Some questions about *ep* scattering and the proton radius

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Questions

Fact: different analysis (choice of data, range, method of analysis..) lead to different results: 'small' (0.84 fm) or 'large' (0.87 fm) radius

- limit to small Q^2 of a (logarithmic) derivative:
  (the error blows up - Δq^2 at the denominator)

- *the scattering formalism is derived for a two-body process: extrapolation to 'compound nucleus' ?*
  (electroproduction versus photoproduction)

- at $Q^2 << 10^{-3}$ GeV^2, the wavelength of the photon > 15 fm

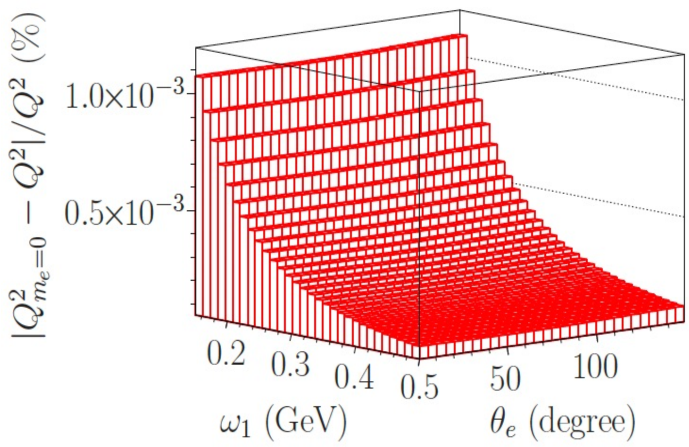
Kinematical variable Q2

The elastic *ep* cross section diverges as $1/(Q^2)^2$ when $Q^2 \to 0$

When $Q^2 \to 0$? $Q^2 = -4EE'\sin^2(\theta/2)$ [neglects lepton mass]

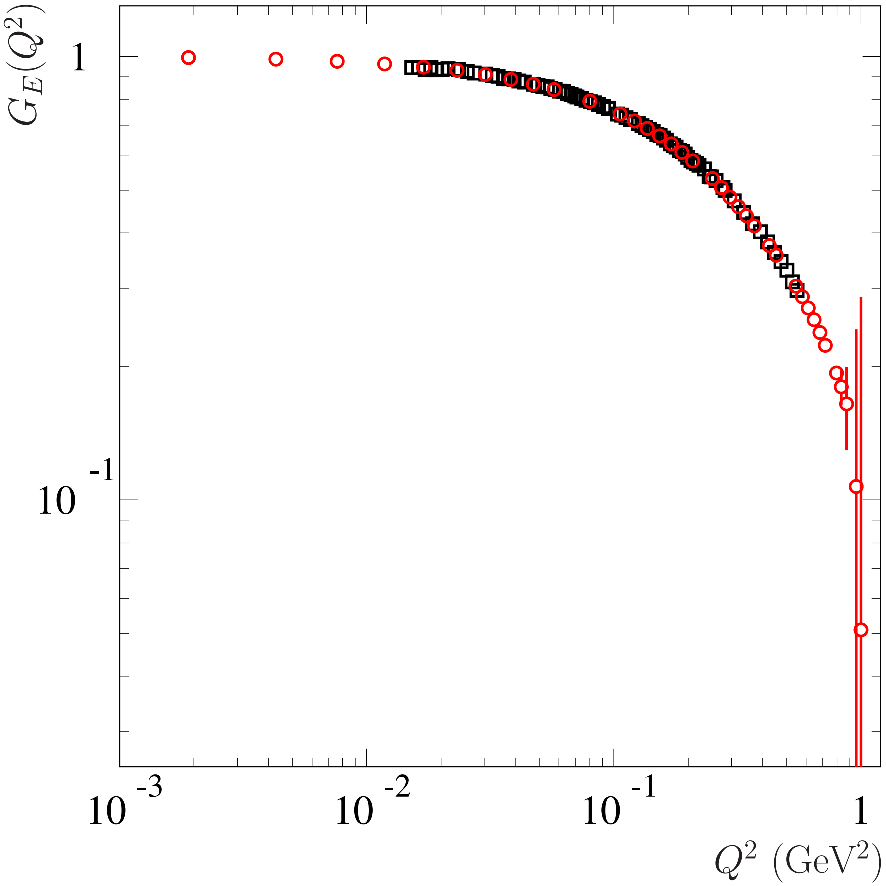
1) $E'=0$: capture process, compound hydrogen atom -> <u>*the scattering formalism does not apply*</u>

2) $\theta=0$: the incident electron does not 'see' the target



In general the extrapolation of electron to photon induced processes is not straightforward

Mainz ep elastic scattering



Spline: Q2>0.0005 GeV^2

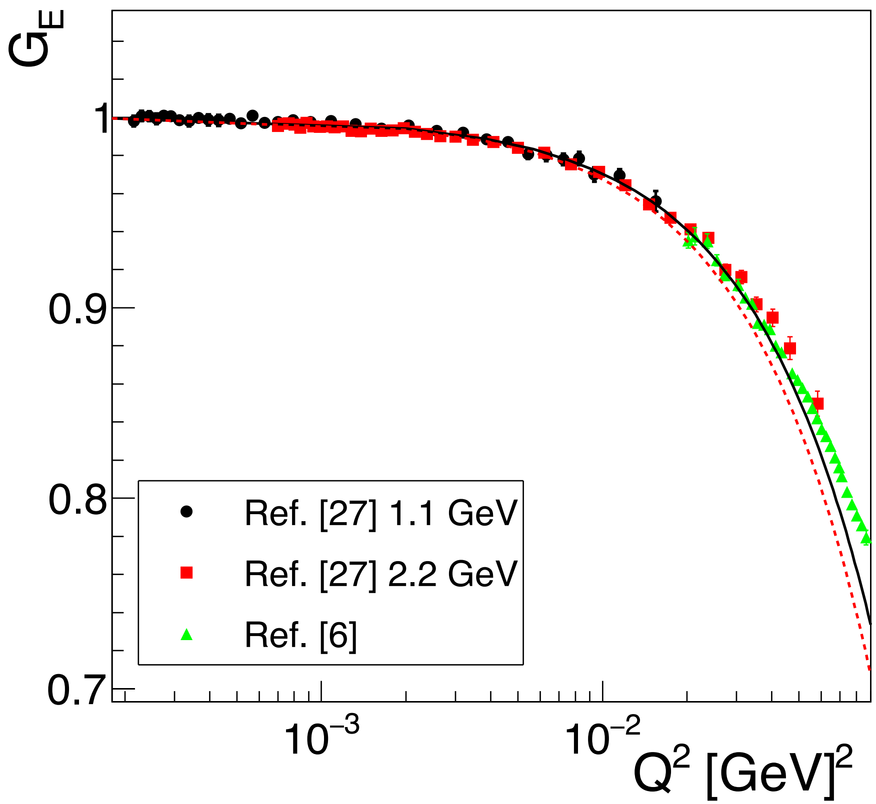
G_E from a global fit of s(Q^2,e), based on a pre-defined function

