

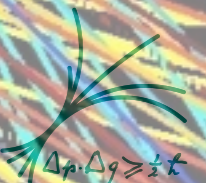
# Towards full NLO for Higgs pair production in gluon fusion

*Gudrun Heinrich*

*Max Planck Institute for Physics,  
Munich*

MITP workshop HPPC

Johannes Gutenberg Universität Mainz, April 29, 2015

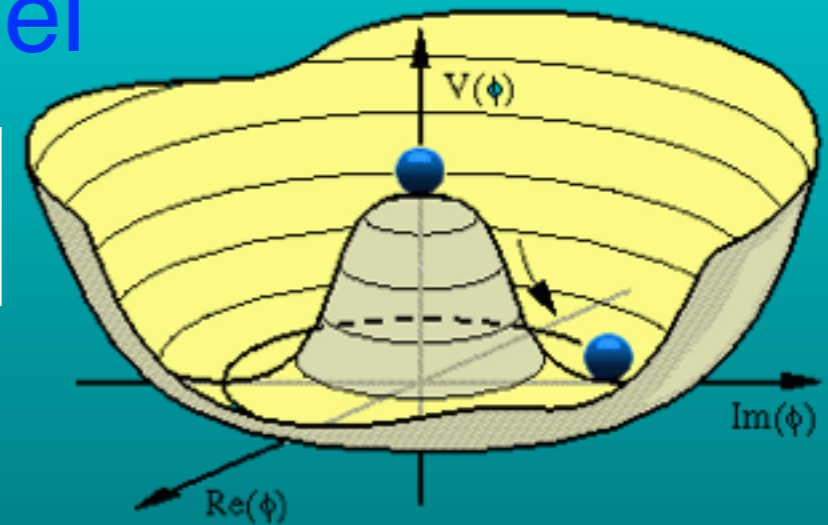


# Motivation

we need to find out whether the mechanism of EW symmetry breaking is the one predicted by the Standard Model

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) , \quad V(\phi) = \frac{1}{2} \mu^2 \phi^2 + \frac{1}{4} \lambda \phi^4$$

$$\phi \rightarrow v + \sigma$$



$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - (-\mu^2) \sigma^2 - \sqrt{-\mu^2 \lambda} \sigma^3 - \frac{\lambda}{4} \sigma^4 + \text{const.}$$

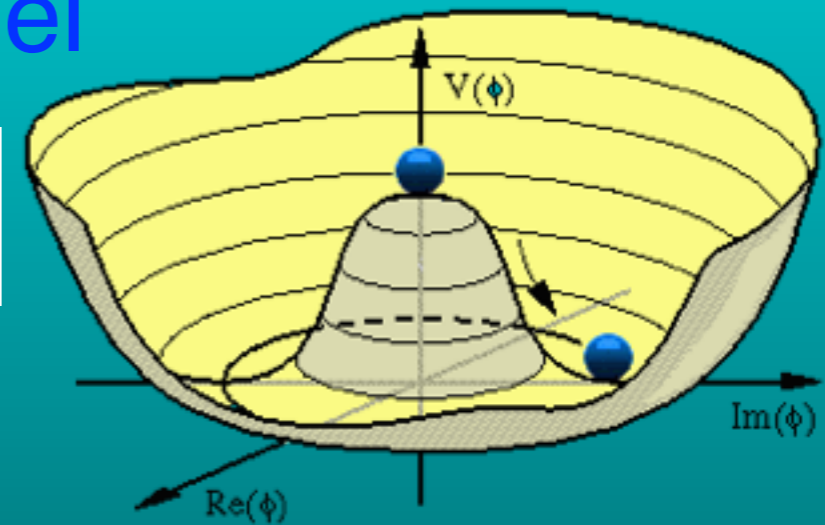


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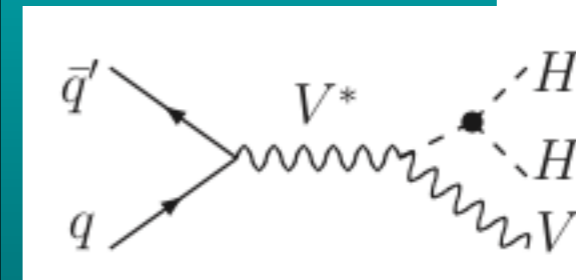
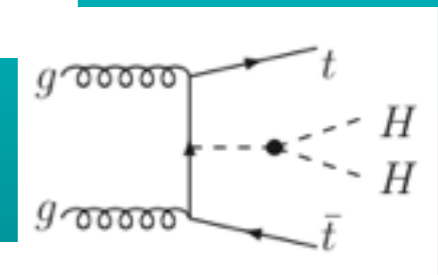
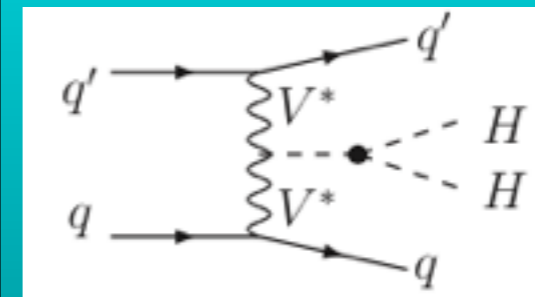
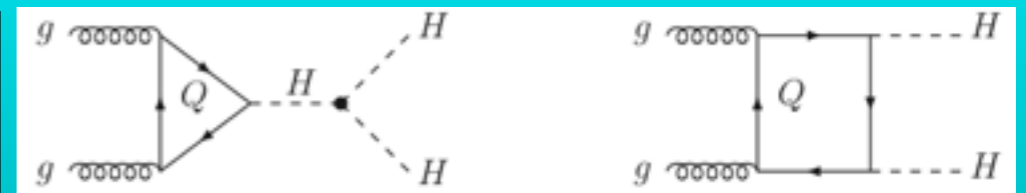
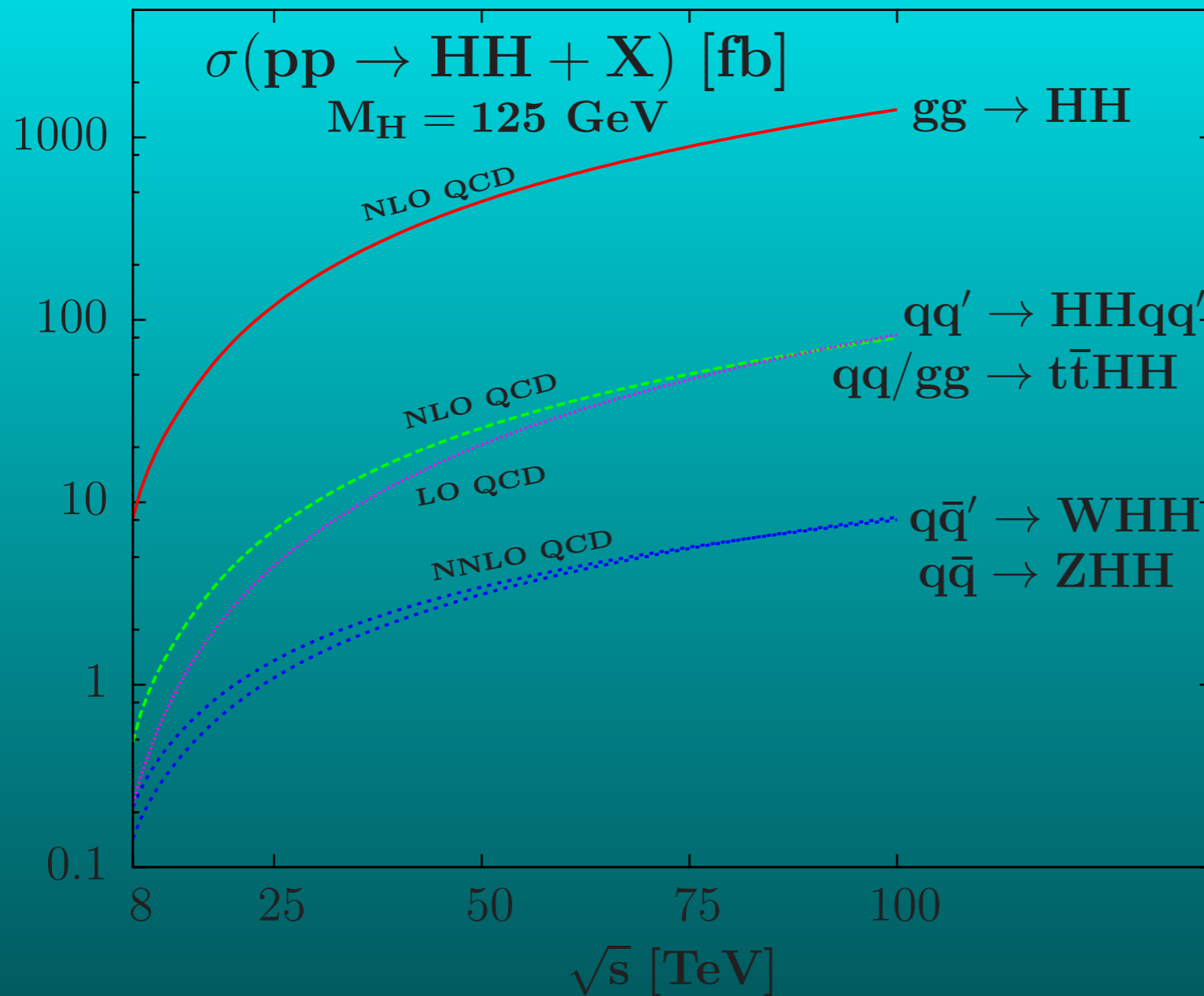


