Massless Scalar & Scalar Condensate from the Quantum Conformal Anomaly

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Outline

Effective Theory of Low Energy Gravity: Role of the Trace Anomaly

- Massless Scalar Poles in Flat Space Amplitudes
- General Form of Effective Action of the Anomaly
- Effective Massless Scalar Degree of Freedom in Low Energy Macroscopic Gravity
- Couplings to Photons, Gluons
- Scalar Condensate
- Scalar ‘Particle’ w. Effects Similar to Axions
Effective Field Theory & Quantum Anomalies

- Expansion of Effective Action in Local Invariants assumes Decoupling of UV from Long Distance Modes
- But Massless Modes do not decouple
- Chiral, Conformal Symmetries are Anomalous
- Special Non-local Additions to Local EFT
- IR Sensitivity to UV degrees of freedom
- Conformal Symmetry & its Breaking controlled by the Conformal Trace Anomaly
- Macroscopic Effects in Black Hole Physics, Cosmology
Chiral Anomaly in QCD

- QCD with $N_f$ massless quarks has an apparent $U(N_f) \otimes U_{ch}(N_f)$ Symmetry
- But $U_{ch}(1)$ Symmetry is Anomalous
- Effective Lagrangian in Chiral Limit has $N_f^2 - 1$ (not $N_f^2$) massless pions at low energies
- Low Energy $\pi_0 \rightarrow 2 \gamma$ dominated by the anomaly

\[ \partial_{\mu} j^{\mu 5} = e^2 N_c F_{\mu \nu} \tilde{F}^{\mu \nu} / 16 \pi^2 \]

- No Local Action in chiral limit in terms of $F_{\mu \nu}$ but Non-local
  - IR Relevant Operator that violates naïve decoupling of UV
- Measured decay rate verifies $N_c = 3$ in QCD
  - Anomaly Matching of IR $\leftrightarrow$ UV
- Coupling to gluons as well (related to $\theta$ term, CP violation, axions)
2D Gravity

\[ S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda) \]

has no local degrees of freedom in 2D, since

\[ g_{ab} = \exp(2\sigma)\bar{g}_{ab} \rightarrow \exp(2\sigma)\eta_{ab} \]

(all metrics conformally flat) and

\[ \sqrt{g}R = \sqrt{g}\bar{R} - 2\sqrt{g} \Box \sigma \]

gives a total derivative in \(S_{cl}\)

Quantum Trace or Conformal Anomaly

\[ \langle T^a \rangle = -\frac{c_m}{24\pi}R \]

\[ c_m = N_S + N_F \] for massless scalars or fermions

Linearity in \(\sigma\) in the variational eq.

\[ \frac{\delta \Gamma_{WZ}}{\delta \sigma} = \sqrt{g} \langle T^a \rangle \]

determines the Wess-Zumino Action by inspection
2D Anomaly Action

• Integrating the anomaly linear in $\sigma$ gives

$$\Gamma_{WZ}[\bar{g}, \sigma] = \frac{c_m}{24\pi} \int d^2 x \sqrt{\bar{g}} \left( -\sigma \Box \sigma + \bar{R} \sigma \right)$$

• This is local but **non-covariant**. Note **kinetic** term for $\sigma$

• By solving for $\sigma$ the WZ action can be also written

$$\Gamma_{WZ}[\bar{g}, \sigma] = S_{anom}[g = e^{2\sigma} \bar{g}] - S_{anom}[\bar{g}]$$

• Polyakov form of the action is covariant but **non-local**

$$S_{anom}[g] = -\frac{c}{96\pi} \int d^2 x \sqrt{g} \int d^2 x' \sqrt{g'} \ R_x \left( \Box^{-1} \right)_{x,x'} \ R_{x'}$$

• A covariant local form implies a **dynamical scalar** field

$$S_{anom}[g; \varphi] = \frac{c}{96\pi} \int d^2 x \sqrt{g} \left[ g^{ab} (\nabla_a \varphi)(\nabla_b \varphi) + 2R \varphi \right]$$

$$-\Box \varphi = R \quad \varphi \leftrightarrow 2\sigma$$
Ward Identity and Massless Poles

Effects of Anomaly may be seen in flat space amplitudes

\[ \Pi_{abcd}(x, x') = \langle T_{ab}(x)T_{cd}(x') \rangle \]

Conservation of \( T_{ab} \) Ward Identity in 2D

\[ \Pi_{abcd}(k) = (\eta_{ab}k^2 - k_ak_b)(\eta_{cd}k^2 - k_ck_d) \Pi(k^2) \]