Rosenbluth: Q2>0.0152 GeV^2

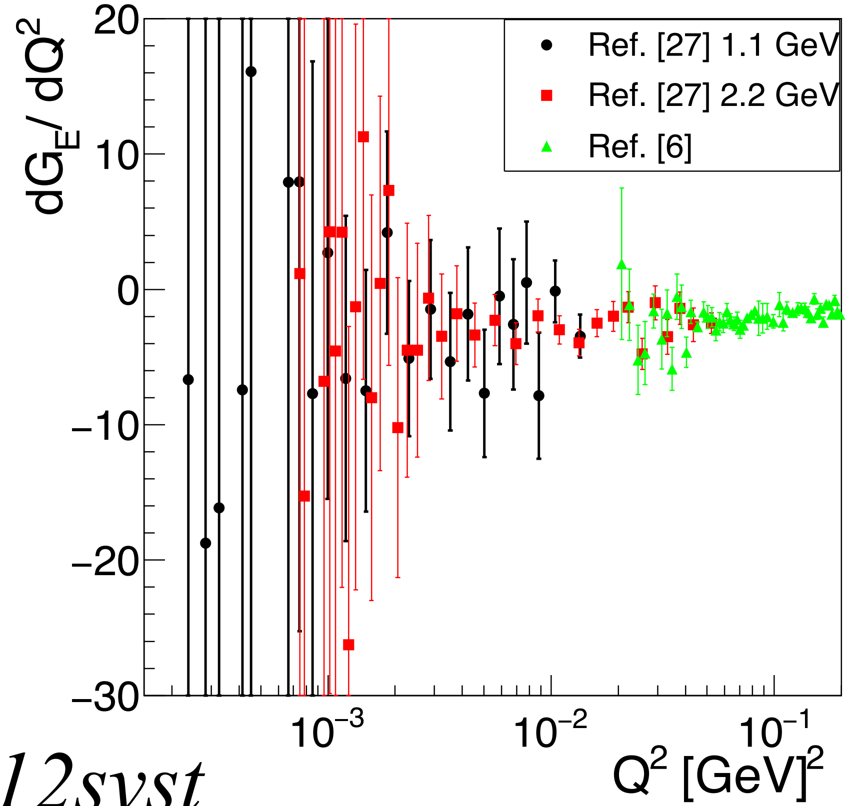
G_E and G_M from the slope and intercept of s_{red} (e), at fixed (Q^2,e). (larger errors, Q^2 interval)

The choice of a pre-defined function imposes serious constraints to the radius through the derivative!

Mainz & CLAS11



Plateau? log scale!

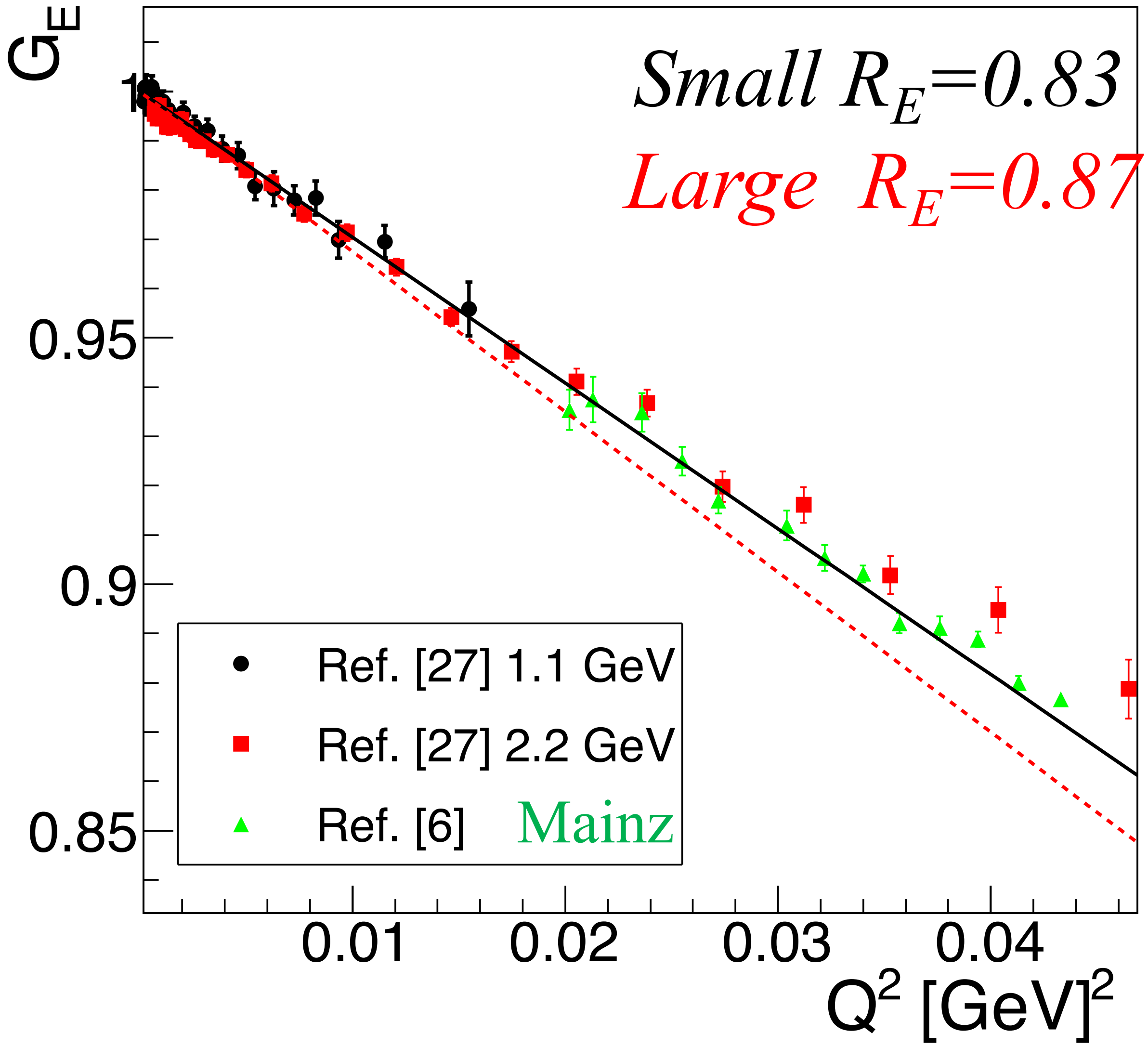


CLAS 11: Small radius!

