Experiences with (solvers for) Boriçi - Creutz fermions

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Novel Lattice Fermions and their Suitability for HPC and Perturbation Theory
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Boriçi - Creutz fermions

*[Michael Creutz JHEP04(2008)017] & [Artan Boriçi Phys. Rev. D **78**, 074504 (2008)]*

- Minimally doubled fermions
  - Preserve exactly the chiral symmetry
  - Strictly local
  - 2 flavours (species of opposite chirality)

- Boriçi – Creutz fermionic action with the free Dirac operator (in the momentum space):

$$D(p) = \sum i\gamma_\mu \sin p_\mu + \sum i\gamma'_\mu \cos p_\mu - 2i\Gamma$$

Two zeros at (0,0,0,0) and $(\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2})$

$$\gamma'_\mu = \sum_\mu \gamma_\mu \Gamma \gamma_\mu$$

$$\Gamma = \frac{1}{2}(\gamma_1 + \gamma_2 + \gamma_3 + \gamma_4) \; and \; \Gamma^2 = 1$$

$$\{\Gamma, \gamma_\mu\} = \{\Gamma, \gamma'_\mu\} = 1.$$

In the 4D space:

$$S_{BC} = \sum_n \Big[\frac{1}{2}\sum_\mu \bar{\psi}_n \gamma_\mu (\psi_{n+\mu} - \psi_{n-\mu}) - \frac{ir}{2}\sum_\mu \bar{\psi}_n (\Gamma - \gamma_\mu)(2\psi_n - \psi_{n+\mu} - \psi_{n-\mu}) + m\bar{\psi}_n\psi_n\Big]$$

Attempts to simulations/calculations

Gauge configurations

$$S_g[U] = -\beta \sum_{\mu\nu i} \frac{1}{3} \mathrm{Re}\, Tr U_{\mu,i} U_{\nu,i+\mu'} U^{-1}_{\mu,i+\nu'} U^{-1}_{\nu,i}$$

$$\sum_{i',\alpha',a'} D^{aa'}_{\alpha\alpha',ii'}[U] S^{a'b}_{\alpha',\beta,i'j}[U] = \delta_{ij}\delta_{\alpha\beta}\delta^{ab}$$

Propagators

$$S^F(y,j,b;x,i,a) = D^{-1}\,{}^{y,j,b}_{x,i,a}$$

Solver?

$$G_{t,t_0} \simeq \frac{1}{2} c_1 \cosh am_1(t - t_0 - L/2) + \frac{1}{2} c_2 \cosh am_2(t - t_0 - L/2)$$

Effective masses

$$\frac{G_{t+1,1}}{G_t} = \frac{\cosh am(t - L/2)}{\cosh am(t - 1 - L/2)}, t = 1, \ldots\ldots L$$

$$\frac{G_{t+1,1}}{G_t} = \frac{\sinh am(t - L/2)}{\sinh am(t - 1 - L/2)}, t = 1, \ldots\ldots L$$

Possible solvers

Algorithm CGNE

Require: *Bounded linear operator* $T : \mathcal{X} \longrightarrow \mathcal{Y}$.

Require: *Perturbed right-hand side* y^δ.

$x_0^\delta := 0$

$d_0 := y^\delta$

$p_1 := s_0 := T^* d_0$

for $k = 1, 2, \ldots,$ *unless* $s_{k-1} = 0$ ***do***

    $q_k := T p_k$

    $\alpha_k := \|s_{k-1}\|_{\mathcal{X}}^2 / \|q_k\|_{\mathcal{Y}}^2$

    $x_k^\delta := x_{k-1}^\delta + \alpha_k p_k$

    $d_k := d_{k-1} - \alpha_k q_k$

    $s_k := T^* d_k$

    $\beta_k := \|s_k\|_{\mathcal{X}}^2 / \|s_{k-1}\|_{\mathcal{X}}^2$

    $p_{k+1} := s_k + \beta_k p_k$

Possible solvers

Algorithm ¯ Standard BiCGStab

function BICGSTAB(A, b, x_0)
  $r_0 := b - Ax_0$; $p_0 := r_0$
  for $i = 0, 1, 2, \ldots$ **do**
    computation $s_i := Ap_i$
    begin reduction (r_0, s_i) **end reduction**
    $\alpha_i := (r_0, r_i) / (r_0, s_i)$
    $q_i := r_i - \alpha_i s_i$
    computation $y_i := Aq_i$
    begin reduction (q_i, y_i); (y_i, y_i) **end reduction**
    $\omega_i := (q_i, y_i) / (y_i, y_i)$
    $x_{i+1} := x_i + \alpha_i p_i + \omega_i q_i$
    $r_{i+1} := q_i - \omega_i y_i$
    begin reduction (r_0, r_{i+1}) **end reduction**
    $\beta_i := (\alpha_i / \omega_i) (r_0, r_{i+1}) / (r_0, r_i)$
    $p_{i+1} := r_{i+1} + \beta_i (p_i - \omega_i s_i)$
  end for
end function

