Quasi-Elastic mechanism within Valencia model

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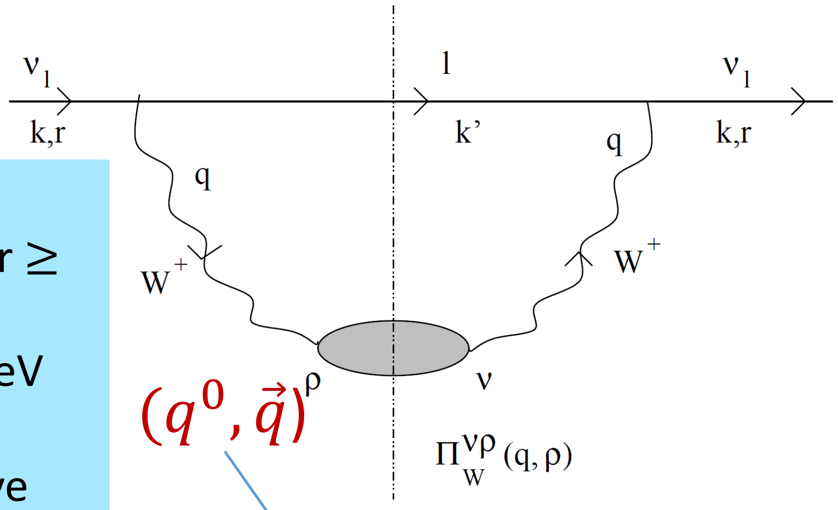
STRONG-2020
European Commission

Theoretical (many body) approach



$(q^0, \vec{q})$

- Energy transfer ≥ 50 – 100 MeV
- (semi-) inclusive processes

$$\frac{d^2\sigma}{d\Omega(\hat{k'})dE'} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}L_{\mu\sigma}W^{\mu\sigma}$$

For instance, charged current process

$$L_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma}k\cdot k' + \mathrm{i}\epsilon_{\mu\sigma\alpha\beta}k'^\alpha k^\beta$$

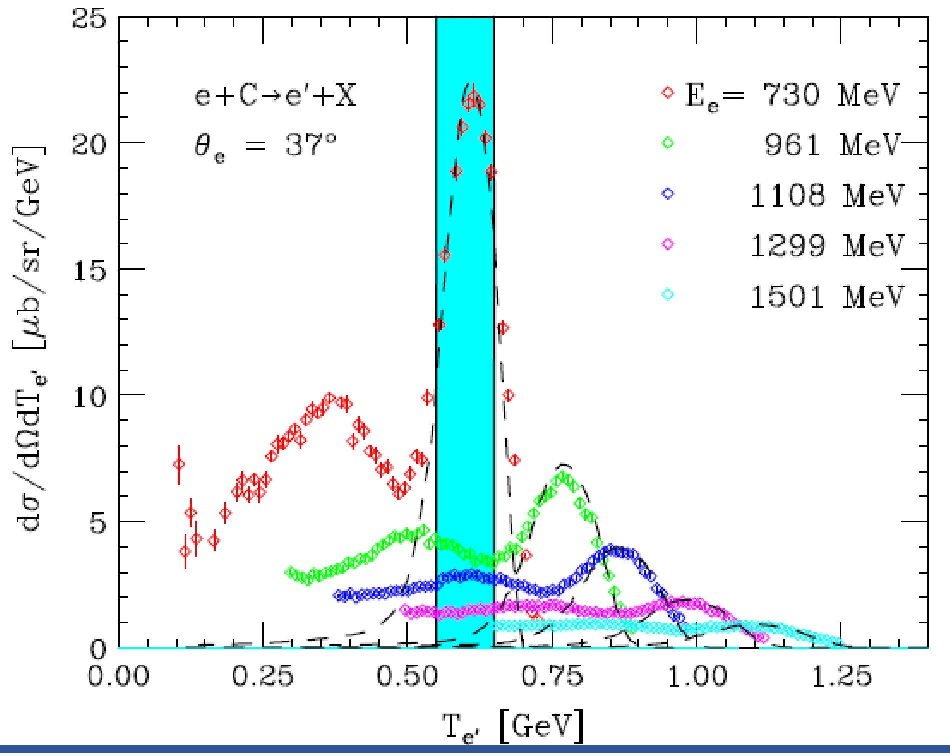
$$W^{\mu\sigma} = W_s^{\mu\sigma} + \mathrm{i}W_a^{\mu\sigma}$$

$$W_s^{\mu\sigma} \propto \int \frac{d^3r}{2\pi}\,\mathrm{Im}\,\left\{\Pi_W^{\mu\sigma}(q,\rho) + \Pi_W^{\sigma\mu}(q,\rho)\right\}\Theta(q^0)$$

$$W_a^{\mu\sigma} \propto \int \frac{d^3r}{2\pi}\,\mathrm{Re}\,\left\{\Pi_W^{\mu\sigma}(q,\rho) - \Pi_W^{\sigma\mu}(q,\rho)\right\}\Theta(q^0)$$

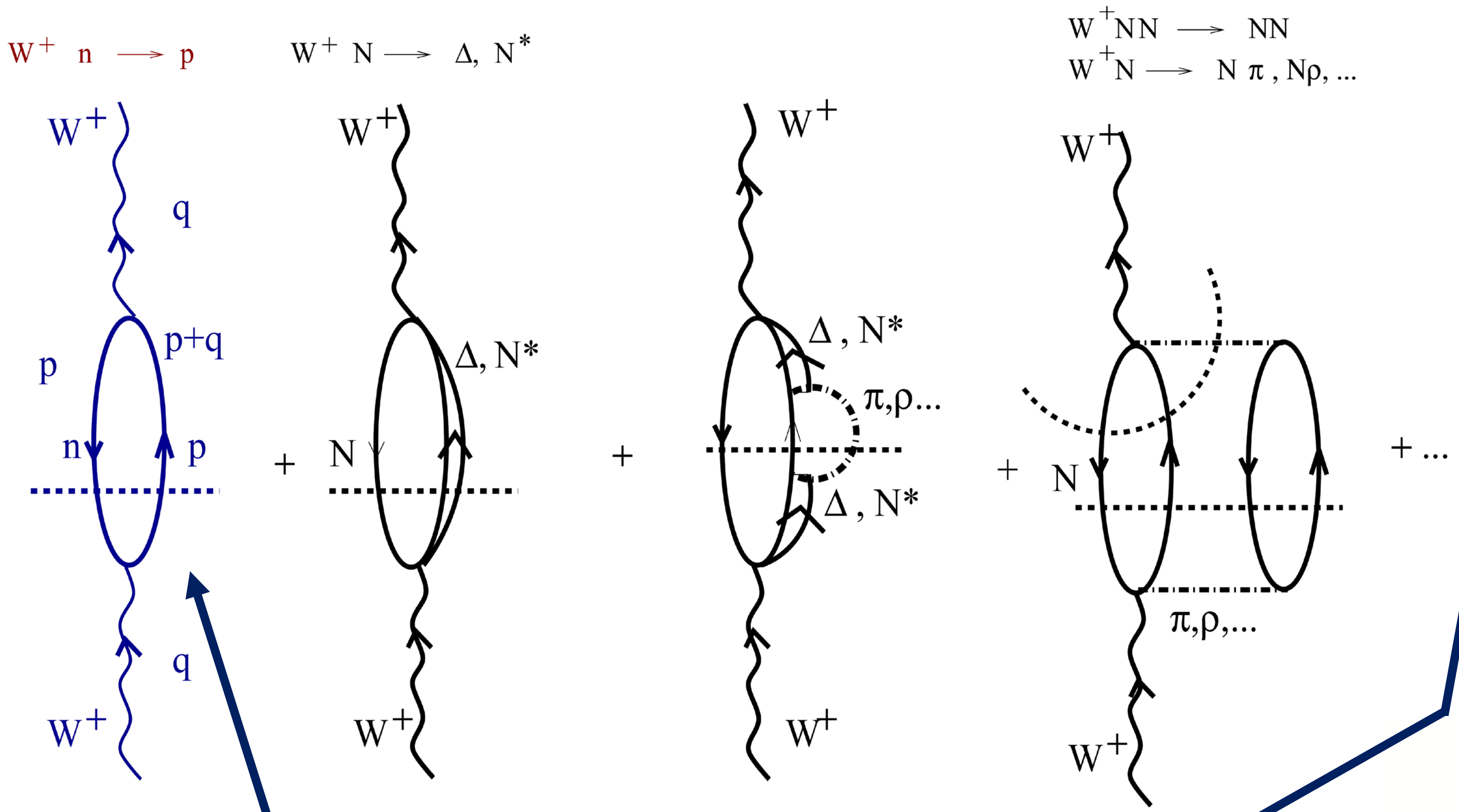
Basic object $\Pi^{\nu\rho}_{W,Z^0,\gamma}(q,\rho)$ $\equiv$ Selfenergy of the Gauge Boson $(W^\pm, Z^0, \gamma)$ inside of the nuclear medium. Perform a Many Body expansion, where the relevant gauge boson absorption modes should be systematically incorporated: absorption by one N, or NN or even 3N, real and virtual (MEC) meson $(\pi,\ \rho,\ \cdots)$ production, Δ excitation, etc...

O. Benhar@NuFacT11: [arXiv : 1110.1835] measured electron-carbon scattering cross sections for a fixed outgoing electron angle $\theta = 37^o$ and different beam energies $\in [730, 1501]$ GeV, plotted as a function of E_e,



CONSISTENT MICROSCOPIC DESCRIPTION of the QE, DIP AND Δ, REGIONS BECOMES FUNDAMENTAL BECAUSE NEUTRINO BEAMS ARE NOT MONOCHROMATIC

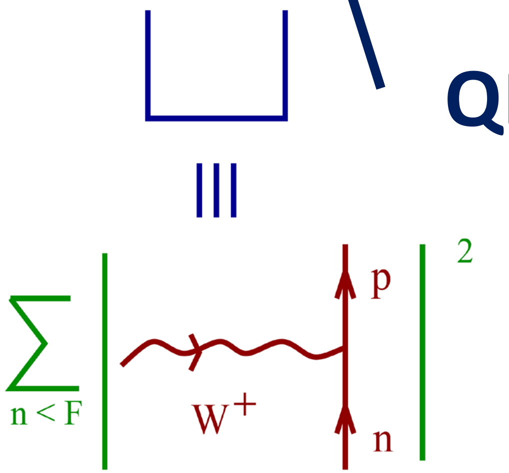
The energy bin corresponding to **the top of the QE peak at E_e = 730 MeV receives significant contributions from** cross sections corresponding to different beam energies and **different mechanisms!**



$$p^2 \approx (p+q)^2$$

$$2pq + q^2 \approx 0$$

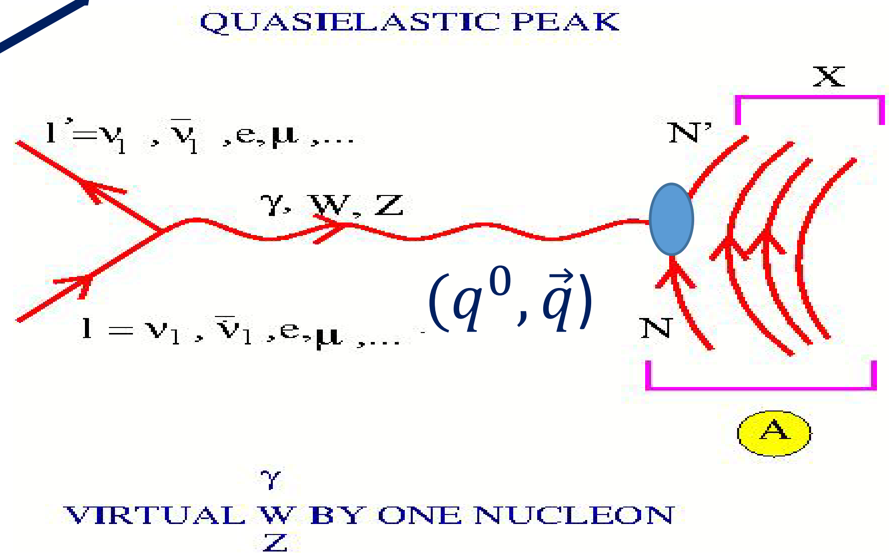
$$q^0 \approx -\frac{q^2}{2M} = \frac{|\vec{q}|^2 - (q^0)^2}{2M}$$



QE (1p1h) contribution

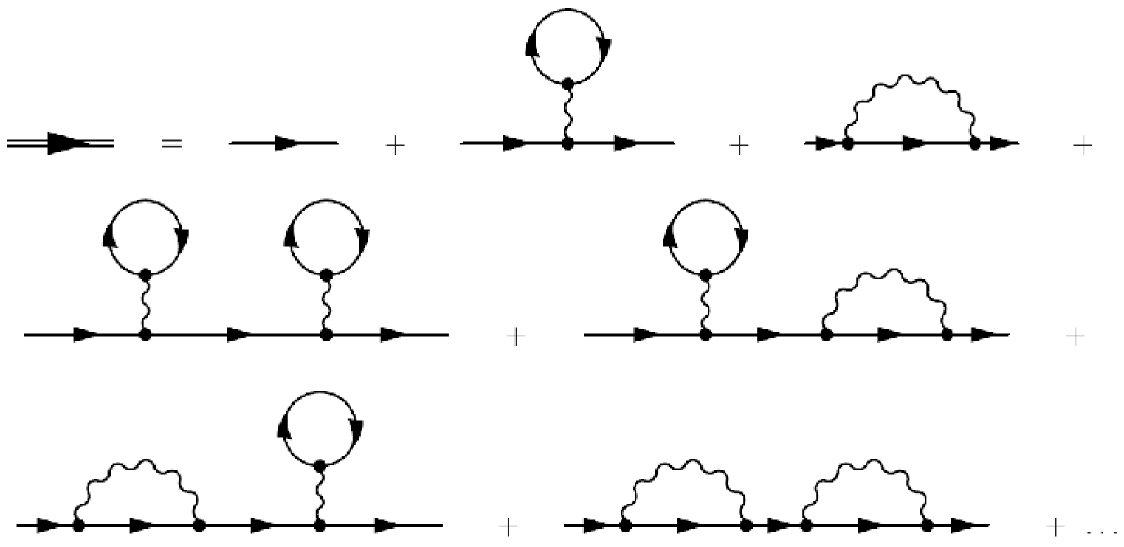
first ingredient $W^{\pm}NN'$ (or Z^0NN or γ^*NN) in **vacuum, after nuclear corrections should be included…..**

+NUCLEAR CORRECTIONS

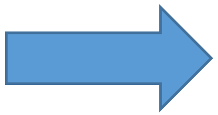


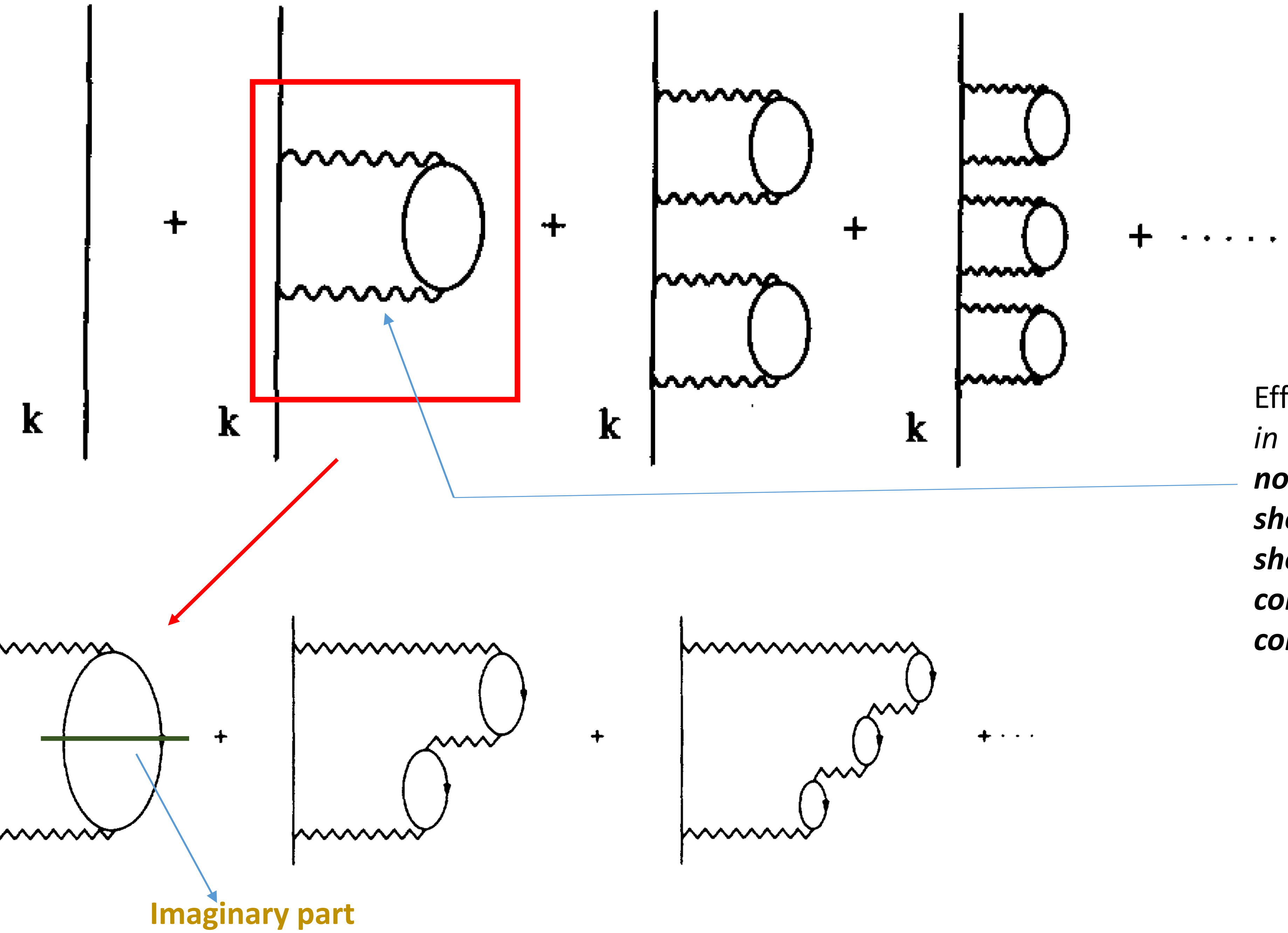
QE nuclear corrections

- Spectral Functions: dressing the nucleon lines in the medium

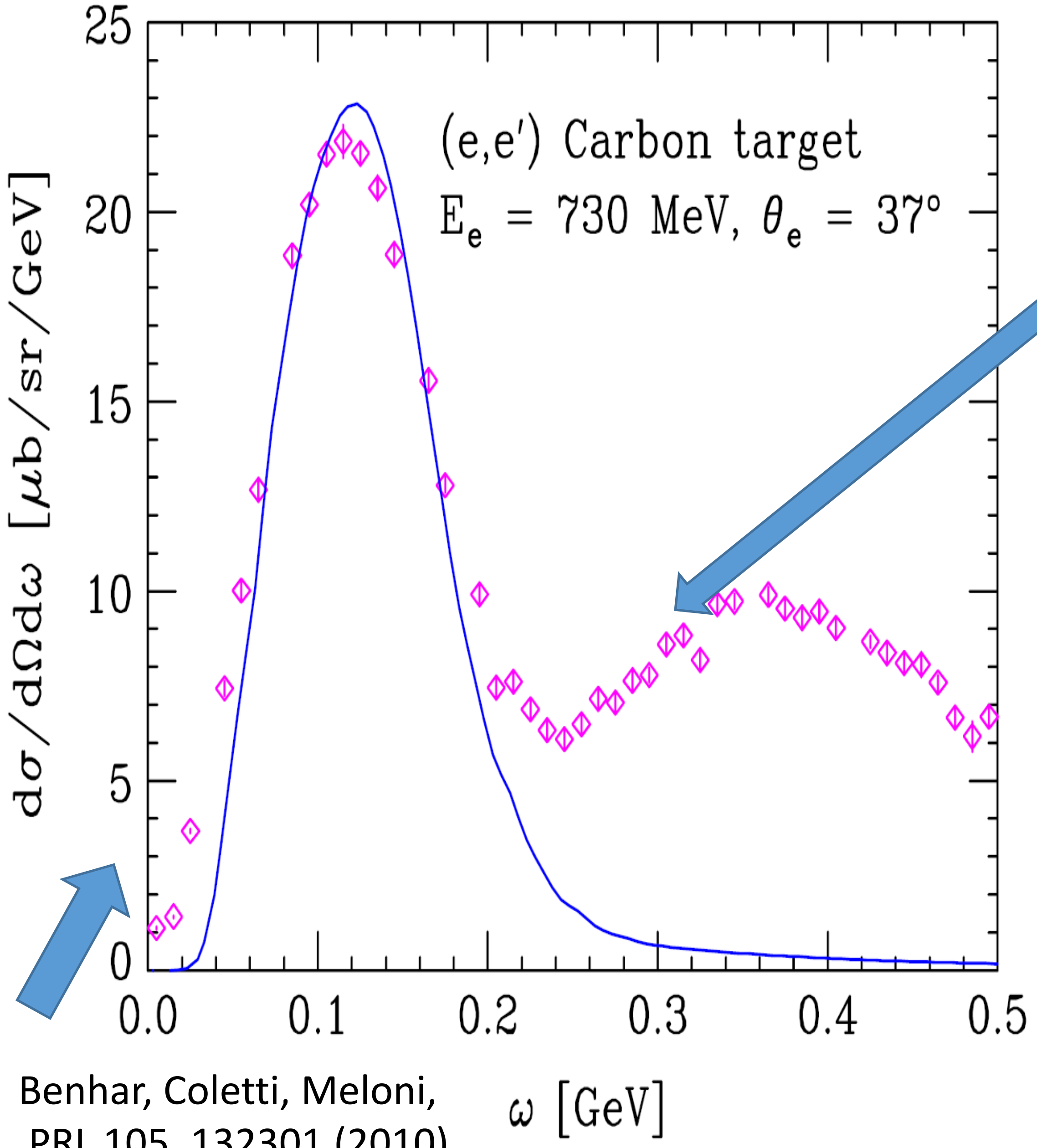


Beyond the Hartree-Fock approximation





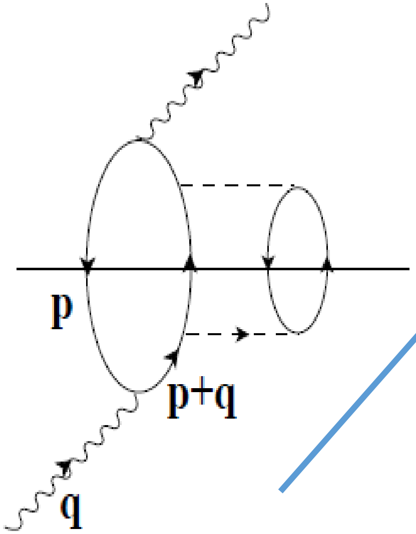
Effective *NN interaction in the medium.* ***It is not just a pion and should account for short-range-correlations and RPA corrections***



Benhar, Coletti, Meloni, PRL 105, 132301 (2010)

Spectral Function (SRC) do not populate the **dip region**

- Spectral Function (SF) + Final State Interaction (FSI): dressing up the nucleon propagator of the hole (SF) and particle (FSI) states in the *ph* excitation



- Change of nucleon dispersion relation:
  - hole ⇒ Interacting Fermi sea (SF)
  - particle ⇒ Interaction of the ejected nucleon with the final nuclear state (FSI)

$$G(p) \to \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\vec{p}\,)}{p^0-\omega-\mathrm{i}\epsilon} + \int_{\mu}^{+\infty} d\omega \frac{S_p(\omega,\vec{p}\,)}{p^0-\omega+\mathrm{i}\epsilon}$$

The hole and particle spectral functions are related to nucleon self-energy $\boxed{\Sigma}$ in the medium,

$$G(p) = \frac{n(\vec{p}\,)}{p^0-\varepsilon(\vec{p})-i\epsilon} + \frac{1-n(\vec{p}\,)}{p^0-\varepsilon(\vec{p})+i\epsilon}$$

$$S_{p,h}(\omega,\vec{p}) = \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\omega,\vec{p}\,)}{[\omega^2-\vec{p}^2-M^2-\mathrm{Re}\Sigma(\omega,\vec{p})]^2+[\mathrm{Im}\Sigma(\omega,\vec{p})]^2}$$

with $\omega \geq \mu$ or $\omega \leq \mu$ for S_p and S_h, respectively (μ is the chemical potential).

Spectral Functions: dressing the nucleon lines in the medium

Basic object: nucleon selfenergy in the medium: Σ (from realistic NN interactions in the medium).

Spectral Functions: modification of the dispersion relation of the nucleons inside of the nuclear medium

This nuclear effect is additional to those due to RPA (long range) correlations !!

The simplest description ⇒ relativistic Fermi Gas with non interacting fermions $\boxed{\Sigma = 0}$,

$$S_p(\omega,\vec{p}) = \frac{\theta(|\vec{p}\,| - k_F)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

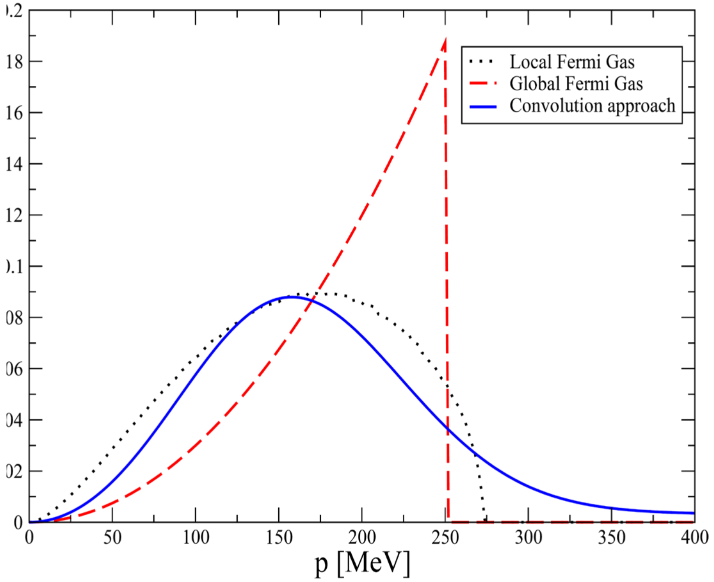
$$S_h(\omega,\vec{p}) = \frac{\theta(k_F - |\vec{p}\,|)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

and only Pauli blocking is incorporated!!

Local vs Global Fermi Gas ?

$$k_F^{p,n}(r) = \left[3\pi^2\rho^{p,n}(r)\right]^{1/3} \text{ vs } k_F^{p,n} = \text{cte ?}$$

$p^2 n(p)/A$ [fm]



Local vs Global Fermi Gas ?

$$k_F(r) = \left[3\pi^2\rho(r)/2\right]^{1/3} \textbf{ vs } k_F \textbf{= cte ?}$$

$$S_h(\omega,\vec{p}) = \delta(\omega - E(\vec{p}))\theta(k_F - |\vec{p}\,|)/2\omega$$

$$n^{\rm RgFG}(|\vec{p}\,|) = \frac{4V}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = \frac{3A}{4\pi k_F^3}\theta(k_F - |\vec{p}\,|)$$

$$n^{\rm LDA}(|\vec{p}\,|) = 4\int\frac{d^3r}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = 4\int\frac{d^3r}{(2\pi)^3}\theta(\mathbf{k_F}(\mathbf{r}) - |\vec{p}\,|)$$

$$\left(\int d^3p\, n(|\vec{p}\,|) = A\right)$$

Convolution approach: C. Ciofi degli Atti, S. Liuti, and S. Simula, PRC 53, 1689 (1996), provide realistic distribution due to short-range correlations !

PHYSICAL REVIEW C VOLUME 46, NUMBER 5 NOVEMBER 1992

Semiphenomenological approach to nucleon properties in nuclear matter

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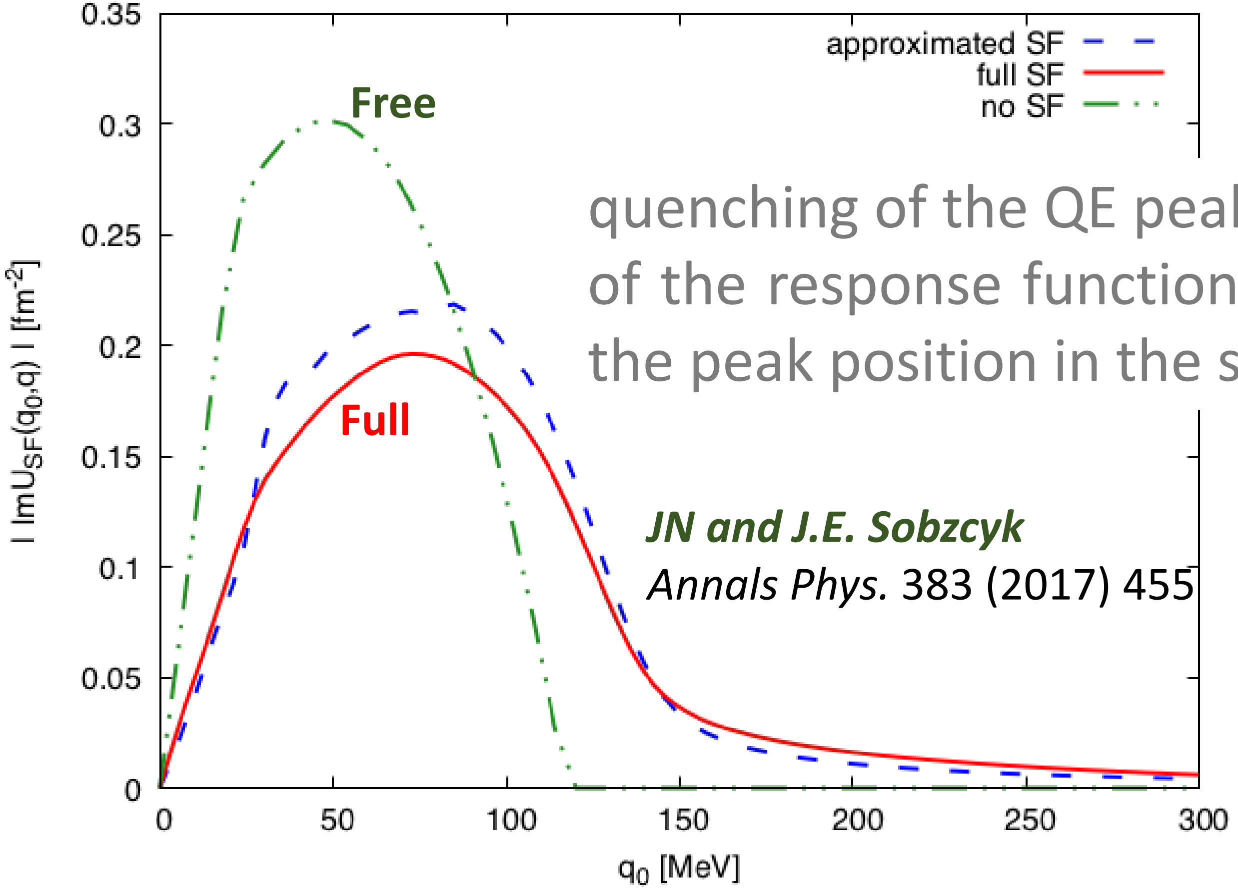
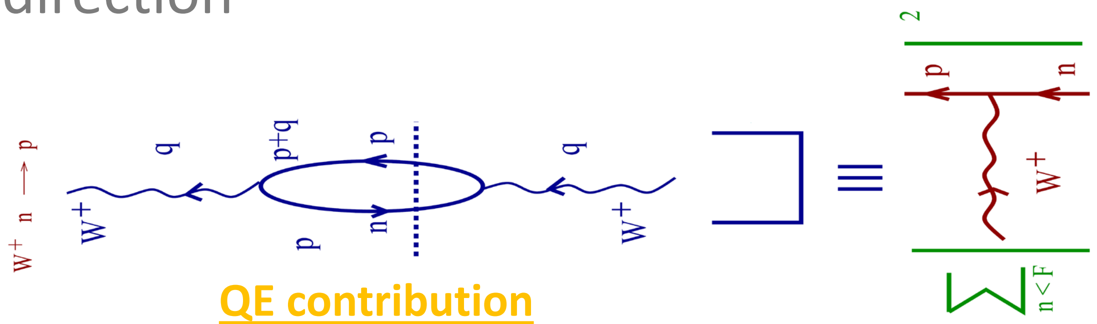
(Received 20 April 1992)

SF from effective NN interaction in the medium constructed from the experimental NN cross section + some medium corrections



quenching of the QE peak, produce a spreading of the strength of the response functions to higher energy transfers and shift the peak position in the same direction

JN and J.E. Sobzcyk
Annals Phys. 383 (2017) 455



QE contribution

Imaginary part of the *ph* propagator (Lindhard function)

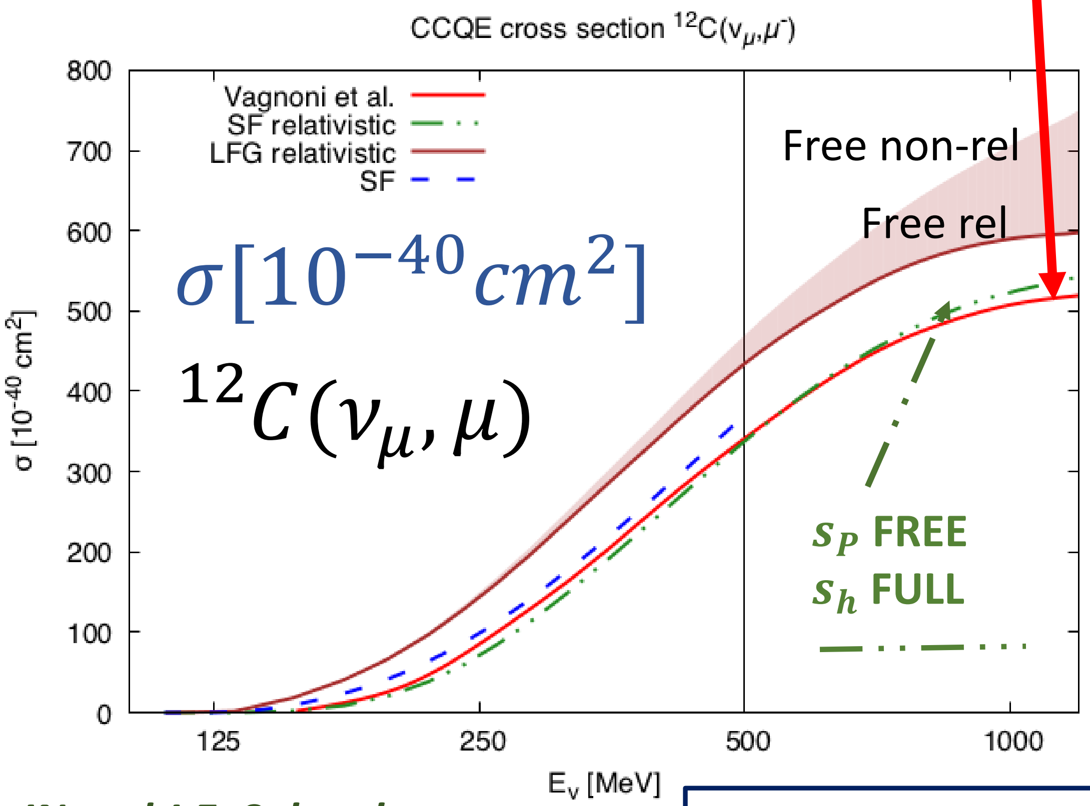
reasonable agreement !

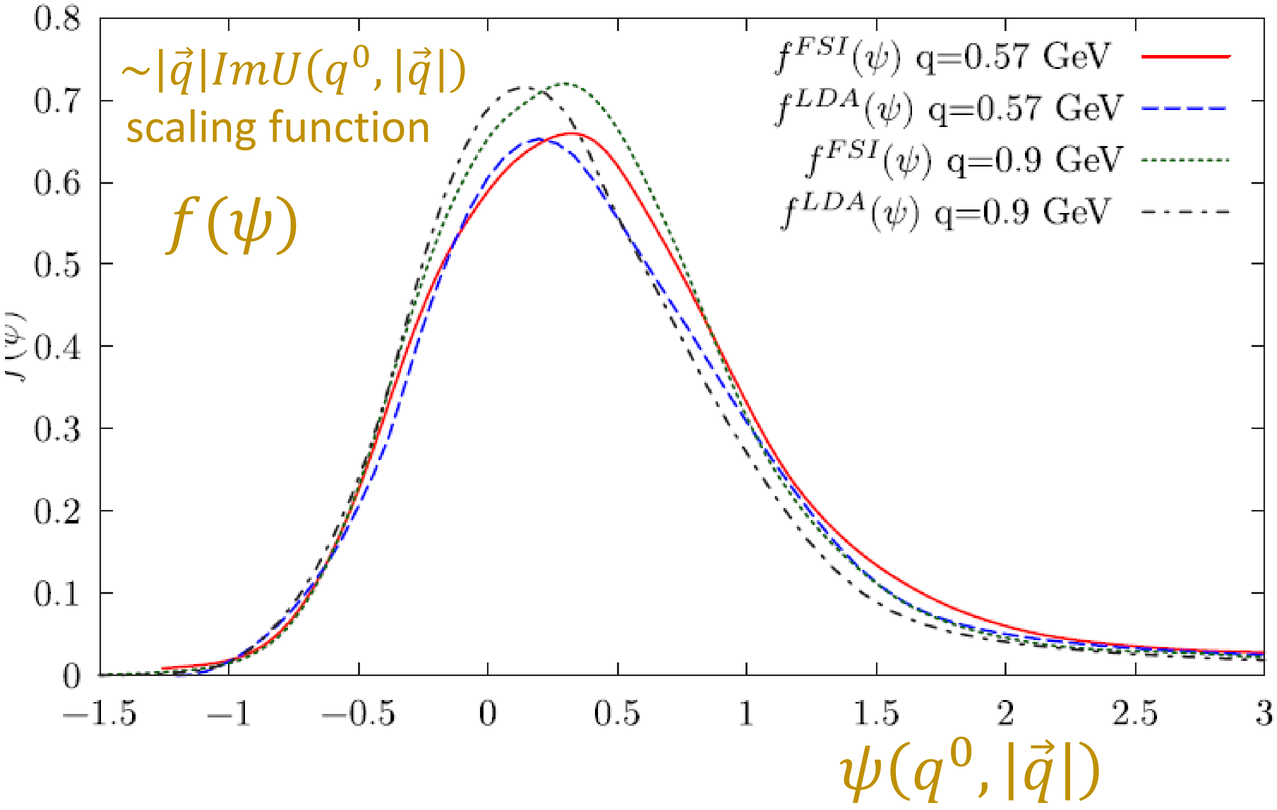
PRL 118, 142502 (2017) PHYSICAL REVIEW LETTERS week ending 7 APRIL 2017

Inelastic Neutrino-Nucleus Interactions within the Spectral Function Formalism

Erica Vagnoni,[1,*] Omar Benhar,[2,3,†] and Davide Meloni[1,‡]

J. E. Sobczyk, N. Rocco, A. Lovato and JN
PRC 97 (2018) 035506





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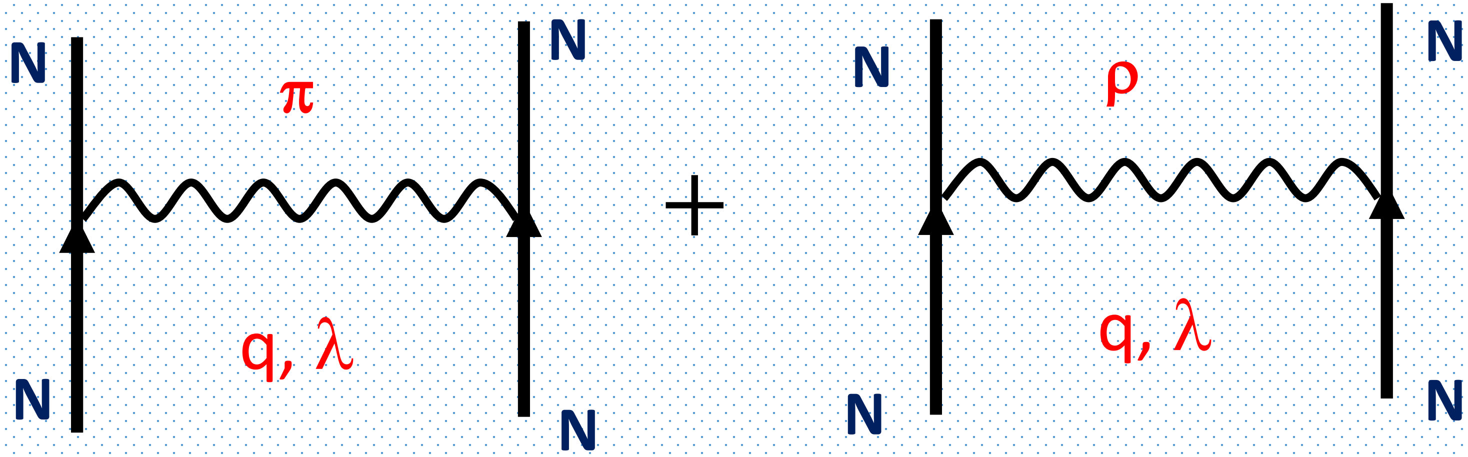
SF from effective NN interaction in the medium constructed from the experimental NN cross section + some medium corrections

Technical parenthesis

π+ρ exchange



$$\hat{V}_\pi(\mathbf{q}) = \sigma_1^i \sigma_2^j \,\vec{\tau}_1 \vec{\tau}_2 V_{ij}^\pi(q), \qquad V_{ij}^\pi(q) = \left(\frac{f}{m_\pi}\right)^2 F_\pi^2(q)\, \vec{q}^2 \, D_\pi(q)\, \hat{q}_i \, \hat{q}_j$$

longitudinal !!

$$\hat{V}_\rho(\mathbf{q})\mathbf{q} = \sigma_1^i \sigma_2^j \,\vec{\tau}_1 \vec{\tau}_2 V_{ij}^\rho(q), \quad V_{ij}^\rho(q) = \left(\frac{f}{m_\pi}\right)^2 F_\rho^2(q)\, \vec{q}^2 \, D_\rho(q) C_\rho \left(\delta_{ij} - \hat{q}_i \, \hat{q}_j\right)$$

transversal !!

$$F_{\pi,\rho}(q) = \frac{\Lambda_{\pi,\rho}^2 - m_{\pi,\rho}^2}{\Lambda_{\pi,\rho}^2 - q^2}, \qquad q^2 = {q^0}^2 - \vec{q}^2, \quad \Lambda_\pi = 1250\ \mathrm{MeV} = \Lambda_\pi^*, \quad \Lambda_\rho = 2500\ \mathrm{MeV} = \Lambda_\rho^*$$

(off-shell behavior) !!

Because $C_\rho = C_\rho^*, \Lambda_\pi = \Lambda_\pi^*$ and $\Lambda_\rho = \Lambda_\rho^*$, the former potentials also describe the

$$\boldsymbol{\Delta N \rightarrow NN,\ NN \rightarrow \Delta N \text{ and } \Delta\Delta \rightarrow NN}$$

interactions with the following replacements

$$\frac{f}{m_\pi}\boldsymbol{\sigma\tau} \rightarrow \frac{f}{m_\pi}\boldsymbol{S\,T} \text{ or } \frac{f}{m_\pi}\boldsymbol{S^\dagger T^\dagger}$$

Note $\boldsymbol{V_{ij}^{\pi}(q) \perp V_{ij}^{\rho}(q)}$

Short range correlations: Attributed to the exchange of the ω meson

$\frac{1}{m_\omega}$ defines the range of the correlations

Correlated potential in coordinate space: $\widetilde{V(r)} = V(r)g(r);\ \ g(r) = 1 - j_0(q_c r), q_c \sim m_\omega \sim 783 \text{ MeV}$

Correlated potential in momentum space: $\widetilde{V(q)} = \int \frac{d^3k}{(2\pi)^3}\, g\left(\vec{k} - \vec{q}\right) V(\vec{k})$

$$g\left(\vec{k}\right) = (2\pi)^3\,\delta^3\left(\vec{k}\right) - 2\pi^2 \frac{\delta\left(\left|\vec{k}\right| - q_c\right)}{q_c^2}$$

NN potential $V(q)= c_0\{f_0(\rho) + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2\} + \vec{\tau}_1\vec{\tau}_2 \sum_{i,j=1}^{3} \sigma_1^i \sigma_2^j V_{ij}^{\sigma\tau}$

zero range Landau force
J. Speth et al., Phys. Rep. 33 (1977) 127

$$V_{ij}^{\sigma\tau} = (\hat{q}_i\hat{q}_j V_l(q) + (\delta_{ij} - \hat{q}_i\hat{q}_j)V_t(q))$$

with $\hat{q}_i = {}^{q_i}/_{|\vec{q}|}$

V_π

$$V_l(q^0,\vec{q}) = \frac{f^2}{m_\pi^2}\left\{\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2}\right)^2 \frac{\vec{q}^2}{q^2 - m_\pi^2} + g_l'(q)\right\},$$

V_ρ

$$\frac{f^2}{4\pi} = 0.08,\ \Lambda_\pi = 1200\ \text{MeV},$$

$$V_t(q^0,\vec{q}) = \frac{f^2}{m_\pi^2}\left\{C_\rho\left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2}\right)^2 \frac{\vec{q}^2}{q^2 - m_\rho^2} + g_t'(q)\right\},$$

$$C_\rho = 2,\ \Lambda_\rho = 2500\ \text{MeV},\ m_\rho = 770\ \text{MeV}.$$

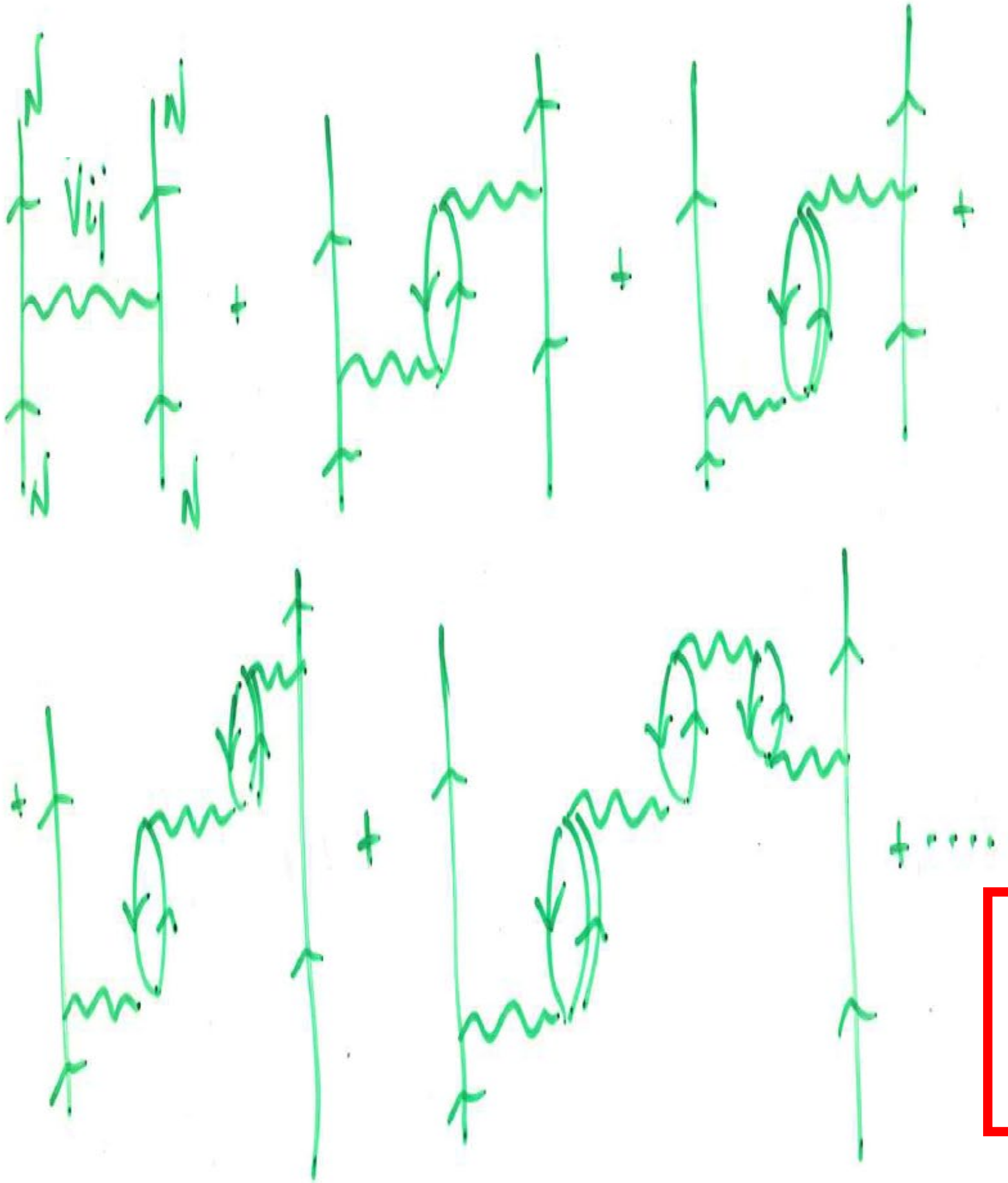
SRC

The $N\Delta$ and the $\Delta\Delta$ potentials are obtained from V_l and V_t by replacing

$$\vec{\sigma} \to \vec{S},\ \vec{\tau} \to \vec{T}.$$

$$f \to f^*$$

The spin-isospin part of the interaction, taking into account the propagation of the mesons through the medium



$$\hat{q}_i\hat{q}_j \perp (\delta_{ij} - \hat{q}_i\hat{q}_j)$$

$$W_{\sigma\tau}(q) = \sigma_1^i\sigma_2^j\,\vec{\tau}_1\vec{\tau}_2 V_{ij}^{\sigma\tau}(q) +$$
$$\sigma_1^i\sigma_2^j\,\vec{\tau}_1\vec{\tau}_2\{V_{ik}^{\sigma\tau}(q)\,U(q)V_{kj}^{\sigma\tau}(q)\} +$$
$$\sigma_1^i\sigma_2^j\,\vec{\tau}_1\vec{\tau}_2\{V_{ik}^{\sigma\tau}(q)\,U(q)V_{km}^{\sigma\tau}(q)U(q)V_{mj}^{\sigma\tau}(q)\} +$$
$$\ldots = \sigma_1^i\sigma_2^j\,\vec{\tau}_1\vec{\tau}_2 W_{ij}^{\sigma\tau}(q)$$

$$U(q) = U_N(q) + U_\Delta(q)$$

(direct + crossed terms)

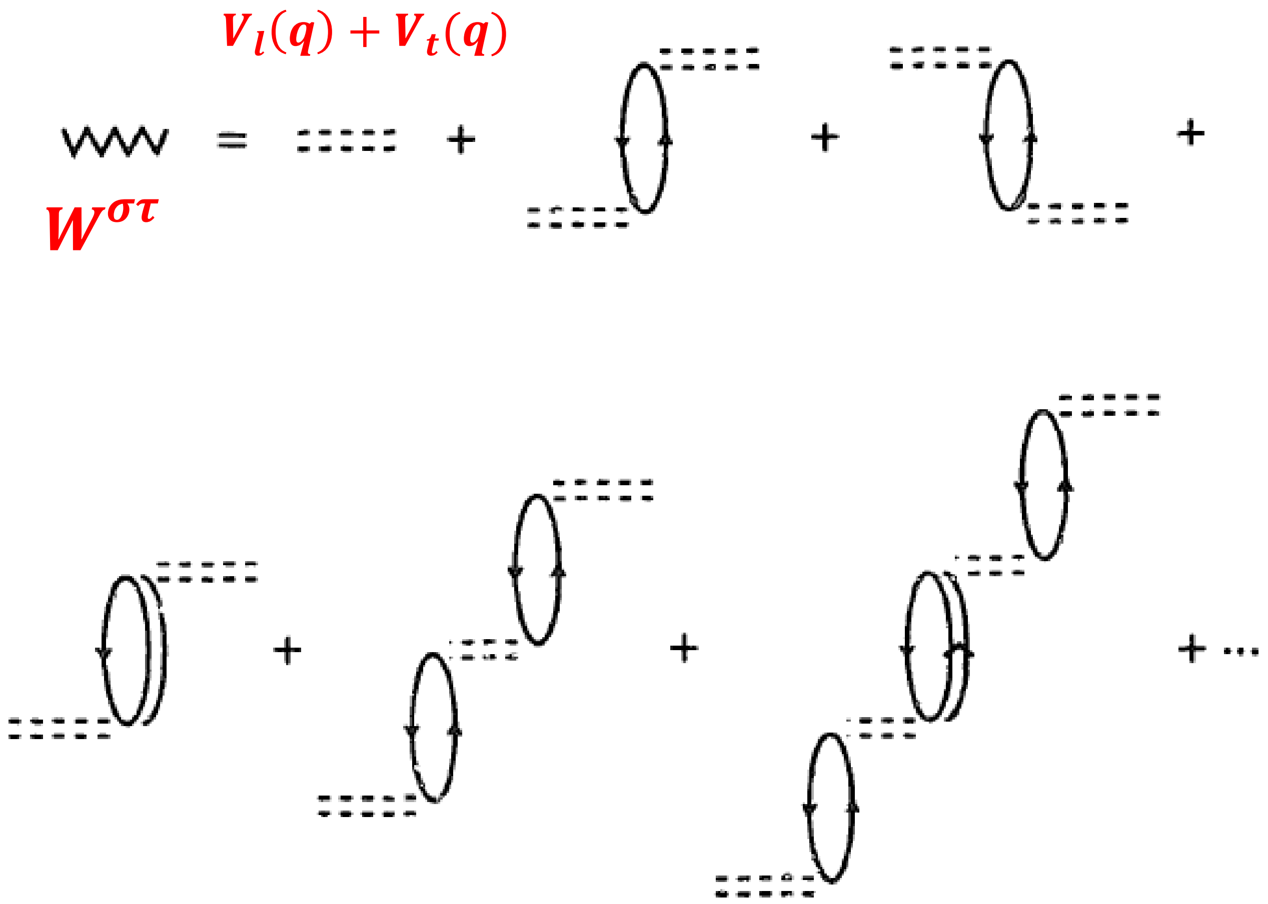
$$W_{ij}^{\sigma\tau}(q) = \frac{V_l(q)}{1-U(q)V_l(q)}\,\hat{q}_i\hat{q}_j + \frac{V_t(q)}{1-U(q)V_t(q)}\,(\delta_{ij} - \hat{q}_i\hat{q}_j)$$

Induced spin-isospin NN interaction in a nuclear medium

Diagrammatically,

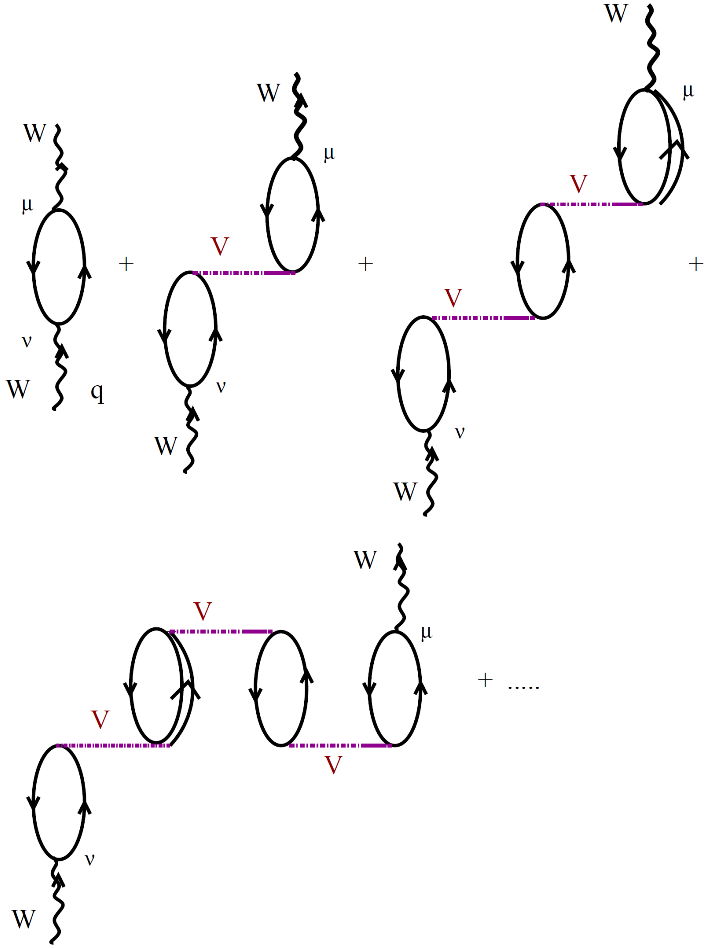
$$W_{ij}^{\sigma\tau}(q) = \frac{V_l(q)}{1-U(q)V_l(q)}\ \hat{q}_i\hat{q}_j\ + \frac{V_t(q)}{1-U(q)V_t(q)}\left(\delta_{ij} - \hat{q}_i\hat{q}_j\right)$$

From the spin-isospin interaction, we construct the induced interaction by exciting ph and Δh components in a RPA sense



QE nuclear corrections: RPA: long range correlations

- Polarization (RPA) effects. Substitute the ph excitation by an RPA response: series of ph and Δh excitations.