$$R_E = 0.831 \pm 0.007 stat \pm 0.012 syst$$

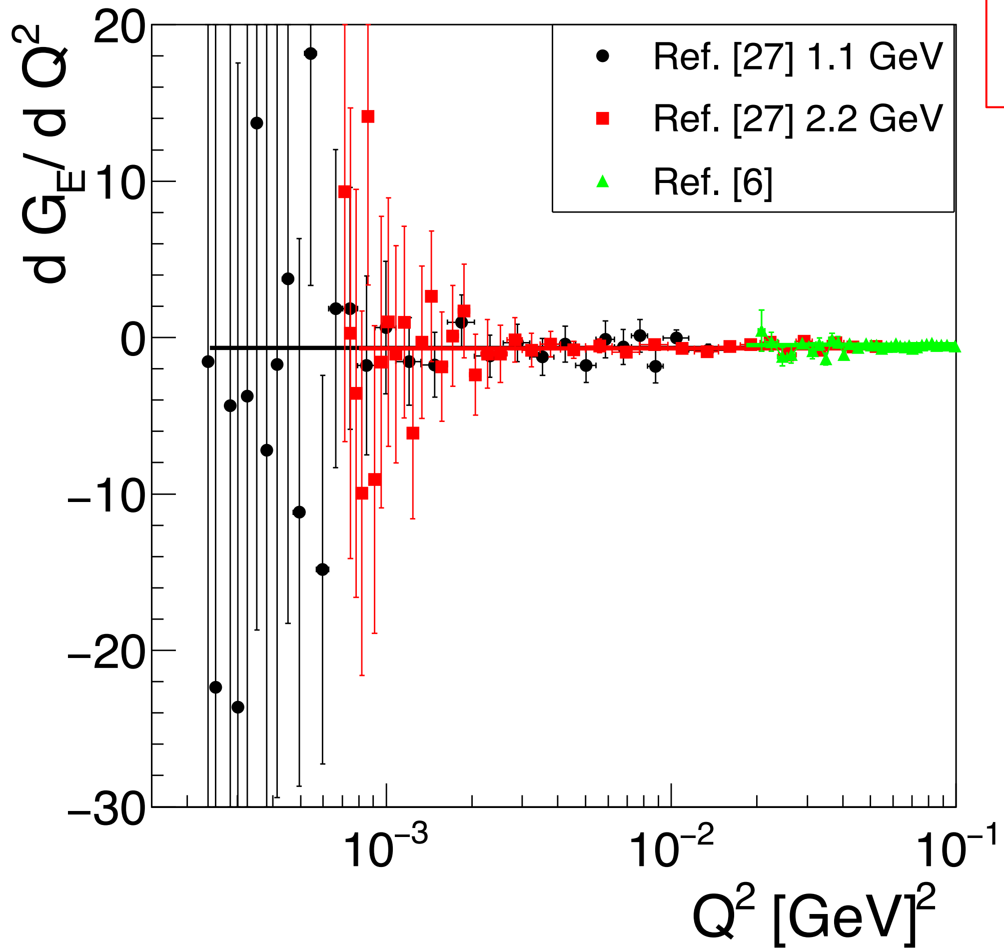
Smaller Q2, larger the error on the derivative

Mainz & CLAS11 Constrained Linear Fit



Mainz & CLAS11- at first sight

Rough estimation from a constrained linear fit



$$\left\langle r^2_{E/M} \right\rangle = -\frac{6\hbar^2}{G_{E/M}(0)} \frac{\mathrm{d}G_{E/M}(Q^2)}{\mathrm{d}Q^2}\bigg|_{Q^2=0}$$

$R_E = 0.81 \pm 0.08$

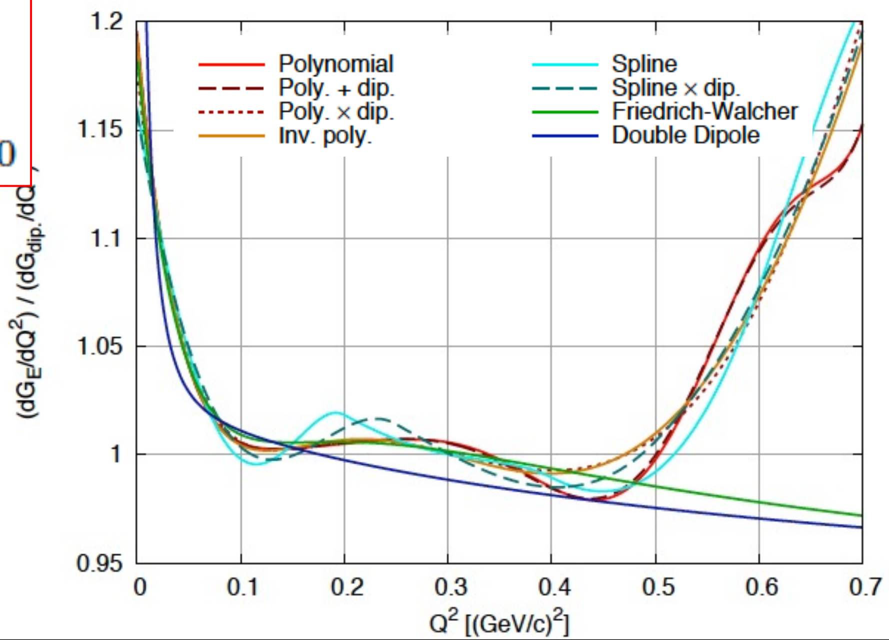
$R_E = 0.82 \pm 0.09$

and from Mainz data:

