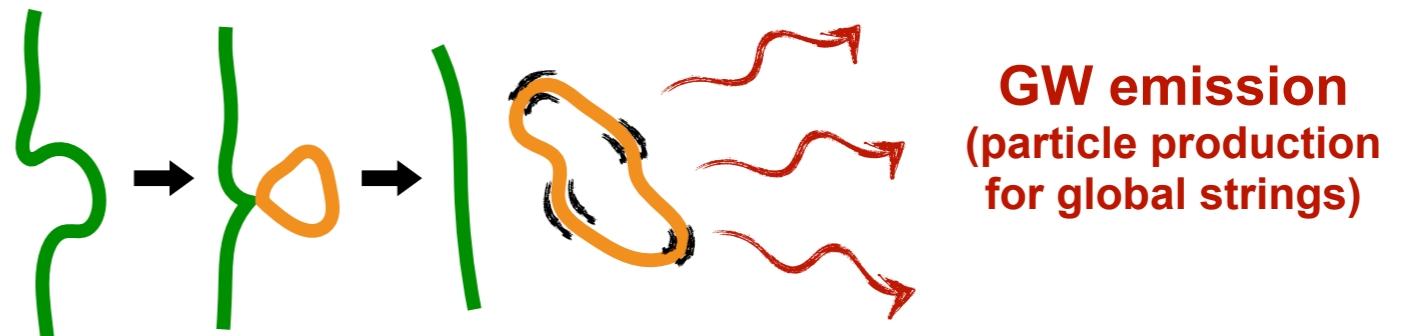
Assuming Standard Model radiation era at high energies

Gravitational Waves from cosmic strings.



Network of cosmic strings
[Allen & Shellard, 1990]

**Cosmic strings:
Long-lasting source of GW**

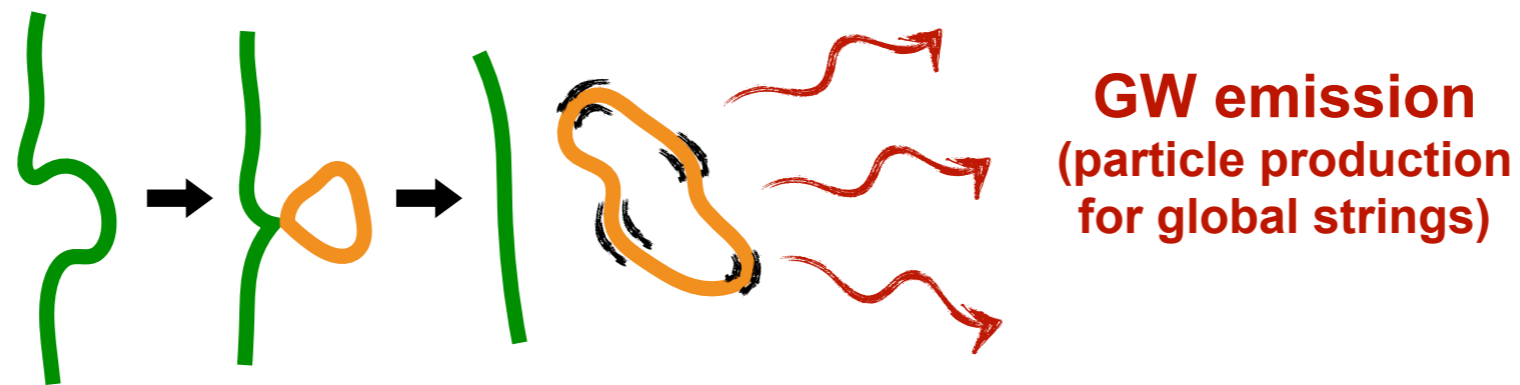


string tension: $\mu \sim \eta^2$
 $\eta = f_a$

recent reviews: [1909.00819, 1912.02569]

Loop formation & scaling regime.

String intercommutation: **loop formation** depletes energy from the network.



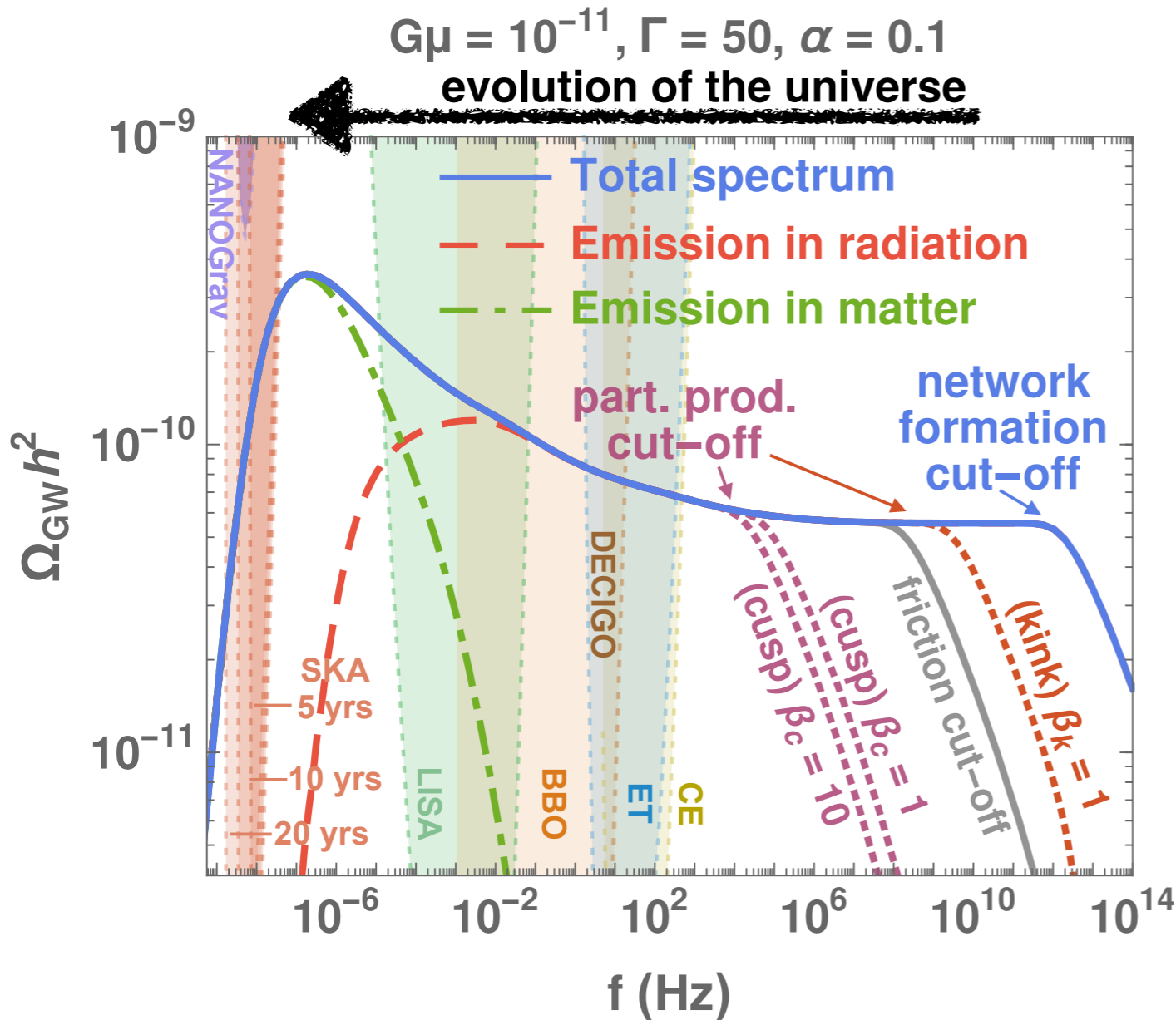
Cosmic strings do not overclose the universe.

Scaling regime:

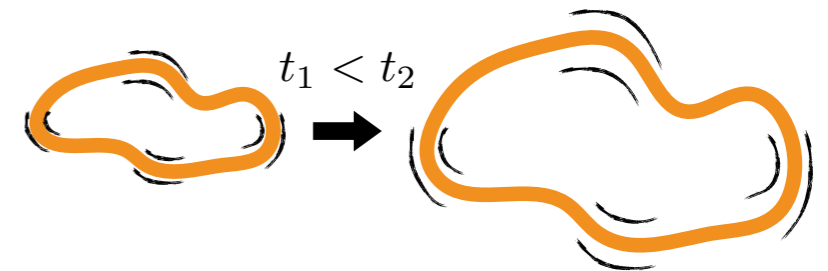
$$\rho_{\infty} \propto t^{-2} \propto \begin{cases} a^{-3} & \text{for matter} \\ a^{-4} & \text{for radiation} \\ a^{-6} & \text{for kination} \end{cases}$$

Gravitational Waves from Cosmic strings.

(long-lasting sources).



Higher $f \Leftrightarrow$ Earlier emission



smaller loop \Leftrightarrow higher oscillation f

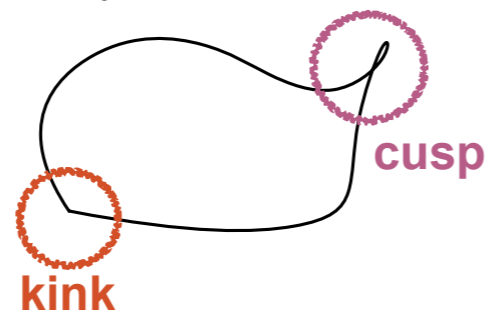
@ earlier t_i

more GW from more loops
 but more red-shift

\Rightarrow Flat during radiation

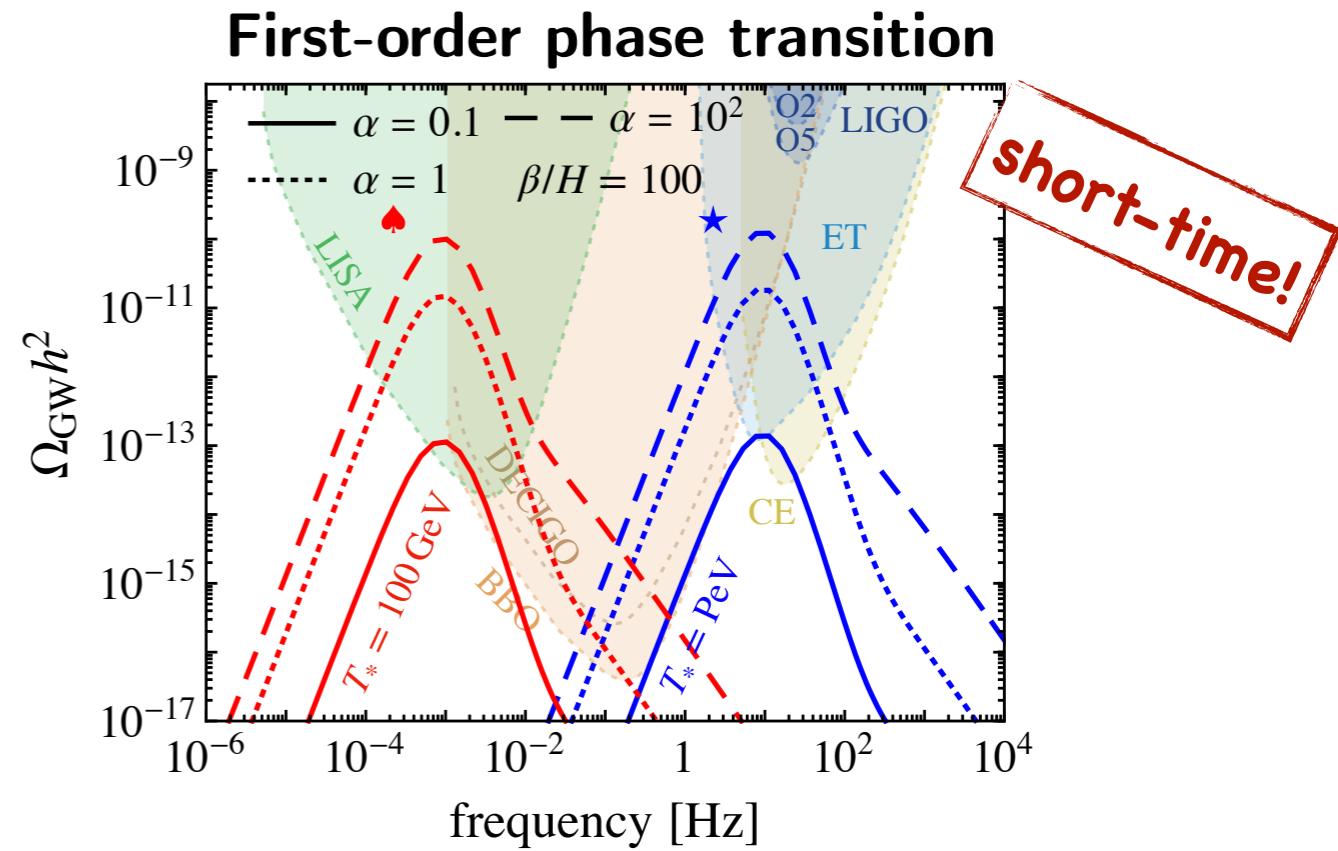
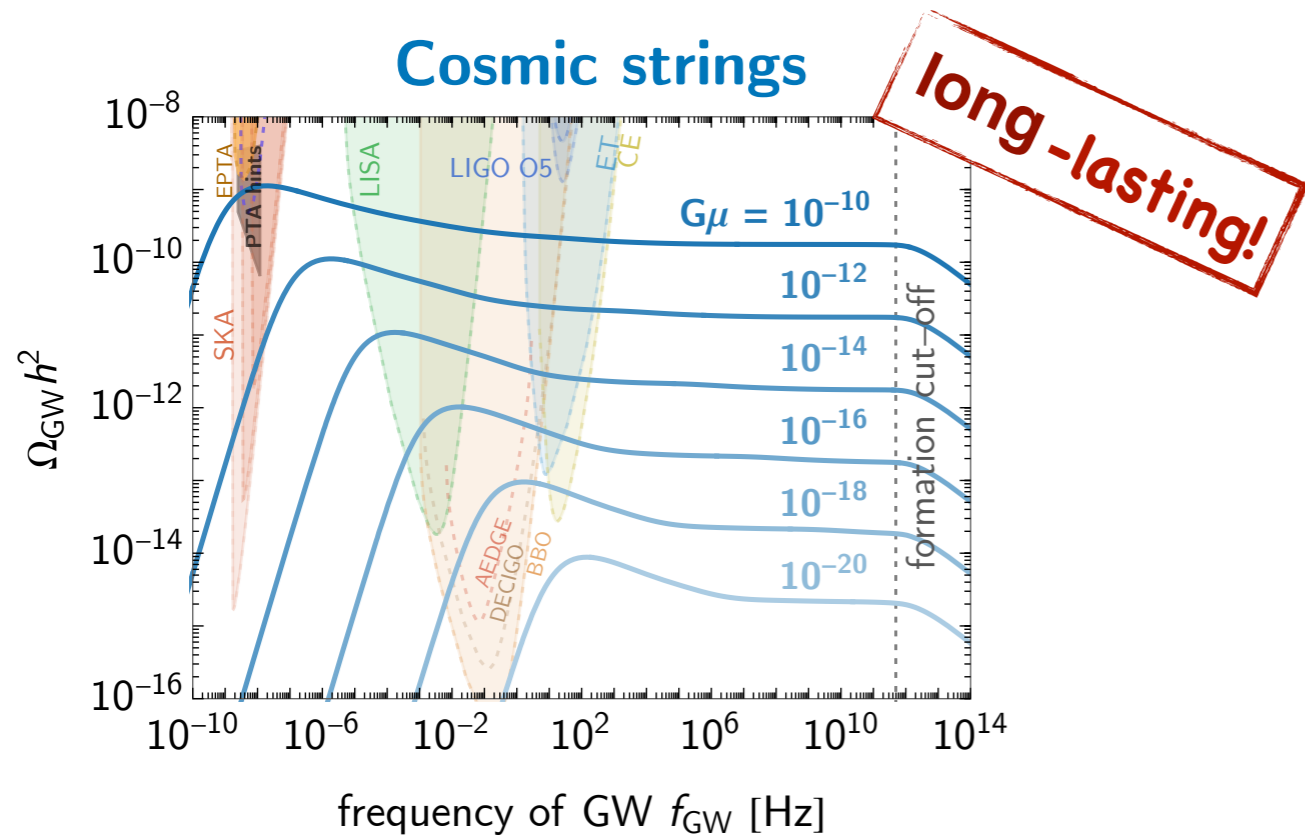
[1912.02569]

singular structures on loop
 (beyond NG approx.)

