



# Fundamental Physics with Nuclei

## Neutrino Scattering at Low and Intermediate Energies

30 June 2023

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<https://physics.wustl.edu/quantum-monte-carlo-group>

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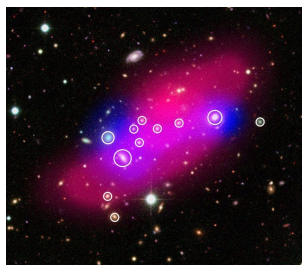
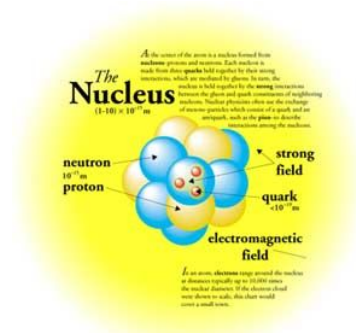
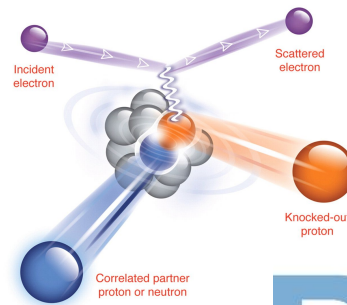
Lorenzo Andreoli (PD) Jason Bub (GS) Graham Chambers-Wall (GS) Garrett King (GS)

Anna McCoy (FRIB TA Fellow)

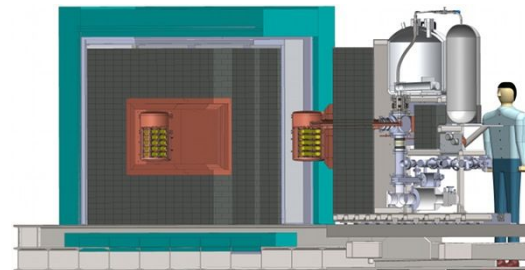
Maria Piarulli and Saori Pastore

Computational Resources awarded by the DOE ALCC, INCITE and SciDAC programs

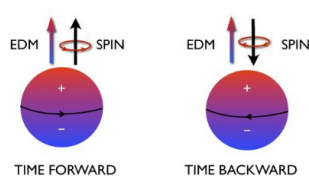
# Understand Nuclei to Understand the Cosmos



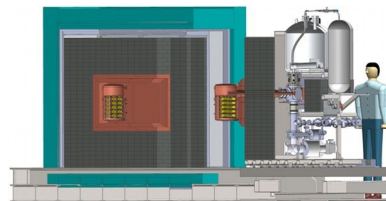
ESA, XMM-Newton, Gastaldello, CFHTL



Ground States'  
Electroweak Moments,  
Form Factors, Radii



Neutrinoless Double  
Beta Decay,  
Muon-Capture



Accelerator Neutrino  
Experiments,  
Lepton-Nucleus XSecs

$(\omega, q) \sim 0$  MeV

$\omega \sim \text{few MeVs}$   
 $q \sim 0$  MeV

$\omega \sim \text{few MeVs}$   
 $q \sim 10^2$  MeV

$\omega \sim \text{tens of MeVs}$

$\omega \sim 10^2$  MeV



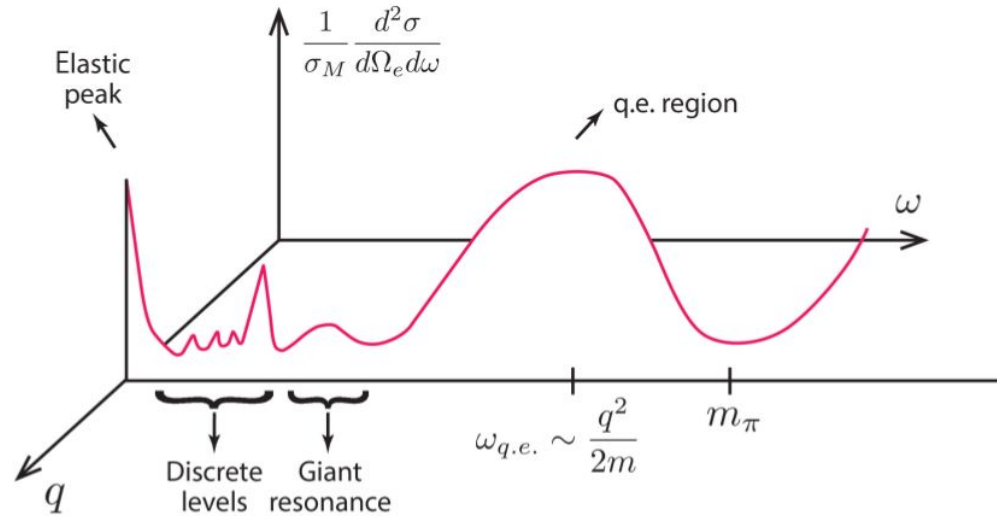
Electromagnetic  
Decay, Beta Decay,  
Double Beta Decay &  
inverse processes



Nuclear Rates for  
Astrophysics



# Electron-Nucleus Scattering Cross Section



Energy and momentum transferred ( $\omega, q$ )

Current and planned experimental programs rely on theoretical calculations at different kinematics

# Strategy

**Validate the Nuclear Model against available data for strong and electroweak observables**

- Energy Spectra, Electromagnetic Form Factors, Electromagnetic Moments, ...
- Electromagnetic and Beta decay rates, ...
- Muon Capture Rates, ...
- Electron-Nucleus Scattering Cross Sections, ...

**Use attained information to make (accurate) predictions for BSM searches and precision tests**

- EDMs, Hadronic PV, ...
- BSM searches with beta decay, ...
- Neutrinoless double beta decay, ...
- Neutrino-Nucleus Scattering Cross Sections, ...
- ...

# Microscopic (or *ab initio*) Description of Nuclei

**Comprehensive theory** that describes quantitatively and predictably nuclear structure and reactions

## Requirements:

- Accurate understanding of the interactions/correlations between nucleons in **pairs, triplets, ... (two- and three-nucleon forces)**
- Accurate understanding of the electroweak interactions of external probes (electrons, neutrinos, photons) with nucleons, correlated nucleon-pairs, ... (**one- and two-body electroweak currents**)
- **Computational methods** to solve the many-body nuclear problem of strongly interacting particles



Erwin Schrödinger

$$H\Psi = E\Psi$$

# Many-body Nuclear Problem

Nuclear Many-body Hamiltonian

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A, s_1, s_2, \dots, s_A, t_1, t_2, \dots, t_A)$$

$\Psi$  are spin-isospin vectors in  $3A$  dimensions with  $2^A \times \frac{A!}{Z!(A-Z)!}$  components

Develop Computational Methods to solve (numerically) exactly or within approximations that are under control the many-body nuclear problem



<http://exascaleage.org/np/>

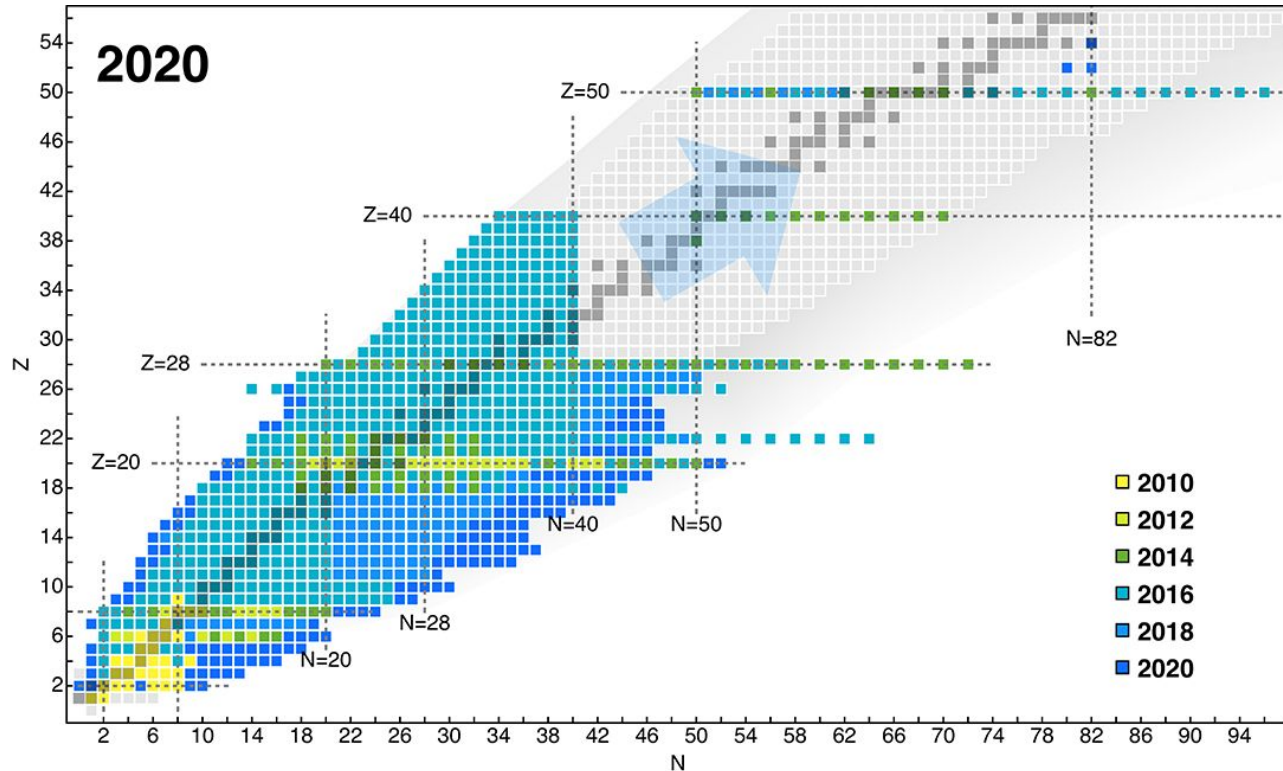
${}^4\text{He}$  : 96

${}^6\text{Li}$  : 1280

${}^8\text{Li}$  : 14336

${}^{12}\text{C}$  : 540572

# Current Status



H. Hergert  
Front. Phys.  
07 October 2020

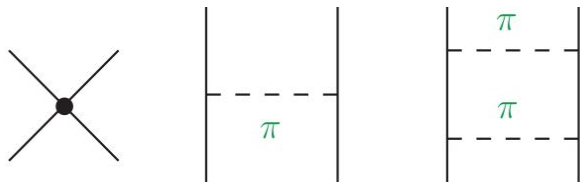


# Many-body Nuclear Interactions

## Many-body Nuclear Hamiltonian

$$H = T + V = \sum_{i=1}^A t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

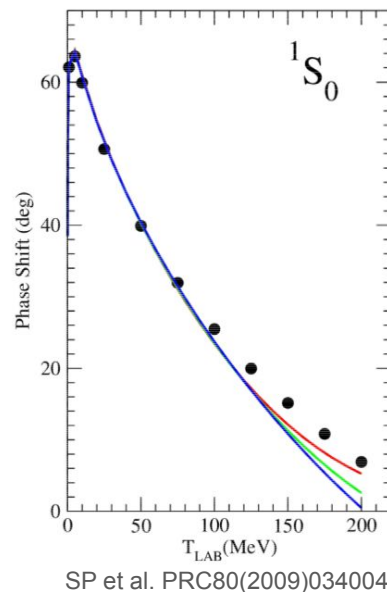
$v_{ij}$  and  $V_{ijk}$  are **two-** and **three-**nucleon operators based on experimental data fitting; fitted parameters subsume underlying QCD dynamics