Anomalous Trace Ward Identity in 2D implies

\[ k^2 \Pi(k^2) \neq 0 \quad \text{at} \quad k^2 = 0 \quad \text{massless pole} \]
Quantum Effects of 2D Anomaly Action

- **Modification** of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a_b \rangle$
- Metric **conformal factor** $e^{2\sigma}$ (was constrained) becomes **dynamical** & itself fluctuates freely
- Gravitational ‘Dressing’ of critical exponents: **long distance/IR** macroscopic physics
- Additional **non-local Infrared** Relevant Operator in $S_{\text{EFT}}$

New **Massless Scalar** Degree of Freedom at low energy
Quantum Trace Anomaly in 4D Flat Space

Massless QED in an External E&M Field

\[ \langle T_a^a \rangle = e^2 F_{\mu\nu} F^{\mu\nu} / 24\pi^2 \]

Triangle Amplitude as in Chiral Case

\[ \Gamma^{abcd} (p,q) = (k^2 g^{ab} - k^a k^b) (g^{cd} p \cdot q - q^c p^d) F_1(k^2) + \ldots \]

In the limit of massless fermions, \( F_1(k^2) \) must have a massless pole:

\[ k = p + q \]

Corresponding Imag. Part Spectral Fn. has a \( \delta \) fn

This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

M. Giannotti & E. M. (2009)
\( <TJJ> \) Triangle Amplitude in QED

Spectral Representation and Finite Sum Rule

\[
F_1(k^2; p^2, q^2) = \frac{1}{3k^2} \int_0^\infty \frac{ds}{k^2 + s - i\epsilon} \left[ (k^2 + s)\rho_T - m^2 \rho_m \right]
\]

Numerator & Denominator cancel here

\[ \text{Im } F_1(k^2 = -s): \text{Non-anomalous, vanishes when } m=0 \]

\[
\rho_T(s; p^2, q^2) = \frac{e^2}{2\pi^2} \int_0^1 dx \int_0^{1-x} dy (1 - 4xy) \delta \left( s - \frac{(p^2x + q^2y)(1 - x - y) + m^2}{xy} \right)
\]

\[
\int_0^\infty ds \rho_T(s; p^2, q^2) = \frac{e^2}{6\pi^2}
\]

and as \( p^2, q^2, m^2 \to 0^+ \)

\[
F_1(k^2) \to \frac{e^2}{18\pi^2k^2}
\]

\[ \rho_T(s) \to \frac{e^2}{6\pi^2} \delta(s) \]

Massless scalar intermediate two-particle state analogous to chiral limit of QCD
Massless Anomaly Pole

For \( p^2 = q^2 = 0 \) (both photons on shell) and \( m_e = 0 \) the pole at \( k^2 = 0 \) describes a massless \( e^+ e^- \) pair moving at \( v = c \) colinearly, with opposite helicities in a total spin-0 state

\[
\h_{\mu\nu} \left( \partial^\mu \partial^\nu - \eta^{\mu\nu} \square \right) \square \varphi
\]

a massless scalar \( 0^+ \) state (‘Cooper pair’) which couples to gravity

\[
\sim \int d^4x \sqrt{g} \left[ -\varphi \square^2 \varphi - \frac{2}{3} \varphi \square R - \frac{e^2}{48\pi^2} \varphi F^{\mu\nu} F_{\mu\nu} \right]
\]

Effective Action special case of general form

Effective vertex
Scalar Pole in Gravitational Scattering

• In Einstein’s Theory only transverse, tracefree polarized waves (spin-2) are emitted/absorbed and propagate between sources $T^{\prime\mu\nu}$ and $T^{\mu\nu}$

• The scalar parts give only non-propagating constrained interaction (like Coulomb field in E&M)

• But for $m_e = 0$ there is a scalar pole in the $<TJJ>$ triangle amplitude coupling to photons

• This scalar wave propagates in gravitational scattering between sources $T^{\prime\mu\nu}$ and $T^{\mu\nu}$

• Couples to trace $T^{\prime\mu\mu}$

• $<TTT>$ triangle of massless photons has pole

• At least one new scalar degree of freedom in EFT
Trace Anomaly in Curved Space

$$\langle T^a_a \rangle = b C^2 + b' (E - \frac{2}{3} \Box R) + b'' \Box R + c F^2$$  
(for $m_e = 0$)

$\langle T_{ab} \rangle$ is the Stress Tensor of Conformal Matter

- $\langle T^a_a \rangle$ is expressed in terms of Geometric Invariants $E$, $C^2$
- One-loop amplitudes similar to previous examples
- State-independent, independent of $G_N$
- No local effective action in terms of curvature tensor

