

SPECTRAL THEORY, ALGEBRAIC GEOMETRY, AND STRINGS  
MITP TOPICAL WORKSHOP, JUNE 19-23, 2023  
FINAL REPORT

The aim of this workshop was to bring physicists and mathematicians together under the umbrella of the purported Topological String – Spectral Theory (TS/ST) correspondence. Work on the physics side has provided a conjectural correspondence between enumerative invariants of noncompact Calabi-Yau geometries and the spectra of quantum curves; while recent work on the mathematics side recasts this connection in terms of zeroes and limits of generalized periods arising from algebraic  $K$ -theory of the curves. Other interpretations and applications abound, in contexts ranging from integrable systems to resurgence theory, and we aimed to feature “both sides” of the story (in numerous ways) in an effort to familiarize researchers in diverse fields with the full landscape of ideas and to spark new collaborations.

The workshop began with two talks by Jie Gu, who gave an introduction to resurgence theory and Borel resummation, which can be seen as providing the connection between the earlier perturbative (asymptotic series) approach to problems in topological string theory and the more recent emphasis on exact, non-perturbative solutions. This was supplemented by a talk by one of the organizers (M. Mariño) under the aegis of PRISMA in the middle of the week.

The talks by Gu were followed by a first look at the mathematical perspective on TS/ST, given by Soumya Sinha Babu on his joint work with two of the organizers (Doran and Kerr). This focused on the discovery that certain special points in the moduli of a family of  $K_2$ -classes on curves “are” eigenvalues of the corresponding quantum curve/operator, which confirms a conjecture by organizers Mariño and Grassi at  $\hbar = 2\pi$ . On Tuesday morning, Lukas Schimmer gave a talk on the asymptotic behavior of these eigenvalues, which is related to the converse question (of whether all eigenvalues arise from these special points in moduli). Sinha Babu’s second talk, on Thursday, featured brand new results arising from his collaboration with P. Bousseau, relating the TS/ST conjecture in the ‘t Hooft limit to dimer models and Harnack curves.

The talk by Hollands focused on the four-dimensional Sieberg-Witten theory and the resolved conifold as exemplars of features of the non-perturbative topological string partition function. This involved both piecewise-constant dependence on a additional phase and an interpretation of jumping phenomena via BPS states.

Tanzini explained the role of isomonodromy in the study of topological string partition functions through a Painlevé hierarchy (both usual and  $q$ -difference PVI). The oper limit of the flat connection of the Hitchin system associated with gauge theories was the starting point of a refined picture for the topological string that included recent joint work with Bonelli and Grassi.

Qianyu Hao explained how a limiting case of the TS/ST conjectures for a particular family of geometries can be proved by using the connection between the corresponding quantum operators and the classical Toda equations.

The talk by Albrecht Klemm focused on the connection between the special geometry of Calabi–Yau manifolds and amplitudes in quantum field theory.

There were various talks on complex Chern–Simons theory. This is connected to the main topic of the workshop since it shares many of the features of the TS/ST correspondence and it provides a simpler setting for resurgence ideas and techniques. The talk by Kashaev presented a generalization of quantum invariants from complex Chern–Simons invariants to

local fields, and the talk by Garoufalidis analysed in detail the behaviour of these quantum invariants under multi-covering. The talk by Fei Yan presented a quantum field theory explanation for conjectures of Garoufalidis, Gu and Mariño on the resurgent structure of quantum knot invariants. Andy Neitzke gave in his talk a quantum field theory argument, based on line defects, to explain some properties of skein algebras at roots of unity. Finally, the talk by Ioana Coman discussed the connections between the  $Z$ -hat invariants of three-manifolds and mock modular forms.

Here is the list of talks that took place during the course of the week.