Contact term: short-range

Two-pion range: intermediate-range  $r \propto (2m_\pi)^{-1}$

One-pion range: long-range  $r \propto m_\pi^{-1}$



Hideki Yukawa

**AV18+UIX**; **AV18+IL7**

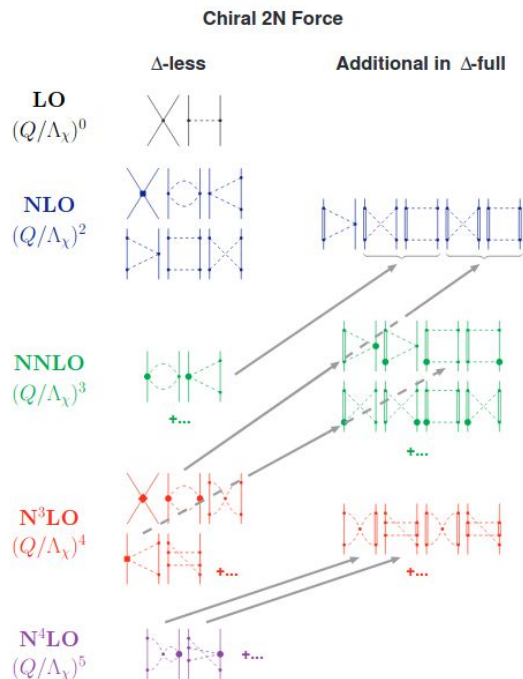
Wiringa, Schiavilla, Pieper  
*et al.*

chiral  $\pi N\Delta$

**N3LO+N2LO** Piarulli *et al.*

**Norfolk Models**

# Norfolk Two- and Three-body Potentials

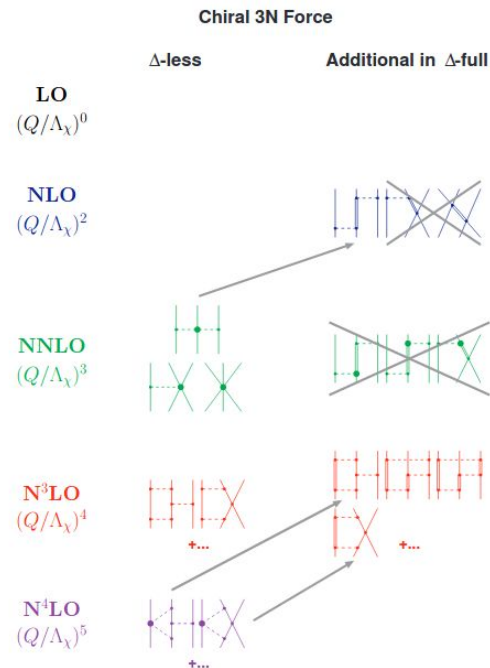


Norfolk Chiral Potentials

NV2+3

developed in Piarulli *et al.*  
PRC91(2015)024003  
PRC94(2016)054007

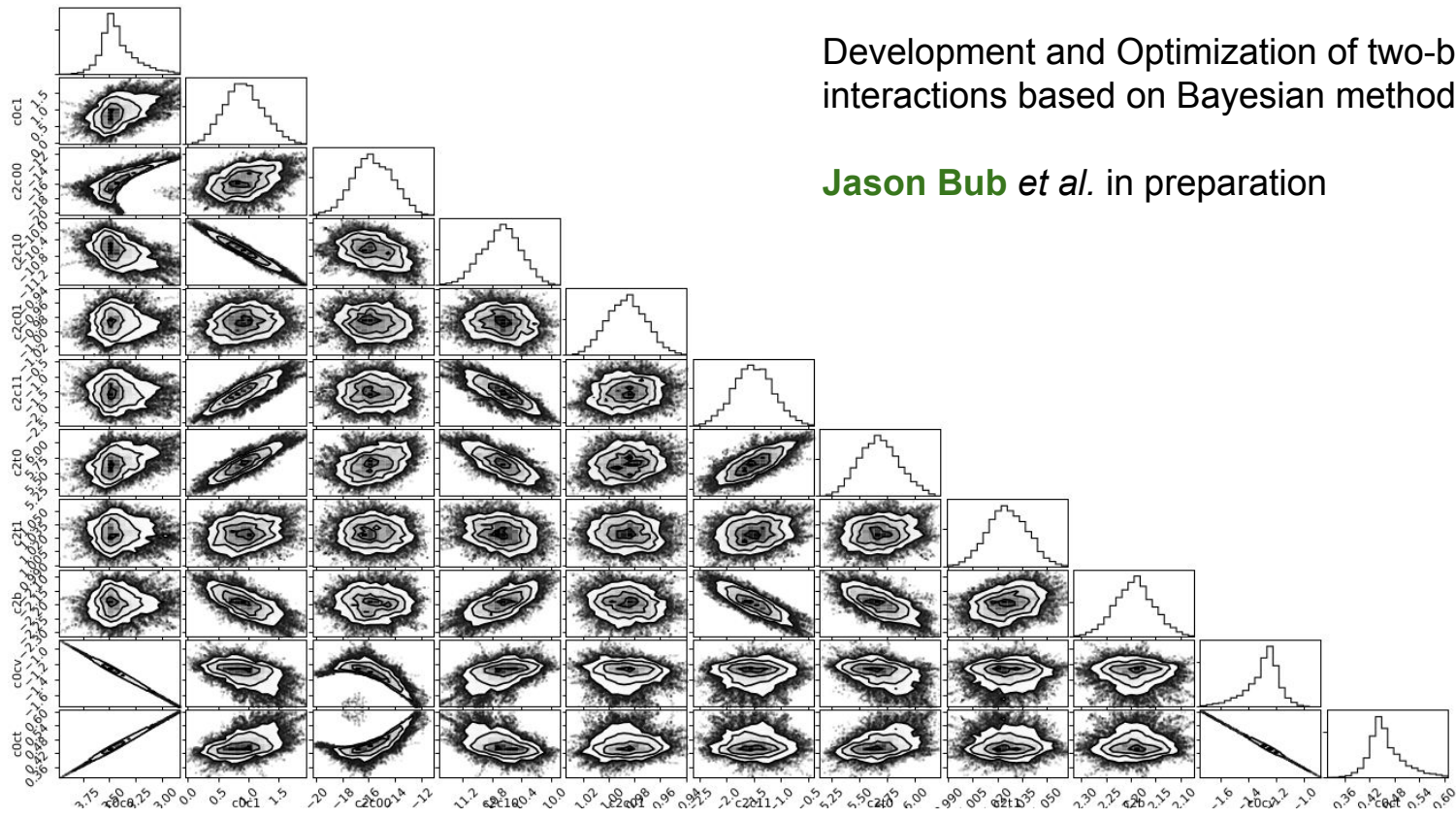
26 LECs fitted to np and pp  
Granada database  
(2700-3700 data points;  
125-200 MeV) with a  
chi-square/datum  $\sim 1$



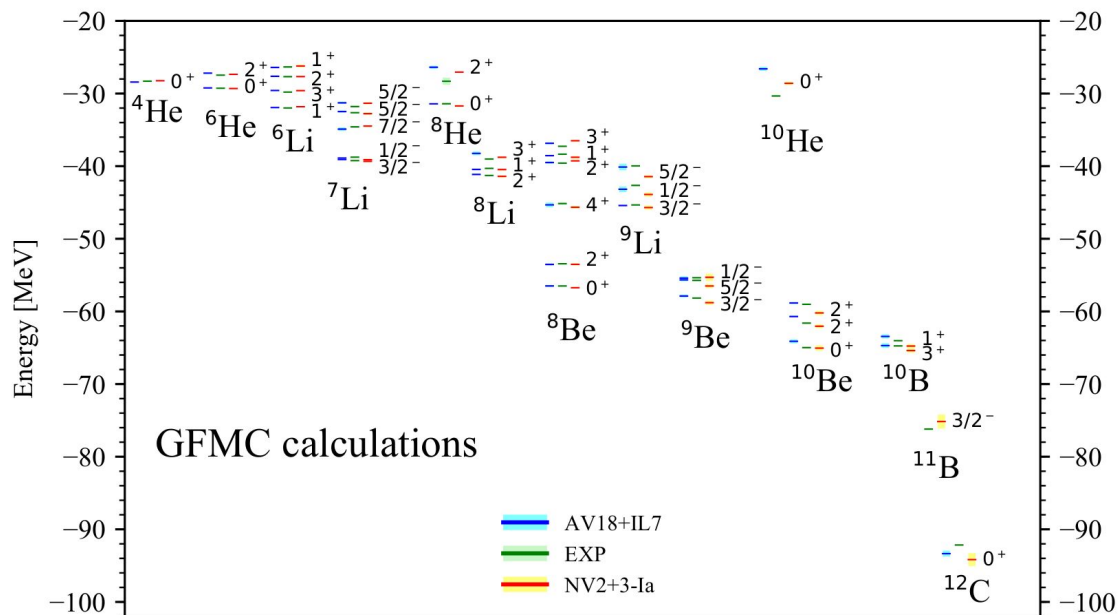
# Optimization of Nuclear Two-body Interactions

Development and Optimization of two-body interactions based on Bayesian methods

Jason Bub *et al.* in preparation

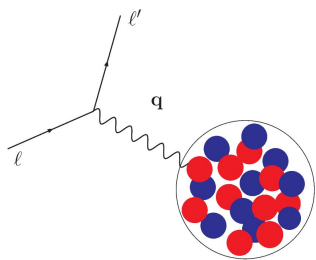


# Energies

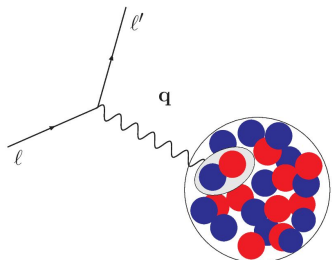


Piarulli *et al.* PRL120(2018)052503

# Many-body Nuclear Electroweak Currents



one-body



two-body

- Two-body currents are a manifestation of two-nucleon correlations
- Electromagnetic two-body currents are required to satisfy current conservation

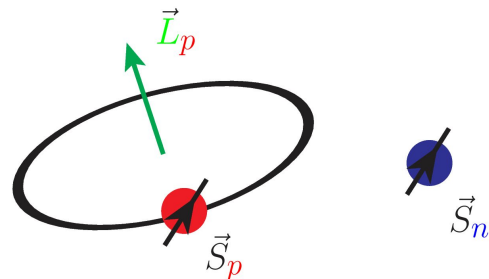
$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

Nuclear Charge Operator

$$\rho = \sum_{i=1}^A \rho_i + \sum_{i<j} \rho_{ij} + \dots$$

Nuclear (Vector) Current Operator

$$\mathbf{j} = \sum_{i=1}^A \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$



Magnetic Moment: Single Particle Picture

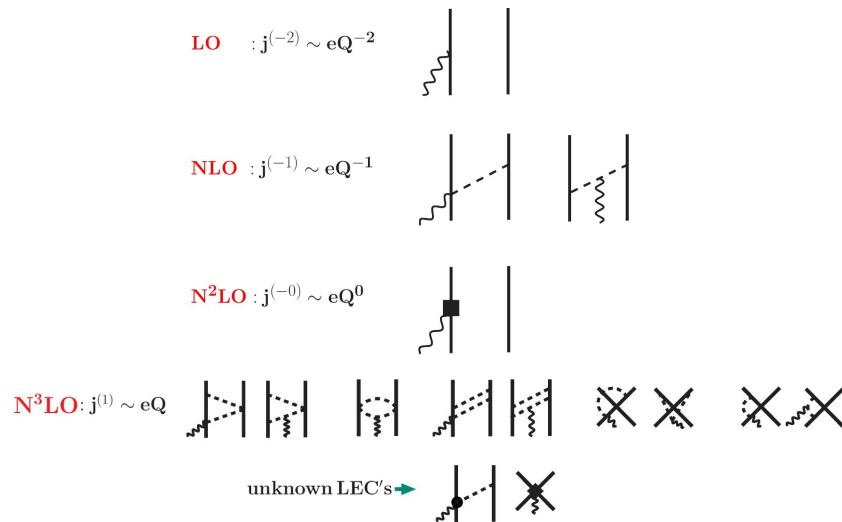
# Many-body Currents

- **Meson Exchange Currents (MEC)**

Constrain the MEC current operators by imposing that the current **conservation relation is satisfied with the given two-body potential**

- **Chiral Effective Field Theory Currents**

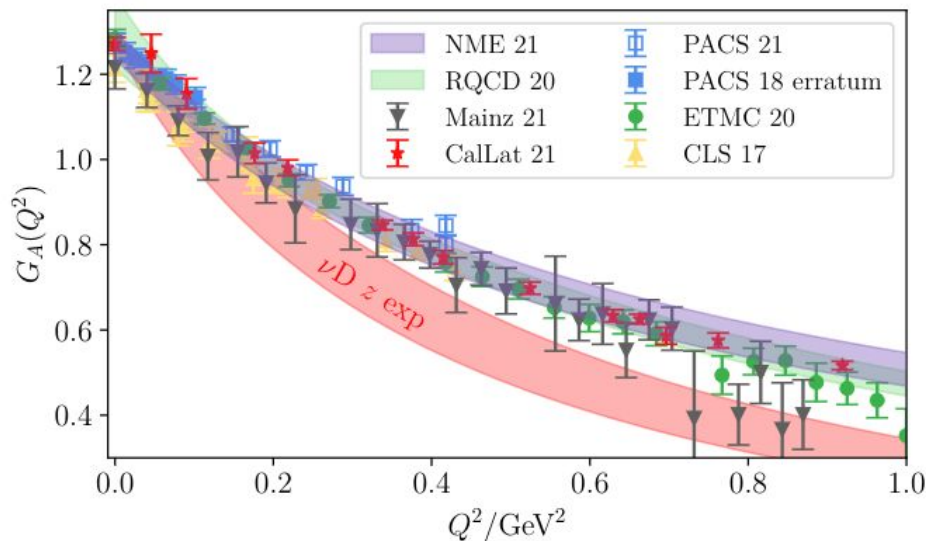
Are constructed consistently with the two-body chiral potential; Unknown parameters, or Low Energy Constants (**LECs**), need to be **determined by either fits to experimental data or by Lattice QCD calculations**



Electromagnetic Current Operator

SP *et al.* PRC78(2008)064002, PRC80(2009)034004,  
 PRC84(2011)024001, PRC87(2013)014006  
 Park *et al.* NPA596(1996)515, Phillips (2005)  
 Kölling *et al.* PRC80(2009)045502 & PRC84(2011)054008

