Charged-current stopped-pion neutrino scattering off ^{127}I and ^{133}Cs and the quenching of g_{A}

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Motivation

- The quenching of g_{A} has an important effect on the accuracy of the theoretical calculations on several weak processes, including neutrino-nucleus scattering.
- Accurate theoretical calculations of neutrino scattering cross sections are themselves important for the design of future experiments and for interpreting the obtained measurement data.
- The quenching of g_{A} in neutrino scattering reactions is therefore of notable importance.

Background

- Terrestrial-based neutrino scattering measurement results for low and intermediate energies (≤ 100 MeV) exist for only seven nuclear targets[1,2,3].
- Recently the cross section for the charged-current scattering reaction $\nu_e + {}^{127}\mathrm{I} \longrightarrow e^- + {}^{127}\mathrm{Xe}$ was measured.
- Previous theoretical studies for this reaction have found a relatively wide range of results for the cross section.
- The topic of this talk is to present new theoretical results for said cross section and discuss the role of the weak axial-vector coupling g_{A} and its quenching in the calculations. Results for the cc-scattering off ^{133}Cs are also presented.

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[1] J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. **84**, 1307 (2012)
[2] R. Maschuw et al., Prog. Part. Nucl. Phys. **40**, 183 (1998)
[3] P. An et al. (COHERENT), arXiv:2212.11295 (2022)

Previous calculations

- Cross sections for charged-current (cc) neutrino-nucleus scattering off ^{127}I have been experimentally measured and theoretically computed in the past.
- The theoretical research is mostly quite old (15+ or even nearly 30 years).
- ^{127}I and ^{133}Cs are both computationally expensive nuclei in the shell model formalism, so theoretical studies into the former have often utilized certain methods to avoid the explicit construction of final states, such as the closure approximation and Fermi gas models.
- New systematic studies on the quenching of g_{A} have also been conducted relatively recently and more information on the choice of $g_{\mathrm{A,eff}}$ is available.

General considerations

- The goals of the research were to compute realistic up-to-date scattering cross sections for charged-current stopped-pion and supernova neutrinos off the nuclei ^{127}I and ^{133}Cs, and compare these to the latest experimental results for ^{127}I.
- The plan was to use data from β-decay studies to choose the values of $g_{\mathrm{A,eff}}$ and judge the validity of this choice for ^{127}I by the aforementioned comparison.
- The scattering calculations were done by utilizing the Donnelly-Walecka formalism with the initial and final nuclear states being modelled by using the Microscopic quasiparticle-phonon model (MQPM).

The Donelly-Walecka method

- At relatively low momentum transfer q neutrino-nucleus scattering can be modelled as a current-current interaction in the lowest order with the effective Hamiltonian matrix element[4][5]

$$\langle f|H_{\text{eff}}^{\nu/\overline{\nu}}|i\rangle = \frac{G}{\sqrt{2}} \int \mathrm{d}^3\mathbf{x}\, e^{-i\mathbf{q}\cdot\mathbf{x}} l_\mu^{\mp} \langle f|\mathcal{J}^{\pm,\mu}(\mathbf{x})|i\rangle \,, \qquad (1)$$

  where $l_\mu^{\mp}$ is the lepton matrix element and $\mathcal{J}^{\pm,\mu}(\mathbf{x})$ the nuclear current.
- The idea is to express the above matrix element in terms of spherical tensor operators and apply the Wigner-Eckart theorem.
- The nuclear current contains both vector and axial-vector parts $\mathcal{J}^{\pm,\mu}(\mathbf{x}) = J_{\mathrm{V}}^{\pm,\mu}(\mathbf{x}) - J_{\mathrm{A}}^{\pm,\mu}(\mathbf{x})$ and their matrix elements can be considered separately.