- Monday, June 19
  - Jie Gu: Resurgence theory and its application in topological string theory (2 talks)
  - Soumya Sinha Babu:  $K_2$  and quantum curves, I
- Tuesday, June 20
  - Lukas Schimmer: Eigenvalue asymptotics for operators associated to mirror curves
  - Alessandro Tanzini: Refined Painlevé/gauge theory correspondence
  - Rinat Kashaev: Generalized 3d TQFTs from local fields
- Wednesday, June 21
  - Andrew Neitzke: Line defects and skein algebras at roots of unity
  - Qianyu Hao: TS/ST correspondence via non-autonomous Toda equations
  - Marcos Mariño: Resurgence and non-perturbative physics (external PRISMA talk)
- Thursday, June 22
  - Lotte Hollands: Non-perturbative partition functions for supersymmetric QFTs
  - Stavros Garoufalidis: Asymptotically multiplicative quantum invariants
  - Soumya Sinha Babu:  $K_2$  and quantum curves, II
- Friday, June 23
  - Ioana Coman: Mock modular forms as Z-invariants
  - Albrecht Klemm: Amplitudes and quantum volumes
  - Fei Yan: 3d indices and Chern-Simons thimbles

There were not too many talks, as we wanted to leave time for collaborations among participants. We are aware of several:

- Doran, Pioline, and Schimmanek collaborated on the application of Doran’s work on classification of certain high Picard rank K3 surface fibered Calabi-Yau threefolds to the derivation of modular generating functions for Noether-Lefschetz numbers and D4-D2-D0 invariants for Calabi-Yau threefolds fibered by Picard rank 1 K3 surfaces.
- Doran, Kerr, and Vanhove discussed possible future work on Feynman integrals, including the full asymptotic expansion in the dimensional-regularization parameter
- Mariño and Pioline discussed the connections between resurgence in topological strings and the work of Alexandrov, Pioline and collaborators on the geometry of the hypermultiplet moduli space in Calabi–Yau compactifications.

Unfortunately, intended participants M. Bershtein, A. Goncharov, M. Mazzocco, and C. Rella were unable to attend. The final list of participants is included below, 5 of whom are female. According to affiliations, there were 14 mathematicians and 14 physicists. Speakers are marked with a “ $\diamond$ ”, and organizers with a “†”.

- (1) Murad Alim, Hamburg University (math)
- (2) Giulio Bonelli, SISSA (physics)
- (3) Andrea Brini, University of Sheffield and CNRS (math)

- (4) Ioana Coman, Kavli IPMU (physics)
- (5) † Charles Doran, University of Alberta and CMSA (math)
- (6) ◇ Stavros Garoufalidis, SUST-Shenzhen/MPI-Bonn (math)
- (7) Pavlo Gavrylenko, SISSA (math)
- (8) † Alba Grassi, CERN and University of Geneva (physics)
- (9) ◇ Jie Gu, Nanjin Southeast University (physics)
- (10) ◇ Qianyu Hao, University of Geneva (physics)
- (11) Yasuyuki Hatsuda, Rikkyo University (physics)
- (12) ◇ Lotte Hollands, Heriot-Watt University, Edinburgh (math)
- (13) Min-Xin Huang, USTC-Hefei (physics)
- (14) † Hans Jockers, JGU-Mainz (physics)
- (15) ◇ Rinat Kashaev, University of Geneva (math)
- (16) † Matt Kerr, Washington University in St. Louis (math)
- (17) ◇ Albrecht Klemm, University of Bonn (physics)
- (18) Marcos Mariño, University of Geneva (physics)
- (19) ◇ Andy Neitzke, Yale University (math)
- (20) Boris Pioline, CNRS/ and Sorbonne Université (physics)
- (21) Volodya Roubtsov, University of Angers (math)
- (22) Thorsten Schimannek, CNRS (physics)
- (23) ◇ Lukas Schimmer, Loughborough University (math)
- (24) ◇ Soumya Sinha Babu, University of Georgia (math)
- (25) ◇ Alessandro Tanzini, SISSA-Trieste (math)
- (26) Pierre Vanhove, CEA-Saclay (physics)
- (27) Duco van Straten, JGU-Mainz (math)
- (28) ◇ Fei Yan, Rutgers University (physics)