1. Effective Landau-Migdal interaction **(SRC)**

$$V(\vec{r}_1,\vec{r}_2) = c_0\delta(\vec{r}_1-\vec{r}_2)\left\{\boxed{f_0(\rho)} + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + \boxed{g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2} + g_0'(\rho)\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2\right\}$$

Isoscalar terms $\boxed{}$ do not contribute to CC

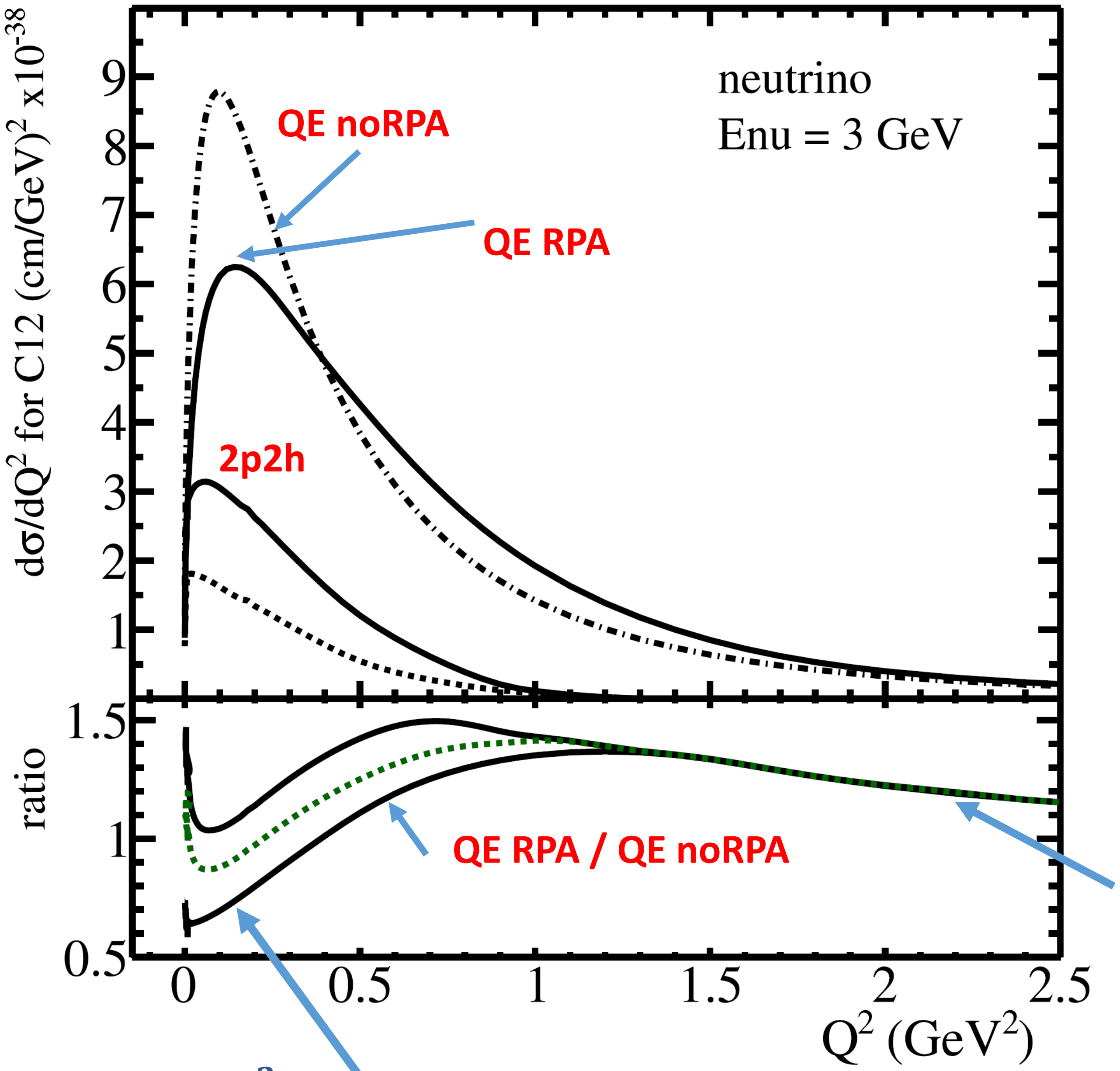
2. $S=T=1$ channel of the ph–ph interaction → s longitudinal (π) and transverse (ρ) + **SRC**

$$g_0'\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2 \rightarrow \left[V_l(q)\hat{q}_i\hat{q}_j + V_t(q)(\delta_{ij}-\hat{q}_i\hat{q}_j)\right]\sigma_1^i\sigma_2^j\vec{\tau}_1\vec{\tau}_2$$

(SRC)

$$V_{l,t}(q) = \frac{f_{\pi NN,\rho NN}}{m^2_{\pi,\rho}}\left(F_{\pi,\rho}(q^2)\frac{\vec{q}^{\,2}}{q^2-m^2_{\pi,\rho}} + g'_{l,t}(q)\right)$$

3. Contribution of Δh excitations important

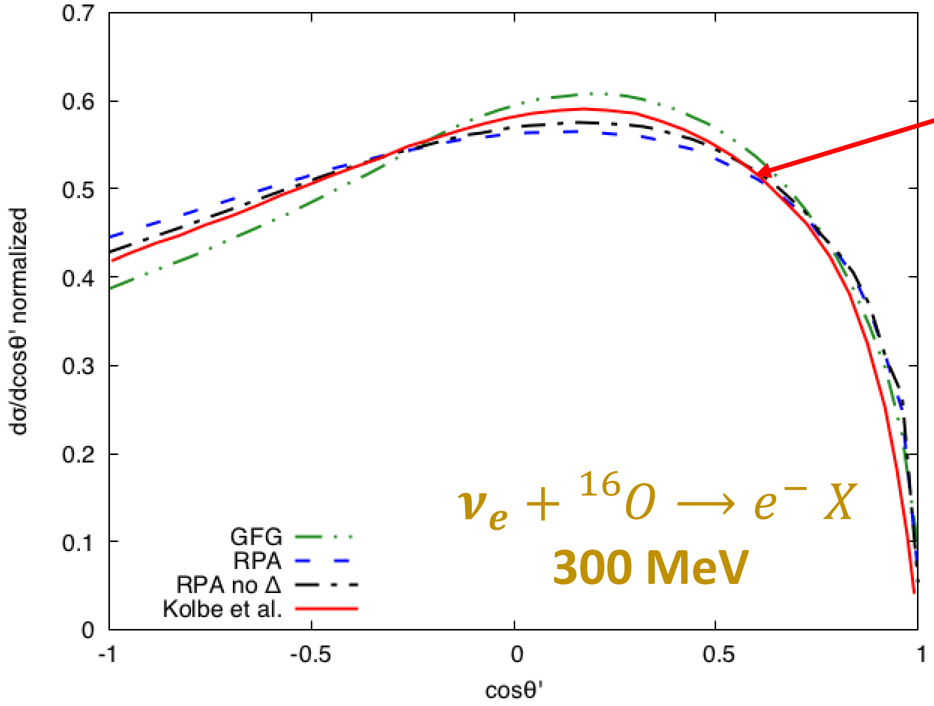


deplection at low Q^2 values

Gran+JN+Sánchez+Vicente-Vacas
*PRD*88 (2013) 113007

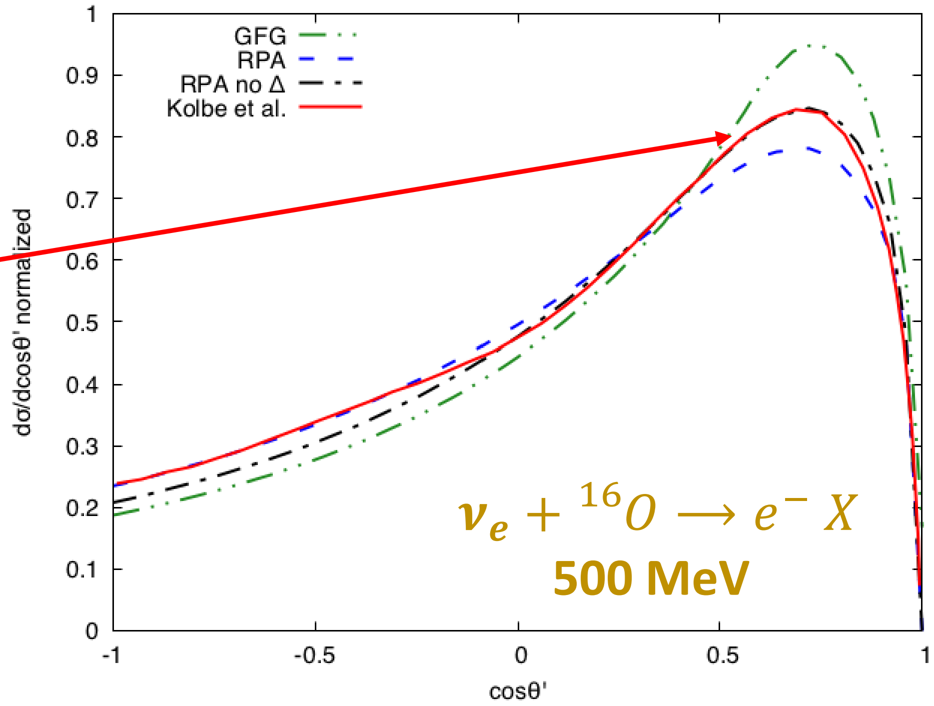
RPA (long range correlations) the weak probe interacts with the nucleus as a whole,

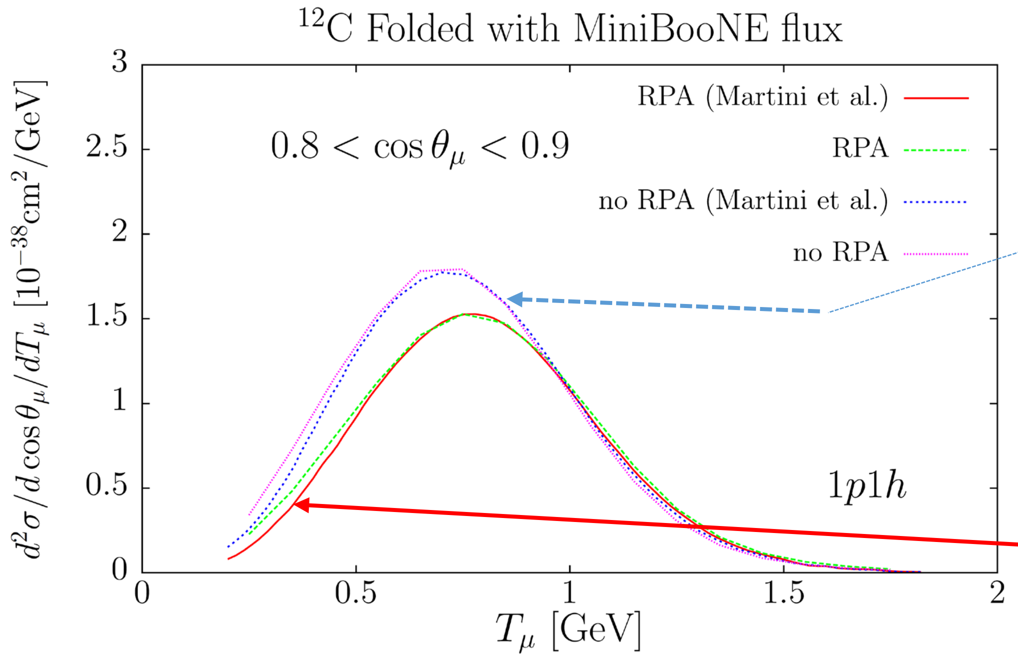
RPA effects → 0, when $^1/_{\sqrt{Q^2}}$ ≪ <u>nuclear radius</u>, since then the probe would see the individual nucleons or even the partons



Kolbe, Langanke, Martinez-Pinedo, Vogel, J. Phys. G29, 2569 (2003)

$d\sigma / dcos(\theta')$





M. Martini, M. Ericson, and G. Chanfray, Phys.Rev. C84, 055502 (2011)

reasonable agreement !

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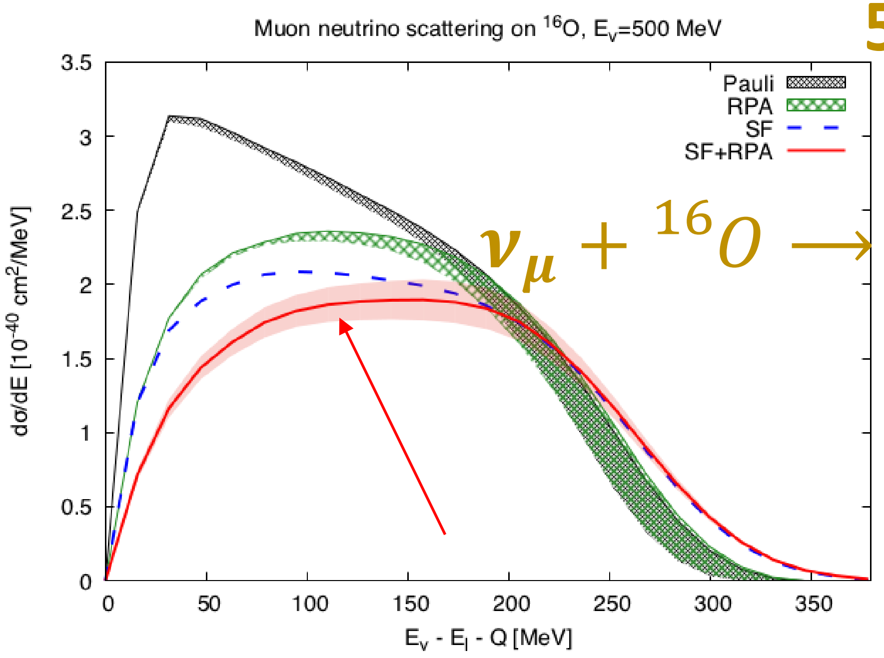
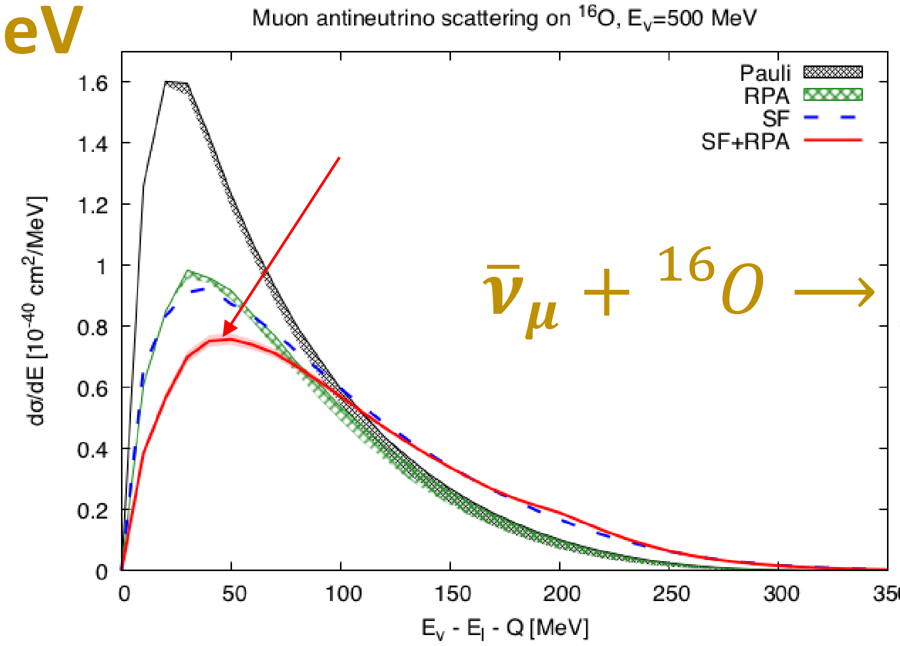
500 MeV





QE: 1p1h

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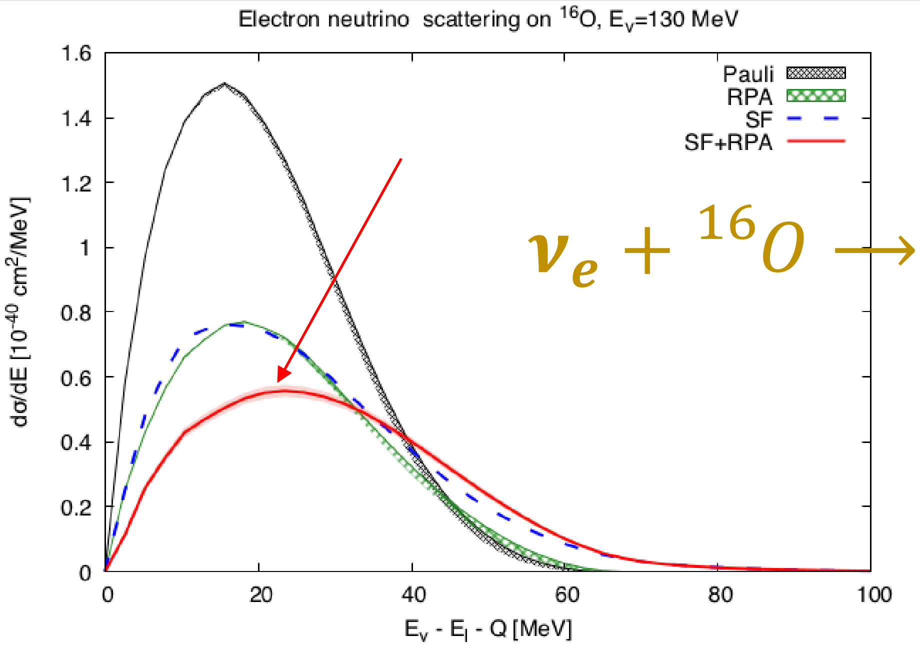
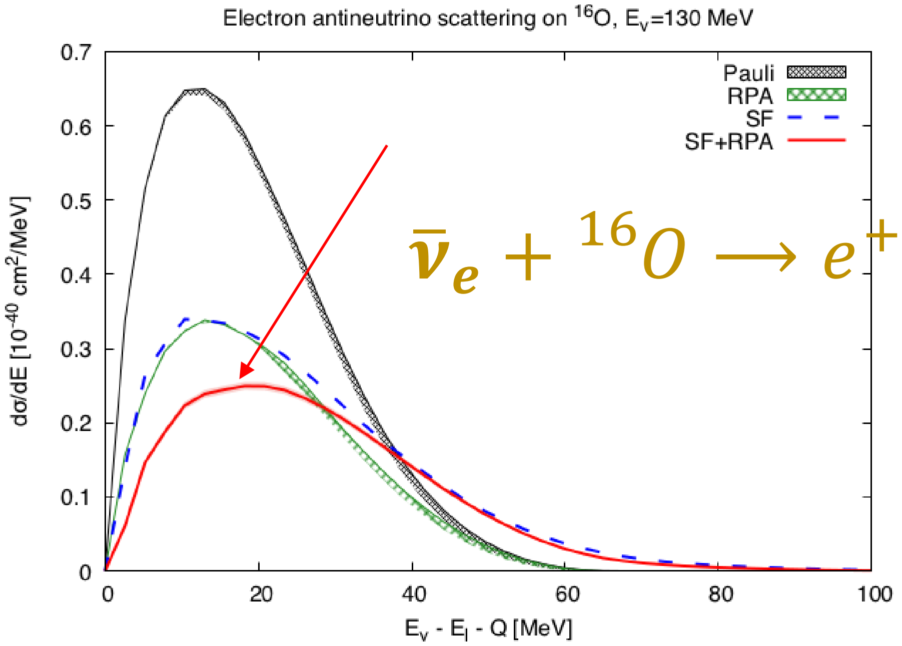
RPA effects in integrated decay rates or cross sections become significantly smaller when SF corrections are also considered



130 MeV

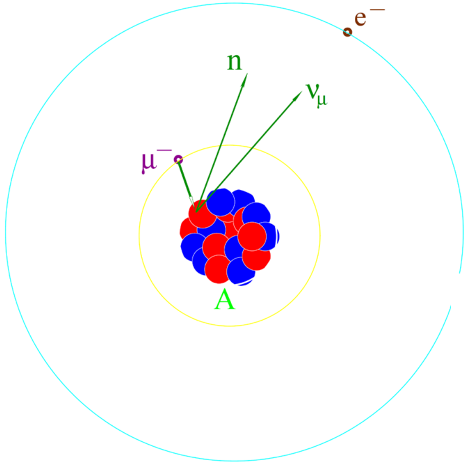


SF+RPA

SF+RPA

Free

Free + RPA



Inclusive Muon Capture: $\Gamma\left[(A_Z - \mu^-)^{1s}_{\text{bound}}\right]$

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Nucleus	Pauli (10^4 s^{-1})	RPA (10^4 s^{-1})	SF (10^4 s^{-1})	SF+RPA (10^4 s^{-1})	Exp. (10^4 s^{-1})
^{12}C	5.76	3.37 ± 0.16	3.22	3.19 ± 0.06	3.79 ± 0.03
^{16}O	18.7	10.9 ± 0.4	10.6	10.3 ± 0.2	10.24 ± 0.06
^{18}O	13.8	8.2 ± 0.4	7.0	8.7 ± 0.1	8.80 ± 0.15
^{23}Na	64.5	37.0 ± 1.5	30.9	34.3 ± 0.4	37.73 ± 0.14
^{40}Ca	498	272 ± 11	242	242 ± 6	252.5 ± 0.6

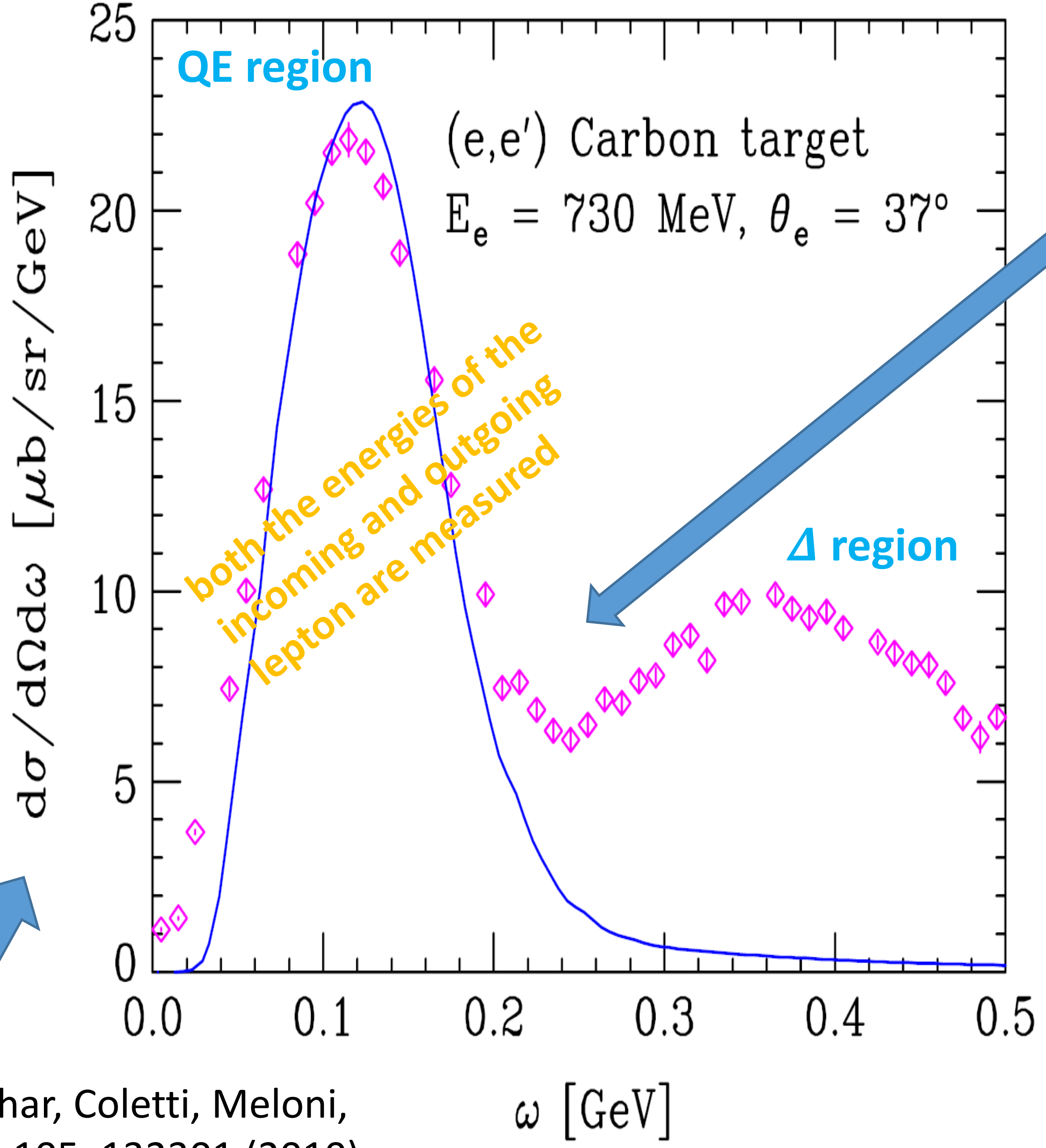
The inclusive $^{12}\mathrm{C}(\nu_\mu, \mu^-)X$ *and* $^{12}\mathrm{C}(\nu_e, e^-)X$ *reactions near threshold*

10^{-40} cm^2
Flux-averaged cross sections

	Pauli	RPA	SF	SF+RPA	SM	SM	CRPA	Experiment		
					[125]	[44]	[45]	LSND [115]	LSND [116]	LSND [117]
$\bar\sigma(\nu_\mu, \mu^-)$	23.1	13.2 ± 0.7	12.2	9.7 ± 0.3	13.2	15.2	19.2	$8.3 \pm 0.7 \pm 1.6$	$11.2 \pm 0.3 \pm 1.8$	$10.6 \pm 0.3 \pm 1.8$
								KARMEN [120]	LSND [118]	LAMPF [119]
$\bar\sigma(\nu_e, e^-)$	0.200	0.143 ± 0.006	0.086	0.138 ± 0.004	0.12	0.16	0.15	$0.15 \pm 0.01 \pm 0.01$	0.15 ± 0.01	0.141 ± 0.023

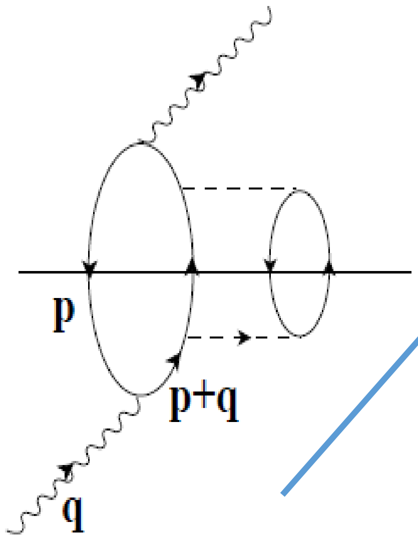
[125]: Hayes & Towner, PRC61, 044603;
[44]: Volpe et al., PRC62, 015501; [45]: Kolbe et al., J. Phys. G29, 2569

Spectral Functions (SRC) populate neither the **dip** nor the Δ regions



Benhar, Coletti, Meloni, PRL 105, 132301 (2010)

- Spectral Function (SF) + Final State Interaction (FSI): dressing up the nucleon propagator of the hole (SF) and particle (FSI) states in the *ph* excitation



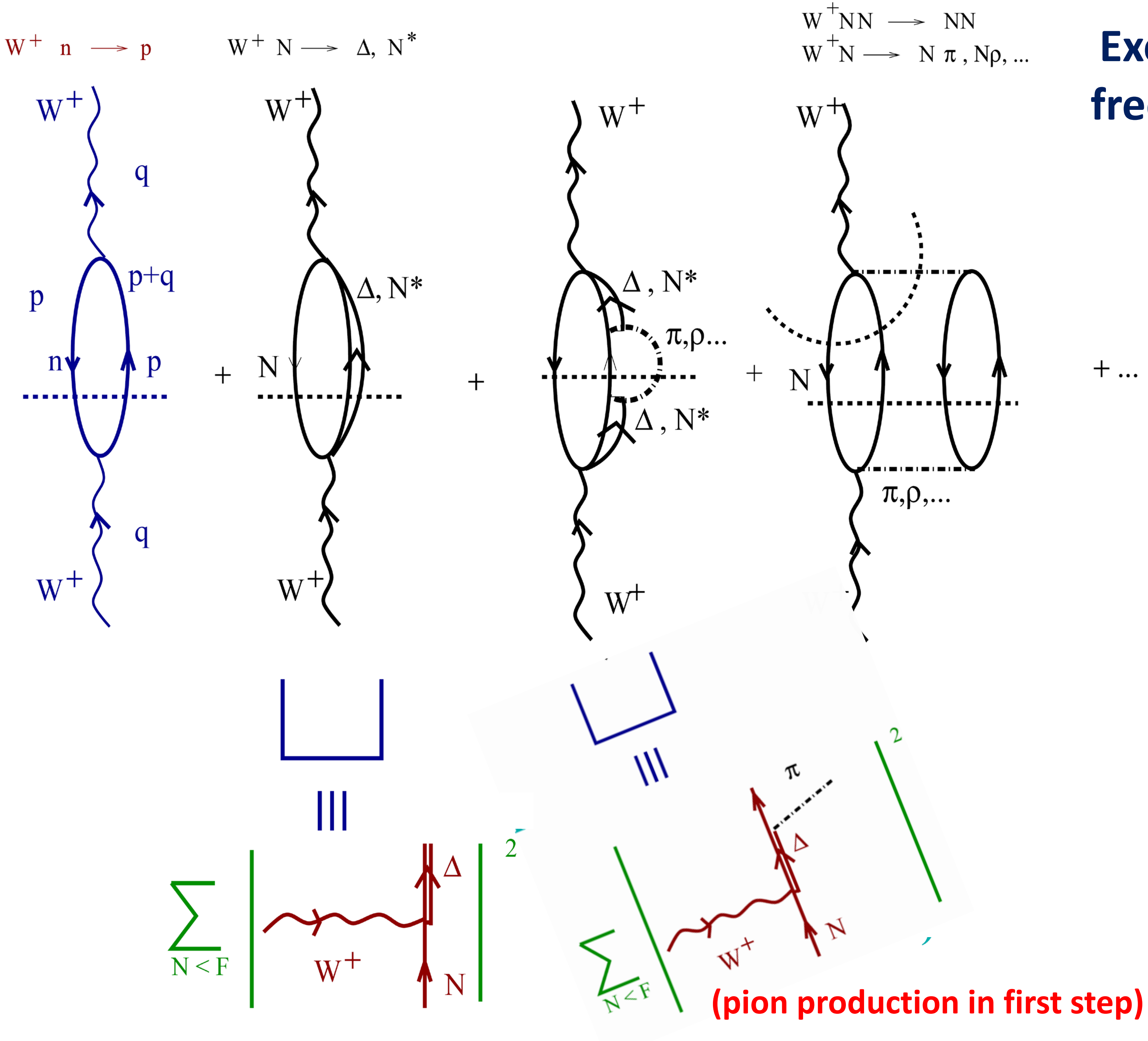
- Change of nucleon dispersion relation:
  - hole ⇒ Interacting Fermi sea (SF)
  - particle ⇒ Interaction of the ejected nucleon with the final nuclear state (FSI)

$$G(p) \to \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\vec{p}\,)}{p^0-\omega-i\epsilon} + \int_{\mu}^{+\infty} d\omega \frac{S_p(\omega,\vec{p}\,)}{p^0-\omega+i\epsilon}$$

The hole and particle spectral functions are related to nucleon self-energy $\boxed{\Sigma}$ in the medium,

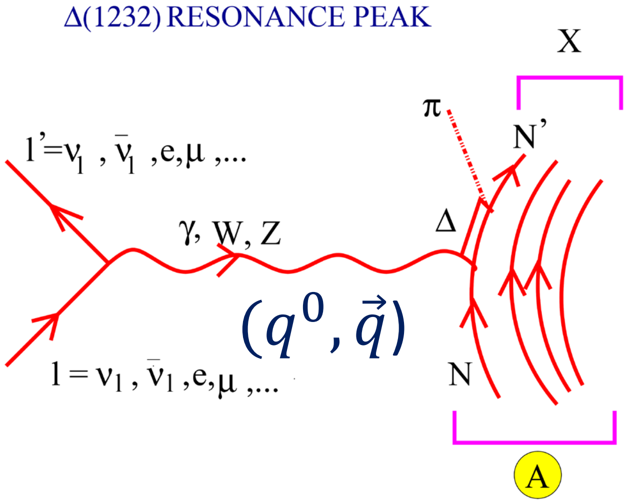
$$G(p) = \frac{n(\vec{p}\,)}{p^0-\varepsilon(\vec{p})-i\epsilon} + \frac{1-n(\vec{p}\,)}{p^0-\varepsilon(\vec{p})+i\epsilon}$$

Excitation of $\Delta(1232)$ degrees of freedom, $T = 3/2$ and $J^P = 3/2^+$



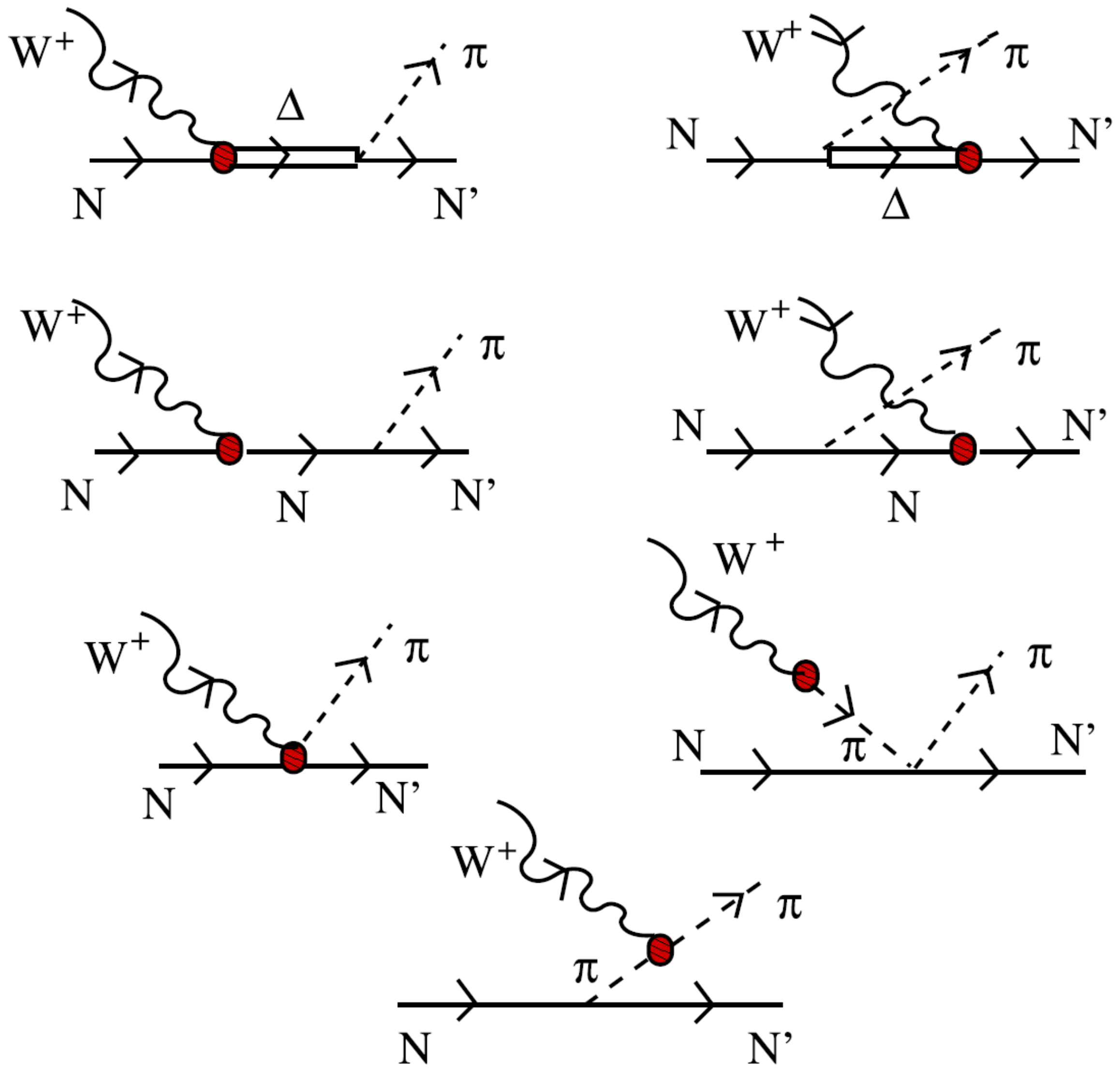
(pion production in first step)

- energy transfer should be sufficiently large…
- because of the large $\pi N \Delta$ coupling, the <u>properties of pion and Δ inside of a nuclear medium become important</u>



Δh contribution

first ingredient $W^{\pm}N \longrightarrow N'\pi$ (or $Z^0 N \longrightarrow N'\pi$ or $\gamma N \longrightarrow N'\pi$) in **vacuum, after nuclear corrections should be included…..**

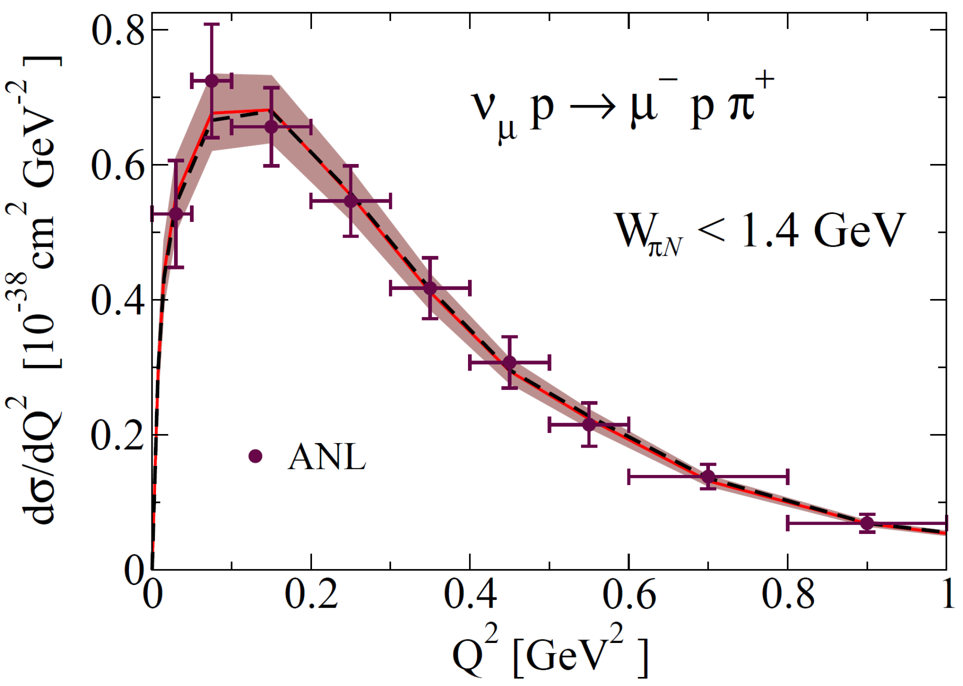


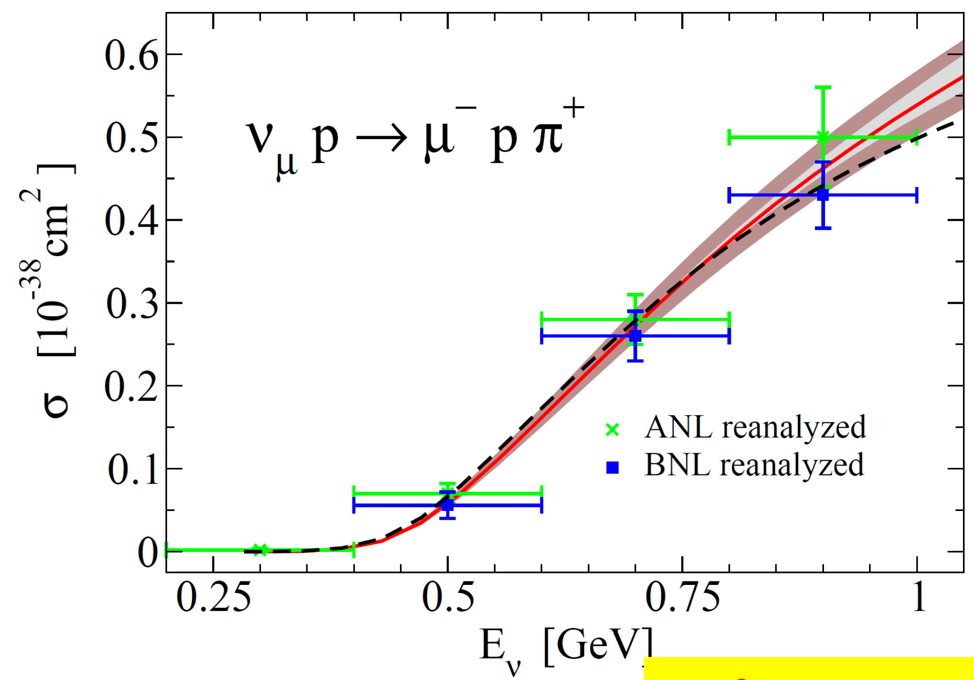
EFT involving pions and nucleons which implements:

- **non-resonant background determined by chiral symmetry and its pattern of spontaneous breaking**
- **unitarity in the dominant multipoles**

+ crossing symmetry+ N(1520)
+ phenomenological q^2 form-factors

Hernández+ JN+Valverde PRD76 (2007) 033005
PRD81 (2010) 085046 (deuteron effects in data)
PRD93 (2016) 014016 (Watson's theorem)
PRD95 (2017) 053007 (local terms and the $n\pi^+$channel)
PRD98 (2018) 073001 (comparison DCC model, T. Sato et al)



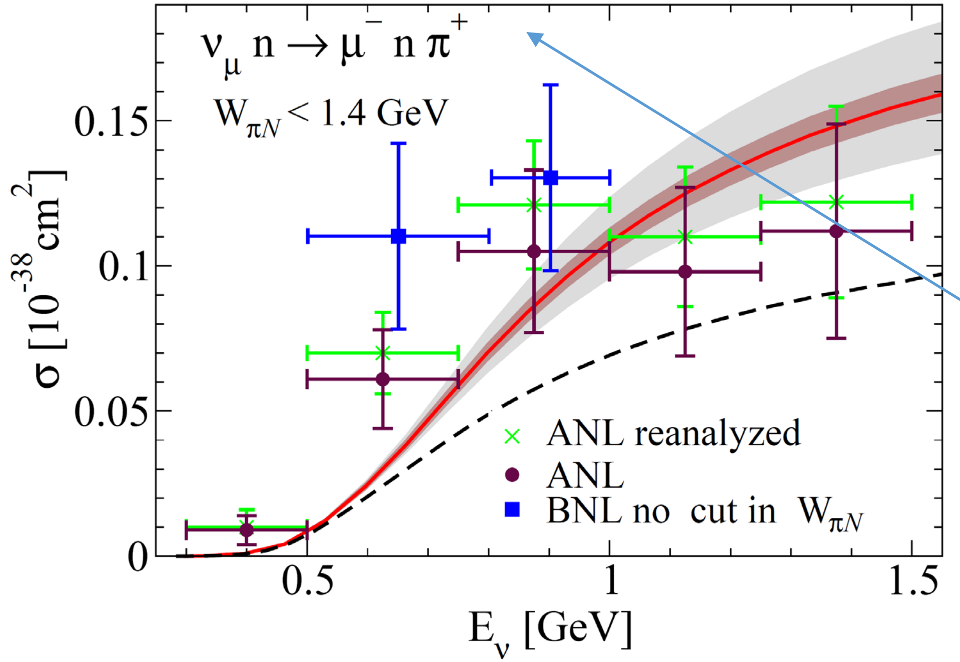


perfect agreement with the PCAC prediction ~1.2

$$C_5^A(q^2) = \underbrace{C_5^A(0)/(1-q^2/M_{A\Delta}^2)^2}$$

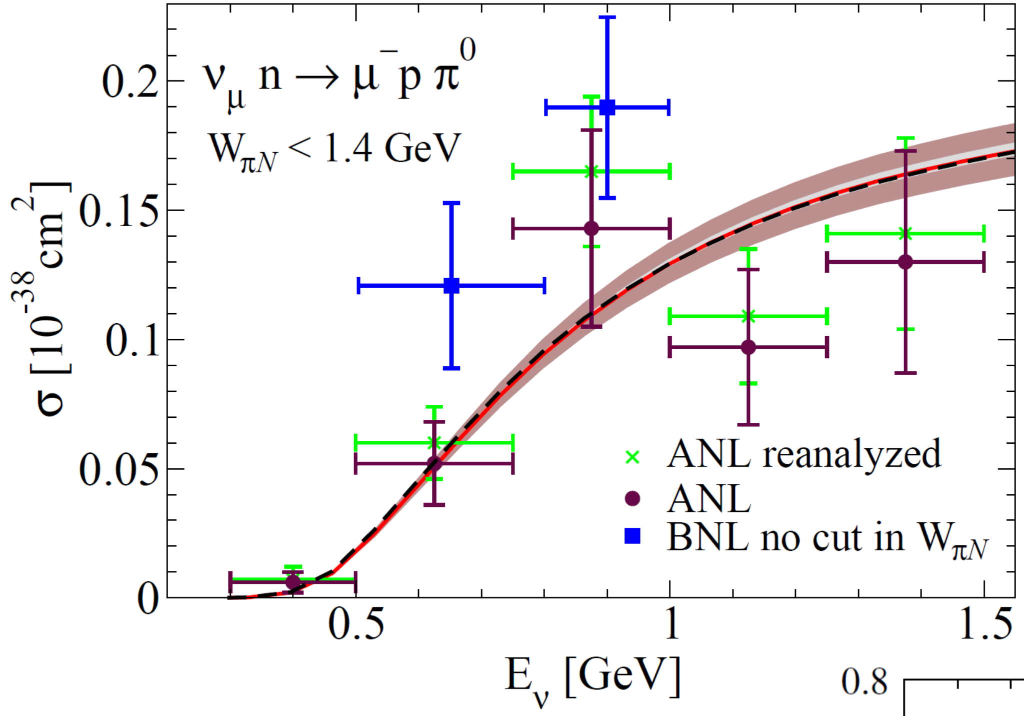
$C_5^A(0)=1.18\pm0.07$ $M_{A\Delta}=950\pm60$ MeV

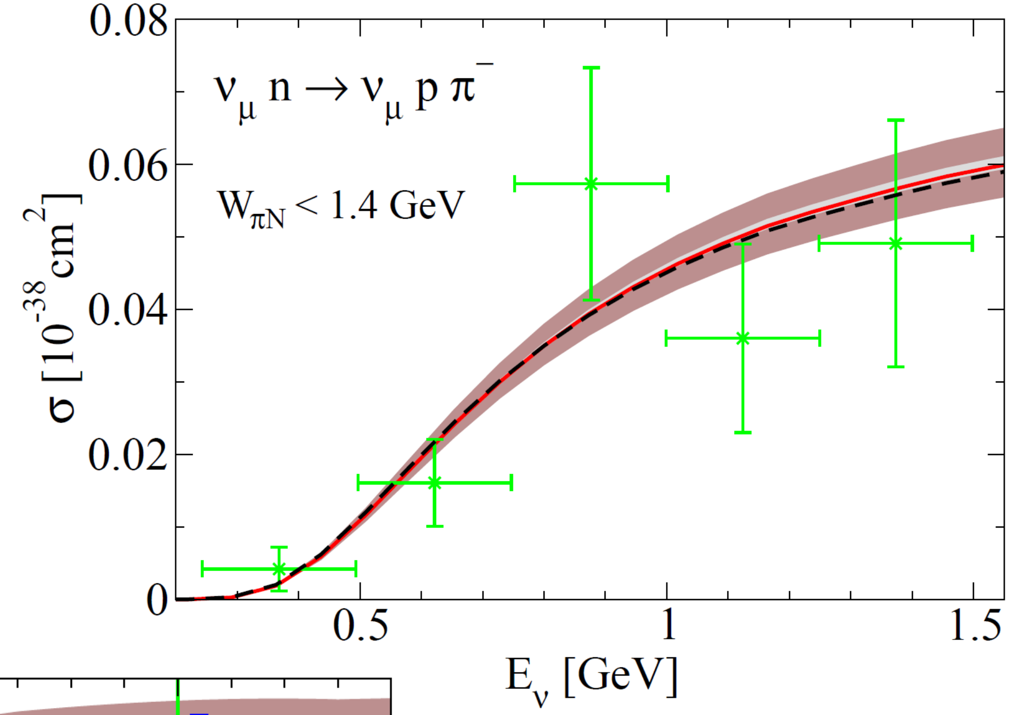
(dominant WNΔ axial coupling)

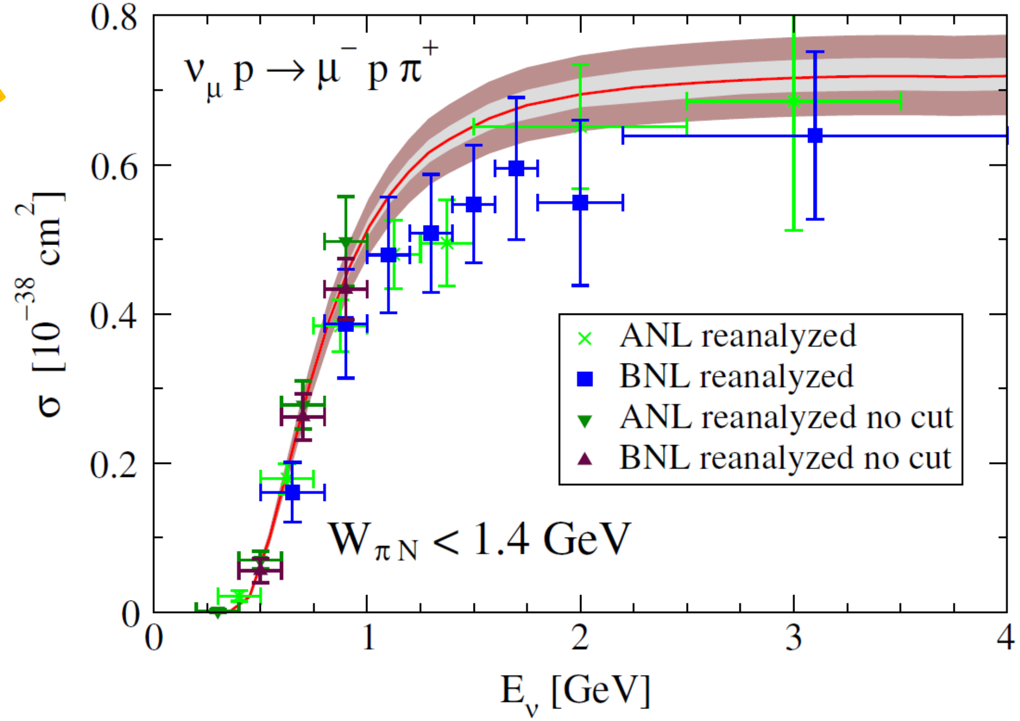


(channel for which the Δ is less dominant)

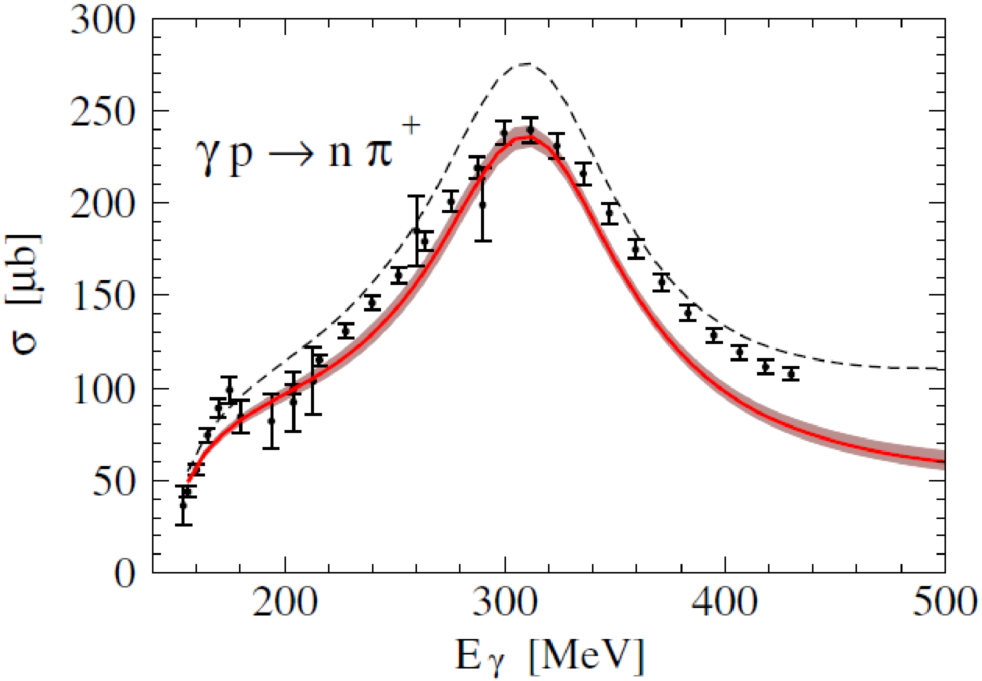
pion neutrino-production off nucleons

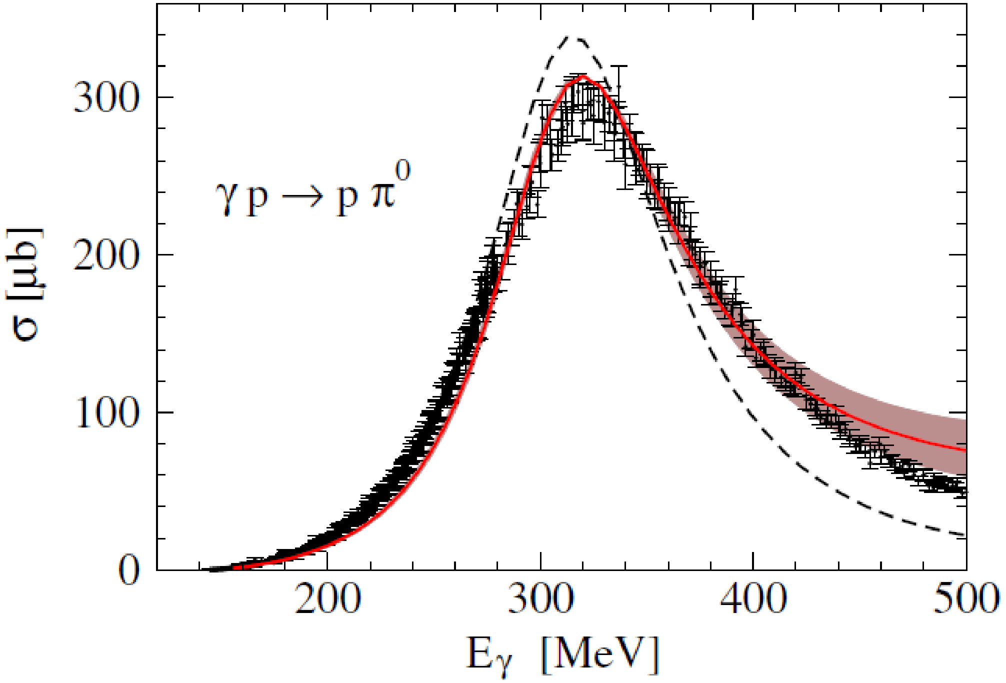


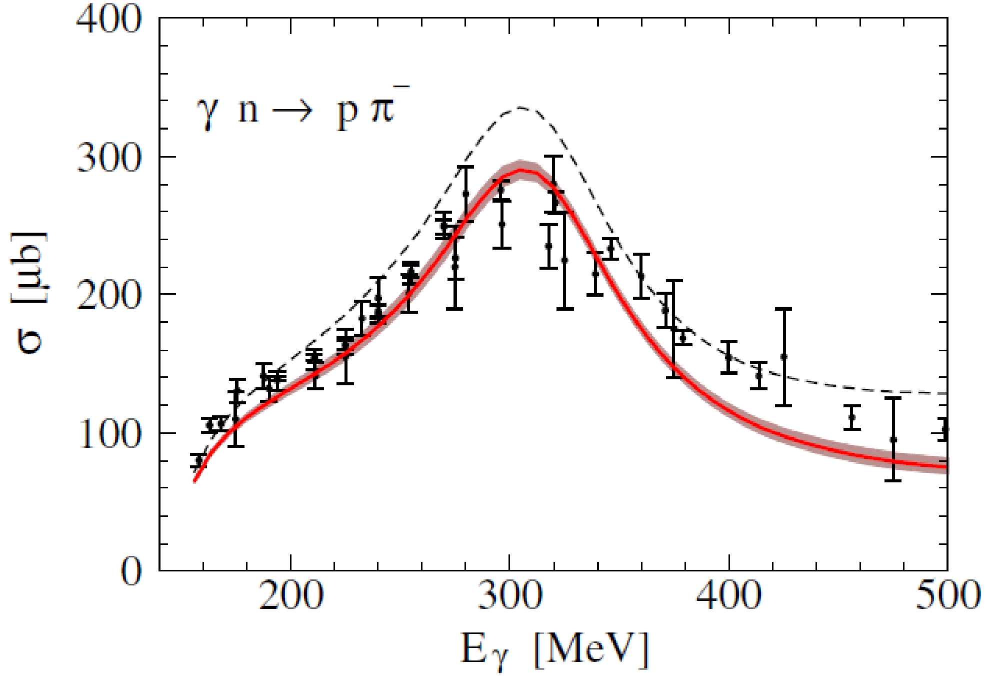




pion neutrino-production off nucleons

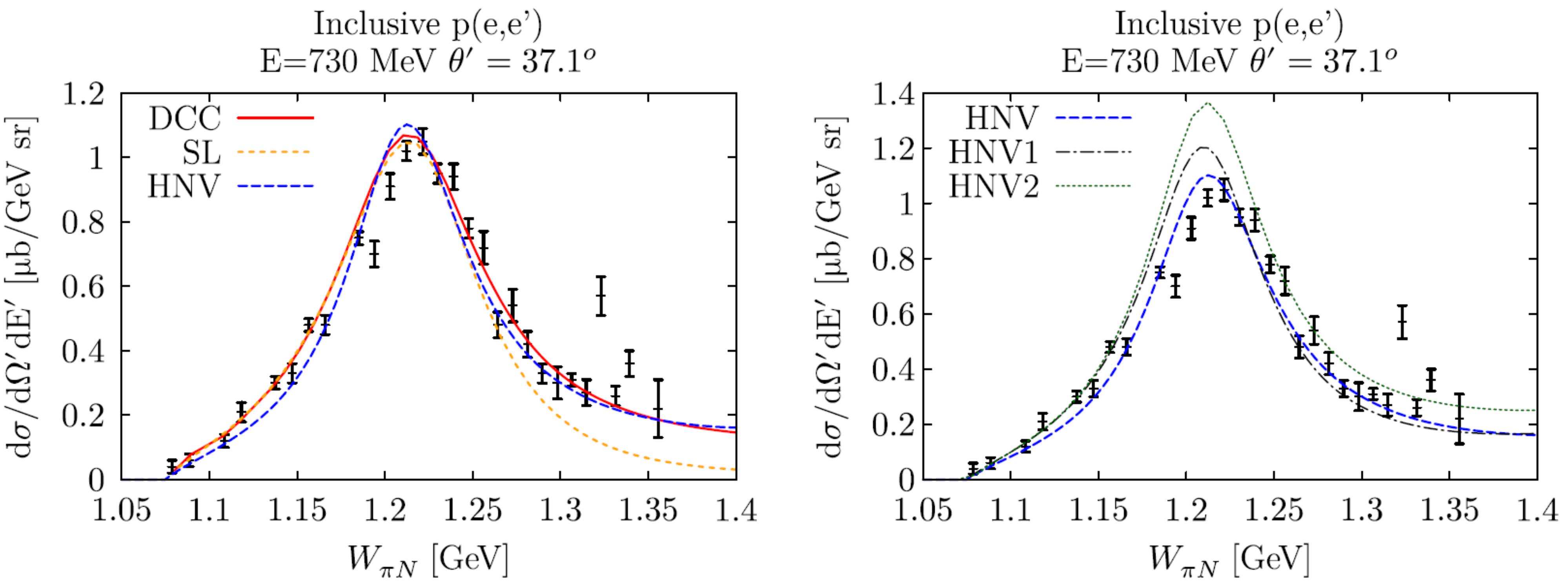


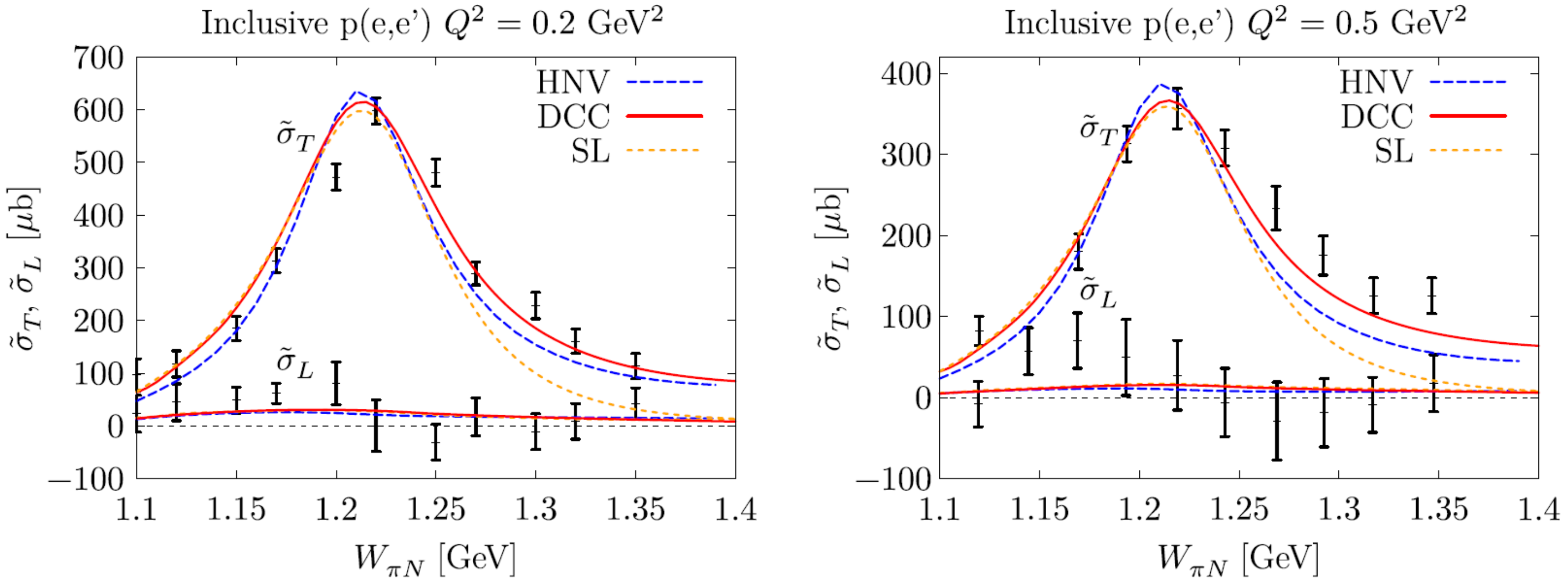




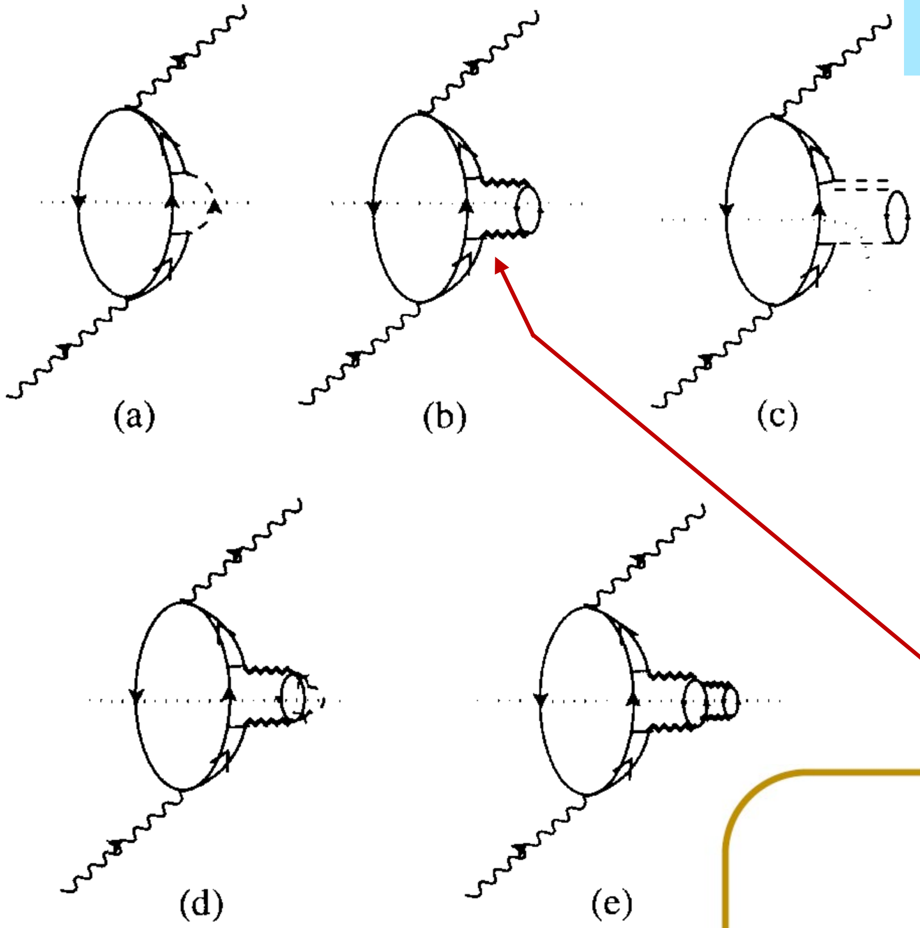
pion photoproduction off nucleons

DCC model: T. Sato et al., (Osaka)