[4] T.W. Donnelly and R.D. Peccei, Phys. Rep. **50**, 1-85 (1979)

[5] J. Walecka, *Theoretical Nuclear and Subnuclear Physics*, Imperial College, London (2004)

The weak axial-vector coupling constant g_{A}

With certain symmetry requirements imposed, the matrix element of the axial-vector part of the nuclear current can be expressed in the basis of free plane wave states in the origin using the Dirac equation as

$$\begin{aligned}
&\langle \mathbf{p}', \sigma' |\, J_{\mathrm{A}}^{(\pm),\mu}(0)\, | \mathbf{p}, \sigma \rangle = \\
&\frac{1}{V} \overline{u}(\mathbf{p}', \sigma') \left[F_{\mathrm{A}}^{\mathrm{CC}}(q^2) \gamma_5 \gamma^\mu + F_{\mathrm{P}}^{\mathrm{CC}}(q^2) \gamma_5 \, q^\mu \right] u(\mathbf{p}, \sigma),
\end{aligned} \tag{2}$$

where the form factor $F_{\mathrm{A}}^{\mathrm{CC}}(q^2)$ has the momentum transfer dependence

$$F_{\mathrm{A}}^{\mathrm{CC}}(q^2) = \frac{g_{\mathrm{A}}}{\left(1 + \frac{q^2}{M_{\mathrm{A}}^2}\right)^2}. \tag{3}$$

The quenching of g_{A}

- The bare value of g_{A} is predicted by the PCAC hypothesis and it can be measured from e.g. the decay of a free neutron. The most recently measured value is $g_{\mathrm{A}} = -1.27641$[6].
- The actual effective value of $g_{\mathrm{A,eff}}$ that is used in practical calculations needs to be quenched wrt the bare value in order to reach acceptable agreement with experimental results (typically β-decay half-lives).
- The reasons for the need for quenching include the limitations in the single-nucleon valence and configuration spaces in numerical calculations and nuclear medium effects, such as the meson-exchange many body currents that are not included in the one-nucleon impulse approximation, and non-nucleonic degrees of freedom (e.g. Δ baryons) that are usually neglected[7].

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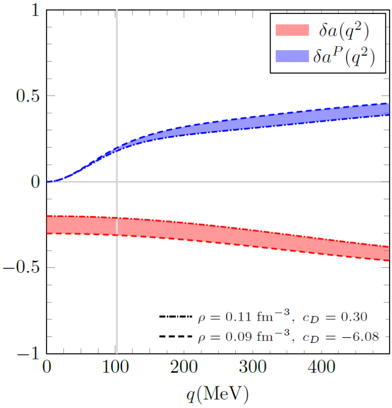
[6] B. Märkisch et al., Phys. Rev. Lett. **122**, 242501 (2019)

[7] J. Suhonen, Front. Phys. 5:55 (2017)

The effect of many-body currents

Many-body currents can have a significant effect on the strength of the quenching of $g_{\rm A}$. These were not included in our calculations as we worked in the impulse approximation. The contribution to the quenching from two-body currents along with the momentum transfer dependence has been studied in e.g. ordinary muon capture. It was found that in the $0-100$ MeV range of q the correction to $g_{\rm A,eff}$ stayed almost constant[a].

[a] P. Gimeno et al., Universe **2023**, 9, 270



2BC contribution δ_a to $g_{\rm A,eff}$ as function of the momentum transfer q. The contributions are approximated by normal-ordered 1B part of the chiral 2BCs.

The choice of the value of $g_{\mathrm{A,eff}}$

- Systematic theoretical studies of β-decays have been conducted in a number of nuclear mass number ranges in which the dependence of $g_{\mathrm{A,eff}}$ on A has been extracted.
- In particular, a study of even-even and odd-odd nuclei in the range $A = 100 - 136$ found the piecewise linear relation[8]

$$|g_{\mathrm{A,eff}}| = \begin{cases} \frac{1}{50}A - \frac{48}{30}, & A \in [100, 120], \\ \frac{1}{60}A - \frac{43}{30} & A \in [122, 136]. \end{cases} \tag{4}$$

- This parametrization was used to obtain the values $g_{\mathrm{A,eff}} \approx -0.68$ and $g_{\mathrm{A,eff}} \approx -0.78$ for ^{127}I and ^{133}Cs, respectively.

