Massless Scalar & Scalar Condensate from the Quantum Conformal Anomaly

E. Mottola, Los Alamos

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Outline

Effective Theory of Low Energy Gravity: Role of the Trace Anomaly

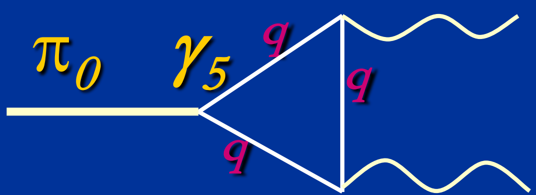
- Massless Scalar Poles in Flat Space Amplitudes
- General Form of Effective Action of the Anomaly
- Effective Massless Scalar Degree of Freedom in Low Energy Macroscopic Gravity
- Couplings to Photons, Gluons
- Scalar Condensate
- Scalar `Particle' w. Effects Similar to Axions

Effective Field Theory & Quantum Anomalies

- Expansion of Effective Action in Local Invariants assumes Decoupling of UV from Long Distance Modes
- But Massless Modes do not decouple
- Chiral, Conformal Symmetries are Anomalous
- Special Non-local Additions to Local EFT
- **IR** Sensitivity to **UV** degrees of freedom
- Conformal Symmetry & its Breaking controlled by the Conformal Trace Anomaly
- Macroscopic Effects in Black Hole Physics, Cosmology

Chiral Anomaly in QCD

- QCD with N_f massless quarks has an apparent $U(N_f) \otimes U_{ch}(N_f)$ Symmetry
- But $U_{ch}(1)$ Symmetry is Anomalous
- Effective Lagrangian in Chiral Limit has $N_f^2 - 1$ (*not* N_f^2) massless pions at low energies
- Low Energy $\pi_0 \to 2\gamma$ dominated by the anomaly



$$\partial_\mu j^{\mu 5} = e^2 N_c F_{\mu\nu} \tilde{F}^{\mu\nu} / 16\pi^2$$

- ***No Local*** Action in chiral limit in terms of $F_{\mu\nu}$ but Non-local IR Relevant Operator that violates naïve decoupling of UV
- Measured decay rate verifies $N_c = 3$ in QCD
  Anomaly Matching of IR ↔ UV
- Coupling to gluons as well (related to θterm, CP violation, axions)

2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \rightarrow \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\,\bar{\Box}\,\sigma$$

gives a total derivative in S_{cl}

Quantum Trace or Conformal Anomaly

$$\langle T^a_{\ a}\rangle = -\frac{c_m}{24\pi}R$$

$c_m = N_S + N_F$ for massless scalars or fermions

Linearity in σ in the variational eq.

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\,\langle T^a_{\ a}\rangle$$

determines the Wess-Zumino Action by *inspection*

2D Anomaly Action

- Integrating the anomaly linear in σ gives

$$\Gamma_{WZ}[\bar{g},\sigma] = \frac{c_m}{24\pi}\int d^2x\sqrt{\bar{g}}\,\left(-\sigma\,\bar{\Box}\,\sigma + \bar{R}\,\sigma\right)$$

- This is local but **non-covariant**. Note kinetic term for σ
- By solving for σ the WZ action can be also written

$$\Gamma_{WZ}[\bar{g},\sigma] = S_{anom}[g = e^{2\sigma}\,\bar{g}] - S_{anom}[\bar{g}]$$

- Polyakov form of the action is covariant but **non-local**

$$S_{anom}[g] = -\frac{c}{96\pi}\int d^2x\sqrt{g}\int d^2x'\sqrt{g'}\;R_x\left(\Box^{-1}\right)_{x,x'}R_{x'}$$