# LCQD inputs for neutrino-nucleus scattering



Building blocks of ab initio nuclear approaches:

Nucleonic form factors

Transition form factors

Pion production amplitudes

Two-nucleon couplings (strong and EW)

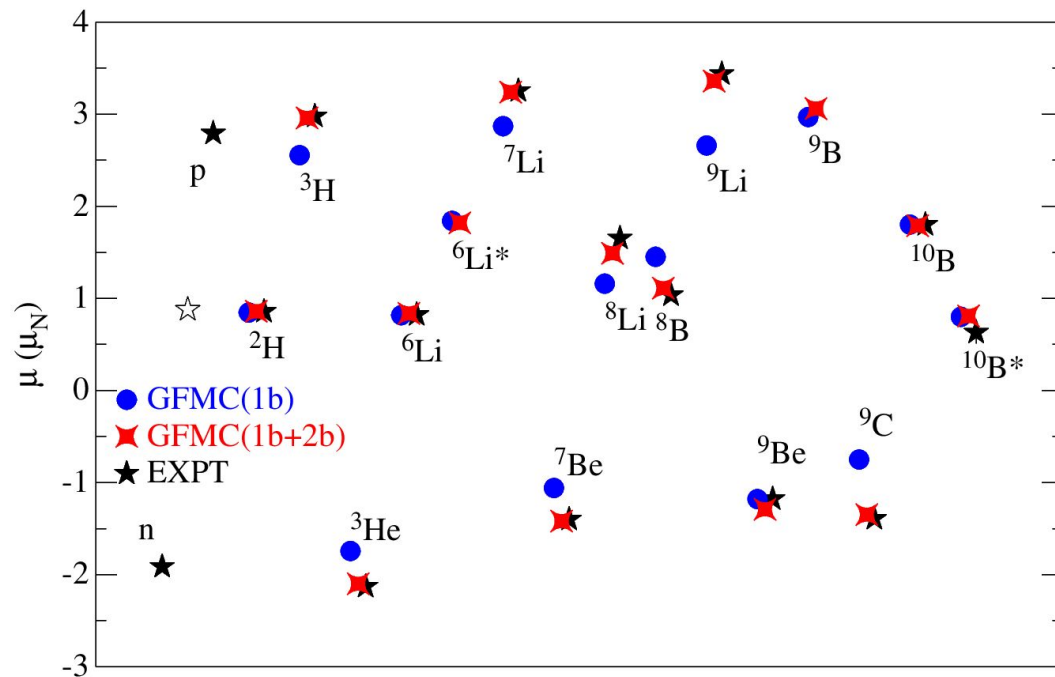
...

Taken from data where available, or from theory

Snowmass WP: Theoretical tools for neutrino scattering: interplay between lattice QCD, EFTs, nuclear physics, phenomenology, and neutrino event generators; [arXiv:2203.09030](https://arxiv.org/abs/2203.09030)

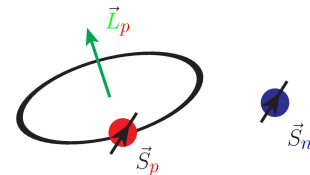
Meyer, Walker-Loud, Wilkinson (2022)

# Magnetic Moments of Light Nuclei



SP *et al.* PRC87(2013)035503

Single particle picture



$$\mu_N(1b) = \sum_i [(L_i + g_p S_i)(1 + \tau_{i,z})/2 + g_n S_i(1 - \tau_{i,z})/2]$$

Small two-body  
current effects

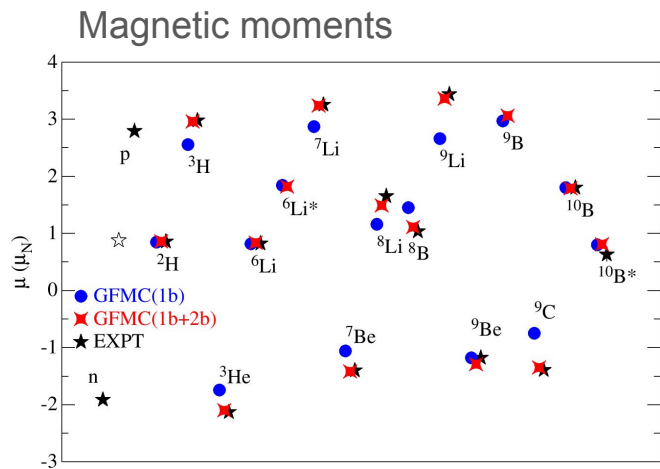


Large two-body  
current effects



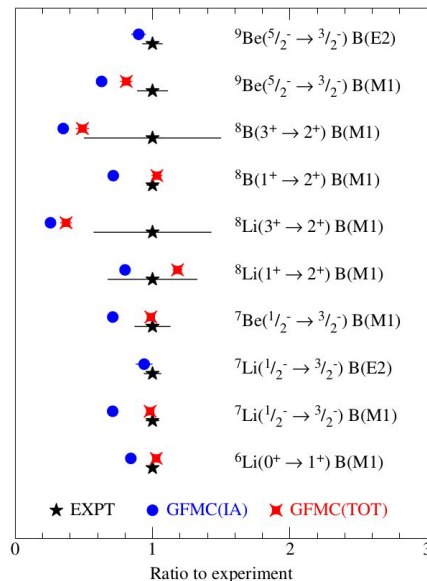


# Electromagnetic Observables

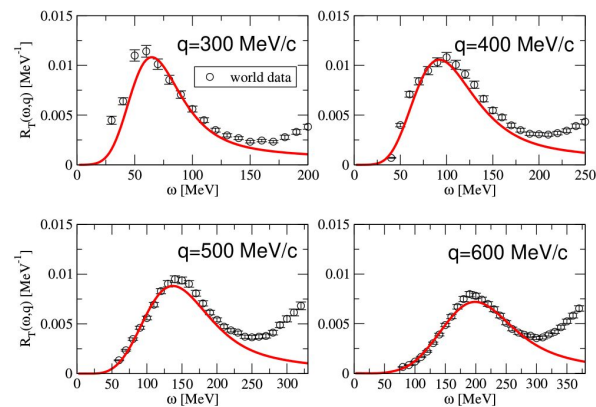


SP *et al.* PRC87(2013)035503,  
 PRC101(2020)044612

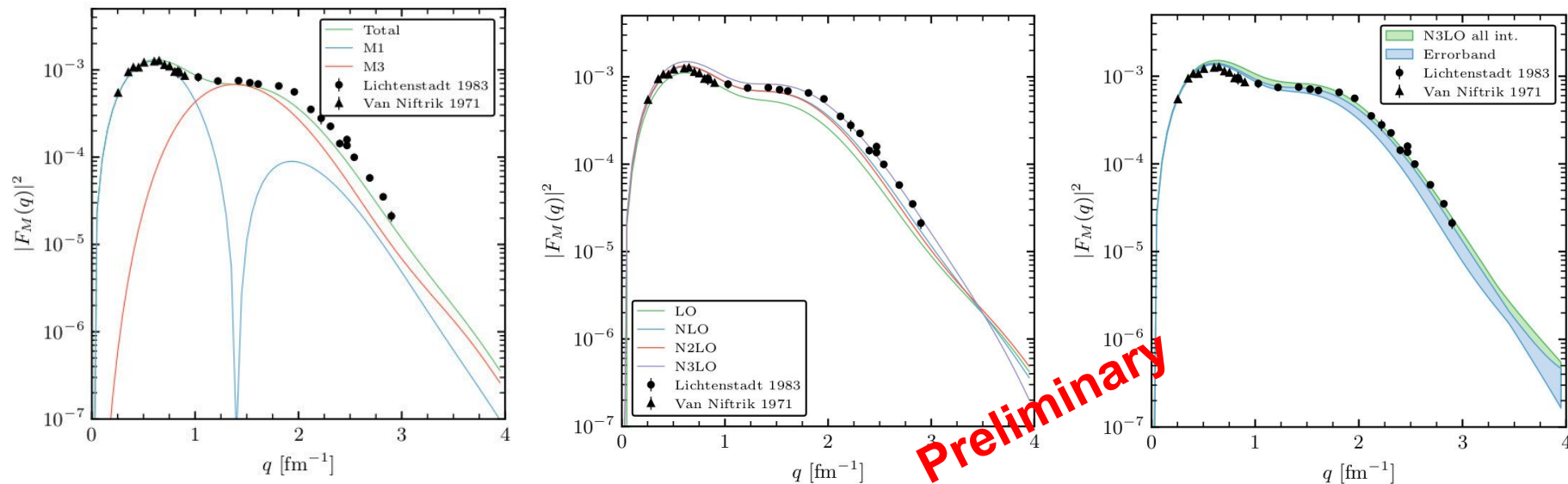
## EM decay



## $e$ - ${}^4\text{He}$ particle scattering

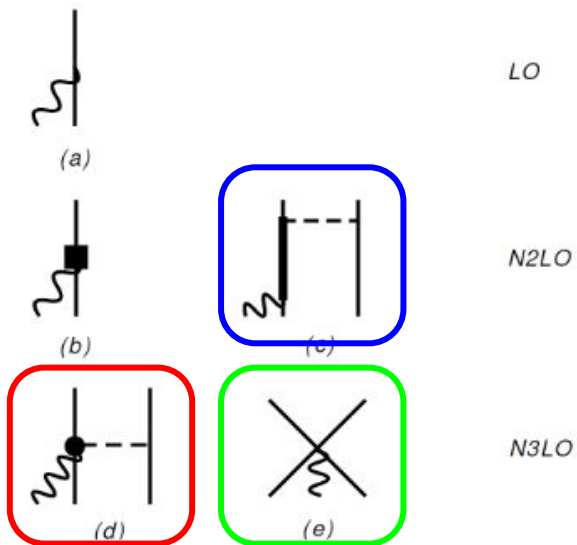


# Electromagnetic form factors from chiEFT



$^7\text{Li}$  magnetic form factor - A. Gnech, G. Chambers-Wall, G. King *et al.* (in preparation)

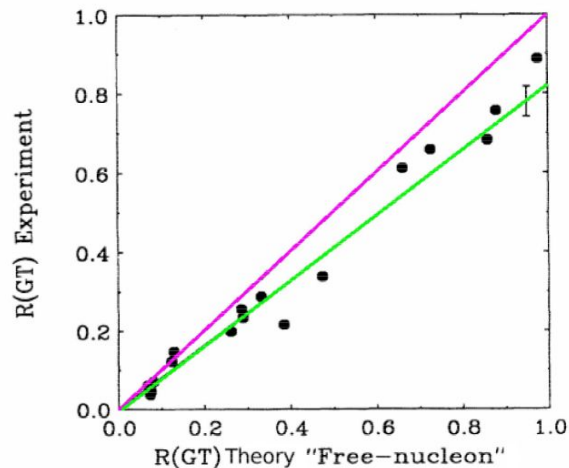
# Axial currents with $\Delta$ at tree-level



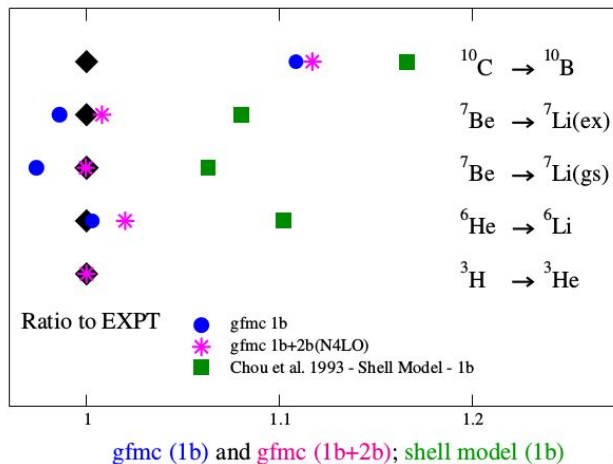
Two body currents of one pion range  
(red and blue) with  $c_3$   $c_4$  from Krebs  
*et al.* Eur.Phys.J.(2007)A32