[8] P. Pirinen and J. Suhonen, Phys. Rev. C **91**, 054309 (2015)

Problems with using the linear fit

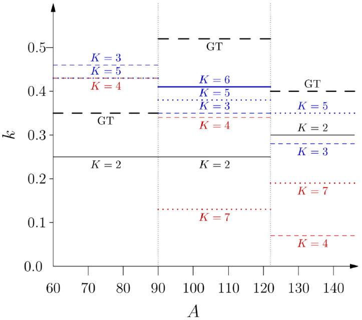
- There are some potential problems with using the linear model to obtain the $g_{\mathrm{A,eff}}$ for neutrino scattering calculations. Firstly, the model represents a least squares best-fit relation rather than an exact result, and there is considerable variance its accuracy for individual nuclei.
- Secondly, the model was obtained by considering Gamow-Teller decays, and the $g_{\mathrm{A,eff}}$ values obtained using it are consequently appropriate in a strict sense for only allowed transitions.
- Despite this, we used the obtained quenched values also for forbidden transitions and used the results for the ^{127}I scattering cross section to judge the accuracy of this choice.

Allowed vs. forbidden transitions

The quenching in forbidden β-decays has also been studied. Analysis of 1^{st} forbidden unique β-decays in the $A = 72 - 136$ range using the pnQRPA framework found the averaged quenched value of $g_{\mathrm{A,eff}} = 0.57$[a]. A theoretical study of the higher forbidden unique decays found even more quenched $g_{\mathrm{A,eff}}$ values[b]. In this sense our choice of using the $g_{\mathrm{A,eff}}$ value derived from GT-decays for all transitions represents a relatively conservative estimate for the total quenching.

[a] Ejiri et al., Phys. Lett. B **729**, 27-32 (2014)

[b] J. Kostensalo and J. Suhonen, Phys. Rev. C **95**, 014322 (2017)



The ratio of matrix elements $k = \overline{M}_{\mathrm{pnQRPA}}/\overline{M}_{\mathrm{qp}}$ for unique β-decays different forbiddenness (K). Comparison with the ratio $k = \overline{M}_{\mathrm{exp}}/\overline{M}_{\mathrm{pnQRPA}}$ can be used to predict the $g_{\mathrm{A,eff}}$ value when no experimental data is available.

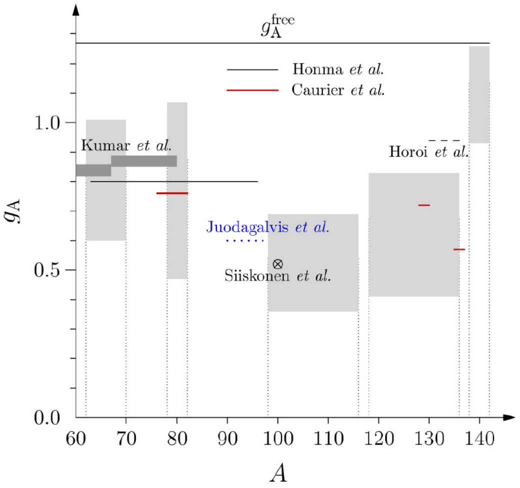
QRPA, MQPM and nuclear model limitations

- The nuclear model used to construct the initial and final nuclear states was the microscopic quasiparticle-phonon model (MQPM)[9], where the states of the odd-A nucleus are constructed by coupling BCS-quasiparticles with quasiparticle random-phase approximation (QRPA) -phonons.
- The main difference between MQPM and QRPA based models in general compared to the interacting shell model (ISM) when it comes to the deficiencies in the many-body treatment of the nucleus is usually the different limitations in the valence and configurations spaces.
- In ISM calculations the valence spaces are usually quite limited due to computational costs, but single-particle configurations in the chosen valence space are typically not. In contrast, the QRPA and MQPM valence spaces are often larger with the trade off that only the simplest of quasiparticle configuartions are included in the computations.

[9] J. Toivonen and J. Suhonen, J. Phys. G **21**, 1491 (1995)

MQPM vs ISM extracted values for $g_{\mathrm{A,eff}}$

Despite the large differences in the nuclear models, the ISM and (pn)QRPA based studies have found that while the $g_{\mathrm{A,eff}}$ values obtained from different models can vary for individual nuclei, the agreement between the $g_{\mathrm{A,eff}}$ values obtained from the different models is on average good, which is a non-trivial result[a].