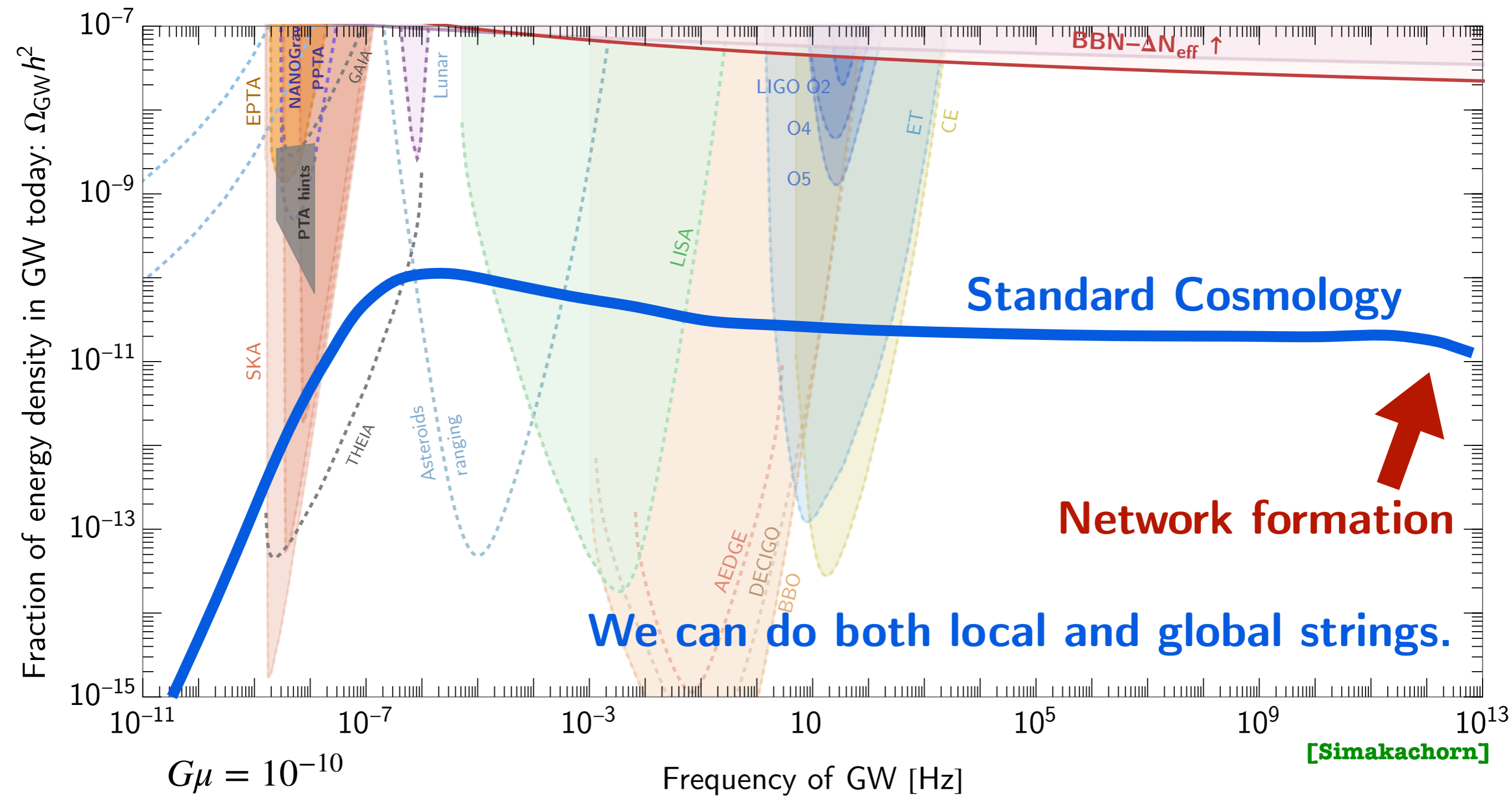


lead to particle emission

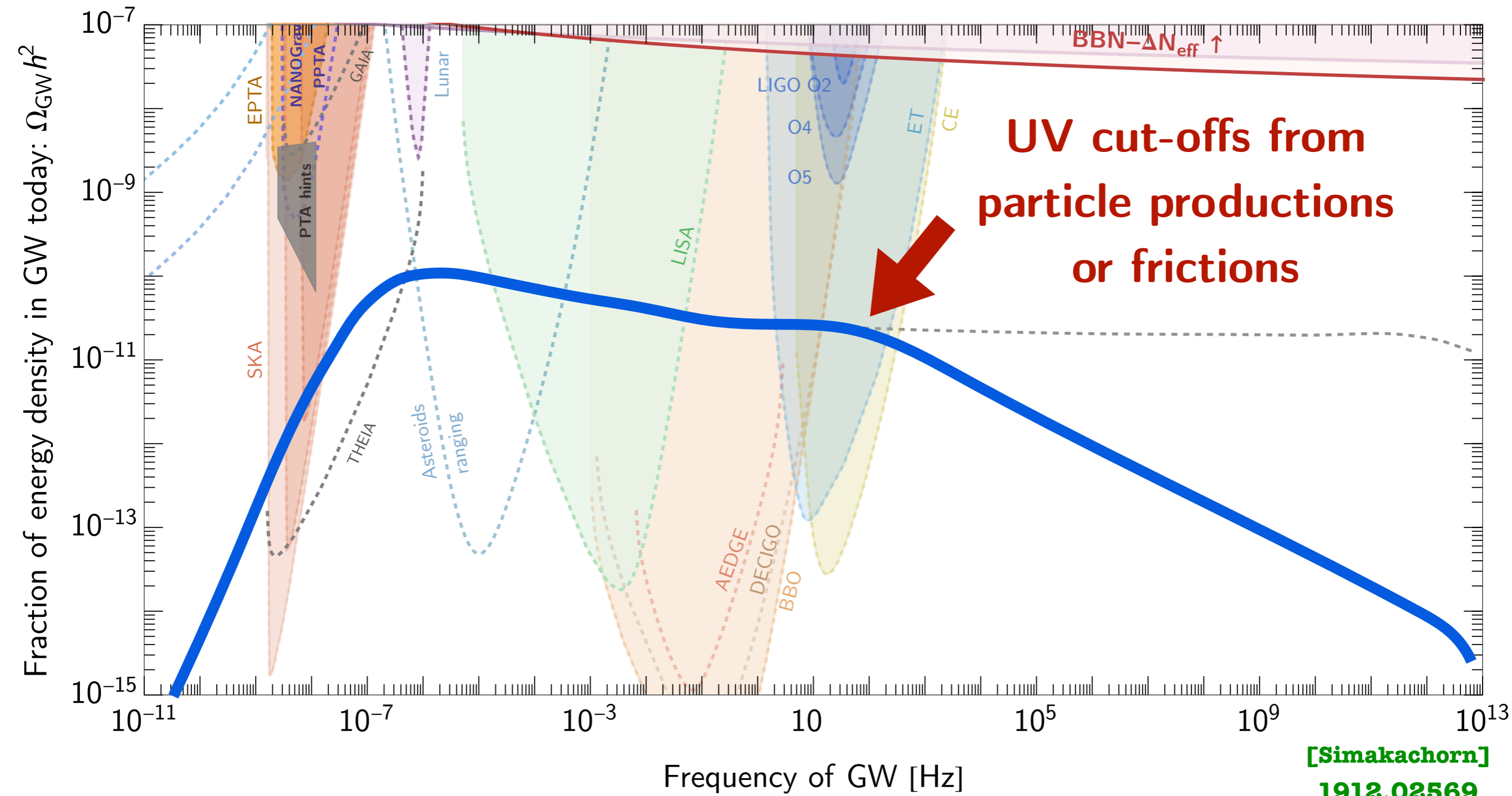
Short-lasting vs long-lasting primordial sources.



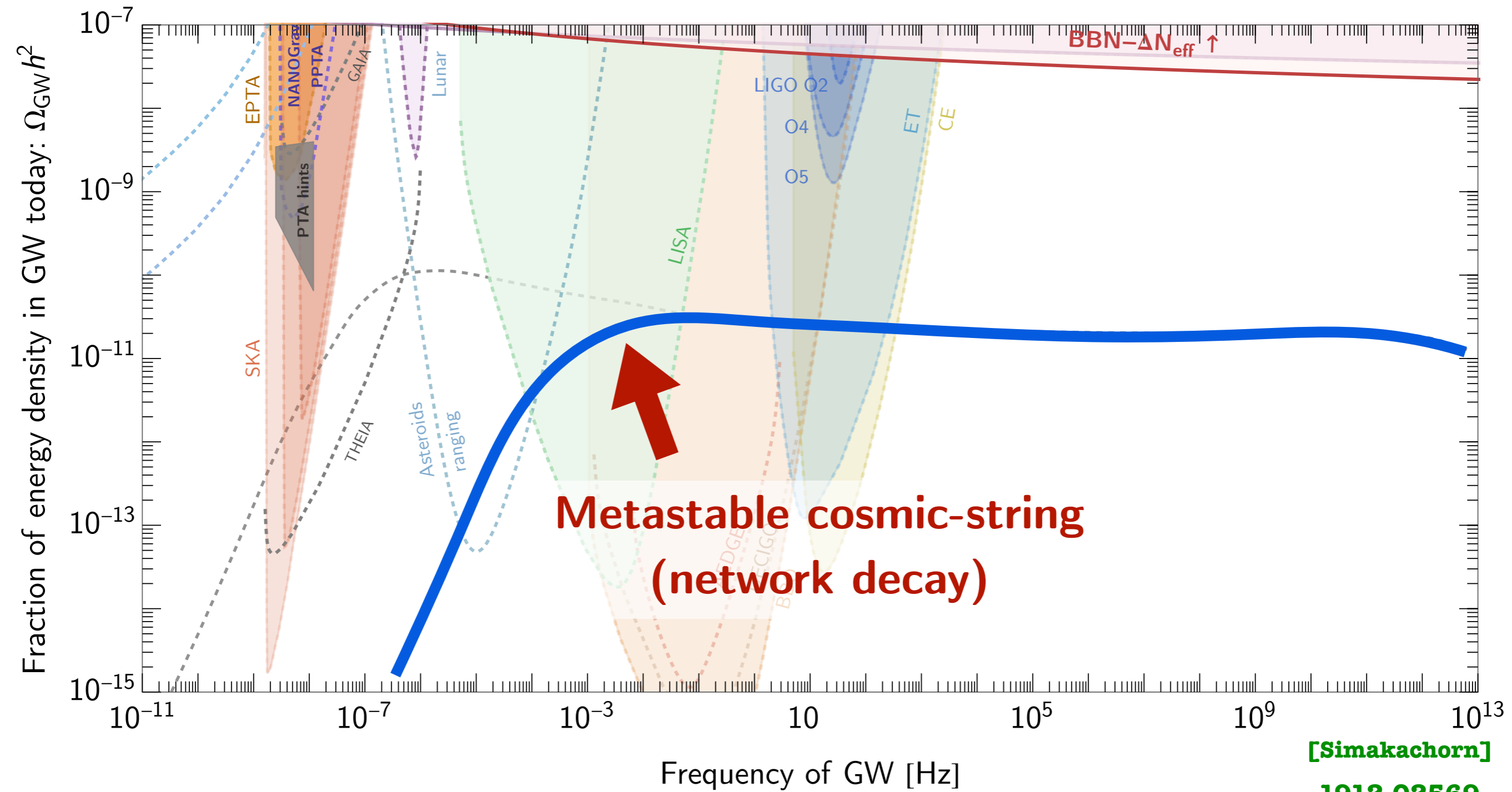
Gravitational Waves from cosmic strings.



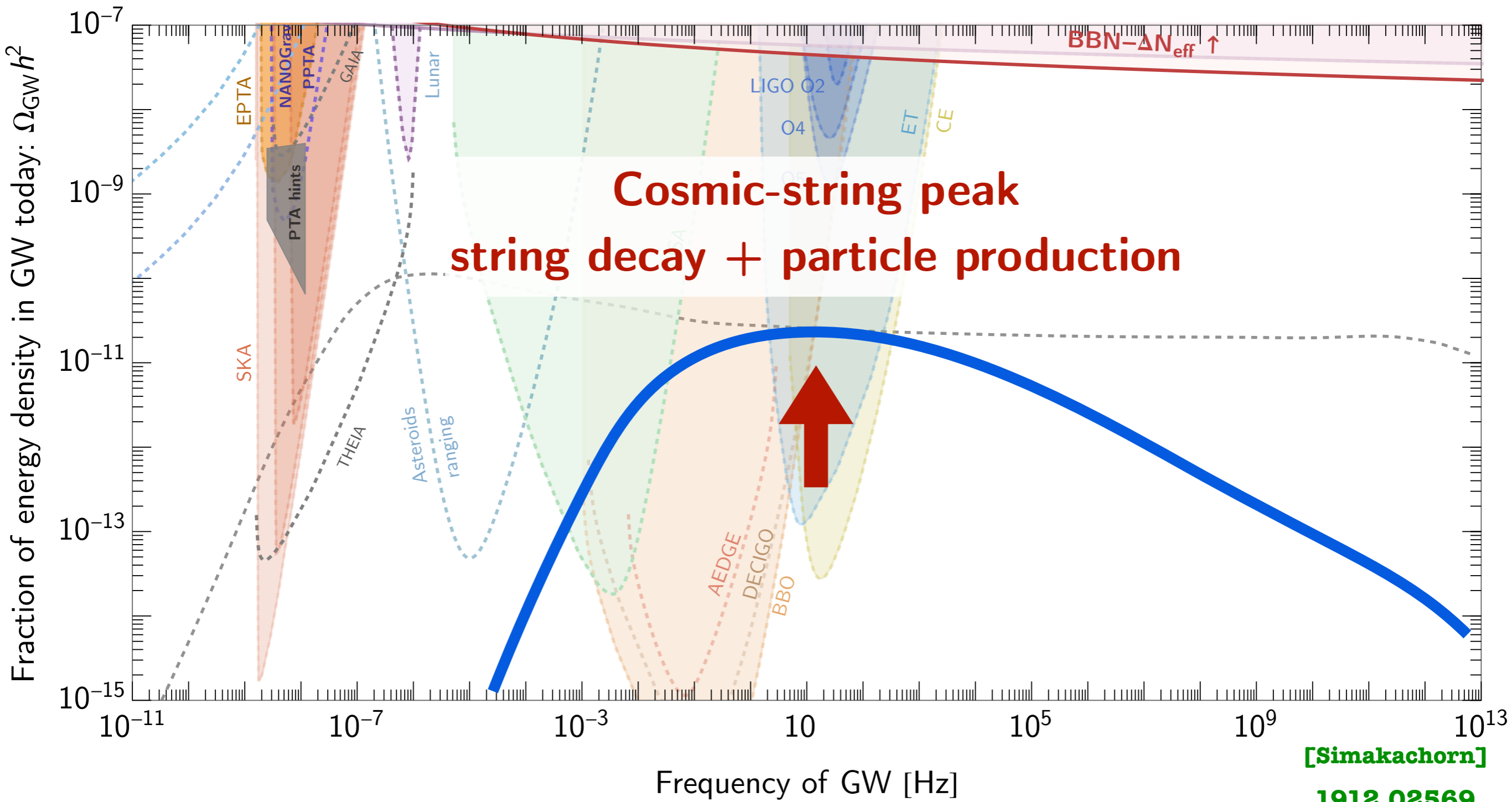
Gravitational Waves from cosmic strings.



Gravitational Waves from cosmic strings.



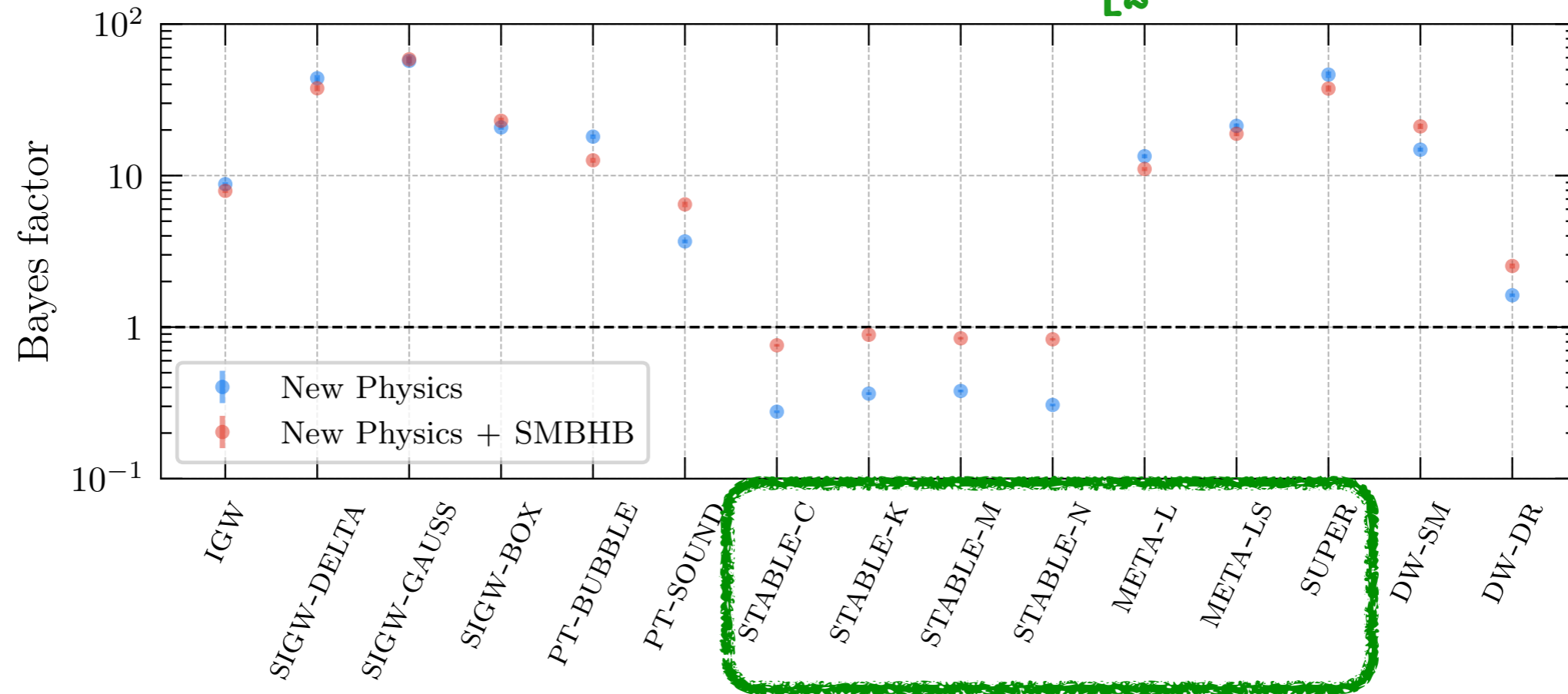
Gravitational Waves from cosmic strings.



LOCAL STRINGS VS GLOBAL STRINGS.

NANOGrav 15-YEAR NEW-PHYSICS SIGNALS

[2]



LOCAL STRINGS

No analysis of global strings in the other PTA papers either.

LOCAL STRINGS VS GLOBAL STRINGS.

See comparison in Appendix F of [1912.02569] .

Loops from global strings : short-lived

Loops from local strings : long-lived.

—> different GW spectra in both frequency and amplitude.

LOCAL STRINGS vs GLOBAL STRINGS.

Global strings: no gauge field, instead massless Goldstone mode, with logarithmically-divergent gradient energy.

Loops quickly decay into axion particles.

GW are mainly produced at the time of the loop production.