$$\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - (-\mu^2) \sigma^2 - \sqrt{-\mu^2 \lambda} \sigma^3 - \frac{\lambda}{4} \sigma^4 + \text{const.}$$

so far we know nothing about  $\lambda$



# Higgs pair production channels



J. Baglio, A. Djouadi, R. Gröber, M. Mühlleitner,  
 J. Quevillon, M. Spira '12

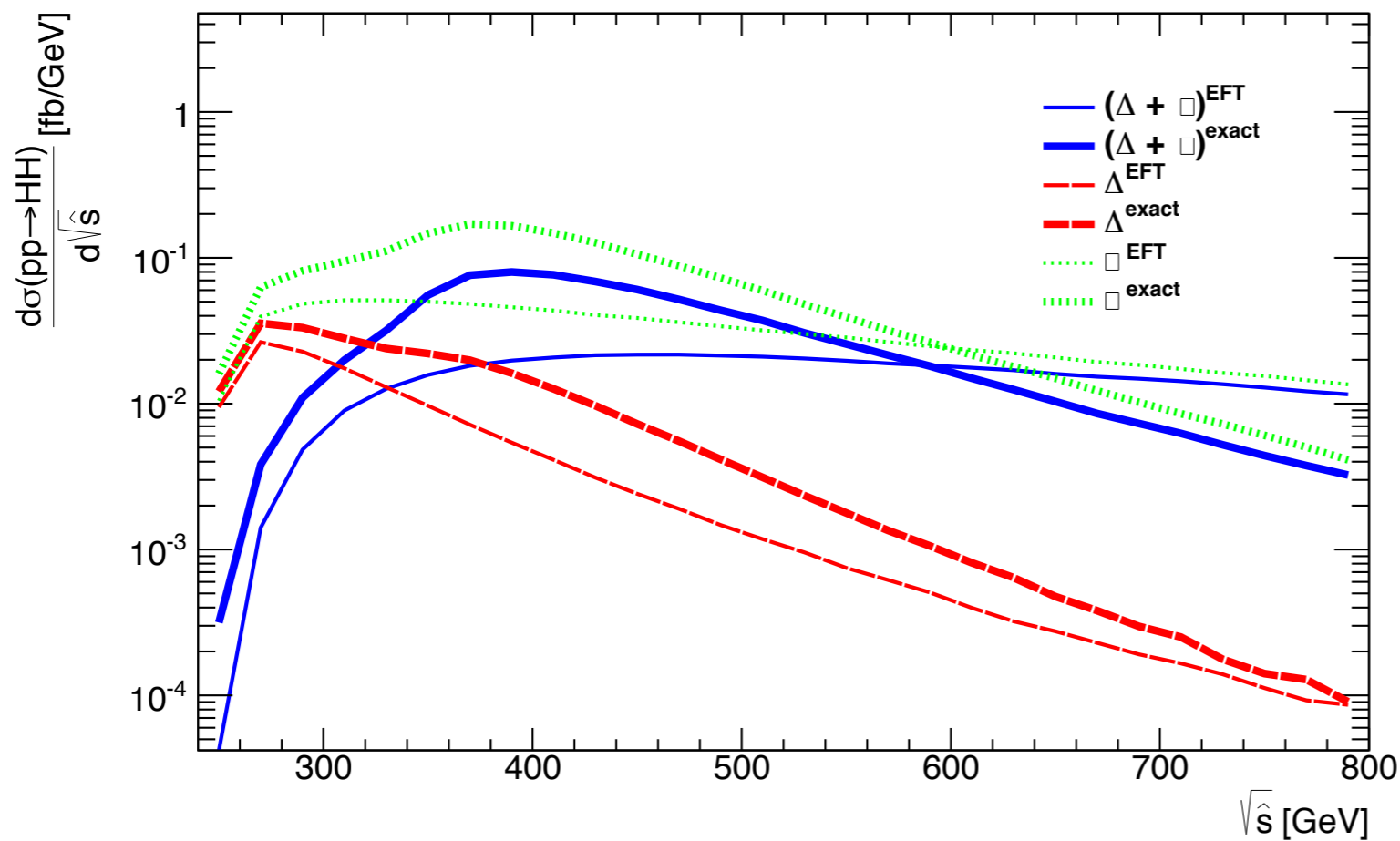
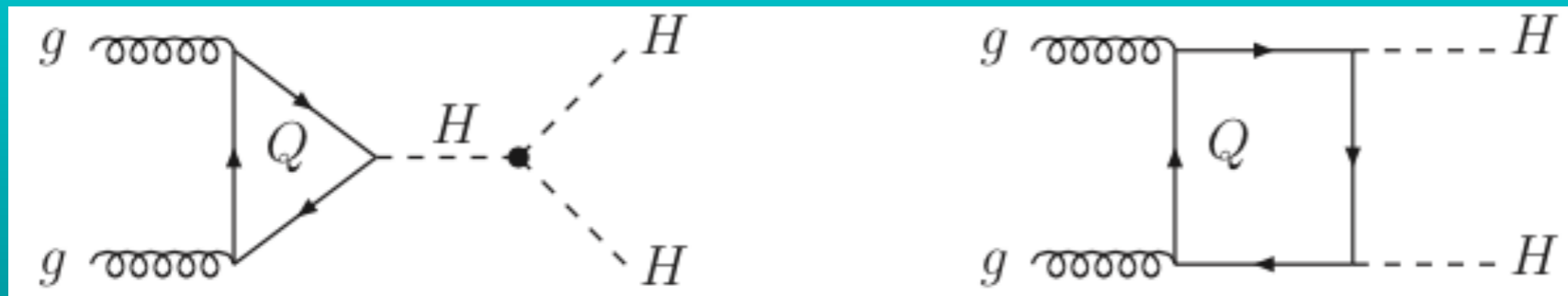
**problem:**  $\sigma_{ggHH} \sim 10^{-3} \sigma_{ggH}$



