Overlap staggered fermions

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with Aleksi Kurkela & Marco Panero

inspired by David Adams

Motivation

- Light u, d quarks needed to simulate correct physics $\rightarrow$ expensive
- Cost-saving: **staggered fermions** (1/4 d.o.f.)

$$S_F = \sum_x \bar{\chi}(x) \sum_\mu \eta_\mu(x) (U_\mu(x)\chi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\chi(x-\hat{\mu})) + m_q \sum_x \bar{\chi}(x)\chi(x)$$

$$\eta_\mu = \pm 1;\ \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu} \ \rightarrow\ \prod_4 \eta = -1 \text{ around any plaquette}$$

- Drawback: $N_f = 4$ (degenerate when $a = 0$) "tastes" $\rightarrow \sqrt{\det(D_{st})}$

"rooting is evil" **Mike Creutz**

- non-locality?
- 't Hooft vertex, $U(1)_A$ breaking?
- staggered fermions don't feel the topology

- No quartet of low-lying eigenvalues $\leftrightarrow$ no index theorem

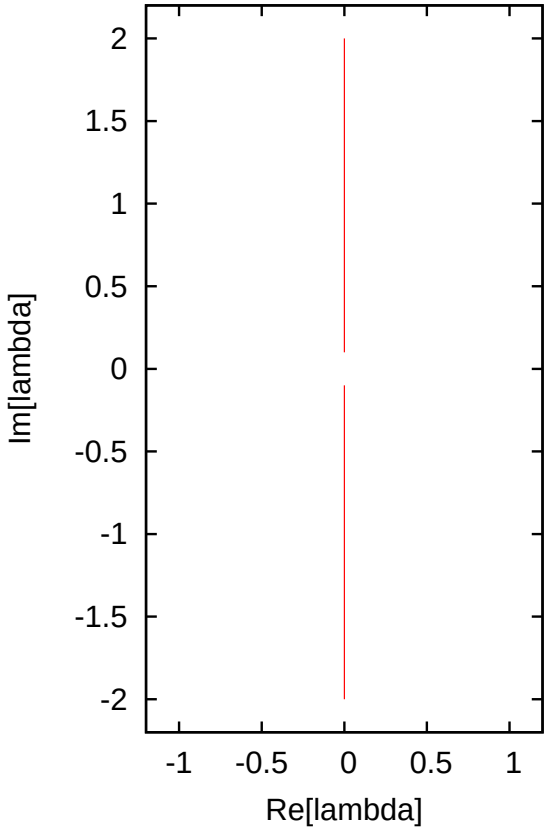
David Adams to the rescue: $N_f = 2$ staggered overlap fermions

0912.2850, 1008.2833

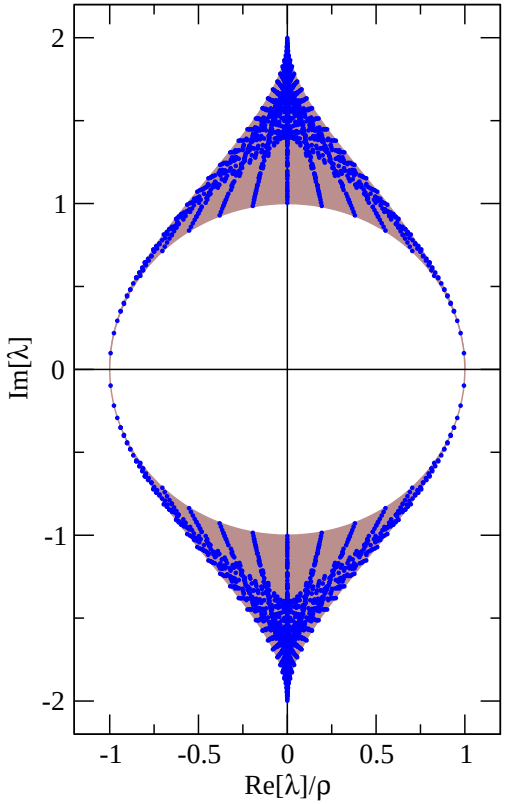
Construction

- Idea # 1: $N_f = 4 \rightarrow 2$

Include taste-dependent mass term: $\pm\rho$ for left-/right-handed *tastes*



D_{st}



$D_{Adams} \equiv D_{st} + \rho(\mathbf{1} \otimes \gamma_5)$

Then add mass (ie. shift spectrum) to make $N_f = 2$ massless flavors!

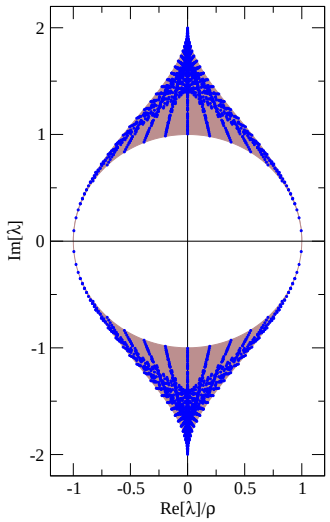
Drawback: additive mass renormalization, ie. fine-tuning for $m_q \rightarrow 0$