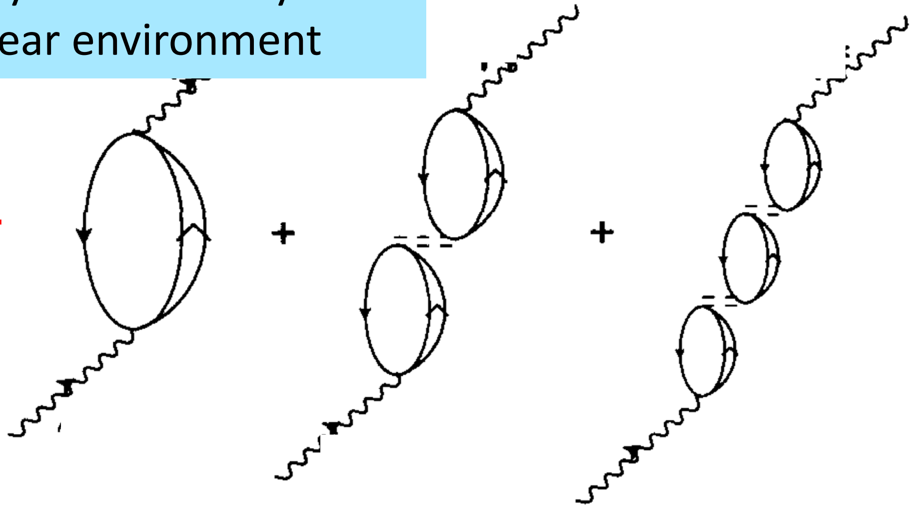
pion electroproduction off nucleons

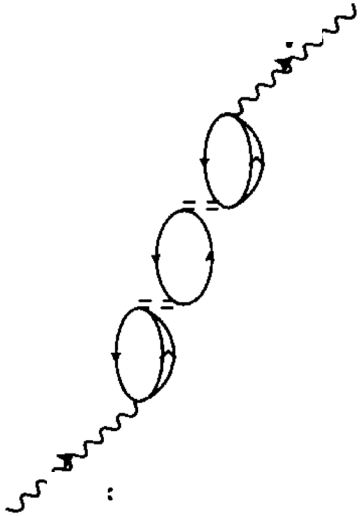


…independent of the probe. They are driven by hadron properties inside of nuclear environment

nuclear corrections

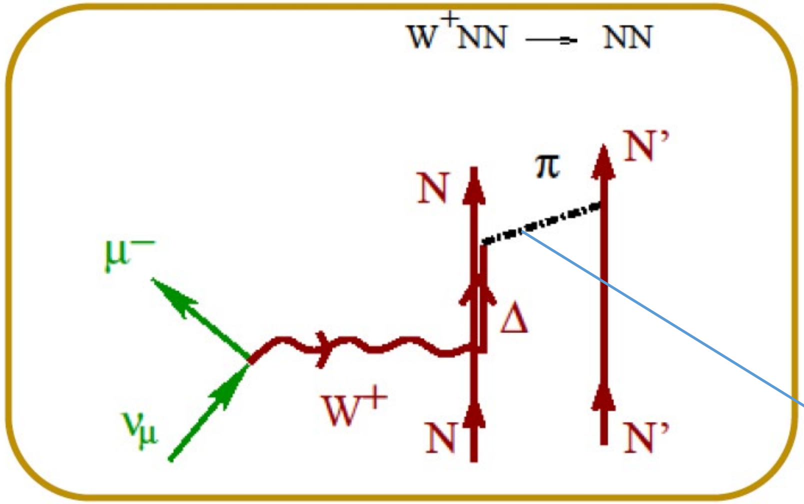
- Pauli blocking
- many body Δ decay modes: $\Delta N \rightarrow NN$
- RPA
- …..





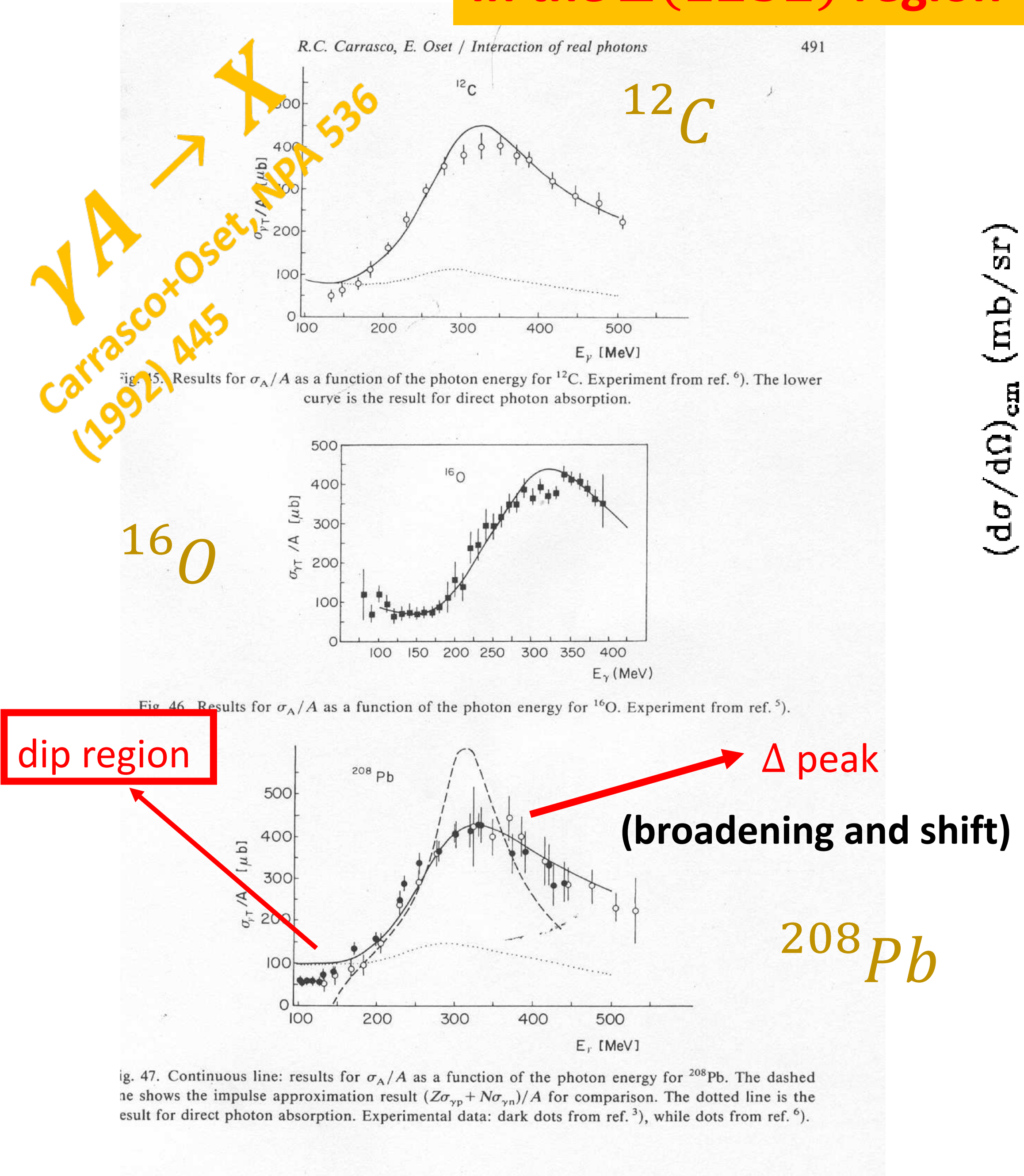
RPA corrections driven by an effective in medium ph-ph, ph-Δh and Δh-Δh interaction that includes SRC

Δ-selfenergy
Oset+Salcedo
NPA 468 (1987) 631



VIRTUAL pion
not only π: $\pi + \rho + \mathrm{SRC} + \mathrm{RPA} + \cdots$
(Effective NN interaction in the medium)

In the $\Delta(1232)$ region

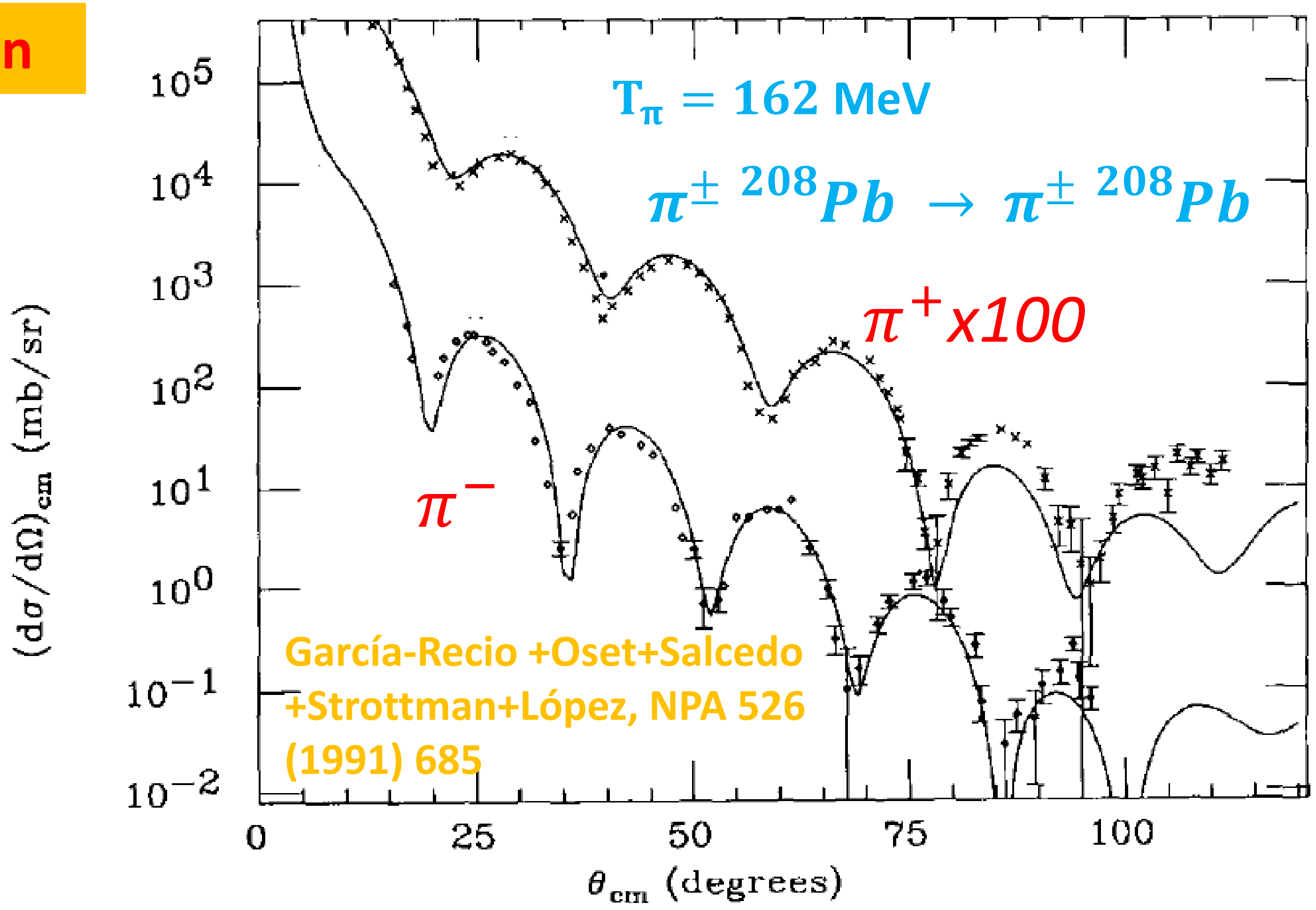


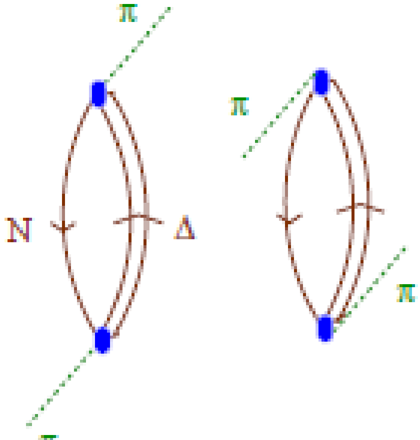
R.C. Carrasco, E. Oset / Interaction of real photons 491

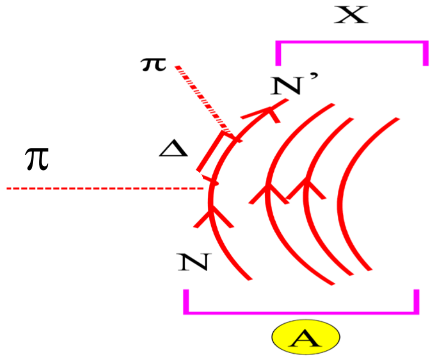
Fig. 45. Results for σ_A/A as a function of the photon energy for ^{12}C. Experiment from ref. [6]). The lower curve is the result for direct photon absorption.

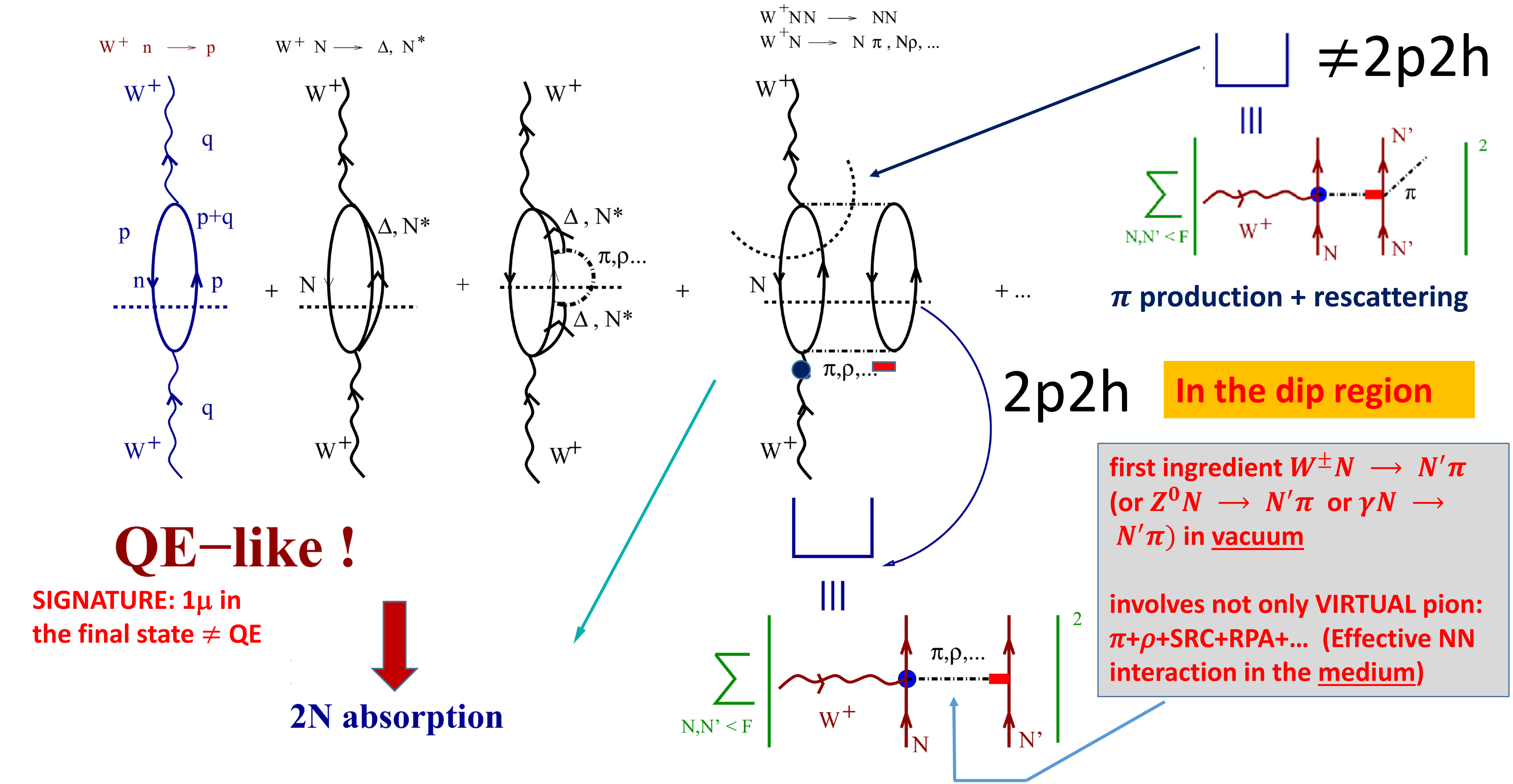
Fig. 46. Results for σ_A/A as a function of the photon energy for ^{16}O. Experiment from ref. [5]).

Fig. 47. Continuous line: results for σ_A/A as a function of the photon energy for ^{208}Pb. The dashed line shows the impulse approximation result $(Z\sigma_{\gamma p}+N\sigma_{\gamma n})/A$ for comparison. The dotted line is the result for direct photon absorption. Experimental data: dark dots from ref. [3]), while dots from ref. [6]).



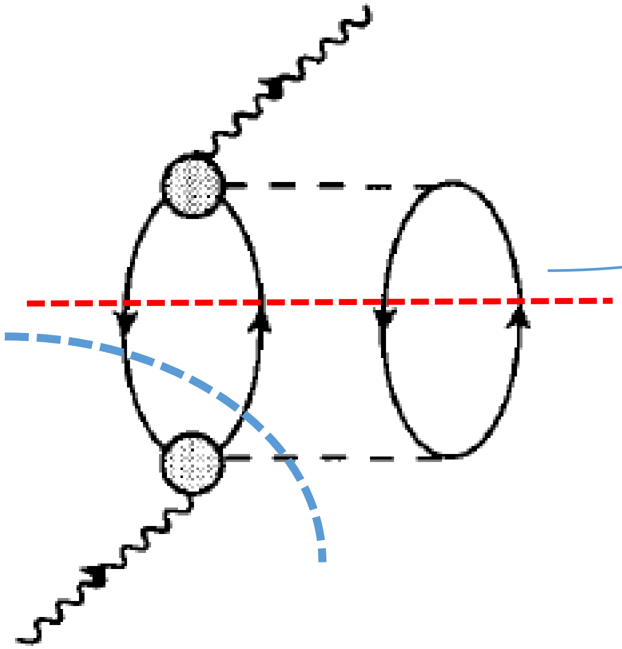


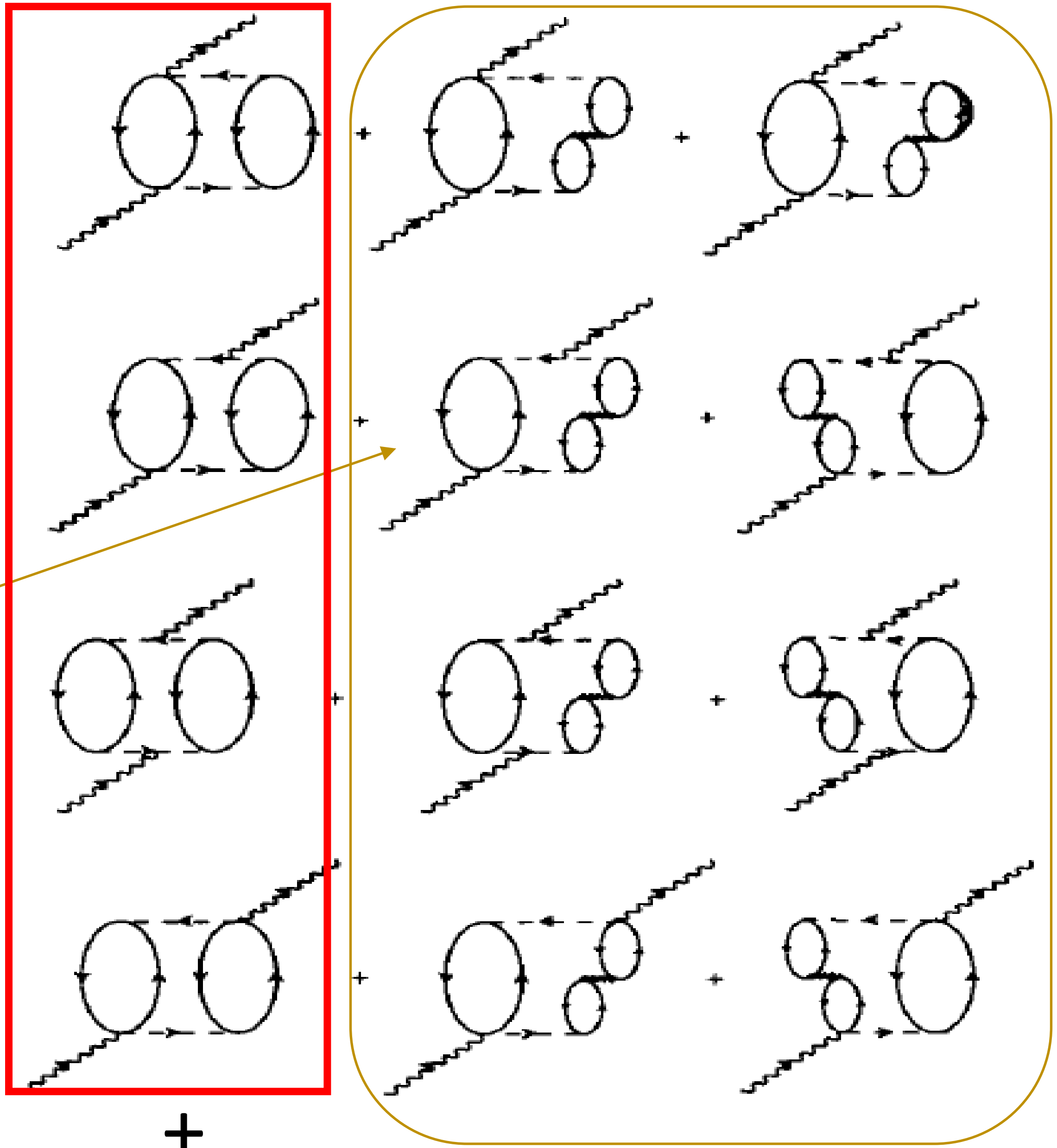




nuclear effect: populates the dip region and not dominated by the $\Delta(1232)$ driven mechanisms

2p2h (two body absorption) contributions





+.....

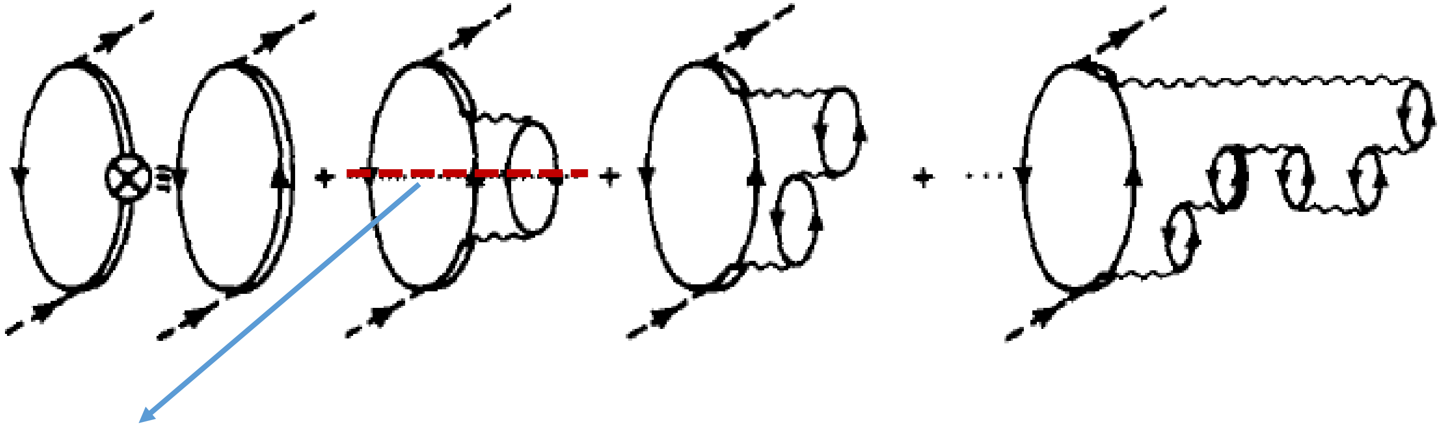
RPA corrections to 2p2h contributions

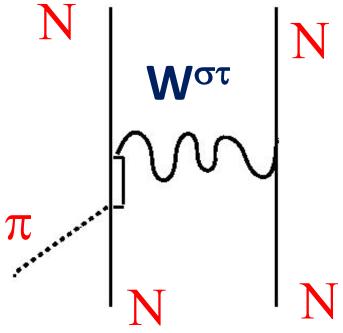
Two cuts: $\gamma^* NN \rightarrow NN$

$\gamma^* N \rightarrow N\pi$ (dressed)

Gil+Nieves+Oset.,NPA 627 (1997) 543 (extension of Carrasco+Oset NPA 536 (1992) 445 for real photons)

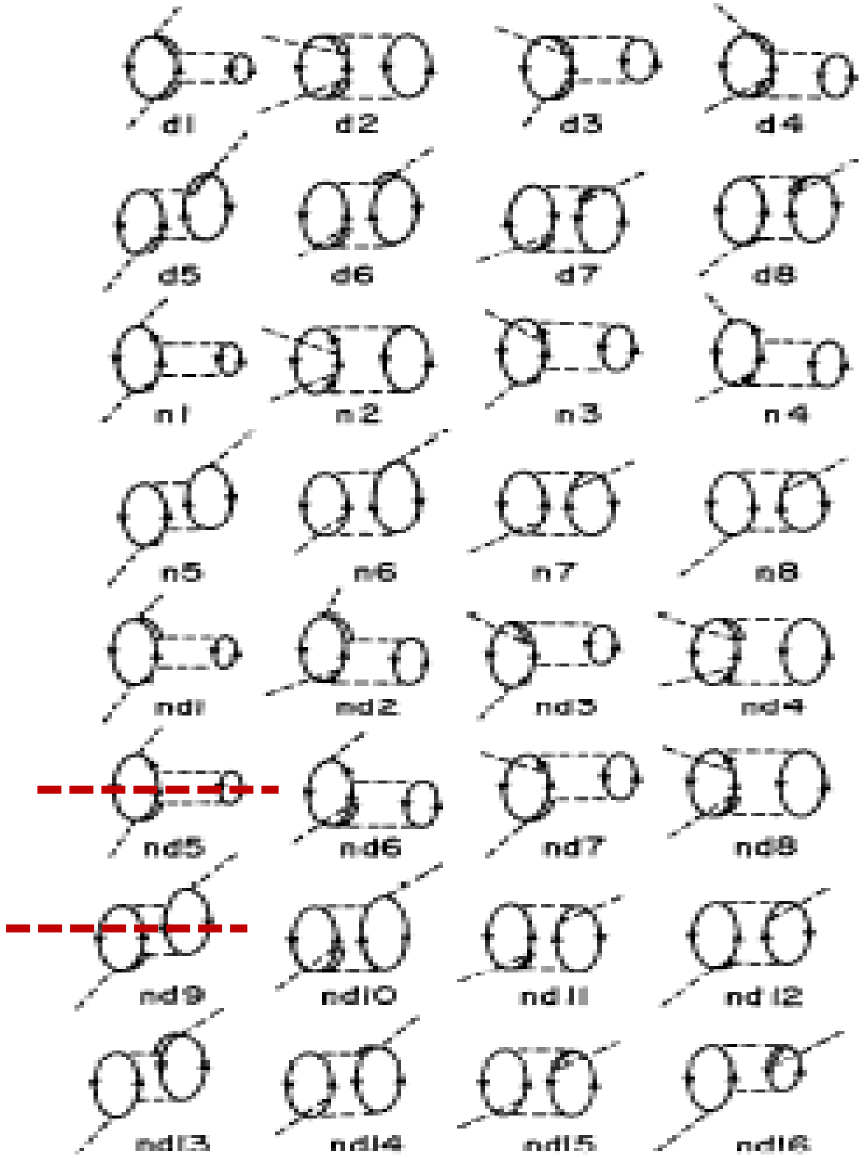
This work is the natural extension of previous work (1985-1993) in pion physics





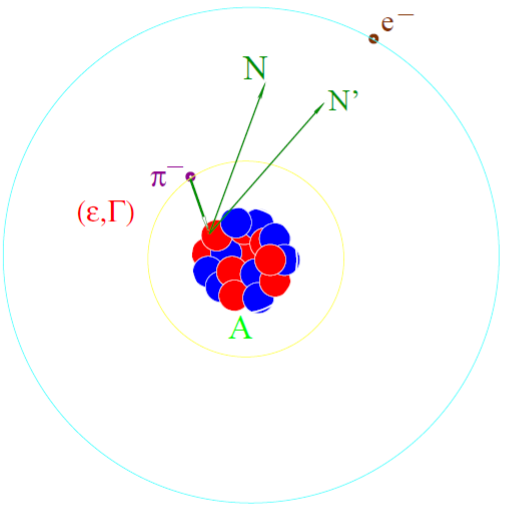
$\pi NN \rightarrow NN$

2p2h



$$2\omega V_1^{(s)}(\mathbf{r}) = -4\pi[(1+\varepsilon)(b_0+\Delta b_0(\mathbf{r}))f(T)\rho + (1+\varepsilon)b_1(\rho_n-\rho_p)$$
$$+ i(\mathrm{Im}\,B_0\,(1+\tfrac{1}{2}\varepsilon)2(\rho_p^2+\rho_p\rho_n)+\mathrm{Im}\,B_0^Q(T)(1+\tfrac{1}{2}\varepsilon)\rho^2)]$$

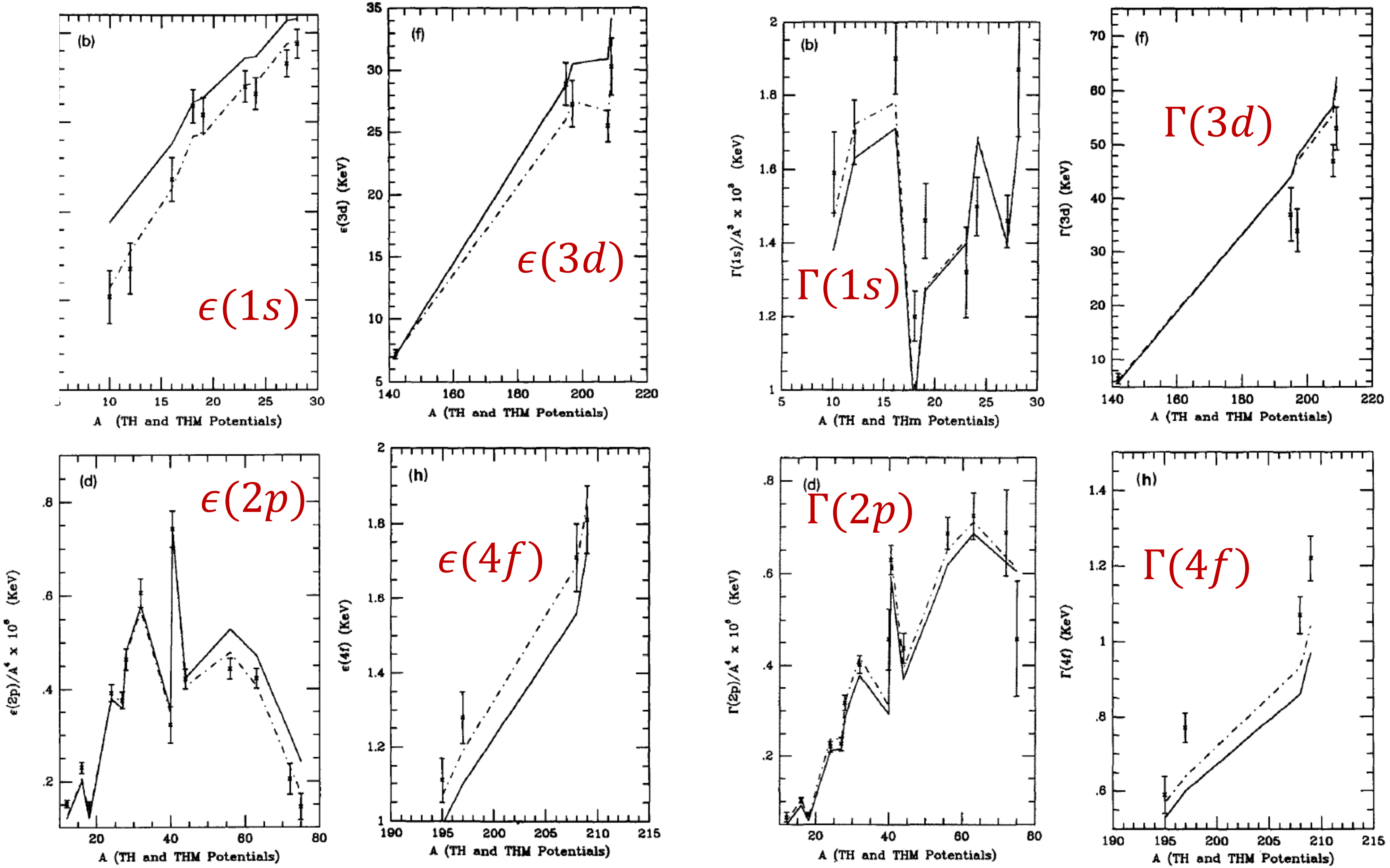
$$2\omega V_{\mathrm{opt}}^{(p)}(\mathbf{r}) = 4\pi\frac{M_N}{s}\left[\nabla\frac{P(\mathbf{r})}{1+4\pi g' P(\mathbf{r})}\nabla - \tfrac{1}{2}\varepsilon\Delta\left(\frac{P(\mathbf{r})}{1+4\pi g' P(\mathbf{r})}\right)\right]$$



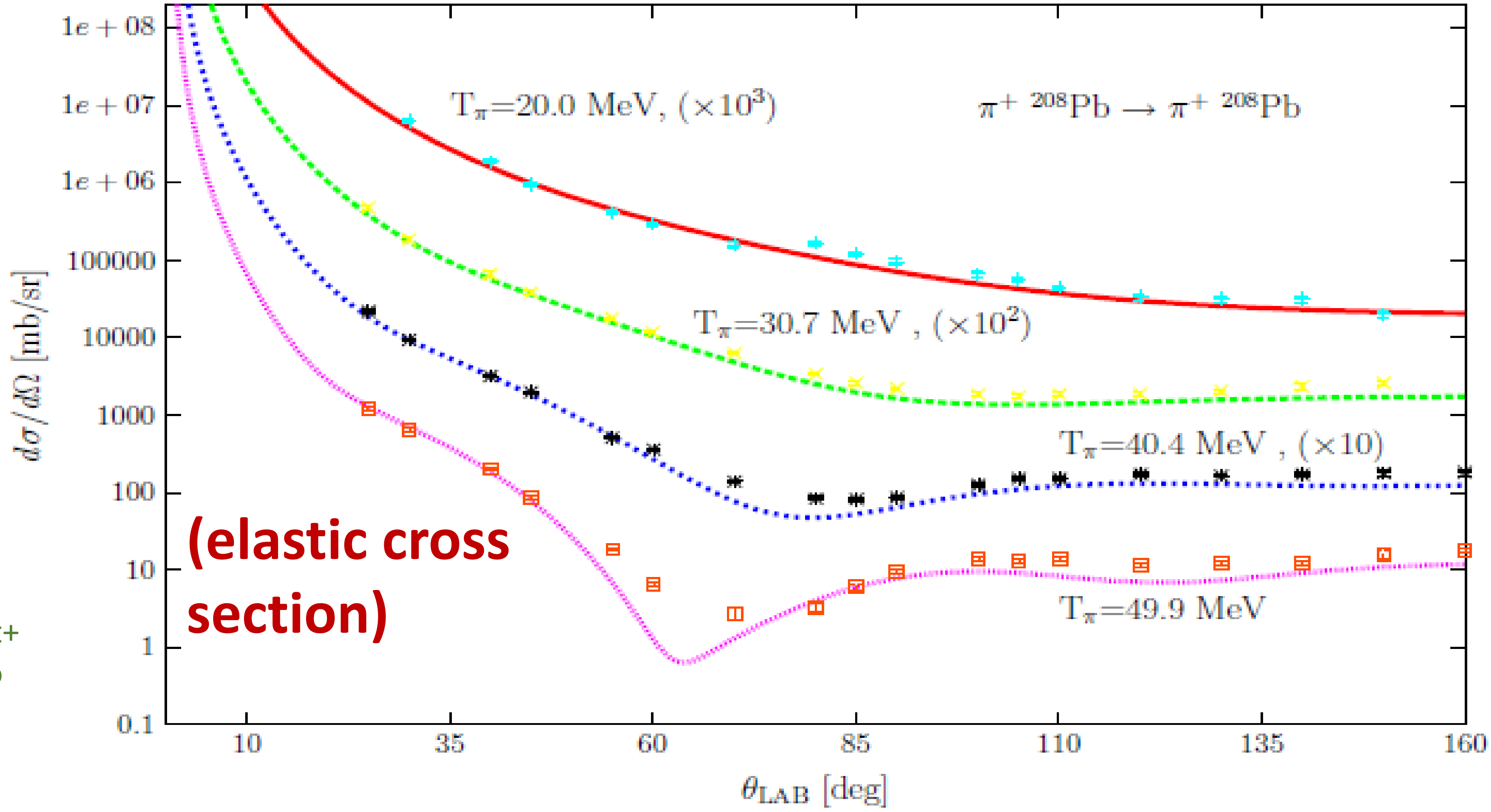
Pionic atoms

Precise experimental measurements : **shifts** $\boldsymbol{\varepsilon = B_{exp} - B_{em}}$ **and widths** $\boldsymbol{\Gamma}$. Information on the pion-nucleus interaction

[Nieves+Oset+García-Recio, NPA 554 (1993) 509]



$\pi^{\pm}$ – nucleus reactions at low energies

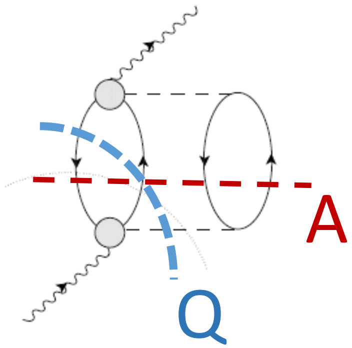


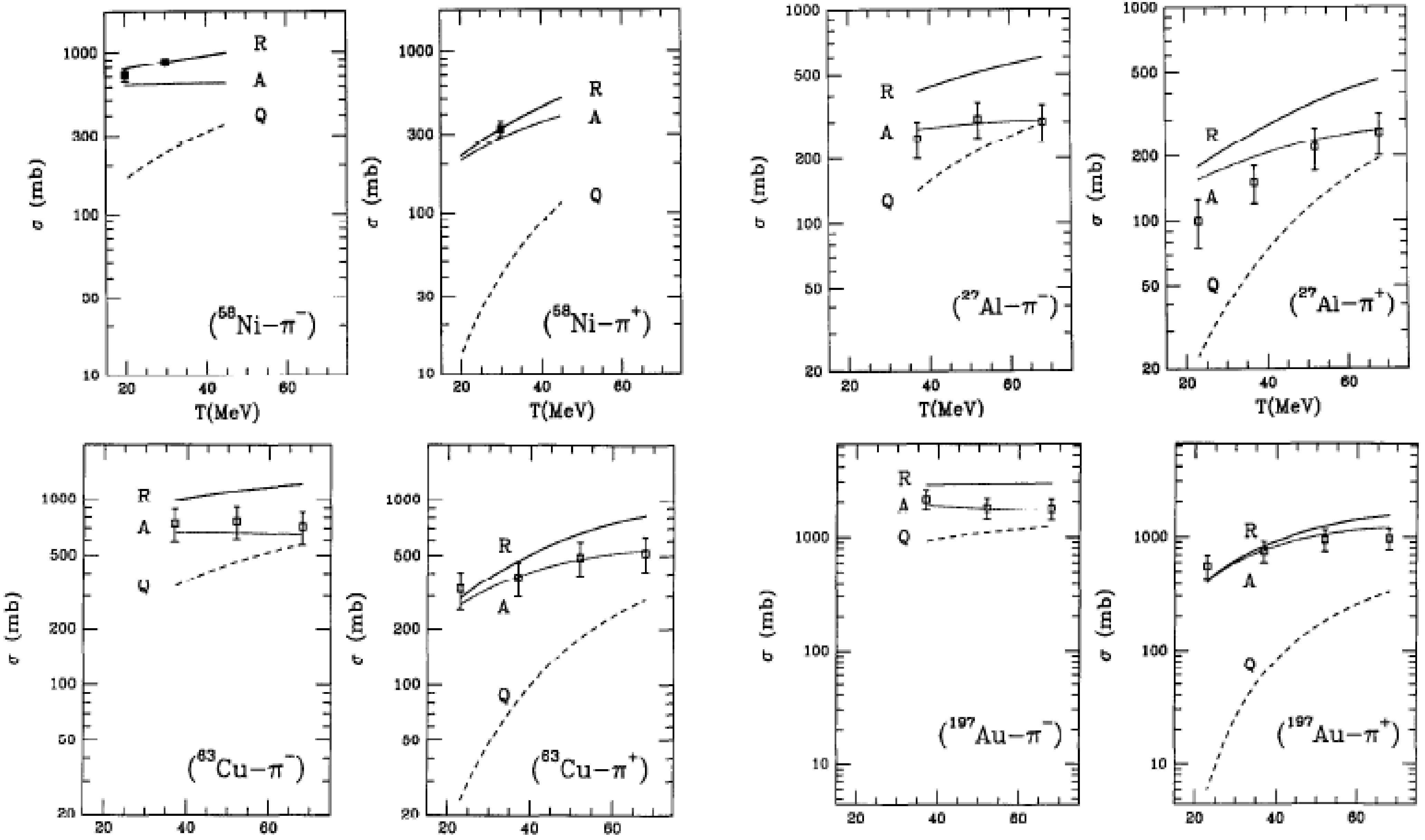
Nieves+Oset+
García-Recio
NPA 554
(1993) 554

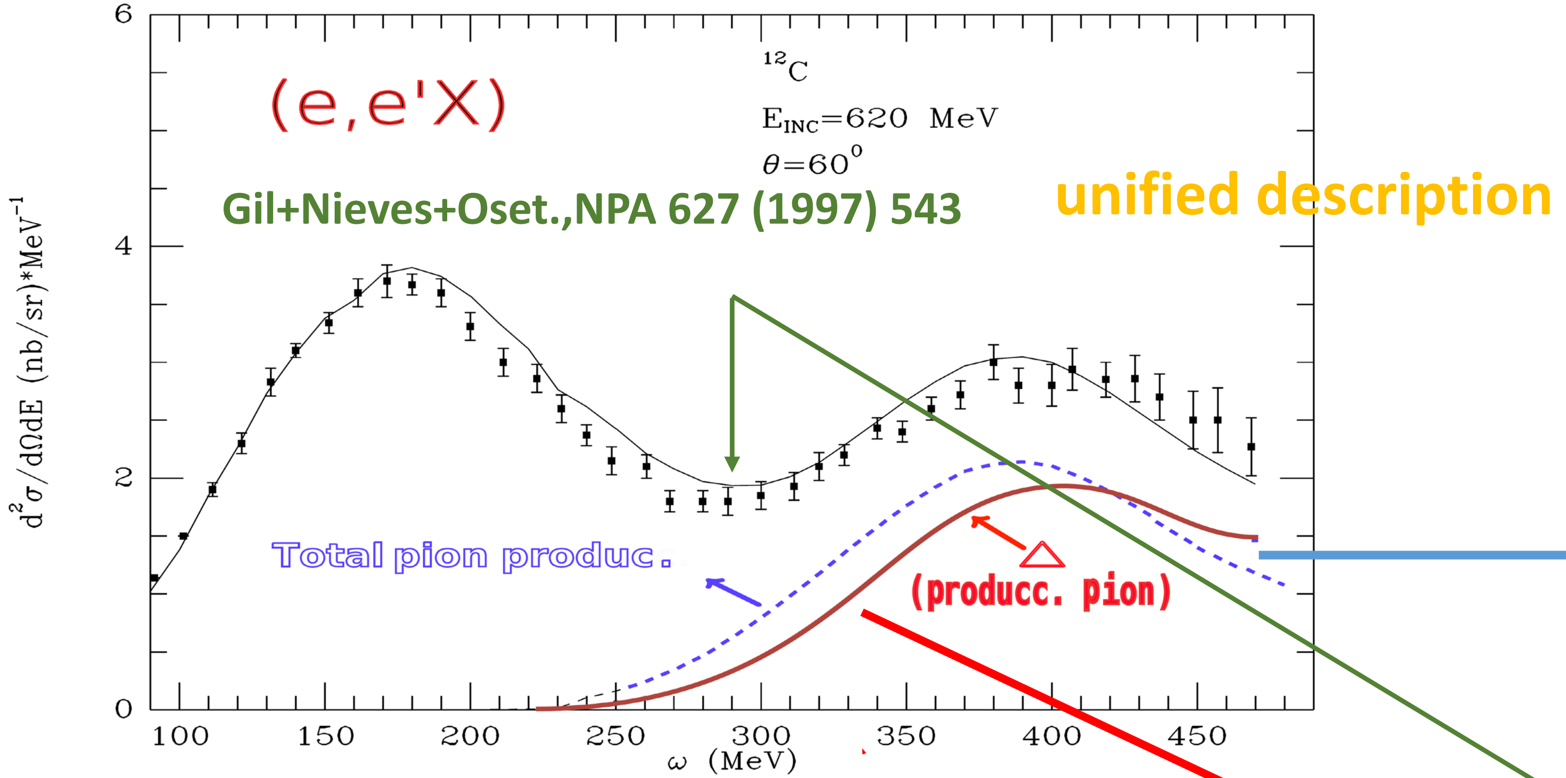
pions at these energies are non-resonant [kinetic energies well below production of $\Delta(1232)$]

Absorption + Quasielastic= Reaction cross section

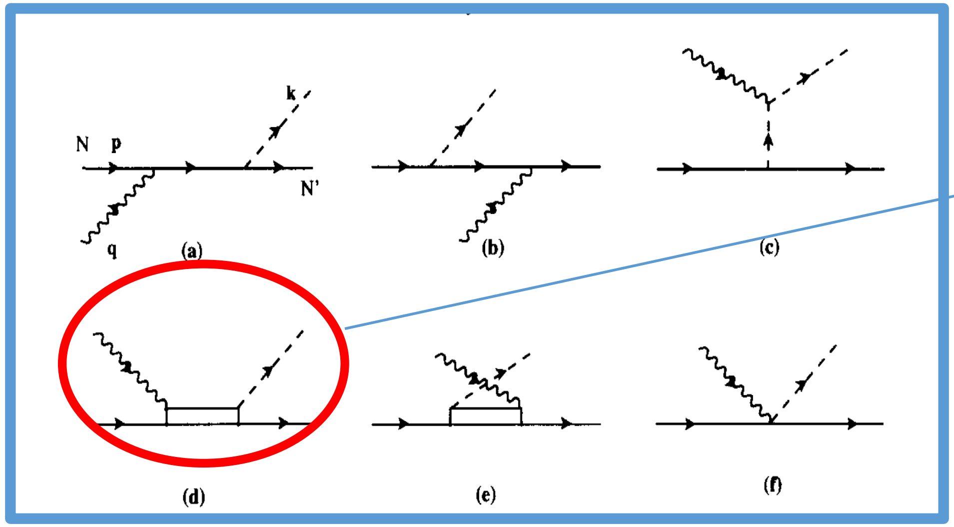
Q= pions which have changed either charge, energy or momentum



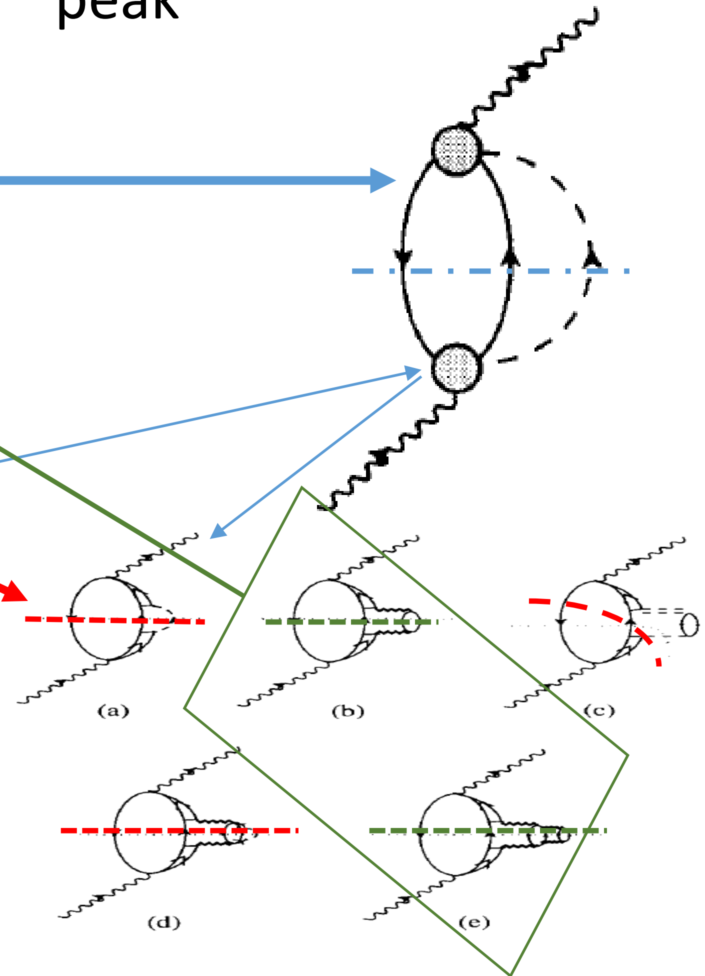




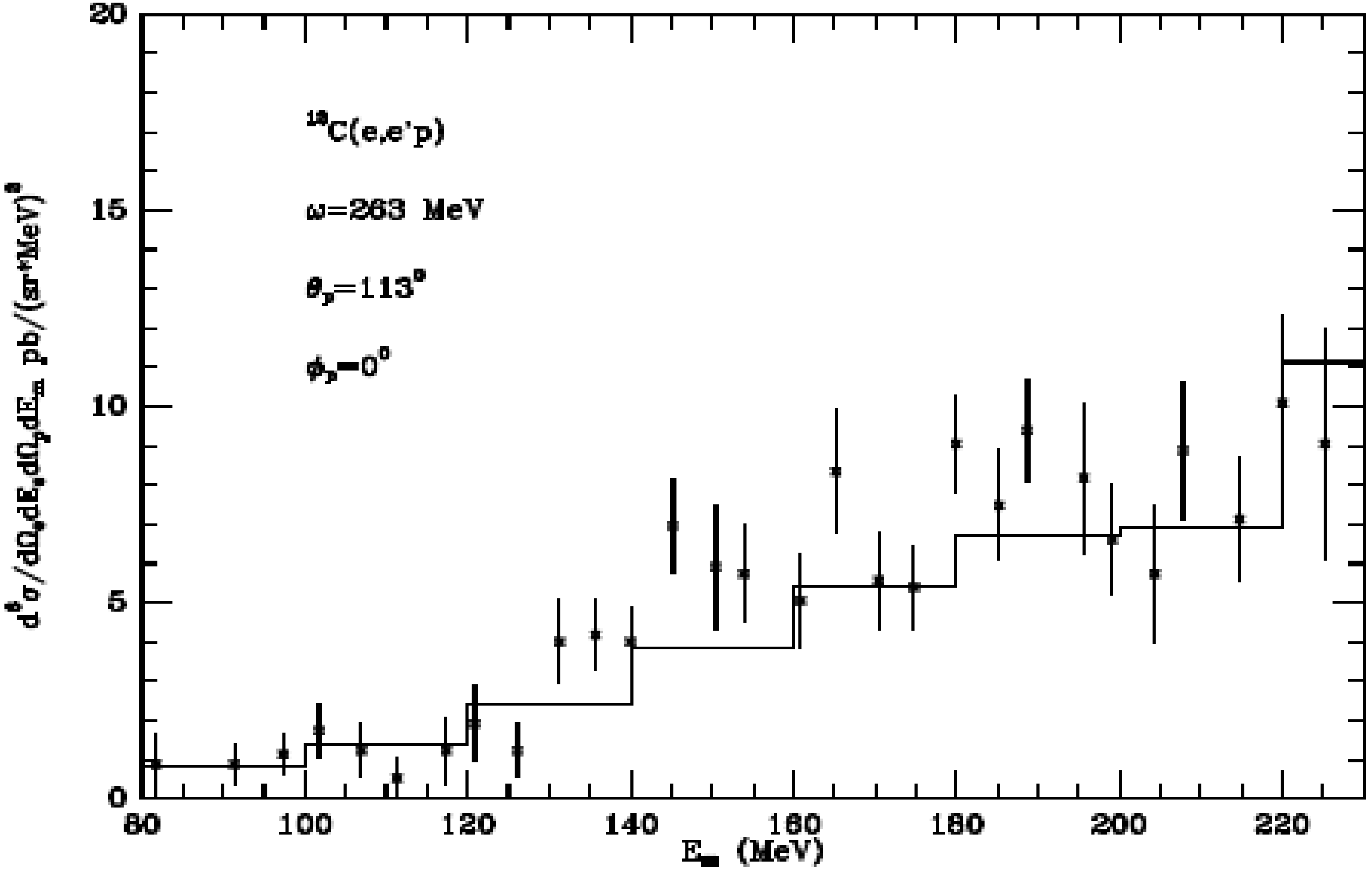
- Δ dominant component of the pion production contribution
- Missing strength both at the dip region and the Δ peak



one of the terms generates the Δ contribution



and by means of a Monte Carlo simulation we obtain cross sections for the processes $(e,e'N)$, $(e,e'NN)$, $(e,e'\pi)$, ...