- A covariant local form implies a **dynamical scalar** field

$$S_{anom}[g;\varphi] = \frac{c}{96\pi}\int d^2x\sqrt{g}\,\left[g^{ab}(\nabla_a\varphi)(\nabla_b\varphi) + 2R\varphi\right]$$

$$-\Box\varphi = R \qquad\qquad \varphi \leftrightarrow 2\sigma$$

Ward Identity and Massless Poles

Effects of Anomaly may be seen in flat space amplitudes

$$\Pi_{abcd}(x,x') = \langle T_{ab}(x) T_{cd}(x') \rangle$$

Conservation of T_{ab} Ward Identity in 2D implies

$$\Pi_{abcd}(k) = (\eta_{ab}k^2 - k_a k_b)(\eta_{cd}k^2 - k_c k_d)\, \Pi(k^2)$$

Anomalous Trace Ward Identity in 2D implies

$k^2\, \Pi(k^2) \neq 0$ at $k^2 = 0$ massless pole

Quantum Effects of 2D Anomaly Action

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes dynamical & itself fluctuates freely
- Gravitational 'Dressing' of critical exponents: long distance/IR macroscopic physics
- Additional non-local Infrared Relevant Operator in S_{EFT}

New Massless Scalar Degree of Freedom at low energy

Quantum Trace Anomaly in 4D Flat Space

Massless QED in an External E&M Field

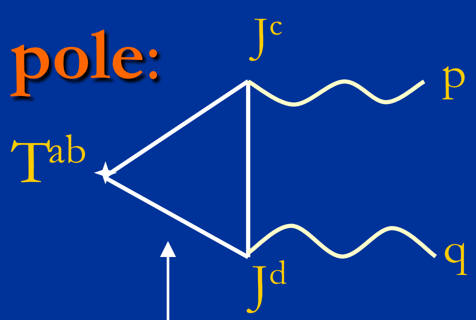
$$\langle T_a{}^a \rangle = e^2 F_{\mu\nu} F^{\mu\nu} / 24\pi^2$$

Triangle Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd}\, p\bullet q - q^c p^d)\, F_1(k^2) + \ldots$$

In the limit of massless fermions, $F_1(k^2)$ must have a **massless pole:**

$k = p + q$



$$F_1 = \frac{e^2}{18\pi^2 k^2}$$

$$\rho(s) = \frac{e^2}{18\pi^2}\delta(s)$$

M. Giannotti & E. M. (2009)

Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

<TJJ> Triangle Amplitude in QED

Spectral Representation and Finite Sum Rule

$$F_1(k^2; p^2, q^2) = \frac{1}{3k^2}\int_0^\infty \frac{ds}{k^2 + s - i\epsilon}\left[(k^2+s)\rho_T - m^2\rho_m\right]$$

Numerator & Denominator cancel here

Im $F_1(k^2 = -s)$: Non-anomalous, vanishes when m=0

$$\rho_T(s; p^2, q^2) = \frac{e^2}{2\pi^2}\int_0^1 dx \int_0^{1-x} dy\ (1-4xy)\ \delta\left(s - \frac{(p^2 x + q^2 y)(1-x-y) + m^2}{xy}\right)$$

$$\int_0^\infty ds\, \rho_T(s; p^2, q^2) = \frac{e^2}{6\pi^2}$$

obeys a finite sum rule independent of p^2, q^2, m^2

and as p^2, q^2, m^2 ➔ 0^+

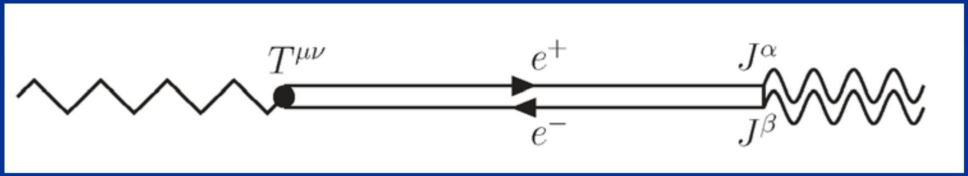
$$\rho_T(s) \to \frac{e^2}{6\pi^2}\,\delta(s)$$