$R_E = 0.7 \pm 0.02$

Mainz ep elastic scattering

$$\left\langle r^2_{E/M} \right\rangle = -\frac{6\hbar^2}{G_{E/M}(0)} \frac{dG_{E/M}(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$



1) Rosenbluth extraction

2) Direct extraction
(assuming a function for FFs)

Spline

$$\left\langle r^2_E \right\rangle^{\frac{1}{2}} = 0.875(5)_{\text{stat.}}(4)_{\text{syst.}}(2)_{\text{model}}\,\text{fm},$$

$$\left\langle r^2_M \right\rangle^{\frac{1}{2}} = 0.775(12)_{\text{stat.}}(9)_{\text{syst.}}(4)_{\text{model}}\,\text{fm}$$

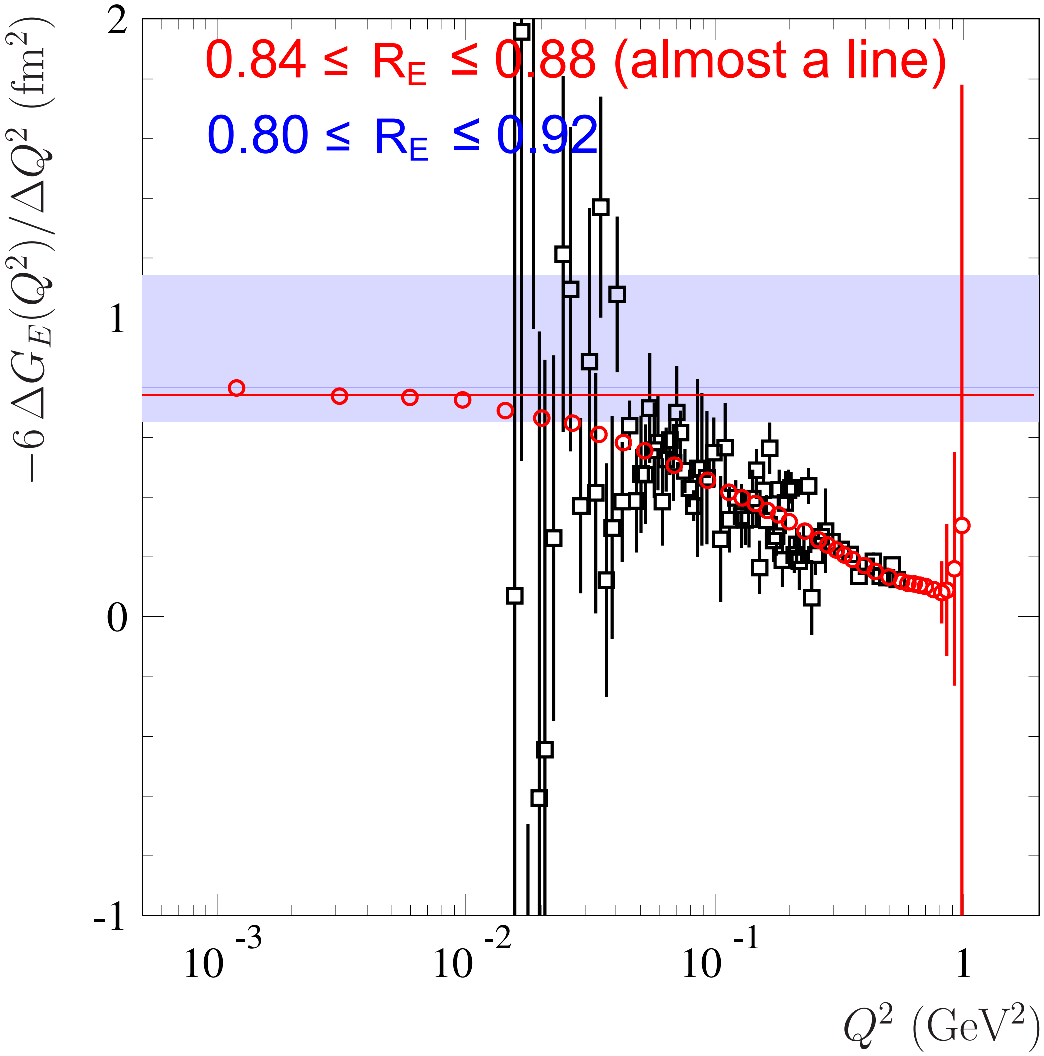
Polynomial

$$\left\langle r^2_E \right\rangle^{\frac{1}{2}} = 0.883(5)_{\text{stat.}}(5)_{\text{syst.}}(3)_{\text{model}}\,\text{fm},$$

$$\left\langle r^2_M \right\rangle^{\frac{1}{2}} = 0.778(^{+14}_{-15})_{\text{stat.}}(10)_{\text{syst.}}(6)_{\text{model}}\,\text{fm}.$$

J.C. Bernauer, PhD,Mainz

Mainz ep elastic scattering-derivative



Rosenbluth

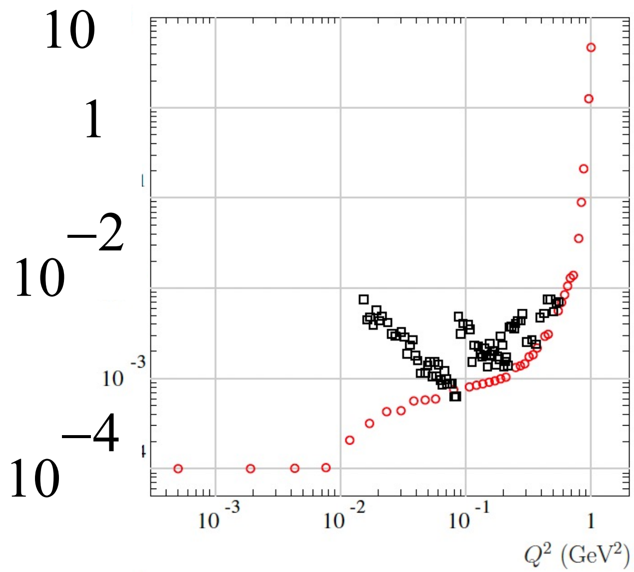
Spline

$$\Delta G^{S,R}_{E,j} = \frac{G^{S,R}_{E,j+1} - G^{S,R}_{E,j}}{Q^{2S,R}_{j+1} - Q^{2S,R}_{j}},$$