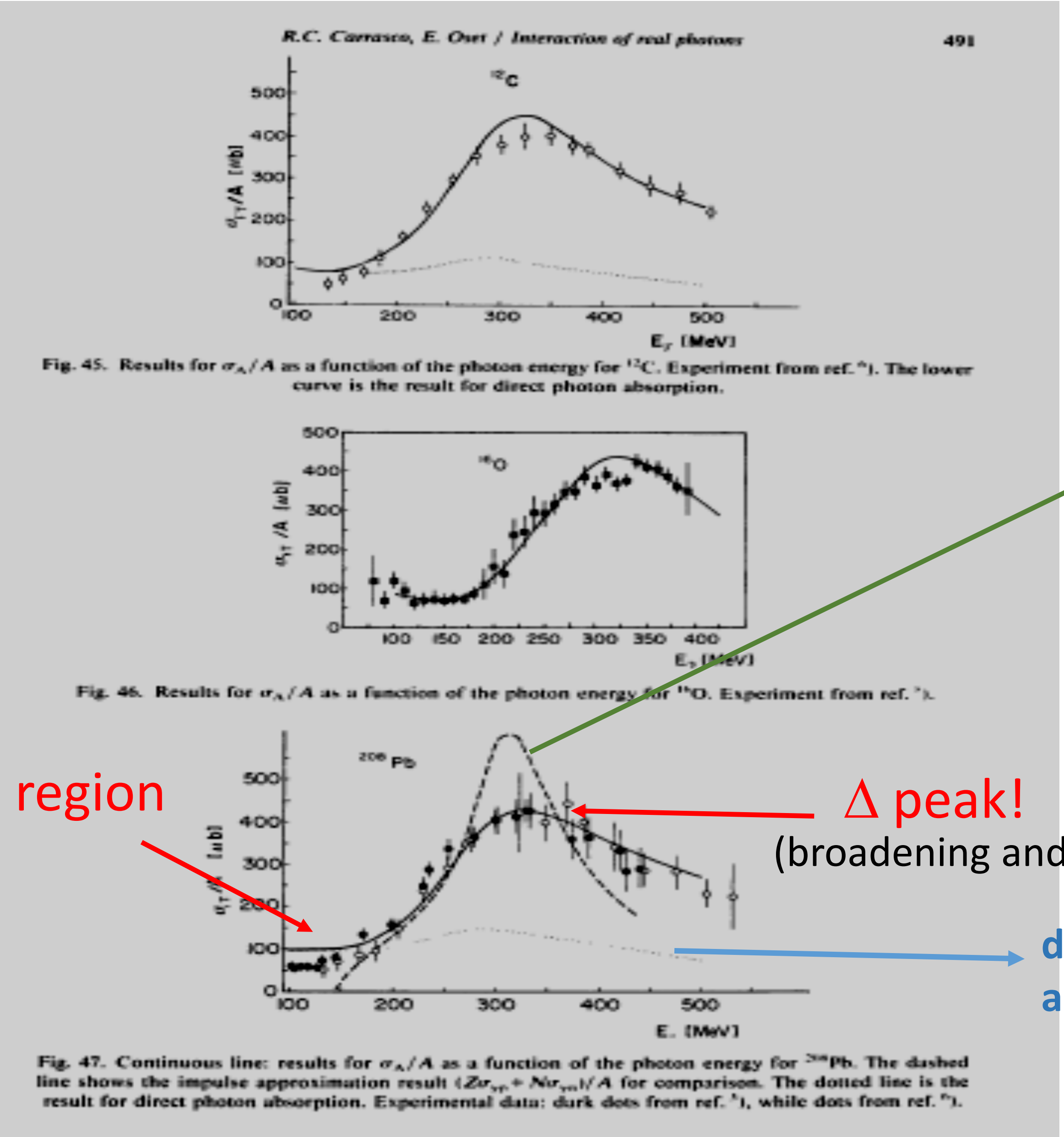


Fig. 45. Results for σ_A/A as a function of the photon energy for ^{12}C. Experiment from ref. [a]). The lower curve is the result for direct photon absorption.

Fig. 46. Results for σ_A/A as a function of the photon energy for ^{16}O. Experiment from ref. [7]).

Fig. 47. Continuous line: results for σ_A/A as a function of the photon energy for ^{208}Pb. The dashed line shows the impulse approximation result $(Z\sigma_{\gamma p} + N\sigma_{\gamma n})/A$ for comparison. The dotted line is the result for direct photon absorption. Experimental data: dark dots from ref. [3]), while dots from ref. [b]).

dip region

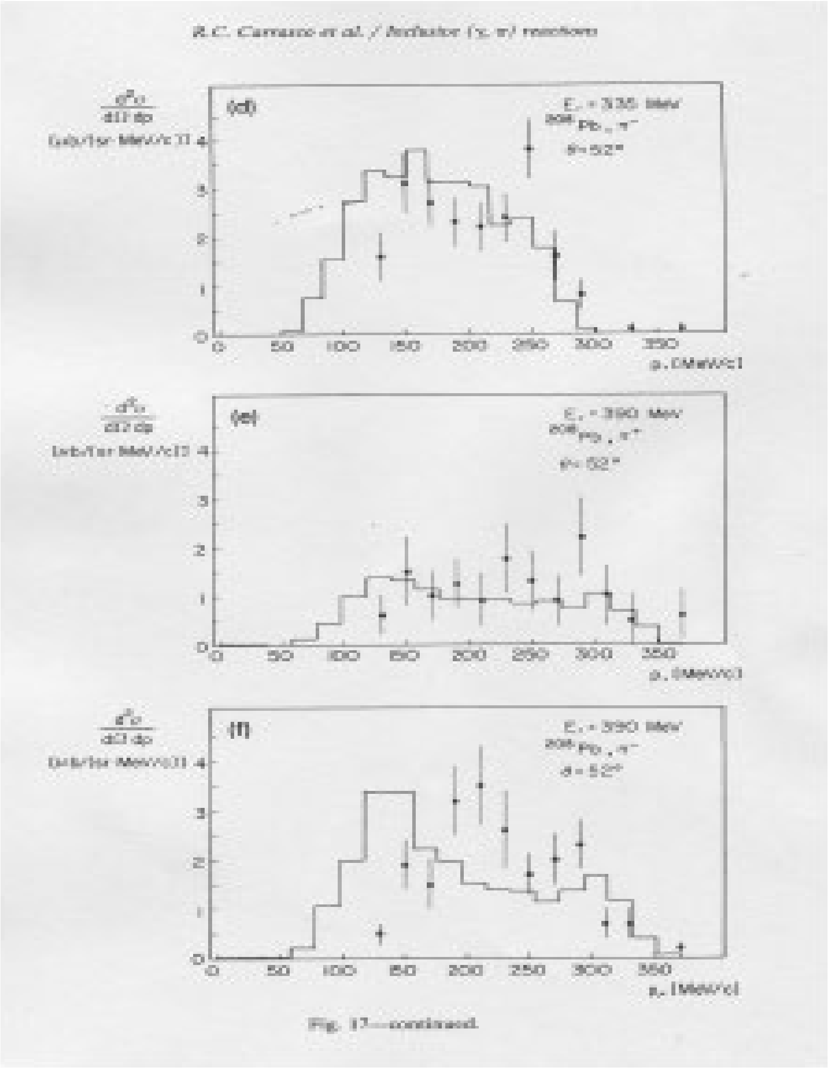
free Δ contribution

Δ peak!

(broadening and shift)

direct photon absorption

$$\gamma + A \to X$$



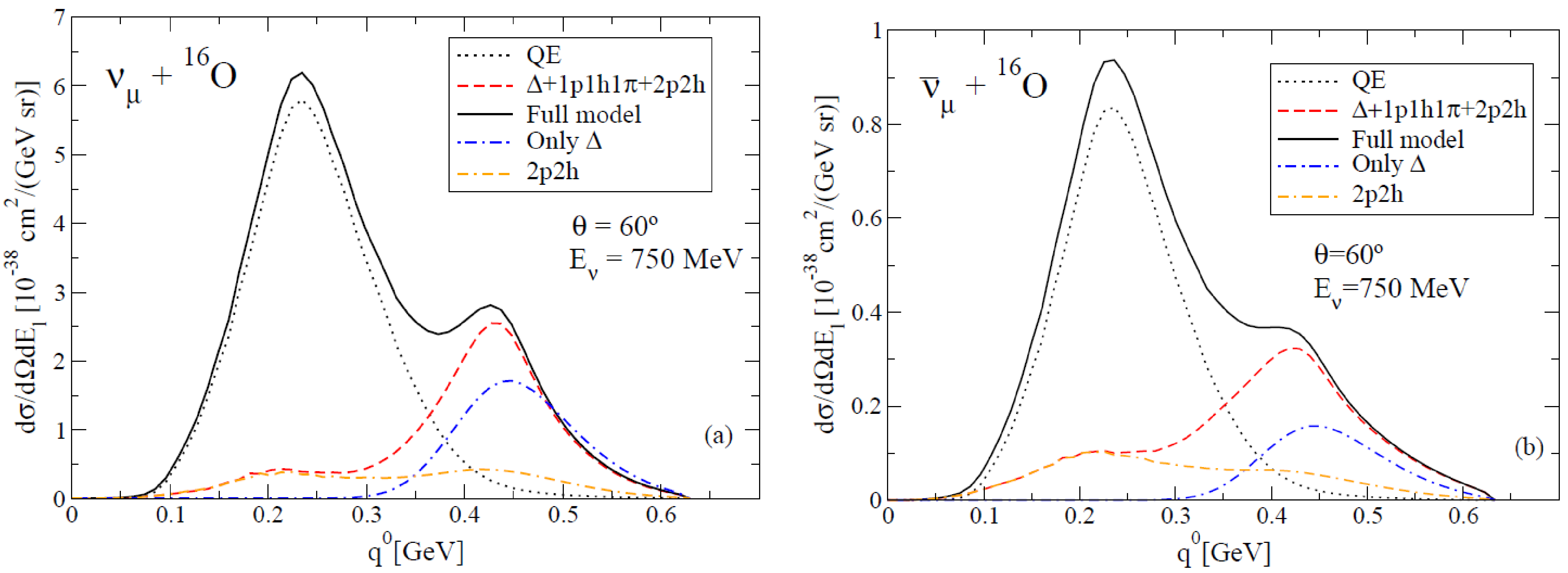
$$\gamma + {}^{208}\mathrm{Pb} \to X + \pi^{\pm}$$

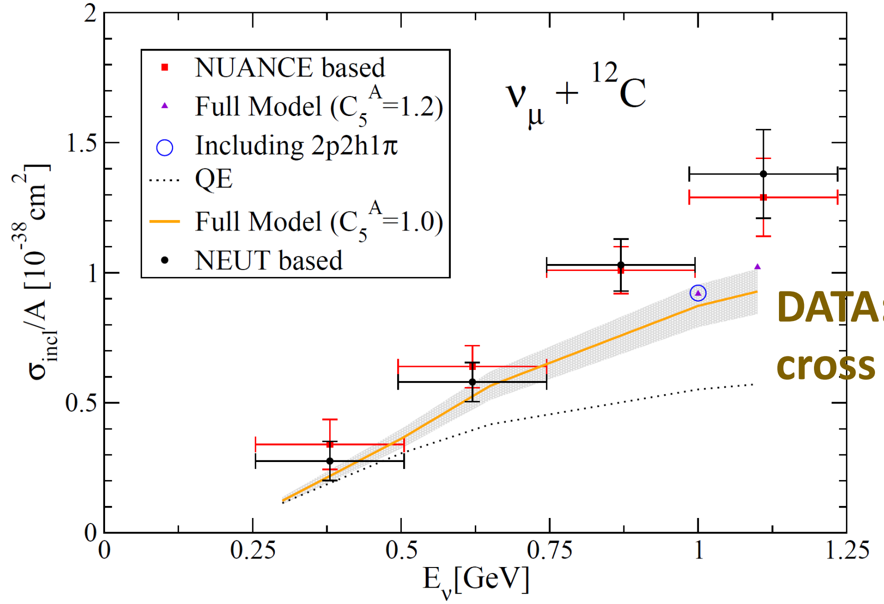
(ν_μ, μ^-) Results

INCLUSIVE CROSS SECTION

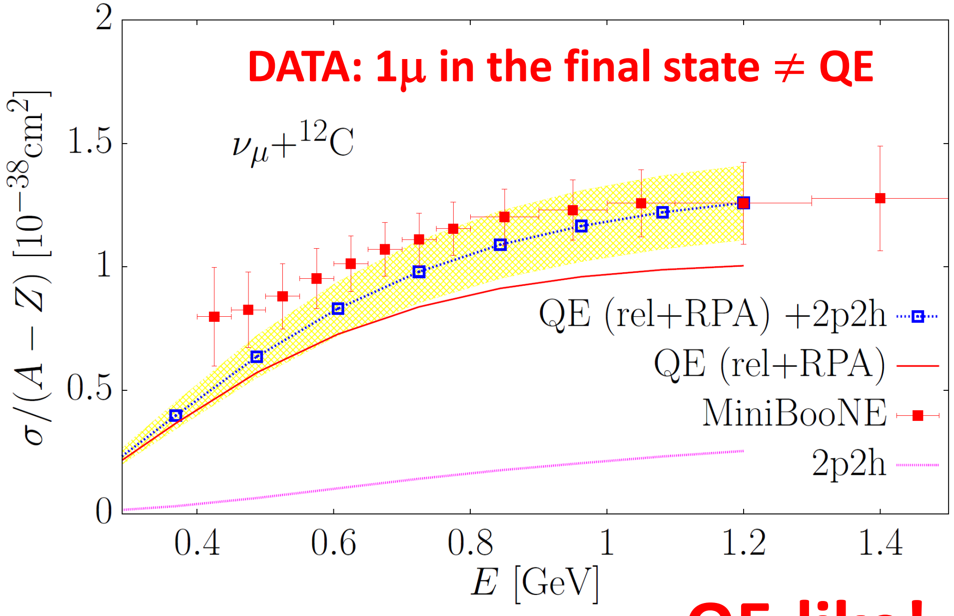
PRC 83 (2011) 045501 [M_A= 1.049 GeV]

MICROSCOPIC MODEL: PREDICTIONS (NO FITTED PARAMETERS) FROM THE QE to the Δ PEAKS, INCLUDING THE DIP REGION





DATA: TOTAL inclusive cross section



QE-like!

MiniBooNE CCQE-like double differential cross section $\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}$

We define a **merit function** and consider our **QE+2p2h results**

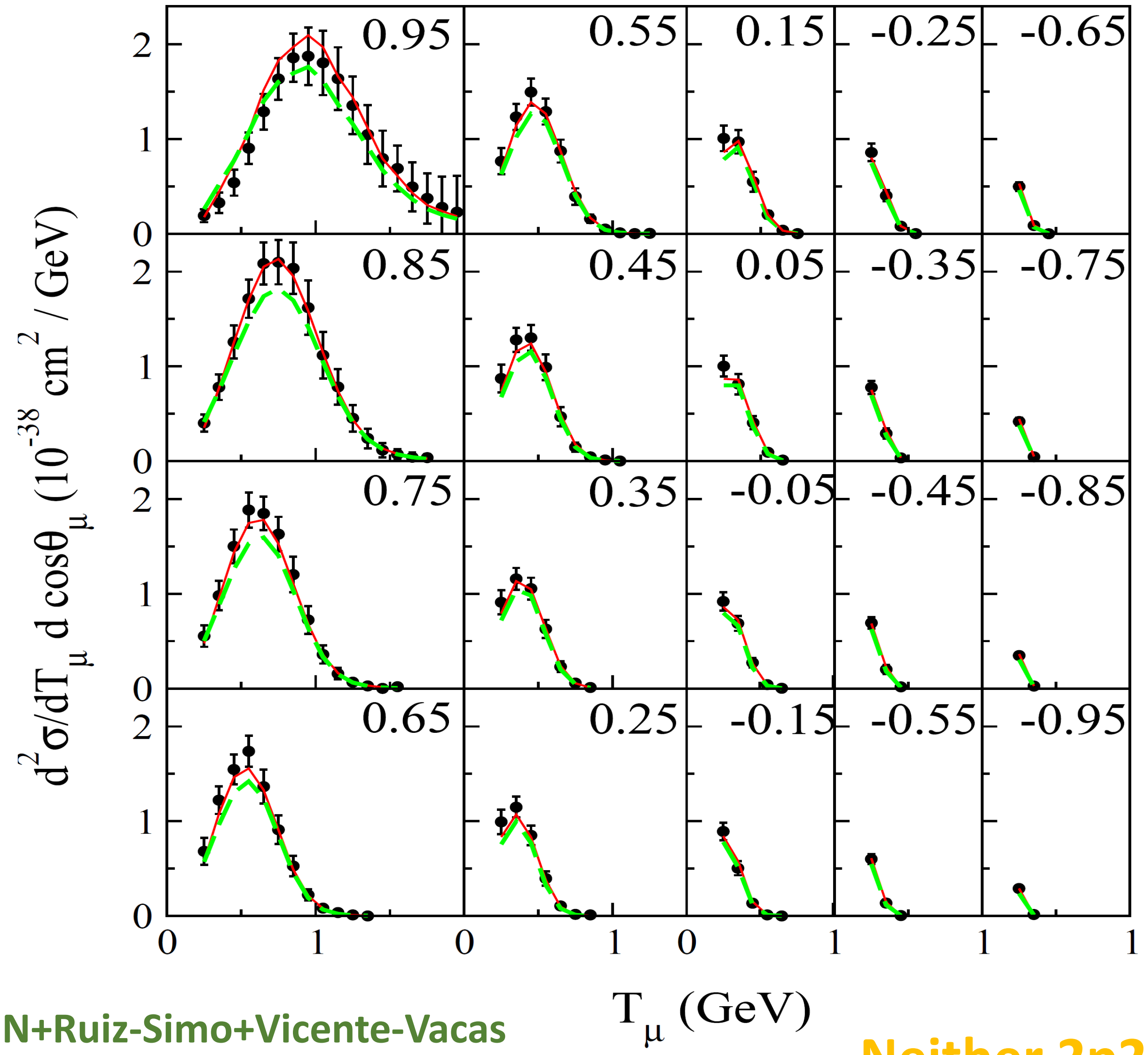
$$\chi^2 = \sum_{i=1}^{137} \left[\frac{\lambda \left(\frac{d^2\sigma^{exp}}{dT_\mu d\cos\theta} \right)_i - \left(\frac{d^2\sigma^{th}}{dT_\mu d\cos\theta} \right)_i}{\lambda\Delta \left(\frac{d^2\sigma}{dT_\mu d\cos\theta} \right)_i} \right]^2 + \left(\frac{\lambda - 1}{\Delta\lambda} \right)^2,$$

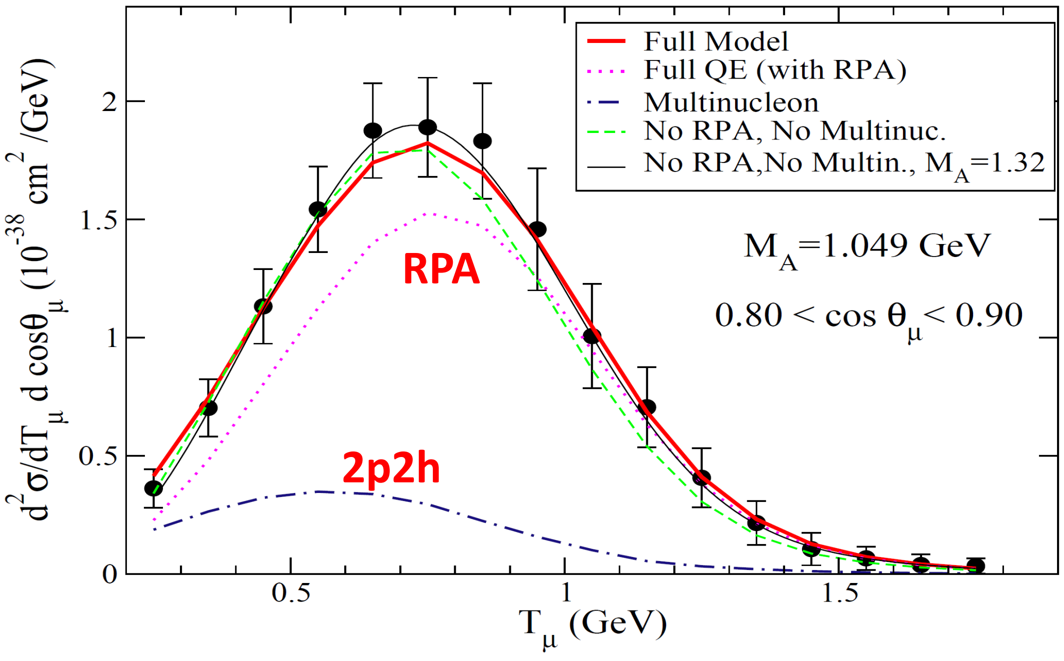
that takes into account the global normalization uncertainty ($\Delta\lambda = 0.107$) claimed by the MiniBooNE collaboration.

We fit λ to data with a fixed value of M_A (=1.049 GeV).

We obtain $\chi^2/\#$ bins =52/137 with $\lambda = 0.89 \pm 0.01$.

The microscopical model, with no free parameters, agrees remarkably well with data! The shape is very good and χ^2 strongly depends on λ, which is strongly correlated with M_A.





Model	Scale	M_A (GeV)	$\frac{\chi^2}{\#bins}$
LFG	0.96±0.03	1.32±0.03	35/137
Full	**0.92±0.03**	**1.08±0.03**	**50/137**
Full $\vert q\vert > 0.4^{\dagger}$ GeV	0.83±0.04	1.01±0.03	30/123

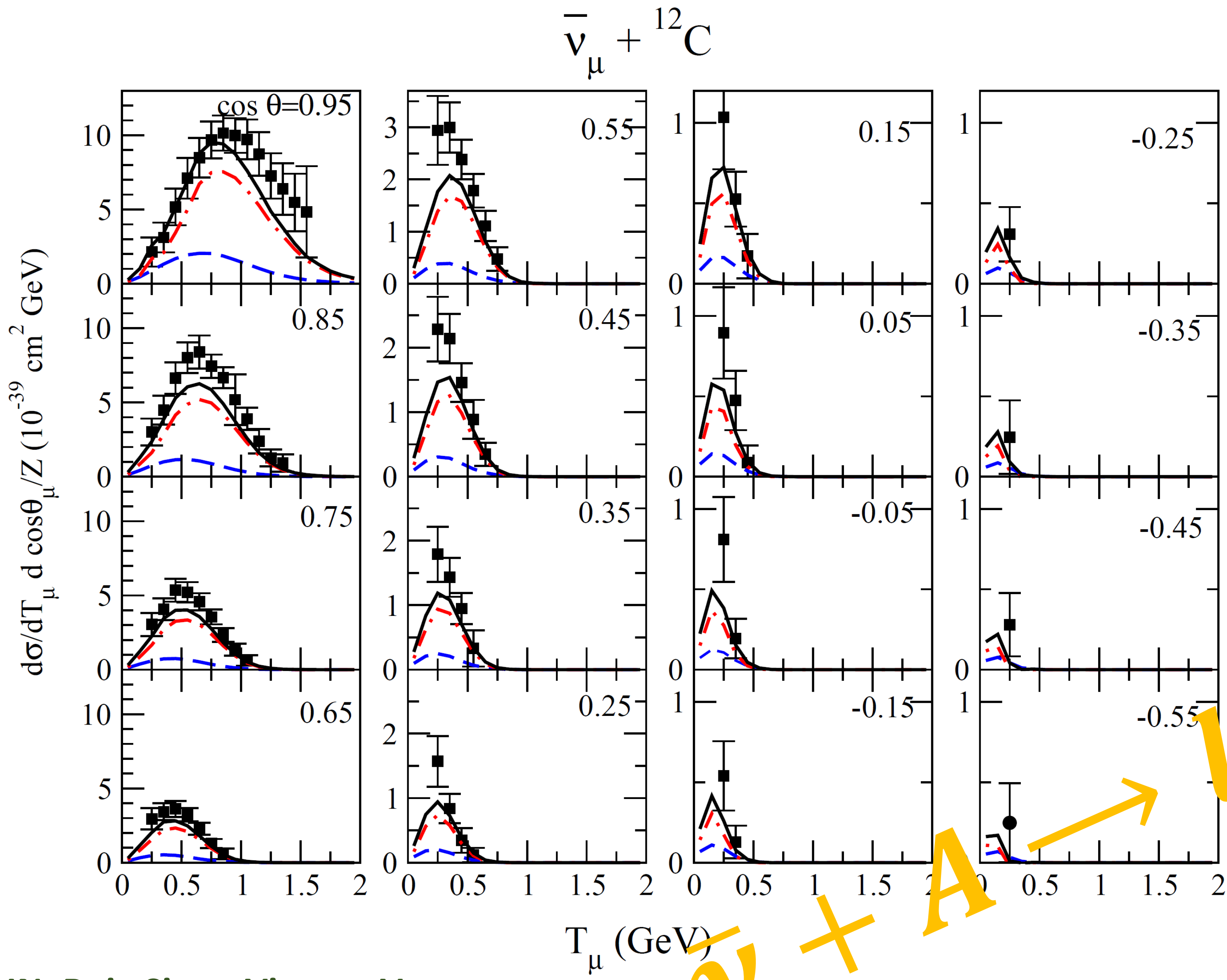
MB estimate of total normalization error 10.7%

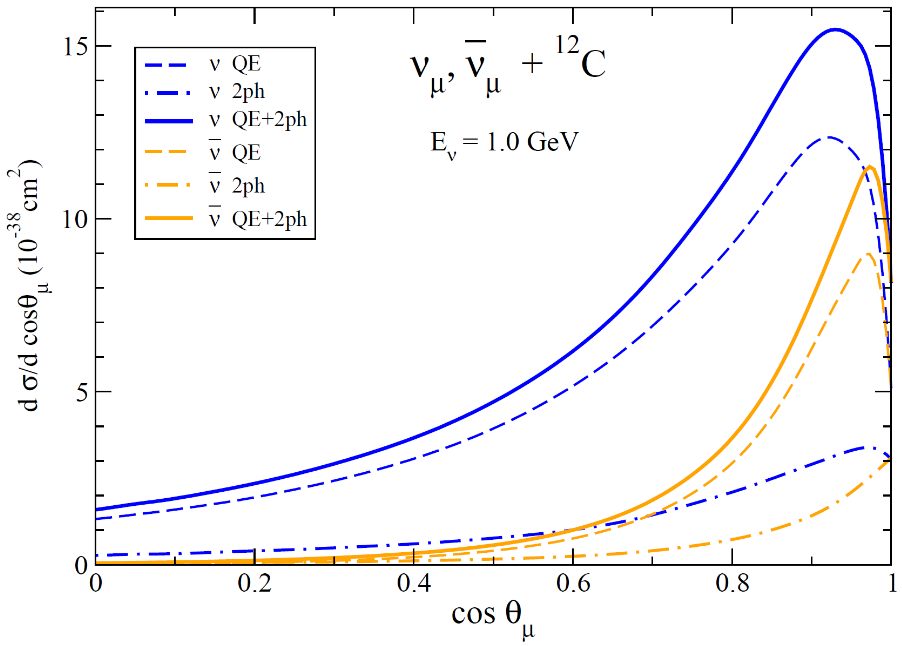
† : As suggested by Sobczyk et al. PRC 82, 045502

JN+Ruiz-Simo+Vicente-Vacas
PLB 707 (2012) 72

Neither 2p2h contributions nor RPA effects alone describe the MB 2D dataset, which is however described by the combination of both nuclear mechanisms!

$M_A \sim 1.03$ GeV

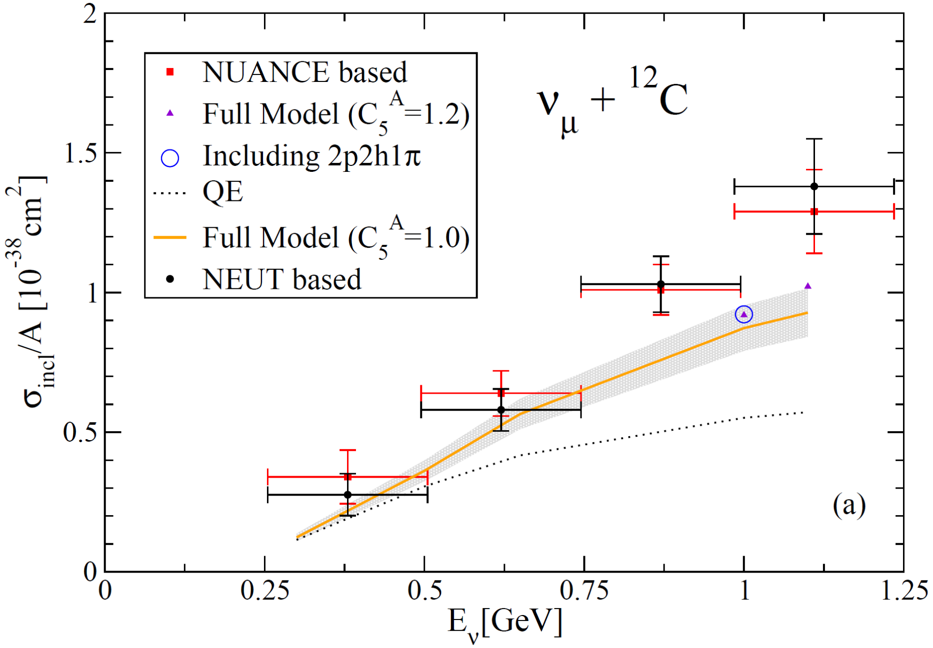




$\bar{\nu} + A \longrightarrow l+X$

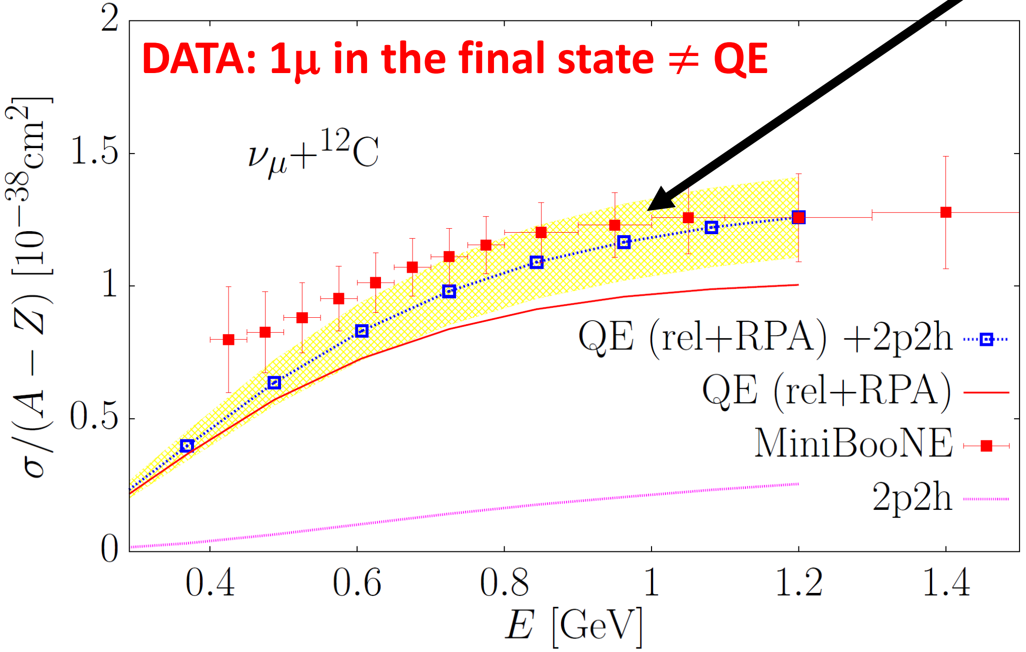
- **Antineutrino distributions are more forward peaked**
- **Relative importance of 2p2h contributions in ν and $\bar{\nu}$ are similar**

JN+Ruiz-Simo+Vicente-Vacas
Phys.Lett.B 721 (2013) 90



Inclusive

some discrepancies between QE+2p2h and MiniBooNE flux-unfolded cross section caused by the **neutrino energy reconstruction procedure used to pass from flux-folded to flux-unfolded data**



MB neutrino and antineutrino 2D dataset is, however, reasonably described by the combination of both nuclear mechanisms

QE-like!

Neutrino Energy Reconstruction:

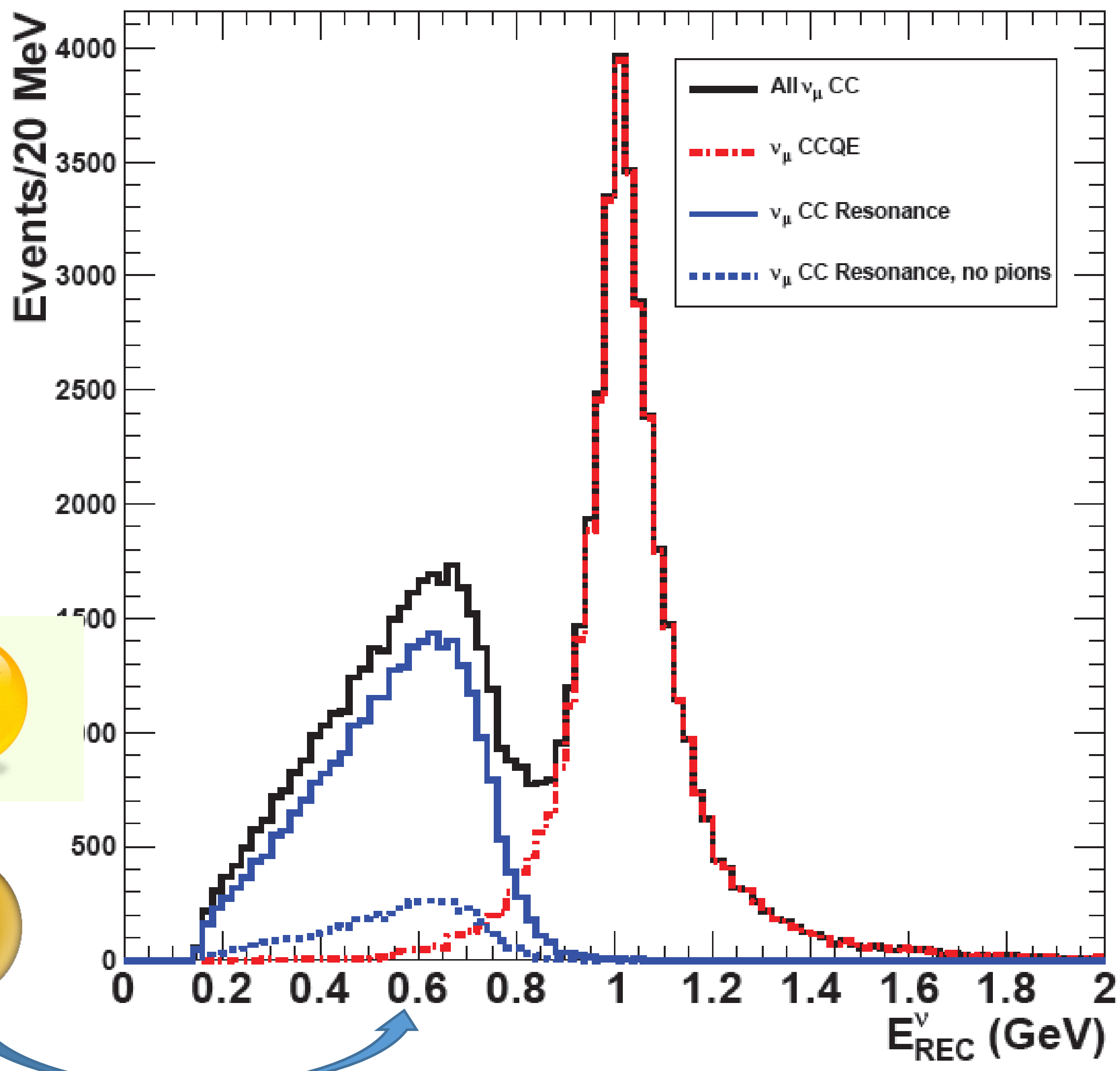
QE: $\nu_\mu + n \longrightarrow p\,\mu^-$ (bound in the nucleus)

$$E_{\rm rec} = \frac{ME_\mu - m_\mu^2/2}{M - E_\mu + |\vec{p}_\mu|\cos\theta_\mu}$$

Exp

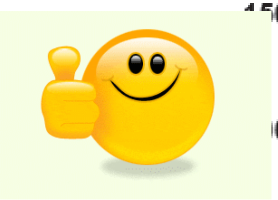
QE-like problem ⇒ absorbed or not detected real pions and…

GENIE E_ν = 1 GeV



exp: only 1μ (from the lepton vertex). But, for instance if pions are produced:

- pion decays and the extra muon is detected (2 muons in the final state)



- pion is absorbed or not detected

(MC corrected if the pion production cross section is well known…)

Neutrino Energy Reconstruction:

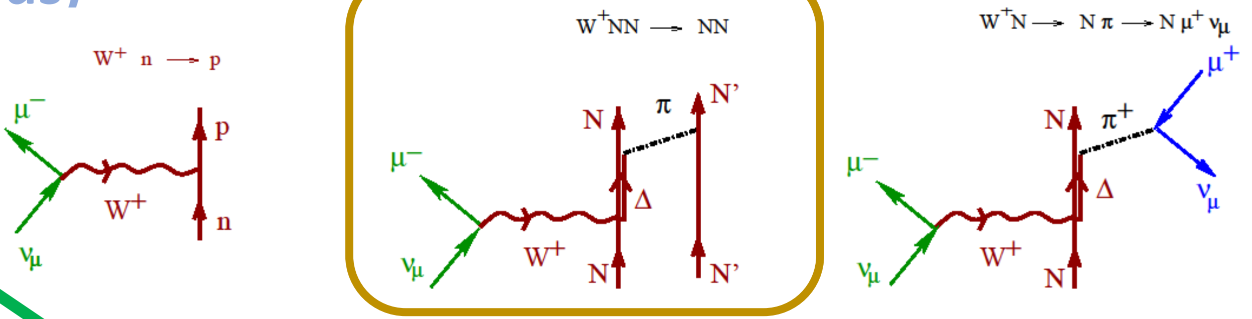
QE: $\nu_\mu + n \longrightarrow p\,\mu^-$ (bound in the nucleus)

$$E_{\rm rec} = \frac{M E_\mu - m_\mu^2/2}{M - E_\mu + |\vec{p}_\mu|\cos\theta_\mu}$$

Exp

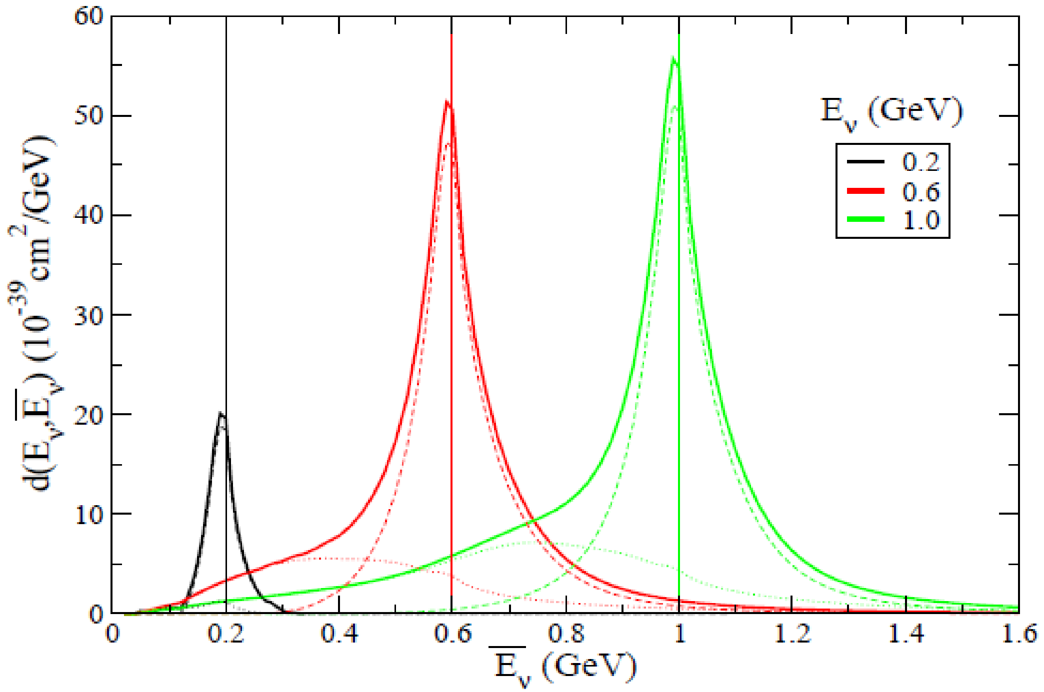
QE-like: problem absorbed or not detected pions and 2p2h (nucl. effect)

M. Martini, M. Ericson, PRD 87 (2013)



QE Energy Reconstruction will be wrong !!

2p2h: $\nu_\mu + NN \longrightarrow N'N''\,\mu^-$



MC correct for this effect: ⟸ cross section

Neutrino energy reconstruction

Neutrino beams ARE NOT monochromatic. For QE-like events, only the charged lepton is observed and the only measurable quantities are then its direction (scattering angle θ_μ with respect to the neutrino beam direction) and its energy E_μ. The energy of the neutrino that has originated the event is unknown. Assuming QE dynamics is defined a "reconstructed" energy

$$E_{\rm rec} = \frac{ME_\mu - m_\mu^2/2}{M - E_\mu + |\vec{p}_\mu| \cos\theta_\mu}$$

(genuine quasielastic event on a nucleon at rest, ie. $E_{\rm rec}$ is determined by the QE-peak condition $q^0 = -q^2/2M$). Note that each event contributing to the flux averaged double differential cross section $d\sigma/dE_\mu d\cos\theta_\mu$ defines unambiguously a value of $E_{\rm rec}$. The actual ("true") energy, E, of the neutrino that has produced the event will not be exactly $E_{\rm rec}$.

Flux-folded $d\sigma/dT_\mu d\cos\theta_\mu \overset{?}{\rightarrow}$ CCQE-like unfolded $\sigma(E)$

Unfolding procedure needs theoretical input!

$$P_{\rm true}(E) = \int dE_{\rm rec} \underbrace{P_{\rm rec}(E_{\rm rec})}_{\rm EXP} \underbrace{P(E|E_{\rm rec})}_{theory!}$$

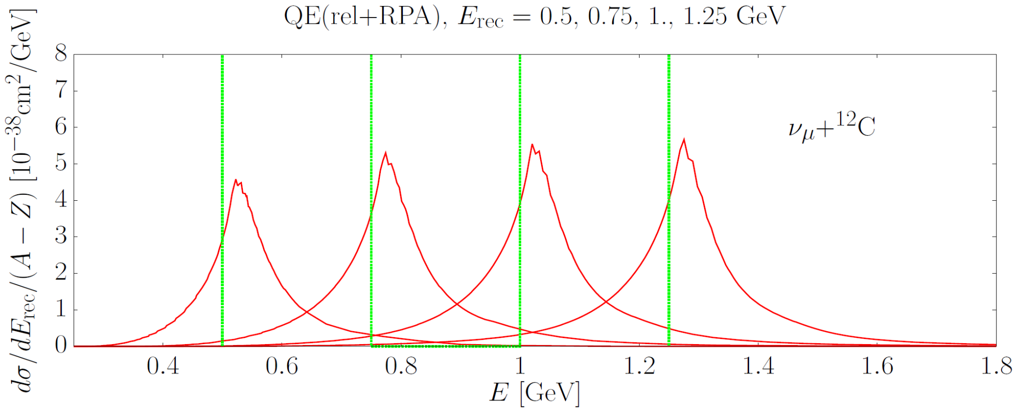
$P_{\rm rec}(E_{\rm rec})$ is the *pd* of measuring an event with reconstructed energy $E_{\rm rec}$. $P(E|E_{\rm rec})$ is, given an event of reconstructed energy $E_{\rm rec}$, the conditional *pd* of being produced by a neutrino of energy E.

...using Bayes's theorem $P(E|E_{\rm rec})$ could be related to

$P(E_{\rm rec}|E)$ is determined by $\dfrac{d\sigma}{dE_{\rm rec}}(E; E_{\rm rec})$

given a true neutrino energy E, there is a distribution of reconstructed energies E_{rec} .....

QE



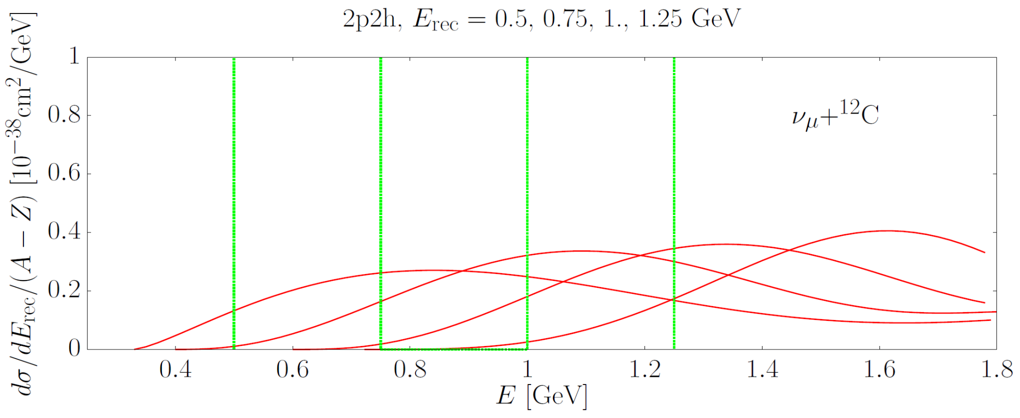
Neutrino Energy Reconstruction and the Shape of the CCQE-like Total Cross Section

(qualitatively in agreement with Martini et al., PRD85 093012)

theory !

$$\frac{d\sigma}{dE_{\rm rec}}(E; E^0_{\rm rec}) = \int_{m_\mu}^{E} dE_\mu \frac{d^2\sigma}{dE_{\rm rec}dE_\mu}(E; E^0_{\rm rec}) = \int_{m_\mu}^{E} dE_\mu \left|\frac{\partial(\cos\theta_\mu)}{\partial E_{\rm rec}}\right| \frac{d^2\sigma}{d(\cos\theta_\mu)dE_\mu}(E; E^0_{\rm rec})$$

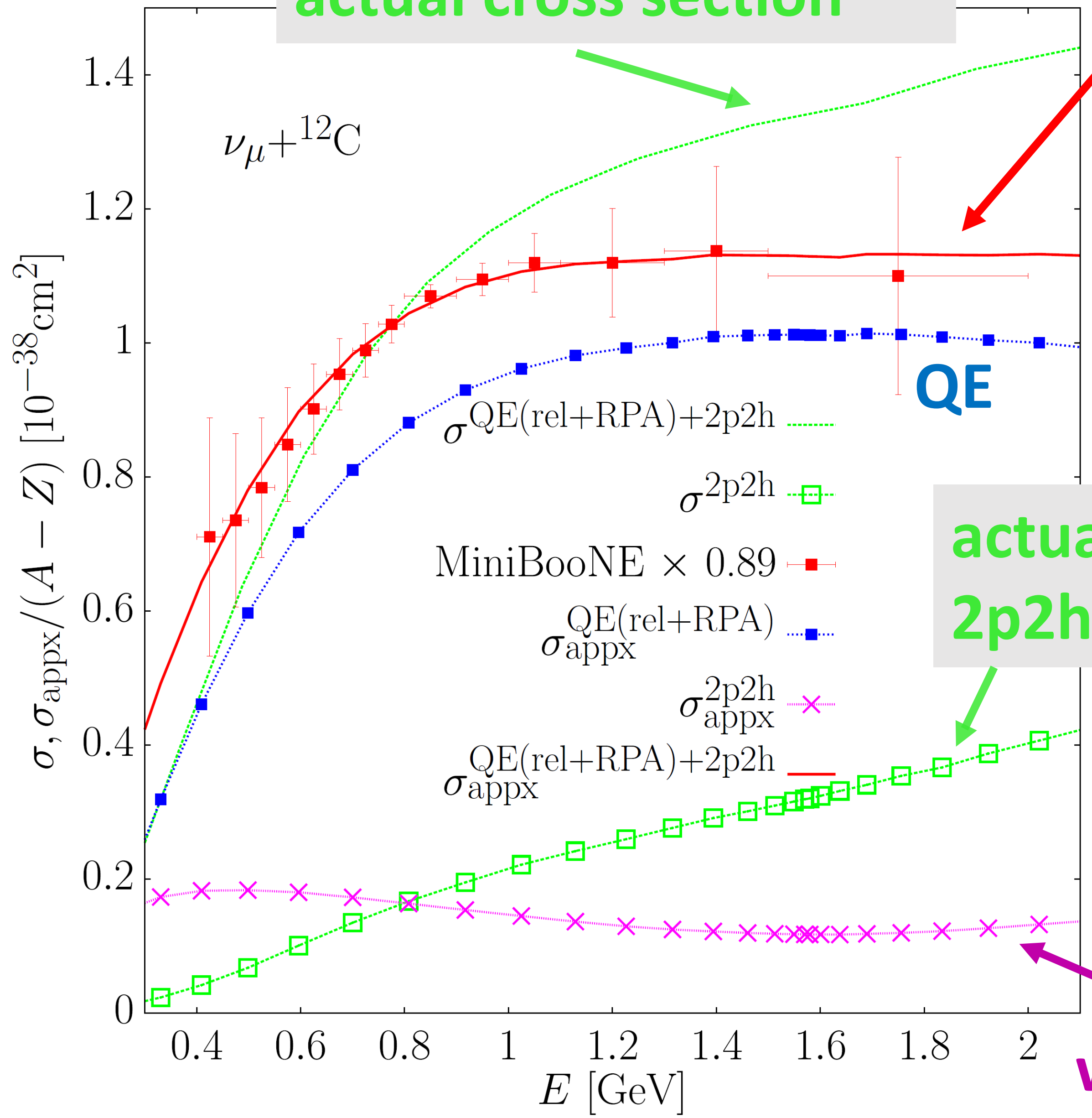
For each $E_{\rm rec}$, there exists a distribution of true neutrino energies that could give rise to events whose muon kinematics would lead to the given value of $E_{\rm rec}$.



2p2h

actual cross section

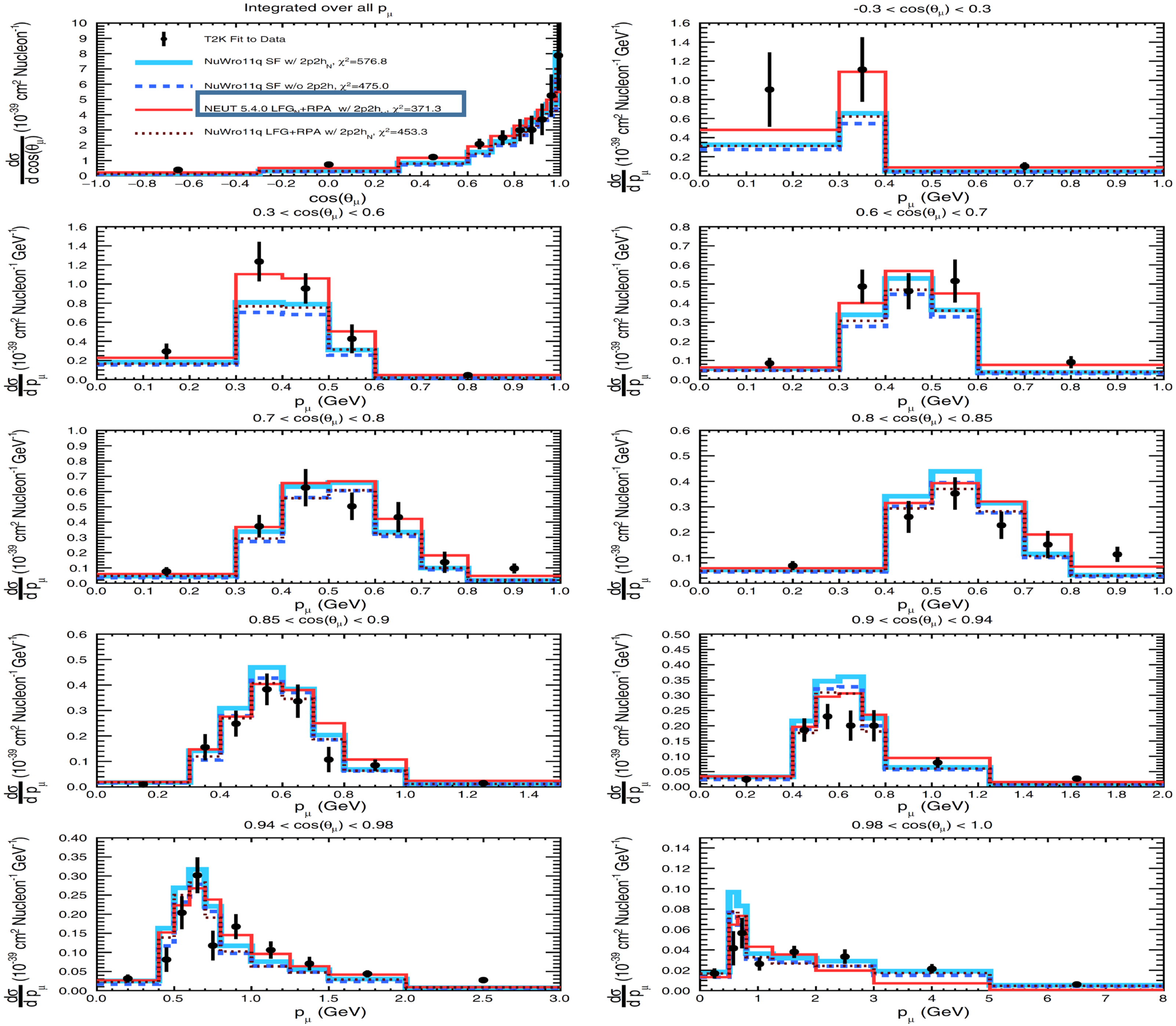
wrongly unfolded cross section



the algorithm used to reconstruct the neutrino energy is not adequate when dealing with quasielastic-like events, and a distortion of the total flux-unfolded cross-section shape is produced. <u>This amounts to a redistribution of strength from high to low energies</u>, which gives rise to a sizable excess (deficit) of low (high) energy neutrinos. This distortion of the shape leads to a good description of the MiniBooNE unfolded charged current quasielastic-like cross sections published by the MiniBooNE Collaboration

wrongly unfolded 2p2h cross section

JN, F. Sánchez, I Ruiz-Simo, M.J. Vicente-Vacas PD85 (2012) 113008

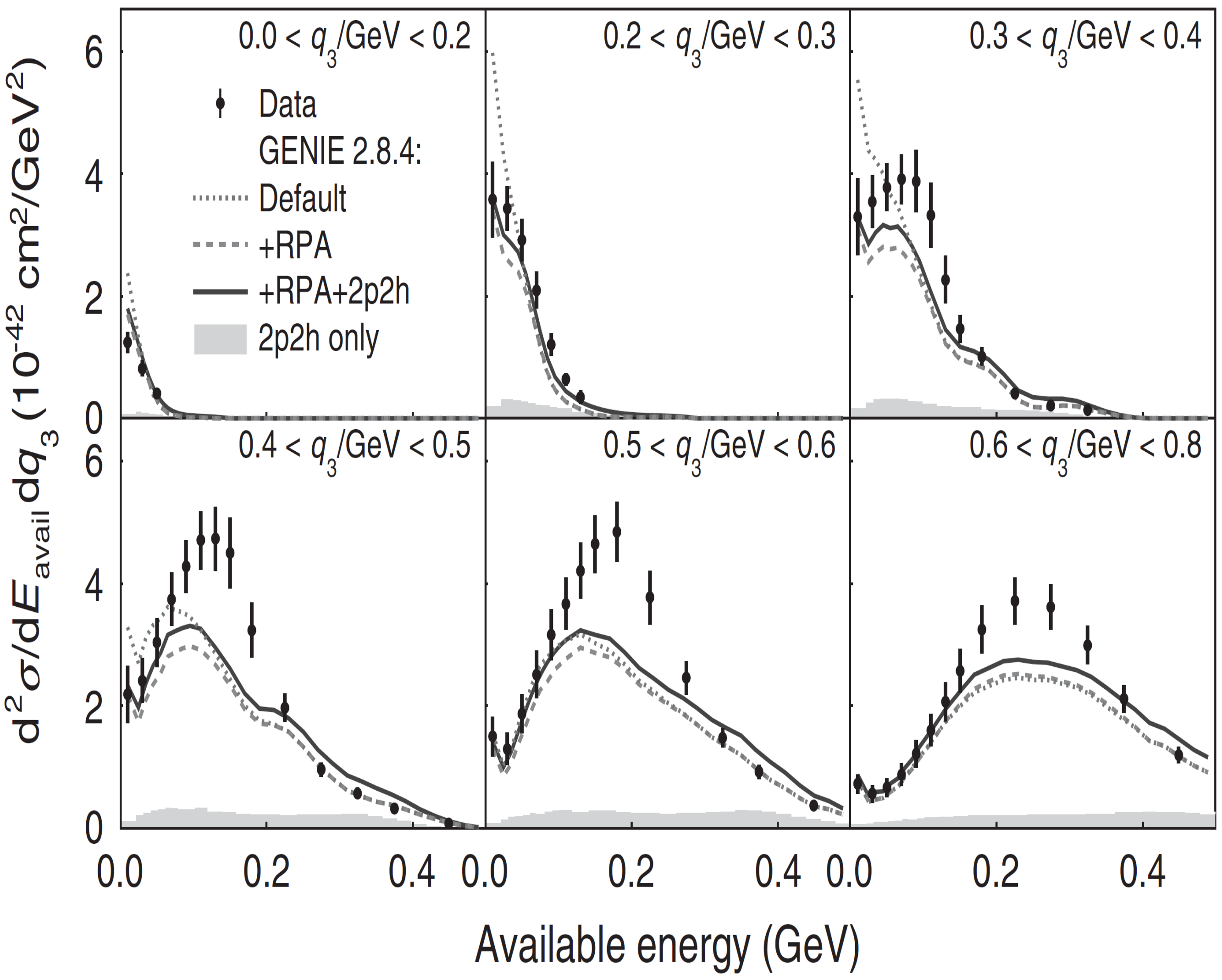


NEUT 5.4.0 LFG$_N$+RPA w/ 2p2h$_N$, χ^2=371.3

Measurement of the cross section as a function of the muon kinematics when there are no protons (with momenta above 500 MeV).

good agreement with T2K data!

T2K: PRD 98 032003 (2018)



The data make clear two distinct multinucleon effects that are essential for complete modeling of neutrino interactions at low momentum transfer. The $2p2h$ model tested in this analysis improves the description of the event rate in the region between QE and Δ peaks, and the rate for multiproton events, but does not go far enough to fully describe the data. Oscillation experiments sensitive to energy reconstruction effects from these events must account for this event rate. The cross section presented here will lead to models with significantly improved accuracy.