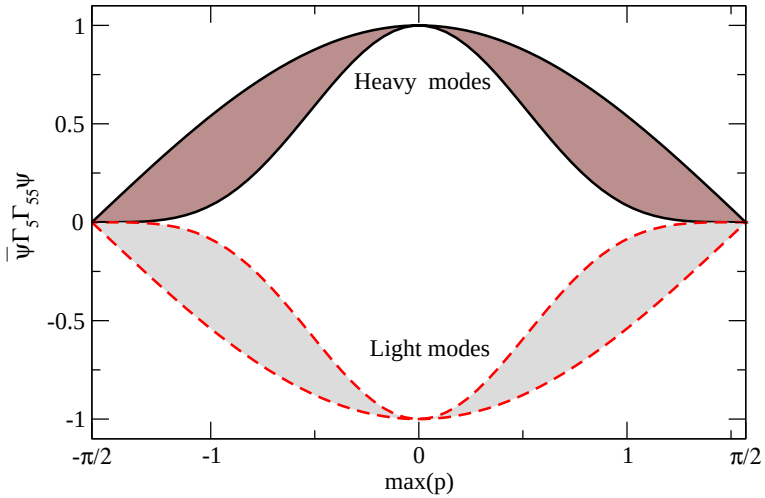
Construction: mass term

- Common lore: two choices for γ_5
  - local: $(-)^{x+y+z+t} \equiv$ "γ_5"
  - non-local: $\frac{1}{4!}(\eta_1 U_{\hat{1}} \eta_2 U_{\hat{2}} \eta_3 U_{\hat{3}} \eta_4 U_{\hat{4}} + \text{perm.}) \equiv$ "Γ_5"
- In fact: "γ_5" $\rightarrow \gamma_5^{Dirac} \otimes \gamma_5^{taste}$

  "Γ_5" $\rightarrow \gamma_5^{Dirac} \otimes \mathbf{1}^{taste}$

So that "γ_5" $\times$ "Γ_5" $\rightarrow \mathbf{1}^{Dirac} \otimes \gamma_5^{taste}$, mass term (4 links)



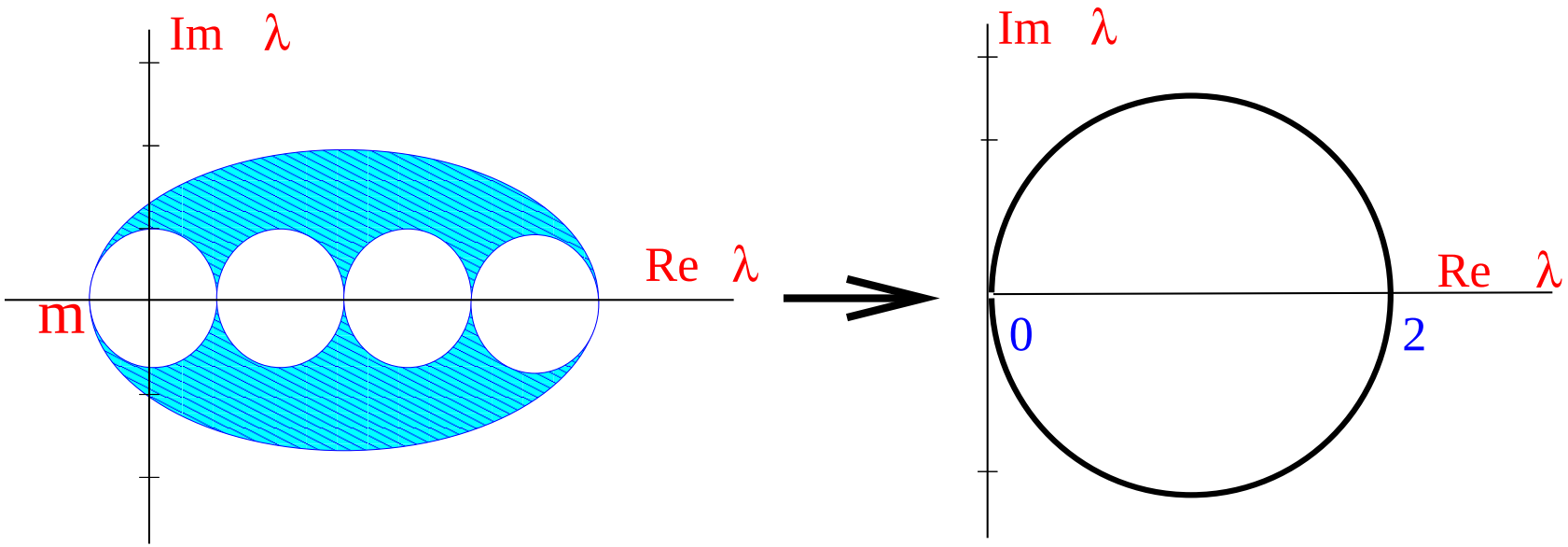


$\langle\bar{\Psi}|$"$\gamma 5$" $\times$ "Γ_5" $|\Psi\rangle$ versus $\max_4 p_\mu$

Construction

- Idea # 2: use as kernel in overlap $D_{ov} = 1 + \frac{D_{Adams}}{\sqrt{D^\dagger_{Adams} D_{Adams}}}$

ie. unitary projection (polar decomp.): no more additive mass renorm.