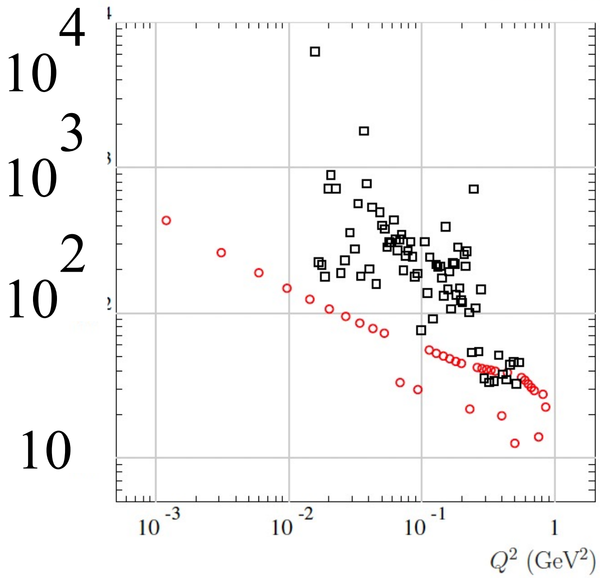
$$\delta\Delta G^{S,R}_{E,j} = \frac{\sqrt{\left(\delta G^{S,R}_{E,j+1}\right)^2 + \left(\delta G^{S,R}_{E,j}\right)^2}}{Q^{2S,R}_{j+1} - Q^{2S,R}_{j}}$$

$$\overline{Q}^{2S,R}_{j} = \frac{Q^{2S,R}_{j+1} + Q^{2S,R}_{j}}{2}$$

Ratio of relative errors



$\delta G_E/G_E$



$$\frac{\delta \Delta GE/GE}{\delta GE/GE}$$

$$\left\langle r^2_{E/M} \right\rangle = -\frac{6\hbar^2}{G_{E/M}(0)} \frac{dG_{E/M}(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

Rosenbluth

Spline

The relative error on the derivative blows up:

1% on σ ->

few % on G_E->

100% ΔG_E -> radius!

Mainz Data – Fitting Procedure

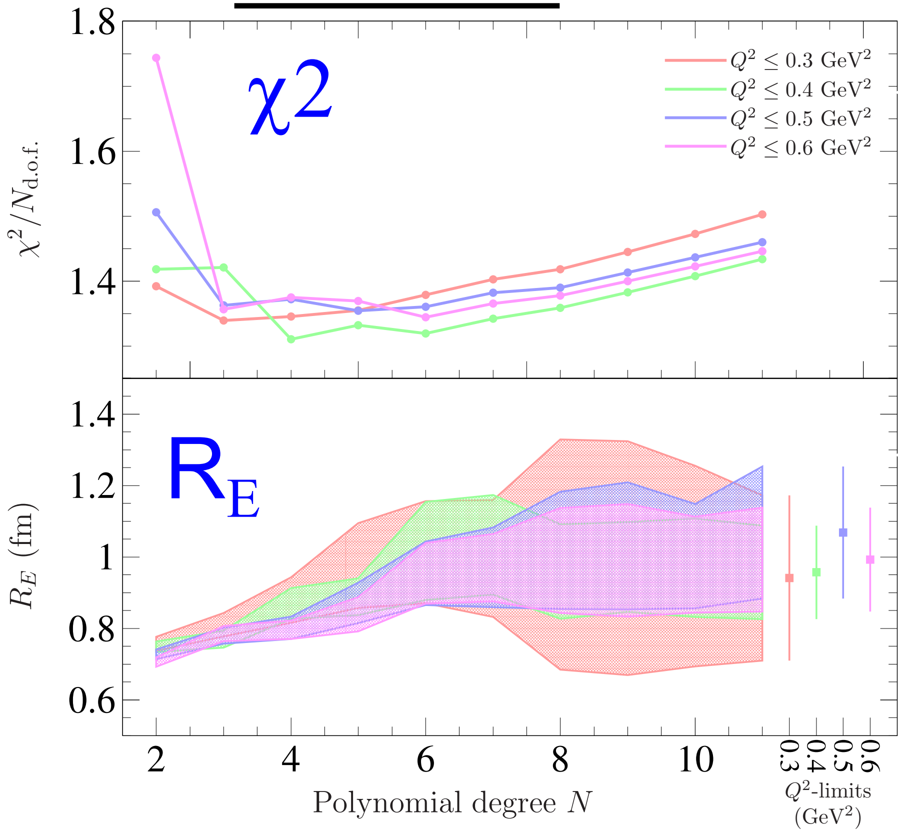
- 4 sets of data:
  - 2 G_E data: Rosenbluth and Spline
  - 2 discrete derivatives

$$\left\{\overline{Q}_j^{2S}, \Delta G_{E,j}^S, \delta\Delta G_{E,j}^S\right\}_{j=1}^{N_S-1} \qquad \left\{\overline{Q}_j^{2R}, \Delta G_{E,j}^R, \delta\Delta G_{E,j}^R\right\}_{j=1}^{N_R-1}$$