Contact current involves the LEC  $c_D$

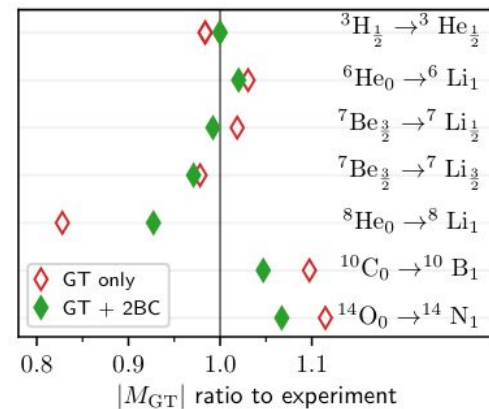
# Beta decay



Chou et al. PRC47(1993)163

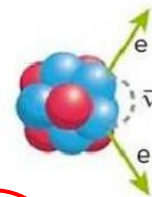


SP et al. PRC97(2018)022501



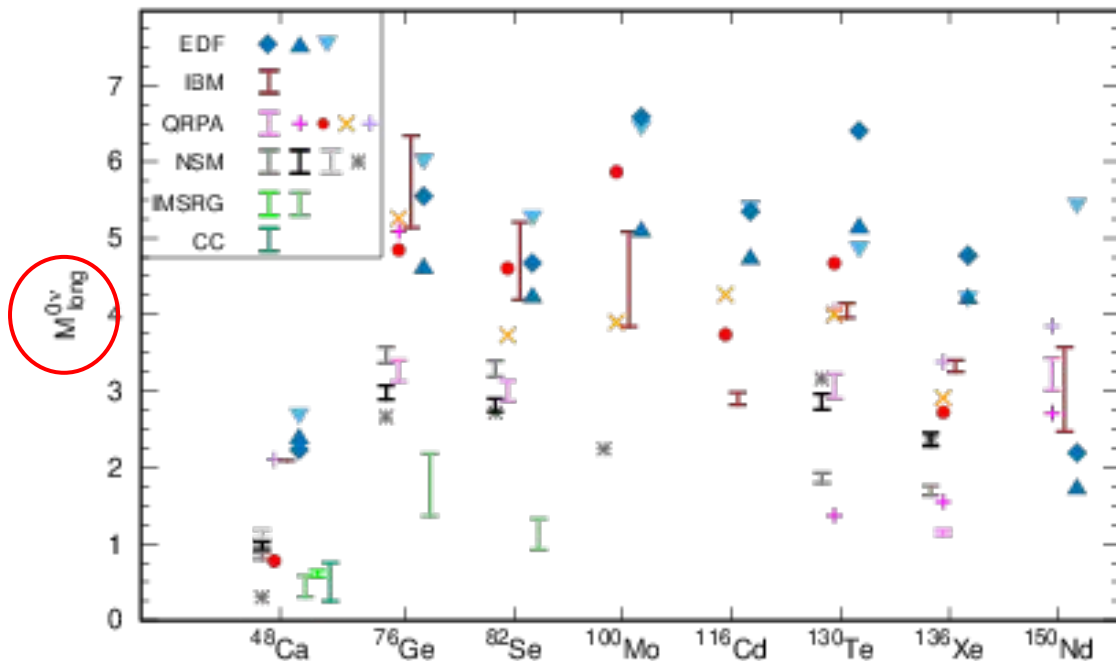
P. Gysbers *Nature Phys.* 15 (2019)

# Neutrinoless Double Beta Decay

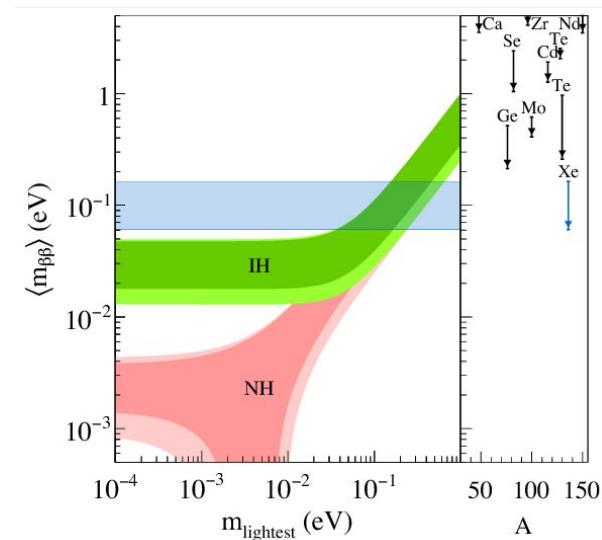


$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q, Z) |M_{0\nu}|^2 m_{\beta\beta}^2$$

$$(Z, N) \rightarrow (Z+2, N-2) + 2e$$



Menéndez et al. *Rev.Mod.Phys.* 95 (2023) 2, 025002



# Partial muon capture rates: VMC calculations

$$\Gamma_{\text{VMC}}(\text{avg.}) = 1495 \text{ s}^{-1} \pm 19 \text{ s}^{-1}$$

$$\Gamma_{\text{expt}} = 1496.0 \text{ s}^{-1} \pm 4.0 \text{ s}^{-1}$$

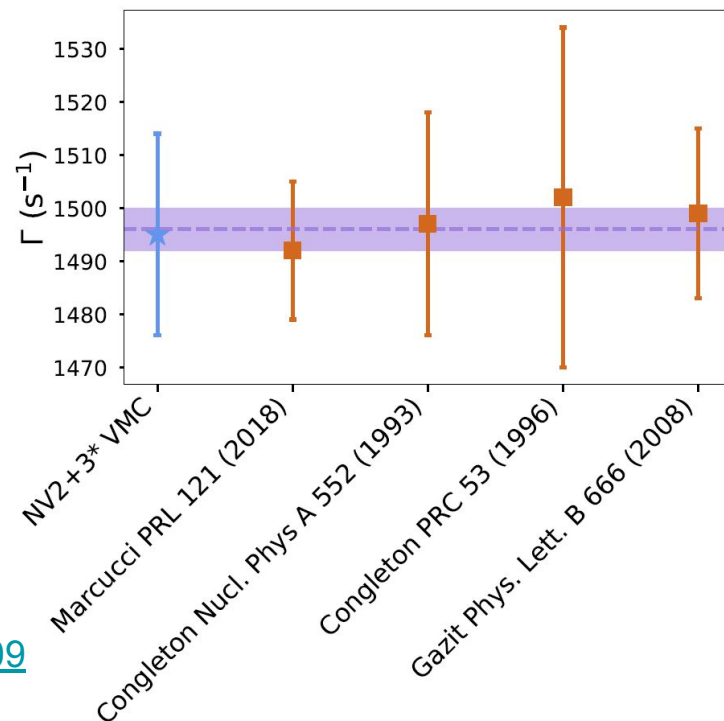
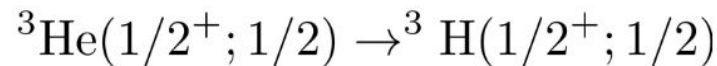
Ackerbauer *et al.* PLB417, 224(1998)

Momentum transfer  **$q \sim 100 \text{ MeV}$**

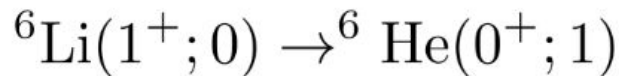
Two-body correction is  $\sim 8\%$  of total rate on average for  $A=3$

**Garrett King** *et al.* PRC2022

Review by Measday [Physics Reports 354 \(2001\) 243–409](#)



# Partial muon capture rates: VMC calculations



$$\Gamma_{\text{VMC}}(\text{avg.}) = 1235 \text{ s}^{-1} \pm 101 \text{ s}^{-1}$$

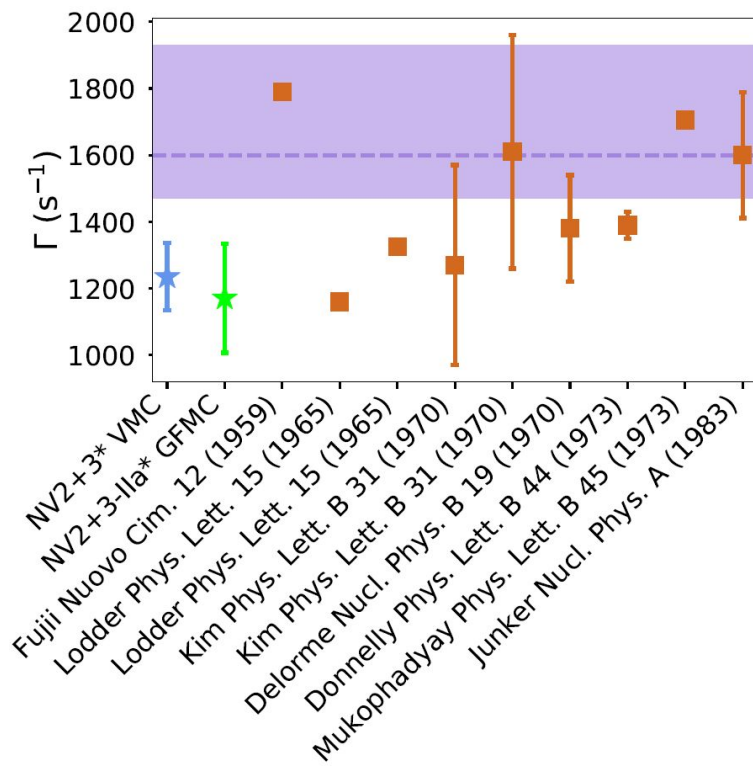
$$\Gamma_{\text{GFMC}}(\text{IIa}^*) = 1171 \text{ s}^{-1} \pm 164 \text{ s}^{-1}$$

$$\Gamma_{\text{expt}} = 1600 \text{ s}^{-1} +330/-129 \text{ s}^{-1}$$

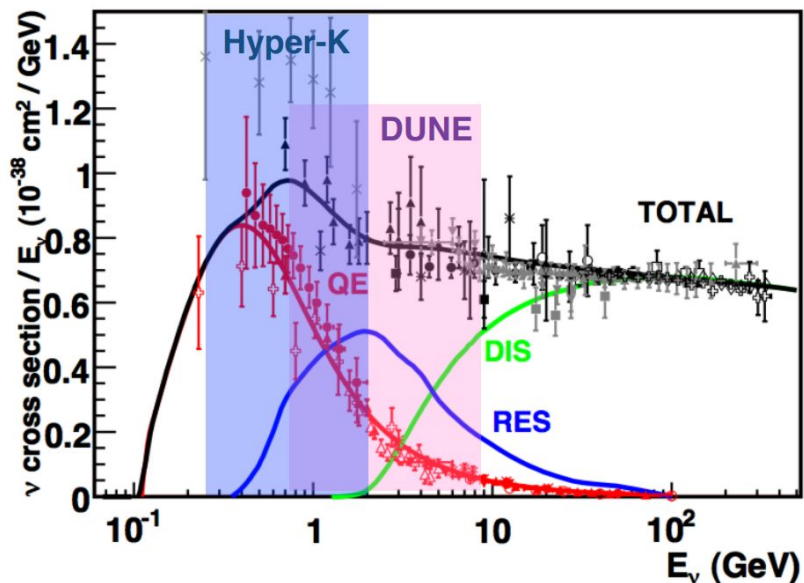
Deutsch *et al.* PLB26(1968)315

**Garrett King** *et al.* PRC2022

With FRIB experimentalist colleagues:  
Gamow-Teller strength in A=11;  
Schmitt *et al.* PRC106(2022)



# Neutrino cross section anatomy



Formaggio & Zeller

**Quasi-elastic:** dominated by single-nucleon knockout

**Resonance:** excitation to nucleonic resonant states which decay into mesons

**Deep-inelastic scattering:** where the neutrino resolves the nucleonic quark content

Each of these regimes requires knowledge of both the **nuclear ground state** and the **electroweak coupling and propagation of the struck nucleons, hadrons, or partons**

A challenge for achieving precise neutrino-nucleus cross-section is **reliably bridging the transition regions which use different degrees of freedom**



# Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

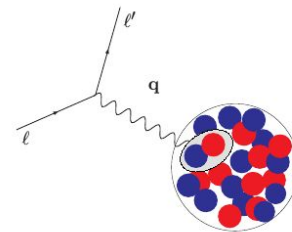
$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by the charge operator  $O_L = \rho$

Transverse response induced by the current operator  $O_T = \mathbf{j}$

5 Responses in neutrino-nucleus scattering

$$\frac{d^2 \sigma}{d\omega d\Omega} = \sigma_M [v_L R_L(\mathbf{q}, \omega) + v_T R_T(\mathbf{q}, \omega)]$$

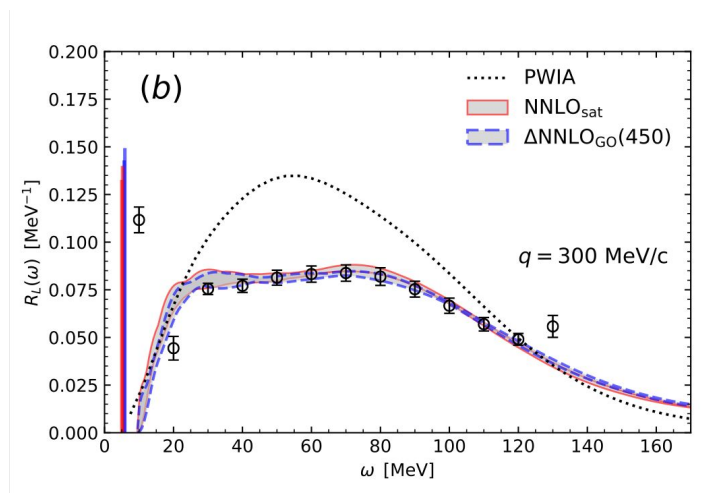


For a recent review on QMC, SF methods see

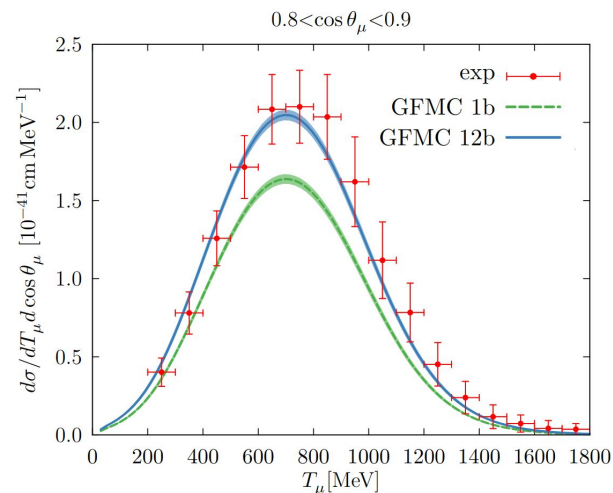
[Rocco Front. In Phys.8 \(2020\)116](#)

# Inclusive Cross Sections with Integral Transforms

Exploit integral properties of the response functions and closure to avoid explicit calculation of the final states (Lorentz Integral Transform **LIT**, **Euclidean**, ...)