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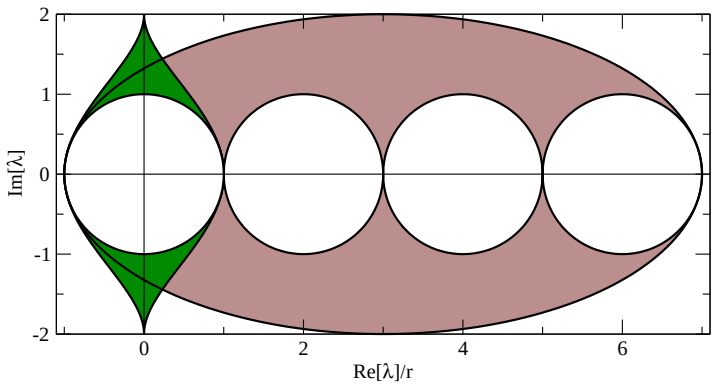
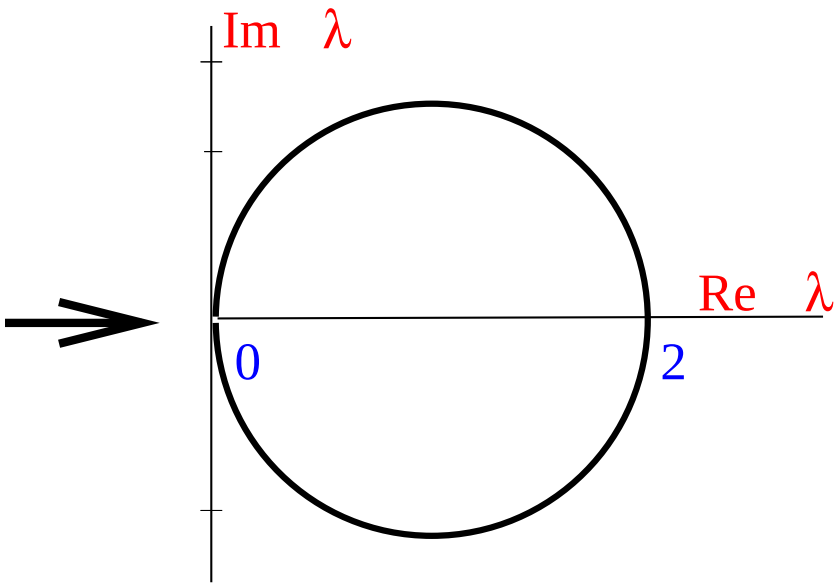




- Added bonus: index theorem, Ginsparg-Wilson symmetry
- Cost? in-between staggered and Neuberger...

Index from eigenvalue flow

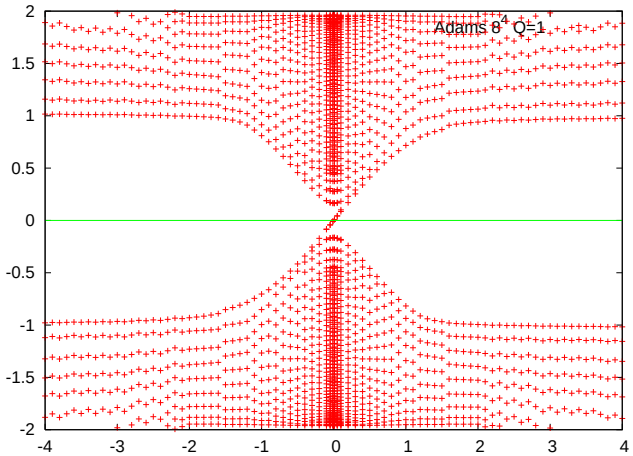
- Index from flow of eigenvalues of $H(m) = \gamma_5(\not{D} + m) = \gamma_5 \not{D} + m\gamma_5$
- Topology comes from gluon field, ie. taste-singlet

$\Longrightarrow$ Need **taste-singlet** γ_5, at least for mass term $\rightarrow$ "Γ_5"

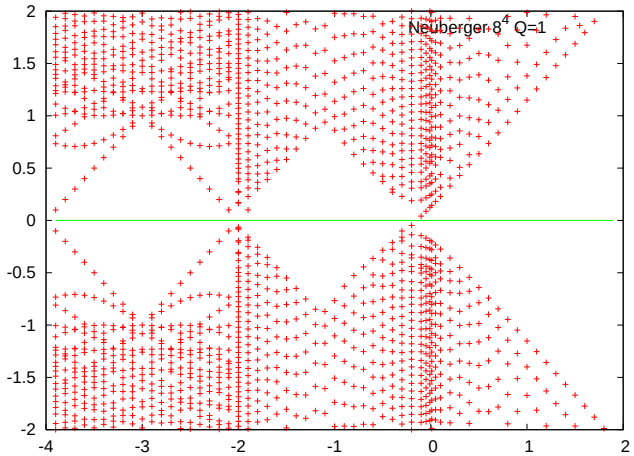
$$H(m) = \text{"}\gamma_5\text{"}\,(D_{st} + m\text{"}\Gamma_5\text{"})$$

$$D_{st} = \tfrac{1}{2}\sum_\mu \eta_\mu(x)(U_\mu(x) - U_\mu^\dagger(x-\hat{\mu}))$$

$\gamma_5 = (-)^{x+y+z+t}$, $\quad \Gamma_5 = \prod_4 \eta_\mu \times \sum$ 4-link transporters