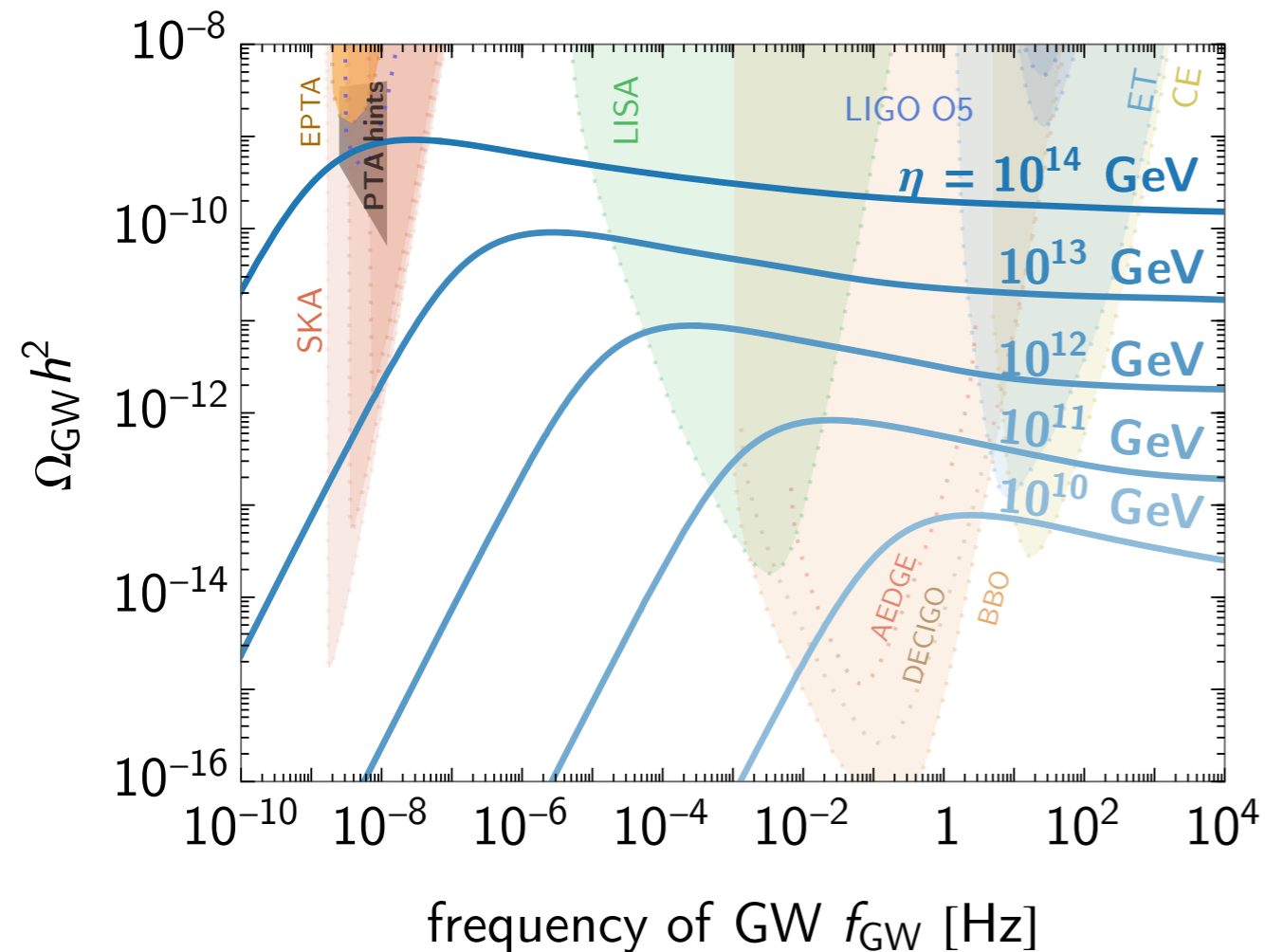


$$\Omega_{\text{GW}}^{\text{local}} \simeq \Omega_r \frac{\eta}{M_{\text{pl}}},$$

$$\Omega_{\text{GW}}^{\text{global}} \simeq \Omega_r \left(\frac{\eta}{M_{\text{pl}}} \right)^4 \log^3 (\eta t_i).$$

$$\eta = f_a$$

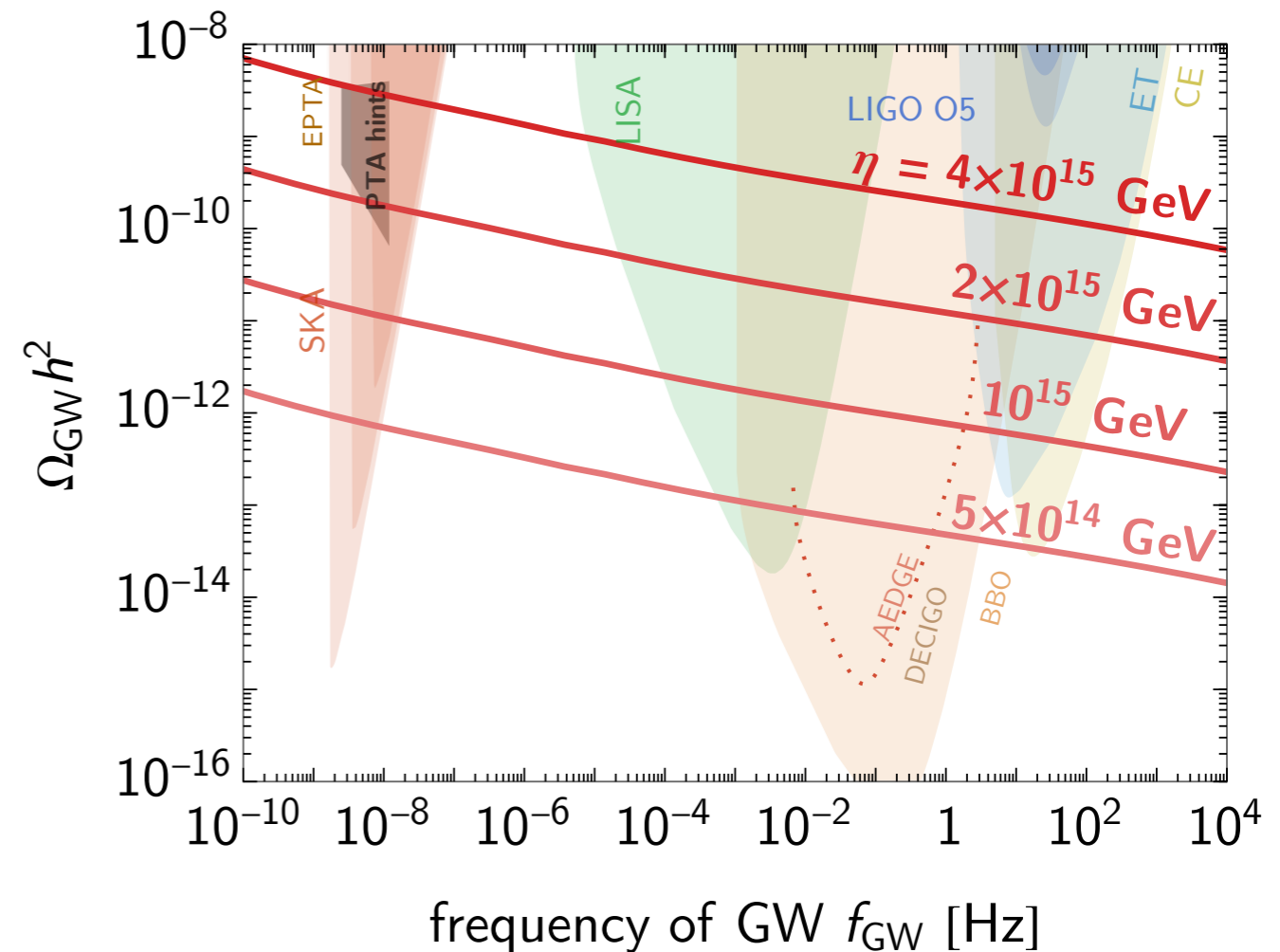
LOCAL STRINGS



**spectral shape
changes with η**

**local loops live longer
before decaying
(& lifetime depends on η)**

versus GLOBAL STRINGS ($m_a=0$)



**global loops
decay fast.**

**To reach the same amplitude as the
local strings, global strings need a
larger η since GW production is not
the leading energy loss.**

Temperature-frequency relation.

A loop population produced at temperature T quickly decays into GW of frequency

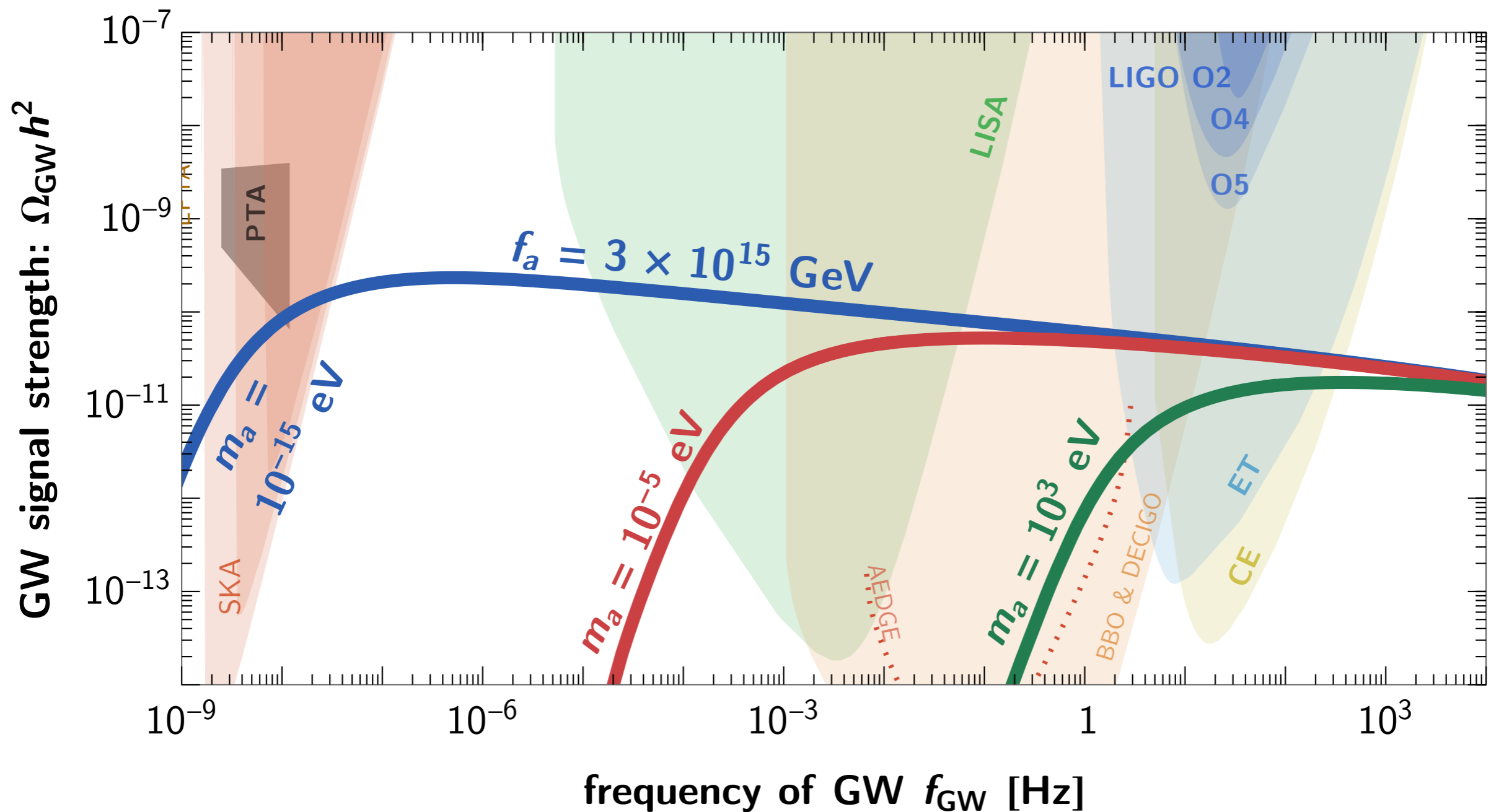
$$f_{\text{GW}}^{\text{CS}}(T) \simeq 63 \text{ nHz} \left(\frac{\alpha}{0.1} \right) \left(\frac{T}{10\text{MeV}} \right) \left[\frac{g_*(T)}{10.75} \right]^{\frac{1}{4}},$$

**α : typical loop
size in units of
Hubble horizon**

IR cutoff of GW spectrum fixed by axion mass

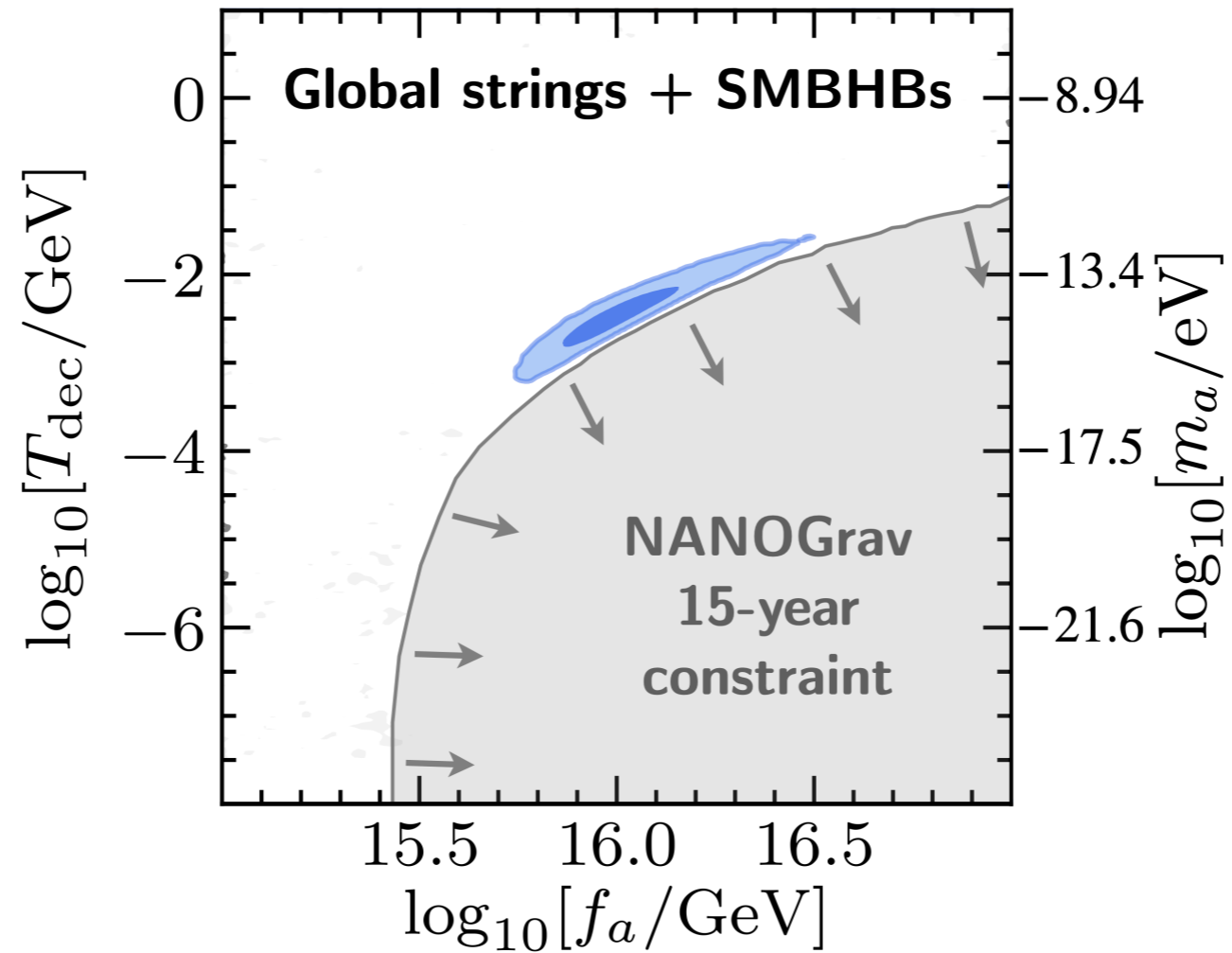
Network decays when $H \sim m_a$

$$f_{\text{GW}}^{\text{CS}}(m_a) \simeq 9.4 \text{ nHz} \left(\frac{\alpha}{0.1} \right) \left(\frac{m_a}{10^{-15} \text{ eV}} \right)^{\frac{1}{2}}.$$



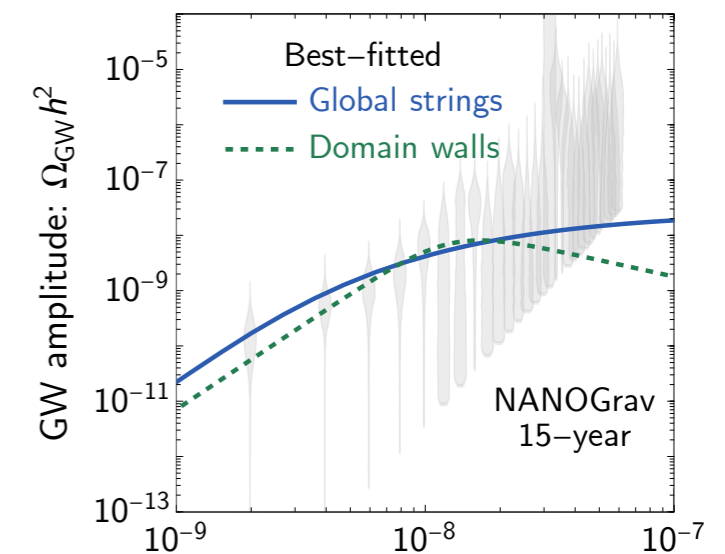
Analysis of 15 year-NANOGrav data

[Servant, Simakachorn, 2307.03121



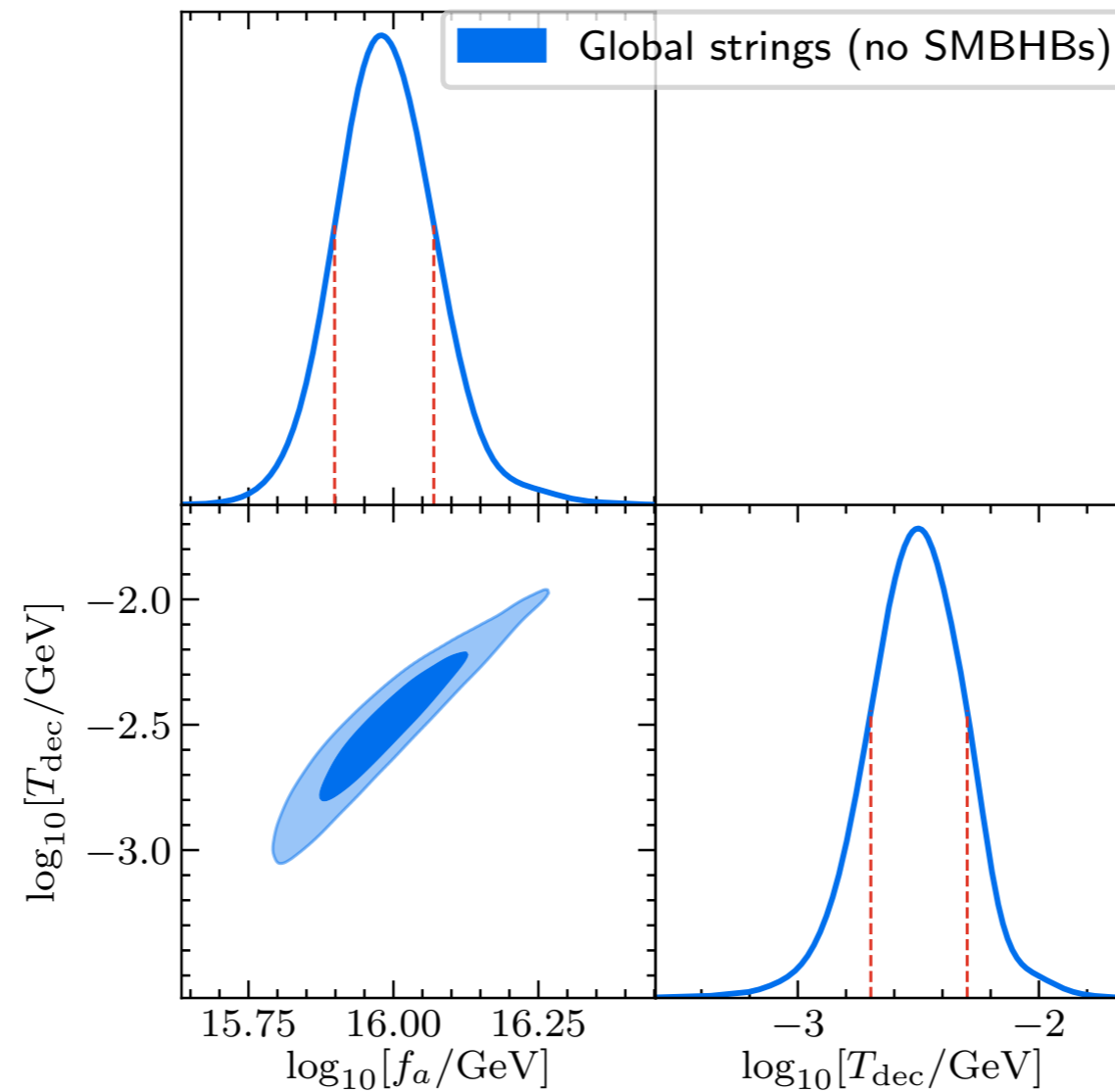
Best-fitted spectra to PTA data:

$$\{f_a, m_a\} \simeq \{9.9 \cdot 10^{15} \text{ GeV}, 4.8 \cdot 10^{-15} \text{ eV}\}$$