Sobczyk et al, PRL127 (2021)



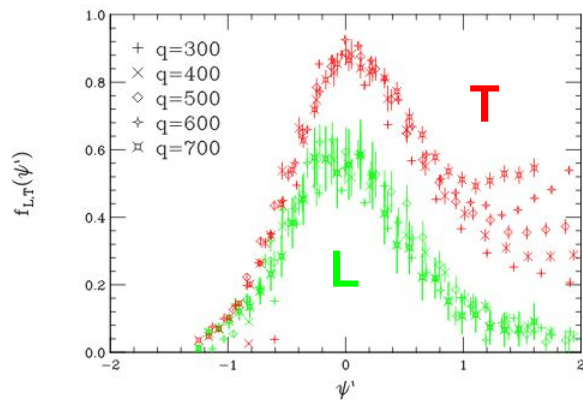
Lovato et al. PRX10 (2020)

# Lepton-Nucleus scattering: Data

## Transverse Sum Rule

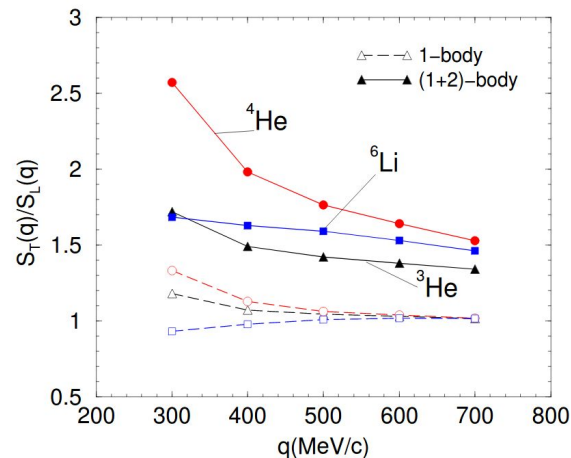
$$S_T(q) \propto \langle 0 | \mathbf{j}^\dagger \mathbf{j} | 0 \rangle \propto \langle 0 | \mathbf{j}_{1b}^\dagger \mathbf{j}_{1b} | 0 \rangle + \langle 0 | \mathbf{j}_{1b}^\dagger \mathbf{j}_{2b} | 0 \rangle + \dots$$

Observed transverse enhancement explained by the combined effect of two-body correlations and currents in the interference term



$^4\text{He}$  Electromagnetic Data  
Carlson *et al.* PRC65(2002)024002

$$\begin{aligned} & \left| \begin{array}{c} \text{wavy line} \\ \text{wavy line} \end{array} \right| \quad \langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{1b} \rangle > 0 \\ & \text{Leading one-body term} \\ & \left| \begin{array}{c} \text{wavy line} \\ \text{wavy line} \end{array} \right| \quad \langle \mathbf{j}_{1b}^\dagger \mathbf{j}_{2b} v_\pi \rangle \propto \langle v_\pi^2 \rangle > 0 \\ & \text{Interference term} \end{aligned}$$

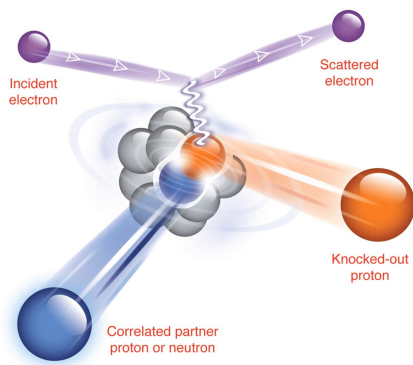


Transverse/Longitudinal Sum Rule  
Carlson *et al.* PRC65(2002)024002

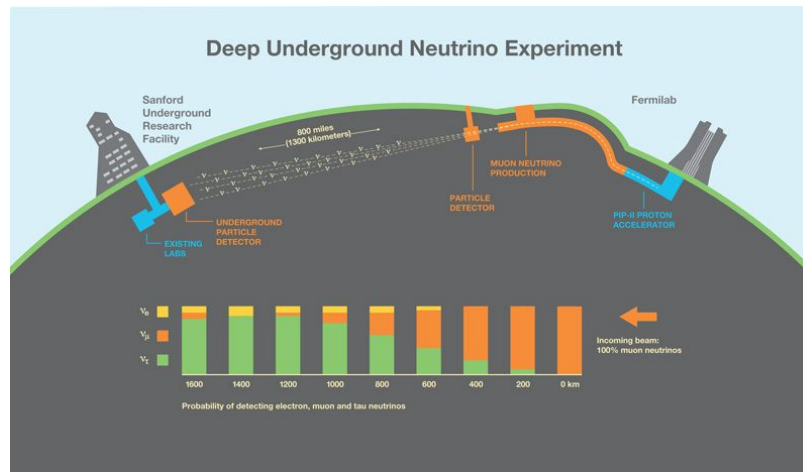
# Beyond Inclusive: Short-Time-Approximation

## Short-Time-Approximation Goals:

- Describe electroweak scattering from  $A > 12$  without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects



Subedi et al. Science320(2008)1475



[Stanford Lab article](#)

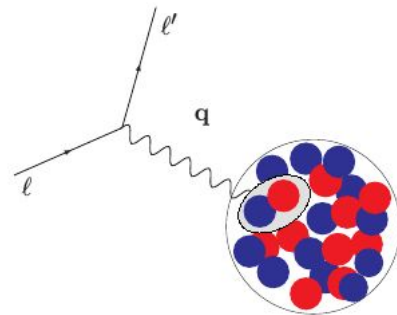
[e4u collaboration](#)



# Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization (for SF see Noemi's talk)
- Retains two-body physics
- Correctly accounts for **interference**



$$R(q, \omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega + E_0)t} \langle 0 | O^\dagger e^{-iHt} O | 0 \rangle$$

$$O_i^\dagger e^{-iHt} O_i + O_i^\dagger e^{-iHt} O_j + \boxed{O_i^\dagger e^{-iHt} O_{ij}} + O_{ij}^\dagger e^{-iHt} O_{ij}$$

$$H \sim \sum_i t_i + \sum_{i < j} v_{ij}$$

# STA: regime of validity

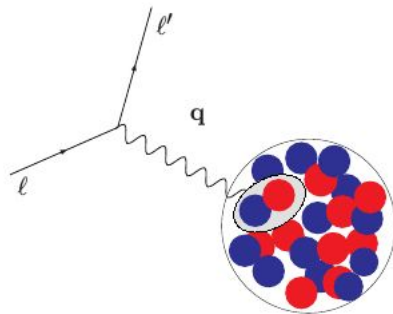
The typical (conservative estimate) energy (time) scale in a nucleus with  $A$  correlated nucleons in pairs is

$$\varepsilon_{\text{pair}} \sim 20 \text{ MeV} \quad (t \sim 1/\varepsilon_{\text{pair}})$$

This sets a natural expansion parameter in the QE region characterized by  $\omega_{\text{QE}}$

$$\varepsilon_{\text{pair}} / \omega_{\text{QE}}$$

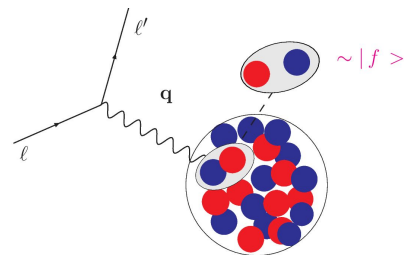
The STA neglects terms of order  $\mathcal{O}(\varepsilon_{\text{pair}} / \omega_{\text{QE}})^2$



# Short-Time-Approximation

Short-Time-Approximation:

- Based on Factorization
- **Retains two-body physics**
- Response functions are given by the **scattering from pairs of fully interacting nucleons** that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides “more” exclusive information in terms of nucleon-pair kinematics via the Response Densities



Response Functions  $\propto$  Cross Sections

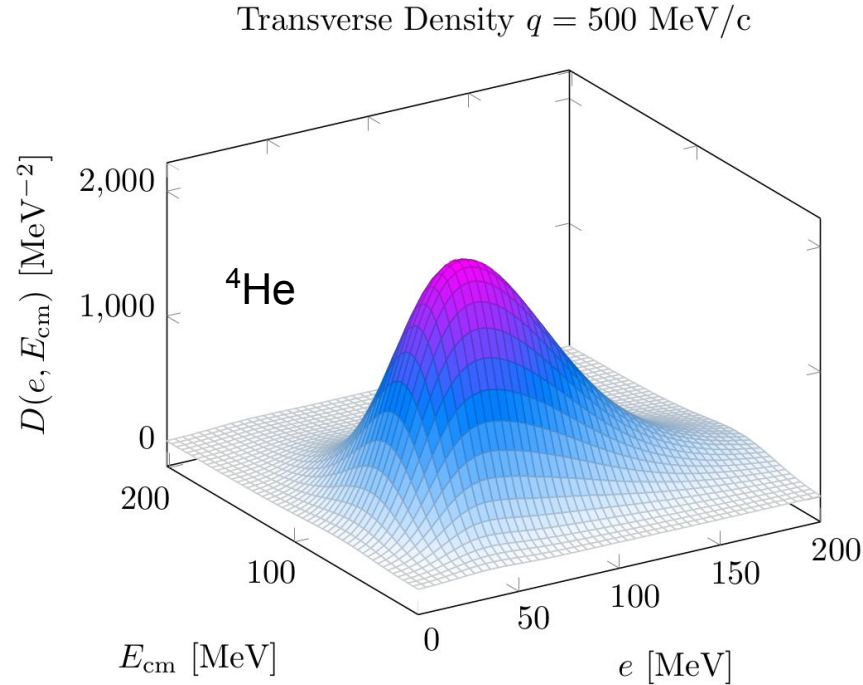
$$R_{\alpha}(q, \omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_{\alpha}(\mathbf{q}) | 0 \rangle|^2$$

Response **Densities**

$$R(q, \omega) \sim \int \delta(\omega + E_0 - E_f) dP' dp' \mathcal{D}(p', P'; q)$$

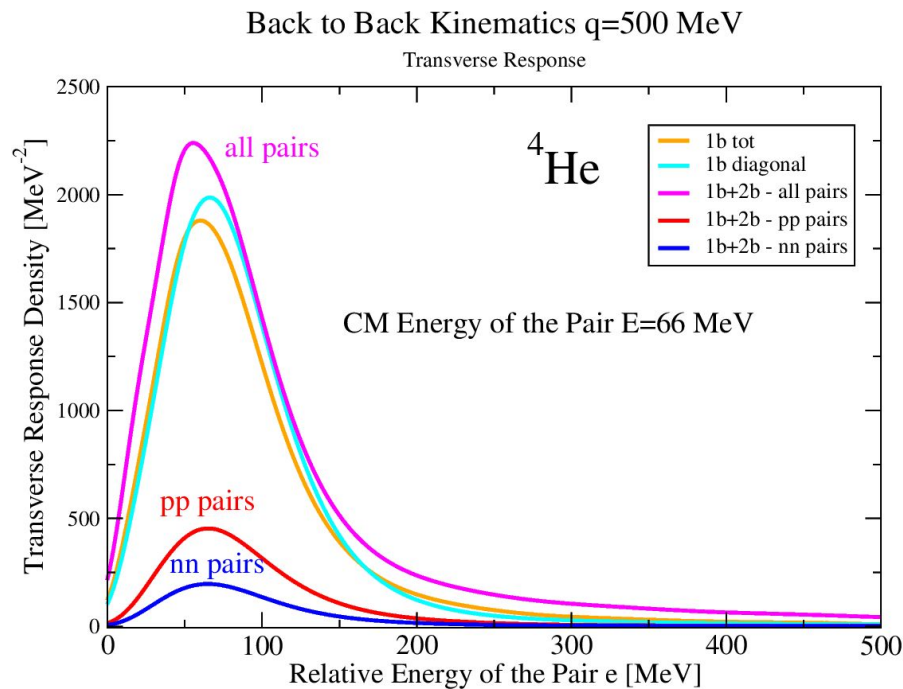
$P'$  and  $p'$  are the CM and relative momenta of the struck nucleon pair