Adams

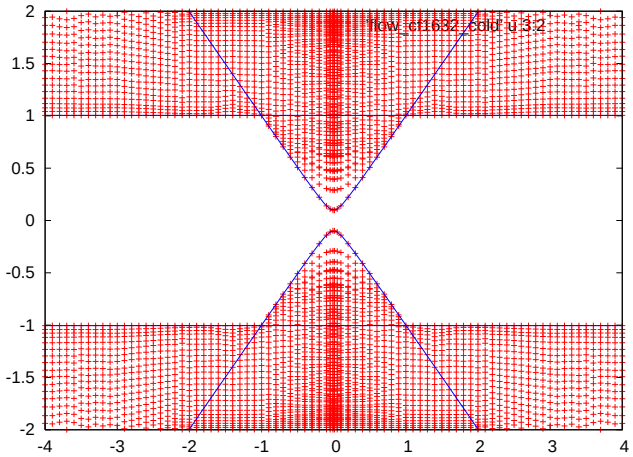


Neuberger

$Q = 1$

More eigenvalue flows

- Cold configuration: agreement with analytic result



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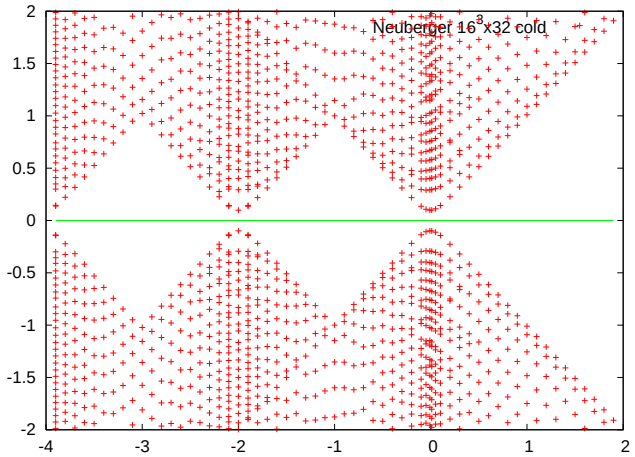


Neuberger

- $\beta = 6.0$: eigenvalue gap closes, but $|m|$ can be *arbit. large* in Adams

Overlap staggered fermions

- Just like Neuberger: $\boxed{D_{ov} = 1 + \gamma_5 \mathrm{sign}(H(-m_0))} = 1 + \frac{D_{Adams}}{\sqrt{D^\dagger_{Adams} D_{Adams}}}$

with $\gamma_5 = (-)^{x+y+z+t}$ (need $\gamma_5^2 = 1$)

- Potential advantages:

  - cheaper (4 times fewer d.o.f. per site)

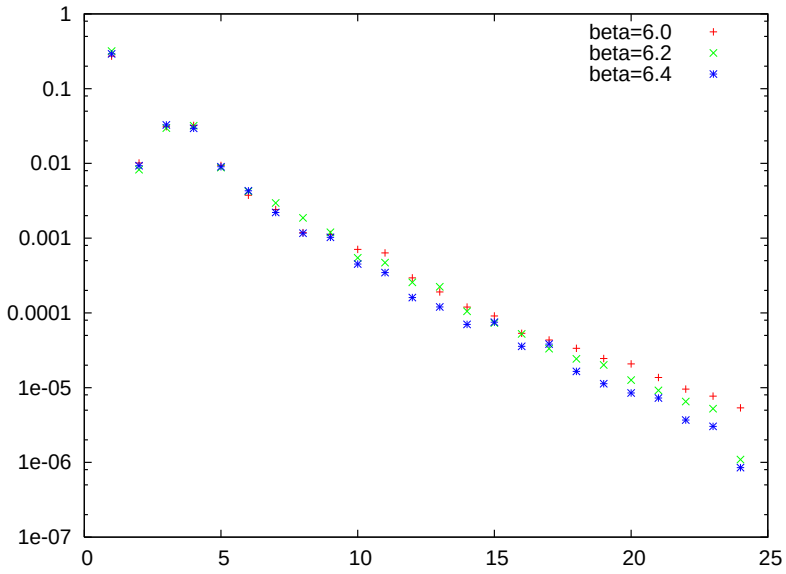
  - more robust ($|m_0|$ can be arbitrarily large)

And reduces $N_f = 4$ to $N_f = 2$ tastes.

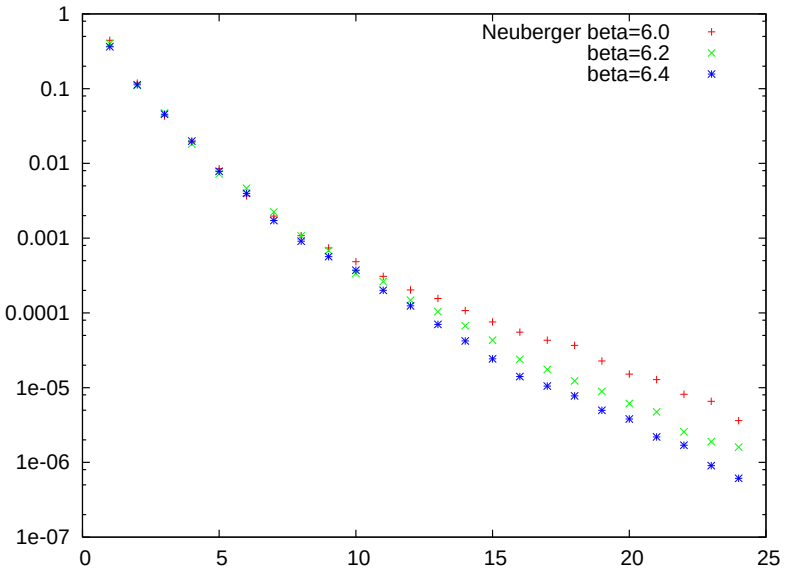
Locality of operator?