# sensitivity to Higgs self coupling

$$gg \rightarrow HH$$

two types of contributions: “triangle ( $\lambda$ )” and “box”

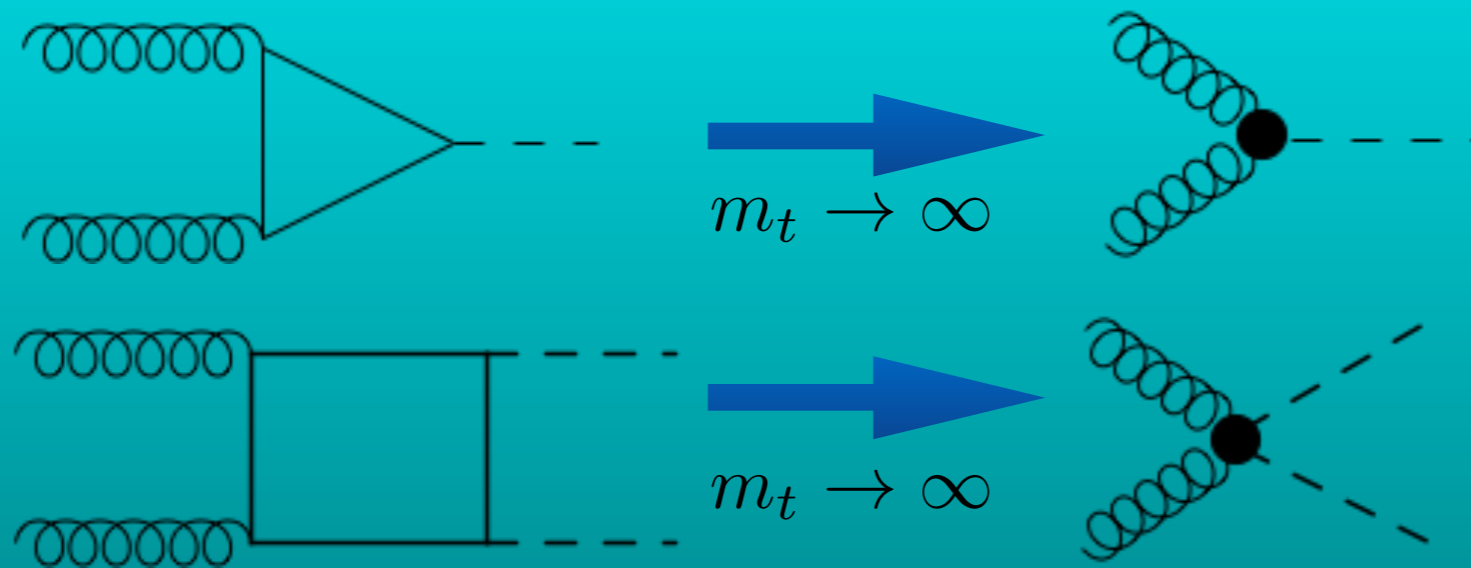


Slawinska,  
van den Wollenberg,  
van Eijk, Bentvelsen '14

(LO)



# heavy top limit (EFT)



strictly only valid for  $\sqrt{\hat{s}} \ll 2m_t$

$\sqrt{\hat{s}} > 2m_H$  for  $gg \rightarrow HH$

→  $m_t \rightarrow \infty$  limit poor beyond threshold

→ need full  $m_t$  dependence → need 2-loop box integrals with  $m_t, m_H$



# current status

**LO (1 loop):** Glover, van der Bij '88 (full heavy quark mass dependence)

**NLO in  $m_t \rightarrow \infty$  limit (EFT):** Plehn, Spira, Zerwas '96; Dawson, Dittmaier, Spira '98 (HPAIR)

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**full mass dependence in NLO  
real radiation part  
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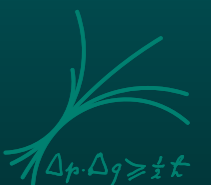
Frederix, Hirschi, Mattelaer, Maltoni, Torrielli, Vryonidou, Zaro '14;  
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**study of BSM effects** Dawson, Furlan, Lewis '12; Goertz, Papaefstathiou, Yang, Zurita '14;  
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**soft gluon resummation NNLL in  $m_t \rightarrow \infty$  limit:** Shao, Li, Li, Wang '13

**+ lots of phenomenological studies**

Baglio, Barr, Dolan, Englert, Ferreira de Lima, Goncalves-Netto, Greiner,  
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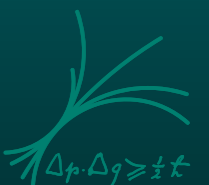
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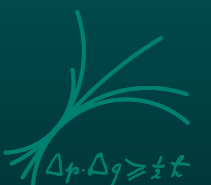
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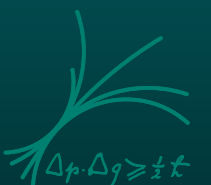
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# terminology

## Born improved HEFT (Higgs Effective Field Theory) at NLO QCD

$$d\sigma_{NLO}^{virt} \approx \frac{d\sigma_{NLO}^{virt}(m_t \rightarrow \infty)}{d\sigma_{LO}^{virt}(m_t \rightarrow \infty)} d\sigma_{LO}^{virt}(m_t) =: d\sigma_{NLO}^{virt, HEFT}$$

HPAIR (Dawson, Dittmaier, Spira)

$$d\sigma_{NLO}^{real} = d\sigma_{NLO}^{real}(m_t \rightarrow \infty)$$

improvements towards “more mass dependence”

$$d\sigma_{NLO}^{virt, HEFT}$$

$$d\sigma_{NLO}^{real}(m_t)$$

Maltoni, Vryonidou, Zaro

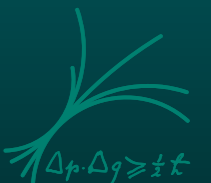
-10%

$$d\sigma_{NLO}^{virt}(m_t) = \sum_n c_n(x) \left( \frac{m_H^2}{m_t^2} \right)^n \quad x = 4m_H^2/s, \quad n_{max} = 6$$

$$d\sigma_{NLO}^{real}(m_t \rightarrow \infty)$$

Grigo, Hoff, Melnikov, Steinhauser

+10%



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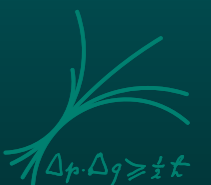
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Grigo, Hoff, Melnikov, Steinhauser

+10%

full mass dependence needed!