- 4 Q^2 ranges,
- polynomes up to 12 degree

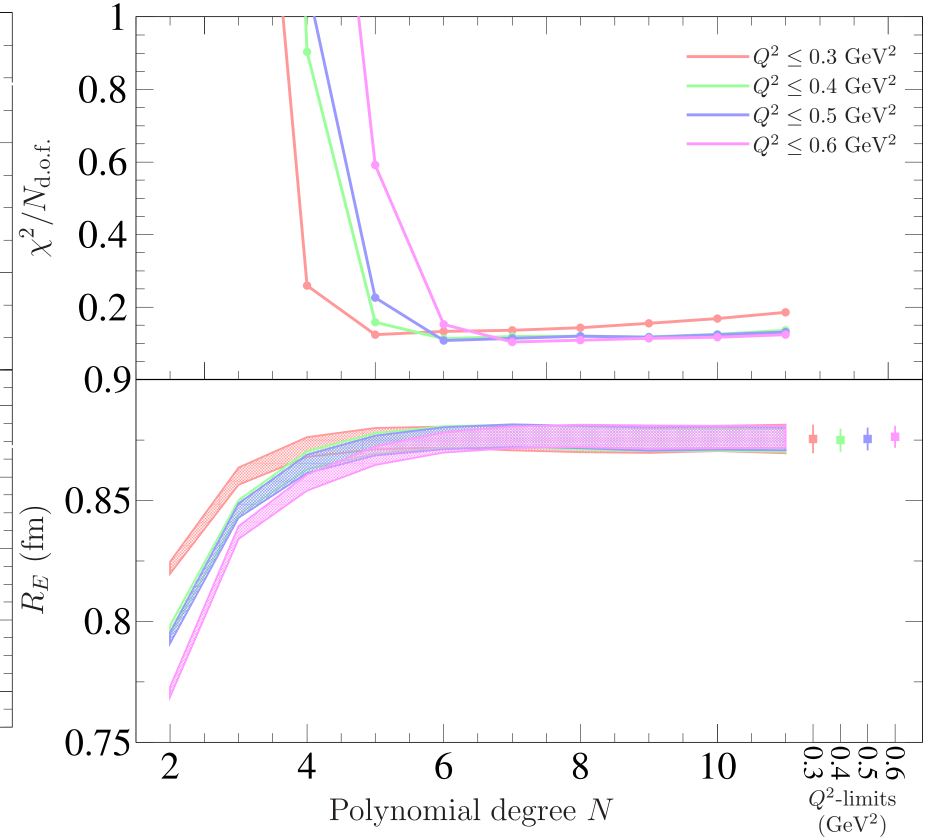
Radius – Fitting discrete derivative dG_E (R &S)

Rosenbluth



Large errors

Spline

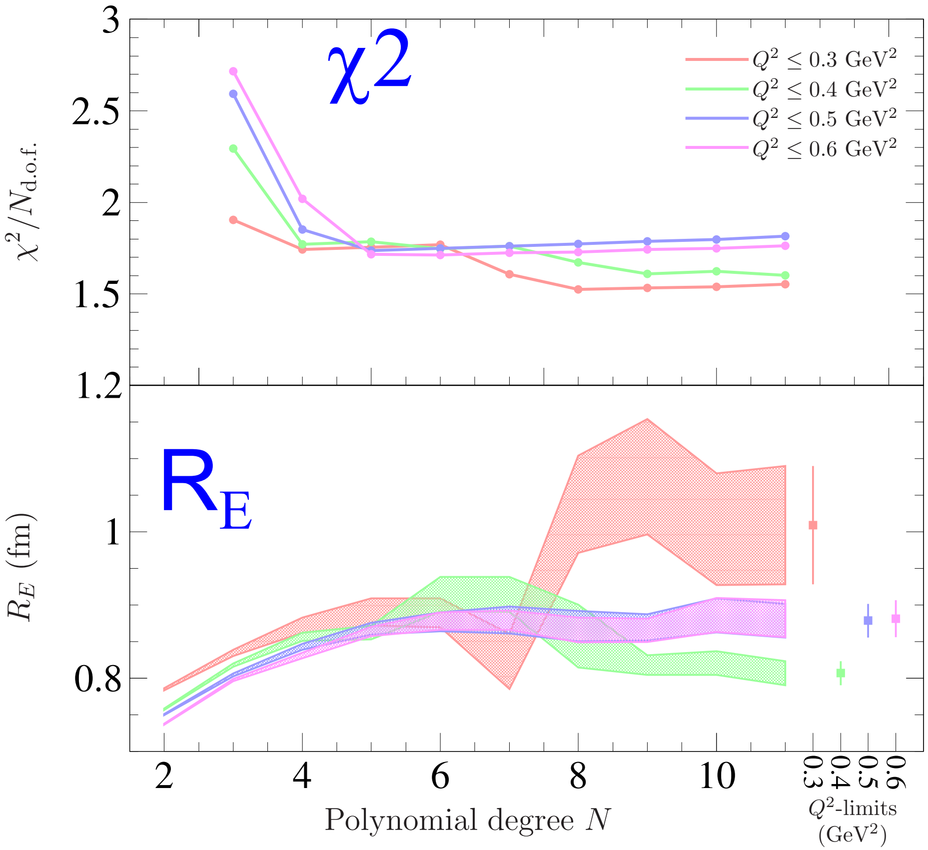


Stability of the results

Very small χ2

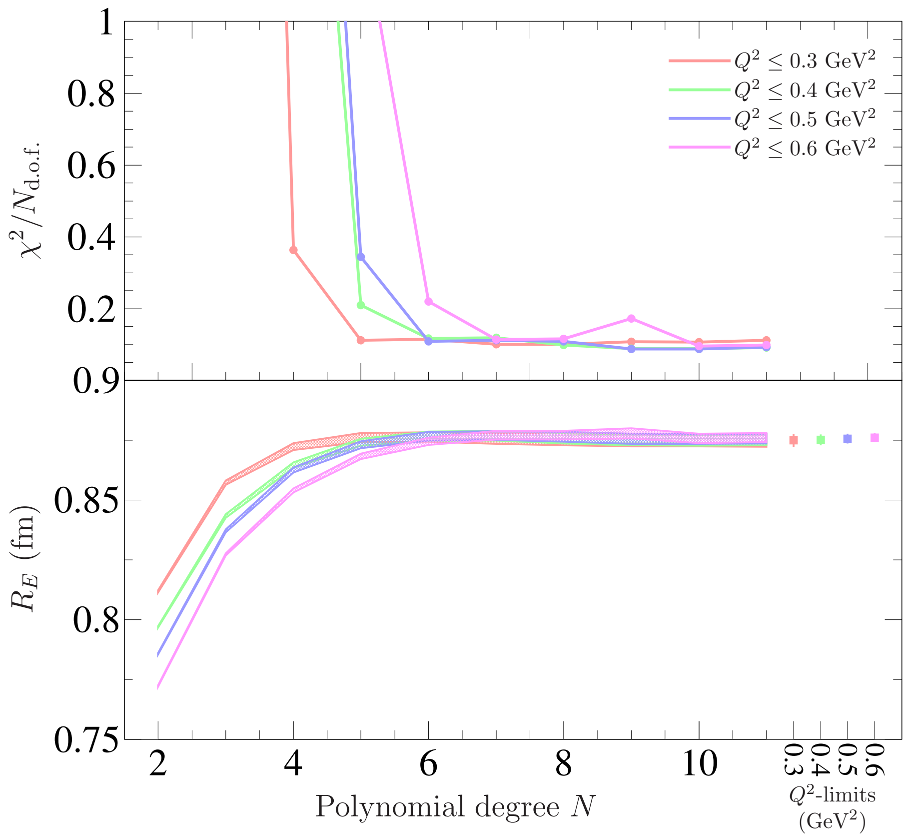
Radius – Fitting G_E & dG_E (R &S)