$$F_1(k^2) \to \frac{e^2}{18\pi^2 k^2}$$

Massless scalar intermediate two-particle state analogous to chiral limit of QCD

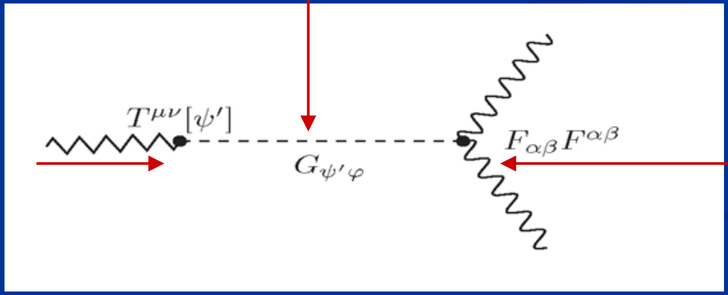
Massless Anomaly Pole

For $p^2 = q^2 = 0$ (both photons on shell) and $m_e = 0$ the pole at $k^2 = 0$ describes a *massless* $e^+ e^-$ pair moving at v=c colinearly, with opposite helicities in a total spin-0 state



a massless scalar 0^+ state ('Cooper pair') which couples to gravity

$$h_{\mu\nu}\,(\partial^\mu\partial^\nu - \eta^{\mu\nu}\,\Box)\,\Box\varphi$$



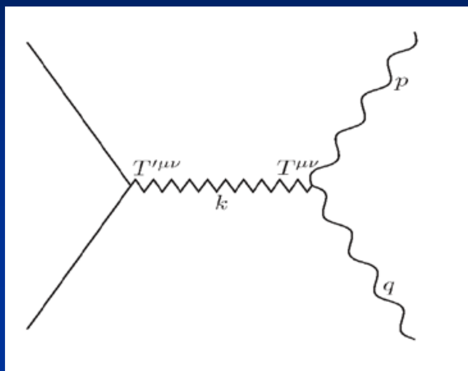
Effective vertex

$$\varphi F^{\mu\nu} F_{\mu\nu}$$

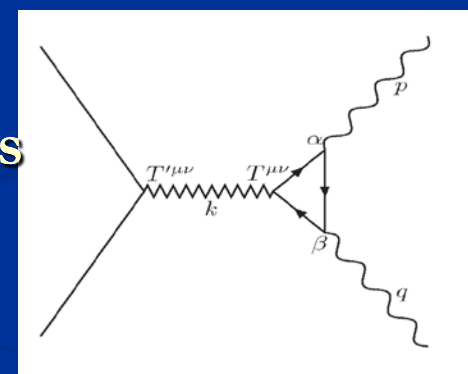
Effective Action special case of general form

$$\sim \int d^4x \sqrt{g}\left[-\varphi\,\Box^2\varphi - \frac{2}{3}\,\varphi\,\Box R - \frac{e^2}{48\pi^2}\,\varphi\,F^{\mu\nu}F_{\mu\nu}\right]$$

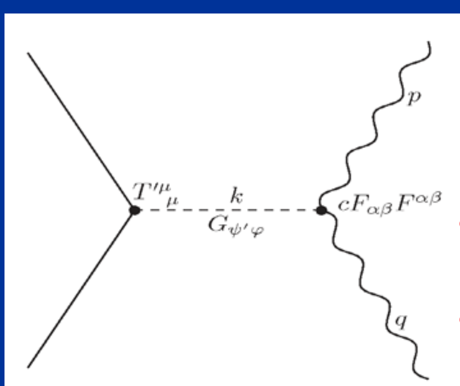
Scalar Pole in Gravitational Scattering



- In Einstein's Theory only transverse, tracefree polarized waves (spin-2) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)



- But for $m_e = 0$ there is a scalar pole in the <TJJ> triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$



- Couples to trace $T'^{\mu}_{\ \mu}$
- <TTT> triangle of massless photons has pole
- At least one new scalar degree of freedom in EFT

Trace Anomaly in Curved Space

$$\langle T_a{}^a \rangle = b\, C^2 + b' \left(E - \tfrac{2}{3}\Box R\right) + b''\, \Box R + cF^2$$

(for $m_e = 0$)

$\langle T_{ab} \rangle$ is the Stress Tensor of Conformal Matter

- $\langle T_a{}^a \rangle$ is expressed in terms of Geometric Invariants E, C^2
- One-loop amplitudes similar to previous examples
- State-independent, independent of G_N
- No local effective action in terms of curvature tensor