# tools development

## GoSam

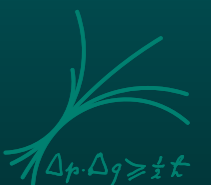
N. Greiner, GH, G. Luisoni, S. Jones, M. Kerner, G. Ossola, P. Mastrolia,  
J. Schlenk, J.F. von Soden-Fraunhofen, T. Zirke

## SecDec

S. Borowka, GH, S. Jones, M. Kerner, J. Schlenk, A. Stoyanov, T. Zirke

## new analytic techniques

P. Mastrolia, T. Peraro, U. Schubert, S. di Vita, V. Yundin



# build on GoSam (one loop)

arXiv:1404.7096

Cullen, Greiner, GH, Luisoni, Mastrolia, Mirabella, Ossola, Peraro, Schlenk, van Deurzen, von Soden-Fraunhofen, Tramontano

program available at

<http://gosam.hepforge.org>

very simple usage

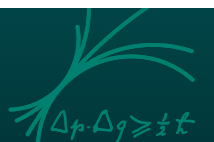
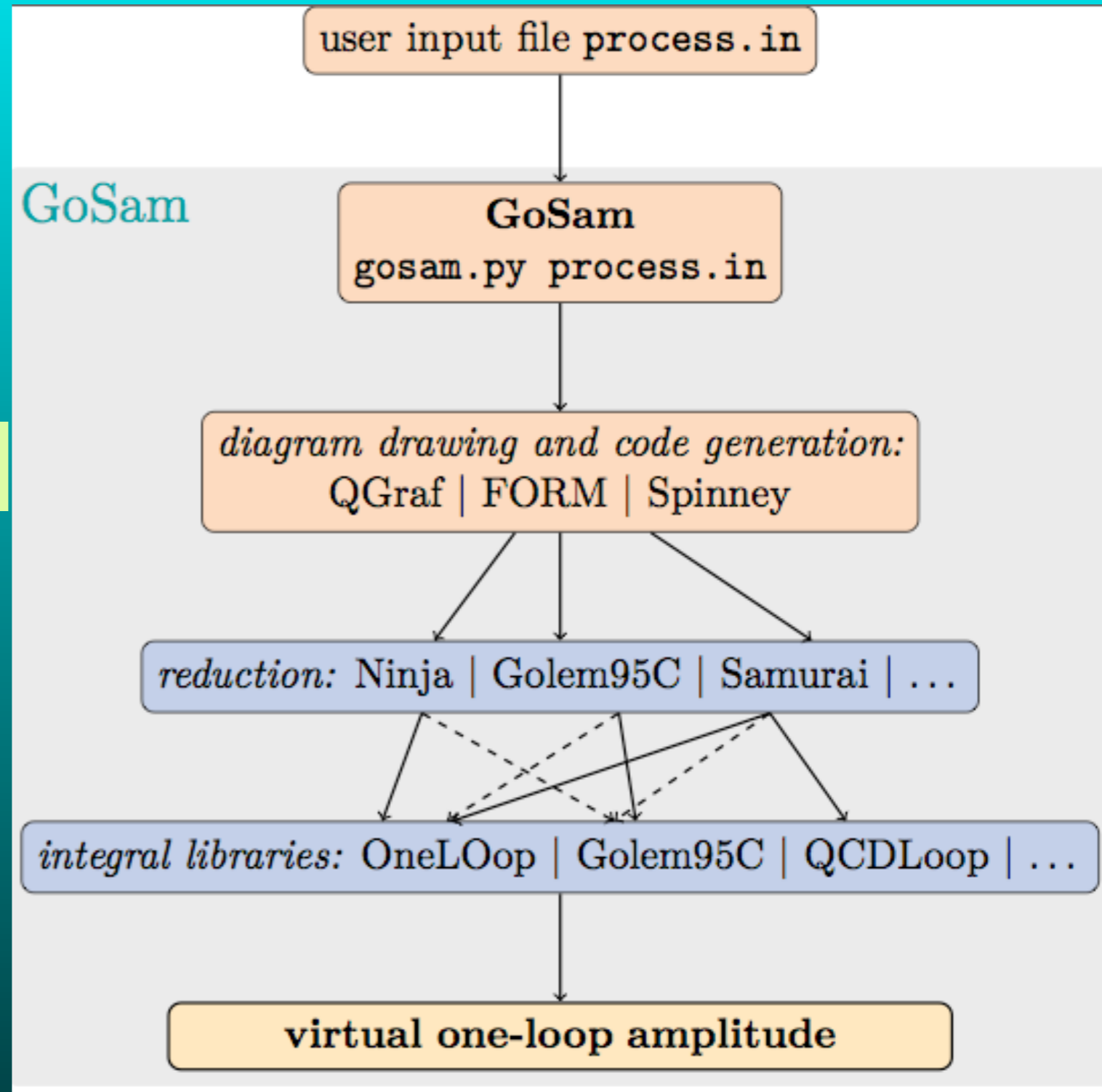
example input file for

$$e^+ e^- \rightarrow t \bar{t}$$

```
process_path=eett
in=      e+, e-
out=     t, t~
order=   gs, 0, 2
```

↑  
LO

↑  
NLO



# GoSam-2Loop

Greiner, GH, Jones, Kerner, Luisoni, Mastrolia, Schlenk, Zirke

generic extension of GoSam, not limited to  $gg \rightarrow hh$

use projectors on tensor structures  
rather than helicity amplitudes

`process.in` for

$gg \rightarrow hh$

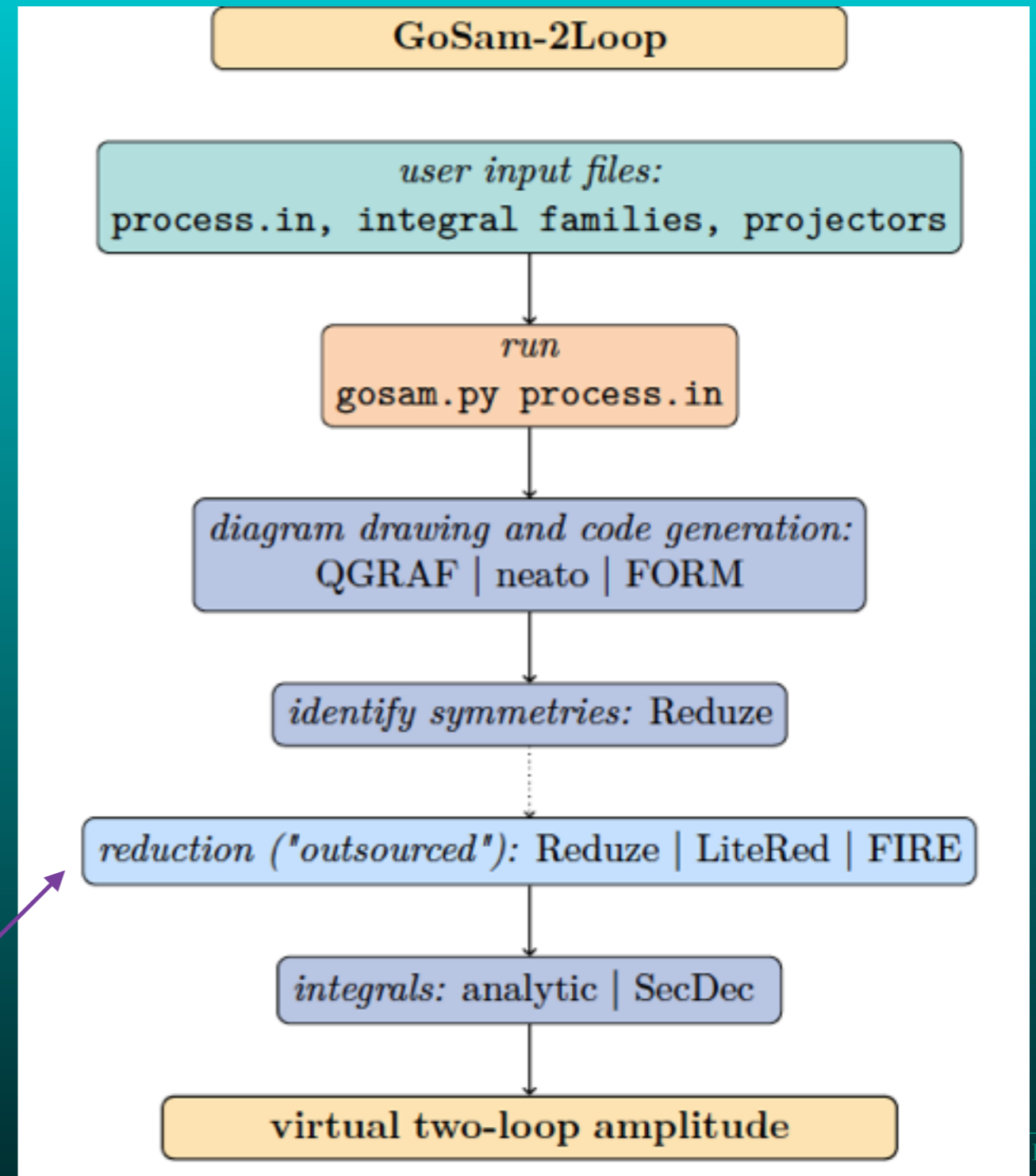
`in=g,g`  
`out=h,h`  
`model=smdiag`  
`order=QCD,none,2,4`