Possible solvers/Convergence

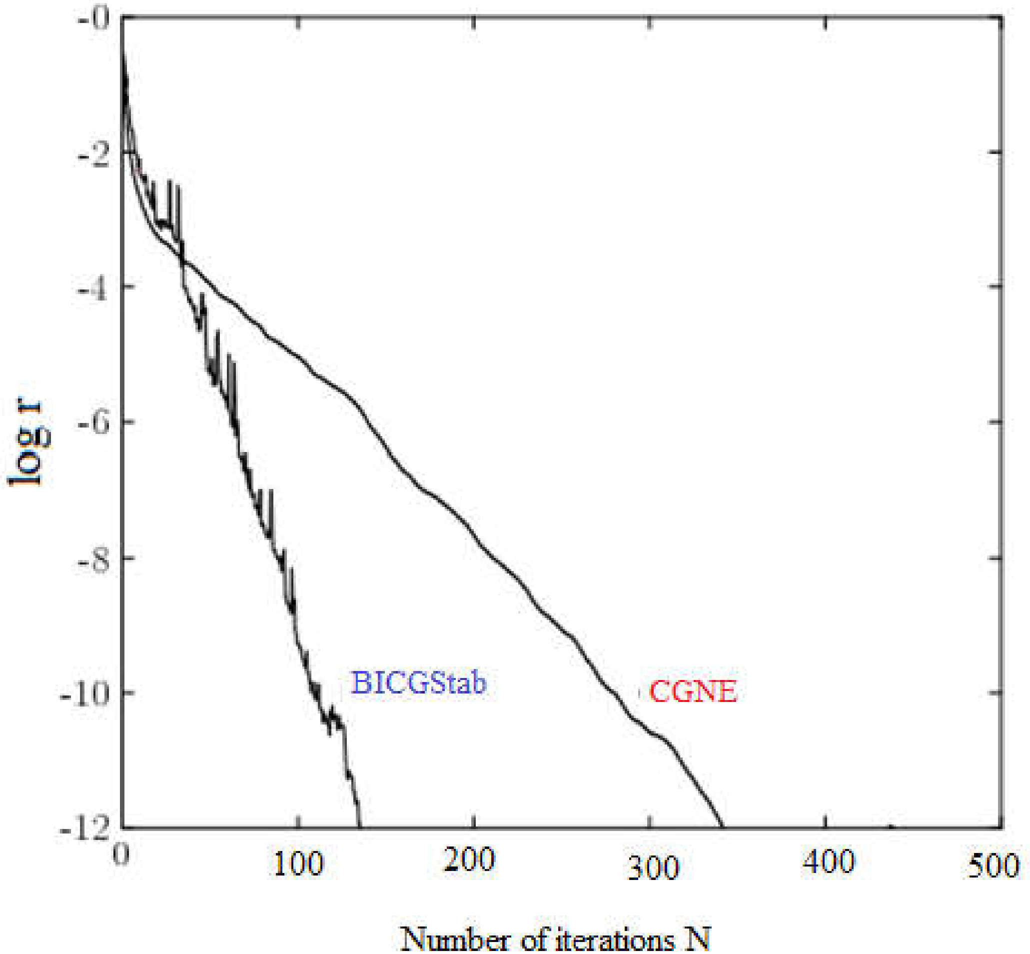


Fig 1. $\log_{10} r$ as function of the number of iteration steps.

Preferable BICGStab

Attempts to simulations/calculations

- Calculation for the pion mass from two different directions

Details of the simulations:

- Lattices 16^4, 28^4 dhe 32^4
- Quenched approximation
- Wilson gauge action
- Boriçi – Creutz action
- **BICGstab** inverter
- Five different quark masses

$$u_x = \frac{1}{4}\sum_p e^{ipx}\left(\sum_{\mu=1}^{4}(1+i\frac{e^{ip_\mu a}-e^{-ip_\mu a}}{2})\psi(p)\right), \text{ prej nga:}$$

$$u_x = \frac{1}{4}\left[4\psi(\mathrm{x})+\frac{i}{2}\sum_{\mu=1}^{4}(U_{x,\mu}\psi(\mathrm{x}+\mathrm{a}\,\mu)-U^{\dagger}{}_{x-\mathrm{a}\mu,\mu}\psi(\mathrm{x}-\mathrm{a}\,\mu)\right] = A\psi(x)$$