But there exists a non-local effective action
which can be rendered local in terms of
a new massless scalar degree of freedom

Macroscopic Quantum Modification of Classical Gravity
4D Anomalous Effective Action

Conformal Parametization

\[ g_{ab} = \exp(2\sigma) \bar{g}_{ab} \]

Since \( \sqrt{g} F = \sqrt{\bar{g}} \bar{F} \) is independent of \( \sigma \), and

\[ \sqrt{g} \left( E - \frac{2}{3} \Box R \right) = \sqrt{\bar{g}} \left( \bar{E} - \frac{2}{3} \Box \bar{R} \right) + 4\sqrt{g} \Delta_4 \sigma \]

is linear in \( \sigma \), the variational eq.,

\[ \frac{\delta \Gamma_{WZ}}{\delta \sigma} = \sqrt{g} \langle T_a \rangle = b \sqrt{g} F + b' \sqrt{g} \left( E - \frac{2}{3} \Box R \right) \]

determines the **Wess-Zumino Action** by inspection:

\[ \Gamma_{WZ} = 2b' \int d^4x \sqrt{g} \sigma \Delta_4 \sigma \]

\[ + \int d^4x \sqrt{g} \left[ b \bar{F} \right. \left. + b' \left( \bar{E} - \frac{2}{3} \Box \bar{R} \right) \right] \sigma , \]

\[ \Delta_4 \equiv \Box^2 + 2R^{ab} \nabla_a \nabla_b - \frac{2}{3} R \Box + \frac{1}{3} (\nabla^a R) \nabla_a \]

\[ F = C_{abcd} C^{abcd} \]

\[ E = R_{abcd} R^{abcd} - 4R_{ab} R^{ab} + R^2 \]
Effective Action for the Trace Anomaly

- **Non-Local Covariant Form**

\[ S_{\text{anom}}[g] = \frac{1}{2} \int_x \int_{x'} \left( \frac{E}{2} - \frac{\Box R}{3} \right)_x \left( \Delta_4^{-1} \right)_{x,x'} \left[ b C^2 + b' \left( \frac{E}{2} - \frac{\Box R}{3} \right) \right]_{x'} + c F^2 + c' G^2 \]

- **Local Covariant Form**

\[ S_{\text{anom}}[g; \varphi] = \frac{b'}{2} \int d^4x \sqrt{g} \left[ -\varphi \Delta_4 \varphi + \varphi \left( E - \frac{2}{3} \Box R + \frac{b}{b'} C^2 + c F^2 + c' G^2 \right) \right] \]

\[ \Delta_4 \varphi = \frac{1}{2} \left( E - \frac{2}{3} \Box R + \frac{b}{b'} C^2 + c F^2 + c' G^2 \right) \]

- **Dynamical Scalar in Conformal Sector**

\[ \Delta_4 = \Box^2 + 2 R^{ab} \nabla_a \nabla_b - \frac{2}{3} R \Box + \frac{1}{3} (\nabla^a R) \nabla_a \]

- **Expectation Value/Classical Field is Scalar Condensate**

- **Condensate Affects Effective QED, QCD Couplings**
IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—\textit{Not given purely in terms of Local Curvature}

\[ S_{EFT}[g, \varphi] = S_{cl}[g] + S_{anom}[g, \varphi] \]

\textit{This is a non-trivial modification of classical General Relativity from quantum effects}

Additional Conformal Scalar Degree of Freedom
Stress Tensor of the Anomaly

Variation of the Effective Action with respect to the metric gives stress-energy tensor

\[ T_{\mu\nu}^{\text{anom}}[g, \varphi] = -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{anom}}}{\delta g^{\mu\nu}} \]

- Quantum Vacuum Polarization in Terms of (Semi-) Classical Scalar ‘Potential’ Condensate
- \( \varphi \) is a scalar degree of freedom in low energy gravity which depends upon the global topology of spacetimes and its boundaries, \textbf{horizons}
Anomaly Scalar in Schwarzschild Space

- General solution of $\phi$ equation as function of $r$ are easily found in Schwarzschild case (Mass $M$)

$$
\frac{d\phi}{dr} \bigg|_S = -\frac{1}{3M} - \frac{1}{r} + \frac{2Mc_H}{r(r-2M)} + \frac{c_\infty}{2M} \left( \frac{r}{2M} + 1 + \frac{2M}{r} \right) + 
q - 2 \left( \frac{r}{2M} + 1 \right) \frac{2M}{r} \ln \left( 1 - \frac{2M}{r} \right) - \frac{q}{6r} \left[ \frac{4M}{r-2M} \ln \left( \frac{r}{2M} \right) + \frac{r}{2M} + 3 \right]
$$