Rosenbluth



Large errors

Spline



Stability of the results

Very small errors

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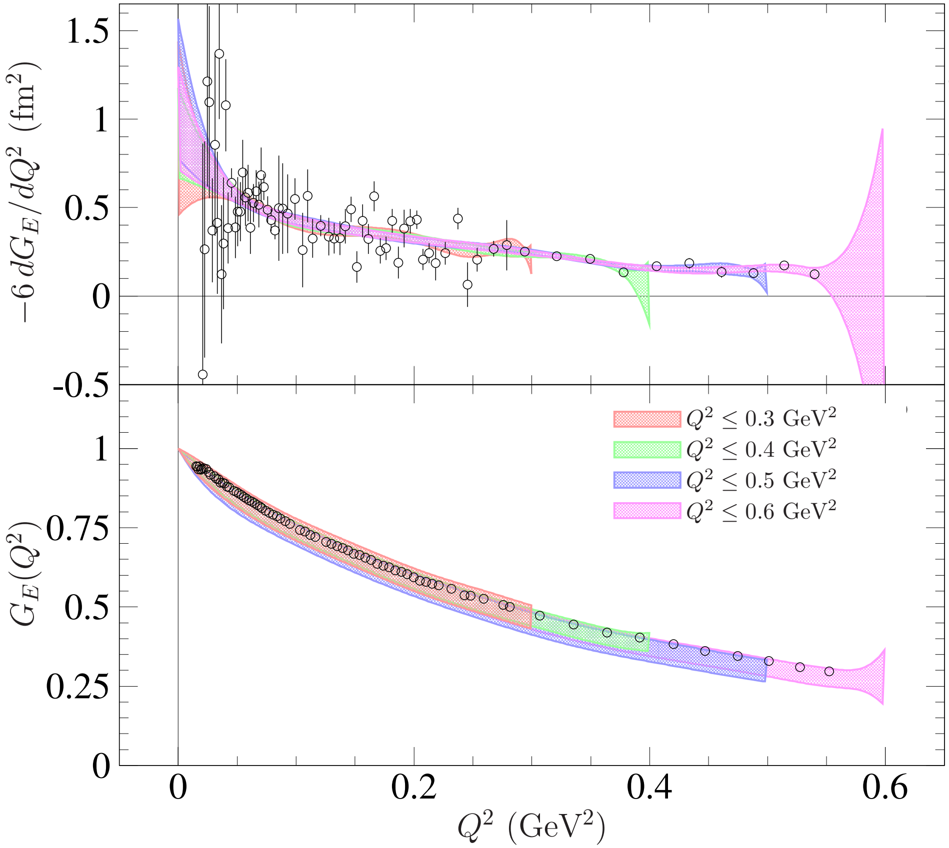
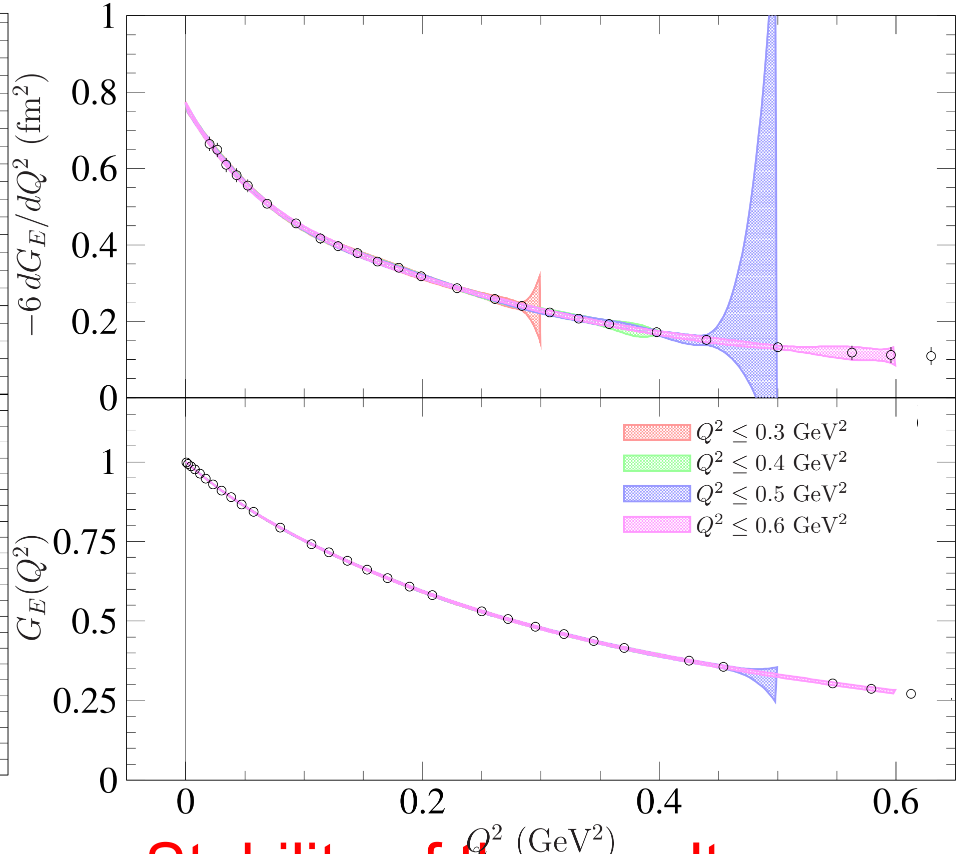
Functions – dG_E (R &S)