# Transverse Response Density: $e$ - $^4\text{He}$ scattering



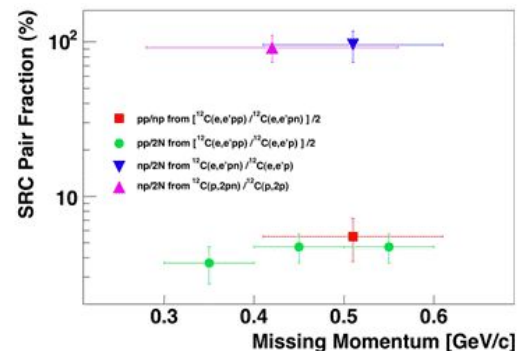


# $e^{-4}\text{He}$ scattering in the back-to-back kinematic



SP *et al.* PRC101(2020)044612

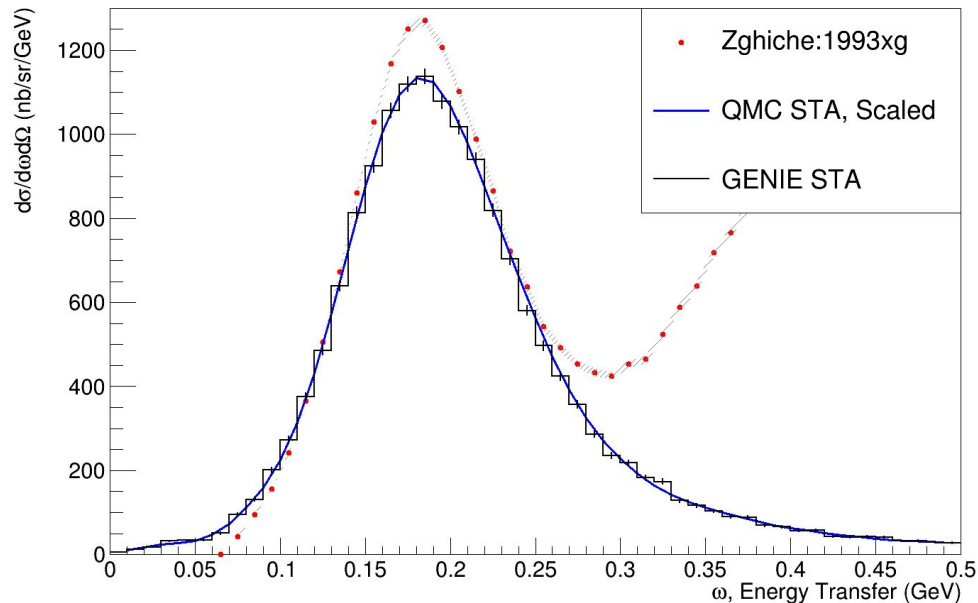
- pp pairs
- nn pairs
- all pairs 1body
- all pairs tot



Subedi *et al.* Science320(2008)1475

# GENIE validation using e-scattering

$Z = 2$ ,  $A = 4$ , Beam Energy = 0.64 GeV, Angle =  $60^\circ \pm 0.25^\circ$

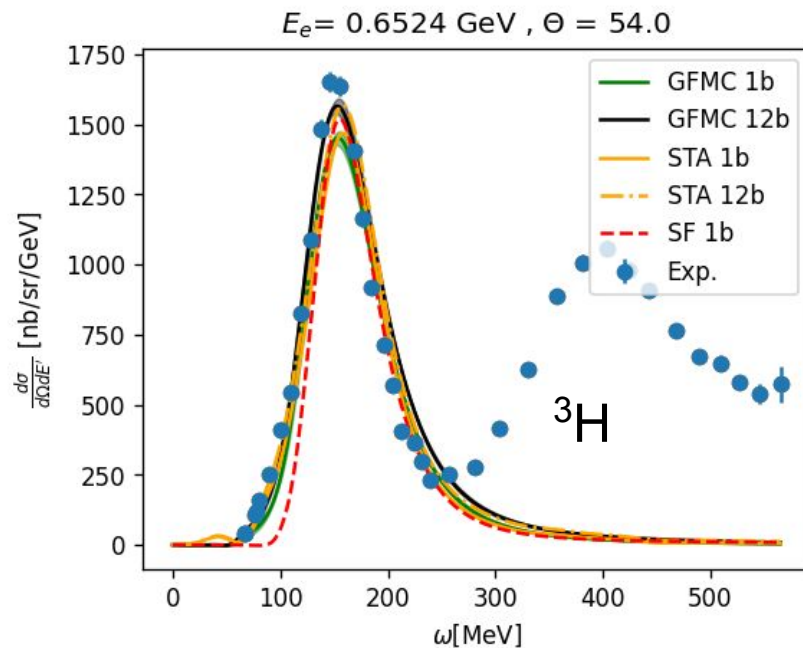
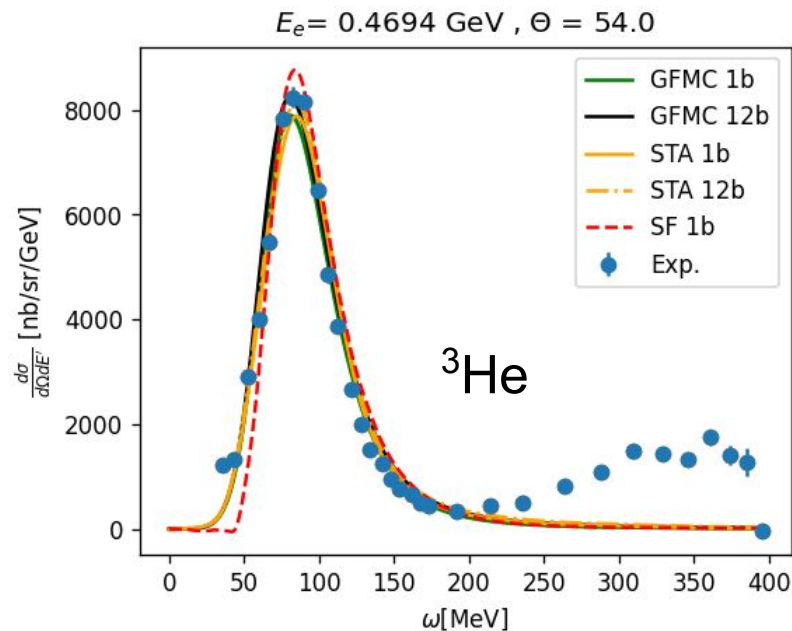


- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE (a Monte Carlo neutrino event generator)
- Here, we use electromagnetic processes (for which data are available) to validate the generator

$$\frac{d^2 \sigma}{d\omega d\Omega} = \sigma_M [v_L R_L(\mathbf{q}, \omega) + v_T R_T(\mathbf{q}, \omega)]$$

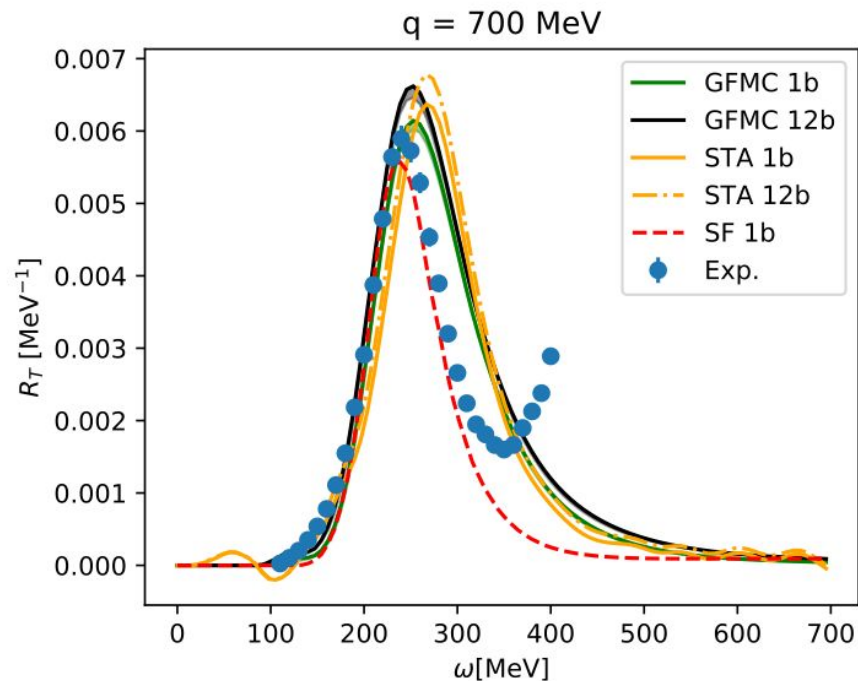
Barrow, Gardiner, SP et al. PRD 103 (2021) 5, 052001

# GFMC SF STA: Benchmark & error estimate



Lorenzo Andreoli, et al. PRC 2021

# Importance of relativistic corrections



Lorenzo Andreoli, *et al.* PRC 2021



# Relativistic corrections

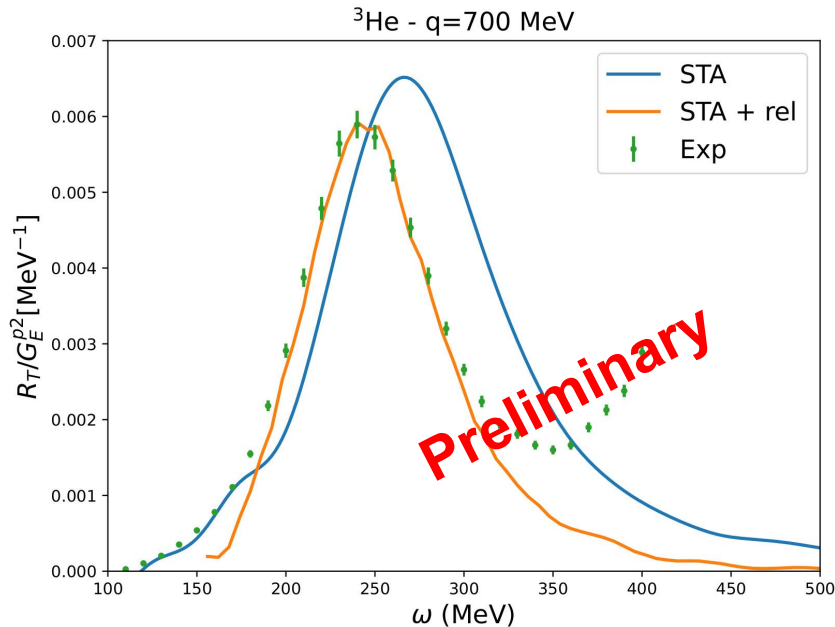
So far we used non relativistic reduction of the single nucleon covarian current (low-momentum expansion both  $p$  and  $p'$ )

With Ronen Weiss

Relativistic corrections obtained expanding the covariant one-nucleon current for high values of momentum transfer  $\mathbf{q}$

$$j^\mu = e\bar{u}(\mathbf{p}'s') \left( e_N \gamma^\mu + \frac{i\kappa_N}{2m_N} \sigma^{\mu\nu} q_\nu \right) u(\mathbf{p}s)$$

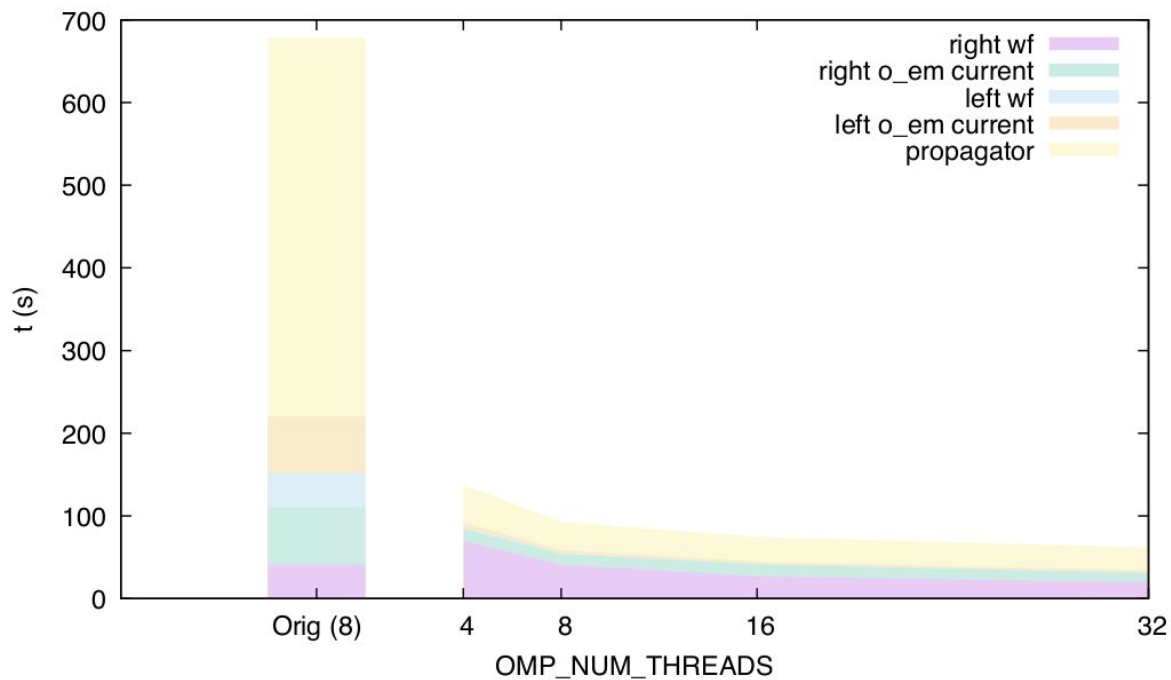
$$\mathbf{p}' = \mathbf{p} + \mathbf{q}$$