Without the SMBHBs

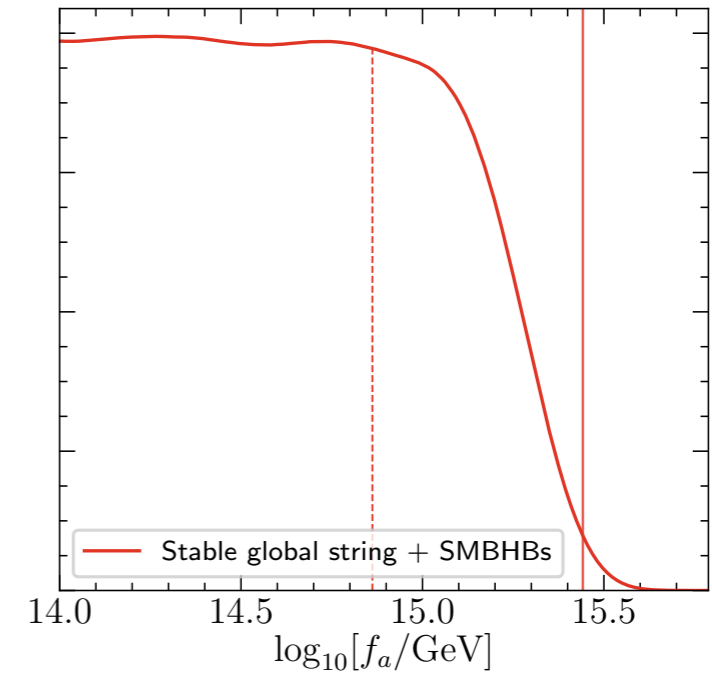
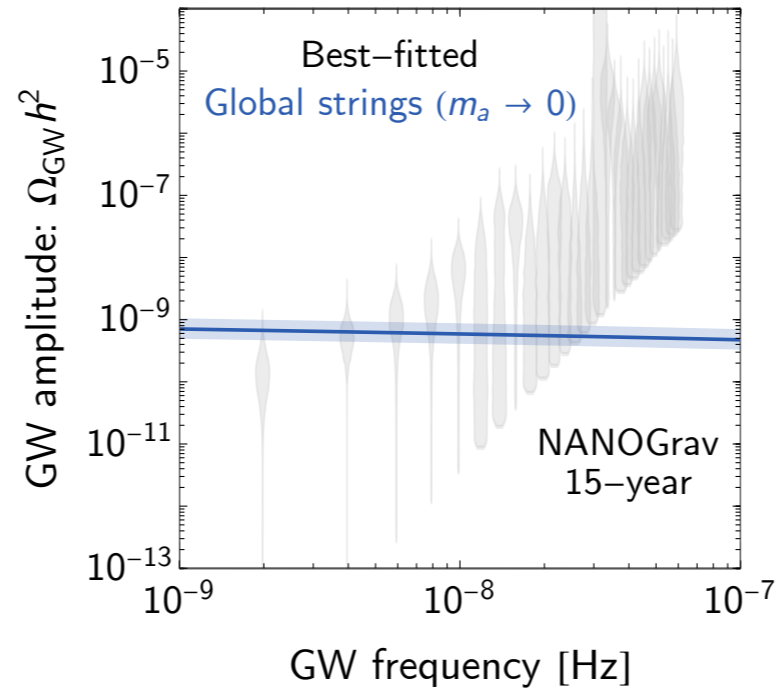
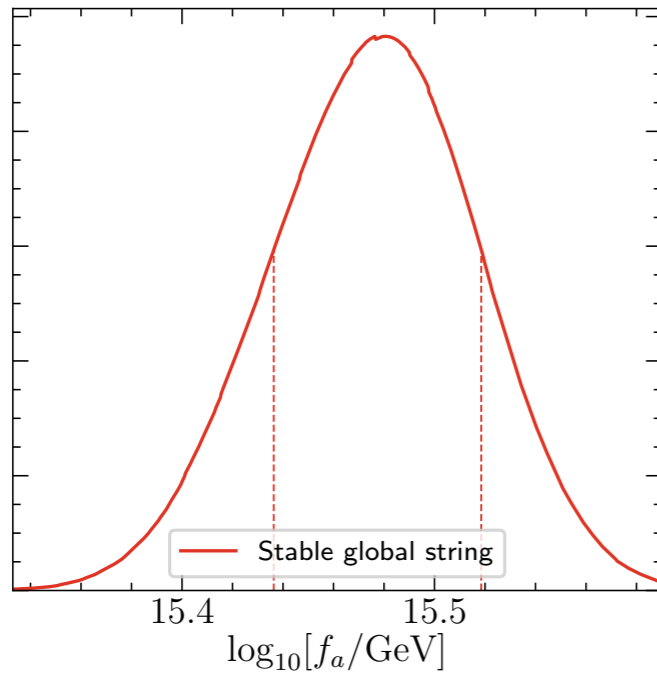
Best-fit does not change much $\{f_a, m_a\} = \{9.55 \cdot 10^{15} \text{ GeV}, 3.89 \cdot 10^{-15} \text{ eV}\}$
 $\text{BF} \simeq 22.8.$



Massless limit .

$$m_a \ll 10^{-16} \text{ eV}$$

Global strings in the limit $m_a \rightarrow 0$

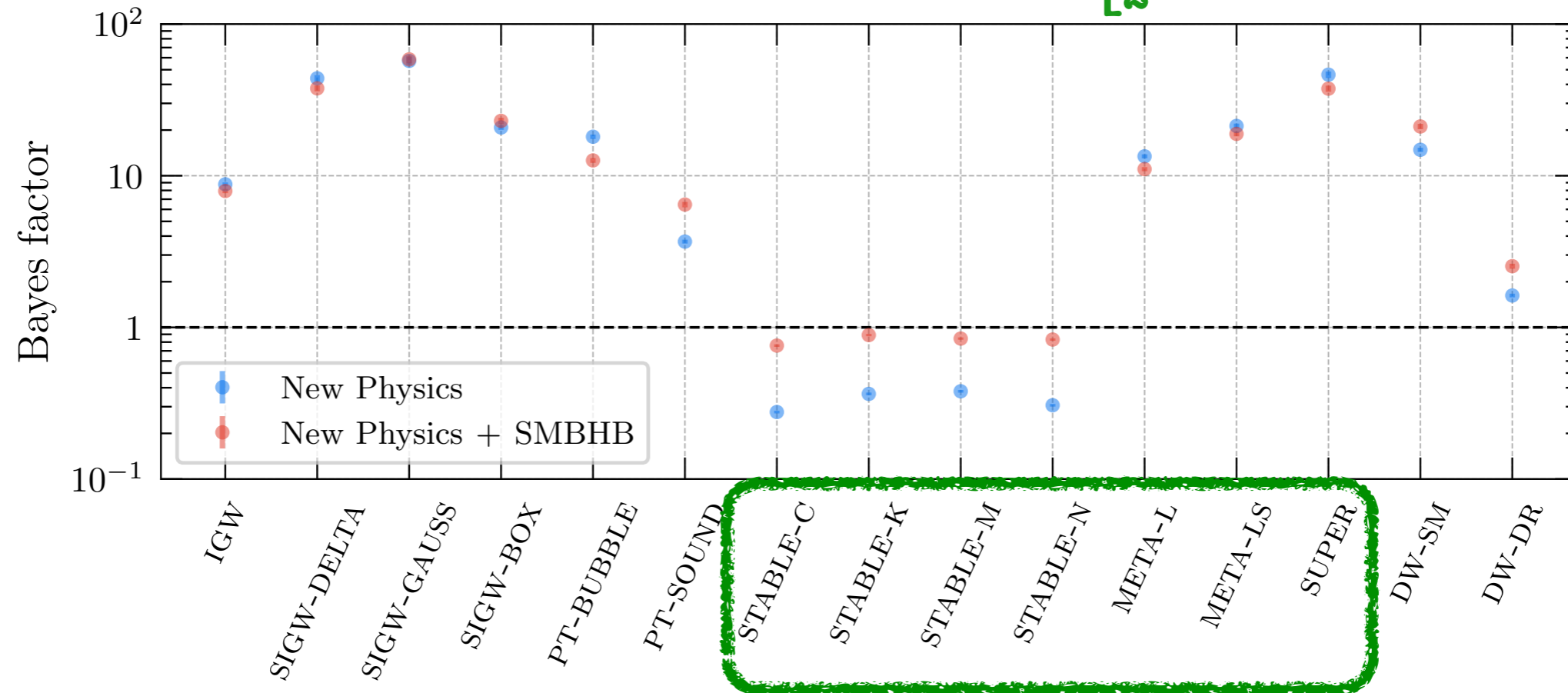


BF = 1.45×10^{-3} (without SMBHBs)

BF = 0.64 (with SMBHBs)

NANOGrav 15-YEAR NEW-PHYSICS SIGNALS

[2]

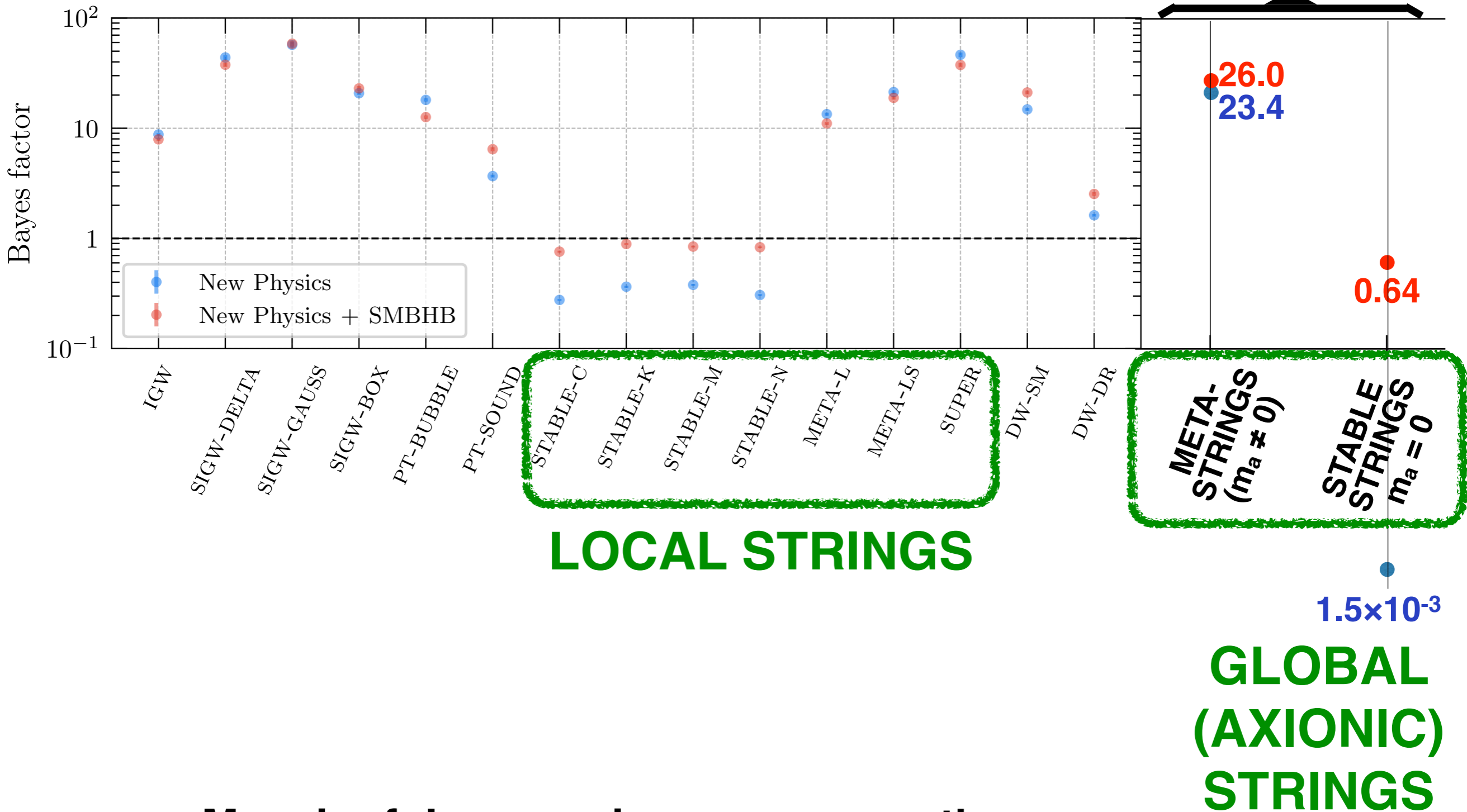


LOCAL STRINGS

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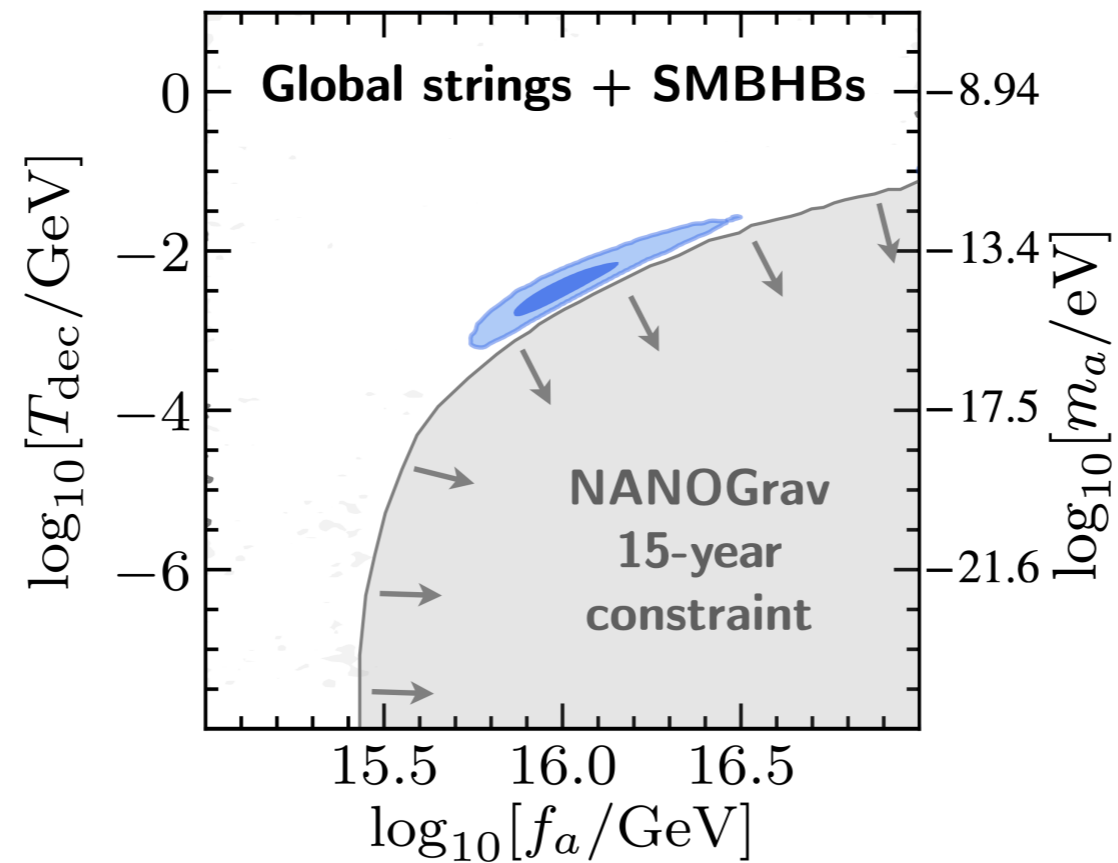
NANOGrav 15-YEAR NEW-PHYSICS SIGNALS

[2306.16219]

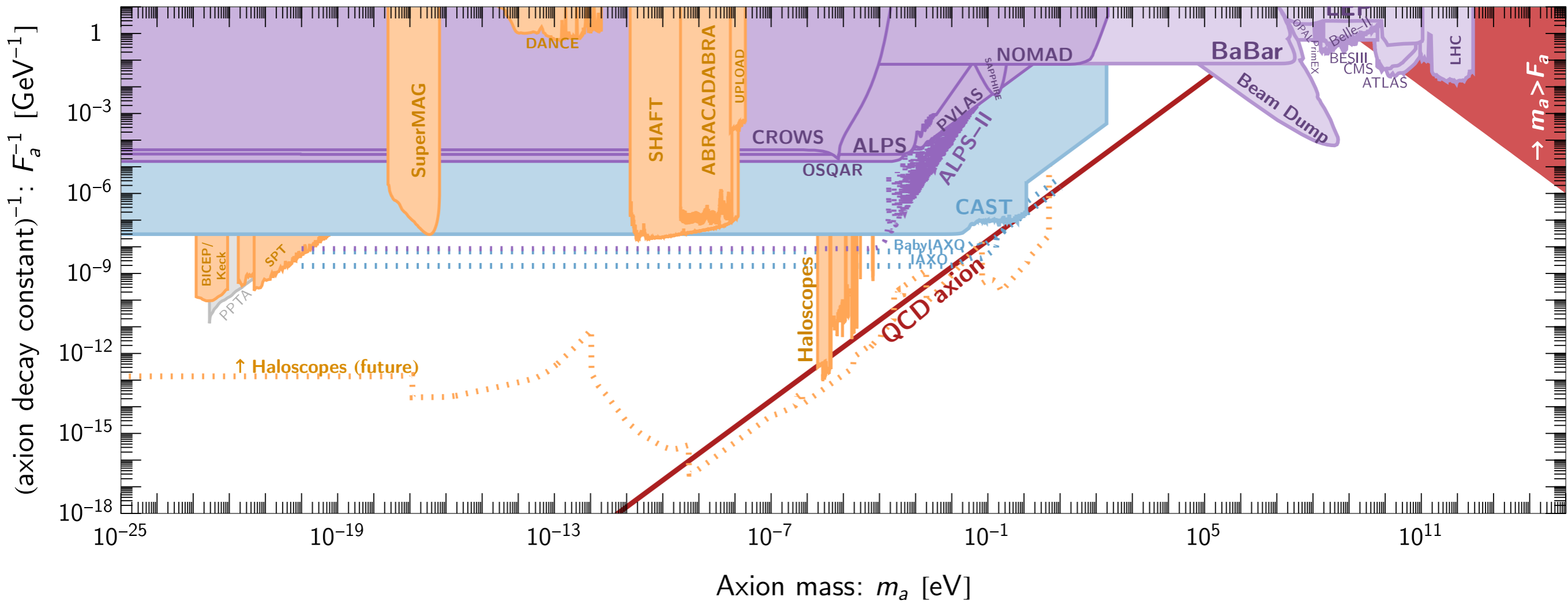


Meaningful comparison as we use the same SMBHB model as in the NG15 paper.