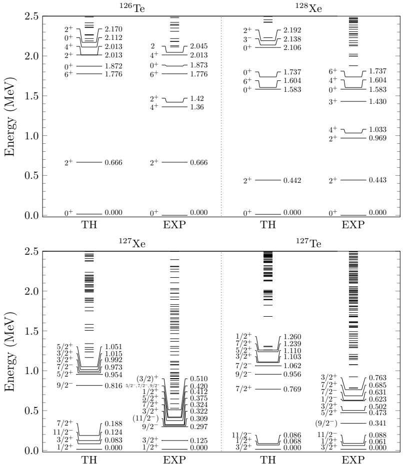


pnQRPA based results for $g_{\mathrm{A,eff}}$ (light gray) plotted with ISM results.

[a] J. Suhonen, Front. Phys. 5:55 (2017)

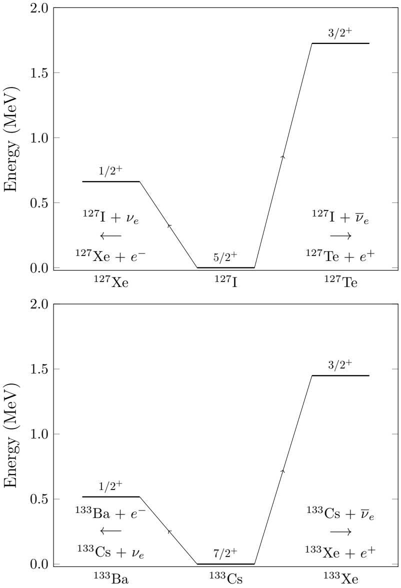
Nuclear spectra

The agreement between experimental and theoretical spectra for the odd-A nuclei of interest was fair. MPQM provides a realistic prediction for the nuclear spectrum up to relatively high (27 MeV and up) excitation energies, but agreement with experiment for any individual state can leave room for improvement.

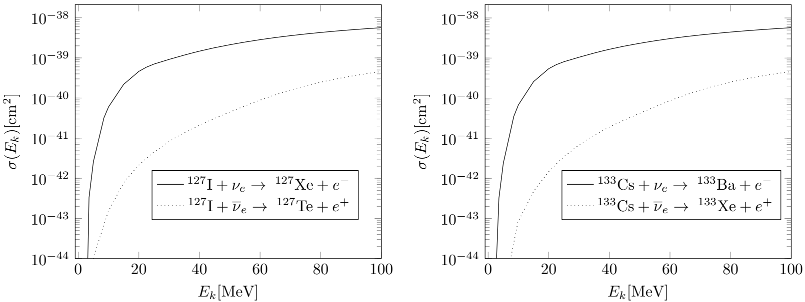


Reaction specifics

The threshold energies for the (anti)neutrino scattering reactions were very similar for both nuclei, and most of the differences in results were expected to come from the different choices in the values of $g_{\mathrm{A,eff}}$ for them.



Cross sections as functions of neutrino energy



Scattering cross sections as functions of incoming (anti)neutrino energy.

Folded cross sections

- Folded cross sections were obtained by folding the energy dependent cross sections over the neutrino spectrum.
- The spectrum used for the stopped-pion electron neutrinos was

$$p_{\mathrm{s.t.}\pi}(E_{\nu_e}) = 96\frac{E_{\nu_e}^2}{m_\mu^4}(m_\mu - 2E_{\nu_e}), \tag{5}$$

  where $E_{\nu_e} \in [0, m_\mu/2]$.
- The results were as follows:

Stopped-pion neutrino results

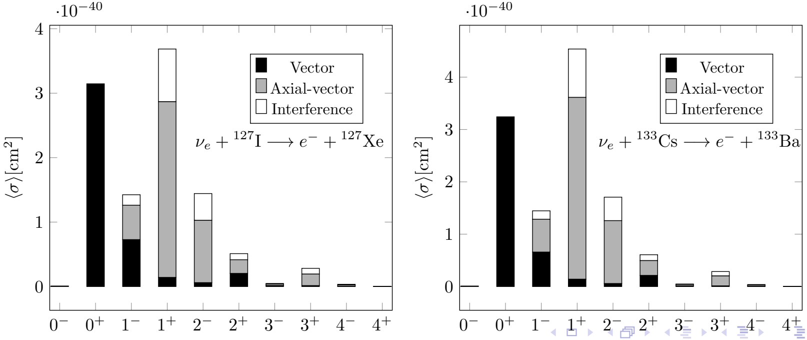
	$^{127}\mathrm{I} + \nu_e \rightarrow\ ^{127}\mathrm{Xe} + e^-$	$^{133}\mathrm{Cs} + \nu_e \rightarrow\ ^{133}\mathrm{Ba} + e^-$
$\langle\sigma\rangle_{\mathrm{MQPM}}$	10.6	11.9
$\langle\sigma\rangle_{\mathrm{Exp.}}$	$9.2^{+2.1}_{-1.8}$	-