MINERνA: CCQE-like (hadron calorimetry)

PRL 116, 071802 (2016)

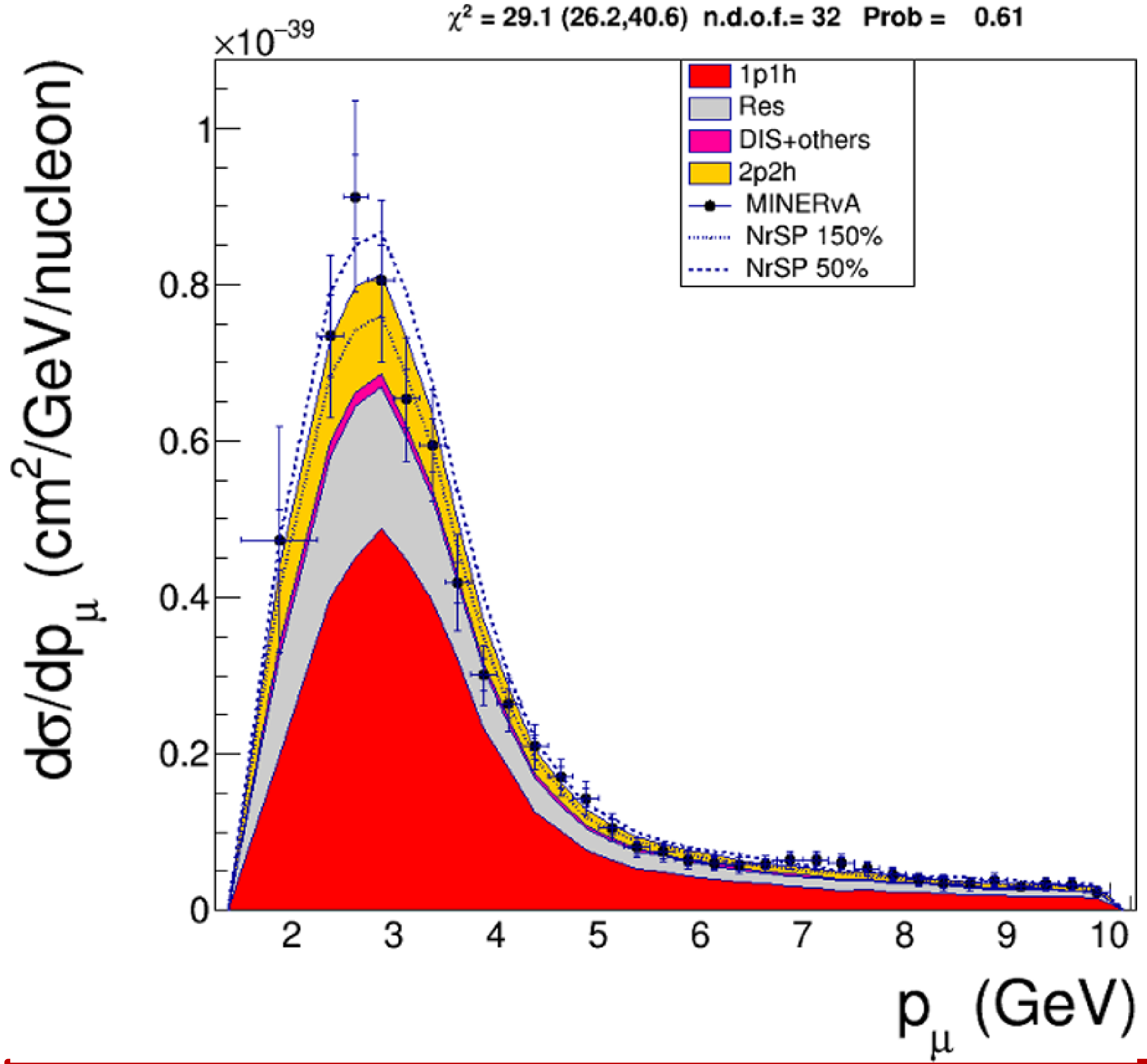
Hadronic energy spectrum: **The IFIC Valencia 2p2h model increases the predicted event rates, but not enough.** This process is increased further with an empirical enhancement based on MINERνA inclusive neutrino data. The additional events are from weighting up the generated 2p2h events according to a two-dimensional Gaussian in true q0, q3, whose six parameters are fit to the neutrino data version of these distributions. This enhancement adds 50% to the predicted 2p2h strength, but it targets the event rate in the kinematic region between the CCQE and Δ peaks where the rate doubles.

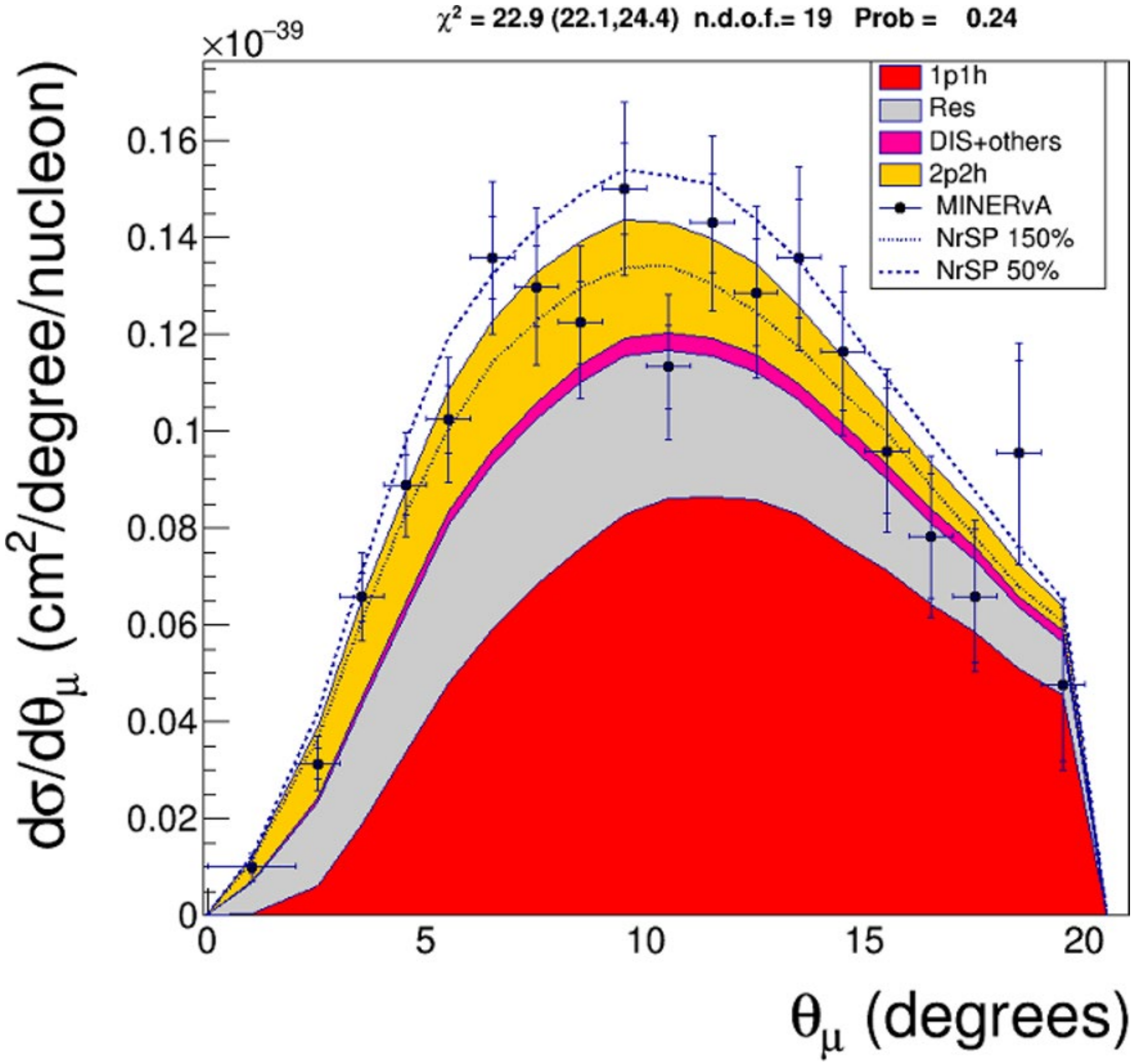
MINERνA (Antineutrino Charged-Current Reactions on Hydrocarbon with Low Momentum Transfer): PRL (2018) 221805

We therefore enhance the 2p2h prediction from the Nieves model in a specific region. Integrated overall phase space the rate of 2p2h is increased by 53%.

MINERνA (Measurement of Quasielastic-Like Neutrino Scattering at $\langle E_\nu \rangle \sim$ 3.5 GeV on a Hydrocarbon Target) Phys.Rev.D 99 (2019) 1, 012004

however……

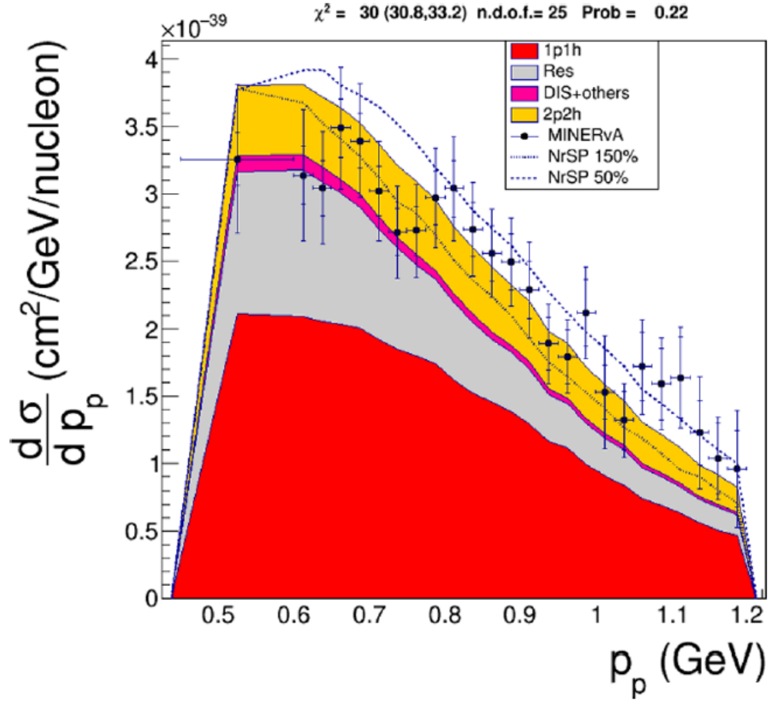


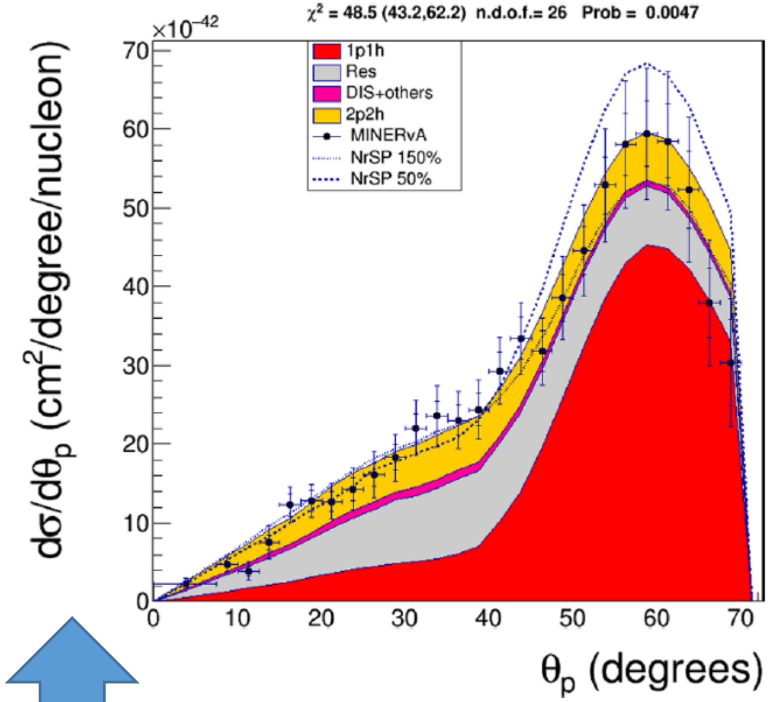


MINERνA CC0π1p

Bourguille B., Nieves J. and Sánchez F.: Inclusive and exclusive neutrino-nucleus cross sections and the reconstruction of the interaction kinematics, JHEP 04 (2021) 004 (results obtained with NEUT)

MINERνA $CC0\pi1p$

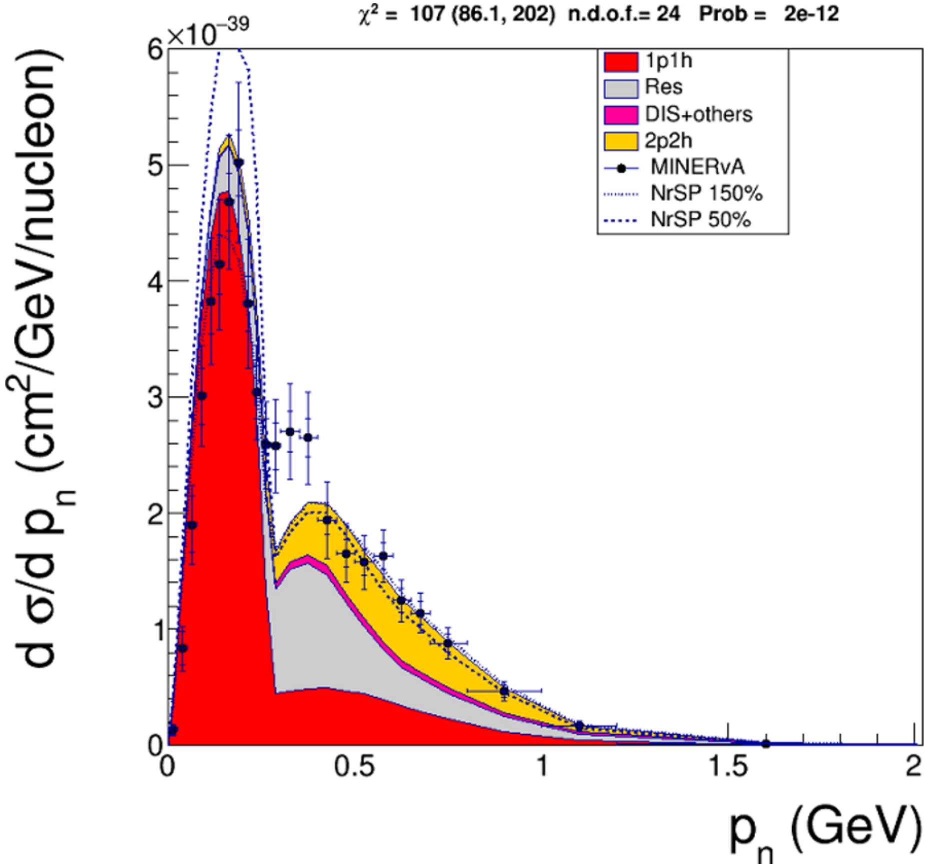




visible proton in the final state

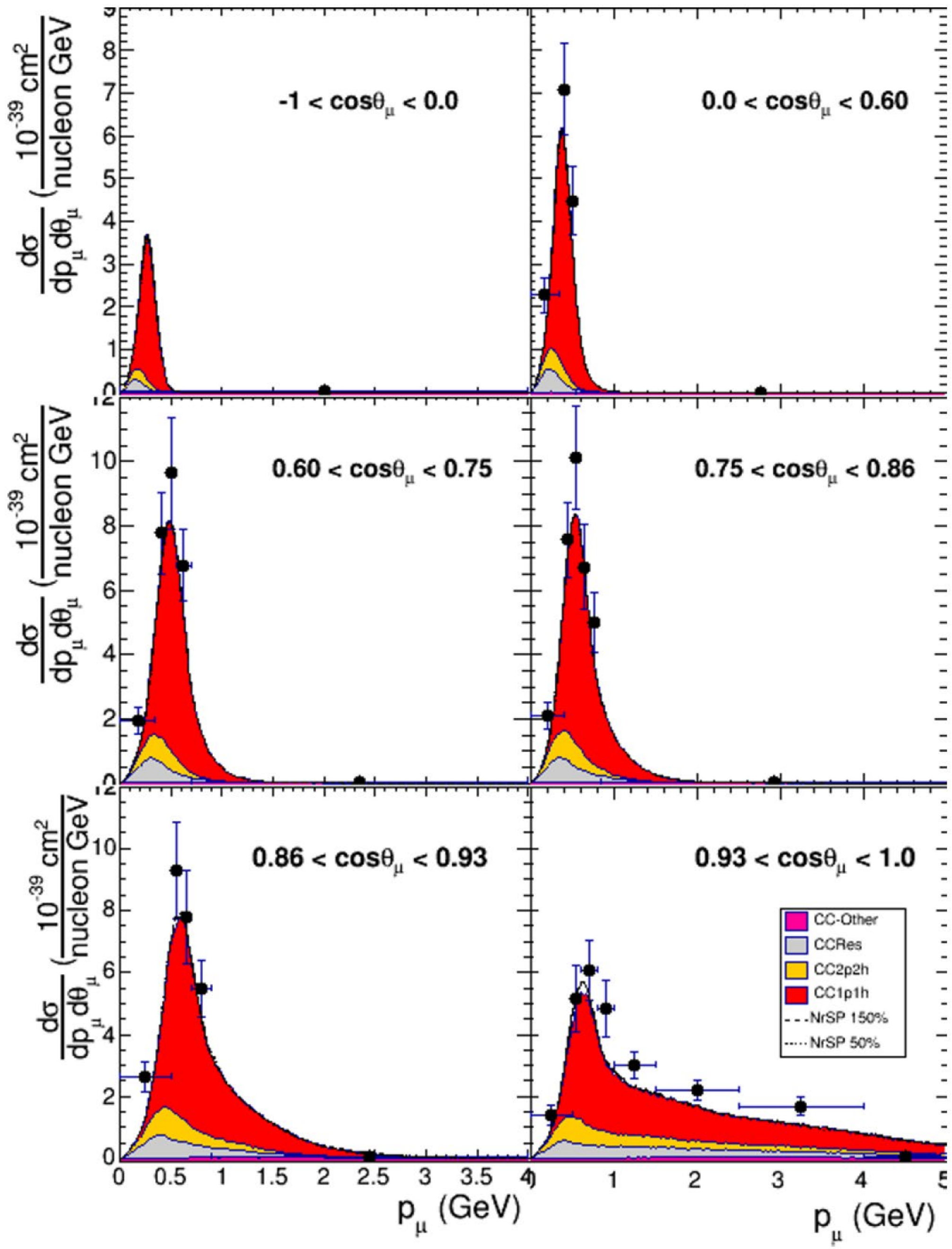
QE: $\nu_\mu + n \longrightarrow p\, \mu^-$

(bound in the nucleus)

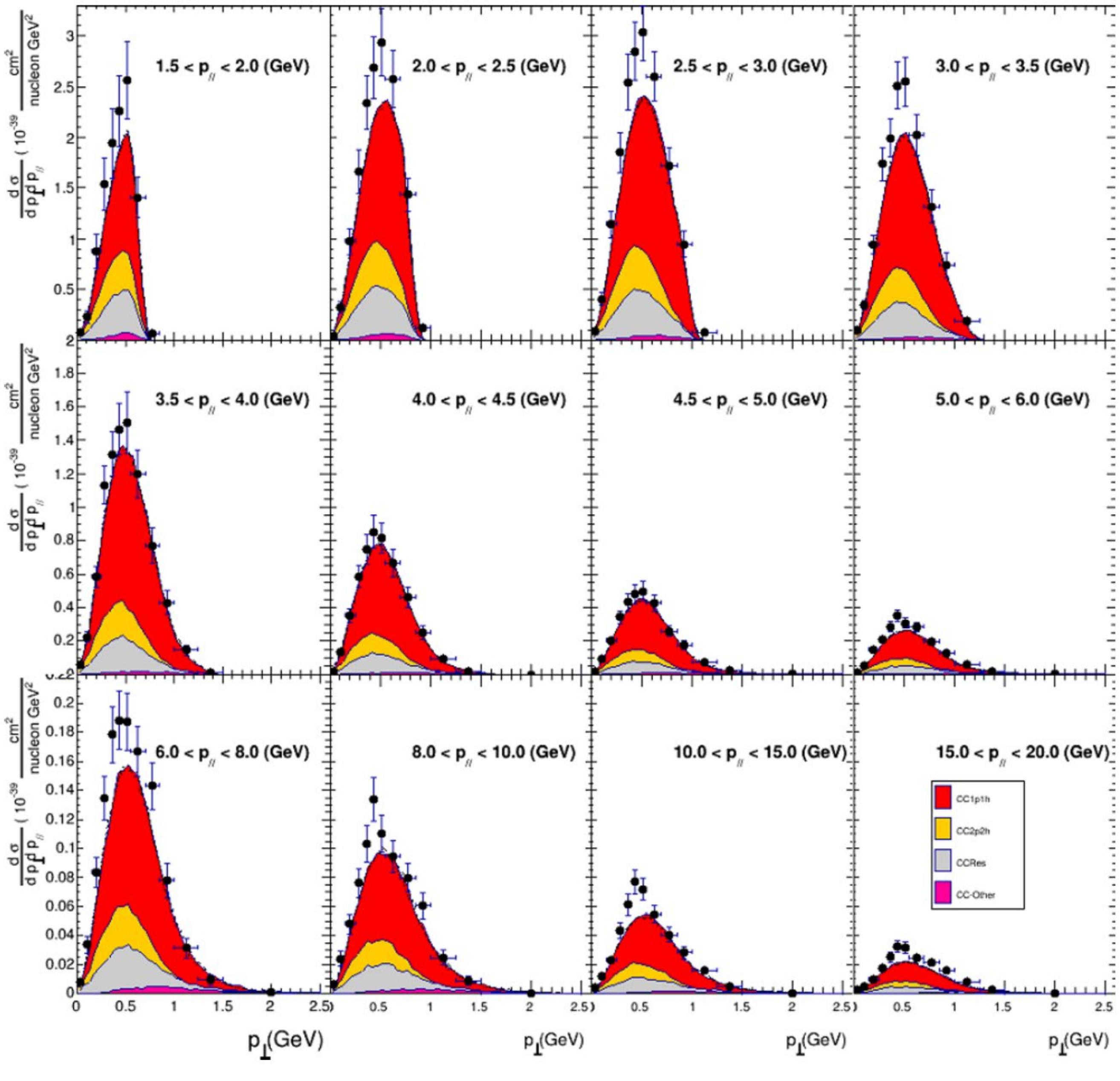


reconstructed neutron in the initial state

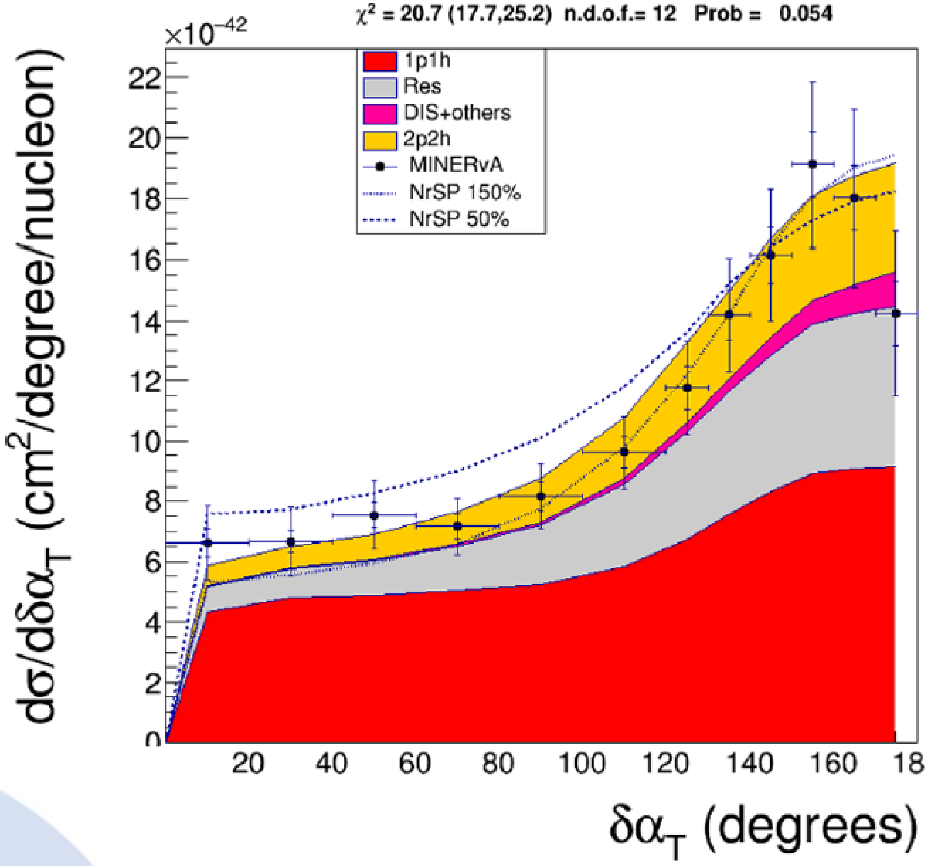
T2K CC0π



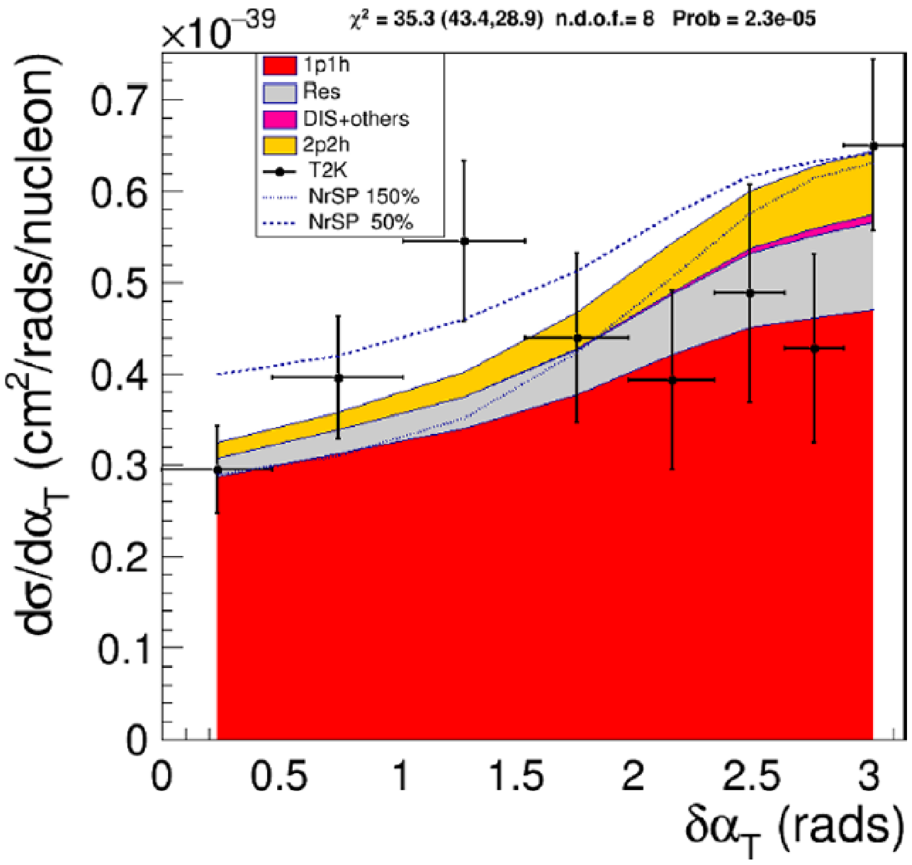
MINERνA CC0π



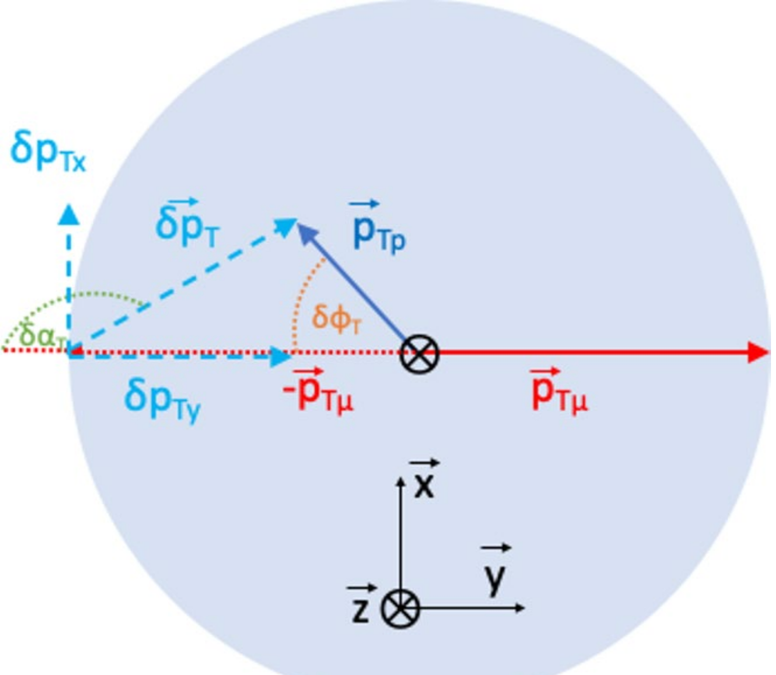
Angular and transverse momentum variables (neutrino perpendicular to the plane).



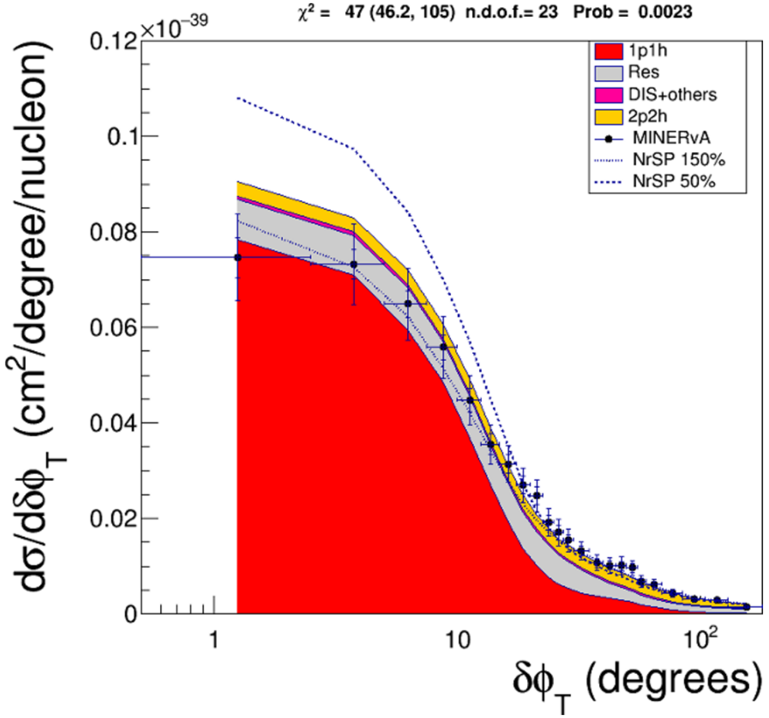
MINERvA CC0π1p

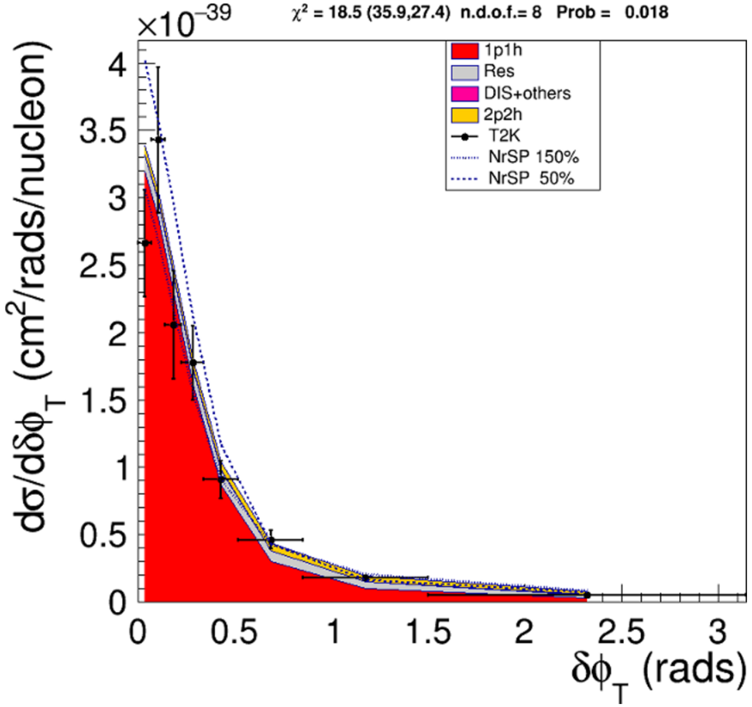


T2K CC0π1p



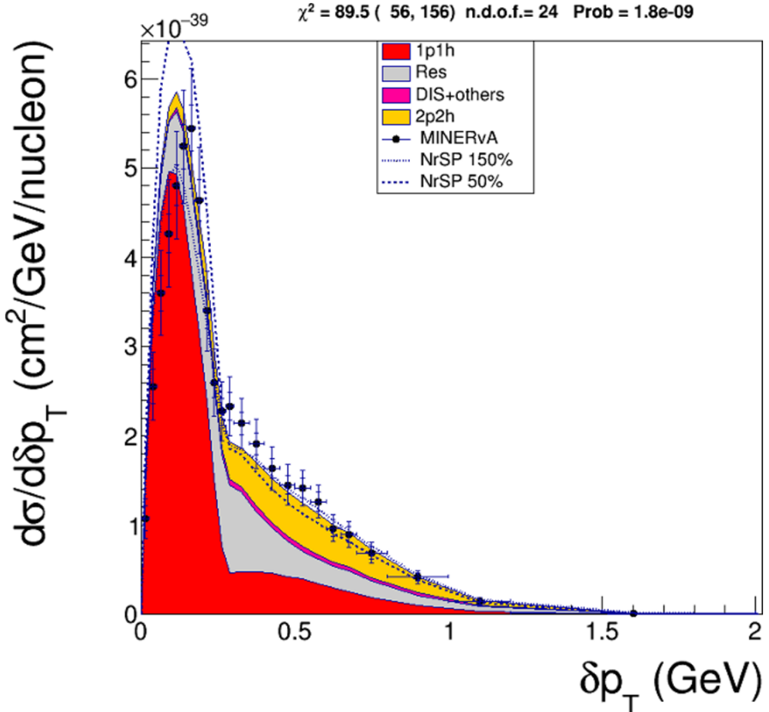
Transverse Kinematic Imbalance (TKI) variables



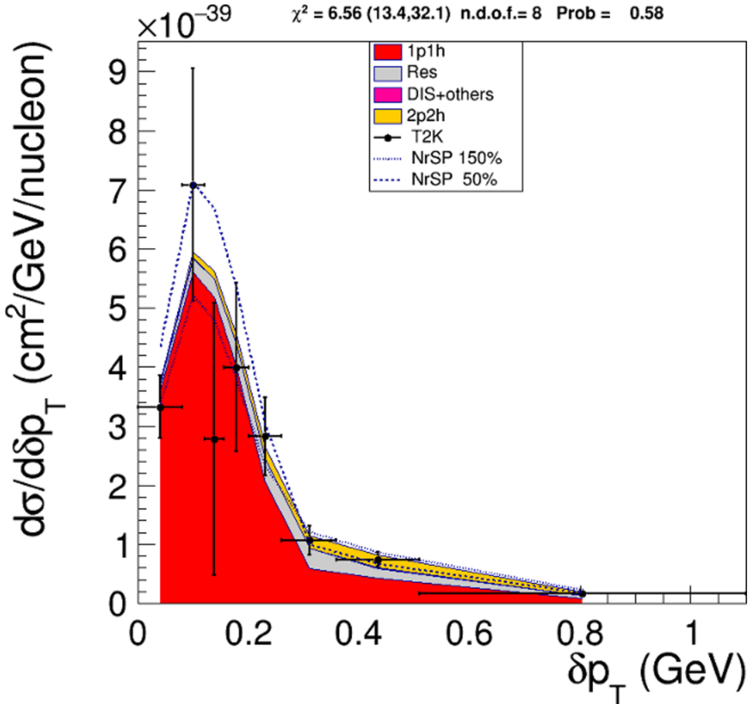


MINERνA & T2K

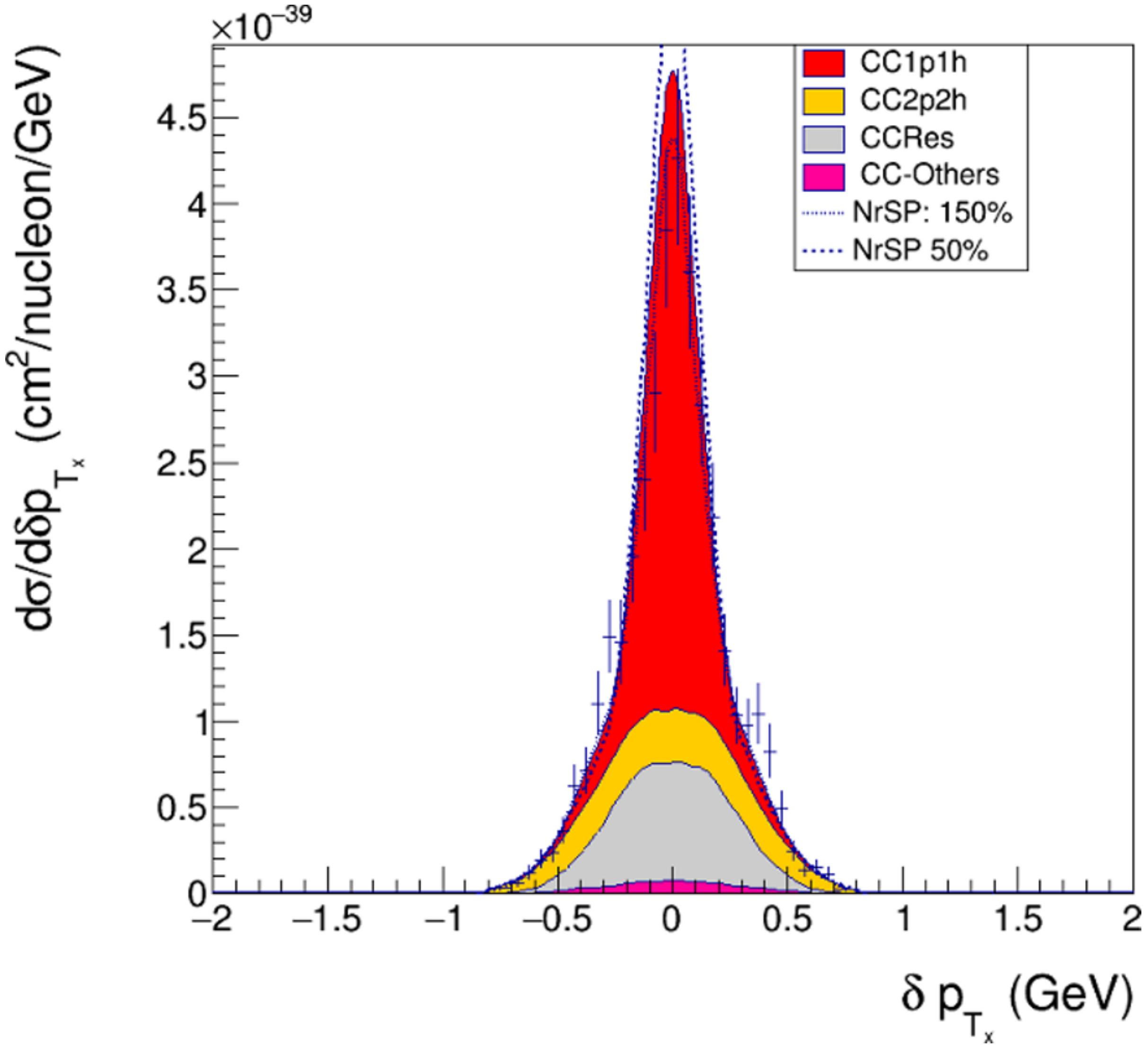
$CC0\pi1p$

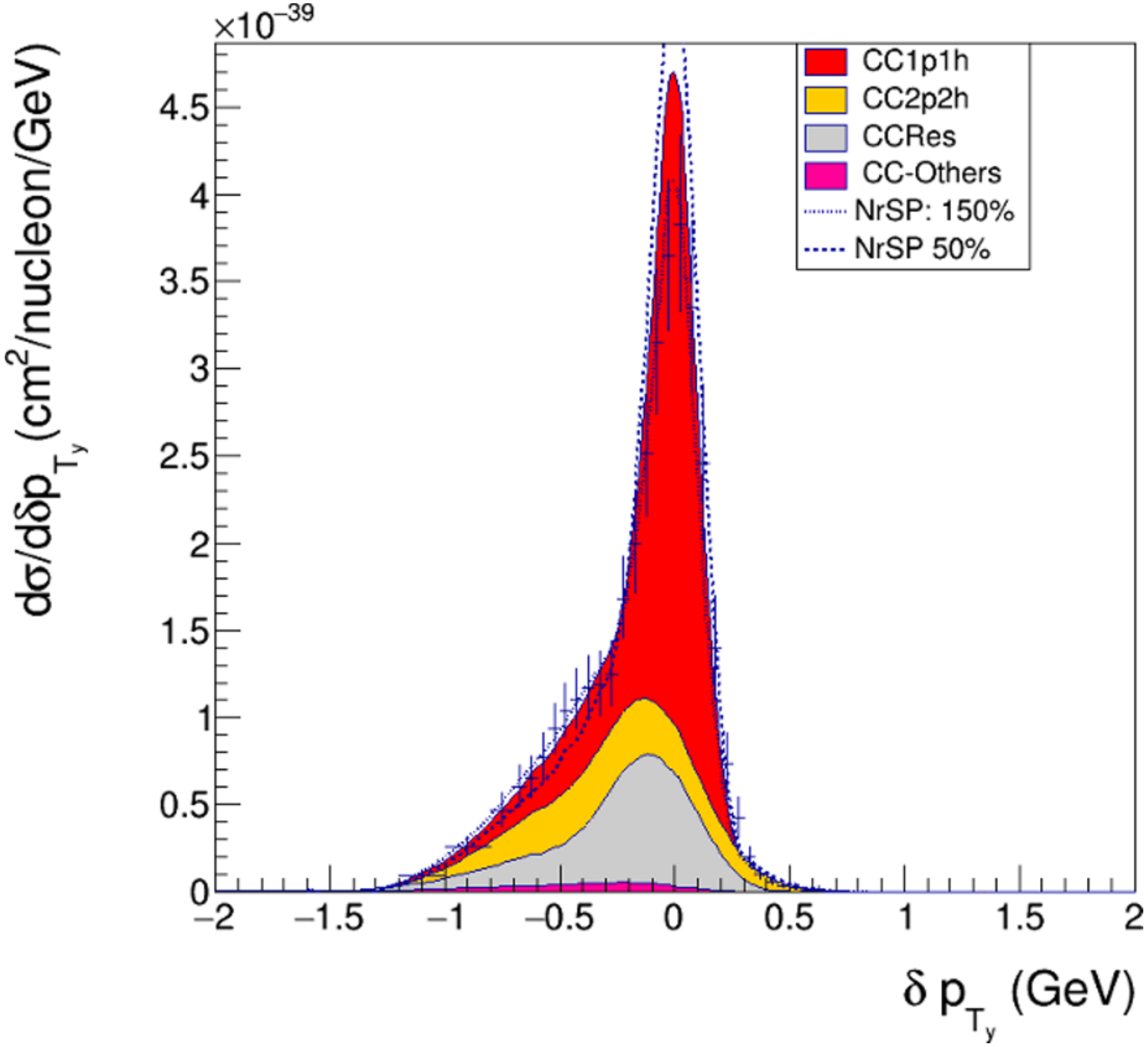


TKI distributions

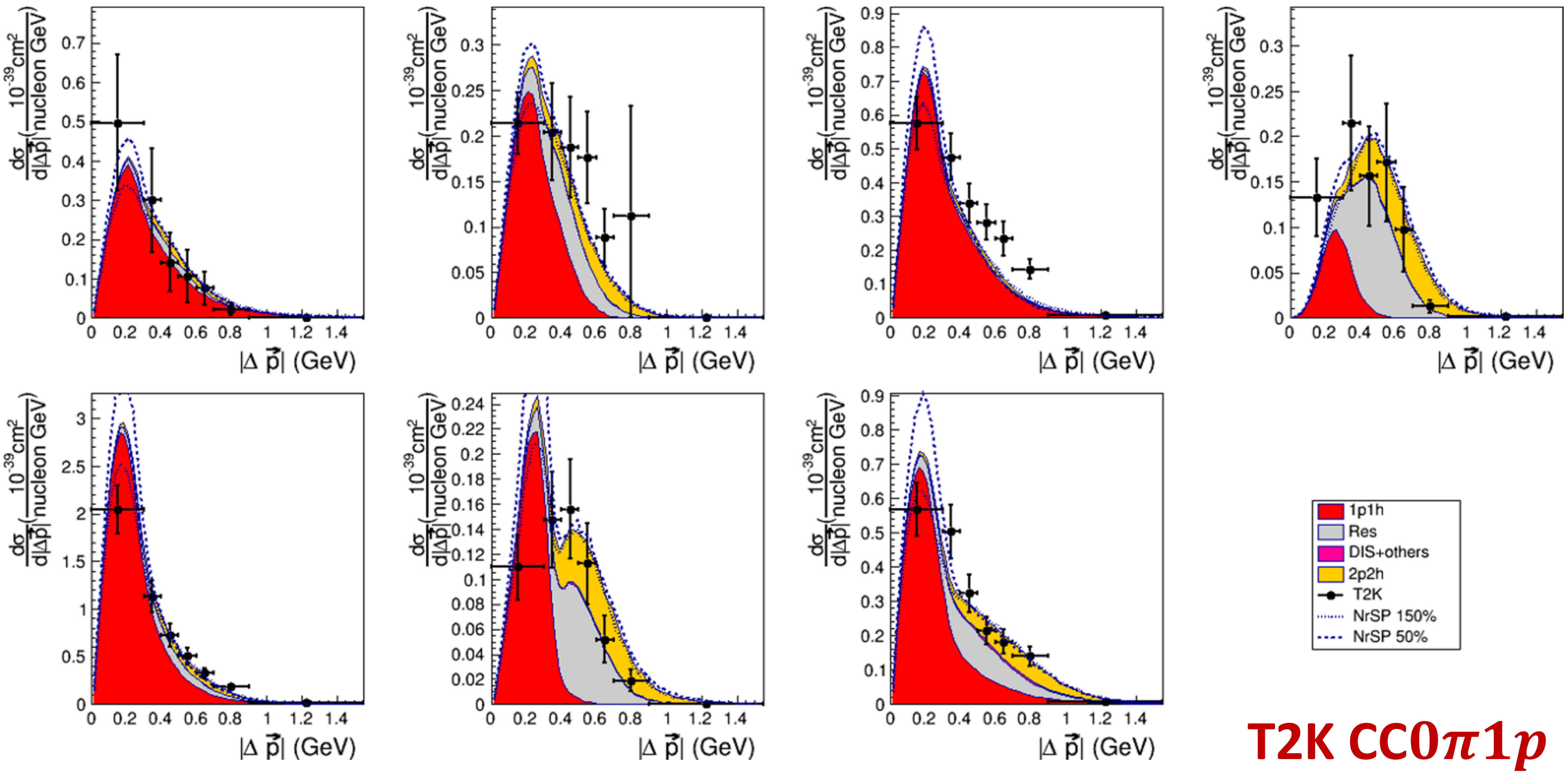


MINERνA CC0π1p





TKI distributions



T2K CC0π1p

$$|\Delta\vec{p}| = |\vec{p}_p^{\,\mathrm{inf}} - \vec{p}_p^{\,\infty}| \qquad \vec{p}_p^{\,\mathrm{inf}} = \vec{p}_\nu^{\,\mathrm{rec}} - \vec{p}_\mu$$

PHYSICAL REVIEW C **102**, 024601 (2020)

previous results are confirmed & distributions of momenta of the outgoing nucleons in the first step

Exclusive-final-state hadron observables from neutrino-nucleus multinucleon knockout

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[2] *Institut für Kernphysik and PRISMA+ Cluster of Excellence, Johannes Gutenberg-Universität Mainz, 55128 Mainz, Germany*

[3] *Université de Genève, Faculté des Sciences, Département de Physique Nucléaire et Corpusculaire (DPNC) 24, Quai Ernest-Ansermet, CH-1211 Genève 4, Switzerland*

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We present results of an updated calculation of the two particle two hole (2p2h) contribution to the neutrino-induced charge-current cross section. We provide also some exclusive observables, interesting from the point of view of experimental studies, e.g., distributions of momenta of the outgoing nucleons and of available energy, which we compare with the results obtained within the NEUT generator. We also compute, and separate from the total, the contributions of 3p3h mechanisms. Finally, we discuss the differences between the present results and previous implementations of the model in MC event generators, done at the level of inclusive cross sections, which might significantly influence the experimental analyses, particularly in the cases where the hadronic observables are considered.

Conclusions

- **CONSISTENT MICROSCOPIC DESCRIPTION of the QE, DIP AND Δ , REGIONS BECOMES FUNDAMENTAL BECAUSE NEUTRINO BEAMS ARE NOT MONOCHROMATIC**

  - ✓ SFs are responsible for the quenching of the QE peak, produce a spreading of the strength of the response functions to higher energy transfers and shift the peak position in the same direction. The overall result is a decrease of the integrated cross sections and a considerable change of the differential shapes.

  - ✓ RPA effects in integrated decay rates or cross sections become significantly smaller when SF corrections are also taken into account, in sharp contrast to the case of a free LFG where they lead to large reductions, even of around 40%.

  - ✓ **2p2h: necessary ingredients i) $W^{\pm}N \longrightarrow N'\pi$ (or $Z^{0}N \longrightarrow N'\pi$ or $\gamma N \longrightarrow N'\pi$) in vacuum and ii) effective NN interaction in the medium: π+ρ+SRC+RPA+…**

  - ✓ **properties of pions and Δ inside of a nuclear medium become essential to describe the resonance region**

- We have analyzed the **MiniBooNE CCQE 2D cross section data using a theoretical model that has proved to be quite successful in the analysis of nuclear reactions with electron, photon and pion probes and contains no additional free parameters.**

- ✓ **RPA and multinucleon knockout have been found to be essential for the description of the data.**

- ✓ MiniBooNE ν and $\bar{\nu}$ CCQE-like data are fully compatible with former determinations of M_A in contrast with several previous analyses. We find, $M_A = 1.08 \pm 0.03$ GeV.

- ✓ The MiniBooNE ν_μ flux could have been underestimated ($\sim$ 10%).

- ✓ **Because of the multinucleon mechanism effects, the algorithm used to reconstruct the neutrino energy is not adequate when dealing with QE-like events.**

- ✓ nucleon-nucleon correlation effects in the RPA series yields a much larger shape distortion toward relatively more high-q^2 interactions, with the 2p2h component filling in the suppression at very low q^2

2018-2021: 2p2h+RPA nuclear model describes fairly well MINERVA-T2K inclusive CC0π data. Problems with MINERνA persist in available hadron energy distributions (2p2h contributions need to be substantially enhanced!), perhaps related with pion production data…

Back up Slides

$\pi^{\pm}$ – nucleus reactions at low energies

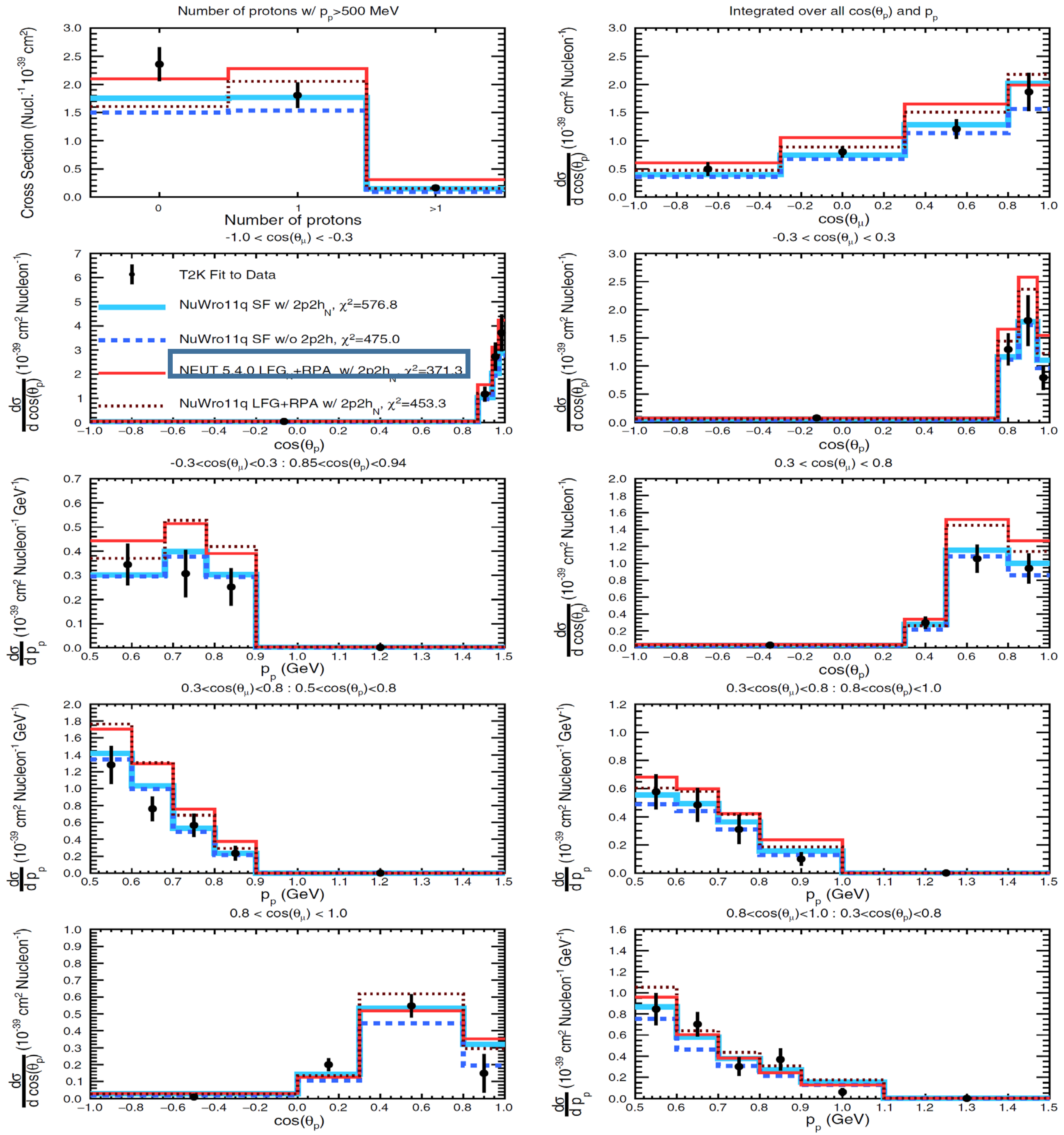
- $\pi^{\pm}$ – nucleus reactions [Nieves+Oset+García-Recio NPA 554 (1993) 554]
  - ✓ $\pi^{\pm} A_Z \rightarrow \pi^{\pm} A_Z$ [elastic]
  - ✓ $\pi^{\pm} A_Z \rightarrow \pi' X$ [quasielastic]
  - ✓ $\pi^{\pm} A_Z \rightarrow X$ (no pions) [absorption]

- Determination of neutron distributions from pionic atom data [García-Recio+Nieves+Oset NPA 547 (1992) 473]
-
- Radiative pion capture [Chiang +Oset+Carrasco+Nieves+Navarro, NPA 510 (1990) 573]

$$(\pi^{-} A_Z)_{\text{bound}} \rightarrow \gamma X$$

- Chiral symmetry restoration [García-Recio+Nieves+Oset PLB 541 (2002) 64]

$$f_\pi(\rho)/f_\pi \rightarrow 0, \rho >> 0$$



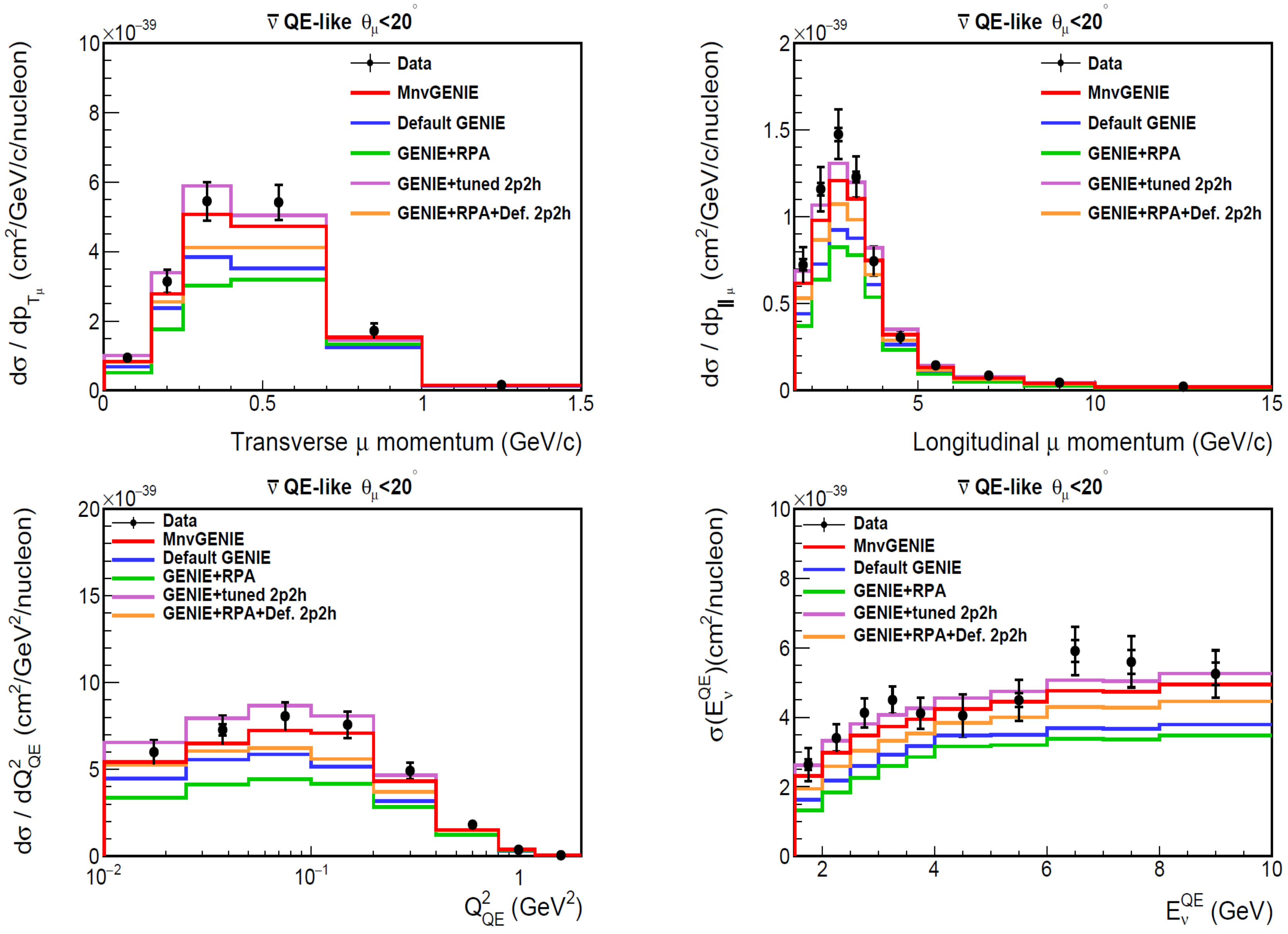
NEUT 5.4.0 LFG$_N$+RPA w/ 2p2h$_N$, $\chi^2=371.3$

Measurement of the cross section as a function of the proton multiplicity (top left) and as a function of proton and muon kinematics where there is exactly one proton (with momentum above 500 MeV).

good agreement with T2K data!