$$d_x = \frac{\Gamma}{4}\sum_p e^{ipx}\left(\sum_{\mu=1}^{4}(1-i\frac{e^{ip_\mu a}+e^{-ip_\mu a}}{2})\psi(p+\frac{\pi}{2})\right), \text{prej nga:}$$

$$d_x = \frac{\Gamma}{4}\sum_{p'} e^{ip'x}e^{-i\frac{\pi}{2}x}\left(\sum_{\mu=1}^{4}(1-i\frac{e^{ip'_\mu a}-e^{-ip'_\mu a}}{2})\psi(p')\right), \text{ ose}$$

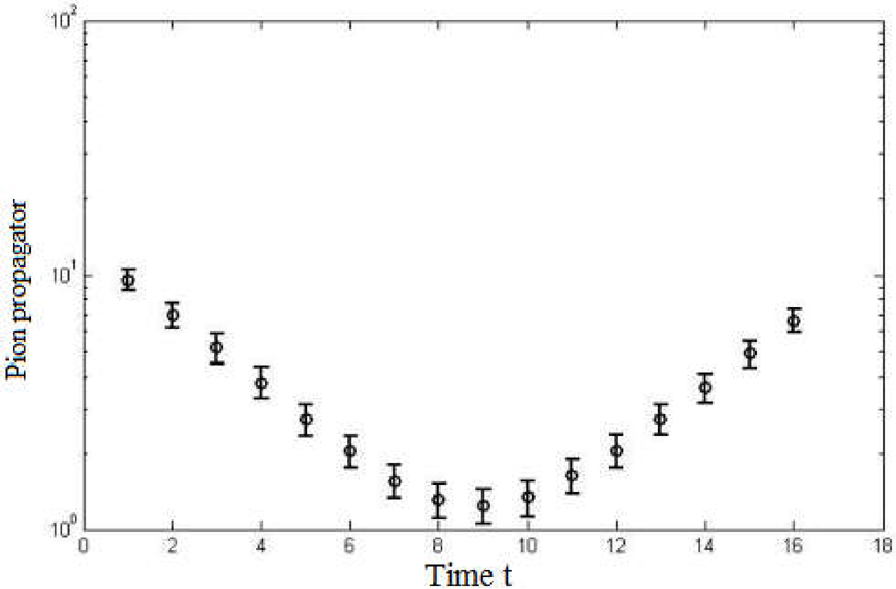
$$d_x = \frac{\Gamma}{4}e^{-i\frac{\pi}{2}(x_1+x_2+x_3+x_4)}\left[4\psi(\mathrm{x})+\frac{i}{2}\sum_{\mu=1}^{4}(U_{x,\mu}\psi(\mathrm{x}+\mathrm{a}\,\mu)-U^{\dagger}{}_{x-\mathrm{a}\mu,\mu}\psi(\mathrm{x}-\mathrm{a}\,\mu)\right]$$

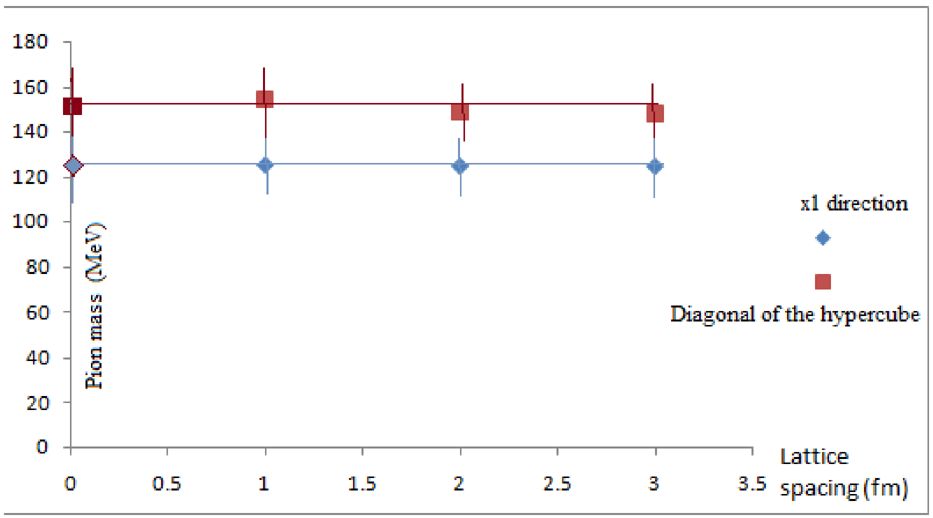
$$d_x = \Gamma e^{-i\frac{\pi}{2}(x_1+x_2+x_3+x_4)}u_x$$