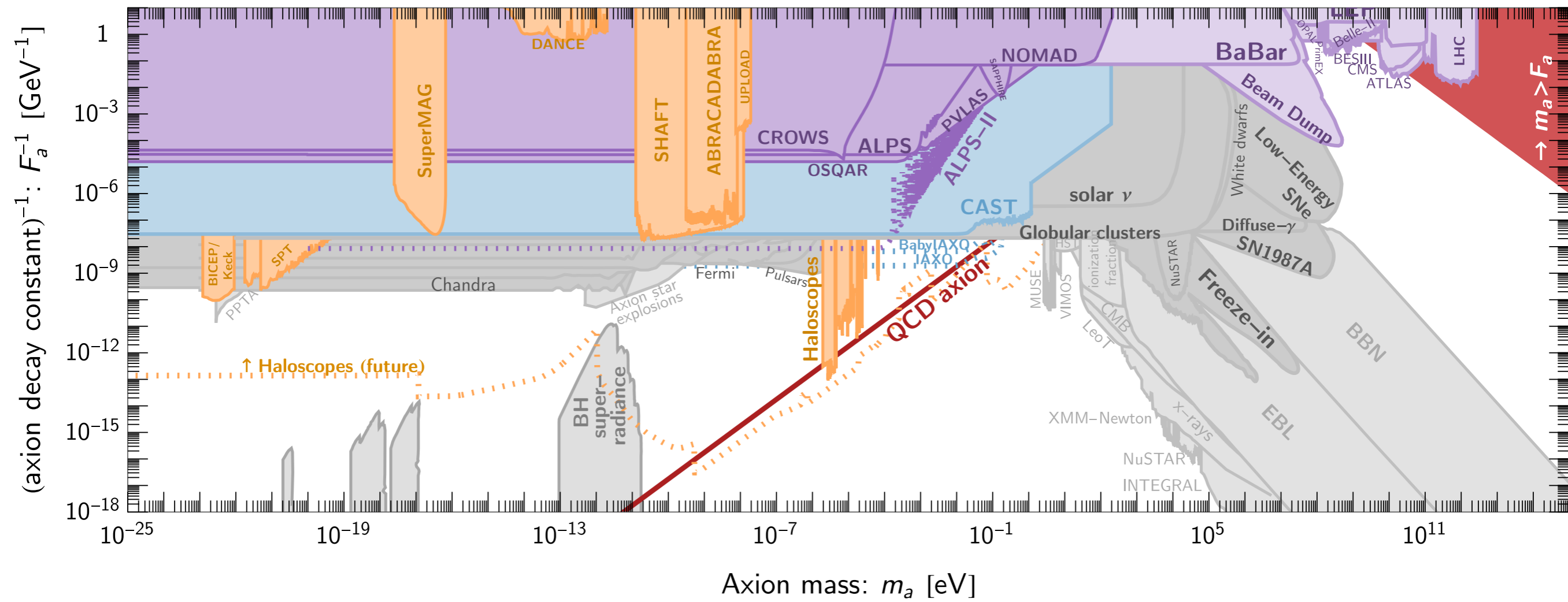
Translating the new constraint in the (m_a, f_a) plane



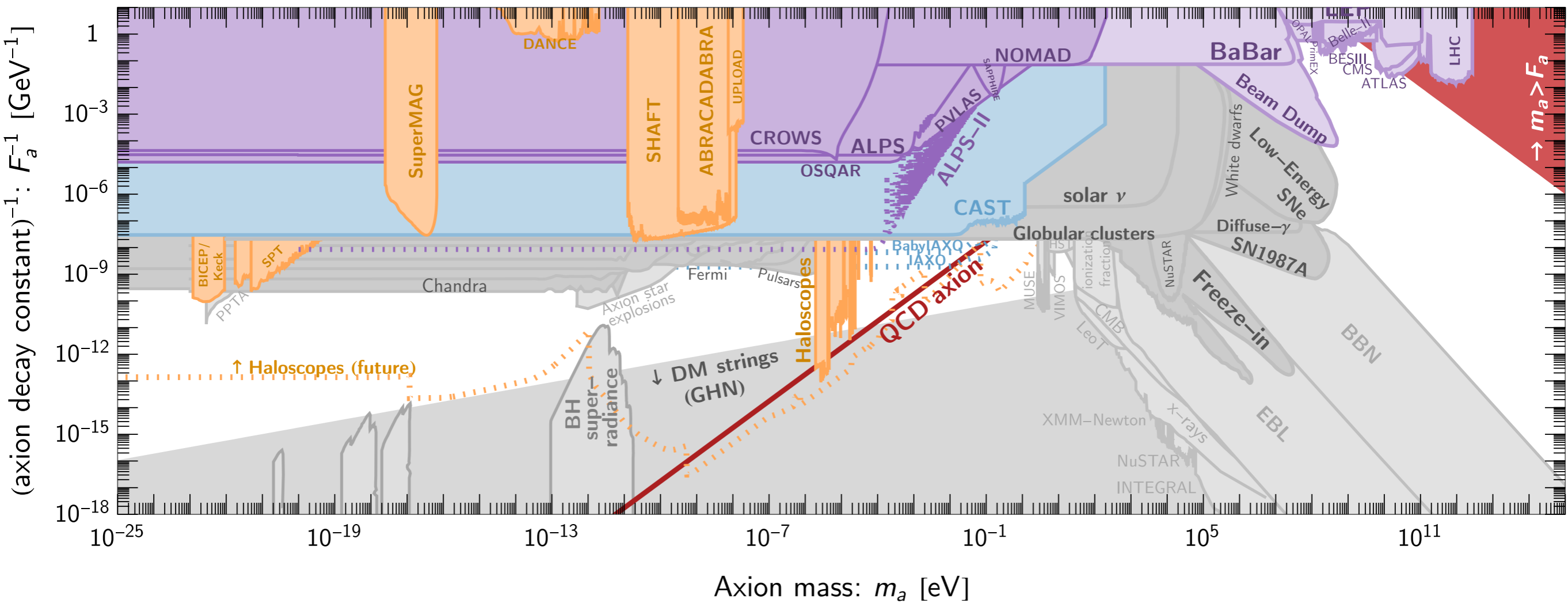
Lab bounds



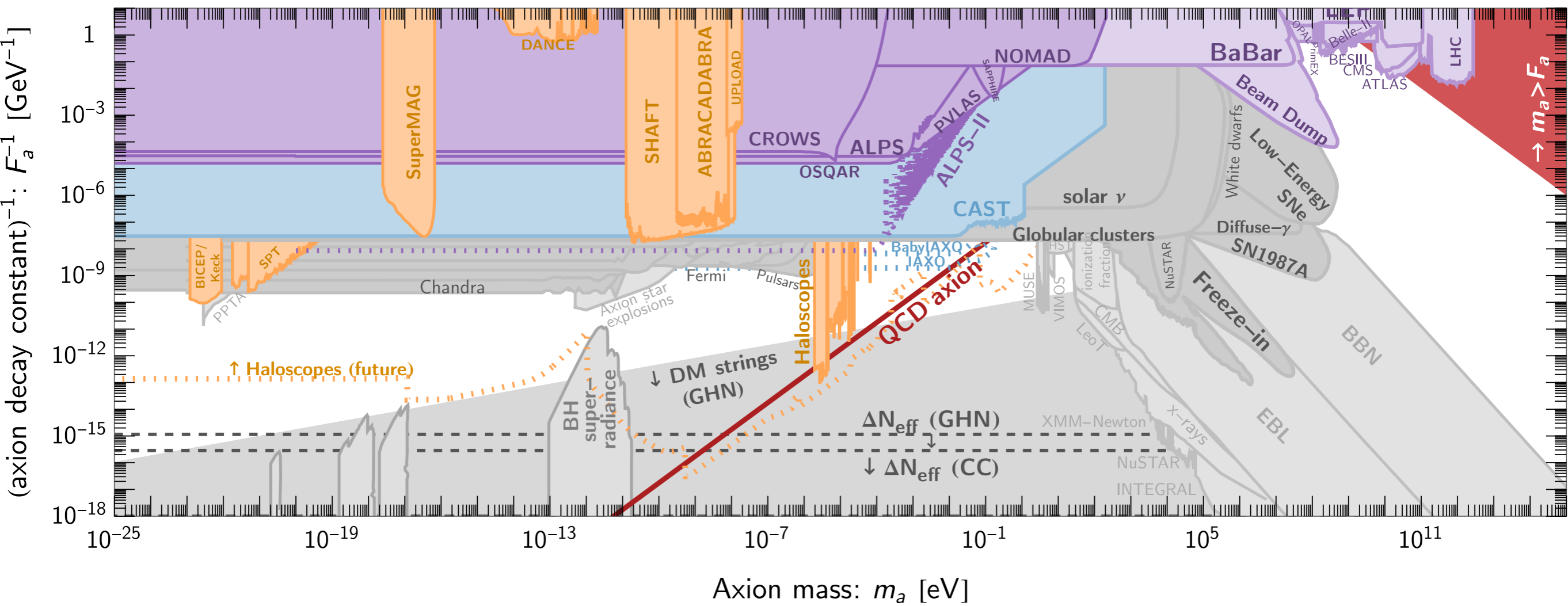
Adding astro bounds

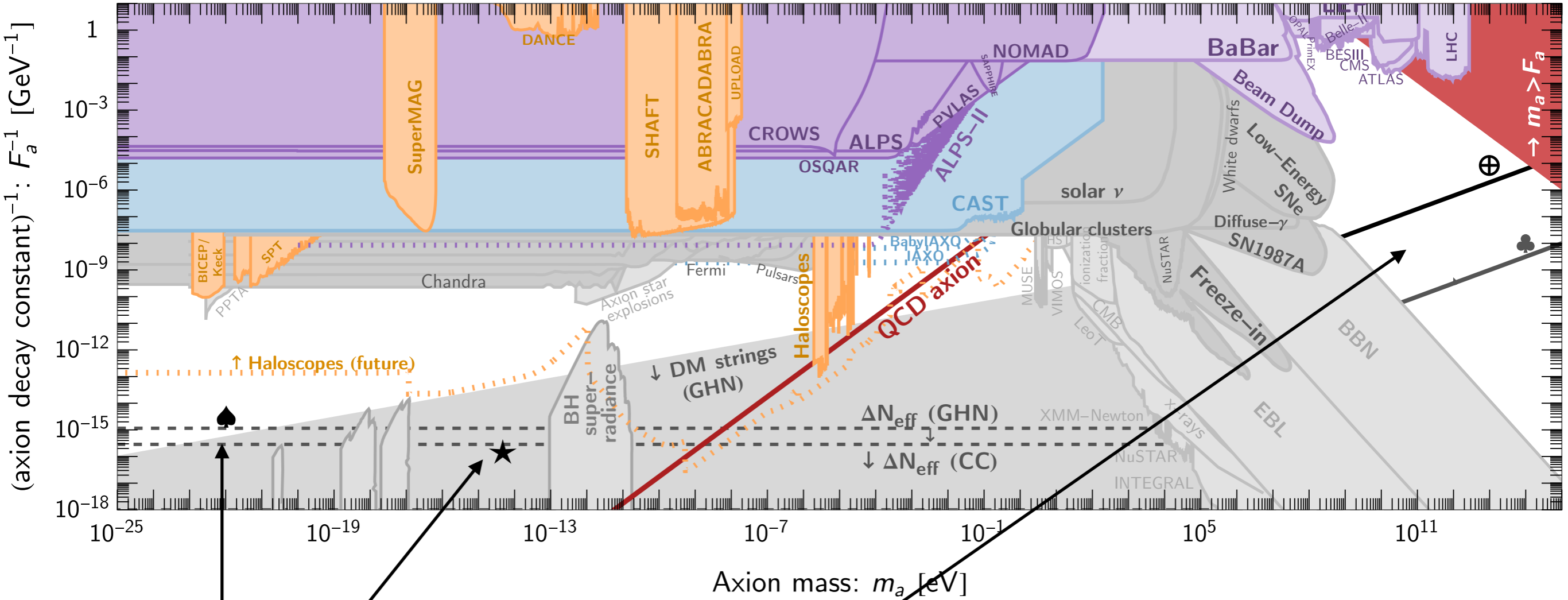


Theoretical bound from ALP abundance

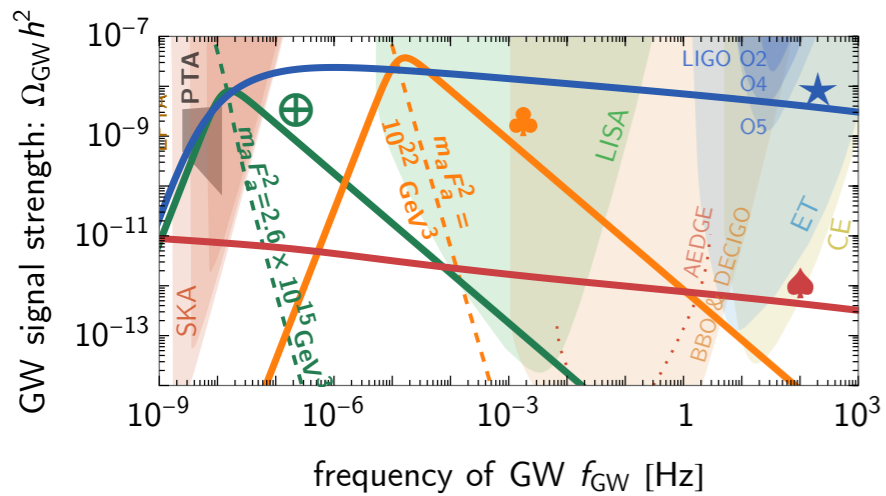


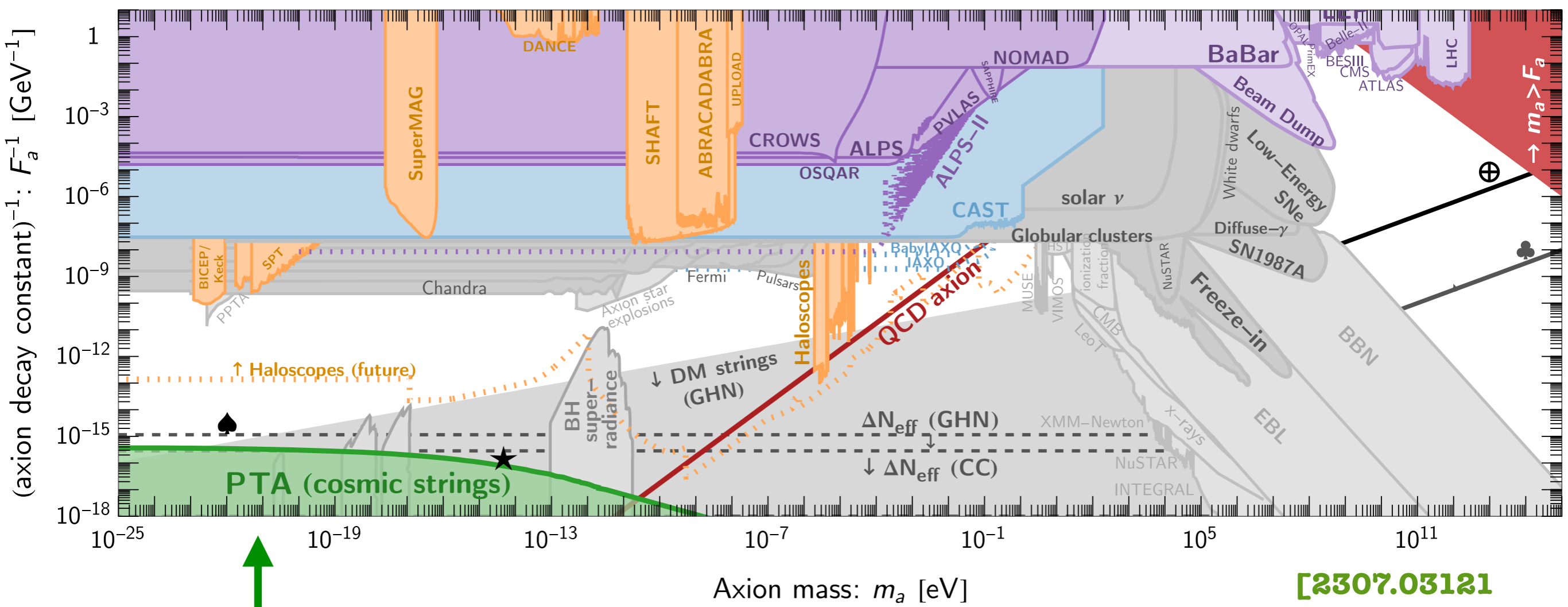
N_{eff} bound from ALP abundance





Benchmark spectra





New PTA constraint

[2307.03121]

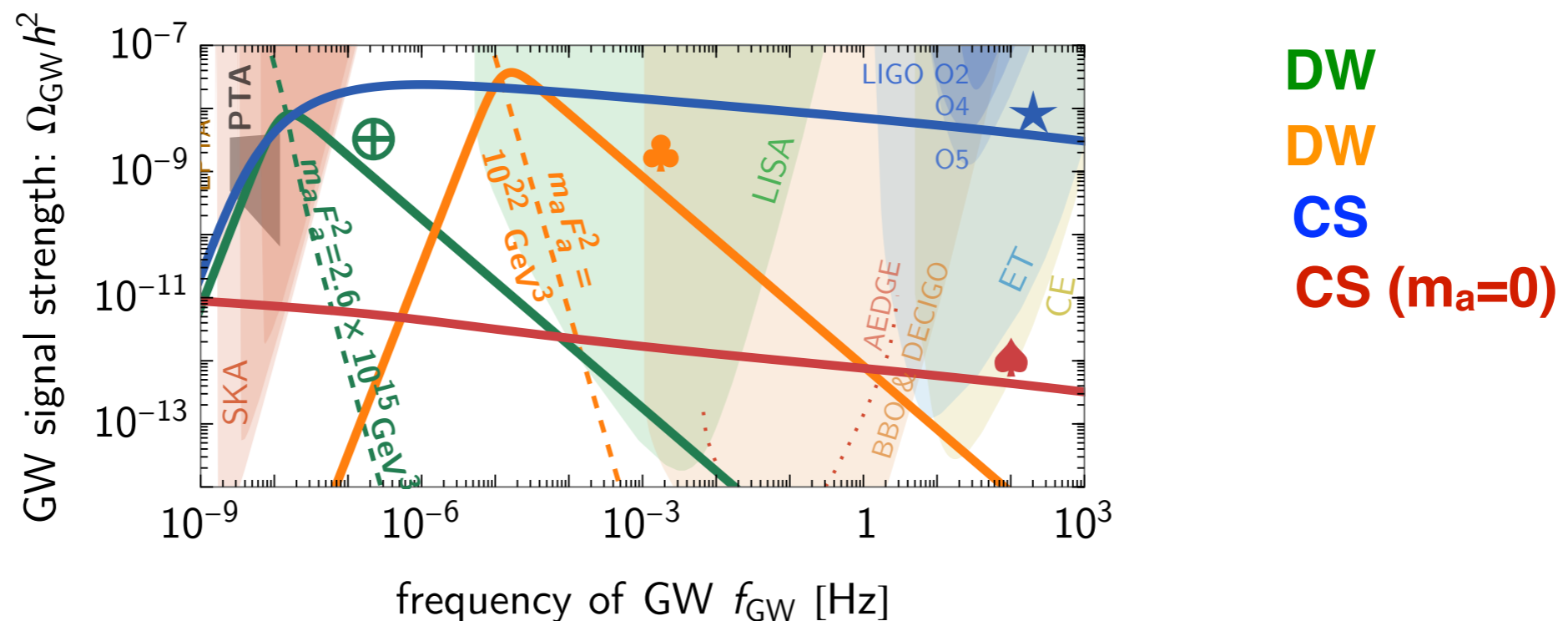
Domain Wall contribution .