R. Weiss & L. Andreoli *et al.* (in preparation)

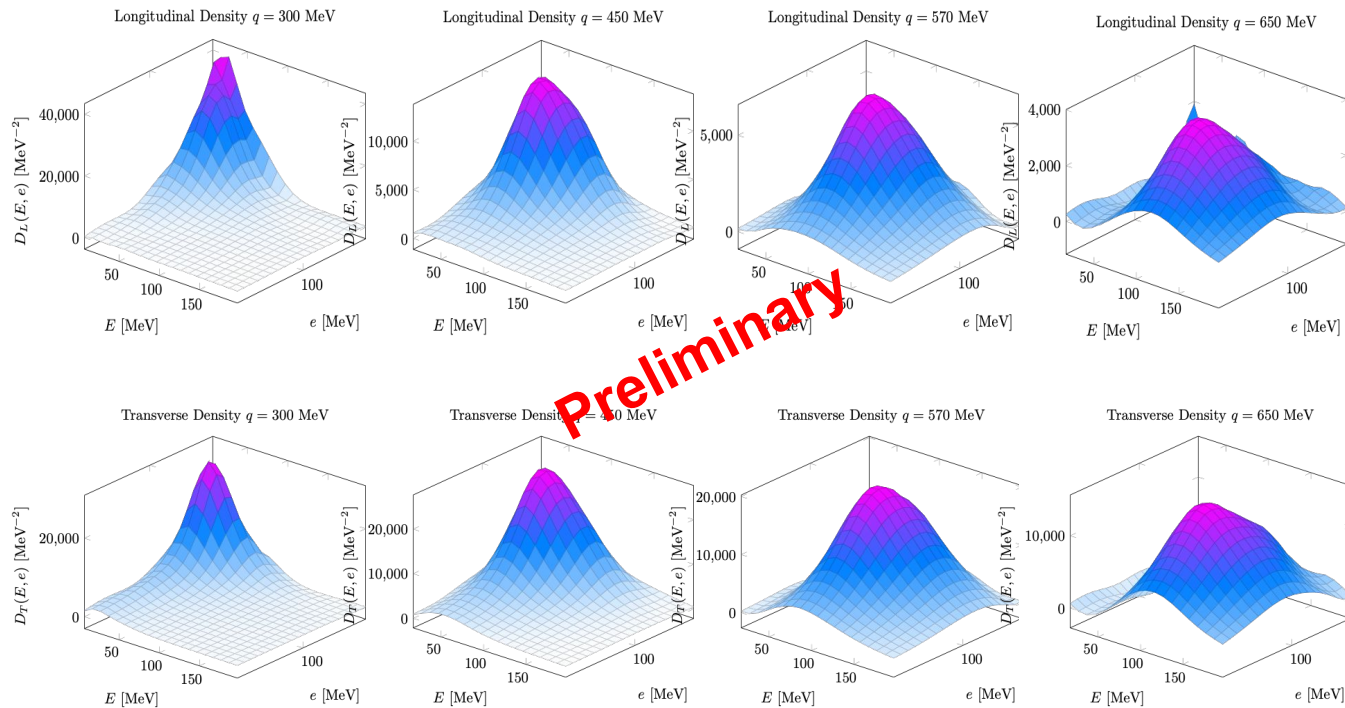
# Towards A=12

**Lorenzo Andreoli**  
*et al.* in preparation



$$R_{\alpha}(q, \omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega + E_i)t} \langle \Psi_i | O_{\alpha}^{\dagger}(\mathbf{q}) e^{-iHt} O_{\alpha}(\mathbf{q}) | \Psi_i \rangle$$

# $^{12}\text{C}$ Response Densities



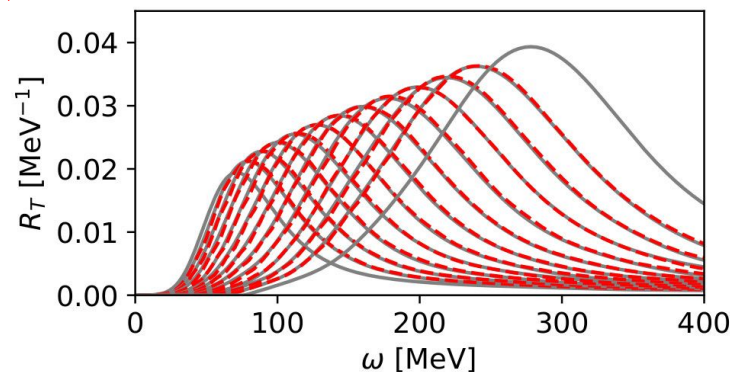
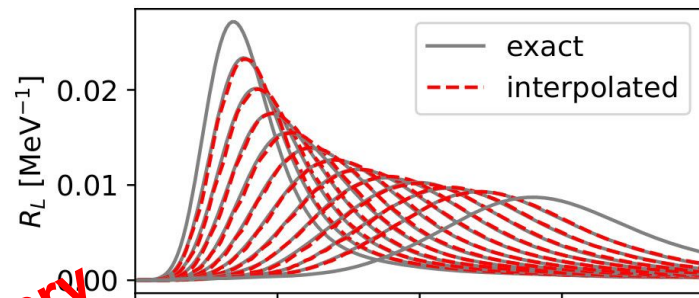
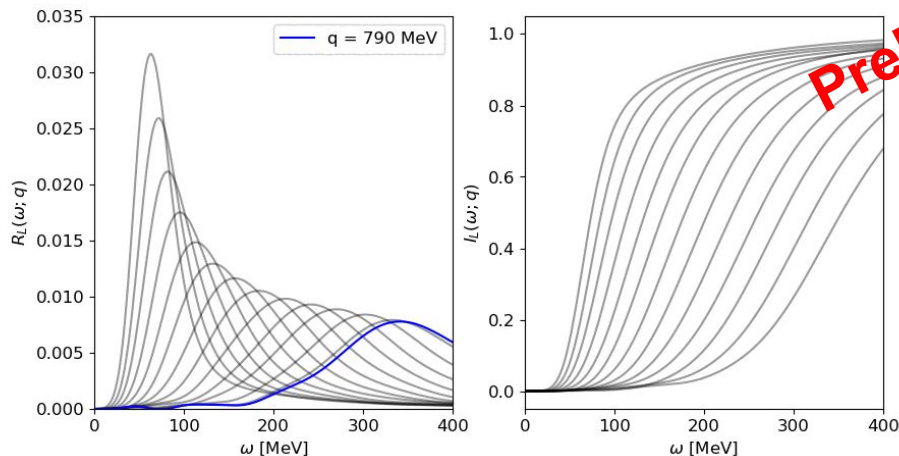
Lorenzo Andreoli *et al.* in preparation

# $^{12}\text{C}$ cross sections: interpolation scheme

We have coarse grid in  $q$  for  $^{12}\text{C}$ . We use an interpolation scheme tested on He4.

$$I_{L/T}(\omega; \mathbf{q}) = \frac{\int_0^\omega R_{L/T}(\omega'; \mathbf{q}) d\omega'}{\int_0^\infty R_{L/T}(\omega'; \mathbf{q}) d\omega'}$$

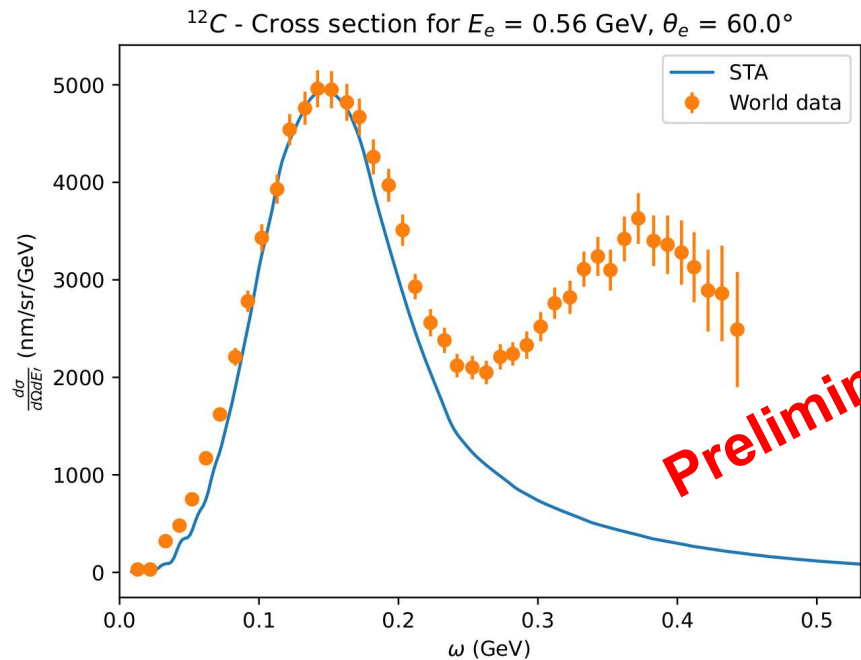
4He, longitudinal response



Lorenzo Andreoli *et al.* in preparation



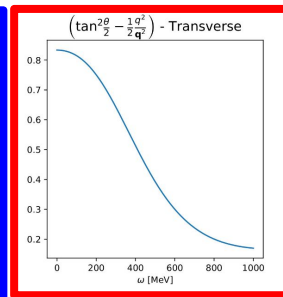
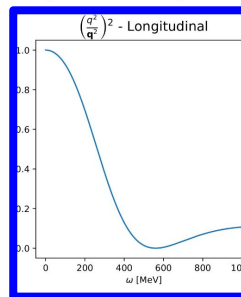
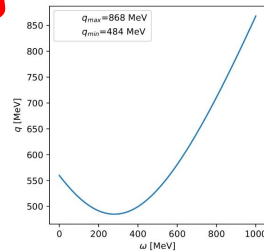
# $^{12}\text{C}$ cross sections



Preliminary

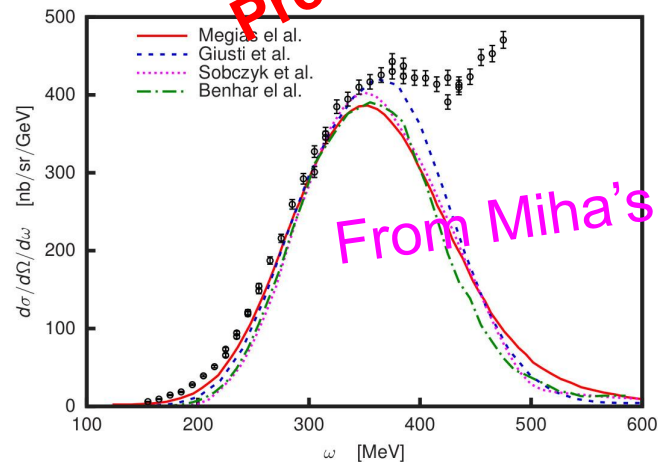
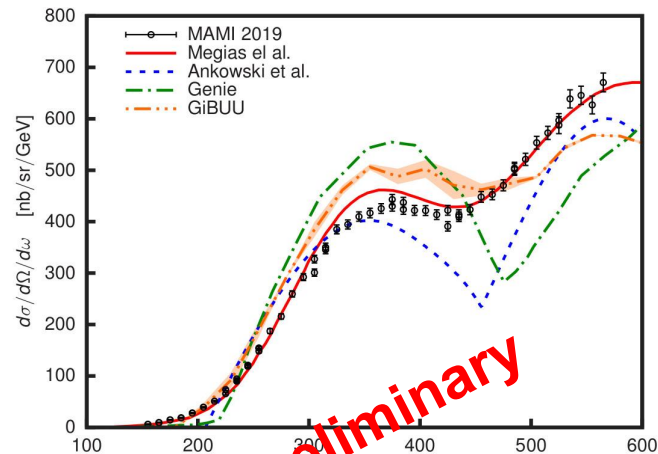
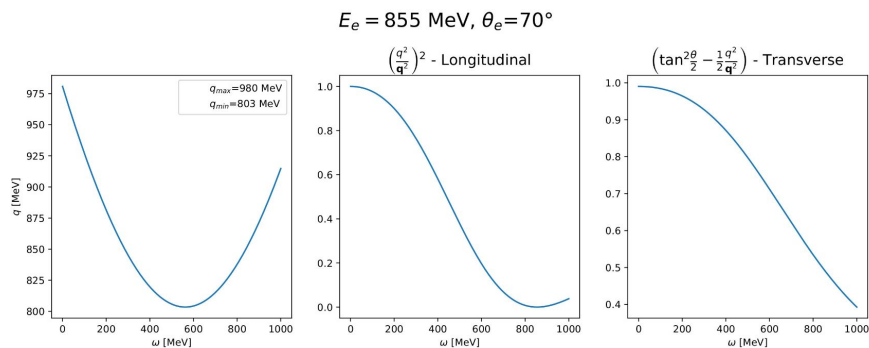
$$\left( \frac{d^2\sigma}{dE' d\Omega'} \right)_e = \left( \frac{d\sigma}{d\Omega'} \right)_M \left[ \left( \frac{q^2}{\mathbf{q}^2} \right)^2 H_L(|\mathbf{q}|, \omega) + \left( \tan^2 \frac{\theta}{2} - \frac{1}{2} \frac{q^2}{\mathbf{q}^2} \right) H_T(|\mathbf{q}|, \omega) \right]$$