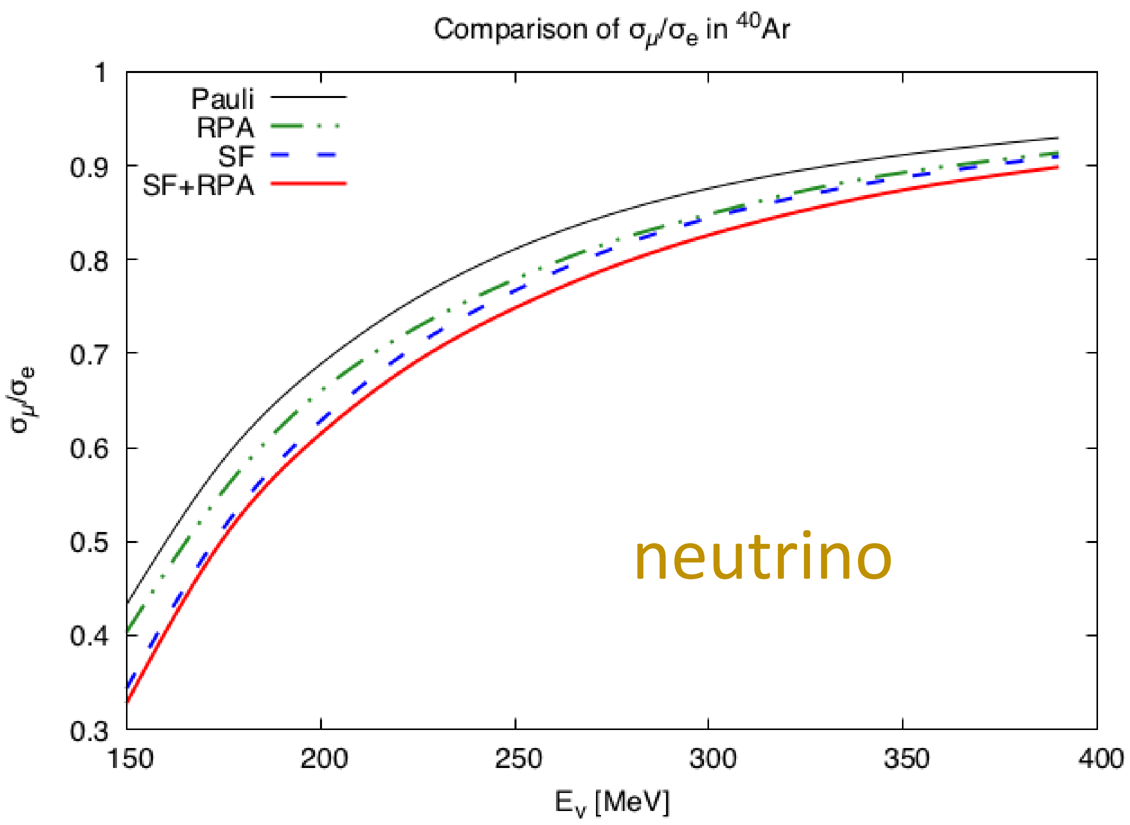
T2K: PRD 98 032003 (2018)

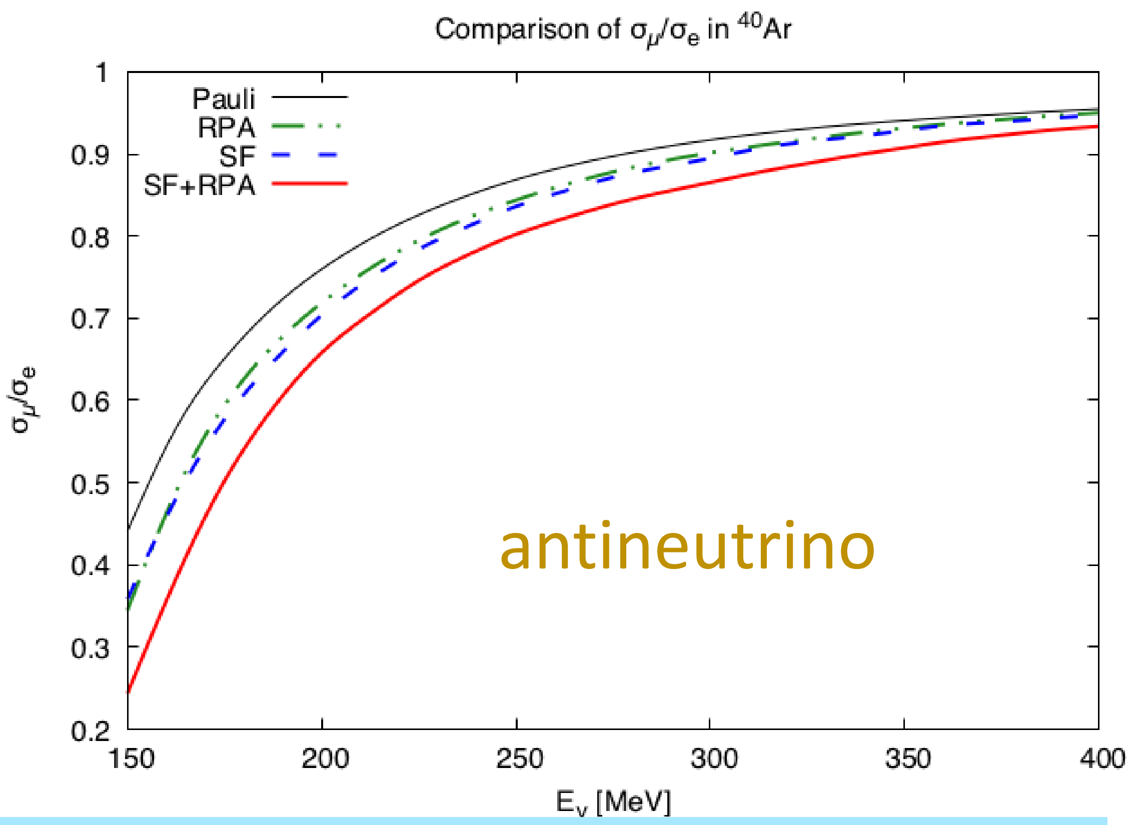
MINERνA: $\bar{\nu}$–CCQE-like



...addition of RPA and 2p2h effects to the simulation substantially improves agreement with the MINERvA QE-like data over default GENIE. Addition of either RPA or 2p2h alone is not sufficient. However, substantial discrepancies between the improved model and data remain, indicating that more model development is needed.

C.E. Patrick et al., PRD97 (2018) 052002





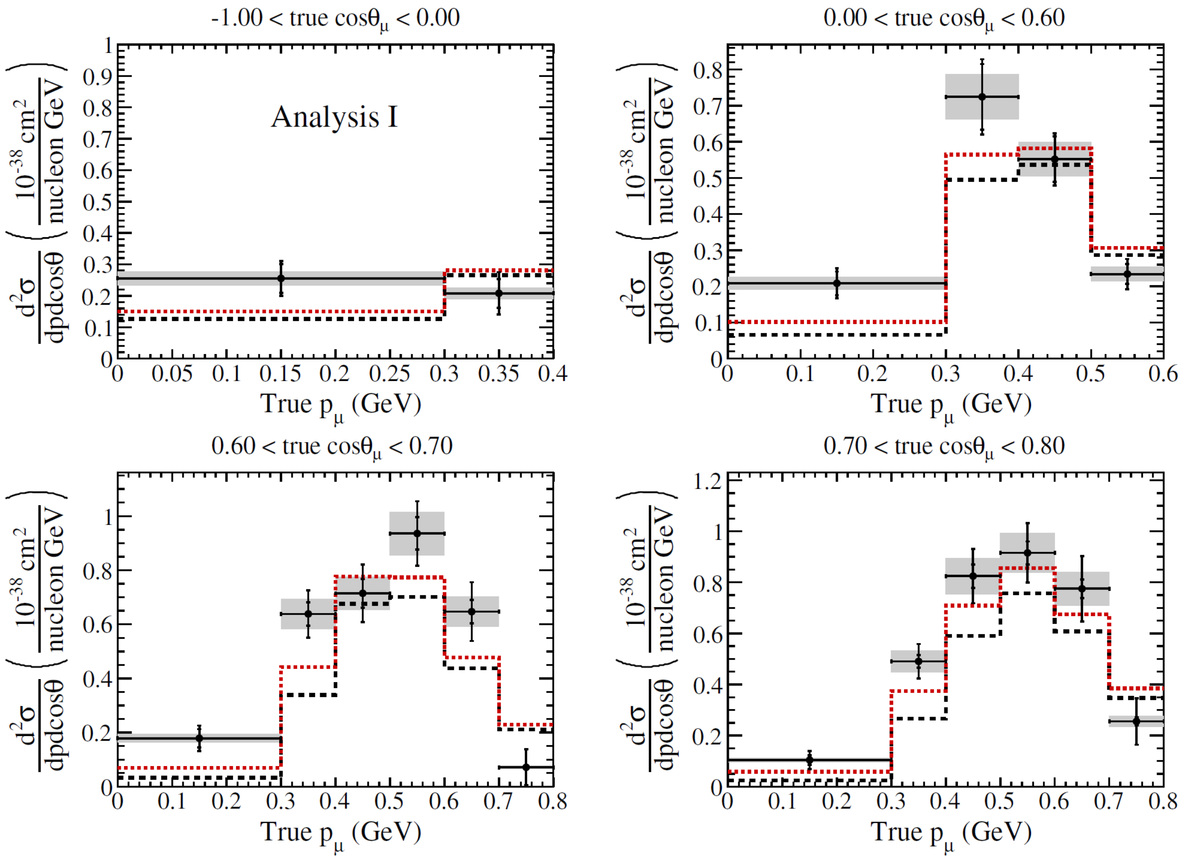
$^{40}Ar \quad {}^{\sigma_\mu}/_{\sigma_e}$

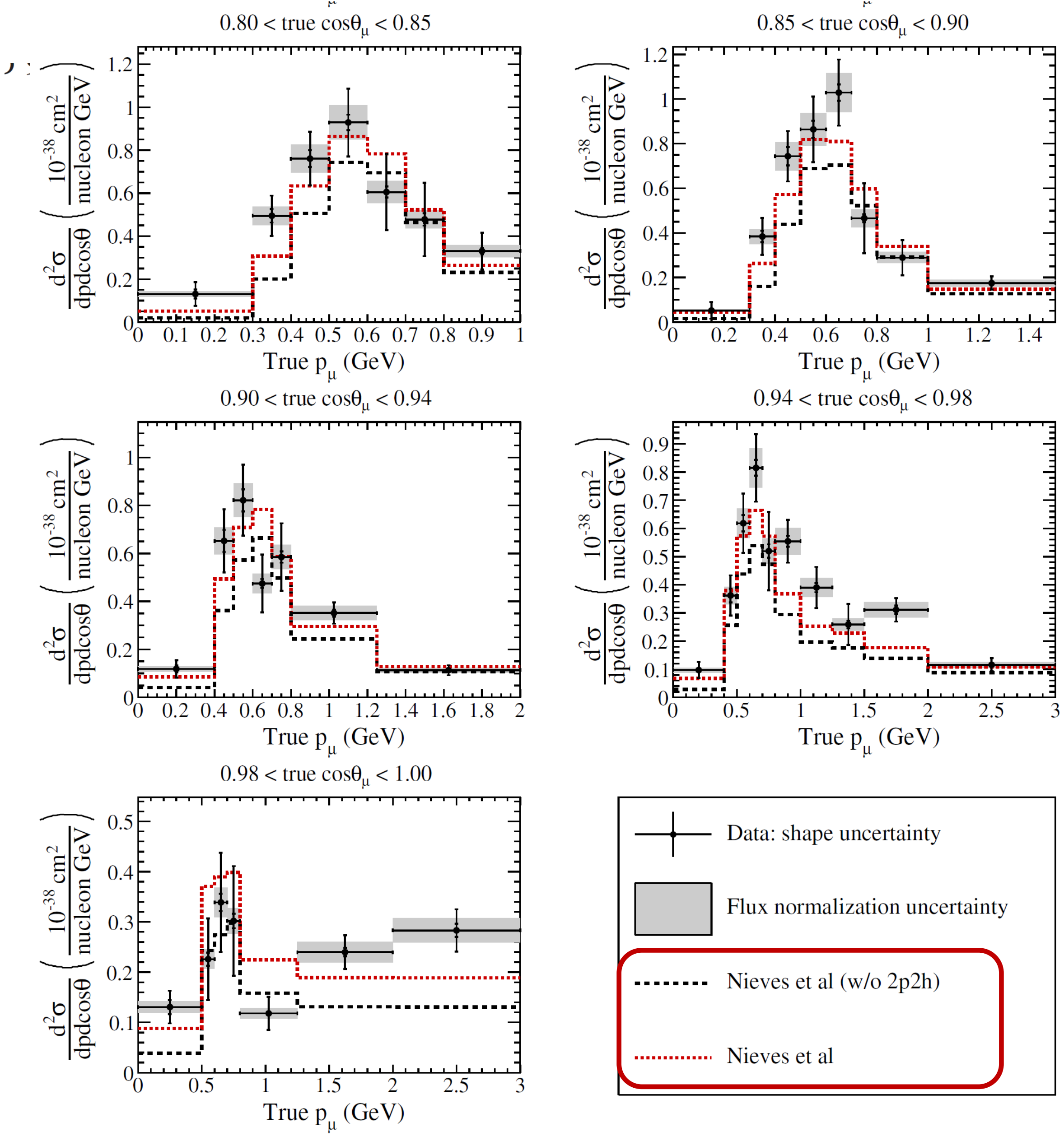
- Important deviations from 1 ($150\ MeV < E_\nu < 400\ MeV$)
- SFs & RPA effects strongly affect the ratio and become essential to perform a correct analysis of appearance neutrino oscillation events in LBE.

Final remark:

Nuclear effects lead to sizable uncertainties on the neutrino nucleus cross sections at low $-q^2 = Q^2 < 1\ \mathrm{GeV}^2$

It is important to incorporate these effects in event generators (GENIE, etc..)



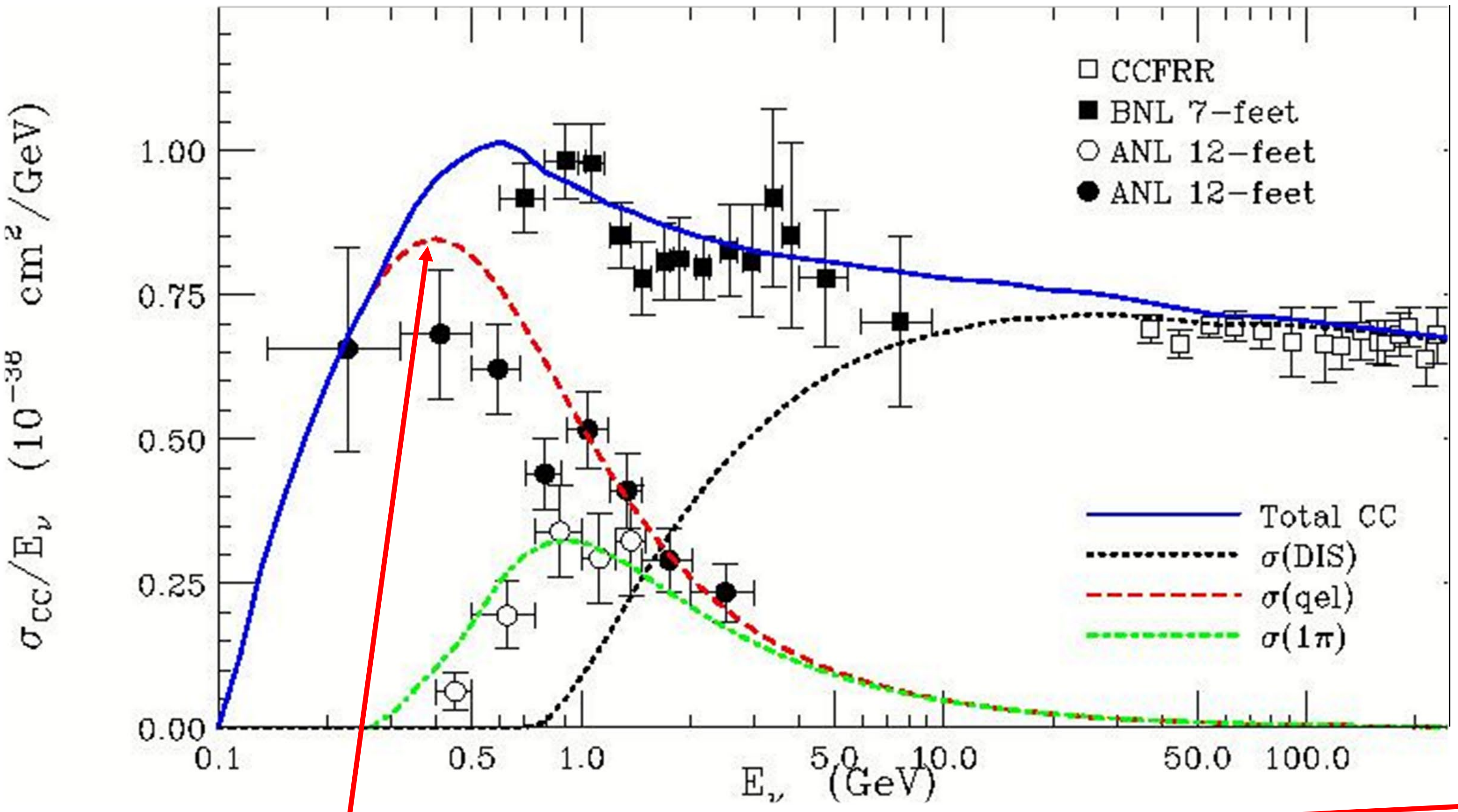


T2K [PRD 93 112012 (2016)]: Results show sizable nuclear effects for all muon kinematics. Models including 2p2h+RPA contributions agree well with the data

T2K: CCQE-like

Motivation: Neutrino oscillations, neutrino detectors and nuclear cross sections

Details on the axial structure of hadrons in the free space and inside of nuclei



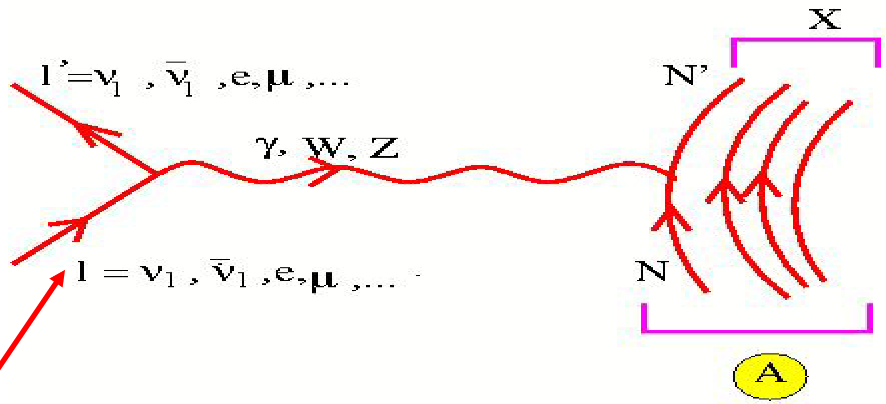
Theoretical knowledge of QE and 1π cross sections is important to carry out a precise neutrino oscillation data analysis...

^{12}C → Liquid scintillators

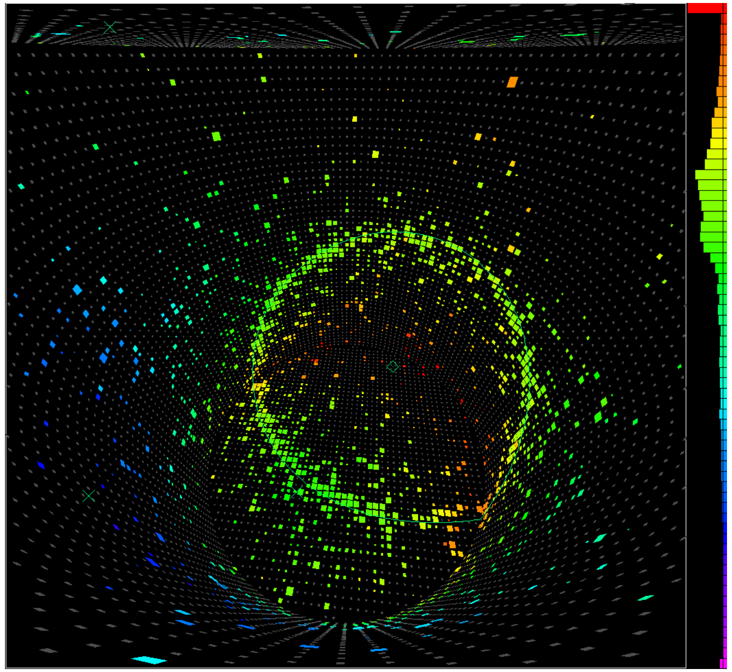
^{16}O → Cerenkov, nuclear emulsion detectors

^{40}A → TPC's (time projection chambers) ....

QUASIELASTIC PEAK

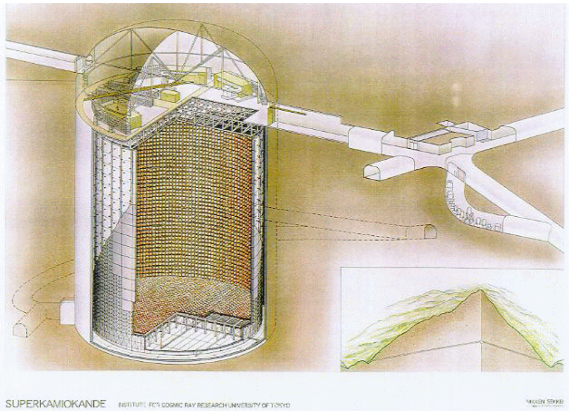


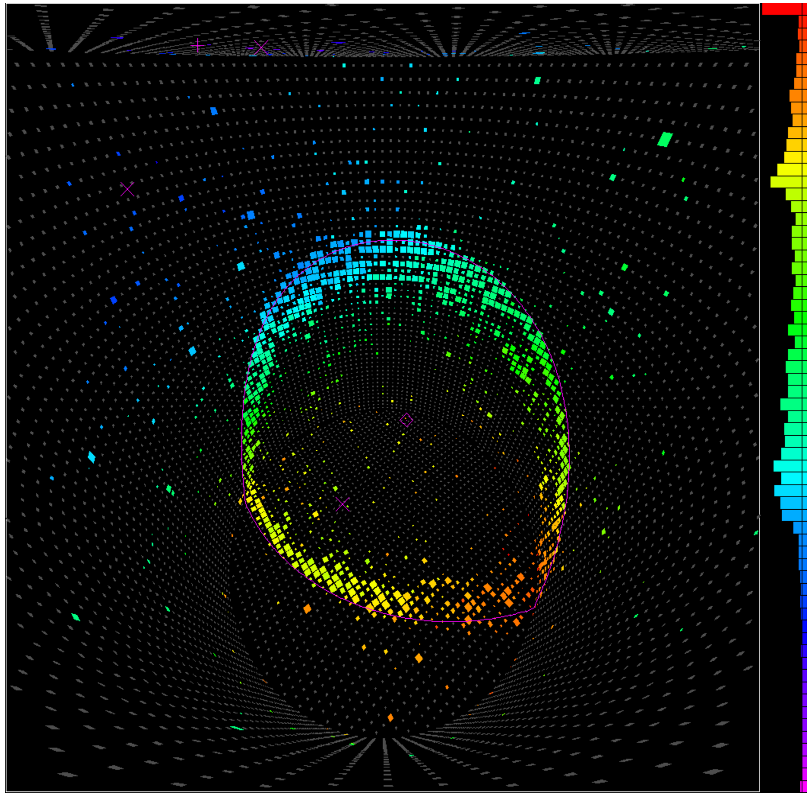
γ
VIRTUAL W BY ONE NUCLEON
Z



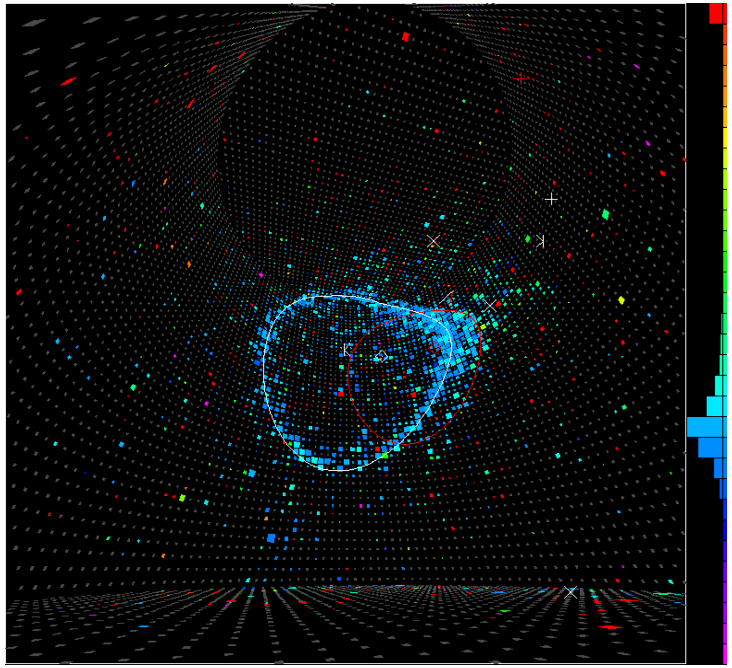
$\nwarrow$ e

SuperKamiokande





$\uparrow$ μ



$\leftarrow \pi^0$

$$\beta > \frac{1}{n} \rightarrow E_{\pi,\mu} > 200 - 300\ \mathrm{MeV}$$

Pion production → misidentification of 1 Cherenkov ring events that are assumed to be produced by Charged Current (CC) QE reactions $\nu_\alpha A \rightarrow l^\alpha A'$

Even distinguishing between μ- and e-like rings

- **Appearance Probability** $P(\nu_\mu \rightarrow \nu_e)$: The CC QE signature $\nu_e A \rightarrow e\, A'$ used to identify ν_e can be confused with the NC 1π production $\nu_\mu A \rightarrow \nu_\mu A' \pi^0$
- **Survival Probability** $P(\nu_\mu \rightarrow \nu_\mu)$: The CC QE signature $\nu_\mu A \rightarrow \mu\, A'$ used to identify ν_μ can be confused with the CC or NC $\nu_{\mu,\tau} A \rightarrow (\nu_{\mu,\tau}\ or\ \mu, \tau) A' \pi$ when only one of the particles emits Cherenkov light. For instance, processes (ν_μ, μ, π) might produce an incorrect reconstruction of the neutrino energy E → L/E analysis ?

Nuclear cross sections are crucial to reduce the systematic errors of oscillation analysis !

Dedicated experiments as MINERνA (FermiLab), which seeks to measure low energy neutrino interactions both in support of neutrino oscillation experiments and also to study the strong dynamics of the nucleon and nucleus that affect these interactions

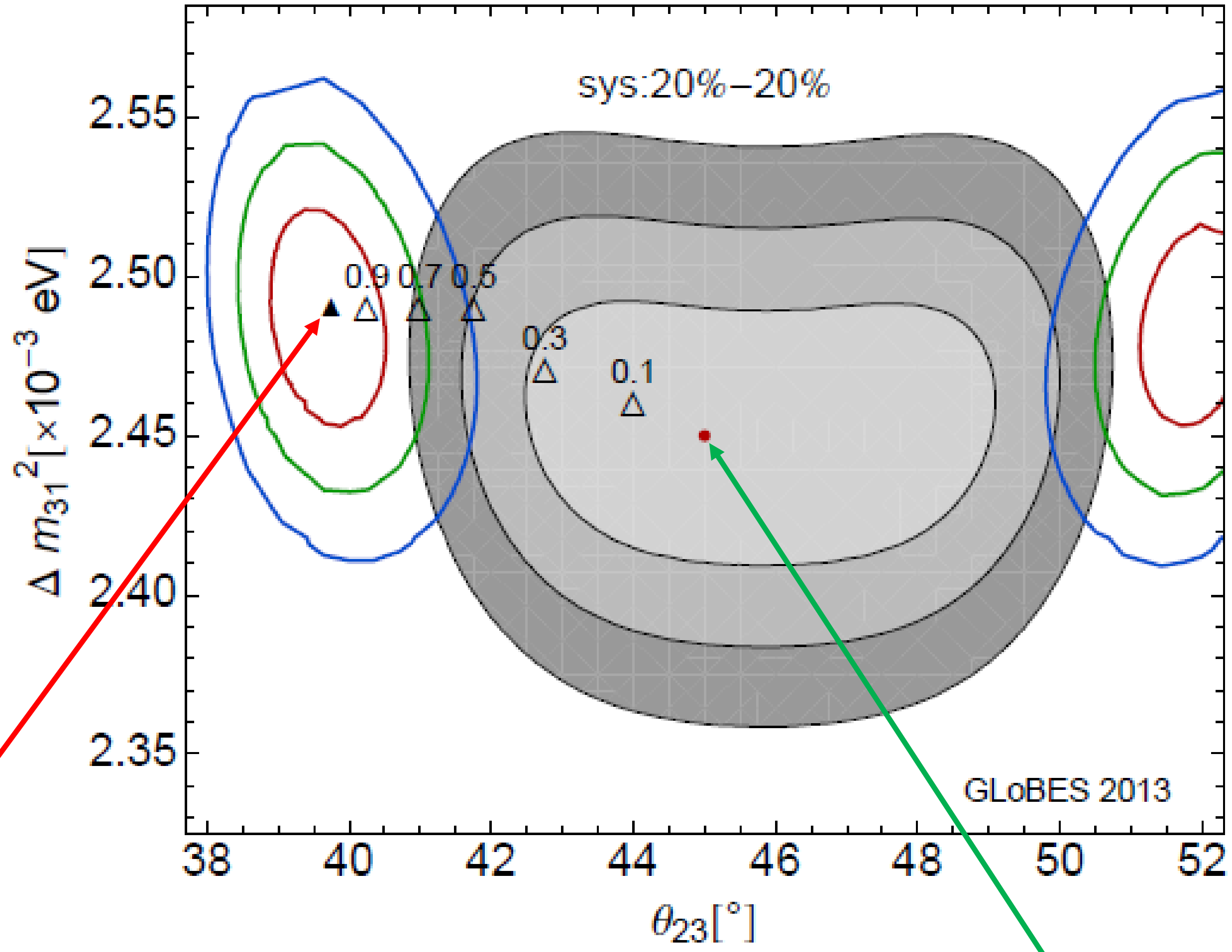
P. Coloma, P. Huber, PRL 111 (2013)

Quantitative impact in the determination of the oscillation parameters

Effects of a simple model for QE-like events ....

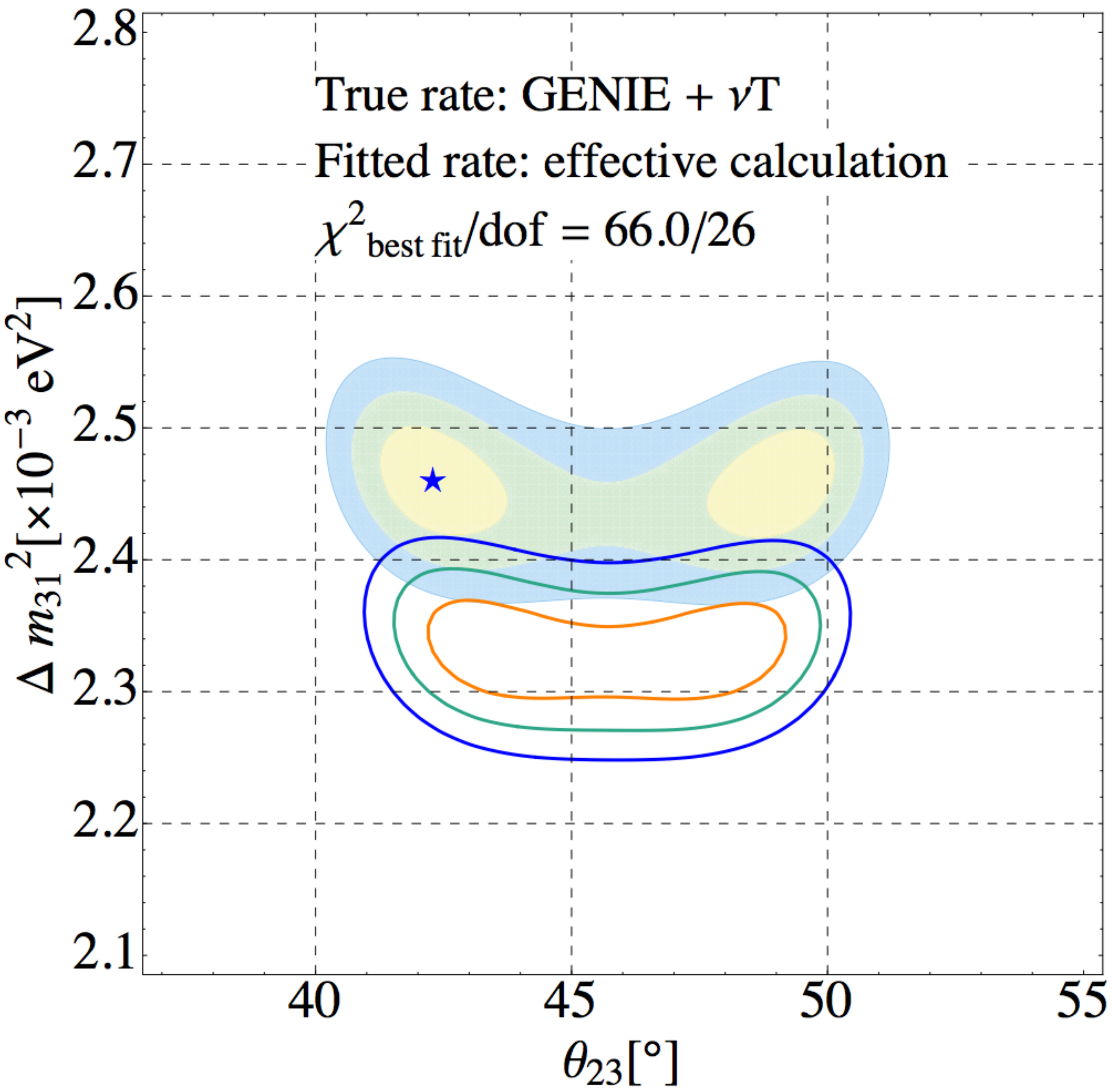
$$N_i^{\rm test}(\alpha) = \alpha \times N_i^{\rm QE} + (1-\alpha) \times N_i^{\rm QE\text{-}like}$$

α parametrizes the fraction of two-nucleon absorption that is neglected in the fit



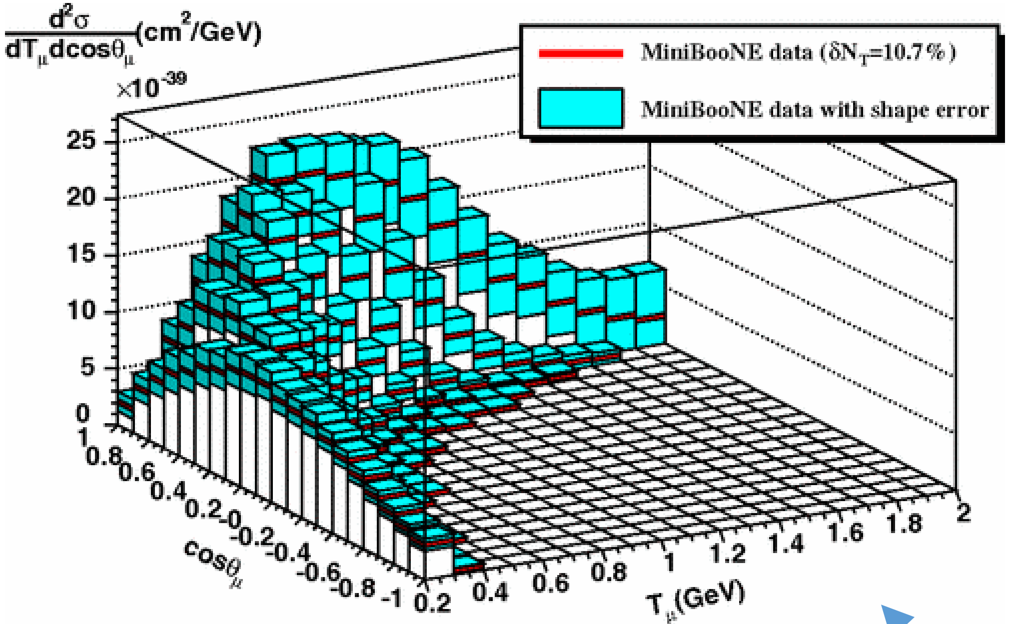
Reconstructed from naive QE dynamics

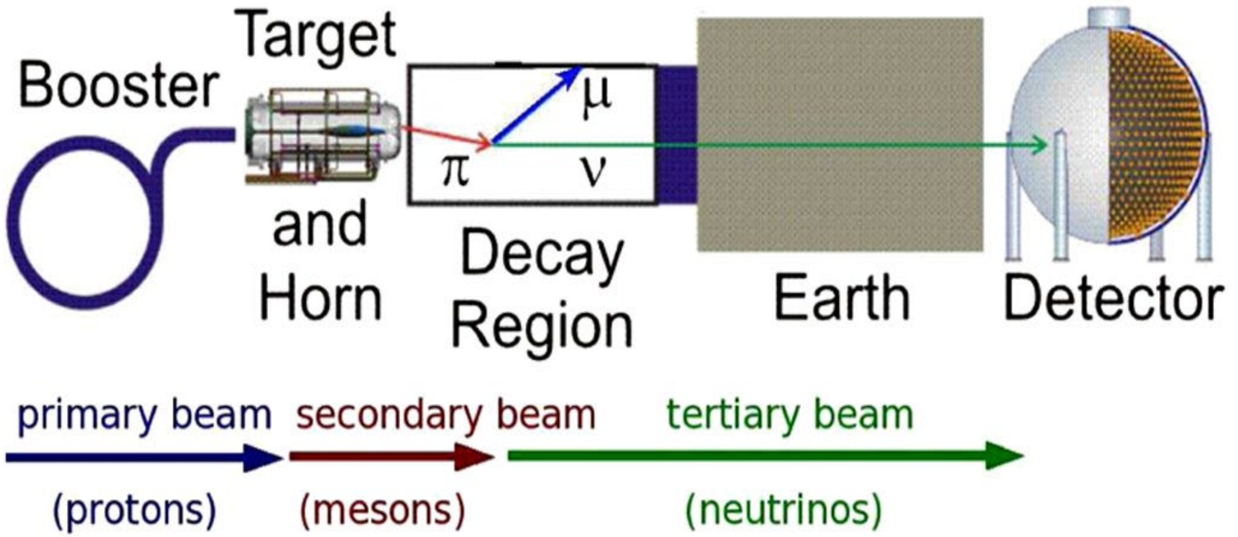
true

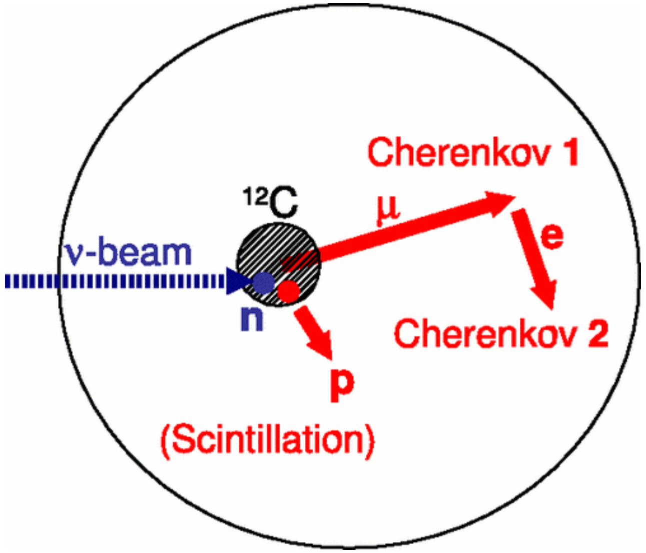


Systematic uncertainties in long-baseline neutrino-oscillation experiments,
Artur M Ankowski and Camillo Mariani,
J.Phys.G 44 (2017) 054001

MiniBooNE CCQE (PRD 81, 092005)







$$1:\ \nu_\mu + n \rightarrow \mu^- + p$$
$$2:\ \hookrightarrow e^- + \bar{\nu}_e + \nu_\mu.$$

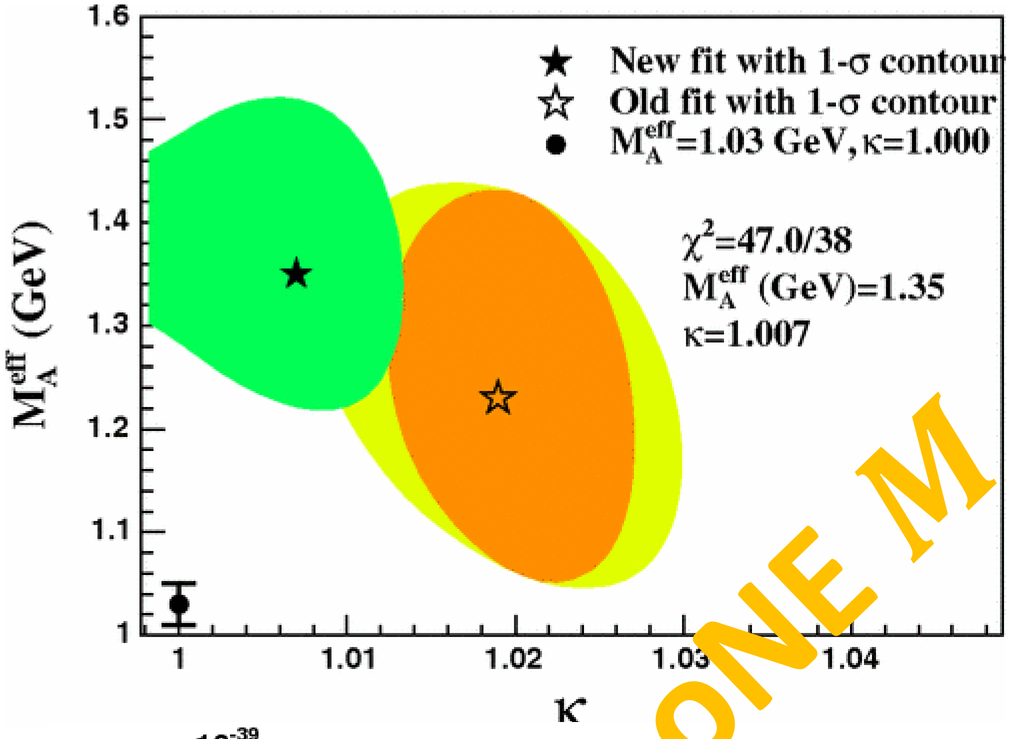
1 muon events !

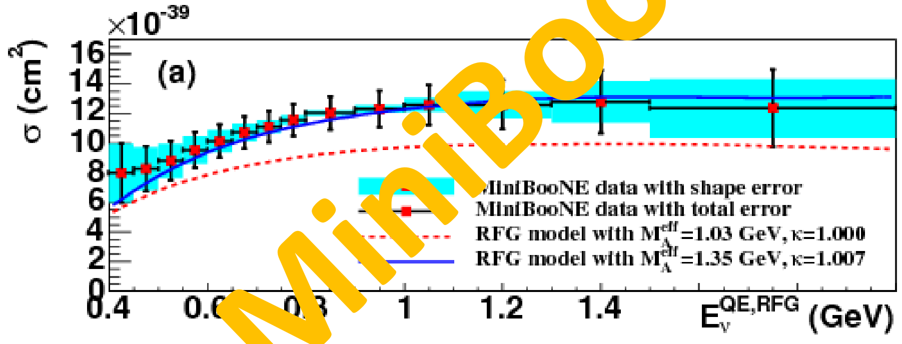
The largest background is from CC single pion production: CC1π^+

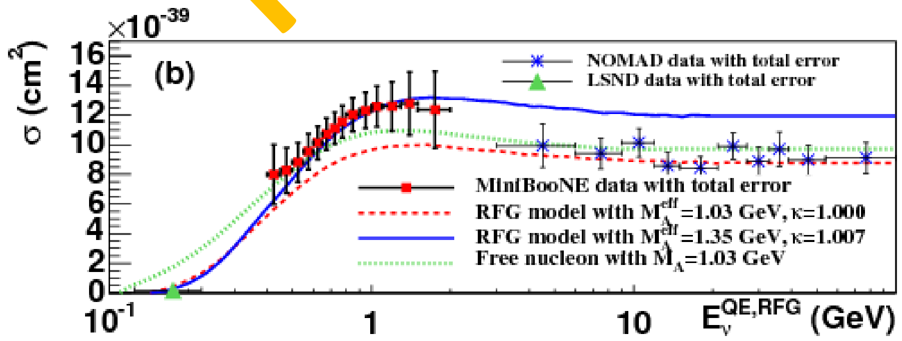
$$1:\ \nu_\mu + p(n) \rightarrow \mu^- + p(n) + \pi^+$$
$$\hookrightarrow \mu^+ + \nu_\mu$$
$$2:\ \hookrightarrow e^- + \bar{\nu}_e + \nu_\mu$$
$$3:\ \hookrightarrow e^+ + \nu_e + \bar{\nu}_\mu.$$

2 muon events

MiniBooNE M_A puzzle





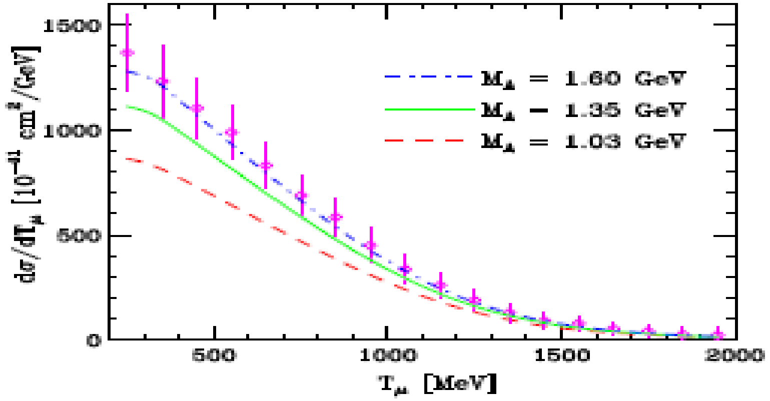


$$M_A^{\rm eff} = 1.35 \text{ GeV}$$

vs

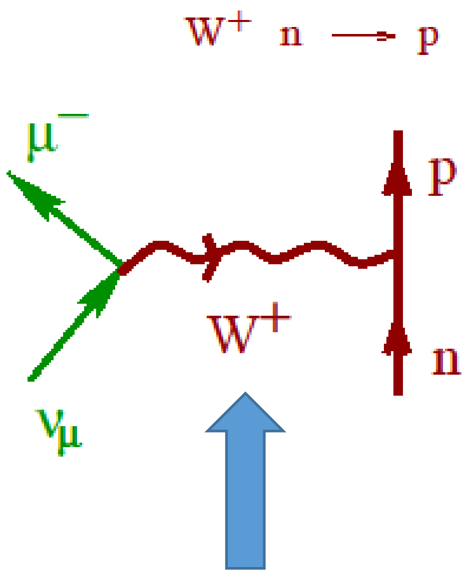
1.03 GeV (world avg)

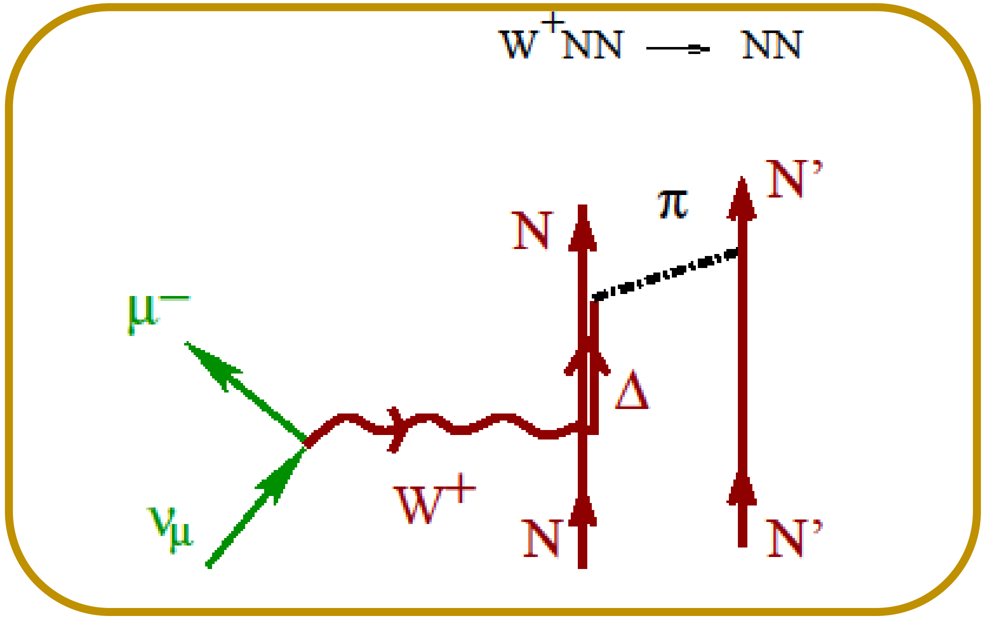
confirmed by many other groups, for instance by Benhar et al. (PRL 105, 132301)

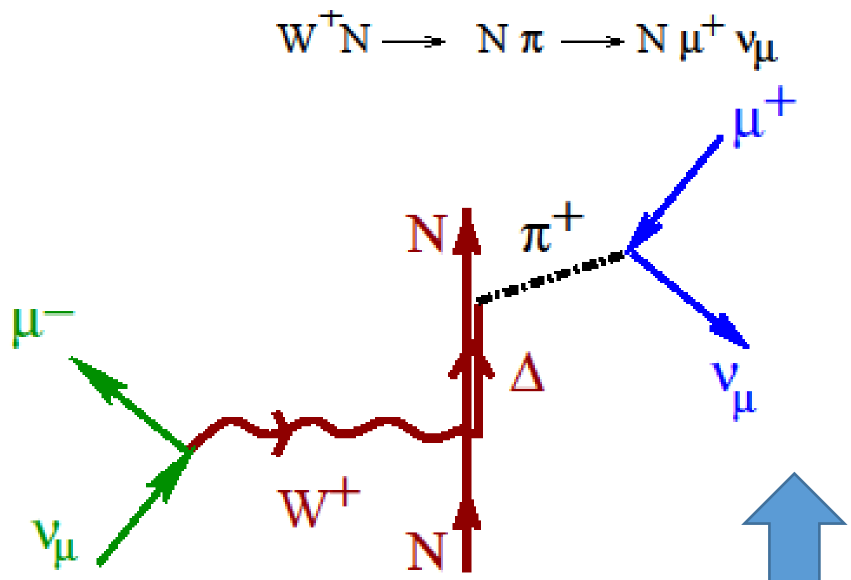


ChPT O(p^3) + single pion electroproduction data: M_A = 1.014± 0.016 GeV (V. Bernard, N. Kaiser, and U. G.Meissner, PRL69, 1877 (1992))

• CCQE measurements on deuterium and, to lesser extent, hydrogen targets is M_A = 1.016 ± 0.026 GeV (A. Bodek, S. Avvakumov, R. Bradford, and H. S. Budd, EPJC 53, 349 (2008))





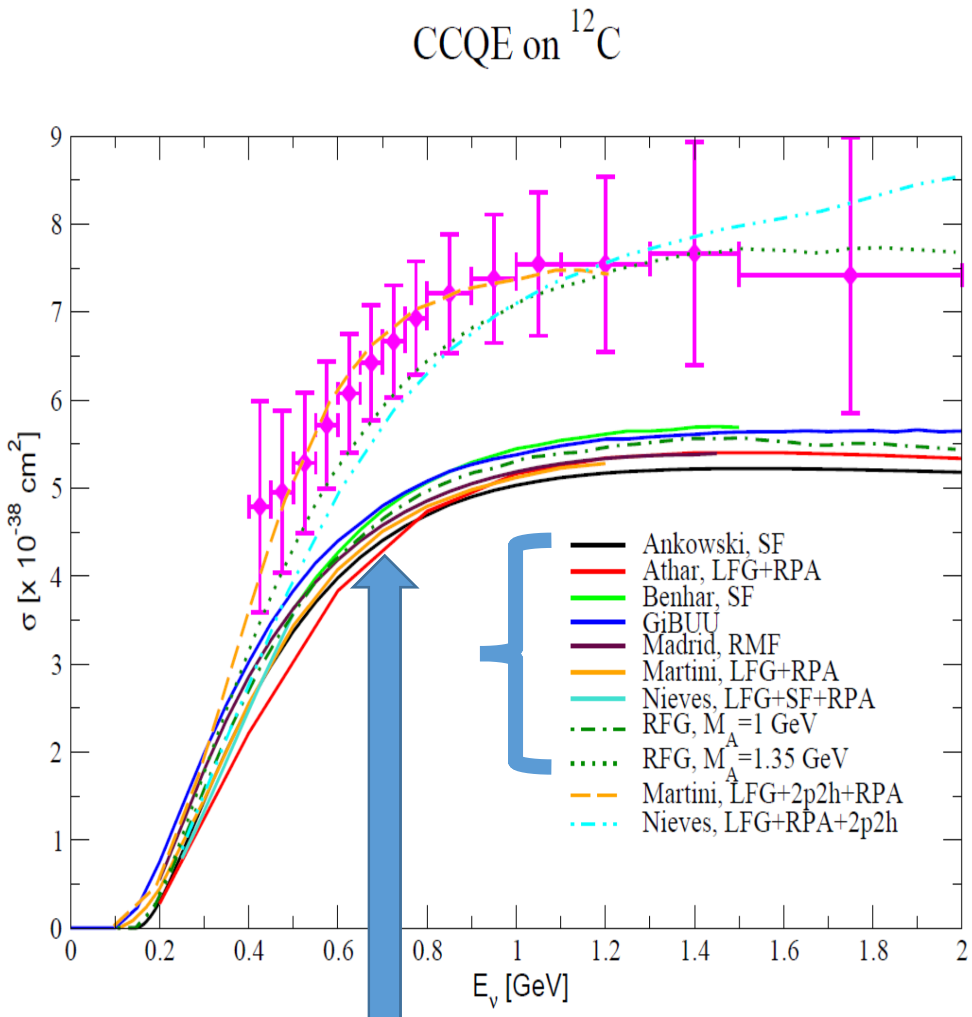


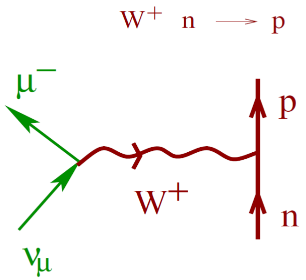
...but key observation (Martini et al., PRC 81, 045502): in most theoretical works QE is used for processes where the gauge boson $W^{\pm}$ or Z^{0} is absorbed by just one nucleon, which together with a lepton is emitted.

However in the recent MiniBooNE measurements, QE is related to processes in which only a muon is detected (ejected nucleons are not detected !) ≡ **CCQE-like**

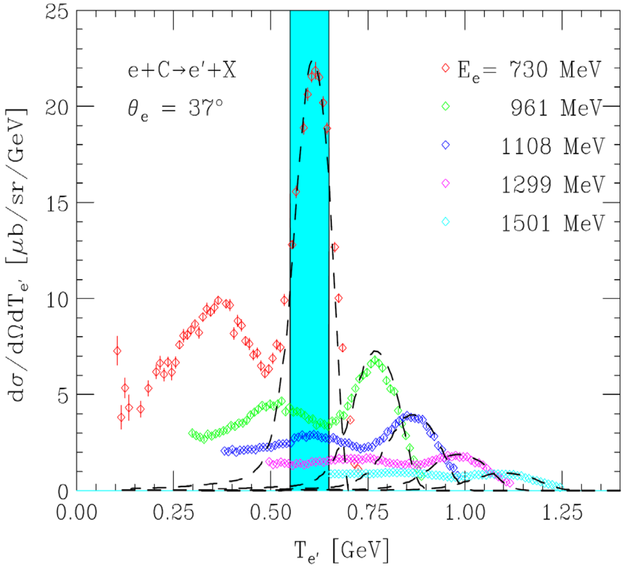
It discards pions coming off the nucleus, since they will give rise to additional leptons after their decay.

It includes multinucleon processes and others like π production followed by absorption (MBooNE analysis Monte Carlo corrects for these latter events).