$\mathrm{Max}_y |M_{xy}|$ versus $|x-y|$ (Manhattan distance) cf. hep-lat/9808010

Adams



Neuberger



Adams comparable to Neuberger although kernel less local (4-link)

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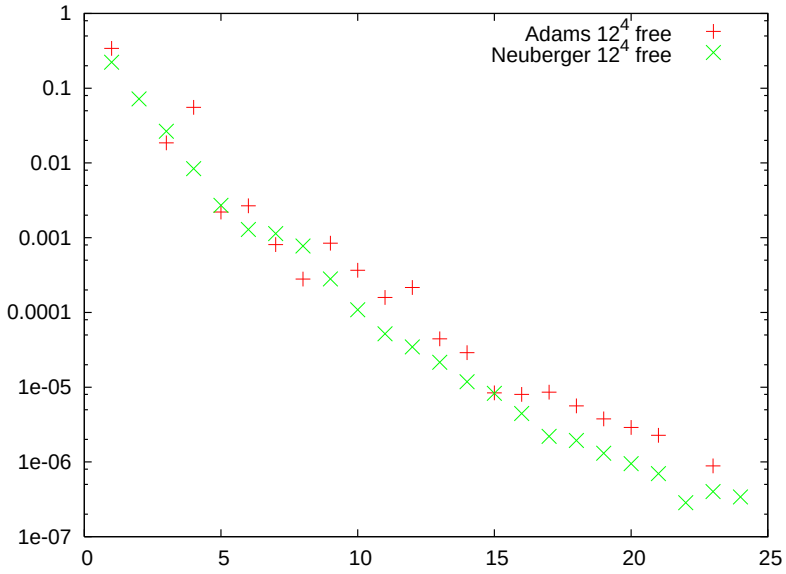
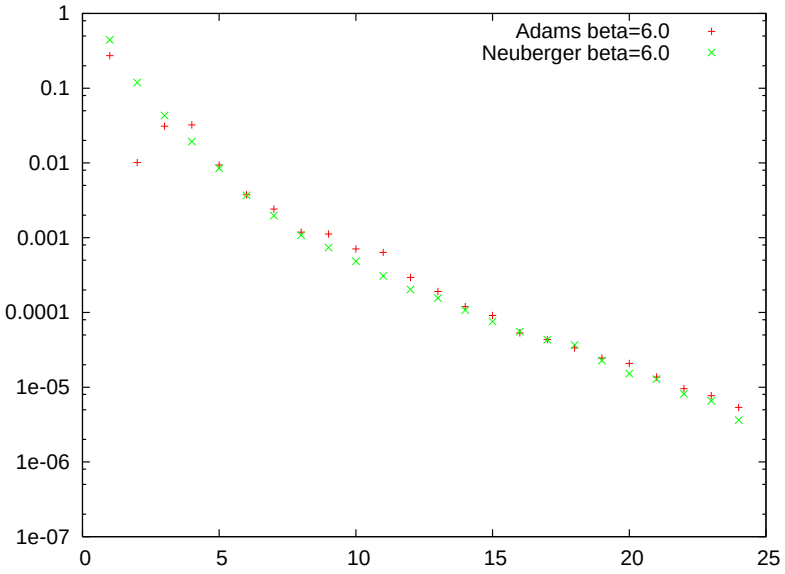
Both, cold



Both, $\beta = 6.0$



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Cost of applying operator

- Multiplication by D: about **2** times faster for Adams (no Dirac indices)
- Sign(H) [using CG, no deflation]:
  - about **8** times faster for Adams on *easy cases*
  - about **2-3** times faster on hard cases

Bag of tricks:

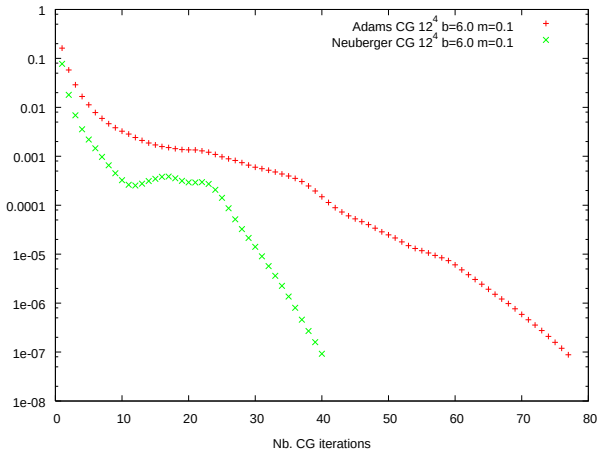
improved operator, link smearing (kinetic and/or mass), deflation, preconditioning, ...