tree

1-loop

2-loop

Manteuffel, Studerus;  
R.N.Lee; A.V.Smirnov



# example $gg \rightarrow hh$

amplitude structure: (independent of #loops)

$$\mathcal{M} = \epsilon_\mu(p_1, n_1) \epsilon_\nu(p_2, n_2) \mathcal{M}^{\mu\nu}$$

$$\mathcal{M}^{\mu\nu} = A_1(s, t, m_H^2, m_t^2, D) T_1^{\mu\nu} + A_2(s, t, m_H^2, m_t^2, D) T_2^{\mu\nu}$$

choose tensor decomposition such that

$$\begin{aligned} \mathcal{M}^{++} &= \mathcal{M}^{--} = -A_1 \\ \mathcal{M}^{+-} &= \mathcal{M}^{-+} = -A_2 \end{aligned}$$

diagrams with trilinear couplings enter only here

$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\nu p_2^\mu}{p_1 \cdot p_2}$$

$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2 (p_1 \cdot p_2)} \left\{ m_H^2 p_1^\nu p_2^\mu - 2 (p_1 \cdot p_3) p_3^\nu p_2^\mu - 2 (p_2 \cdot p_3) p_3^\nu p_1^\mu + 2 (p_1 \cdot p_2) p_3^\nu p_3^\mu \right\}$$

$$p_T^2 = (ut - m_H^4)/s$$



Glover, van der Bij '88





# form factors/reduction

construct projectors  $P_j^{\mu\nu}$  such that

$$\begin{aligned} P_1^{\mu\nu} \mathcal{M}_{\mu\nu} &= A_1(s, t, m_H^2, m_t^2, D) \\ P_2^{\mu\nu} \mathcal{M}_{\mu\nu} &= A_2(s, t, m_H^2, m_t^2, D) \end{aligned}$$

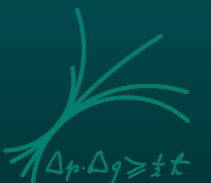
current status:

projectors as input to GoSam-2L

algebra done automatically by GoSam-2L (FORM)

interface to Reduze to identify integral symmetries

reduction: interface to Reduze, LiteRed, FIRE

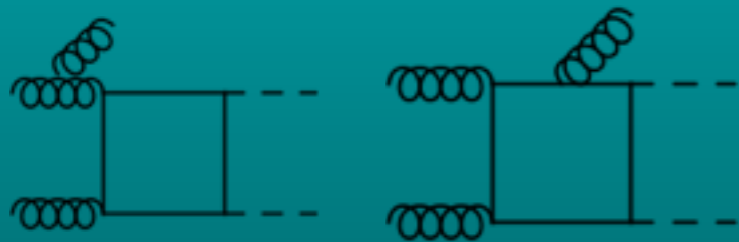


# building blocks

loop diagrams:

	# diagrams
tree	0
1-loop	8
2-loop	122

real radiation:



$$gg \rightarrow hh g$$

$$gq \rightarrow hh q$$

$$g\bar{q} \rightarrow hh \bar{q}$$

$$q\bar{q} \rightarrow hh g$$

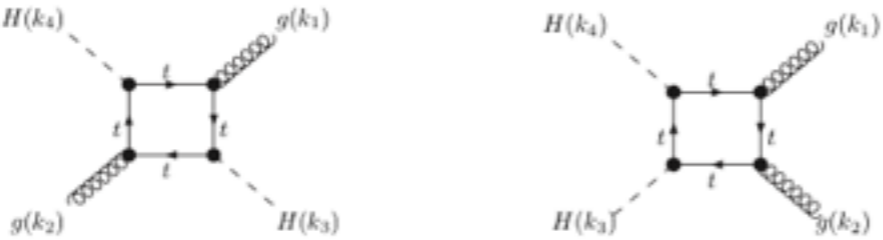
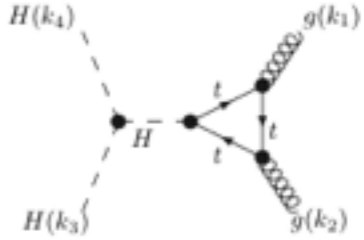
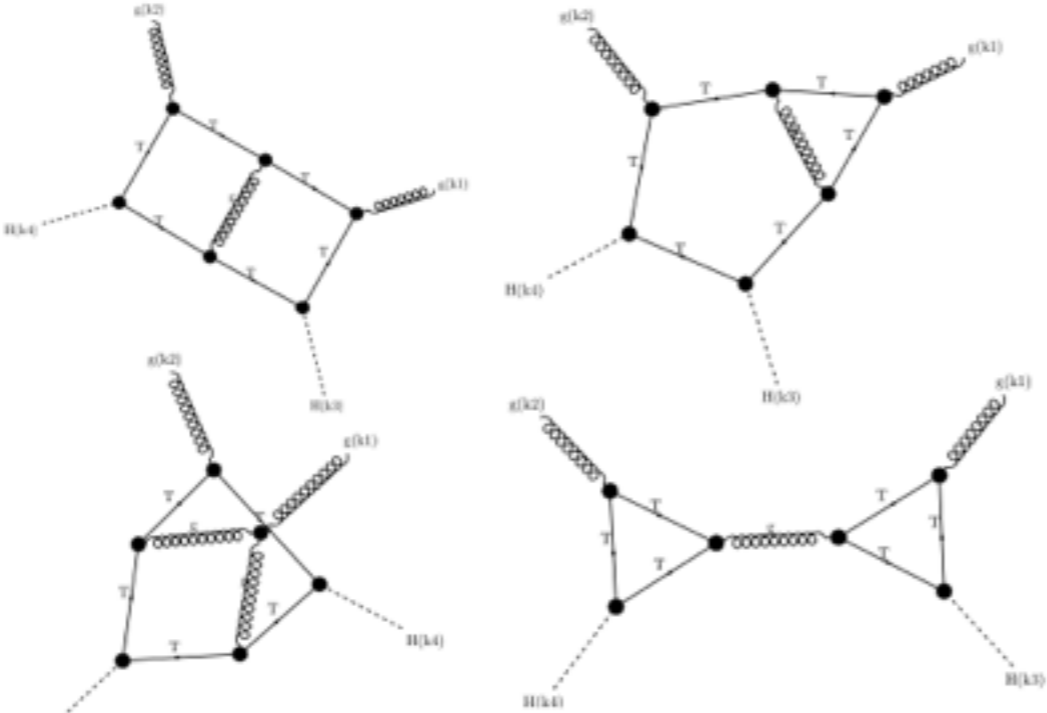
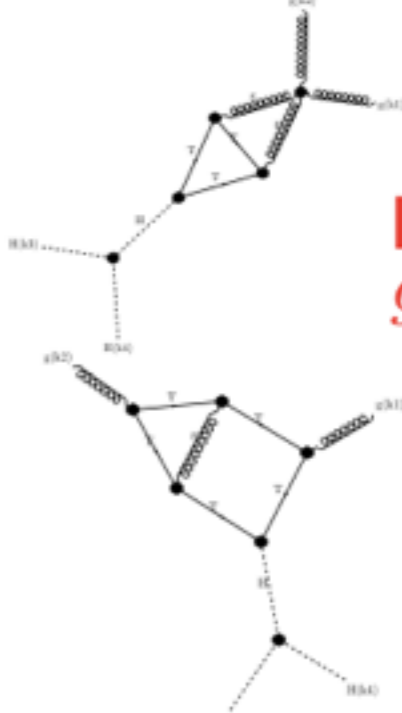
	# diagrams
doubly unresolved (tree)	0
singly unresolved (1-loop)	$54 + 3 \times 8$