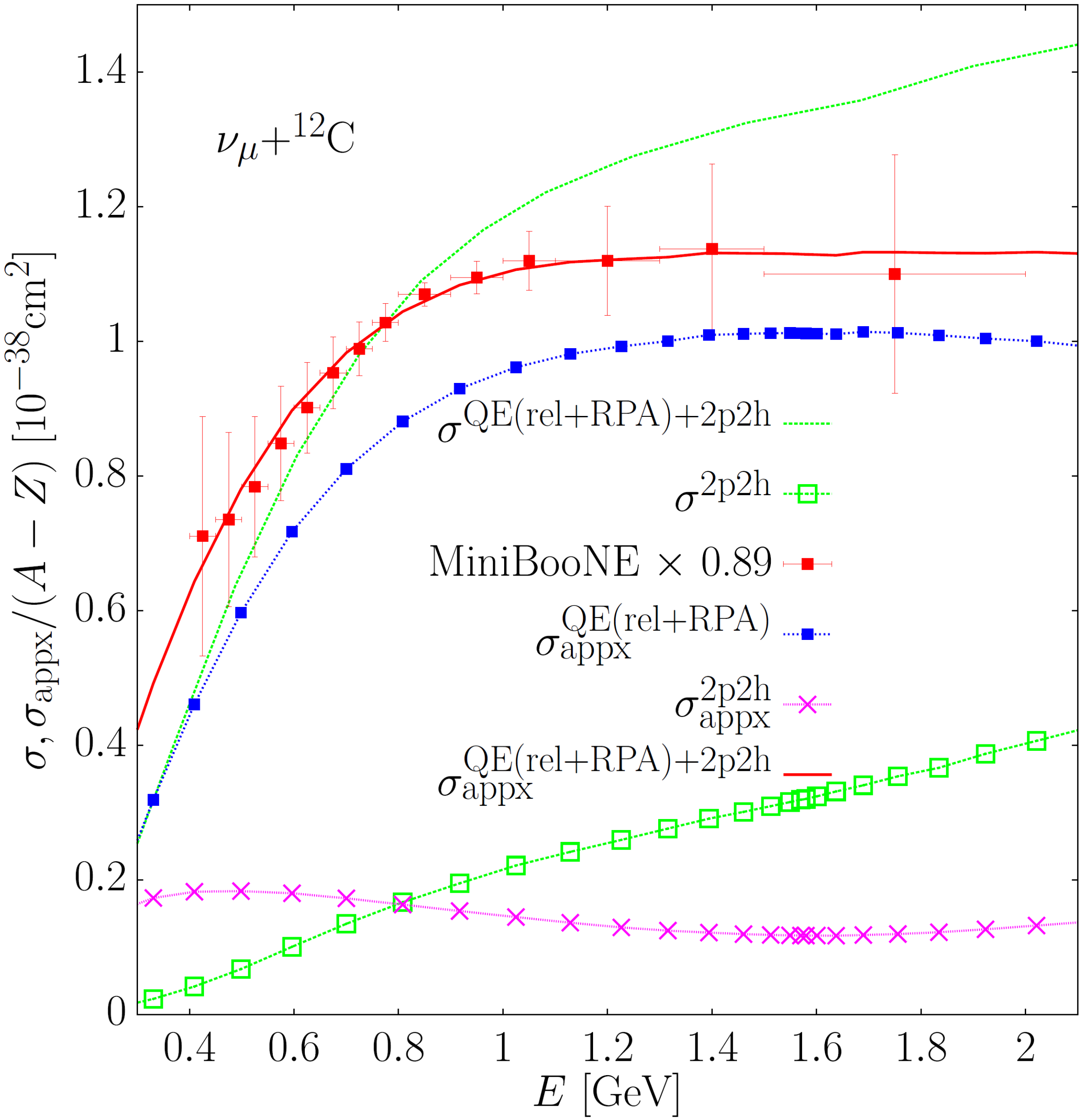


O. Benhar@NuFacT11: [arXiv : 1110.1835] measured electron-carbon scattering cross sections for a fixed outgoing electron angle $\theta = 37^o$ and different beam energies $\in [730, 1501]$ GeV, plotted as a function of E_e,



The energy bin corresponding to **the top of the QE peak at $E_e =$ 730 MeV receives significant contributions from** cross sections corresponding to different beam energies and **different mechanisms!**

problem: incoming neutrino beam is not monochromatic



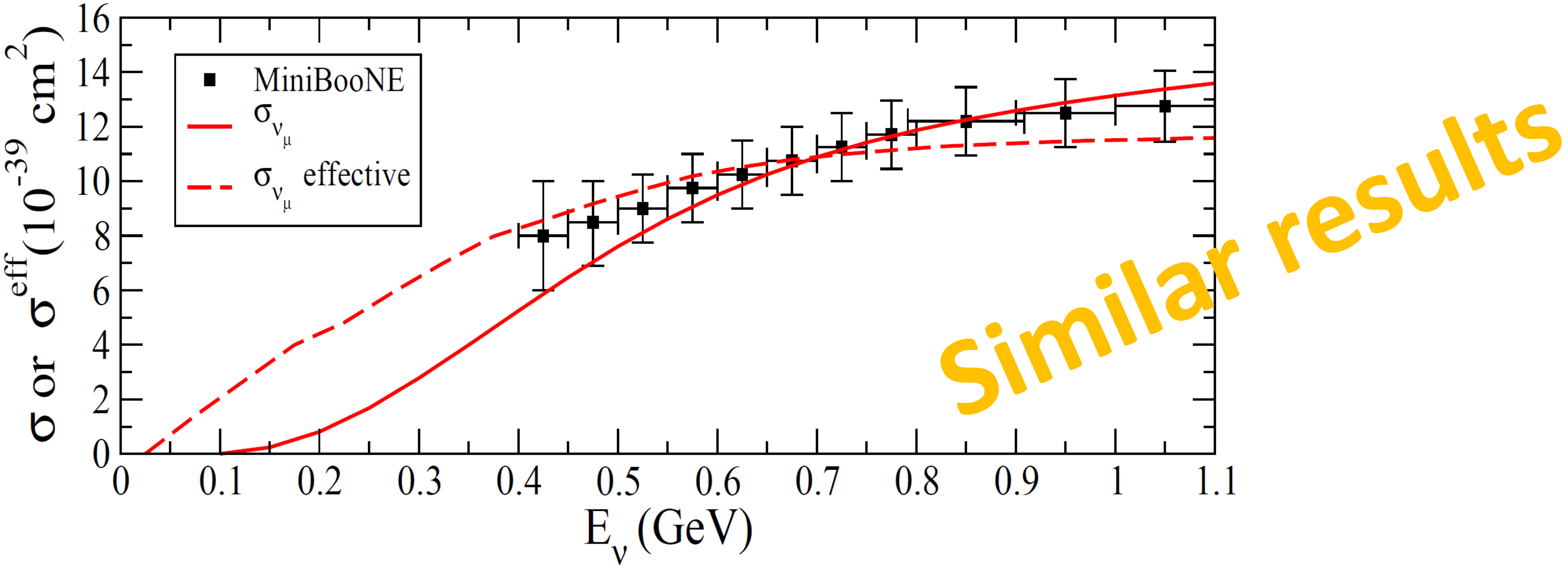
$$\Big[\langle\sigma\rangle P_{\mathrm{rec}}(E_{\mathrm{rec}})\Big]_{\mathrm{Exp}} \sim$$

$$\int \left(\frac{d\sigma}{dE_{\mathrm{rec}}}(E';E_{\mathrm{rec}})\Bigg|^{M_A=1.049\ \mathrm{GeV}}_{\mathrm{QE+RPA,}} + \frac{d\sigma^{\mathrm{2p2h}}}{dE_{\mathrm{rec}}}(E';E_{\mathrm{rec}}) \right) \Phi(E')dE'$$

... and

$$\underbrace{\left[\frac{d\sigma/dE_{\mathrm{rec}}(E;E_{\mathrm{rec}})}{\int dE''\Phi(E'')d\sigma/dE_{\mathrm{rec}}(E'';E_{\mathrm{rec}})}\right]}$$

ONLY QE ,M_A=1.32 GeV **and noRPA**



Martini, Ericson, Chanfray [Phys.Rev. D87 (2013), 013009]

The simplest description ⇒ relativistic Fermi Gas with

non interacting fermions $\boxed{\Sigma = 0}$,

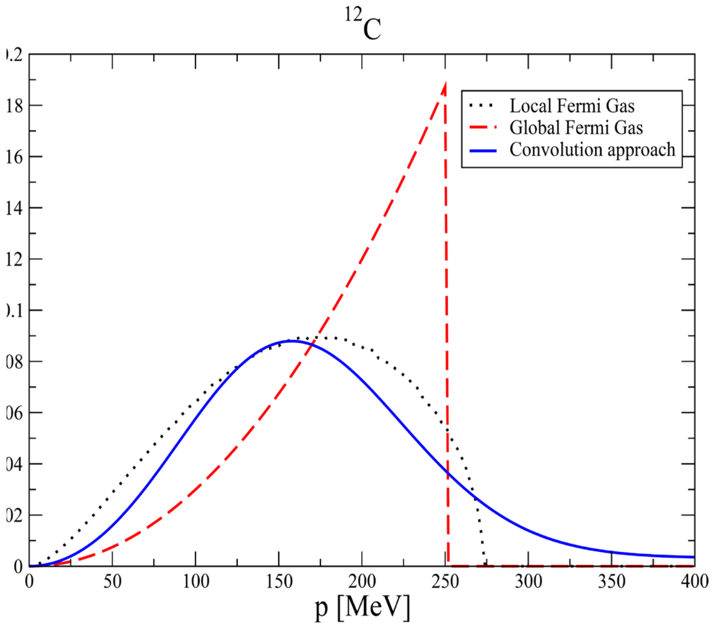
$$S_p(\omega,\vec{p}) = \frac{\theta(|\vec{p}\,| - k_F)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

$$S_h(\omega,\vec{p}) = \frac{\theta(k_F - |\vec{p}\,|)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

and only Pauli blocking is incorporated!!

Local vs Global Fermi Gas ?

$$k_F^{p,n}(r) = \left[3\pi^2 \rho^{p,n}(r)\right]^{1/3} \text{ vs } k_F^{p,n} = \text{cte ?}$$



Local vs Global Fermi Gas ?

$$k_F(r) = \left[3\pi^2\rho(r)/2\right]^{1/3} \textbf{ vs } k_F \textbf{= cte ?}$$

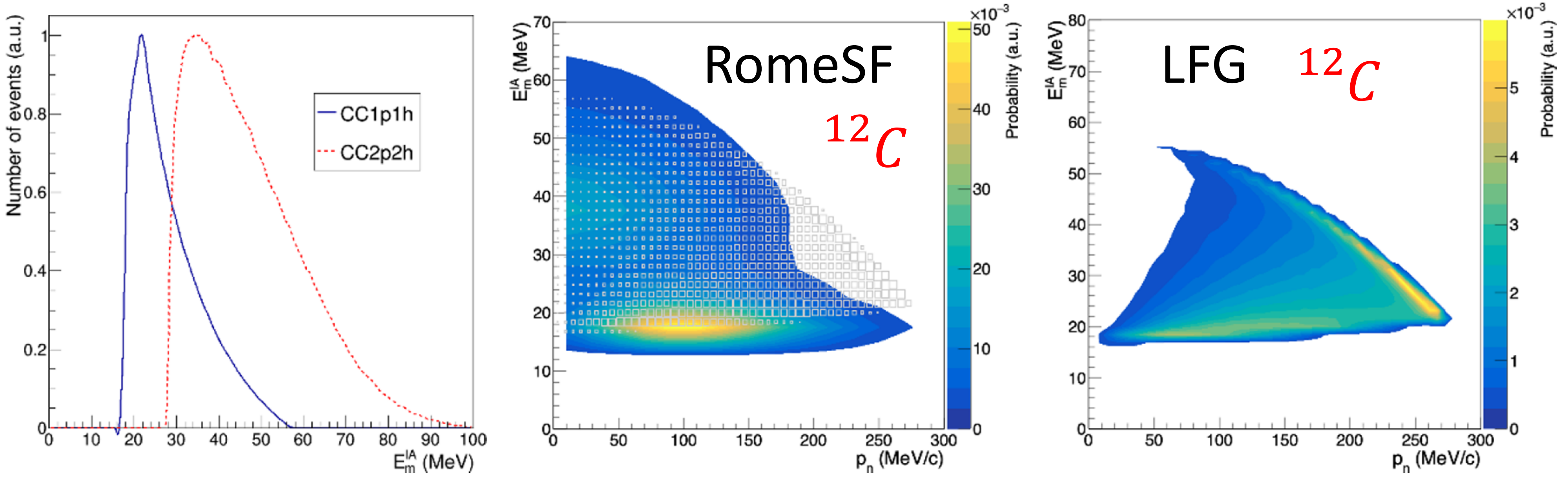
$$S_h(\omega,\vec{p}) = \delta(\omega - E(\vec{p}))\theta(k_F - |\vec{p}\,|)/2\omega$$

$$n^{\rm RgFG}(|\vec{p}\,|) = \frac{4V}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = \frac{3A}{4\pi k_F^3}\theta(k_F - |\vec{p}\,|)$$

$$n^{\rm LDA}(|\vec{p}\,|) = 4\int \frac{d^3r}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = 4\int \frac{d^3r}{(2\pi)^3}\theta(\mathbf{k_F(r)} - |\vec{p}\,|)$$

$$\left(\int d^3p\, n(|\vec{p}\,|) = A\right)$$

Convolution approach: C. Ciofi degli Atti, S. Liuti, and S. Simula, PRC 53, 1689 (1996), provide realistic distribution due to short-range correlations !

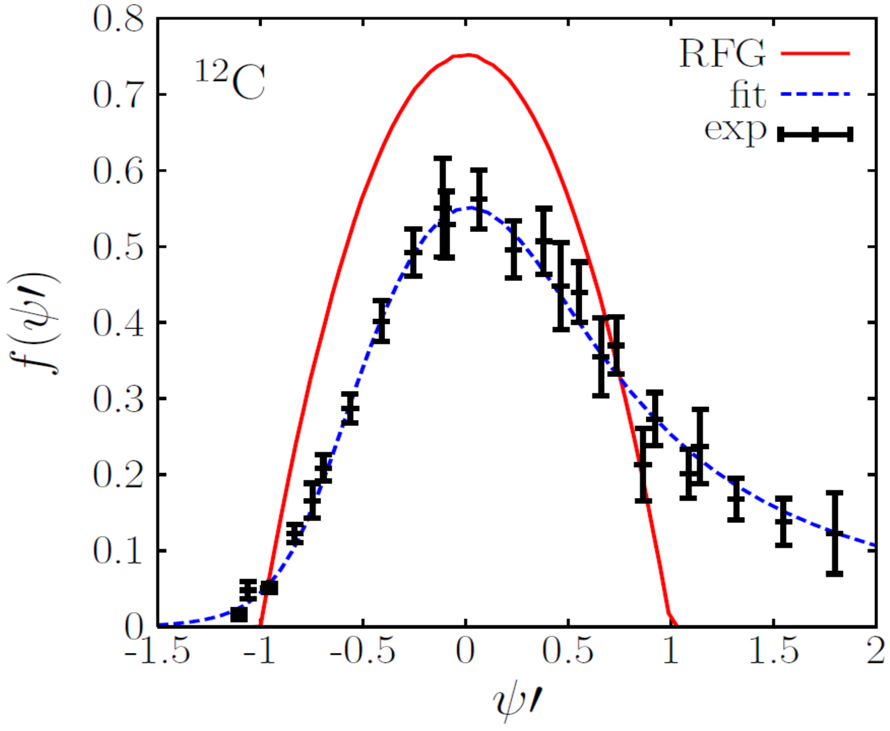


probability to find a neutron in carbon with a momentum (p_n) for a given reaction missing energy E_m^{IA} (T2K flux)

Figure 3. Left: number of events as function of the E_m^{IA} energy for neutrino scattering off ^{12}C within the LFG model. Both the 1p1h (eq. (2.8)) and 2p2h (eq. (3.3)) E_m^{IA} distributions are shown by the blue-solid and red-dashed lines, respectively. The gap between the two distributions is caused by the excitation energy of the two holes in the final state. As in figure 1, results have been folded with the T2K neutrino energy flux. Center: probability to find a neutron in carbon with a momentum (p_n) for a given reaction missing energy (E_m^{IA}) (see eq. (2.8)) as predicted by the SF model [9–11] (contour plot) and for this implementation of the LFG (box plot). Right: LFG predictions corresponding to the box plot displayed in the middle panel. In all cases, the T2K flux [3] is used.

$$E_m^{IA} = E_\nu - E_\mu - T_{N'}^{\infty}$$

$$E_m^{IA}\Big|_{2p2h} = E_\nu - E_\mu - T_{N_1'}^{\infty} - T_{N_2'}^{\infty}$$



Superscaling approach: Inclusive electron scattering data exhibit interesting systematics that can be used to predict (anti)neutrino-nucleus cross sections (T. Donnelly and I. Sick, PRL 82, 3212 (1999)),

$$f = k_F \frac{\frac{d\sigma}{d\Omega' dE'}}{Z\sigma_{ep} + N\sigma_{en}}$$

- **$f = f(\psi')$, with $\psi' = \psi'(q^0, |\vec{q}\,|)$**
- **f is largely independent of the specific nucleus**

Scaling violations reside mainly in R_T: excitation of resonances, meson production, 2p2h mechanisms and even the tail of DIS. An experimental scaling function $f(\psi')$ could be reliably extracted by fitting the data for R_L.

ν QE cross sections can be calculated with the simple RgFG model followed by the replacement $f_{RgFG} \rightarrow f_{\exp}$.

Superscaling function does not take into account **dip region events**

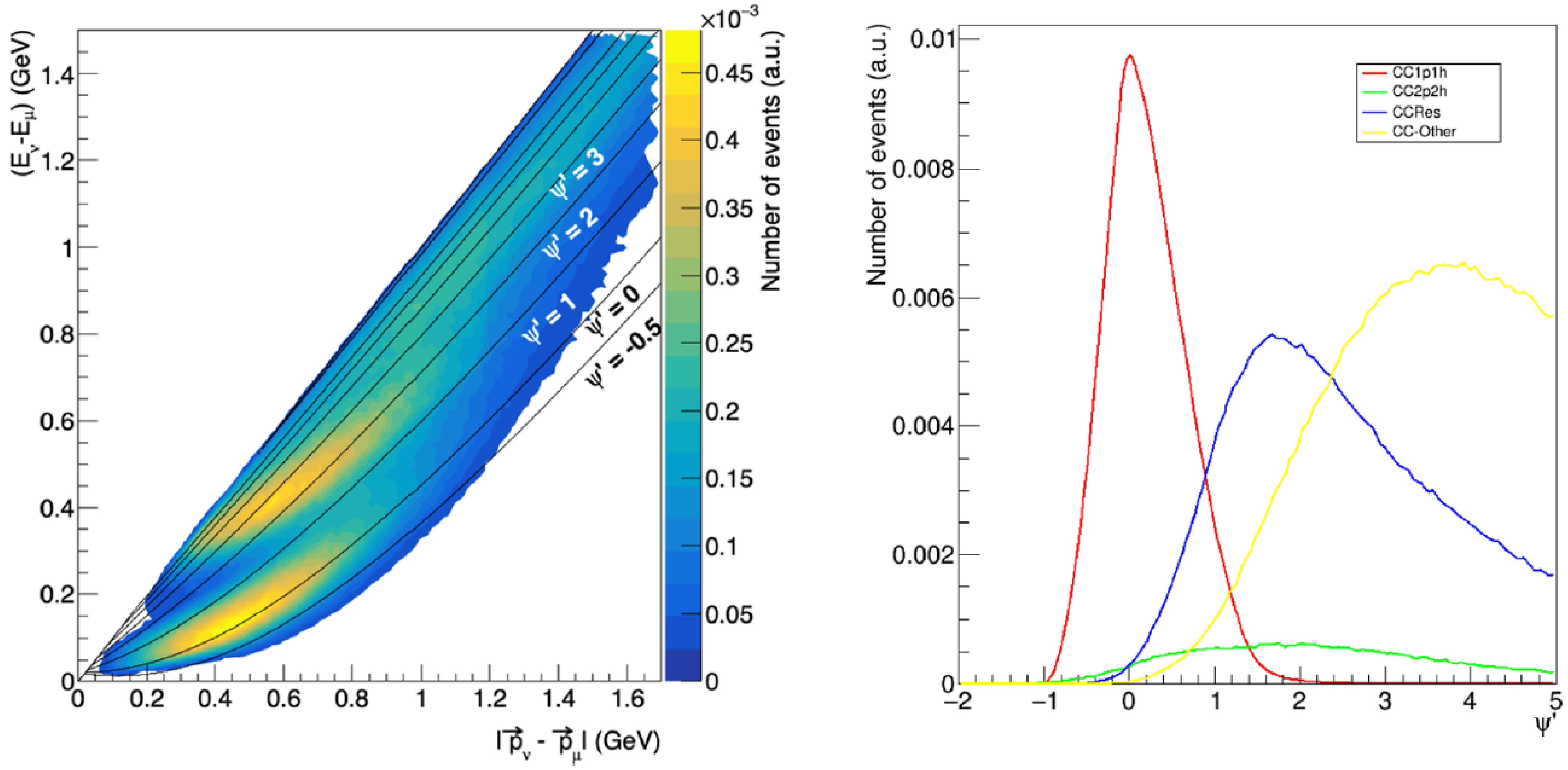
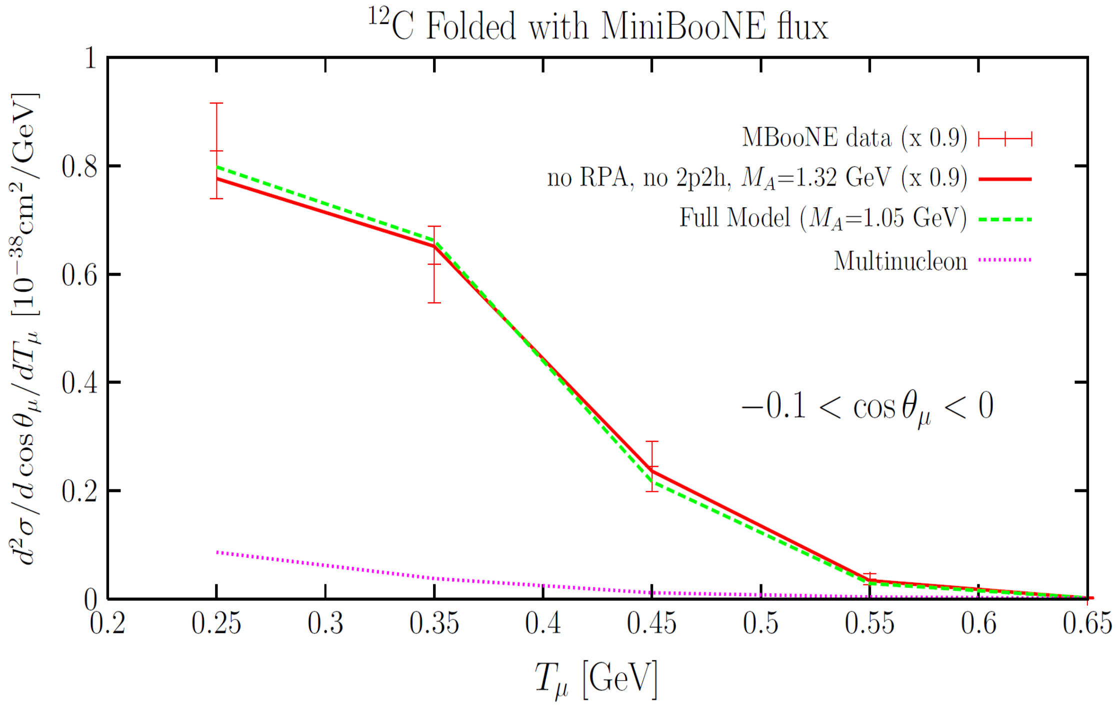
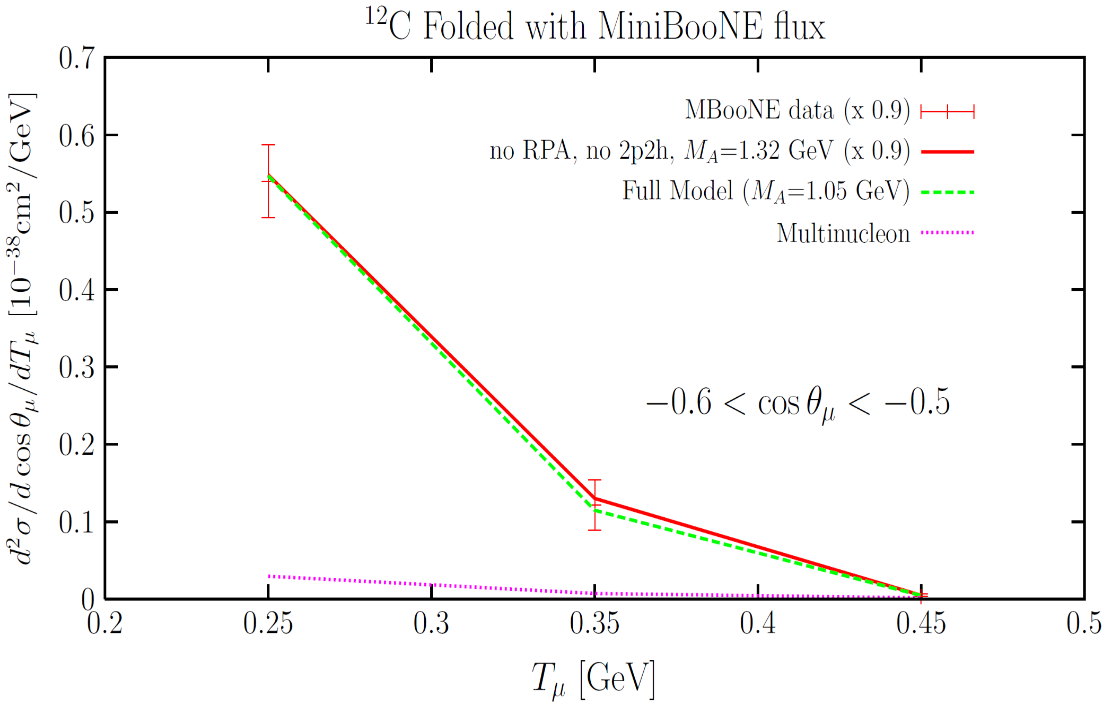
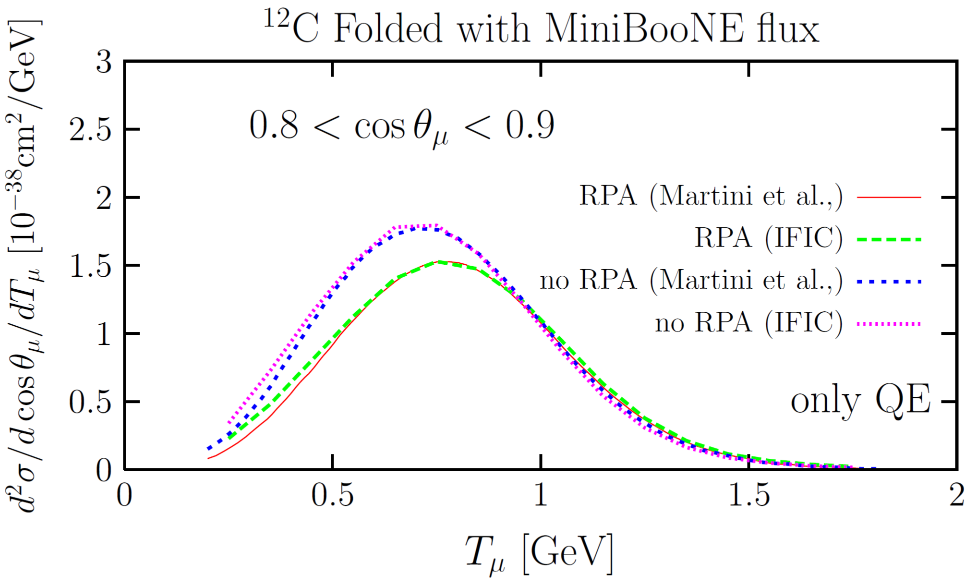


Figure 18. Left: CC inclusive MINERvA $[|\vec{q}| = |\vec{p}_\nu - \vec{p}_\mu|,\ q^0 = (E_\nu - E_\mu)]$ 2D distribution predicted by the NEUT CC inclusive event generator. The black solid lines mark fix ψ' values across the $(q^0, |\vec{q}|)$-plane. Right: ψ' distribution of events obtained from the 2D one shown in the left-panel, and separated by the primary neutrino-nucleon interaction modes.

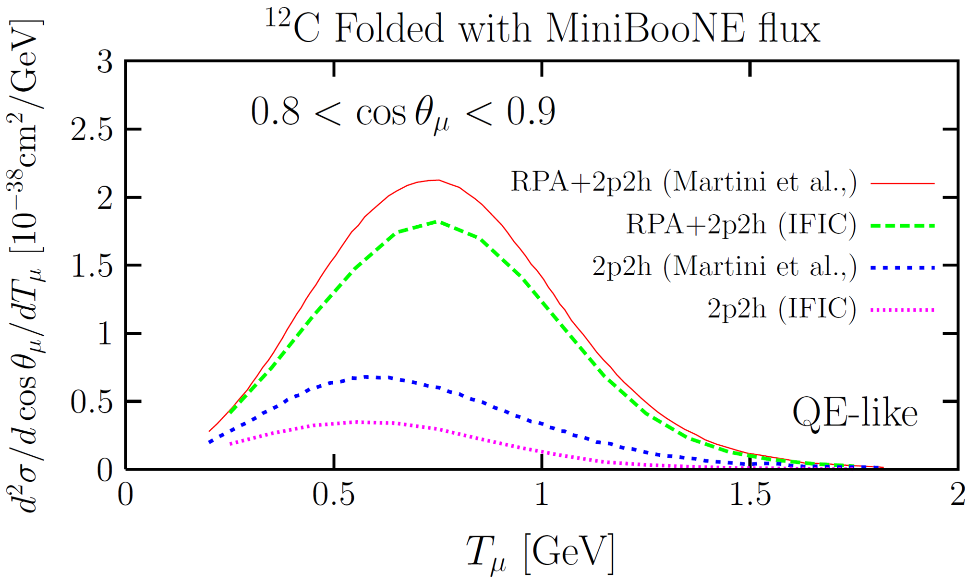
Dependence of the 2p2h contribution on $\cos\theta_\mu$







We compare rather well with Martini et al., PRC 84, 055502 for bare QE and QE+RPA



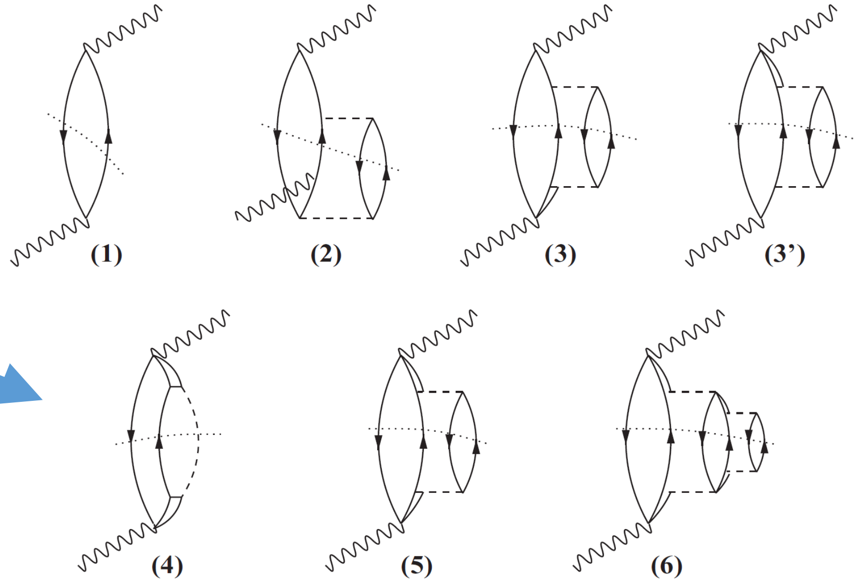
...however our 2p2h contribution is about a factor of 2 smaller!

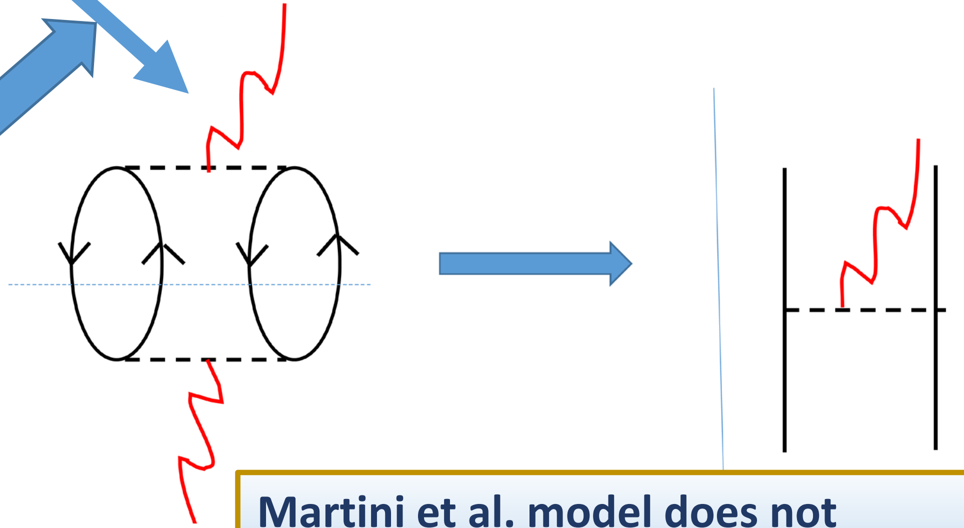
Differences with the work of Martini et al. (PRC80,065501)

1. **Similar for the 2p2h contributions driven by Δh excitation** (both groups use the same model for the Δ−selfenergy in the medium).

2. **Martini et al. do not consider 2p2h contributions driven by contact, pion pole and pion in flight terms.**

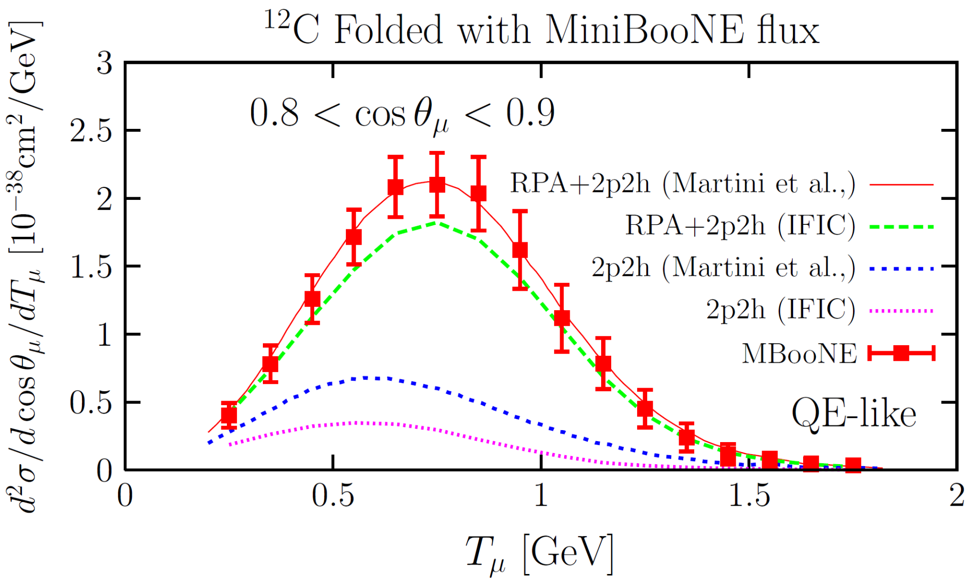
3. **Martini et al. give approximate estimates (no microscopical calculation) for the rest of 2p2h contributions** [relate them to the absorptive part of the p−wave pion-nucleus optical potential at threshold or to a microscopic calculation by Alberico et al. (Annals Phys. 154, 356) specifically aimed at the evaluation of the 2p-2h contribution to the isospin spin-transverse response, measured in inclusive (e, e') scattering].



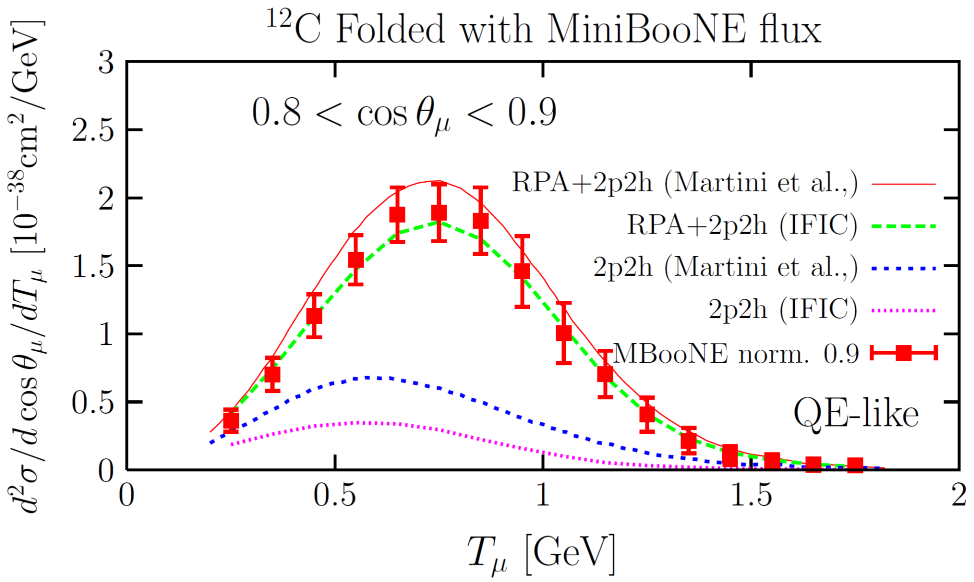


This 2p2h parametrization includes MEC effects driven by the vector current !

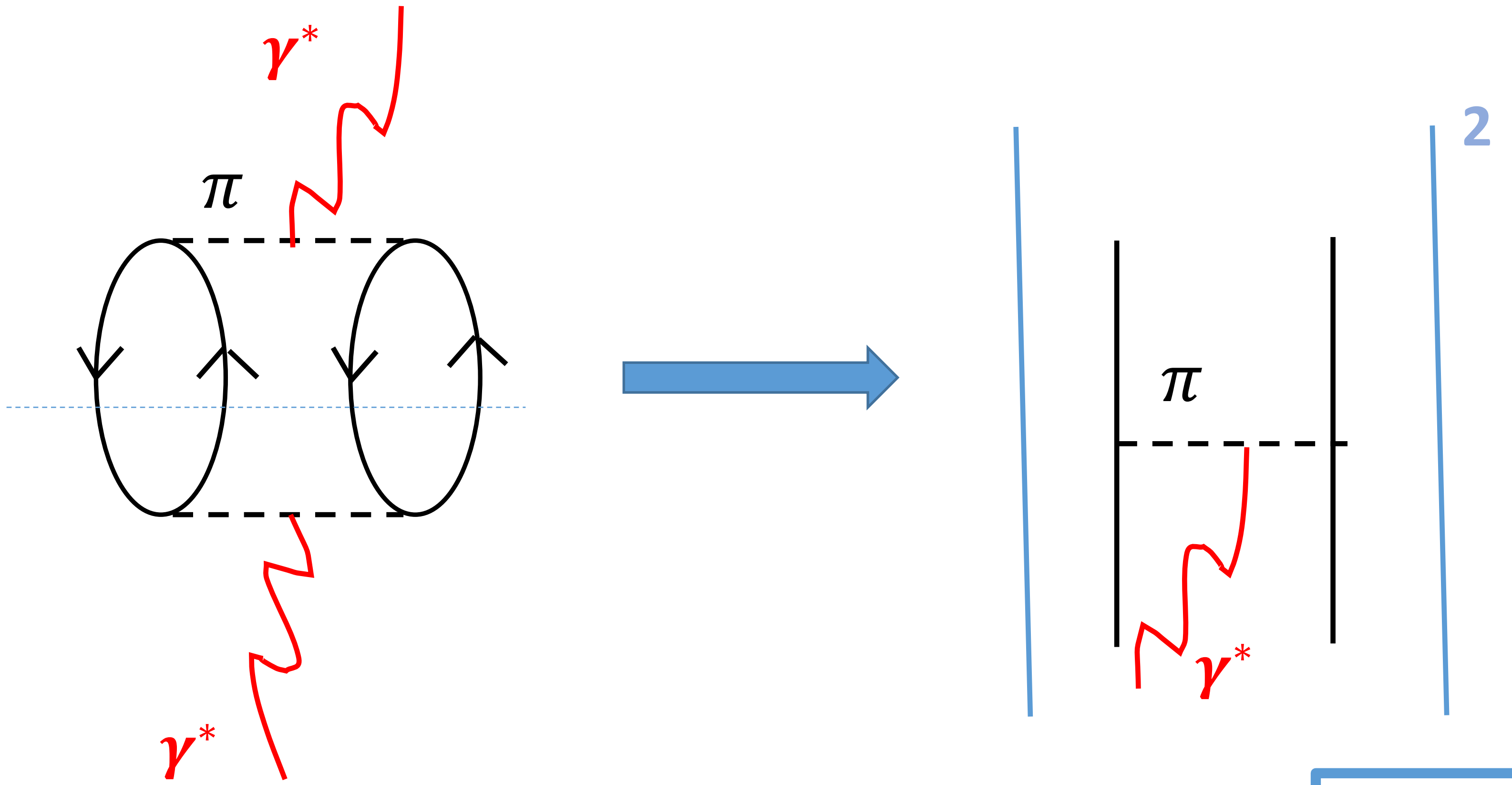
Martini et al. model does not account for all axial and axial-vector interference contributions !



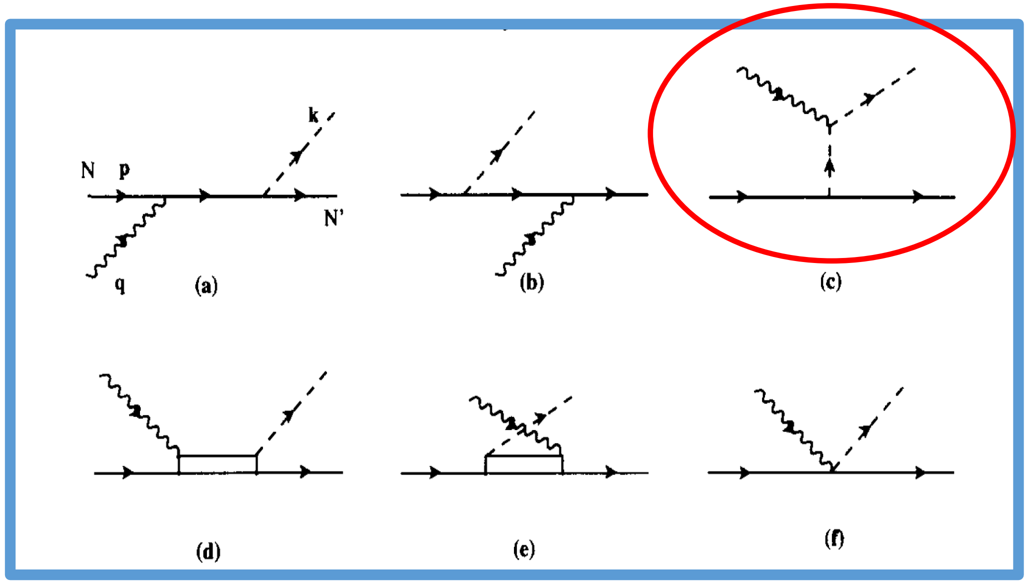
Martini et al., predictions look consistent with MiniBooNE data ..., **but their estimate rely on some computation of the 2p2h mechanisms for (e, e') (Alberico et al.,) $\Rightarrow$ no info on axial part of the interaction!**



...however our predictions for the 2p2h contribution would favor a global normalization scale of about 0.9. This would be consistent with the MiniBooNE estimate of a total normalization error of 10.7%.



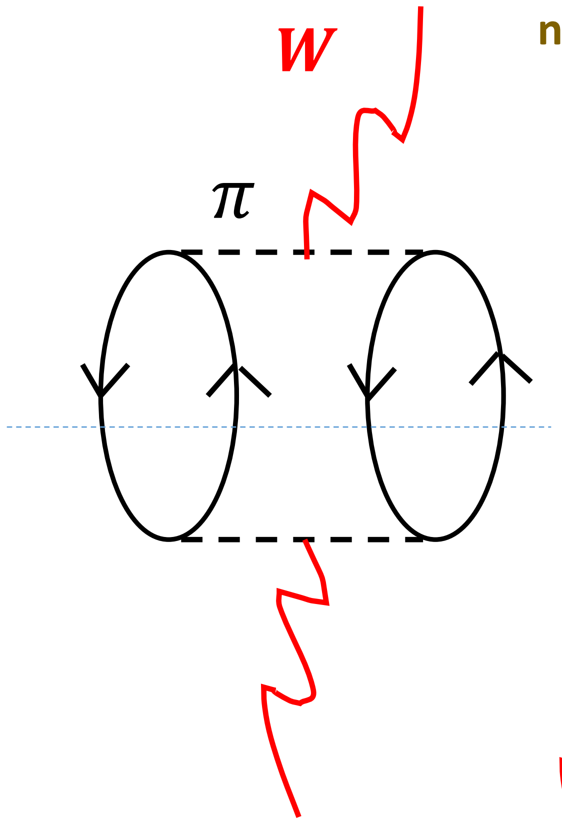
$\gamma^* N \rightarrow \pi N$

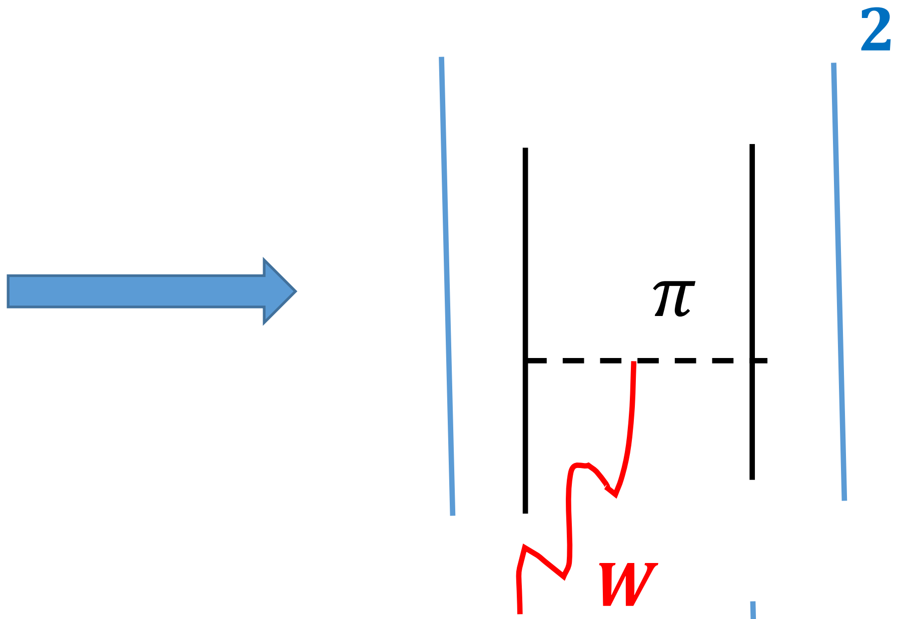


Meson Exchange Contribution

MEC & FSC & ISC

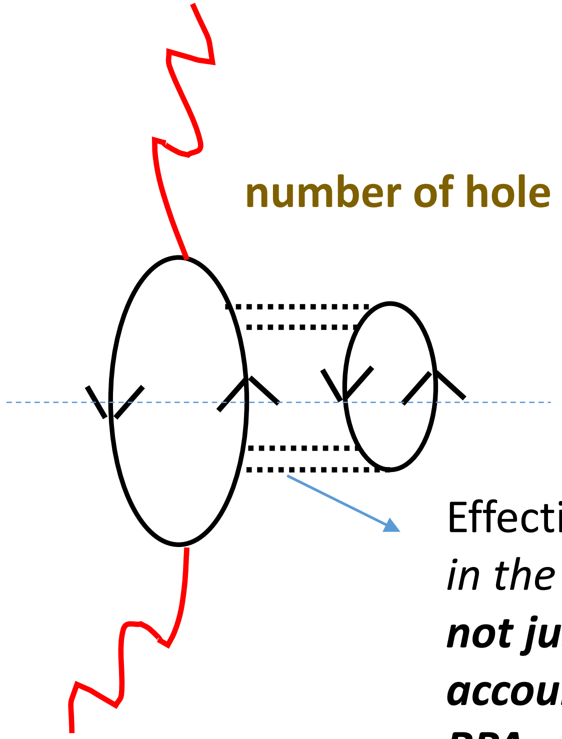
number of hole lines (density) = 2

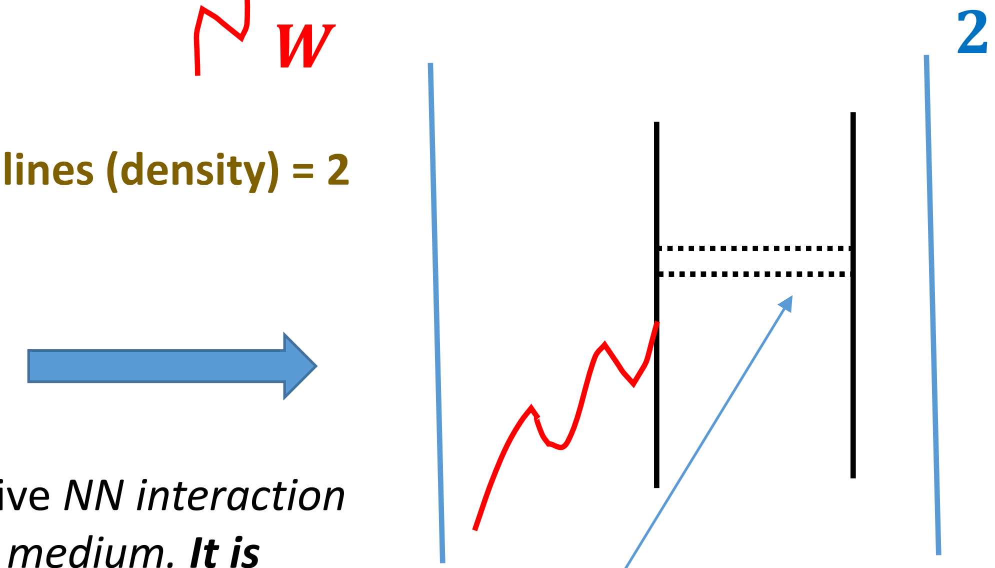




MEC term

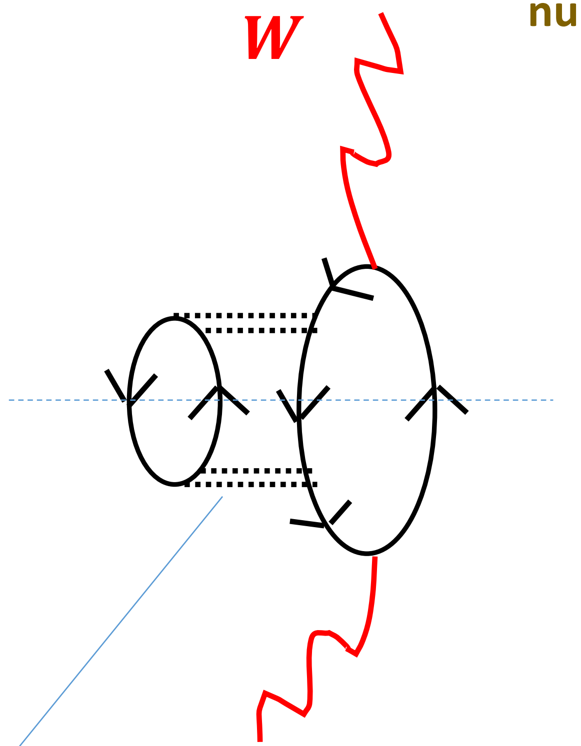
number of hole lines (density) = 2

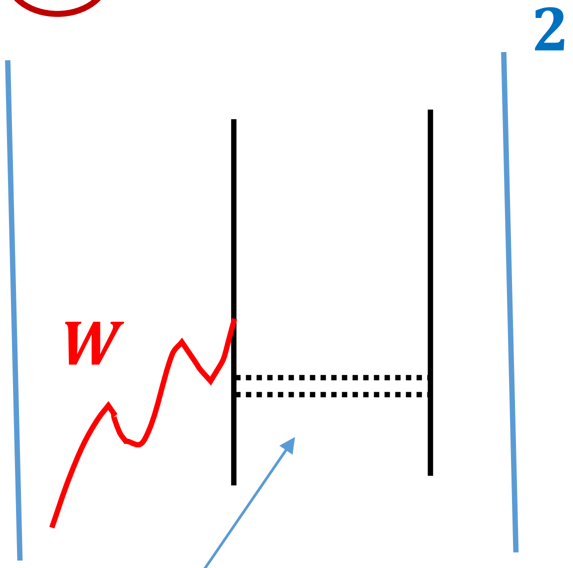




MEC/FSC term

Effective *NN interaction in the medium.* ***It is not just a pion and accounts for SRC and RPA correlations*** number of hole lines (density) = 4

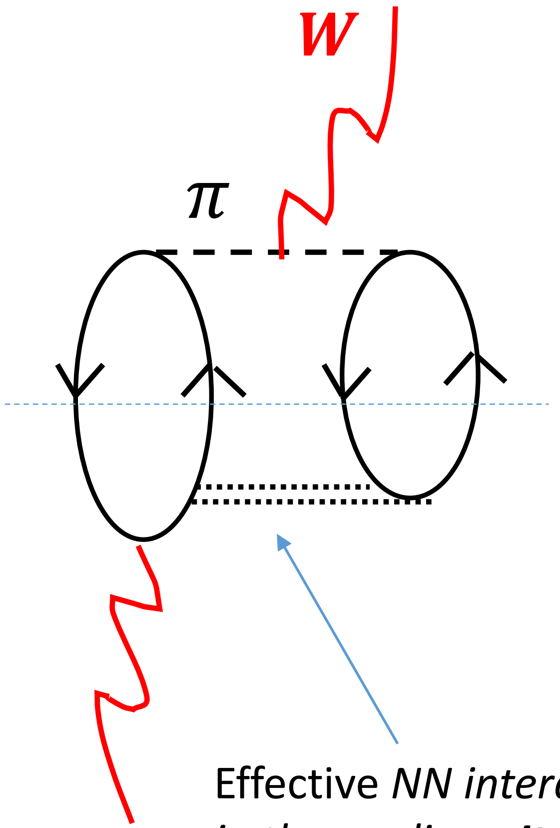


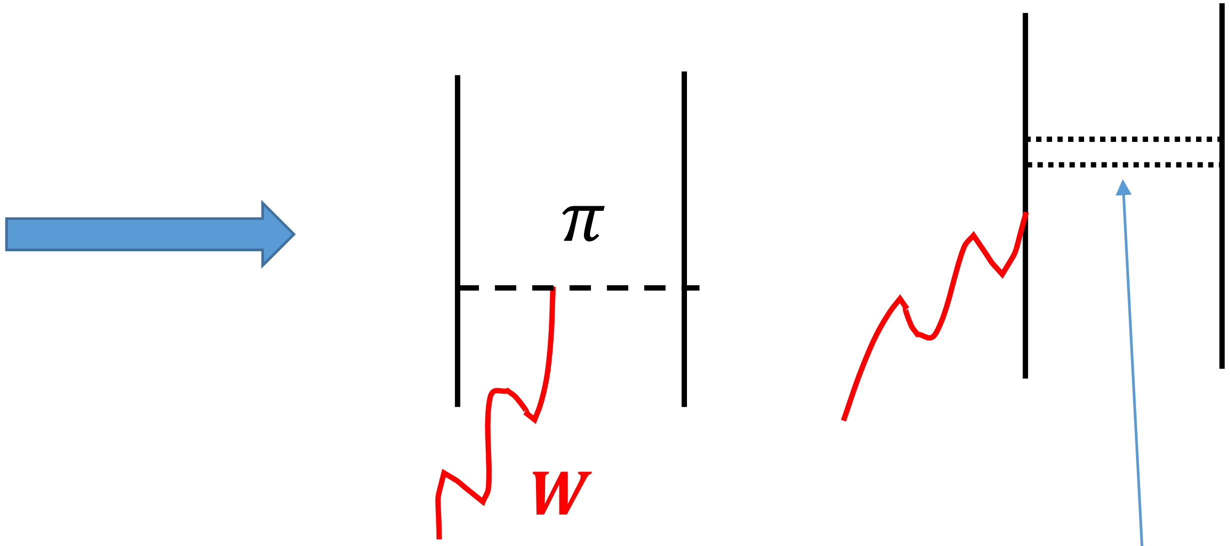


Effective *NN interaction* *in the medium.* ***It is not just a pion and accounts for SRC and RPA correlations***

MEC/ISC term

number of hole lines (density) = 2



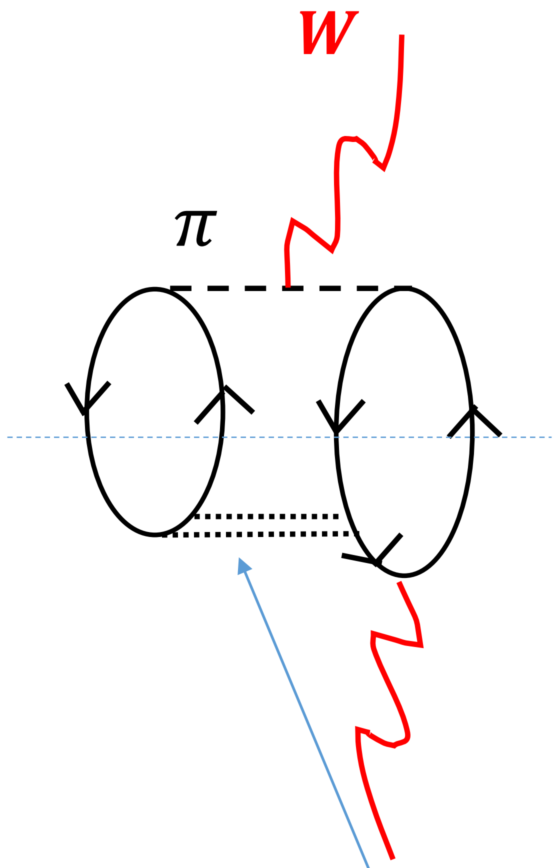


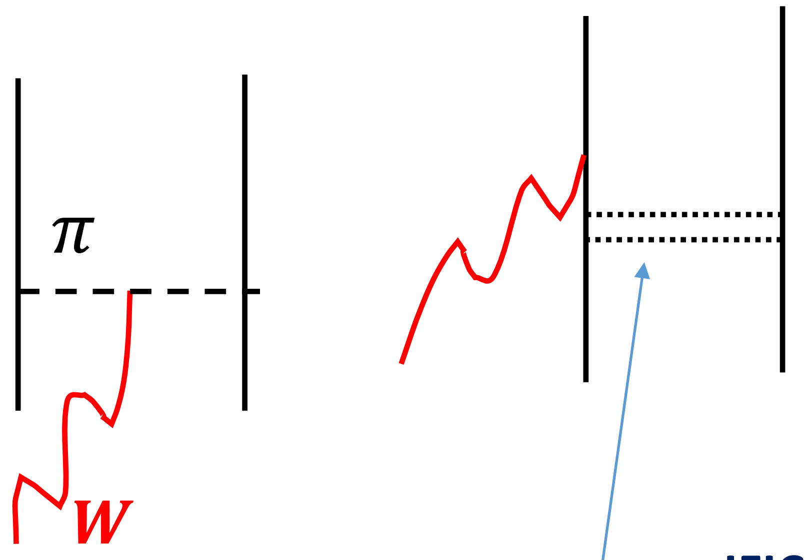
Effective *NN interaction in the medium.* ***It is not just a pion and accounts for SRC and RPA correlations***