Cost of inversion: compare with Neuberger

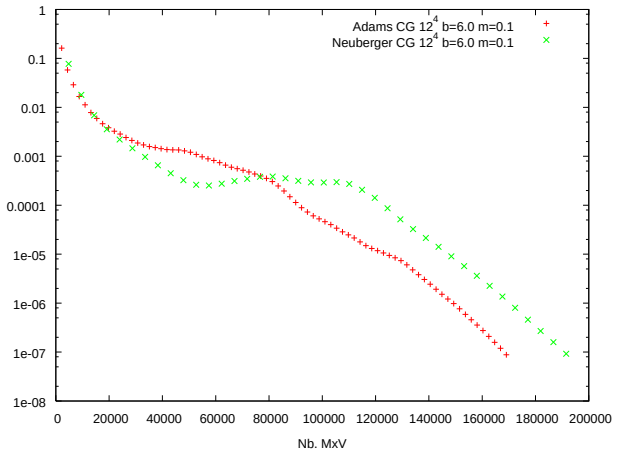
Apples with apples:

- same gauge field ($12^4, \beta = 6.0$)
- same basic algorithm (CG inner, CG outer)

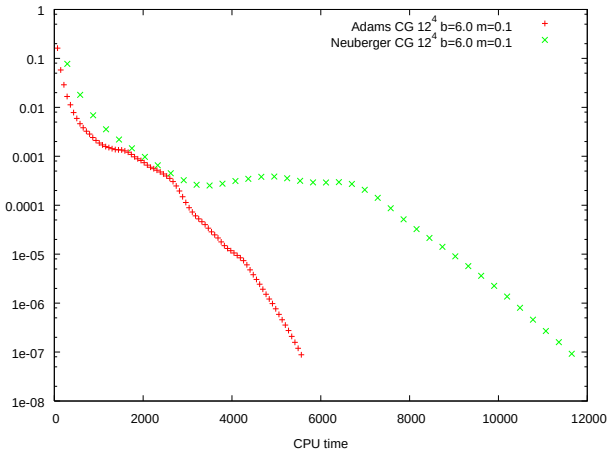
Adams versus Neuberger



Outer CG iter.



MxV

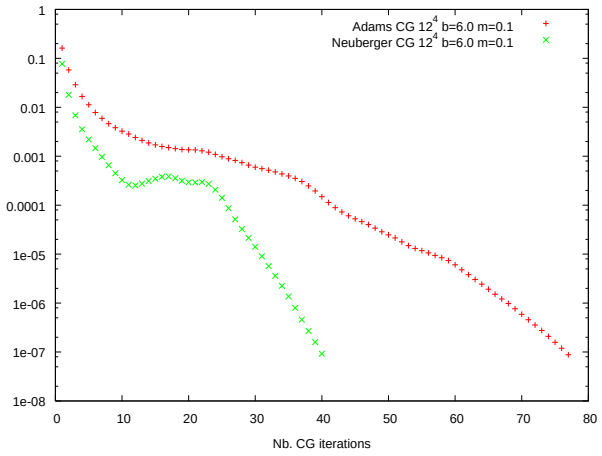


CPU

Net CPU gain: factor 2-3 over Neuberger...

Cost of inversion: compare with Neuberger

Adams versus Neuberger



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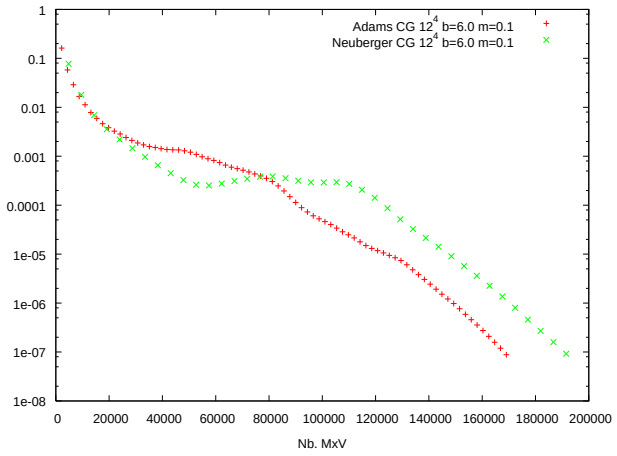
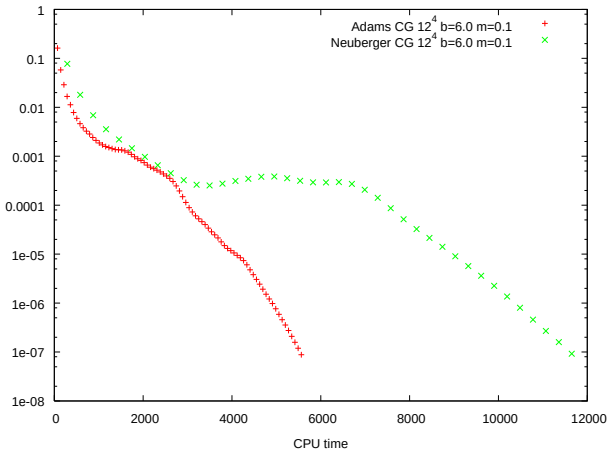