no NNLO-type subtraction needed!  
use GoSam (1-loop) + dipole subtraction



# some loop diagrams (drawn by GoSam-2L)

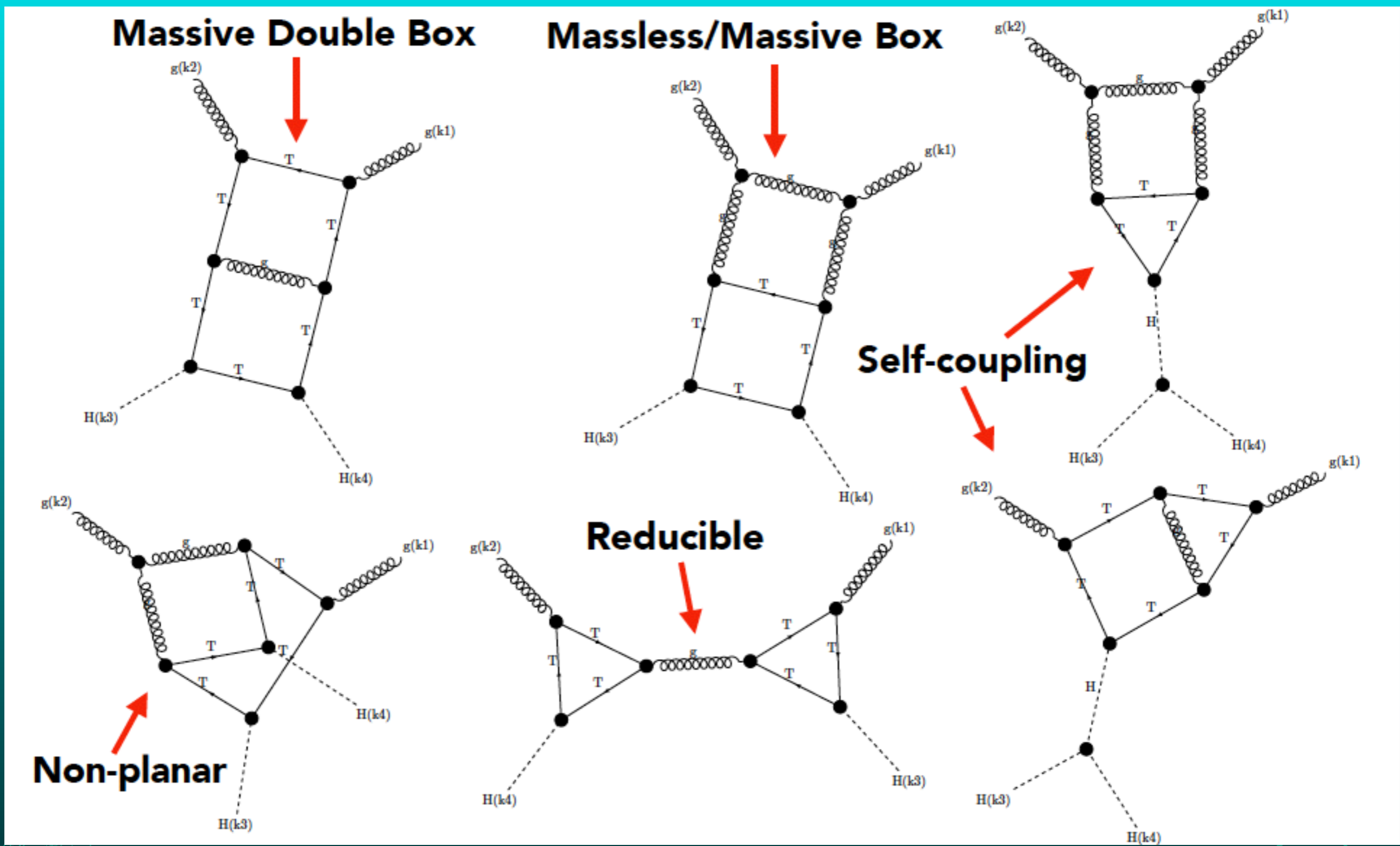
## Boxes & Triangles

	Yukawa only ( $\leq 4$ -point)	Self-coupling (3-point)
LO	 <p>6 Diagrams</p>	 <p>2 Diagrams</p>
NLO	 <p>101 Diagrams</p>	 <p><b>Known</b> <math>gg \rightarrow H</math></p> <p>21 Diagrams</p>

thanks: Stephen Jones



# some types of 2-loop diagrams



thanks: Stephen Jones



# 2-loop integrals

- maximal number of propagators in diagrams obtained from Feynman rules:  $N=7$
- maximal number of scalar products in numerator:  $S=9$

$$S = \frac{l(l+1)}{2} + lm$$

$m$ : number of (independent) external momenta  
 $l$ : number of loops

- define **integral family**: add propagators such that all numerators can be expressed in terms of inverse propagators

➔ input for **Reduze/FIRE5/LiteRed**

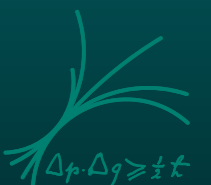
**master integrals:**

**triangle type:** partly known analytically

Bonciani, Mastrolia, Remiddi; Harlander et al; Anastasiou, Beerli et al

**box type:** scales  $s, t, (u), m_H, m_t$  ➔ **SecDec**

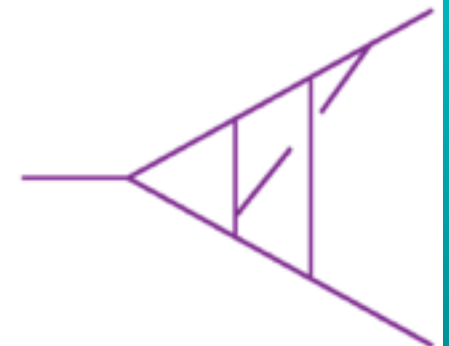
note: **SecDec** can also do integrals with numerators



# SecDec

[http://secdec/hepforge.org](http://secdec.hepforge.org)

- Home
- Subversion
- Tracker
- Wiki



## SecDec

Sophia Borowka, Gudrun Heinrich, Stephen Jones, Matthias Kerner, Johannes Schlenk, Tom Zirke

**A program to evaluate dimensionally regulated parameter integrals numerically**

[home](#)   [download program](#)   [user manual](#)   [faq](#)   [changelog](#)

**NEW:** Version 3.0 of the program can be downloaded as [SecDec-3.0.5.tar.gz](#).

**version 3.0:** arXiv:1502.06595

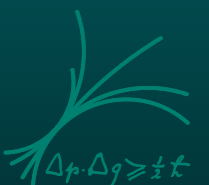
S.Borowka, GH, S.Jones, M.Kerner, J.Schlenk, T.Zirke



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## **new features** (version 3.0):

- implementation of two new decompositions strategies based on a geometric algorithm (guaranteed to stop!)
- improved user interface
- linear propagators can be treated
- option to use numerical integrators from Mathematica (and Cquad)
- propagators with negative indices are possible
- usage on a cluster facilitated