$E_e = 560 \text{ MeV}$ ,  $\theta_e = 60^\circ$

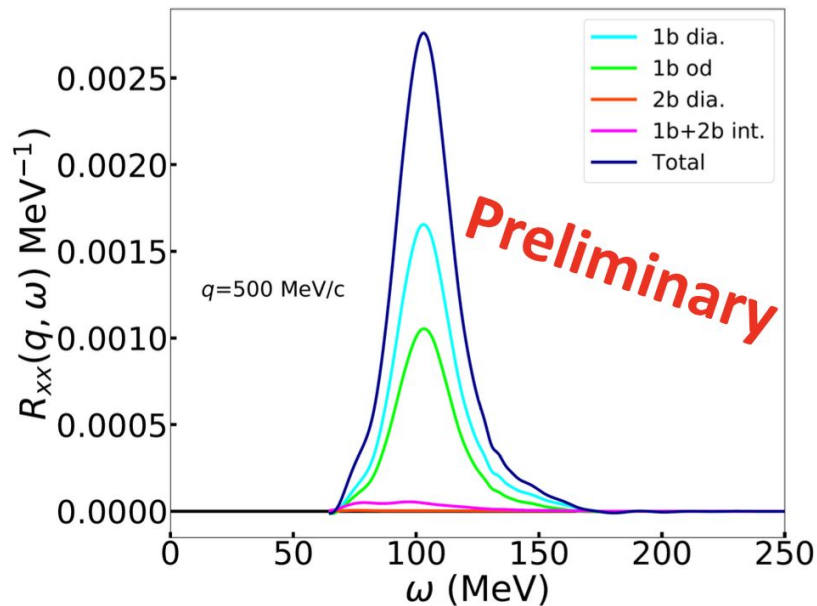


# (Available) Mainz kinematics

$$E_e = 855 \text{ MeV} \quad \theta_e = 70^\circ$$



# NC processes on deuteron with STA

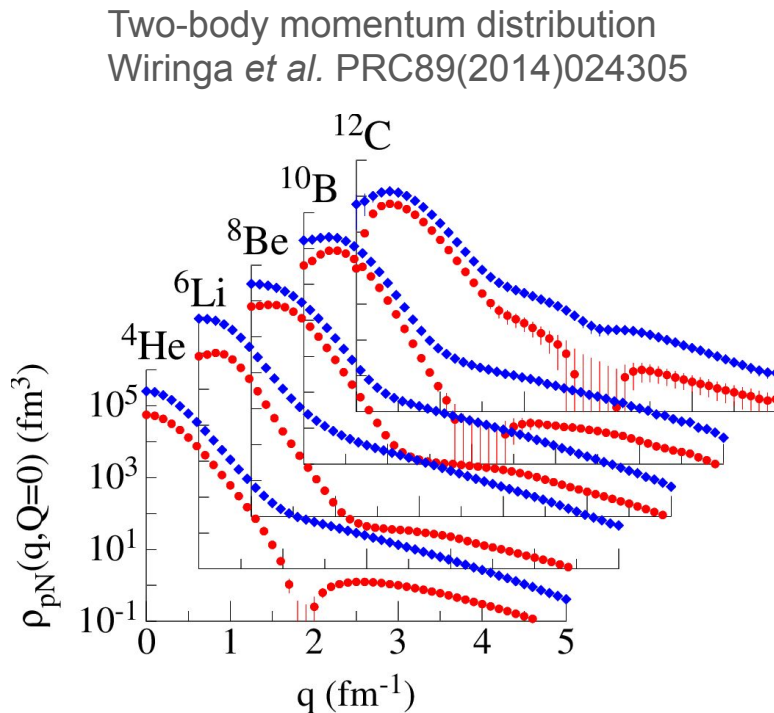


Garrett King *et al.* in preparation

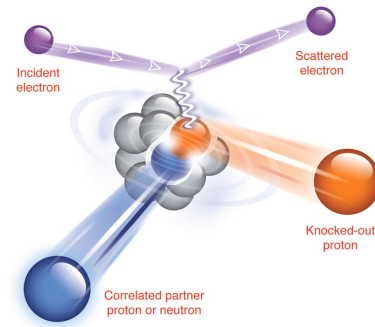
# Momentum distributions



Nuclear properties are strongly affected by two-body correlations and currents in a wide range of energy and momentum transfer



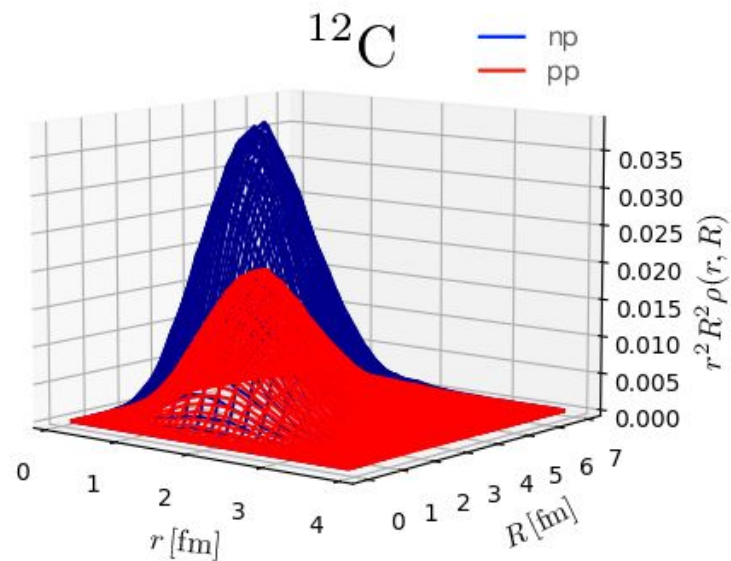
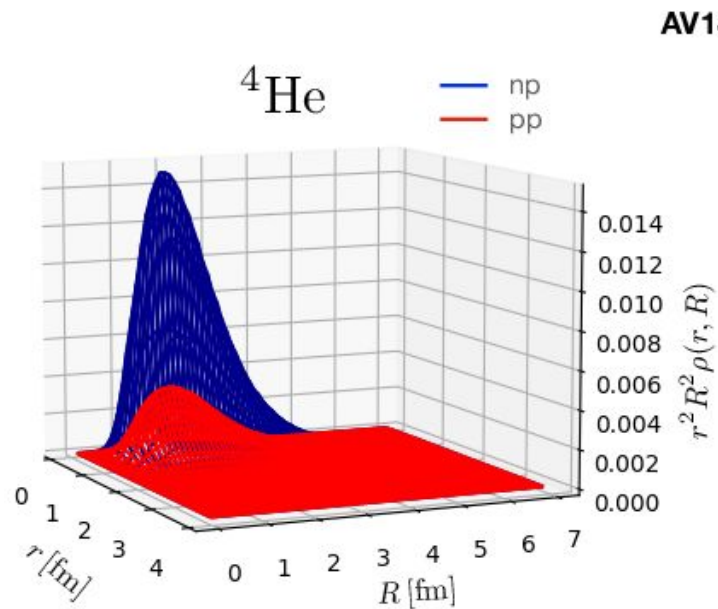
pp-pairs; np-pairs



New two-body momentum distributions from chiEFT potentials

Piarulli, SP, Wiringa PRC (2023)

# Two-body densities

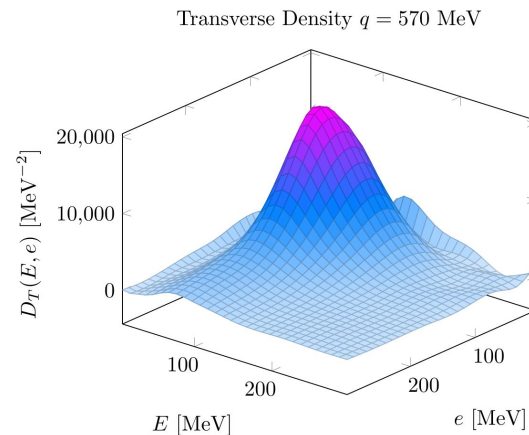
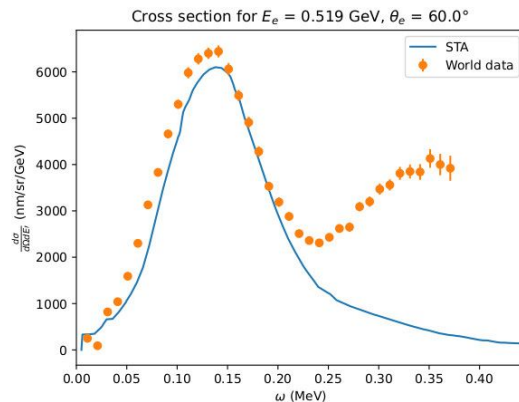
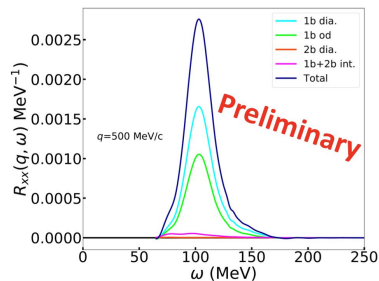
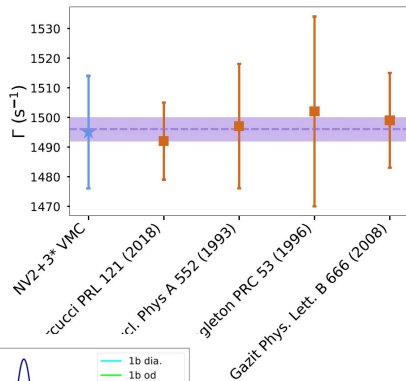


Piarulli, SP, Wiringa PRC (2023)

[data](#)

# Summary

Ab initio calculations of light nuclei yield a picture of nuclear structure and dynamics where **many-body effects play an essential role to explain available data.**



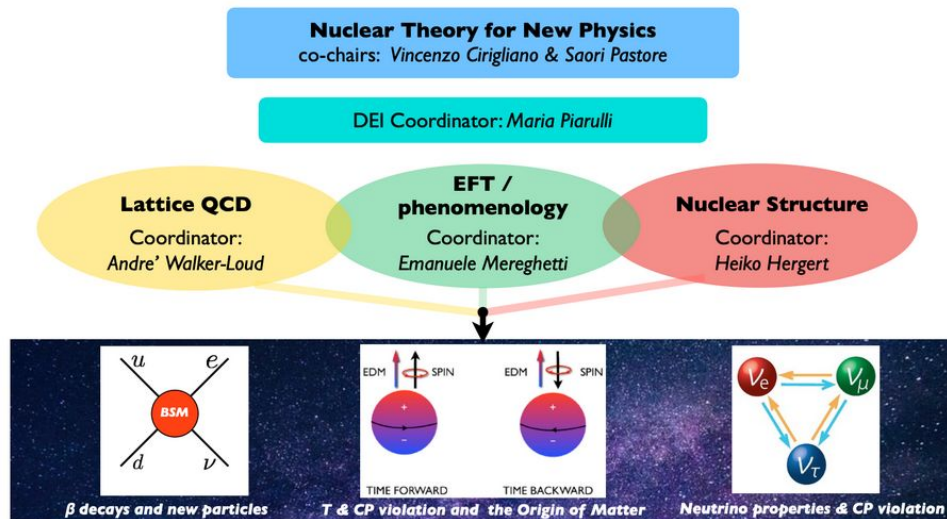
Close **collaborations** between  
**NP, LQCD, Pheno, Hep,  
Comp, Expt, ...**  
are required to progress  
e.g., NP is represented in the  
Snowmass process

It's a very exciting time!

# Nuclear Theory for New Physics NP&HEP TC

## Nuclear Theory for New Physics

- [About Us](#)
- [Commitment to Diversity](#)
- [Funding Acknowledgement](#)



Snowmass:  
Topical groups and  
Frontier Reports,  
Whitepapers, ...

LRP:  
White papers,  
[2301.03975](#), [FSNN](#),  
...

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[NTNP](#)

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Salento U: Girlanda

Huzhou U: Dong Wang

Fermilab: Gardiner Betancourt

MIT: Barrow



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<https://physics.wustl.edu/quantum-monte-carlo-group>



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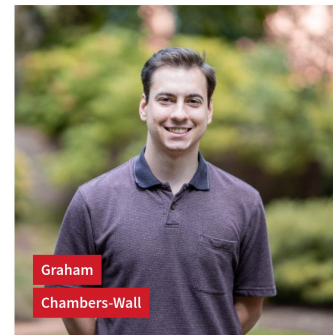
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**Chambers-Wall**