The theoretical cross sections for stopped-pion neutrinos. The agreement with recent the experimental measurement[10] is very good. This already implies that the choice of $g_{\mathrm{A,eff}}$ is relatively accurate.

[10] P. An et al. (COHERENT) arXiv:2305.19594

The effect of the choice of $g_{\mathrm{A,eff}}$

- How much does the choice of the value of $g_{\mathrm{A,eff}}$ affect the results?
- We can look at the contributions to the total folded cross sections from different multipoles and decompose these contributions further into vector, axial-vector and interference parts.
- We have the following multipole profiles for the lowest (most highly contributing) multipoles:



On the proportion of AV contributions, part 1

- Axial-vector contributions consist of 44% of the total folded cross section and 58% of the axial-vector contributions are from the allowed 1^+ multipole alone.
- The forbidden axial-vector contributions thus represent $\sim 18.5\%$ of the total folded cross section.
- Therefore, if the chosen value of $g_{\mathrm{A,eff}}$ for forbidden transitions was highly inaccurate, we would expect it to show in the accuracy of the final result.

On the proportion of AV contributions, part 2

- The proportion of forbidden axial-vector contributions is, however, not large enough that we can make definite conclusions on what the accurate value of $g_{\mathrm{A,eff}}$ for them should be.
- It is also worth noting that the obtained theoretical result is slightly larger than the corresponding experimental result, while the previously mentioned data on the forbidden unique decays points to even more radically quenched g_{A} than what was used in our calculations.
- Therefore it can be expected that the theoretical result would be in even better agreement with the experimental result if the forbidden multipoles were quenched separately from the allowed ones according to said data.

Quenching the forbidden multipoles separately

- To see how much this could improve the obtained result, we have also computed the cross section by quenching the forbidden transitions separately using the value $g_{\mathrm{A,eff}} = 0.57$ obtained from 1^{st} forbidden unique transition.
- This transition is also the most highly contributing non-allowed transition, the other major axial-vector contributor being the 1^- multipole which is a non-unique forbidden transition.
- Due to the more complicated structure of the transition operators and relative scarcity of research into the quenching of non-unique forbidden β-decays in near and in the mass range of interest, we have used the 1^{st} forbidden unique derived $g_{\mathrm{A,eff}}$ value for all the forbidden transitions as a first approximation.

The separately quenched results

- The separately quenched results for the stopped-pion cross sections were $(9.9 \text{ and } 10.6) \times 10^{-40}$ cm^2 for ^{127}I and ^{133}Cs, respectively.
- The more accurately quenched ^{127}I result is closer to the corresponding experimental result, though both it and the more conservatively quenched result are within the uncertainty bounds of the experimental result and the improvement is relatively small.

Supernova neutrinos

- Results were also computed for supernova electron (anti)neutrinos. The spectra used for these was the a modified Fermi-Dirac spectrum

$$p_{\mathrm{s.n.}}(E_\nu) = \frac{1}{T^3 F_2(\alpha)} \frac{E_\nu^2}{e^{E_\nu/T-\alpha}+1}, \qquad (6)$$

  given in terms of the Fermi-Dirac integral $F_n(\alpha) = \int \mathrm{d}x \frac{x^n}{1+e^{x-\alpha}}$ and the degeneracy parameter α, which controls the damping of the high energy tail of the spectrum.
- Parametrized in terms of the average neutrino energy $\langle E_\nu \rangle = \frac{F_2(\alpha)}{F_3(\alpha)} T$, we have used the values $\langle E_{\nu_e} \rangle = 11.5$ MeV and $\langle E_{\overline{\nu}_e} \rangle = 13.6$ MeV and the values $\alpha = 0$ and 3.