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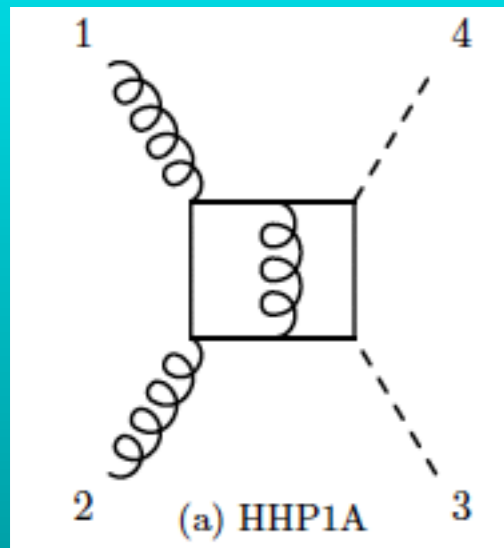
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efficient tool, make it part of GoSam-2Loop



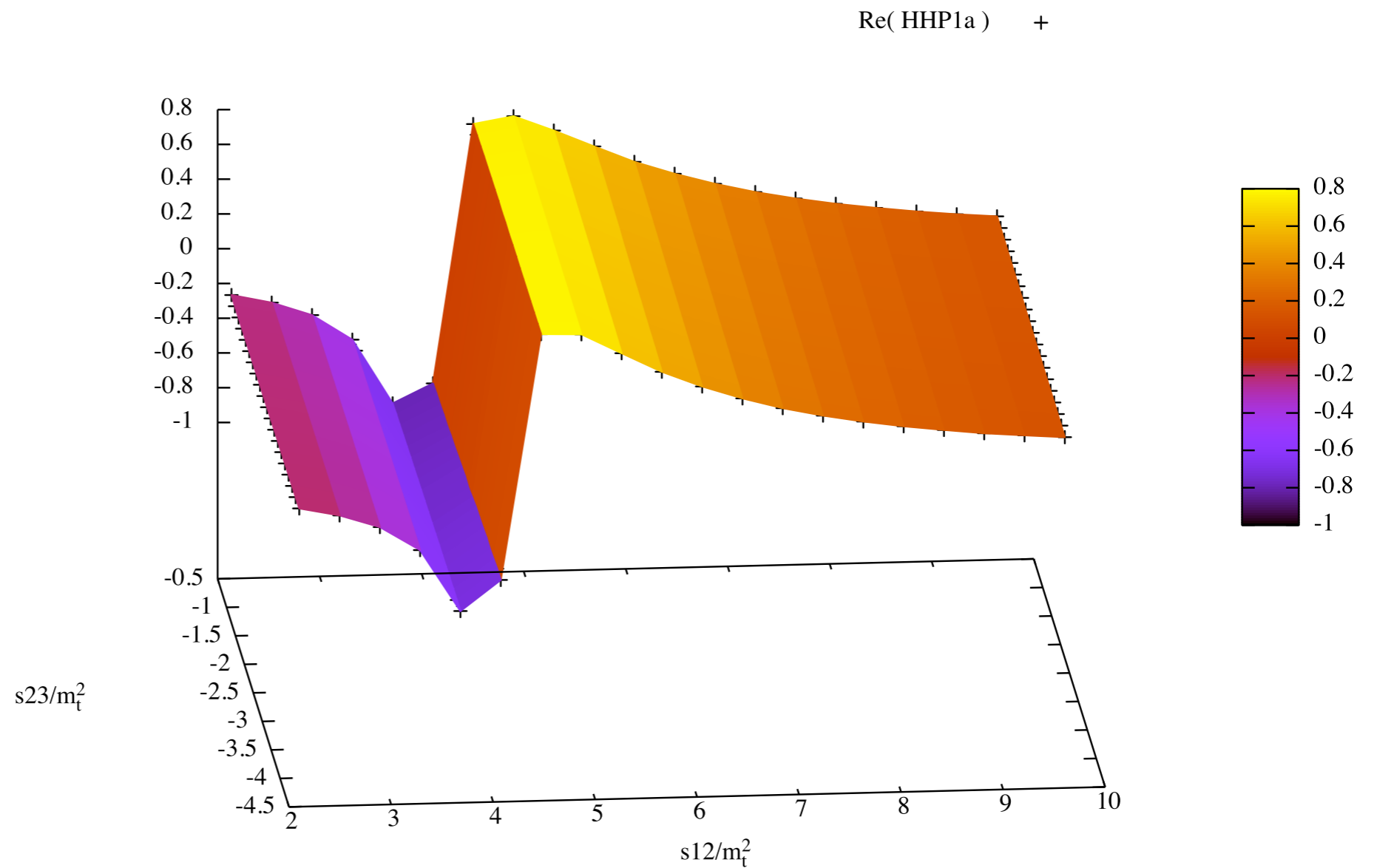


# master integrals: examples

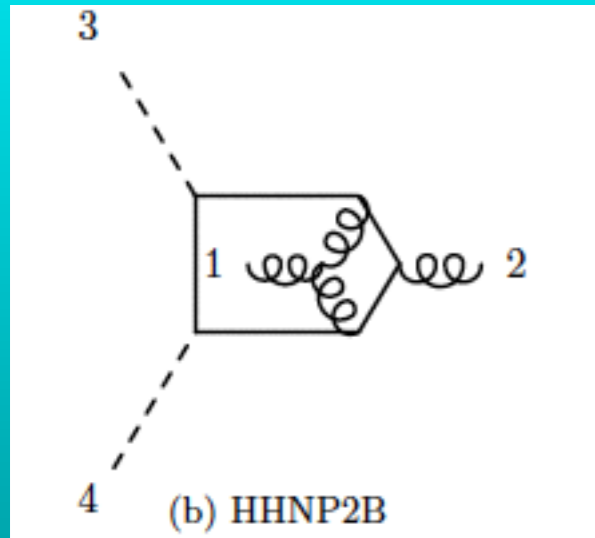


$$m_H = 125 \text{ GeV}$$

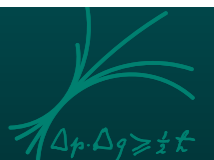
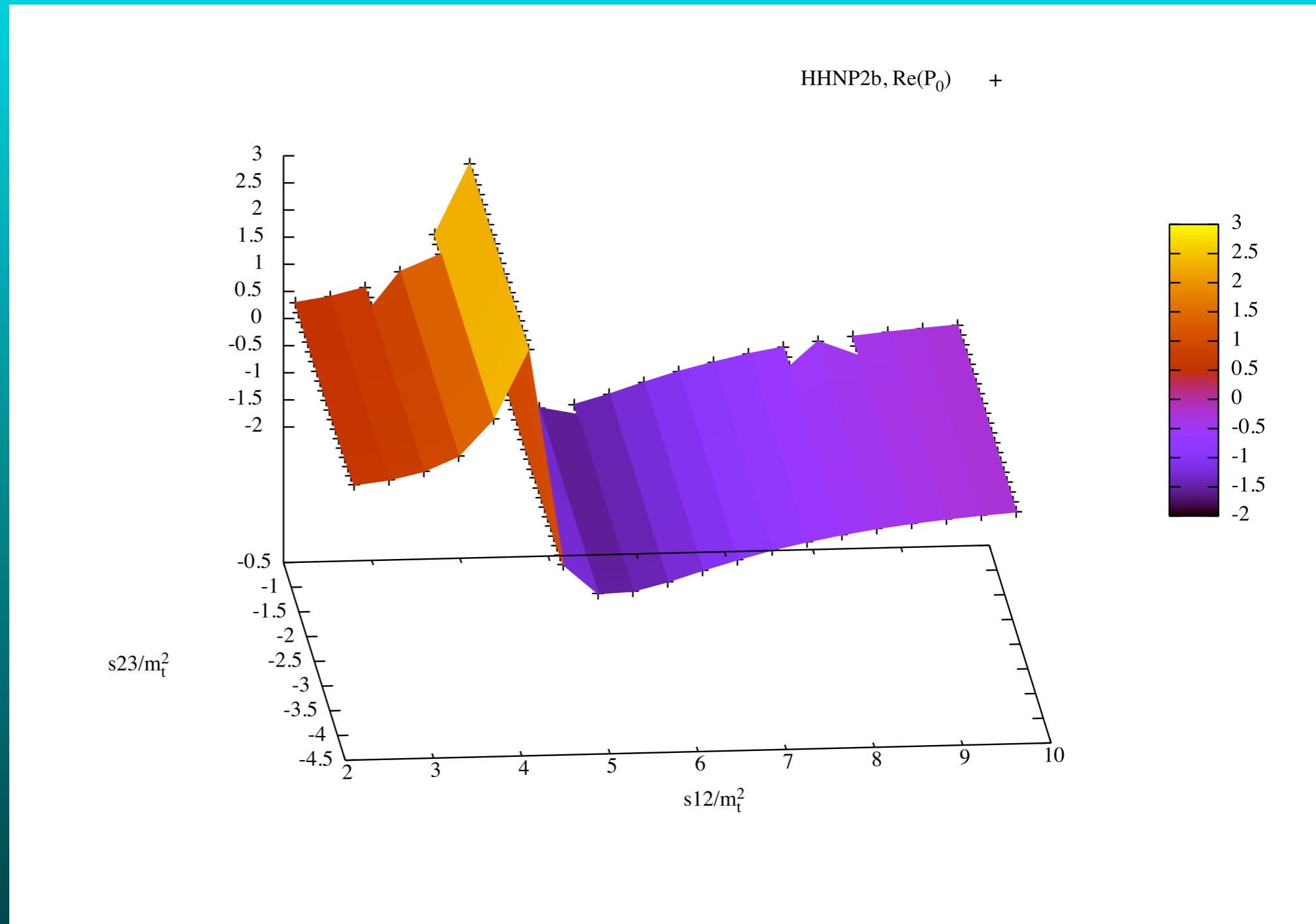
$$m_t = 173 \text{ GeV}$$



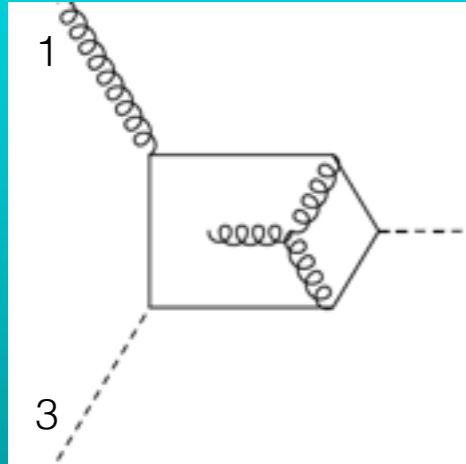
# master integrals: examples



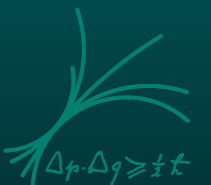