Scalar potential.

$\Phi \equiv \phi \exp(i\theta)$ with ϕ the radial partner

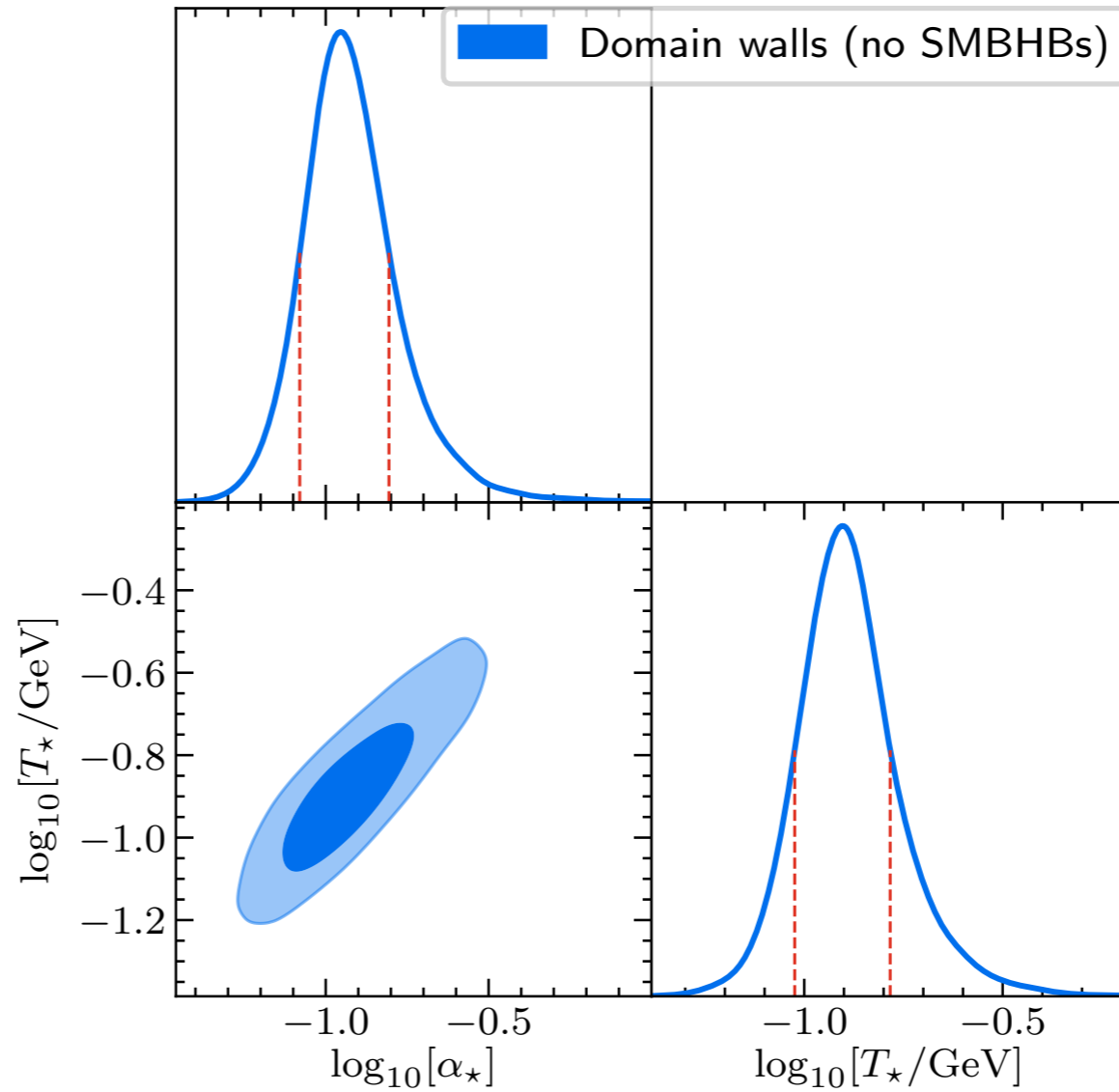
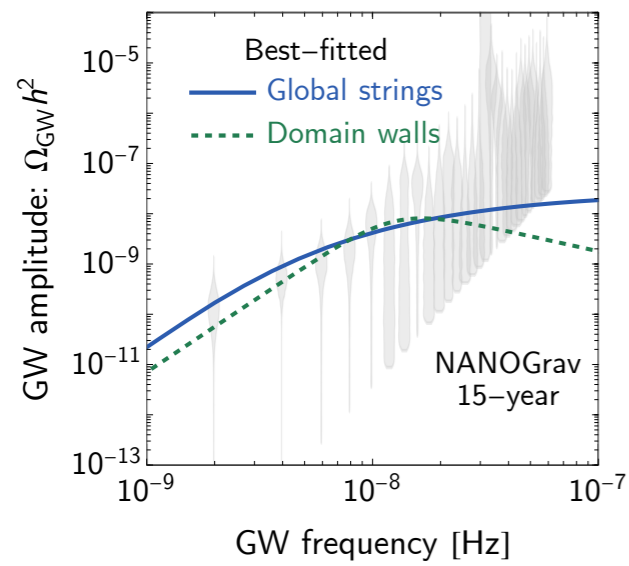
$$V(\Phi) = \underbrace{\frac{\lambda}{2}(\phi^2 - f_a^2)^2}_{\text{cosmic strings}} + \underbrace{\frac{m_a^2 f_a^2}{N_{\text{DW}}^2} [1 - \cos(N_{\text{DW}}\theta)]}_{\text{domain walls}} + V_{\text{bias}},$$

For $N_{\text{DW}} > 1$: Domain wall contribution to the GW bgd.



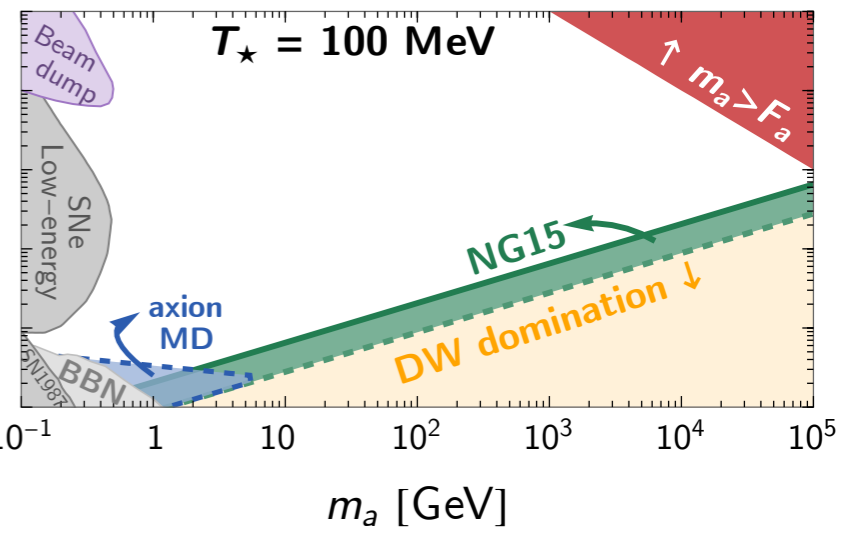
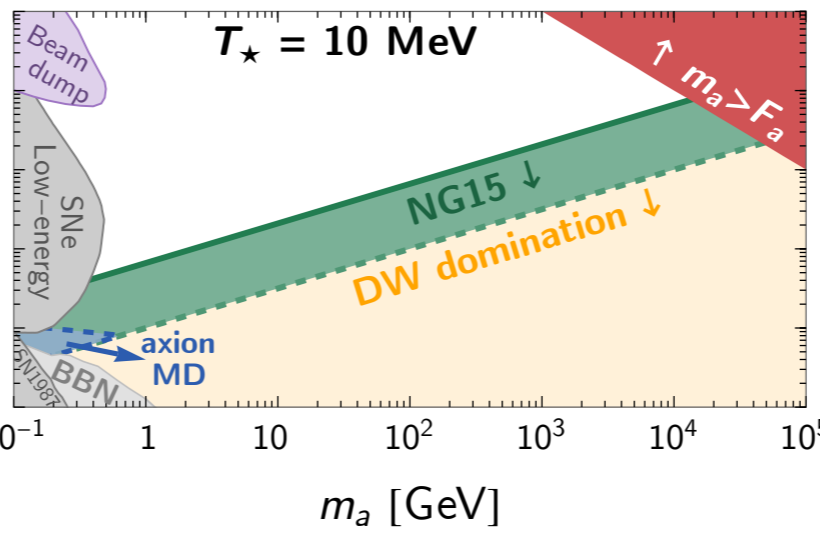
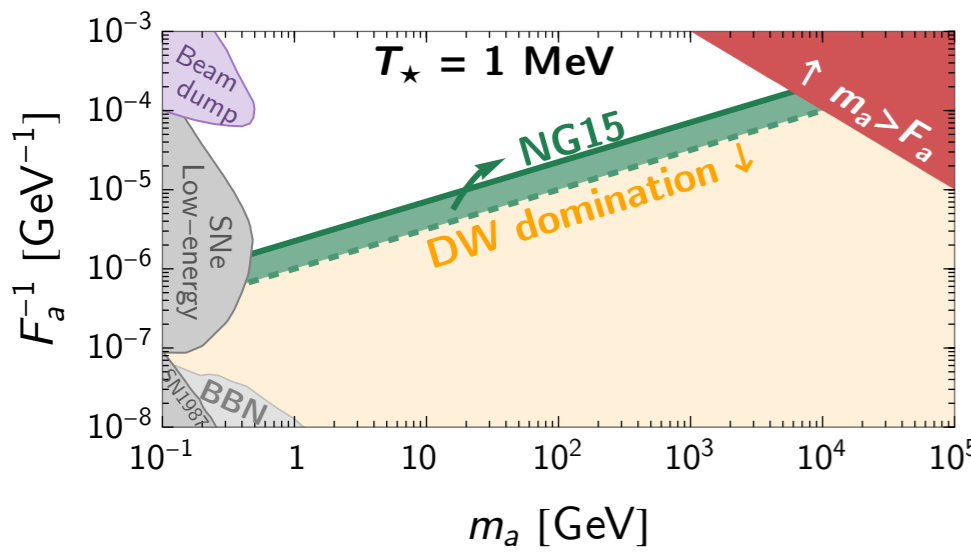
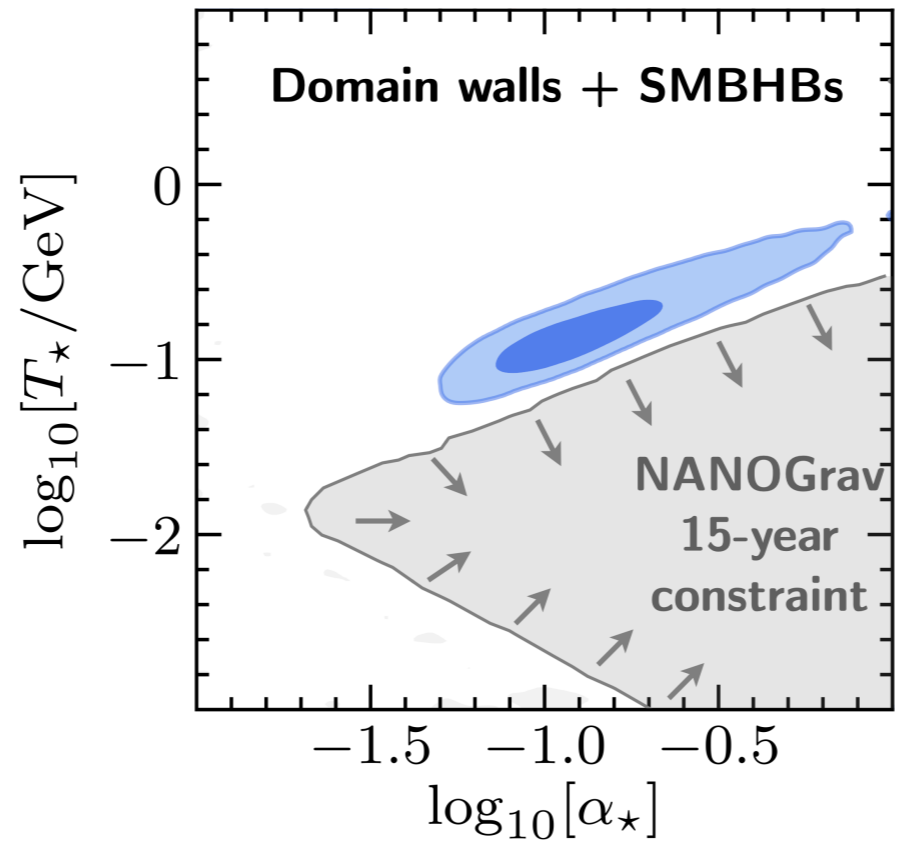
String-wall system collapses at temperature

$$T_{\star} \simeq 53\text{MeV} \left[\frac{10.75}{g_{\star}(T_{\star})} \right]^{\frac{1}{4}} \left[\frac{V_{\text{bias}}^{\frac{1}{4}}}{10\text{MeV}} \right]^2 \left[\frac{\text{GeV}}{m_a} \right]^{\frac{1}{2}} \left[\frac{10^6\text{GeV}}{f_a/N_{\text{DM}}} \right].$$



BF ~ 23.4 without SMBHB

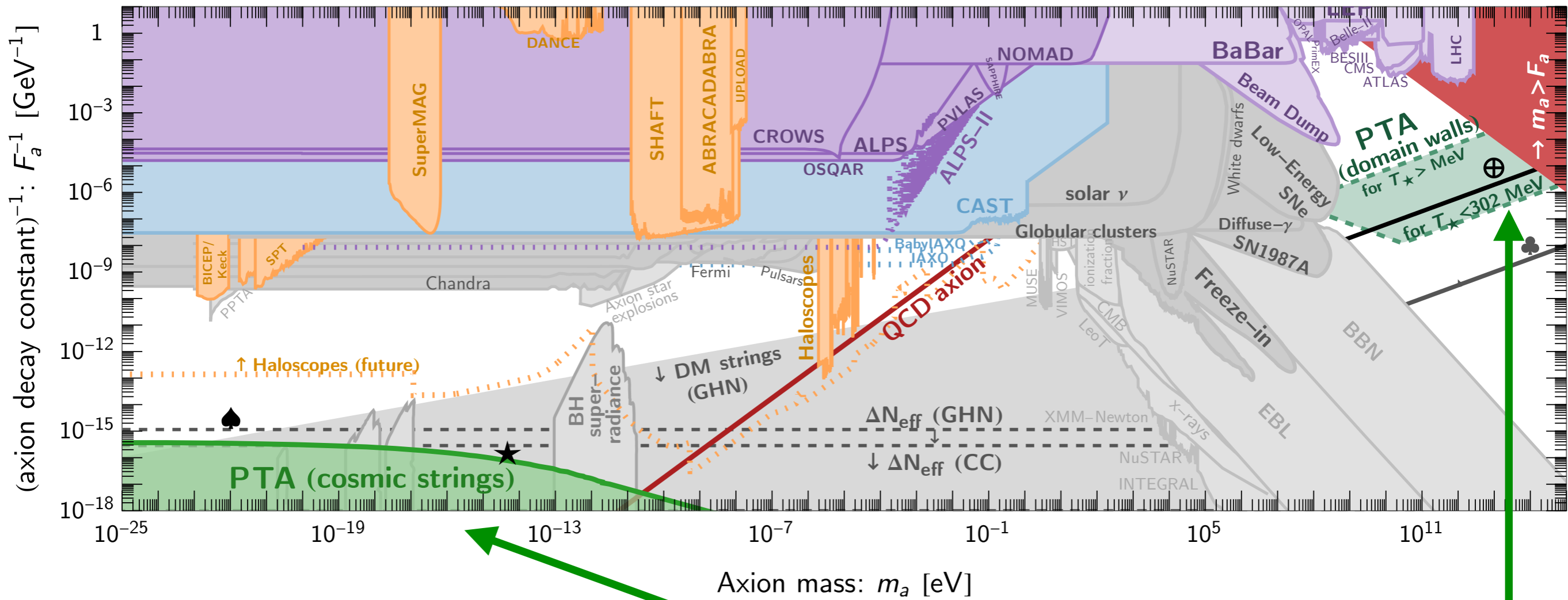
BF ~ 44.7 with SMBHB



[2307.03121]

Constraining post-inflationary axions with Pulsar Timing Arrays

[2307.03121]



New PTA constraints

GWs from axion fragmentation.