MxV

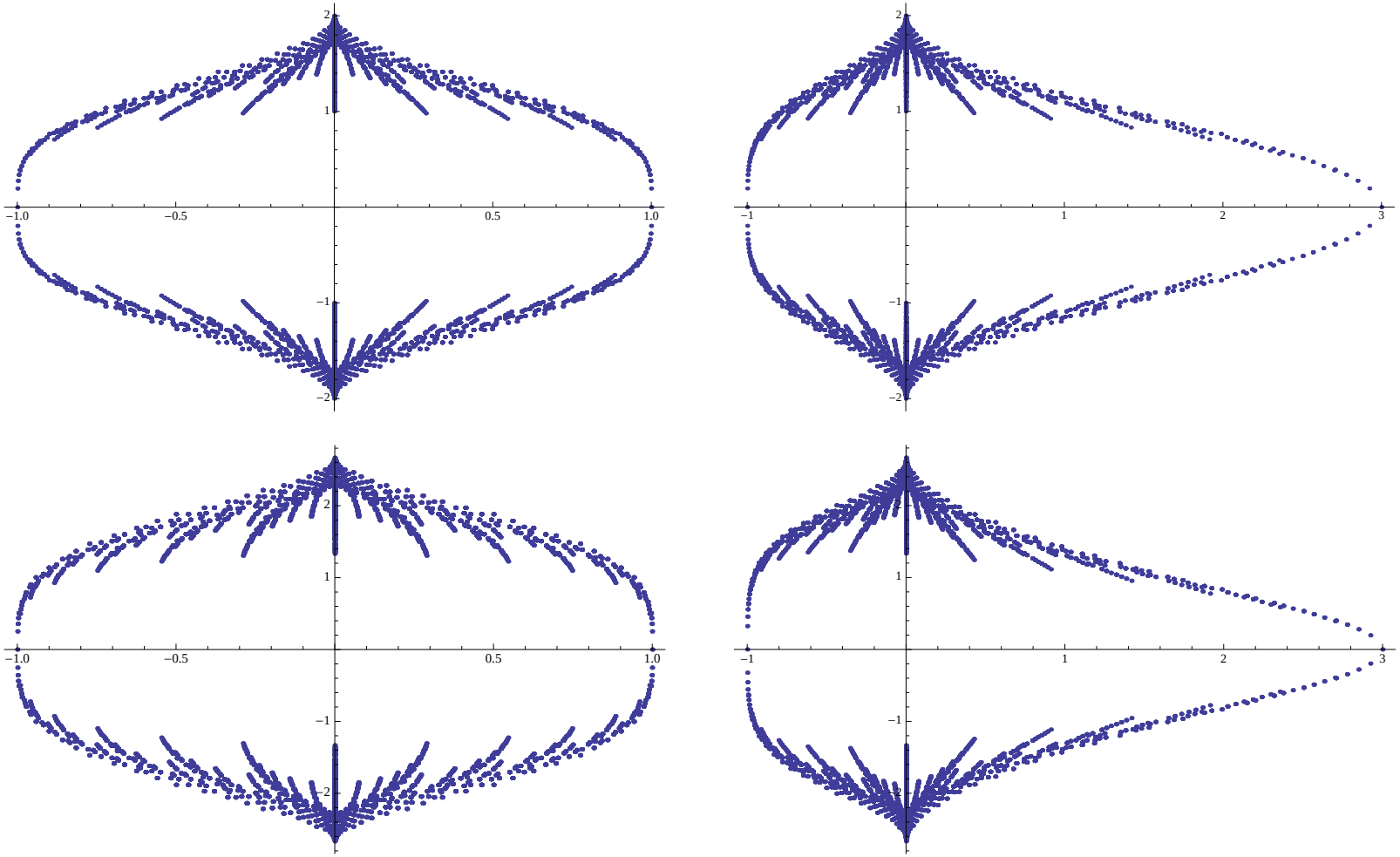


CPU

Free field: now factor 8+ $\rightarrow$ try to keep the free spectrum

How to increase the “hole” in the spectrum?

Strategy I: improve *free* spectrum



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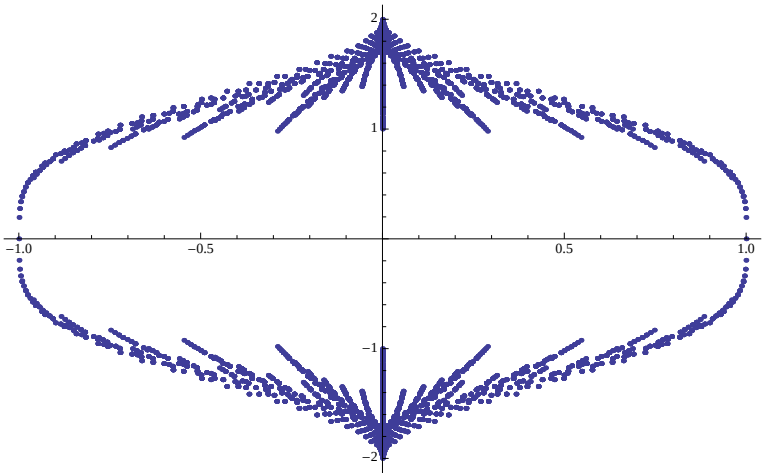

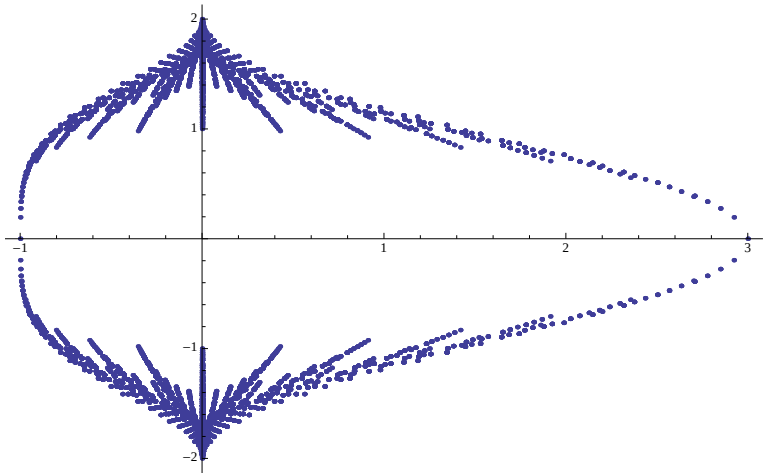

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How to increase the "hole" in the spectrum?