$$\pi_0(x) = \overline{\psi(x)}\gamma_5\tau_3\psi(x) = \overline{\mathrm{u}(x)}\gamma_5\,\mathrm{u}(x) - \overline{\mathrm{d}(x)}\gamma_5\,\mathrm{d}(x)$$

$$\pi_0(x) = \overline{\mathrm{u}(x)}\gamma_5\,\mathrm{u}(x) - \overline{\mathrm{u}(x)}e^{i\frac{\pi}{2}(x_1+x_2+x_3+x_4)}\Gamma\gamma_5\Gamma e^{-i\frac{\pi}{2}(x_1+x_2+x_3+x_4)}u_x = 2\overline{\mathrm{u}(x)}\gamma_5\,\mathrm{u}(x)$$

Naïve evaluation of the broken hypercubic symmetry mass





$$m_{\pi^+(diag)} = 147.54 \pm 5.1 MeV$$

$$m_{\pi^+(x_1)} = 124.59 \pm 5.2 MeV$$

Restoration of hypercubic symmetry of Boriçi – Creutz fermions

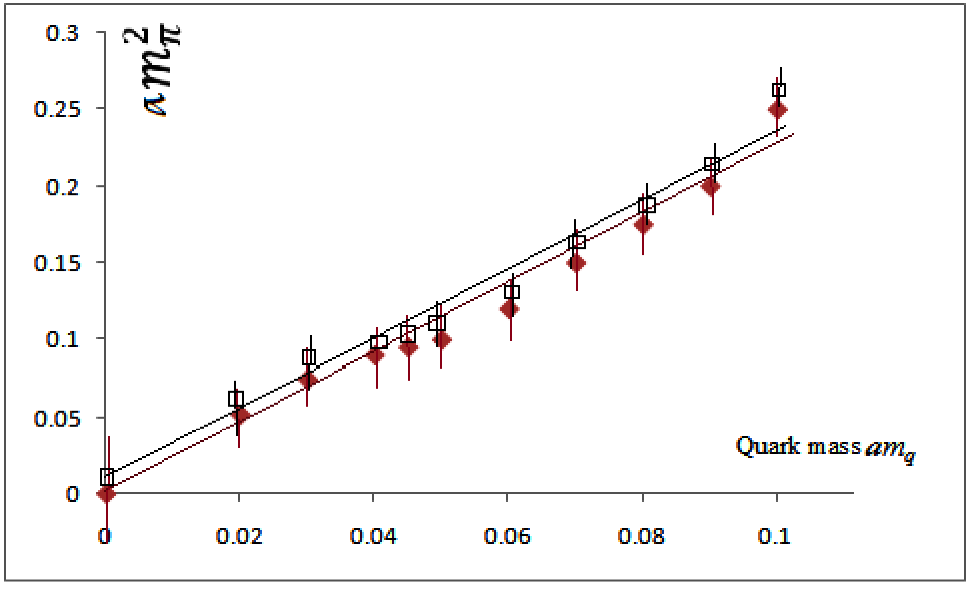
In the 4D space, Boriçi – Creutz action, with the dimension three counteterm added is written as below:

$$\begin{aligned} S_{BC} \;=\; & \sum_n \Big[\frac{1}{2}\sum_\mu \bar{\psi}_n\gamma_\mu(\psi_{n+\mu}-\psi_{n-\mu}) \\ & -\frac{ir}{2}\sum_\mu \bar{\psi}_n(\Gamma-\gamma_\mu)(2\psi_n-\psi_{n+\mu}-\psi_{n-\mu}) \\ & +ic_3\bar{\psi}_n\Gamma\psi_n + m\bar{\psi}_n\psi_n\Big] \end{aligned}$$

How to find non perturbatively the proper counterterms?

Osmanaj talk, Thursday, 9.30

Partially modified BC action



Pion masses in two different directions, in the case of the modified BC action.

Summary

The BiCGStab and the CGNE algorithm are adopted to invert the quark matrix, using BC action.

At small quark masses, both the algorithms perfectly converge, but the CGNE algorithm is quite time-consuming.

BiCGStab turns out to be superior to conjugate gradient on normal equations in the region of small quark masses, which makes it preferable as a solver (not really for KW fermions???)

BC fermions not really adaptable for light hadrons spectroscopy, because point splitting has to be applied and the interpolators(consequently the lattice correlators) have to be written using that method.

In order to have a full restoration, in the quenched approximation, dimension 4 counterterm should be consider and evaluated in further studies.

Thank you!