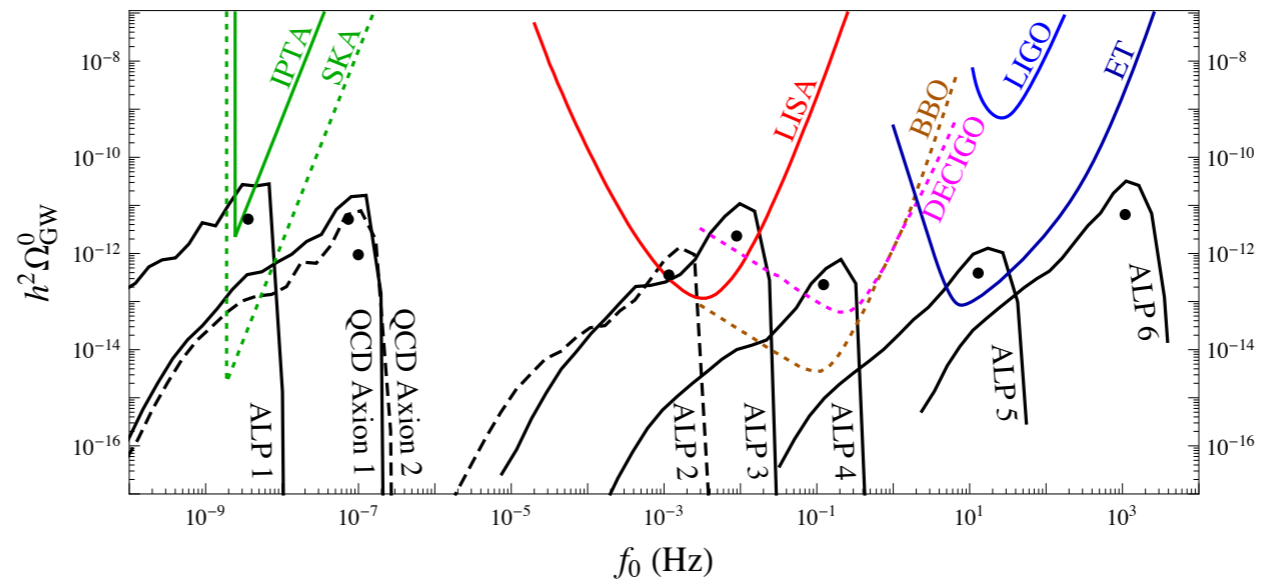
The transfer of energy in the early universe from the homogeneous axion field into axion quantum fluctuations, e.g. axion fragmentation, inevitably produces a stochastic background of gravitational waves of primordial origin with a peak frequency controlled by the axion mass.

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta h_{ij}}{a^2} = \frac{16\pi}{M_{\text{pl}}^2} \Pi_{ij}^{\text{TT}},$$

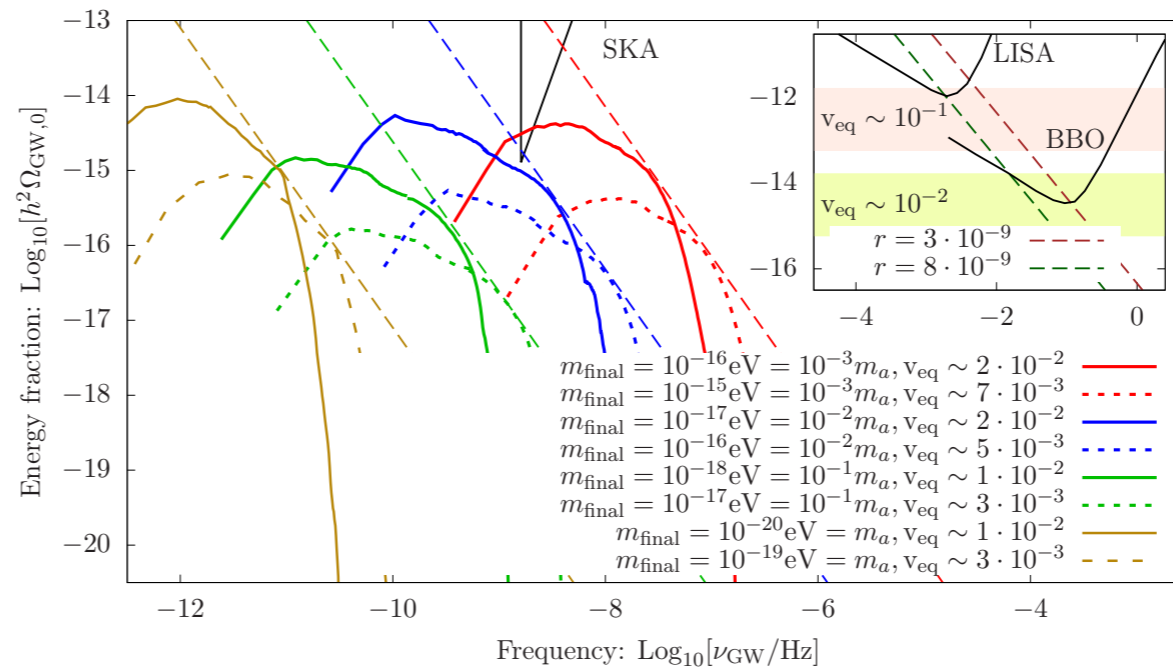
$$\Pi_{ij}^{\text{TT}}(t, \vec{x}) = \frac{1}{a^2} \left[\partial_i \phi(t, \vec{x}) \partial_j \phi(t, \vec{x}) - \frac{1}{3} \delta_{ij} (\partial_k \phi(t, \vec{x}) \partial_k \phi(t, \vec{x})) \right]$$

Examples.

Machado et al,
1811.01950



Chatrchyan, Jaeckel
2004.07844



The signal is generally suppressed when imposing the upper bounds from either the axion dark matter abundance or the axion dark radiation.

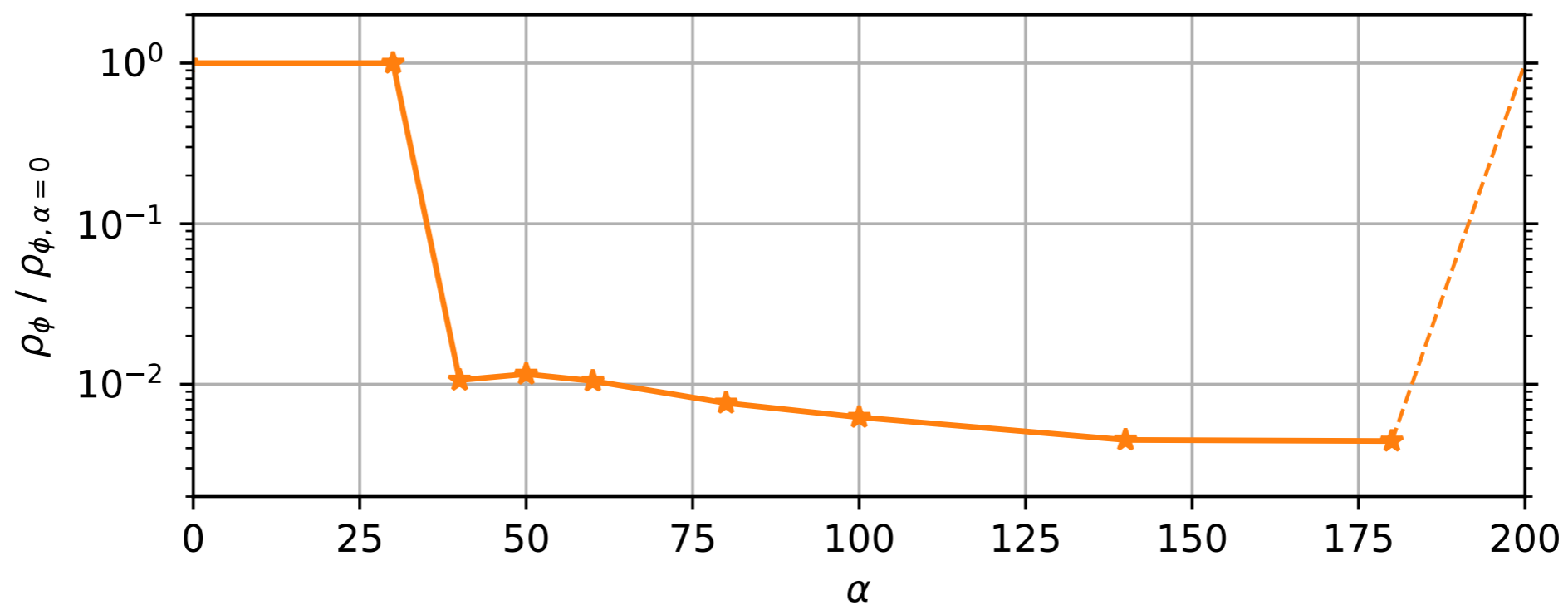
Schwaller et al, 2012.11584 (from coupling to dark photon)

Eroncel et al, 2206.14259

Geller et al, 2307.03724

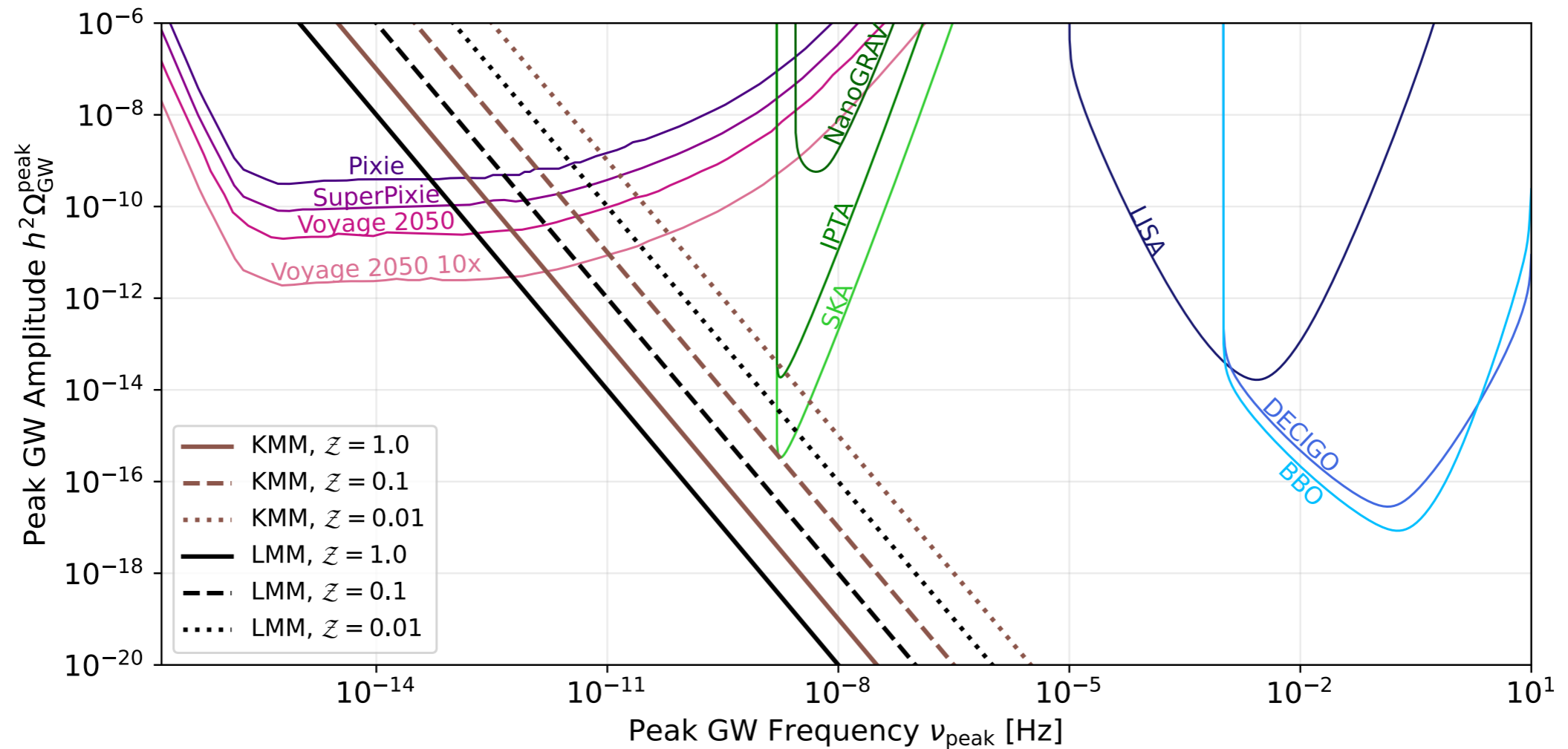
—> Dilution of ALP energy density needed

Achieved dilution factor of ALP energy density



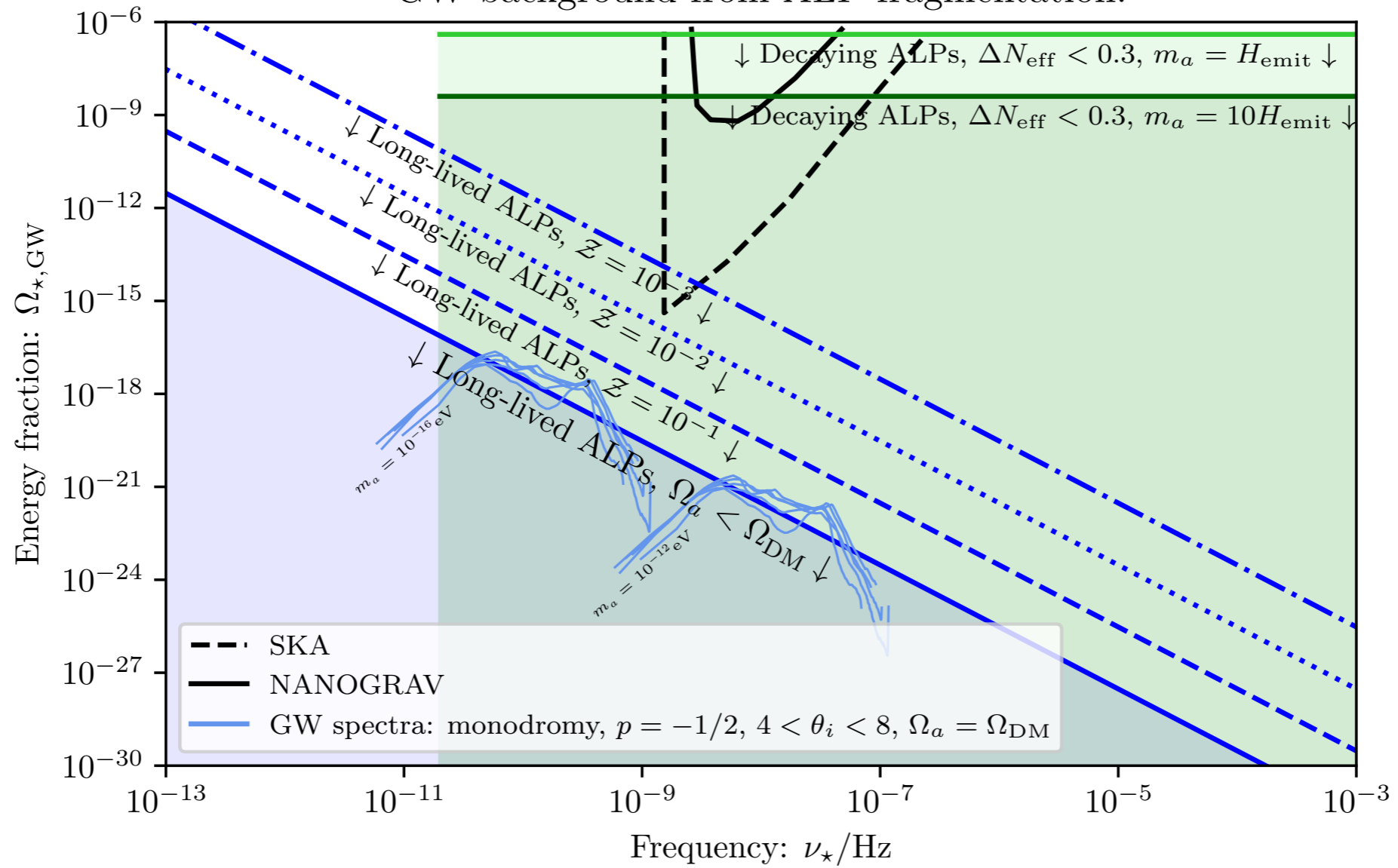
Ratzinger, Schwaller, Stefaneek, 2012.11584

$Z =$ needed dilution factor of ALP energy density

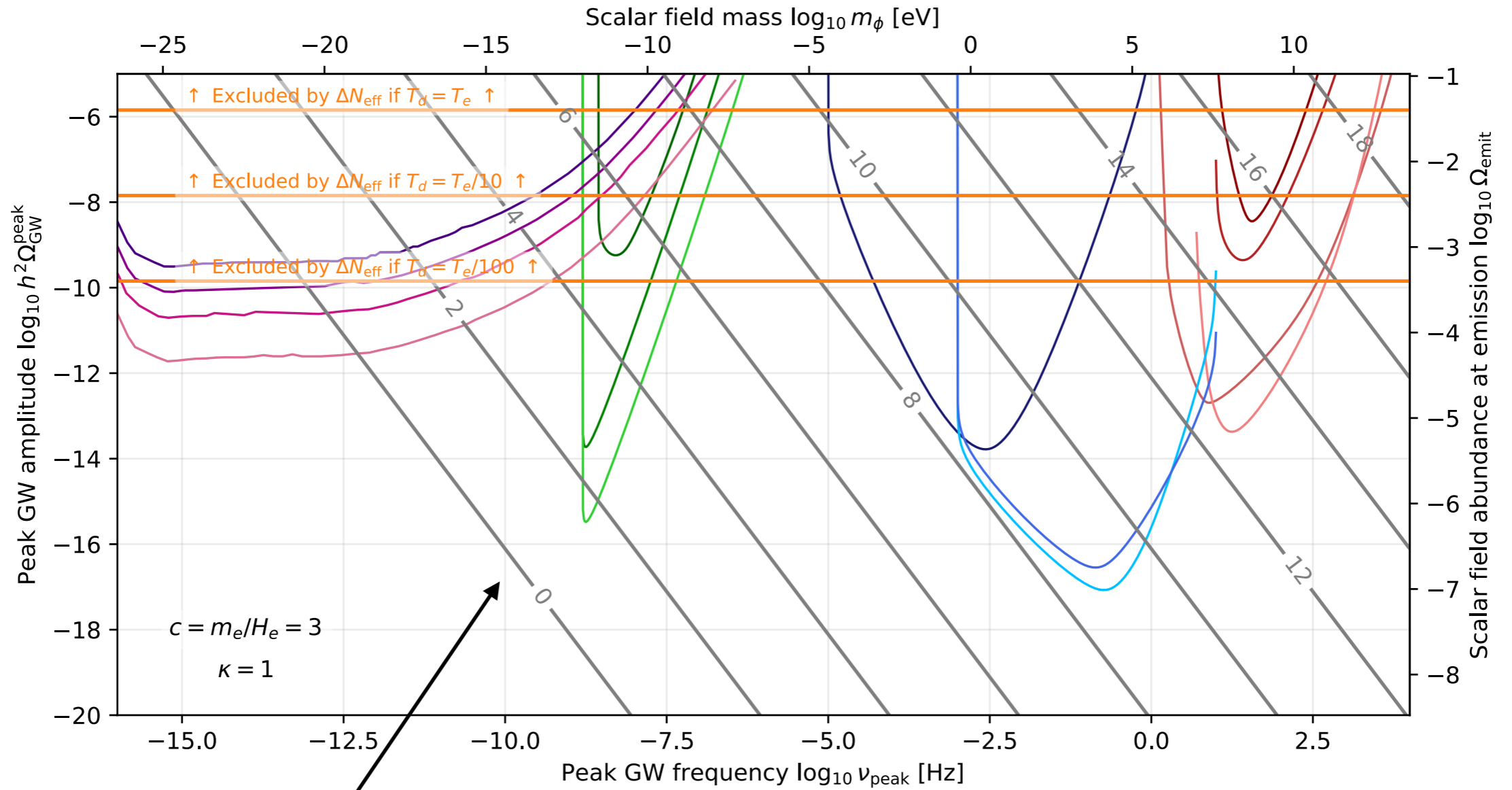


Eroncel et al, 2206.14259

GW background from ALP fragmentation.