But there exists a non-local effective action

which can be rendered local in terms of

a new massless scalar degree of freedom

Macroscopic Quantum Modification of Classical Gravity

4D Anomalous Effective Action

Conformal Parametrization

$$\rightarrow \qquad g_{ab} = \exp(2\sigma)\,\bar{g}_{ab}$$

Since $\sqrt{g}\,F = \sqrt{\bar{g}}\,\bar{F}$

is independent of σ, and

$$\sqrt{g}\,(E - \tfrac{2}{3}\Box R) = \sqrt{\bar{g}}\,(\bar{E} - \tfrac{2}{3}\bar{\Box}\bar{R}) + 4\sqrt{\bar{g}}\bar{\Delta}_4\sigma$$

is linear in σ, the variational eq.,

$$\frac{\delta\Gamma_{WZ}}{\delta\sigma} = \sqrt{g}\langle T_a{}^a\rangle = b\,\sqrt{g}F + b'\sqrt{g}\,(E - \tfrac{2}{3}\Box R)$$

determines the Wess-Zumino Action by inspection:

$$\Gamma_{WZ} = 2b'\int d^4x\sqrt{\bar{g}}\,\sigma\bar{\Delta}_4\sigma$$

$$+\int d^4x\sqrt{\bar{g}}\left[b\bar{F} + b'\left(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\ ,$$

$$\Delta_4 \equiv \Box^2 + 2R^{ab}\nabla_a\nabla_b - \tfrac{2}{3}R\Box + \tfrac{1}{3}(\nabla^a R)\nabla_a$$

$$F = C_{abcd}C^{abcd}$$

$$E = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

Effective Action for the Trace Anomaly

- **Non-Local Covariant Form**

$$S_{anom}[g] = \frac{1}{2}\int_x\int_{x'} \left(\frac{E}{2} - \frac{\Box R}{3}\right)_x \left(\Delta_4^{-1}\right)_{x,x'} \left[b\, C^2 + b'\left(\frac{E}{2} - \frac{\Box R}{3}\right)\right]_{x'}$$

+cF^2 +c'G^2

- **Local Covariant Form**

$$S_{anom}[g;\varphi] = \frac{b'}{2}\int d^4x\sqrt{g}\left[-\varphi\Delta_4\varphi + \varphi\left(E - \frac{2}{3}\Box R + \frac{b}{b'}C^2 + cF^2 + c'G^2\right)\right]$$

$$\Delta_4\varphi = \frac{1}{2}\left(E - \frac{2}{3}\Box R + \frac{b}{b'}C^2 + cF^2 + c'G^2\right)$$

- **Dynamical Scalar in Conformal Sector**

$$\Delta_4 = \Box^2 + 2R^{ab}\nabla_a\nabla_b - \frac{2}{3}R\Box + \frac{1}{3}(\nabla^a R)\nabla_a$$

- **Expectation Value/Classical Field is Scalar Condensate**
- **Condensate Affects Effective QED, QCD Couplings**

IR Relevant Term in the Action

The effective action for the trace anomaly scales logarithmically with distance and therefore should be included in the low energy macroscopic EFT description of gravity—

Not given purely in terms of Local Curvature

$$S_{EFT}[g, \varphi] = S_{cl}[g] + S_{anom}[g, \varphi]$$

This is a non-trivial modification of classical General Relativity from quantum effects

Additional Conformal Scalar Degree of Freedom

Stress Tensor of the Anomaly

Variation of the Effective Action with respect to the metric gives stress-energy tensor

$$T_{\mu\nu}^{anom}[g,\varphi] = -\frac{2}{\sqrt{-g}}\,\frac{\delta S_{anom}}{\delta g^{\mu\nu}}$$

- Quantum Vacuum Polarization in Terms of (Semi-) Classical Scalar 'Potential' Condensate
- φ is a scalar degree of freedom in low energy gravity which depends upon the global topology of spacetimes and its boundaries, horizons

Anomaly Scalar in Schwarzschild Space

- **General solution of φ equation as function of r are easily found in Schwarzschild case (Mass M)**

$$\left.\frac{d\varphi}{dr}\right|_S = -\frac{1}{3M} - \frac{1}{r} + \frac{2Mc_H}{r(r-2M)} + \frac{c_\infty}{2M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right) + \frac{q-2}{6M}\left(\frac{r}{2M} + 1 + \frac{2M}{r}\right)\ln\left(1 - \frac{2M}{r}\right) - \frac{q}{6r}\left[\frac{4M}{r-2M}\ln\left(\frac{r}{2M}\right) + \frac{r}{2M} + 3\right]$$