- $q$, $c_H$, $c_\infty$ are integration constants,

- Only way to have vanishing $\phi$ as $r \to \infty$ is $c_\infty = q = 0$

- But only way to have finiteness on the horizon is $c_H = 0$, $q = 2$

- Topological obstruction to finiteness vs. falloff of stress tensor

- Relevant to Black Hole horizons

- Also gives long range Scalar Condensate potential from any source

- Radial $r$ Dependent Variation of QED, QCD Couplings
Conclusions

- Conformal Anomaly **Predicts** New Massless Scalar
- Classical **Condensate** Potential from Massive Sources
- Gravitational Coupling relevant to BH’s, Dark Energy
- **Scalar** (Breather Mode) **Gravitational Waves**
- Couples also to Two-Photons $F^2$, Two-Gluons $G^2$
- Linear Dependence off $\alpha$, $\alpha_s$
- **Axion-Like Scalar**: HE Scattering off EBL, CMB
- Light through the Wall? Other Terrestrial Tests?
- Dark Matter-like Effects? Time Dependent Condensates?

**Ultra-Light Frontier should include Scalars**
Exact Effective Action & Wilson Effective Action

- Integrating out Matter + ... Fields in Fixed Gravitational Background gives the **Exact** Quantum Effective Action

- The possible terms in $S_{\text{exact}}[g]$ can be classified according to their response to local Weyl rescalings $g \rightarrow e^{2\sigma} g$

  $$S_{\text{exact}}[g] = S_{\text{local}}[g] + S_{\text{anom}}[g] + S_{\text{Weyl}}[g]$$

- $S_{\text{local}}[g] = (1/16\pi G) \int d^4 x \sqrt{g} (R - 2 \Lambda) + \sum_{n \geq 4} M_{\text{Pl}}^{4+n} S^{(n)}_{\text{local}}[g]$

  Ascending series of higher derivative local terms, $n>4$ irrelevant

- Non-local but Weyl-invariant (neutral under rescalings)

  $$S_{\text{Weyl}}[g] = S_{\text{Weyl}}[e^{2\sigma} g]$$

- $S_{\text{anom}}[g]$ special non-local terms that scale linearly with $\sigma$, logarithmically with distance, representatives of non-trivial cohomology under Weyl group

- Wilson effective action captures all **IR** physics

  $$S_{\text{eff}}[g] = S_{\text{HE}}[g] + S_{\text{anom}}[g]$$
Casimir Effect from the Anomaly

In ordinary flat space the relevant tensor is

\[
E_{ab}^{\text{flat}} = -2 (\nabla_{(a} \varphi)(\nabla_{b)} \Box \varphi) + 2 (\Box \varphi)(\nabla_{a} \nabla_{b} \varphi) \\
+ \frac{2}{3} (\nabla_{c} \varphi)(\nabla^{c} \nabla_{a} \nabla_{b} \varphi) - \frac{4}{3} (\nabla_{a} \nabla_{c} \varphi)(\nabla_{b} \nabla^{c} \varphi) \\
+ \frac{1}{6} g_{ab} \left\{ -3 (\Box \varphi)^{2} + \Box (\nabla_{c} \varphi \nabla^{c} \varphi) \right\} - \frac{2}{3} \nabla_{a} \nabla_{b} \Box \varphi ,
\]

Particular Solution:

\[
\varphi = c_{1} \frac{z^{2}}{a^{2}}
\]

Casimir Stress tensor between parallel plates:

\[
T_{ab}^{(\text{anom})} = \frac{C}{a^{4}} \text{diag} (-1, 1, 1, -3)
\]

Other examples (Rindler wedge, de Sitter, Schwarzschild)
Relevance of the Trace Anomaly

- Expansion of Effective Action in *Local* Invariants assumes Decoupling of Short Distance from Long Distance Modes
- But Relativistic Particle Creation is *Non-Local*
- *Massless* Modes do not decouple
- Special Non-local Additions to Local EFT
- *IR* Sensitivity to *UV* degrees of freedom
- QFT Conformal Behavior, Breaking & Bulk Viscosity (analog of conductivity) determined by Anomaly
- Blueshift on Horizons $\Rightarrow$ behavior conformal there
- *Additional Scalar Degree(s) of Freedom* in EFT of Gravity allow & predict *Dynamics of $\Lambda$*