Chatrchyan & al, in prep.



10^a : required dilution factor

Chatrchyan & al, in prep.

Extra material.

SOME REFERENCES

GW from GLOBAL CS:

Chang & Cui, [1910.04781], [2106.09746].

Gouttenoire et al, [1912.02569].

Gorghetto, Hardy & Nicolaescu, [2101.11007].

Ramberg & Visinelli, [1904.05707], [2012.06882].

GW from DW:

T. Hiramatsu, M. Kawasaki and K. Saikawa, On the estimation of gravitational wave spectrum from cosmic domain walls, JCAP 02 (2014) 031 [1309.5001].

R. Zambujal Ferreira, A. Notari, O. Pujolas & F. Rompineve, High Quality QCD Axion at Gravitational Wave Observatories, Phys. Rev. Lett. 128 (2022) 141101 [2107.07542].

K. Saikawa, Gravitational waves from cosmic domain walls: a mini-review, J. Phys. Conf. Ser. 1586 (2020) 012039.

. R. Z. Ferreira, A. Notari, O. Pujolas and F. Rompineve, Gravitational waves from domain walls in Pulsar Timing Array datasets, JCAP 02 (2023) 001 [2204.04228].

. E. Madge, E. Morgante, C. P. Ibáñez, N. Ramberg and S. Schenk, Primordial gravitational waves in the nano-Hertz regime and PTA data – towards solving the GW inverse problem, 2306.14856.

GW from cosmic strings.

GW spectrum

⇒ GW emission from a loop × loop-number density

$$\Omega_{\text{GW}}^{(k)}(f) = \frac{1}{\rho_c} \cdot \frac{2k}{f} \cdot \frac{(0.1)\Gamma^{(k)} G\mu^2}{\alpha(\alpha + \Gamma G\mu)} \int_{t_F}^{t_0} d\tilde{t} \frac{C_{\text{eff}}(t_i)}{t_i^4} \left[\frac{a(\tilde{t})}{a(t_0)} \right]^5 \left[\frac{a(t_i)}{a(\tilde{t})} \right]^3 \Theta(t_i - t_F)$$

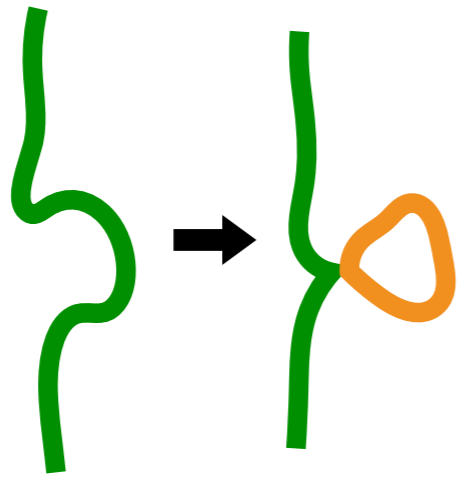
string's nature loop number red-shift

$t_i \equiv$ loop production, $\tilde{t} \equiv$ loop emission

a^{-3} (pointing to the red-shift term)
 $\text{GW: } a^{-4}$, loop size: a^{-1} (pointing to the red-shift term)

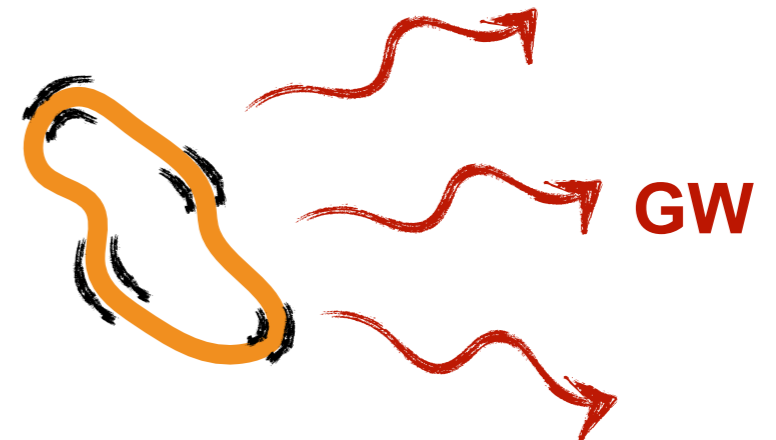
cosmic string (local)

loop production @ t_i



$$t_* \sim t_i / G\mu$$

loop emission @ t_*



Relation between observed frequency & Hubble radius at emission

$$f \approx H_* \left(\frac{a_i}{a_0} \right) \left(\frac{1}{G\mu} \right)^{1/2}$$

With respect to local strings, the GW spectrum from global strings in standard radiation cosmology is:

- suppressed by the shorter Hubble time \tilde{t}_M at the time of GW emission: factor $\tilde{t}_M^{\text{global}} / \tilde{t}_M^{\text{local}} \propto G\mu_{\text{local}} \propto (\eta/M_{\text{pl}})^2$,
- suppressed by the larger GW redshift factor since emission occurs earlier: factor $\left[\frac{a(\tilde{t}_M^{\text{global}})}{a(\tilde{t}_M^{\text{local}})} \right]^4 \propto (\eta/M_{\text{pl}})^4$,
- enhanced by the lower loop redshift factor since GW emission occurs right after loop production: factor $\left(a(\tilde{t}_M^{\text{local}}) / a(\tilde{t}_M^{\text{global}}) \right)^3 \propto (\eta/M_{\text{pl}})^{-3}$,
- increased by the logarithmically-enhanced GW power emission rate: factor $\log^2(\eta t_i)$,
- increased by the logarithmically-enhanced loop lifetime: factor $\log(\eta t_i)$.

**Which of these axions can make
Dark Matter ?**

Axions from the misalignment mechanism.

Start with ALP lagrangian $\mathcal{L} = -\frac{f^2}{2}g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\theta) = -\frac{f^2}{2}g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - m_a^2f^2(1 - \cos\theta).$

Neglecting fluctuations, the homogeneous zero-mode satisfies

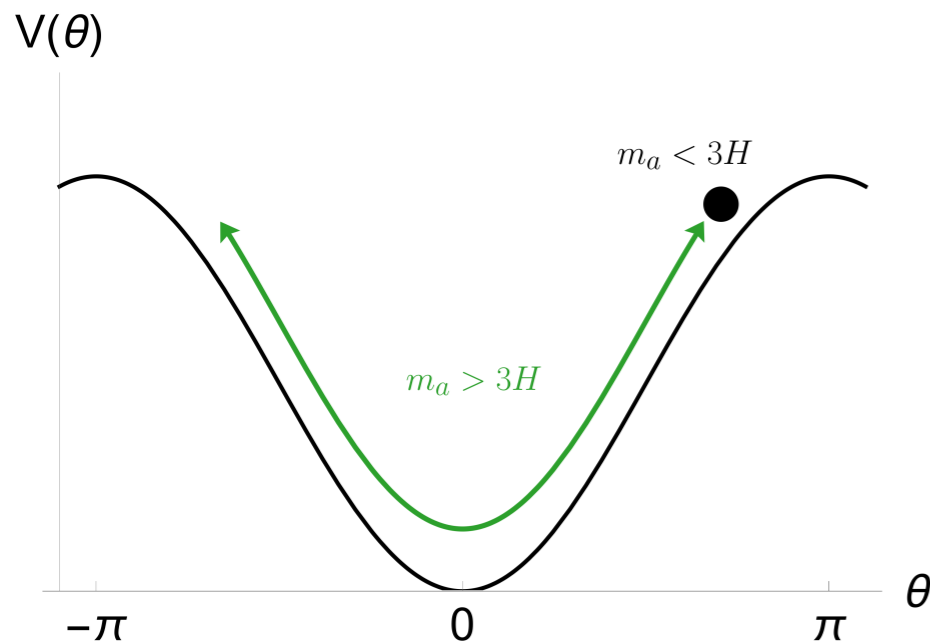
$$\ddot{\Theta} + 3H\dot{\Theta} + m_a^2(T)\sin(\Theta) = 0,$$

$H = \dot{a}/a =$ expansion rate of universe

(here a is the scale factor in the Friedmann-Roberston-Walker metric $ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2d\Omega^2 \right]$!)

With initial conditions:

$$\Theta(t_i) = \Theta_i, \quad \dot{\Theta}(t_i) = 0. \quad \text{standard assumption}$$



> $m_a \ll 3H \iff \rho_a \propto a^0$ (Frozen)

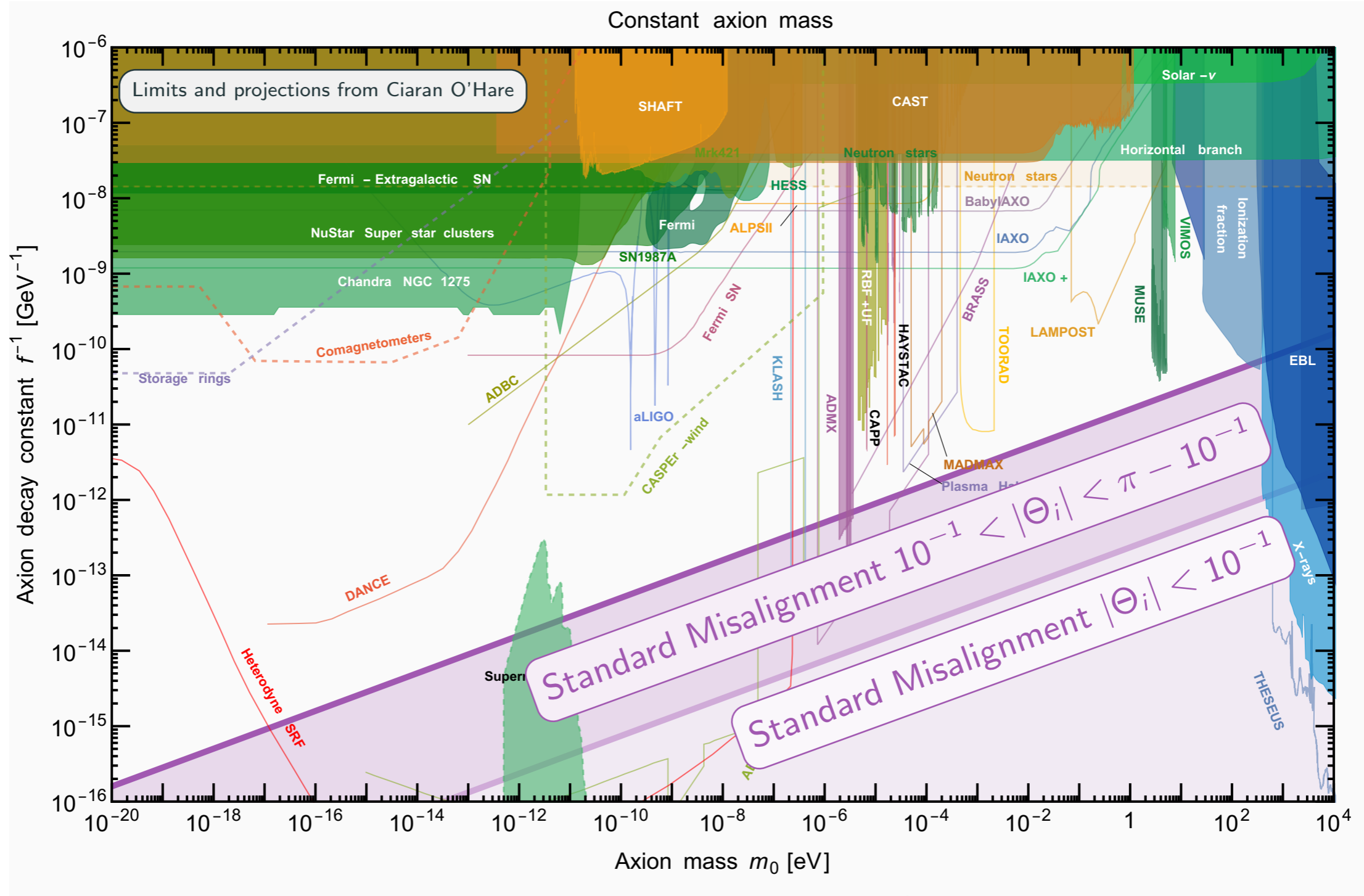
> $m_a \gg 3H \iff \rho_a \propto a^{-3}$ (Oscillating)

→ standard misalignment mechanism

$$\text{For } \Theta_i \sim 1 \quad \rho_{\text{DM}} \sim \rho_{\text{osc}} \left(\frac{a_{\text{osc}}}{a_0} \right)^3 \sim m_a^2 f_a^2 \left(\frac{T_0}{T_{\text{osc}}} \right)^3$$

$$T_{\text{osc}} \sim \sqrt{m_a M_{\text{Pl}}}$$

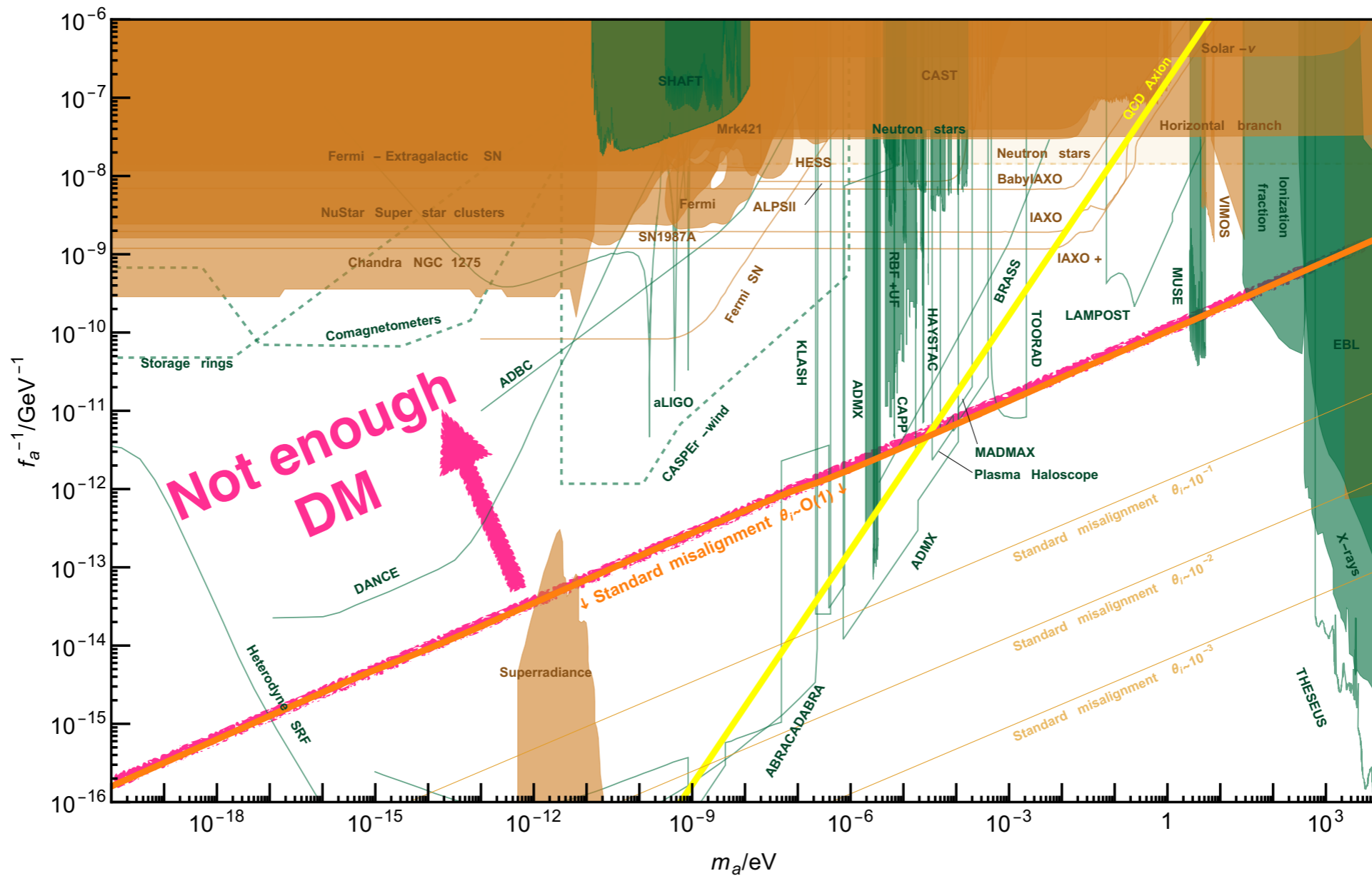
ALP DM parameter space.



(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\text{em}}/2\pi)(1.92/f)$$

Conventional misalignment makes too little DM for low f_a



A way out: switch on initial velocity for the axion

Co, Harigaya et al '19
 Chang, Cui'19
 Eroncel et al, '22