FSC-MEC interference term

MEC-ISC interference term

number of hole lines (density) = 3



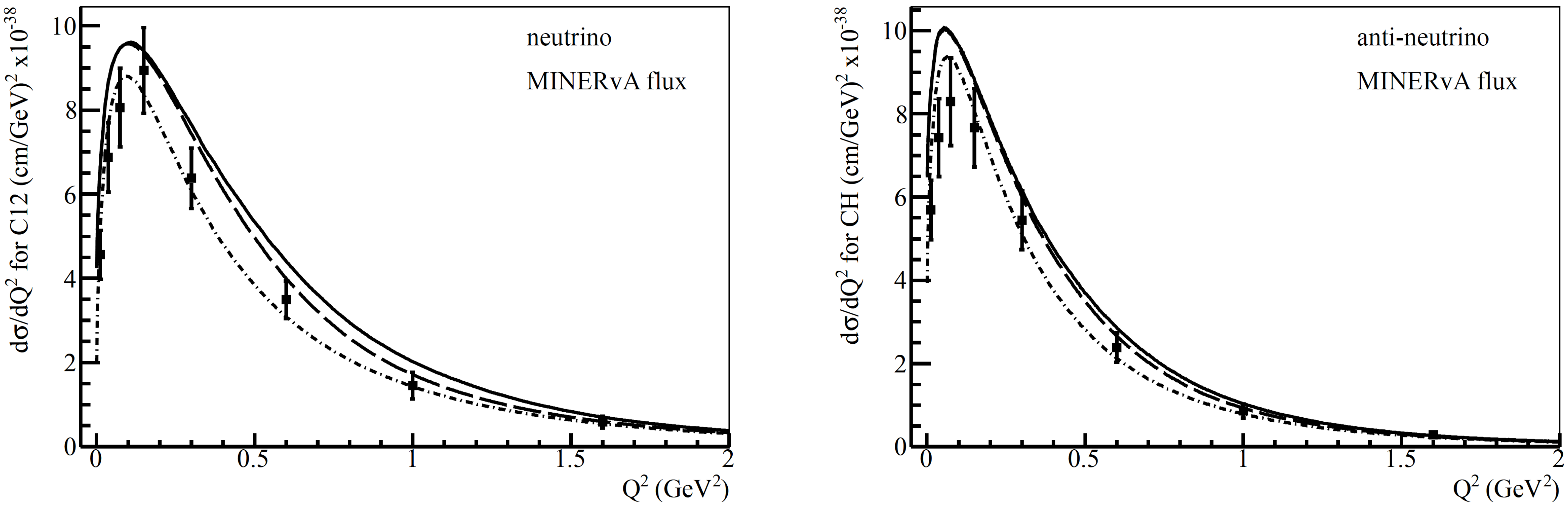


Important ? Benhar, Lovato, Rocco [PRC 92 (2015) 024602]

Effective *NN interaction in the medium.* ***It is not just a pion and accounts for SRC and RPA correlations***

- IFIC 2p2h calculation does not incorporate these terms.
- Martini et al. predictions are based on a 2p2h calculation for $(e, e'X)$ [Alberico et al.,] that accounts for such contributions (only vector current)

MINERνA



PHYSICAL REVIEW D 88, 113007 (2013)

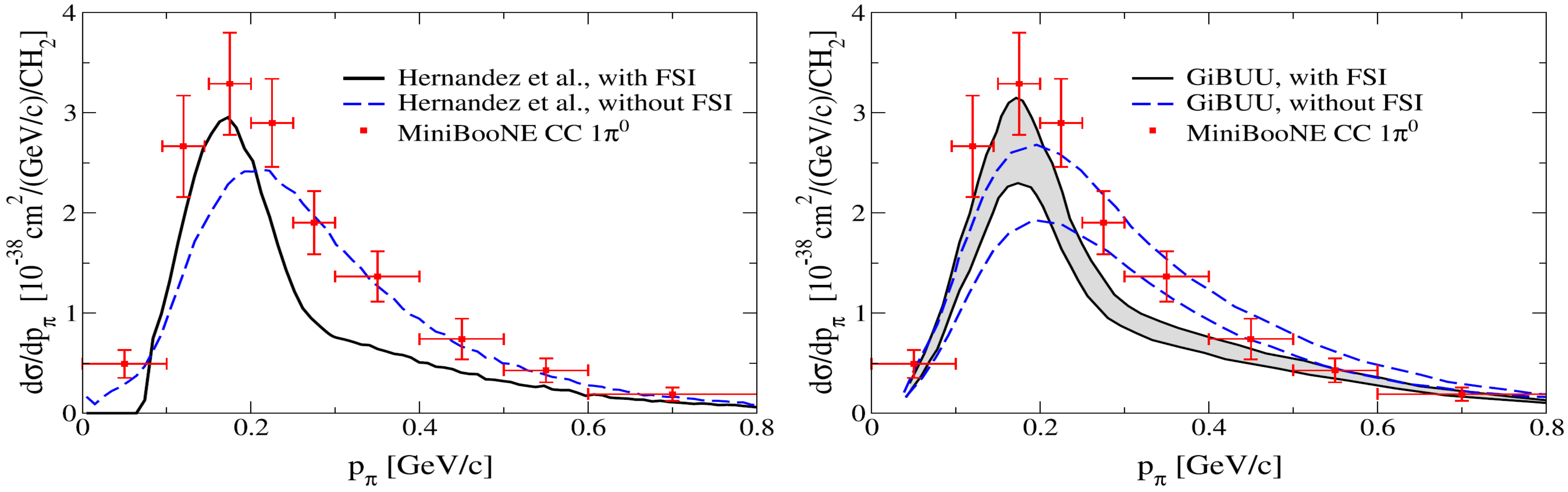
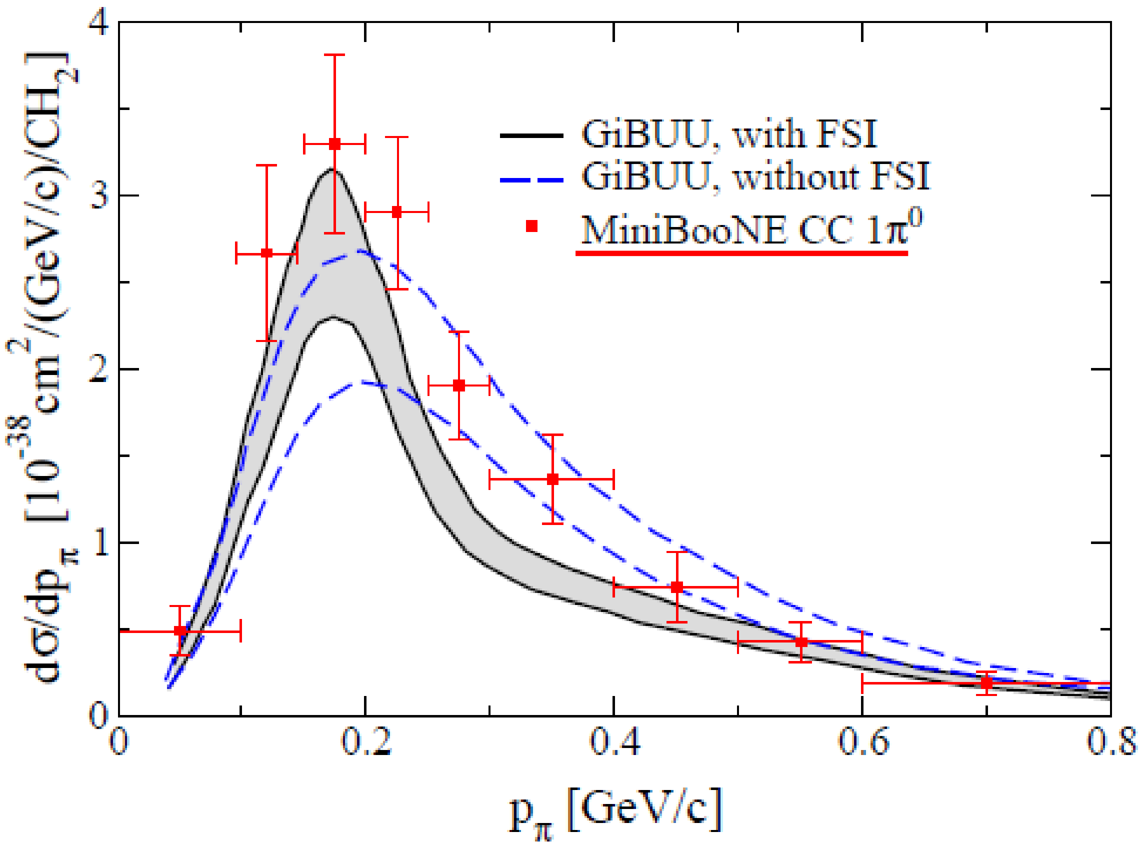


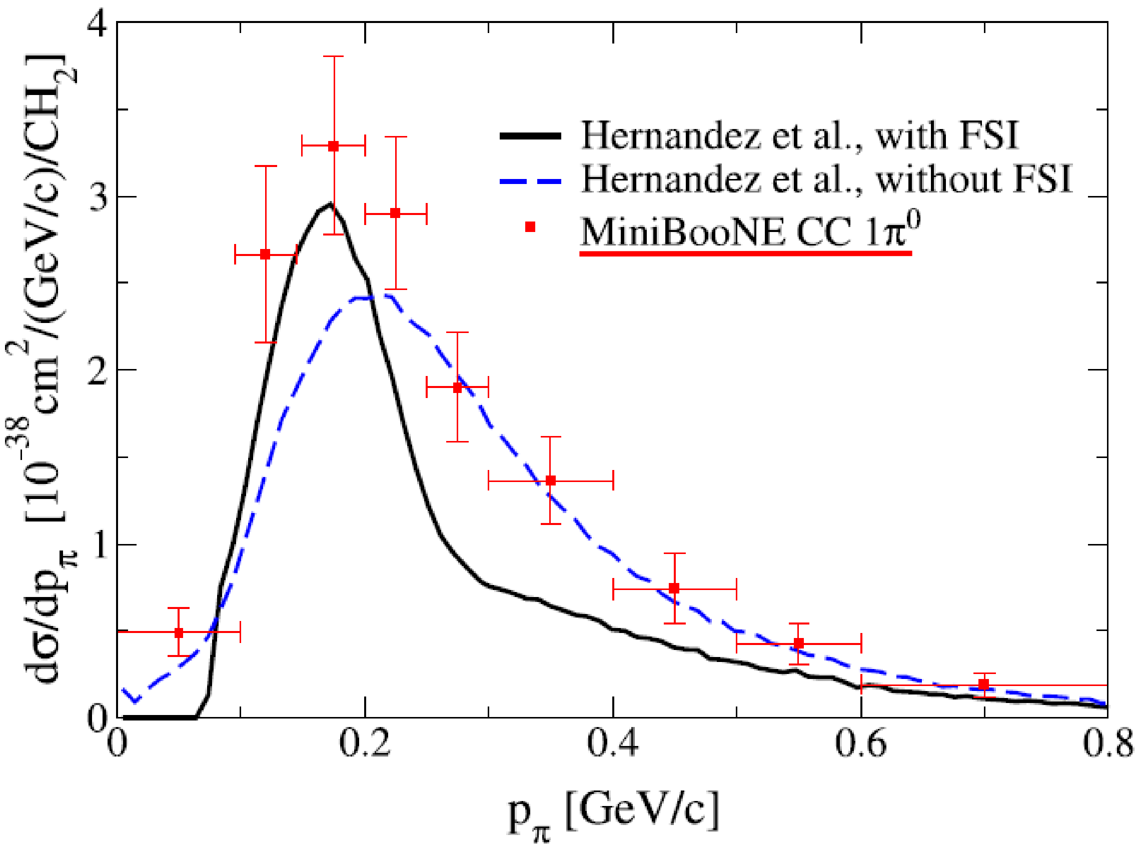
Figure 15. MiniBooNE flux-folded differential $d\sigma/dp_\pi$ cross section for $CC1\pi^0$ production by ν_μ in mineral oil. Data are from [27]. Left: predictions from the cascade approach of [184]. The solid curve corresponds to the full model and the dashed one stands for the results obtained neglecting FSI effects. Right: predictions from the GiBUU transport model of [207]. The dashed curves give the results before FSI, the solid curves those with all FSI effects included. Two different form factors $C_5^A(q^2)$, tuned to the ANL and BNL data-sets have been employed and give rise to the systematic uncertainty bands displayed in the figure.

There exist some discrepancies between theoretical predictions and data!

New J.Phys. 16 (2014) 075015

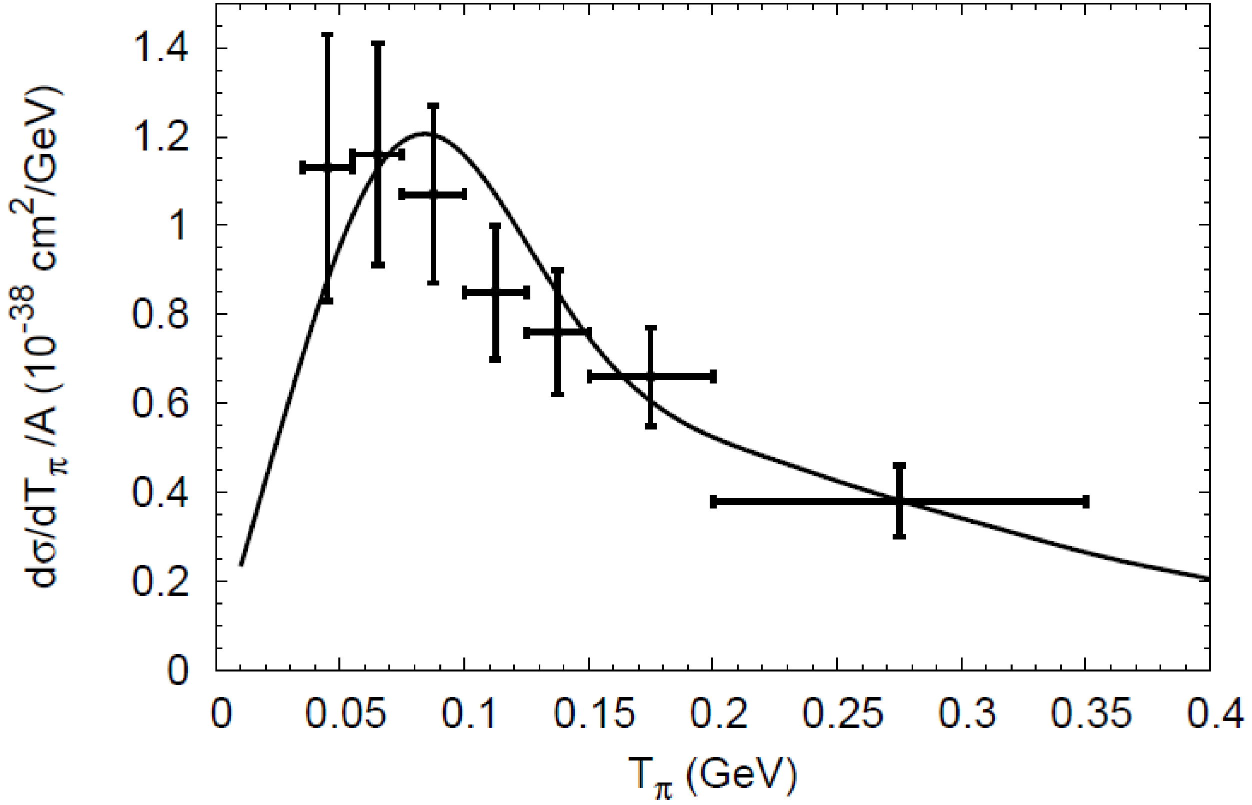


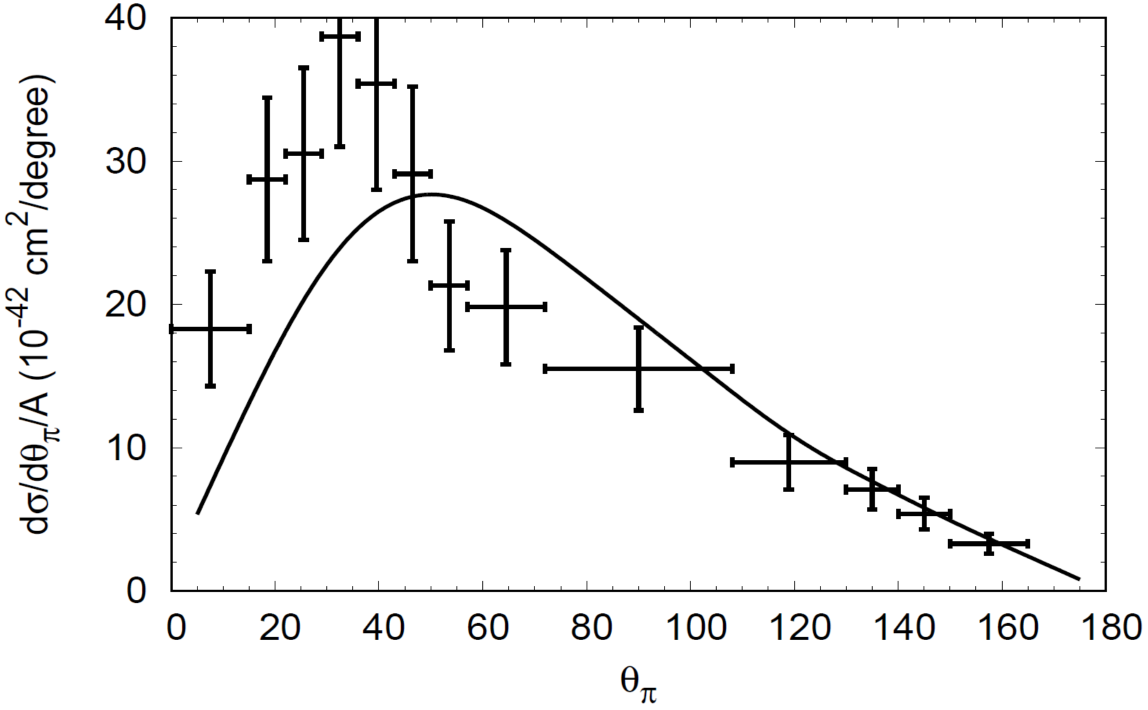
O. Lalakulich, U. Mosel, PRC 87 (2013)



E. Hernandez, J. Nieves M.J. Vicente-Vacas, PRD 87 (2013)

Problems to describe pion production in nuclei (FSI, coherent production ...) ➡ MINERνA and T2K will shed light ….

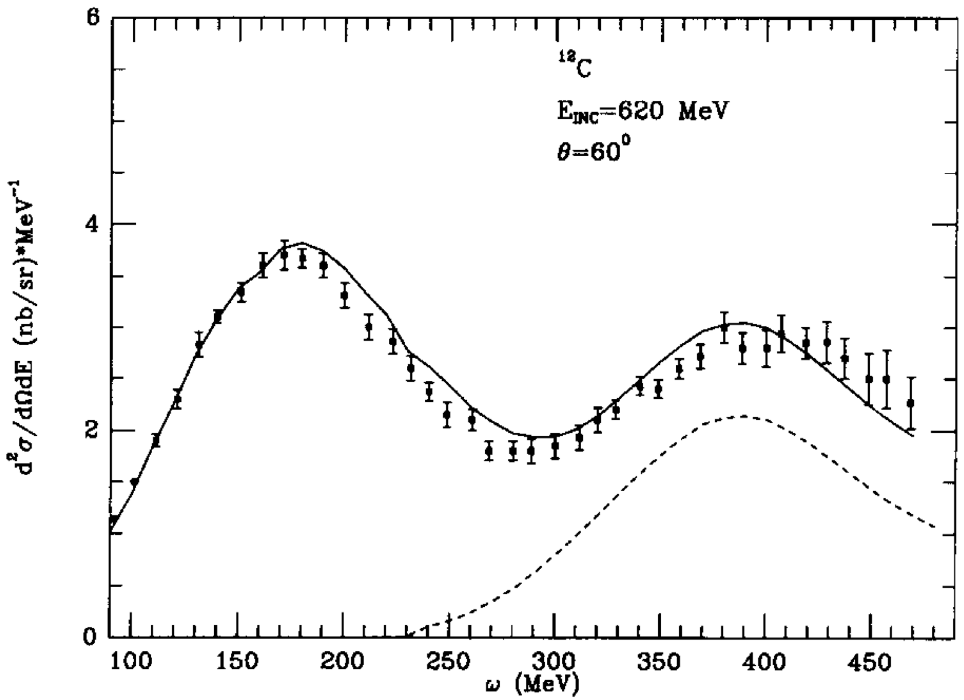




U. Mosel and K. Gallmeister (GiBUU), 1702.04932: Comparison to MINERνA data with $W_{\pi N} < 1.4$ GeV

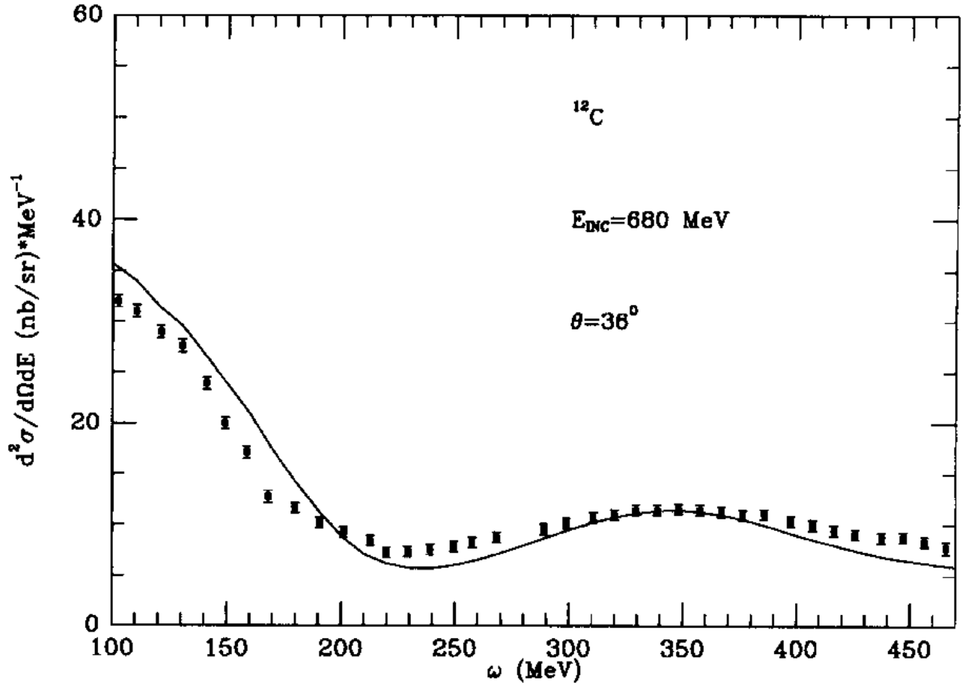
MINERνA and MiniBooNE data compatible?

Juan Nieves, IFIC (CSIC & UV)

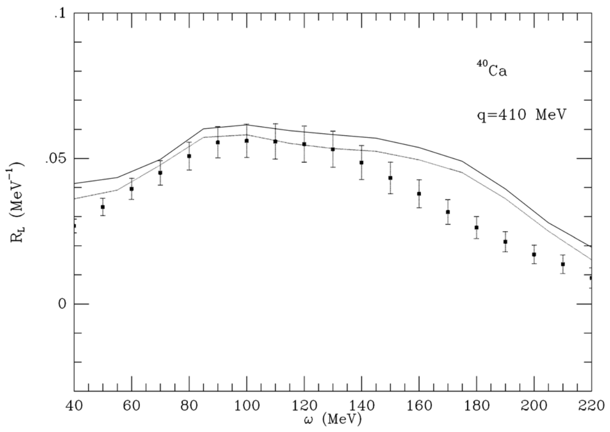


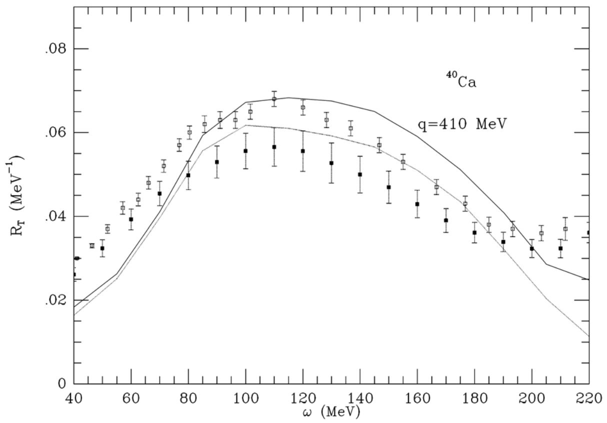
Gil, Nieves, Oset,
NPA 627 (1997) 543

$^{12}C\ (e,e'X)$

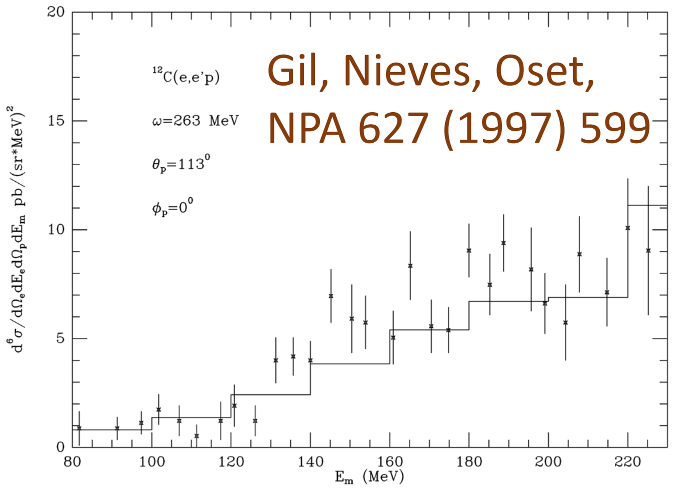


and by means of a Monte Carlo simulation we obtain cross sections for the processes $(e,e'N)$, $(e,e'NN)$, $(e,e'\pi)$, ...





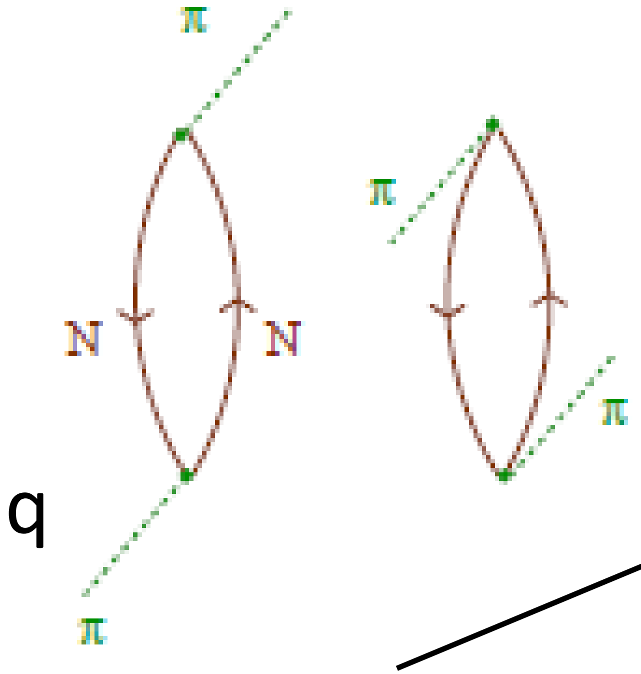
R_L and R_T QE response functions for $e + {}^{40}\mathrm{Ca} \rightarrow e' + X$



Gil, Nieves, Oset,
NPA 627 (1997) 599

$^{12}C\ (e,e'p)$

Pion selfenergy: first approximation πNN vertex



Chiral symmetry

$$\mathcal{L}^{\sigma}_{\rm int} = \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma_\mu\vec{\tau}(\vec{\phi}\times\partial^\mu\vec{\phi})\Psi$$

P-wave   *S-wave (Weinberg-Tomozawa)*

$$f = \frac{m_\pi}{2f_\pi}g_A$$

$\psi(x)$: SU(2) ⇒ bi-spinor : $\begin{bmatrix}\psi_p(x)\\ \psi_n(x)\end{bmatrix}$

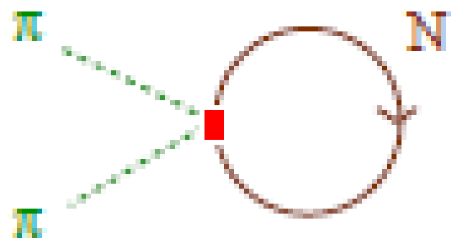
$\phi_+(x) = \phi_-^\dagger(x) = \frac{1}{\sqrt{2}}(\phi_1(x) - i\,\phi_2(x))$; $\phi_0(x) = \phi_3(x)$

$\phi_-^\dagger(x)$ CREATES A π^- AND ANNIHILATES A π^+ MESON.

[J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill, New York, 1965]

$$\Pi(q) = \frac{f^2}{m_\pi^2}\vec{q}^{\,2}\,U_N(q)$$

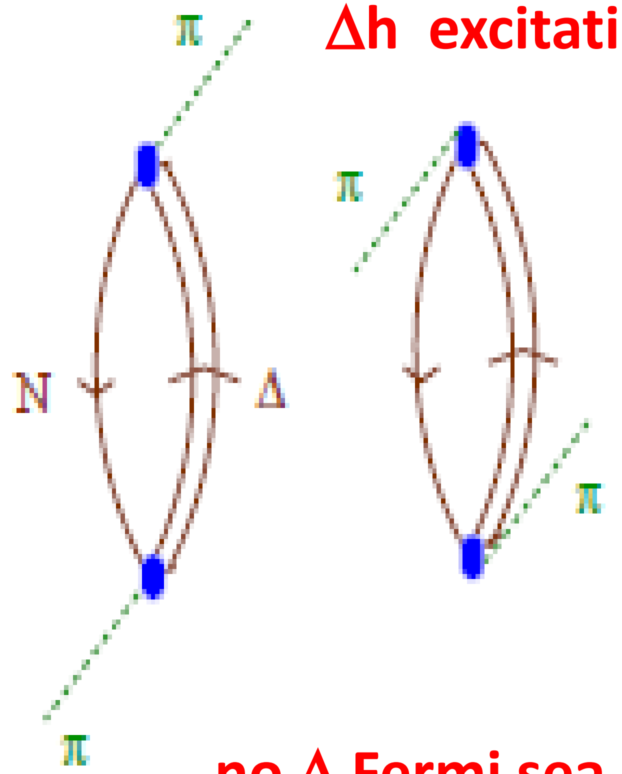
$$U_N(q) = 2U_e(q), \quad (m_e \to m_N)$$



S-wave selfenergy: small for isoscalar nuclei !

The pion cannot only excite nucleons above the Fermi sea, but it can also excite the internal degrees of freedom of the nucleon since it is a composite particle made out of quarks. Hence a nucleon can be converted into a Δ, N^*, Δ^*, $etc.$.

Δh excitation



Δ(1232) [spin 3/2 and isospin 3/2] plays an important role at intermediate energies because of its lower mass and strong coupling → contribution to □

$$\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_\pi} \bar{\Psi}_\mu \vec{T}^\dagger (\partial^\mu \vec{\phi}) \Psi + \text{h.c.}$$

where Ψ_μ is a Rarita-Schwinger $J^\pi = 3/2^+$ field, $\vec{T}^\dagger$ is the isospin transition operator from isospin 1/2 to 3/2, and $f^* = 2.13 \times f = 2.14$.

no Δ Fermi sea !

Some technical aspects about the Δh excitation

- $\left\langle \frac{3}{2}\, M_T \left| T_\nu^\dagger \right| \frac{1}{2}\, m_T \right\rangle = \left(\frac{1}{2}, 1, \frac{3}{2} \middle| m_T, \nu, M_T \right) \left\langle \frac{3}{2} \left\| T_\nu^\dagger \right\| \frac{1}{2} \right\rangle$ (Wigner-Eckart)

  (reduced matrix element struck out → 1)

- Δ propagator (unstable particle)

$P^{\mu\nu}$: spin 3/2 projector

$$G^{\mu\nu}(p_\Delta) = \frac{P^{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta},$$

$$P^{\mu\nu}(p_\Delta) = -(\not{p}_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu\gamma^\nu - p_\Delta^\nu\gamma^\mu}{M_\Delta} \right],$$

$$\Gamma_\Delta(s) = \frac{1}{6\pi}\left(\frac{f^*}{m_\pi}\right)^2 \frac{M}{\sqrt{s}} \left[\frac{\lambda^{1/2}(s, m_\pi^2, M^2)}{2\sqrt{s}} \right]^3 \times \Theta(\sqrt{s} - M - m_\pi), \qquad s = p_\Delta^2,$$

→ $(\pi N\ CM\ momentum)^3$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

Some technical aspects about the Δh excitation: non-relativistic expressions

- πNΔ transition: $$H_{\pi N\Delta} = \frac{f^*}{\mu} \Psi_N^\dagger(x)\, S_i\, \partial_i \phi^\lambda(x)\, T^\lambda\, \Psi_\Delta(x) + \text{h.c.}\,,$$

spin transition operator.

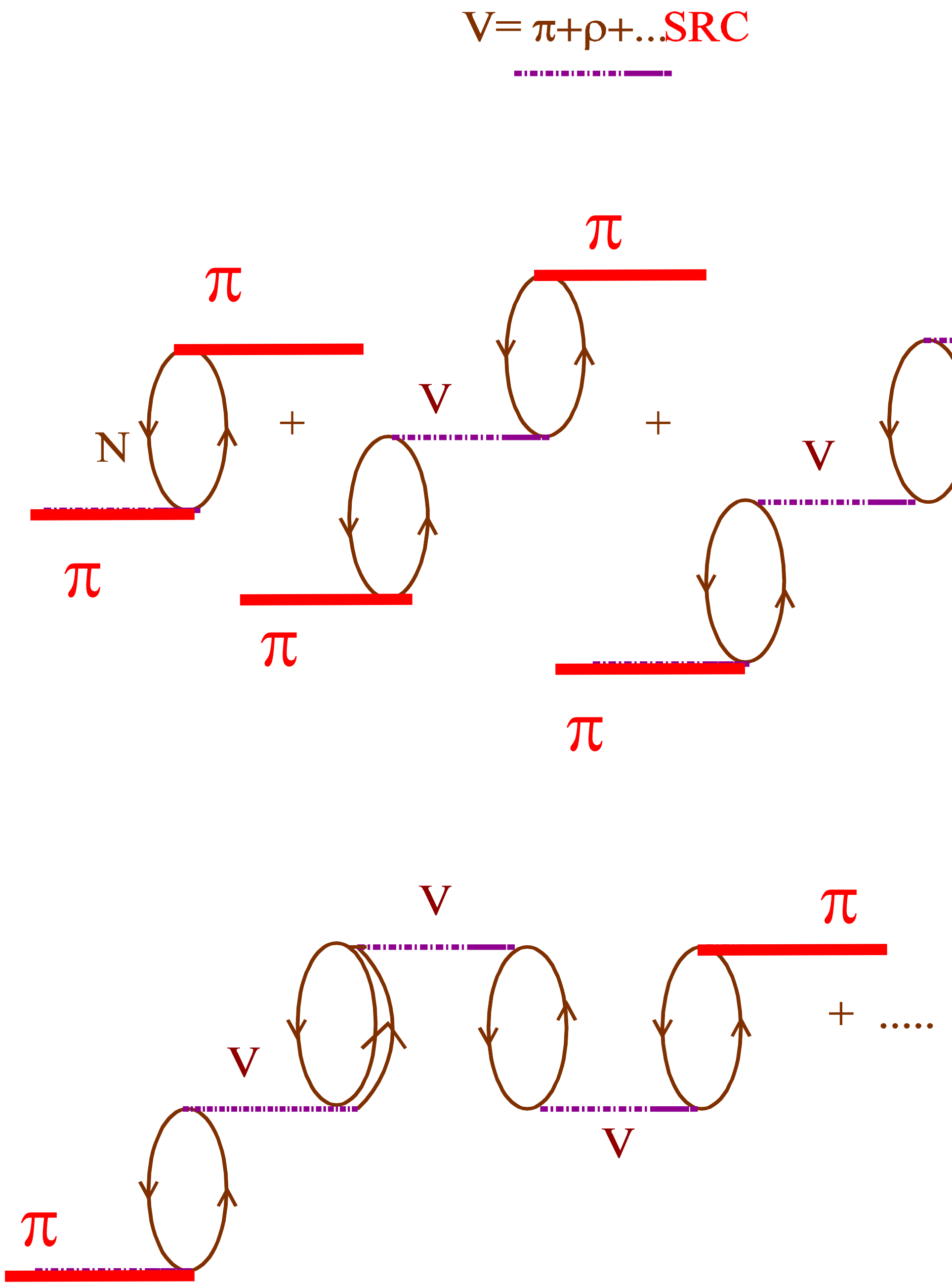
- Δ propagator

$$G_\Delta(k) = \frac{1}{k^0 - w_R - T_\Delta + \frac{1}{2}\mathrm{i}\Gamma_\Delta}$$

with $w_R = M_\Delta - M_N$, T_Δ the Δ kinetic energy a

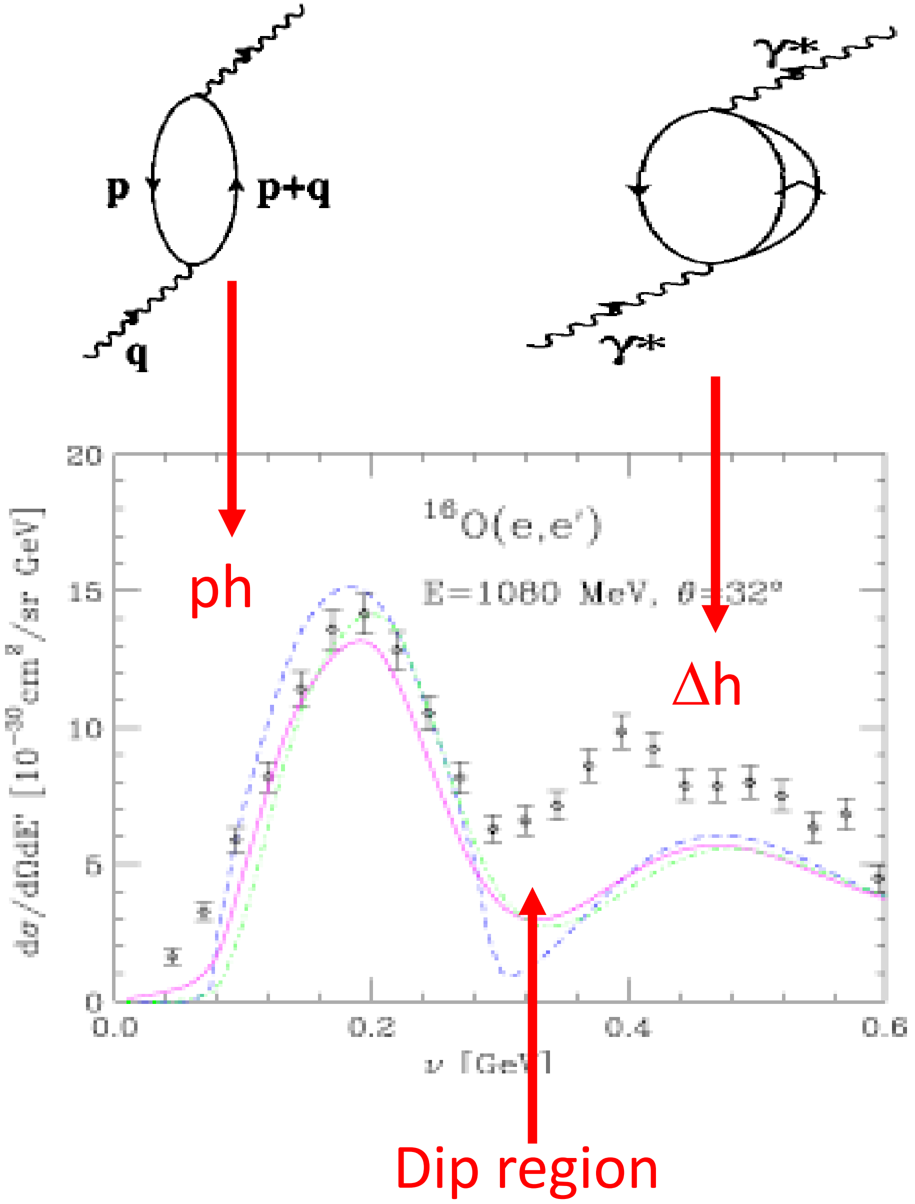
The pion selfenergy reads: $\Pi(q) = \frac{f^2}{m_\pi^2}\, \vec{q}^{\,2}\, U(q)$ ***with*** $U(q) = U_N(q) + U_\Delta(q)$

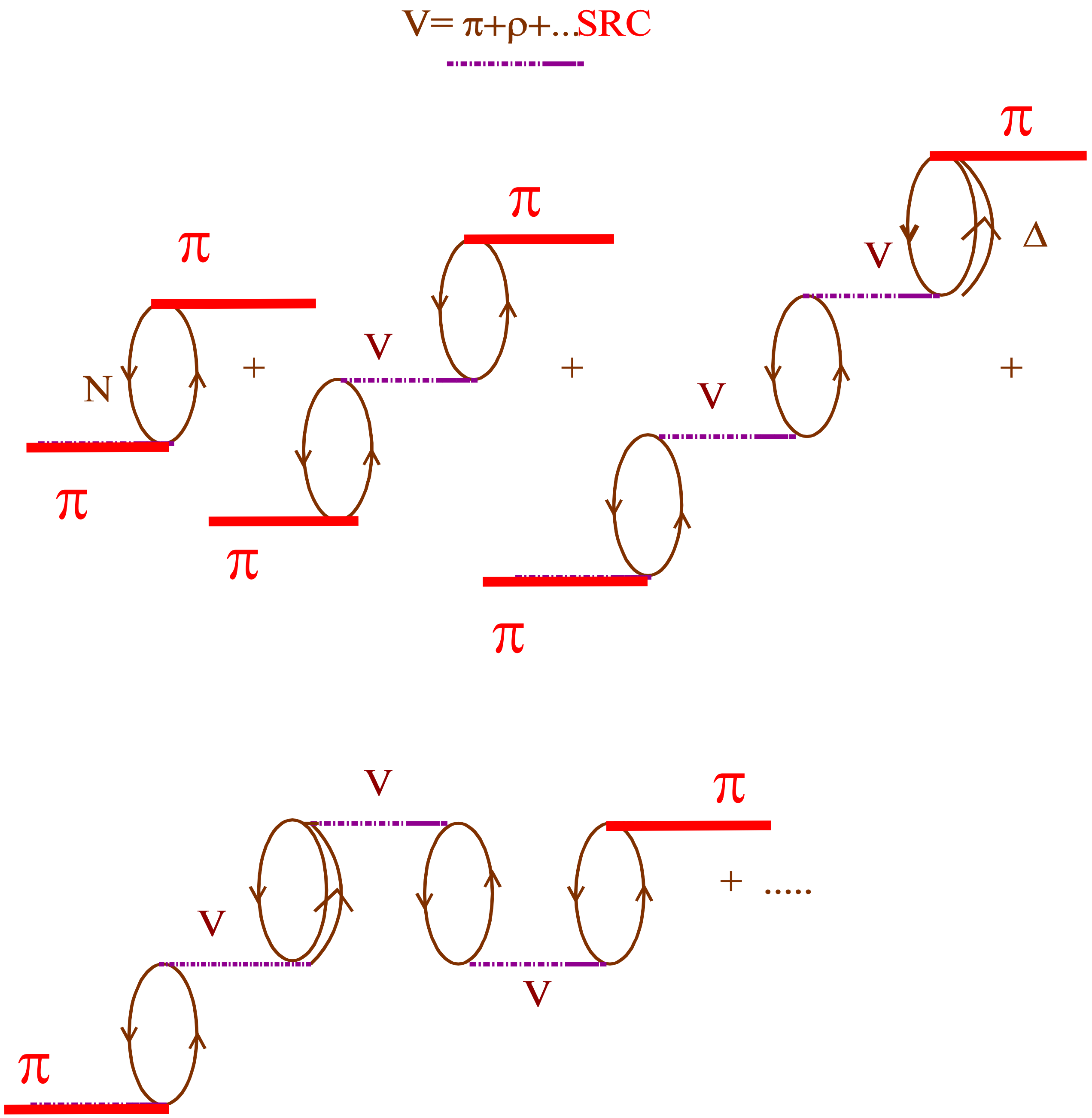
$$U_\Delta(q) = -\mathrm{i}\left(\frac{4}{3}\right)^2 \left(\frac{f^*}{f}\right)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \left[G^0(k)G_\Delta(k+q) + G^0(k)G_\Delta(k-q)\right]$$



Remarks:

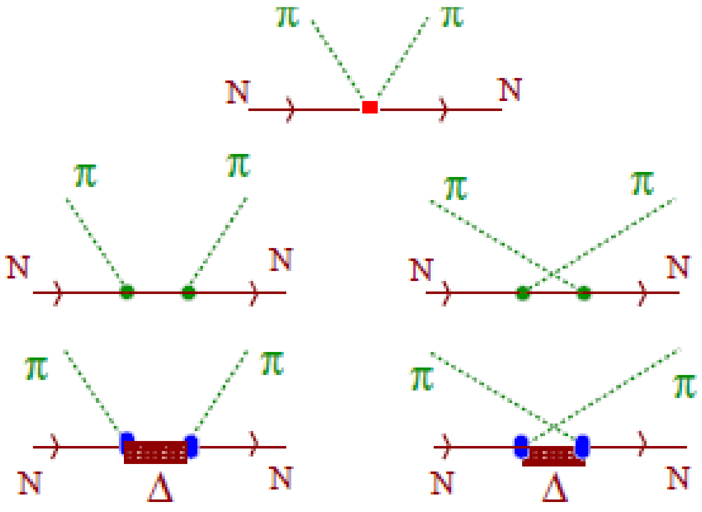
- External γ can also excite Δh components





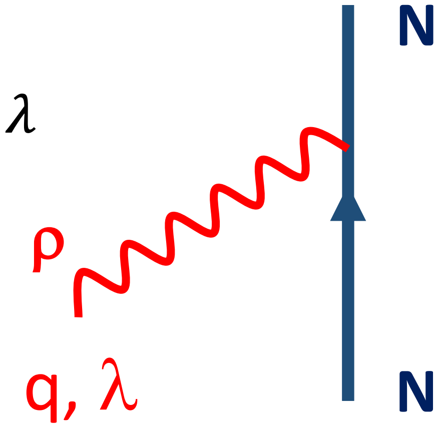
Remarks:

- *V is an effective ph-ph, ph-Δh and Δh-Δh interaction in the nuclear medium*
- *V=π + ρ+ other mesons (Short Range Correlations). Starting point πN → πN in free space (πNN and πNΔ couplings)* [Ericson+Weise, *Pions in nuclei*] *and NN → NN* ***Bonn potential***



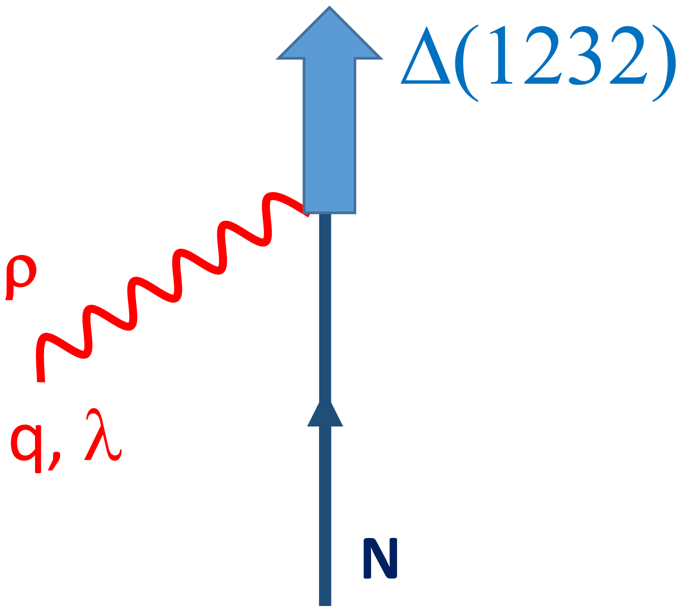
NN Interaction: π+ρ exchange + short range correlations (Oset+Weise 1979)

$$H_{\rho NN}(q) = i\,\frac{f_\rho}{m_\rho}\,(\vec{\sigma} \times \vec{q}\,)\,\vec{\epsilon}\,\tau^{\lambda}$$



$$\left[L_{\rho NN}(x) = -g_{\rho NN}\,\bar{\psi}\left(\gamma^{\mu} - \frac{k}{2M}\,\sigma^{\mu\nu}\partial_{\nu}\right)\vec{\tau}\vec{\rho}_{\mu}\,\psi\right]$$

$$H_{\rho N\Delta}(q) = i\,\frac{f_\rho^*}{m_\rho}\,(\vec{S}^{\dagger} \times \vec{q}\,)\,\vec{\epsilon}\,T^{\dagger\lambda}$$

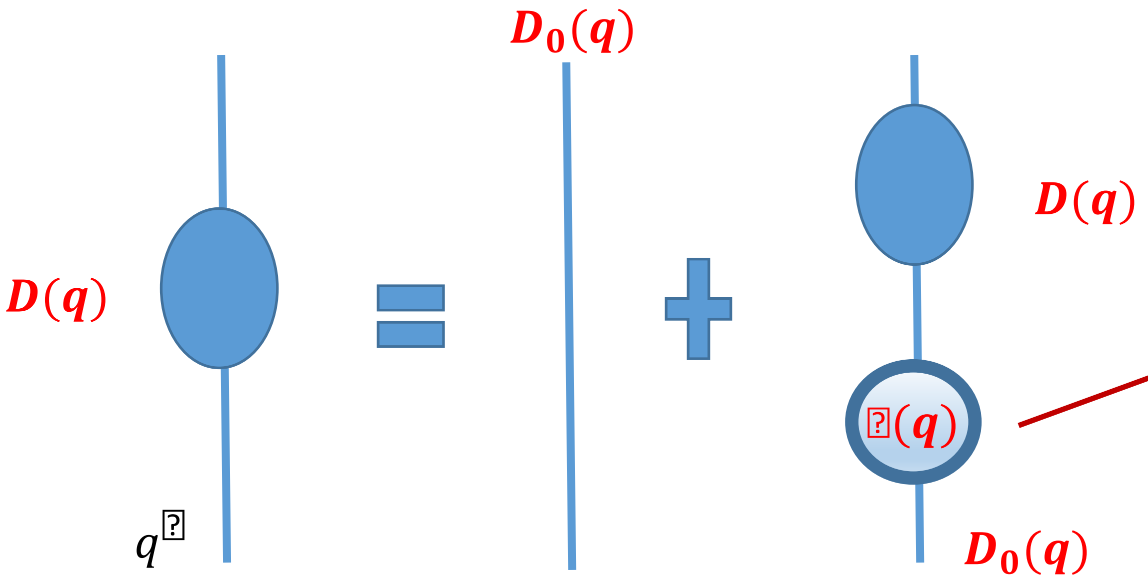


$$C_\rho = \frac{f_\rho^2/m_\rho^2}{f^2/m_\pi^2} \sim 2, \qquad f_\rho^*/f_\rho \sim f^*/f$$

Quark model

Dyson equation

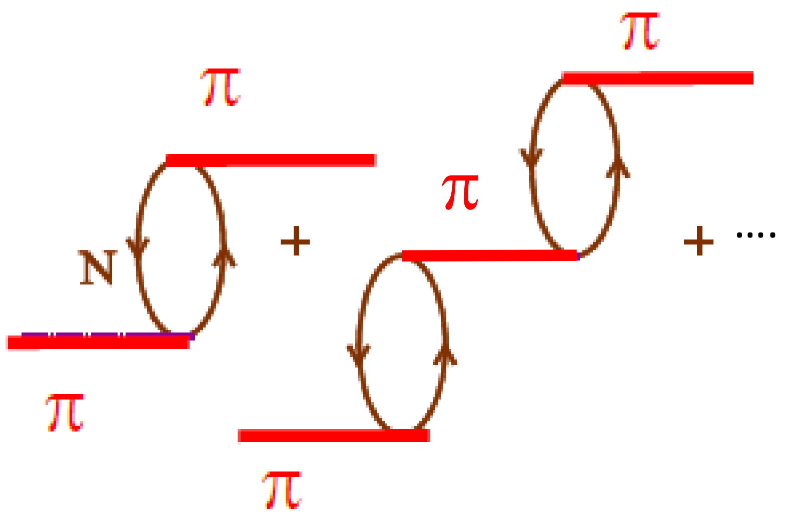
$$iD = iD_0 + iD_0\, i\Pi\, iD \Rightarrow D(q) = \frac{D_0}{1 - D_0\Pi} = \frac{1}{q^2 - m_\pi^2 - \Pi(q)}$$

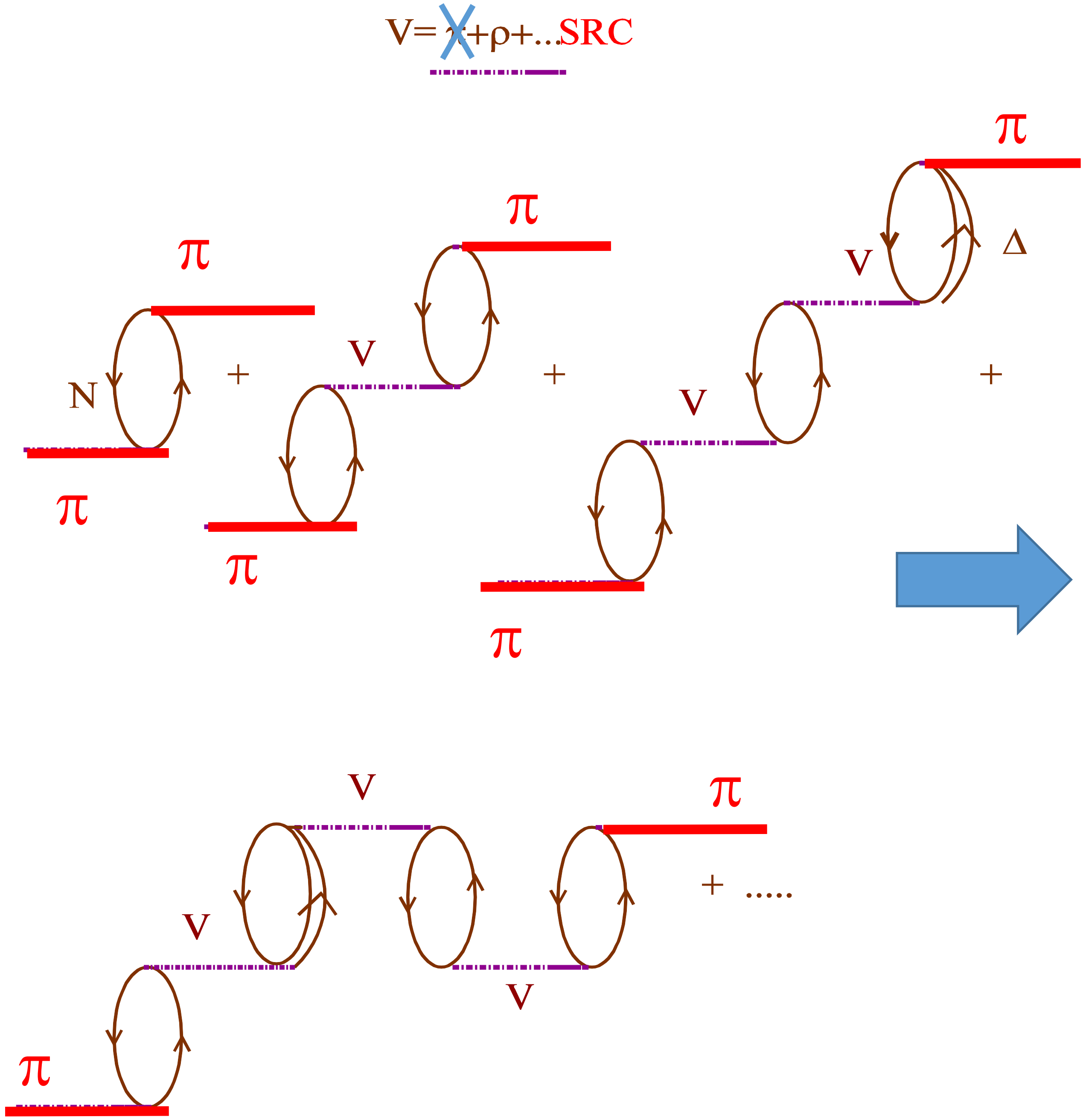


Full pion propagator $D(q)$

Free pion propagator $D_0(q)$

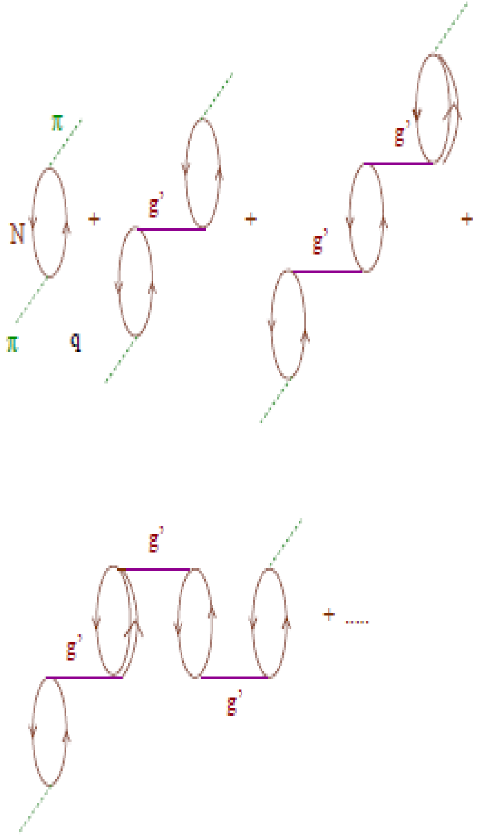
pion selfenergy: contains only irreducible diagrams

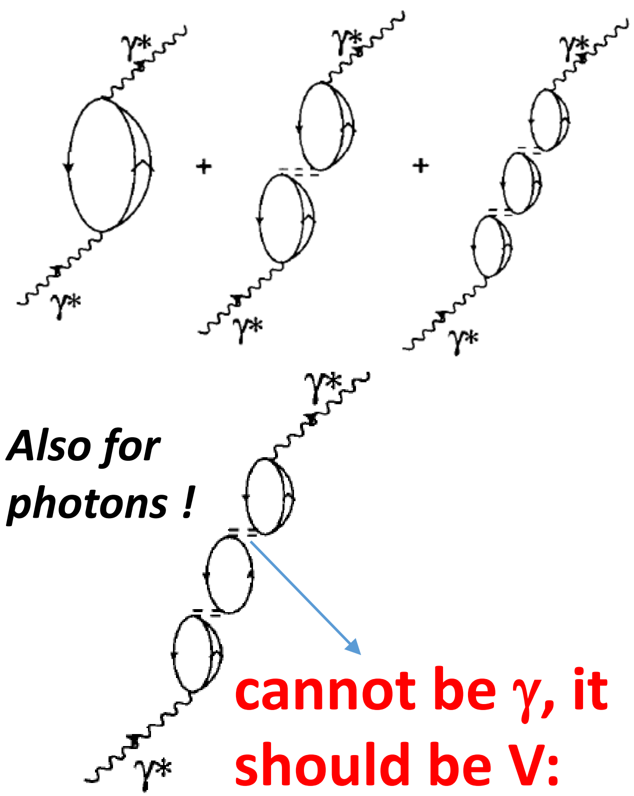




Remarks:

- To compute the pion selfenergy, only irreducible diagrams should be considered: V → ρ + SRC (g’) [the initial pion selects the longitudinal channel]





Also for photons !

cannot be γ, it should be V: ph-ph, ph-Δh interaction