- **q, c_H, c_∞** are integration constants,
- Only way to have vanishing φ as r → ∞ is **$c_\infty = q = 0$**
- But only way to have finiteness on the horizon is **$c_H = 0$, $q = 2$**
- Topological obstruction to finiteness vs. falloff of stress tensor
- Relevant to Black Hole horizons
- Also gives long range Scalar Condensate potential from any source
- Radial r Dependent Variation of QED, QCD Couplings

Conclusions

- Conformal Anomaly Predicts New Massless Scalar
- Classical Condensate Potential from Massive Sources
- Gravitational Coupling relevant to BH’s, Dark Energy
- Scalar (Breather Mode) Gravitational Waves
- Couples also to Two-Photons F^2, Two-Gluons G^2
- Linear Dependence off α , α_s
- Axion-Like Scalar: HE Scattering off EBL, CMB
- Light through the Wall? Other Terrestrial Tests?
- Dark Matter-like Effects? Time Dependent Condensates?

Ultra-Light Frontier should include Scalars

Exact Effective Action & Wilson Effective Action

- Integrating out Matter + … Fields in Fixed Gravitational Background gives the Exact Quantum Effective Action
- The possible terms in $S_{exact}[g]$ can be classified according to their repsonse to local Weyl rescalings $g \to e^{2\sigma} g$

$$S_{exact}[g] = S_{local}[g] + S_{anom}[g] + S_{Weyl}[g]$$

- $S_{local}[g] = (1/16\pi G) \int d^4x \sqrt{g}\, (R - 2\Lambda) + \Sigma_{n\geq 4} M_{Pl}^{4-n}\, S^{(n)}_{local}[g]$

  Ascending series of higher derivative local terms, n>4 irrelevant

- Non-local but Weyl-invariant (neutral under rescalings)

$$S_{Weyl}[g] = S_{Weyl}[e^{2\sigma} g]$$

- $S_{anom}[g]$ special non-local terms that scale linearly with σ, logarithmically with distance, representatives of non-trivial cohomology under Weyl group
- Wilson effective action captures all IR physics

$$S_{eff}[g] = S_{HE}[g] + S_{anom}[g]$$

Casimir Effect from the Anomaly

In ordinary flat space the relevant tensor is

$$E_{ab}\Big|_{flat} = -2\,(\nabla_{(a}\varphi)(\nabla_{b)}\Box\varphi) + 2(\Box\varphi)(\nabla_a\nabla_b\varphi) + \frac{2}{3}(\nabla_c\varphi)(\nabla^c\nabla_a\nabla_b\varphi) - \frac{4}{3}\,(\nabla_a\nabla_c\varphi)(\nabla_b\nabla^c\varphi) + \frac{1}{6}\,g_{ab}\,\{-3\,(\Box\varphi)^2 + \Box(\nabla_c\varphi\nabla^c\varphi)\} - \frac{2}{3}\,\nabla_a\nabla_b\Box\varphi\,,$$

Particular Solution: $\varphi = c_1\,\frac{z^2}{a^2}$

Casimir Stress tensor between parallel plates:

$$T_{ab}^{(anom)} = \frac{C}{a^4}\,\mathrm{diag}\,(-1,1,1,-3)$$

Other examples (Rindler wedge, de Sitter, Schwarzschild)

Relevance of the Trace Anomaly

- Expansion of Effective Action in *Local* Invariants assumes Decoupling of Short Distance from Long Distance Modes
- **But Relativistic Particle Creation is Non-Local**
- ***Massless*** **Modes do not decouple**
- **Special Non-local Additions to Local EFT**
- ***IR*** **Sensitivity to UV degrees of freedom**
- **QFT Conformal Behavior, Breaking & Bulk Viscosity (analog of conductivity) determined by Anomaly**
- **Blueshift on Horizons → behavior conformal there**
- **Additional Scalar Degree(s) of Freedom in EFT of Gravity allow & predict Dynamics of Λ**