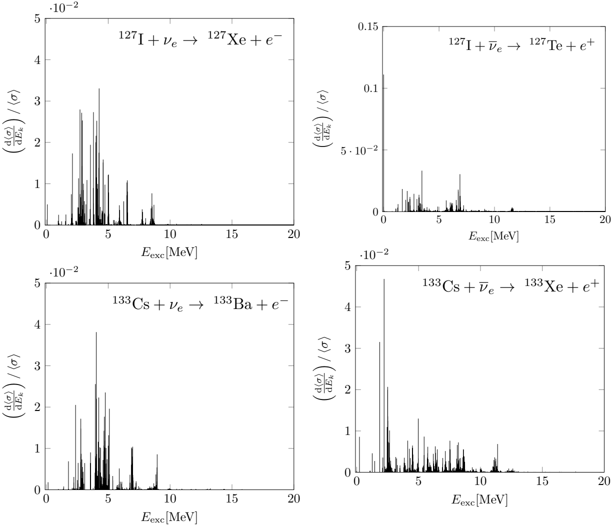
Results for supernova neutrinos

i	^{127}I		^{133}Cs	
$\langle E_{\nu/\overline{\nu}} \rangle$	11.5	13.6	11.5	13.6
f	^{127}Xe	^{127}Te	^{133}Ba	^{133}Xe
$\langle \sigma \rangle_{\alpha=0}$	159.3	1.5	184.9	1.1
$\langle \sigma \rangle_{\alpha=3}$	142.9	1.1	166.2	0.8

The supernova neutrino scattering cross sections are a few orders of magnitude larger than the antineutrino cross sections. The antineutrino results for ^{127}I are larger than the corresponding results for ^{133}Cs despite the lower $g_{\mathrm{A,eff}}$ value. This is explained by looking at the contributions to the total folded cross sections from individual final nuclear states:

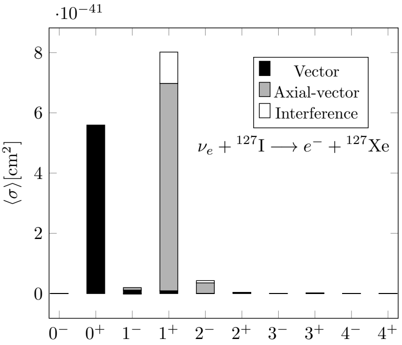
More on the contributions from individual final states

Contributions from individual final states for charged-current scattering of electron supernova (anti)neutrinos on the (right)left off ^{127}I (top) and ^{133}Cs (bottom) with $\alpha = 3$.



More elaborate quenching for supernova neutrinos?

- It is possible to apply the same detailed individual quenching scheme that was used for stopped-pion neutrinos also for supernova neutrino scattering.
- This is, however, not as useful in this case as the neutrino scattering is strongly dominated by allowed contributions and the forbidden contributions to the antineutrino scattering consist of mainly forbidden non-unique transitions.



Multipole profile of supernova neutrino scattering off ^{127}I.

A discrepancy related to the exclusive cross section

- There are also experimental results for the exclusive cross section $\nu_e + {}^{127}\mathrm{I} \longrightarrow e^- + {}^{127}\mathrm{Xe}^*(\text{bound states})$.
- Computing this cross sections in an exact way is challenging, as excitations above the neutron separation energy may not decay into the bound states of ^{127}Xe.
- A lower bound can be easily obtained by including contributions from only the states below the separation energy.
- The result obtained was 6.1×10^{-40} cm^2, which is in agreement with the most recent result of $(5.2^{+3.4}_{-3.1}) \times 10^{-40}$ obtained by COHERENT (though the error bars are large), and in disagreement with the older result of $[2.84 \pm 0.91(\text{stat}) \pm 0.25(\text{syst})] \times 10^{-40}$ obtained by the Los Alamos Meson Physics Facility (LAMPF).

Conclusions

- The results for the inclusive stopped-pion $\nu_e + {}^{127}\mathrm{I} \longrightarrow e^- + {}^{127}\mathrm{Xe}$ cross section obtained using the conservative estimate for $g_{\mathrm{A,eff}}$ is in agreement with the most recent experimental result.
- Quenching the forbidden multipoles separately improves the accuracy, but not considerably.
- More research into the non-unique forbidden quenching is needed

End

Thank you for listening.