Strategy II: suppress eigenvalues in the gap via the measure

- Extra factor: $\frac{\det(D^\dagger_{Adams} D_{Adams})}{\det(D^\dagger_{Adams} D_{Adams} + (a\hat{m})^2)}$, with $a\hat{m} \sim O(1)$

  Neuberger: Fukaya et al.
- Pros: - can modulate suppression by raising to arbitrary power
  - cheap, efficient and robust
  - advantage over Neuberger larger
- Con: freezes topology

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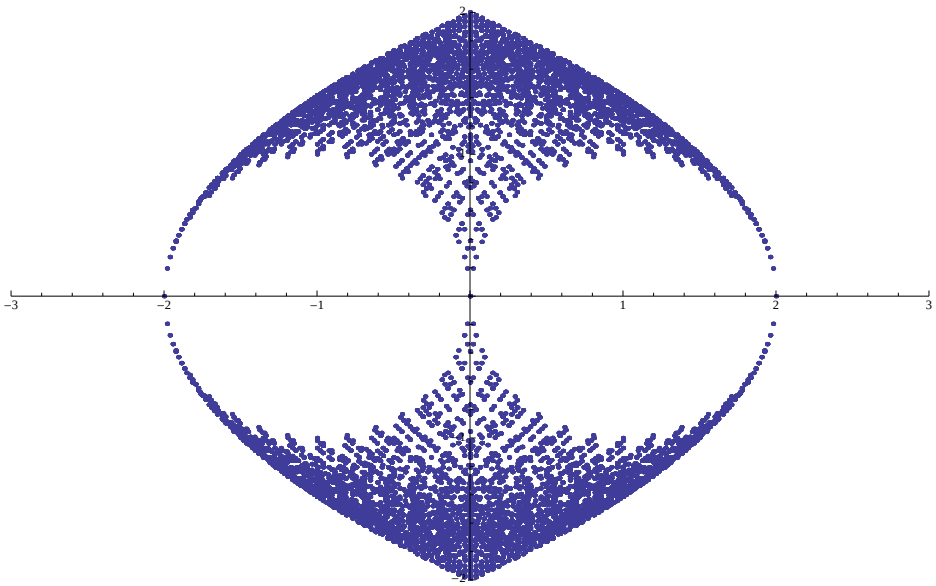
- Variant: $\frac{\det(D^\dagger_{Adams} D_{Adams} + (a\tilde{m})^2)}{\det(D^\dagger_{Adams} D_{Adams} + (a\hat{m})^2)}$, with $a\tilde{m} \ll 1$

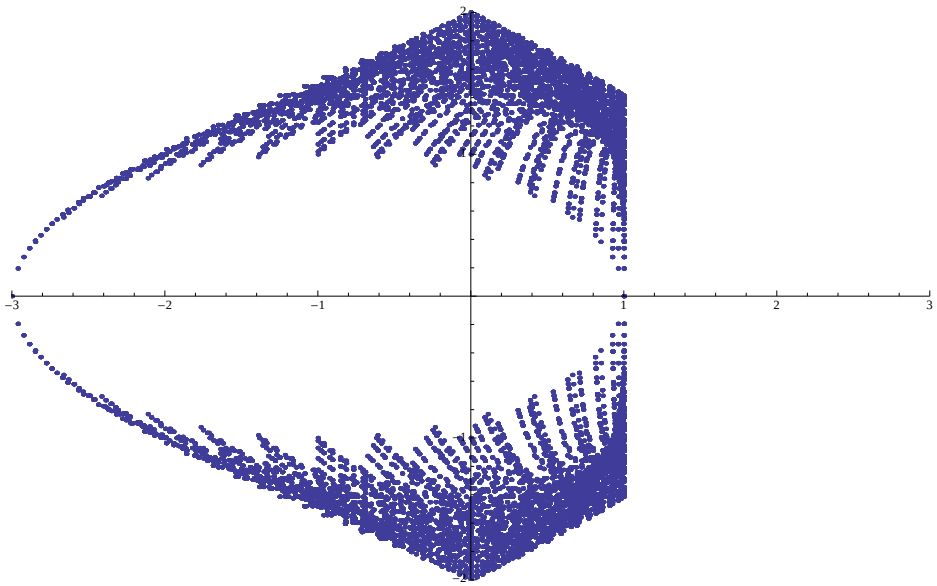
  DWF: N. Christ et al.
- Pros: control topological tunneling via $a\tilde{m}$
- Con: action incl. sign(H_{Adams}) is non-analytic

  $\rightarrow$ must introduce chiral symmetry violation...

Even better? $N_f = 4 \rightarrow N_f = 1$

- Idea: modify mass term to have only ONE branch of free spectrum on the left





Then shift origin to center of the “hole” and apply overlap as before

Conclusions

- Works as advertised: $N_f = 2 \quad \rightarrow \quad$ no more evil rooting!
- Sound approach to chiral & continuum limits

  Compare with Wilson & Wilson-based (Neuberger, Domain-wall)

- How efficient? - cheaper than Neuberger
  - but not dramatically so yet
  - optimization
- Many questions:
  - how to reconstruct $N_f = 2$ Dirac spinors?
  - one massless pion, or three (for $a \neq 0$)?
  - dynamical $N_f = 2$ via $(D^{\dagger}_{N_f=1} D_{N_f=1})$?