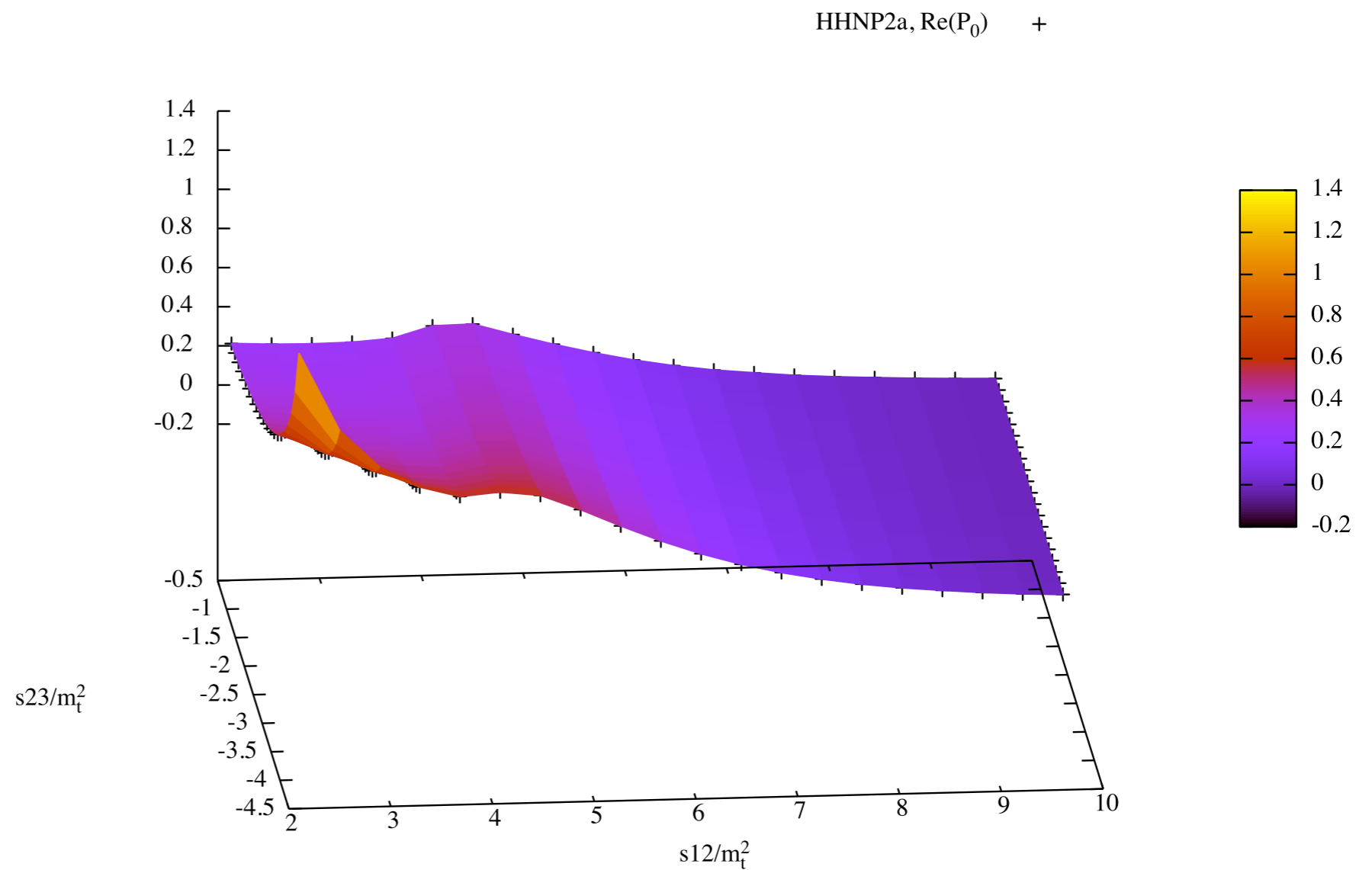
$$I = \frac{P_1}{\epsilon} + P_0$$



# master integrals: examples

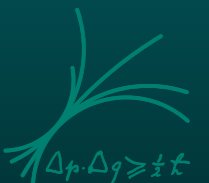


$$I = \frac{P_1}{\epsilon} + P_0$$



# Conclusions/Outlook

- measurement of Higgs pair production vital to understand mechanism of EW symmetry breaking
- heavy top approximation for gg to HH not satisfactory
- NLO with full top mass dependence for HH in gluon fusion well underway
- GoSam-2Loop setup quite generic, can be applied to other processes



# extra slides