Rosenbluth



Spline

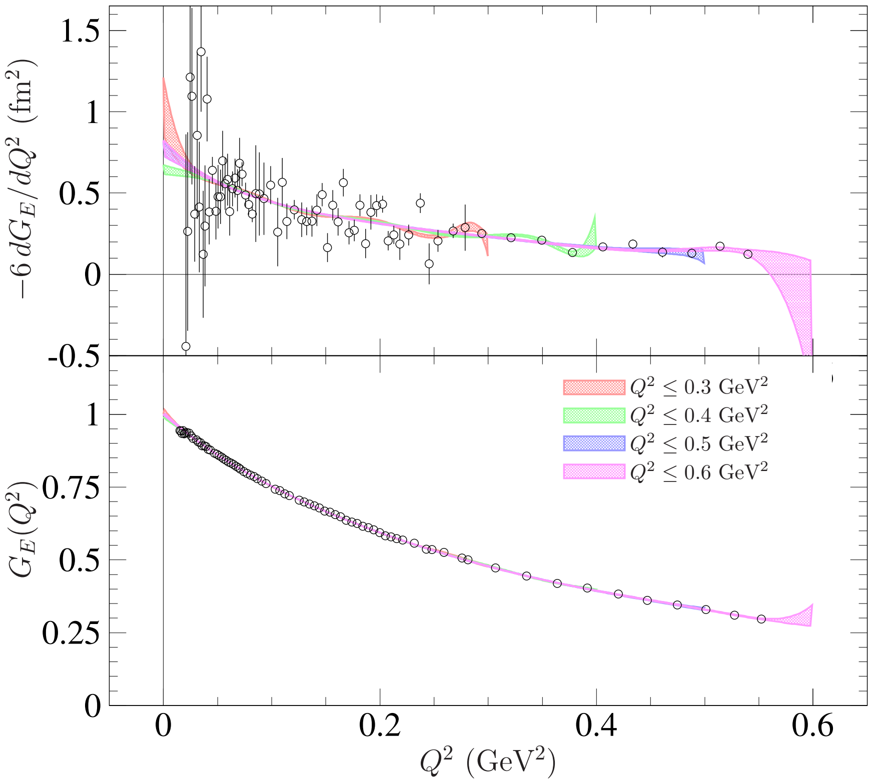


Large errors
Error bands on the data

Stability of the results
Very small errors
Very small $\chi 2$

Functions – Fitting G_E & dG_E(R &S)

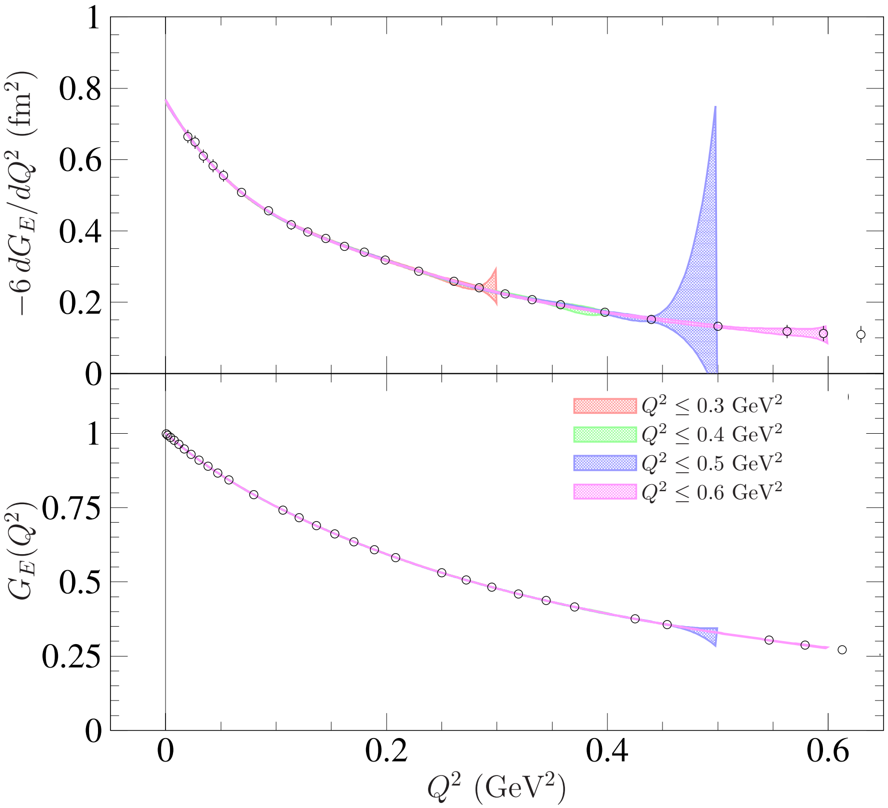
Rosenbluth



Large errors

Error bands on the data

Spline



Stability of the results

Very small $\chi 2$

Mainz – Fitting Procedure

		Rosenbluth		Spline	
		$\chi^2/N_{\mathrm{d.o.f.}}$	R_E (fm)	$\chi^2/N_{\mathrm{d.o.f.}}$	R_E (fm)
$Q^2 \leq 0.3$ GeV^2	dG_E/dQ^2	1.50	0.9411 ± 0.2310	0.19	0.8754 ± 0.0059
	$G_E \cup dG_E/dQ^2$	1.55	1.0088 ± 0.0809	0.11	0.8749 ± 0.0026
$Q^2 \leq 0.4$ GeV^2	dG_E/dQ^2	1.43	0.9568 ± 0.1309	0.14	0.8749 ± 0.0048
	$G_E \cup dG_E/dQ^2$	1.60	0.8070 ± 0.0164	0.09	0.8751 ± 0.0023
$Q^2 \leq 0.5$ GeV^2	dG_E/dQ^2	1.46	1.0681 ± 0.1848	0.13	0.8754 ± 0.0047
	$G_E \cup dG_E/dQ^2$	1.82	0.8786 ± 0.0229	0.09	0.8756 ± 0.0020
$Q^2 \leq 0.6$ GeV^2	dG_E/dQ^2	1.45	0.9927 ± 0.1453	0.12	0.8763 ± 0.0046
	$G_E \cup dG_E/dQ^2$	1.76	0.8811 ± 0.0253	0.10	0.8761 ± 0.0019

- S- Errors << R-data (x 5-10)
- S- Values very stable, R-values depend on fitting scheme
- Discrepancy on the central R- and S- values
- Very small $\chi 2$ and stability of S-results derive from the large constraint due to the pre-imposed function

Our final values from Mainz data $Q^2<0.6$ GeV^2

Rosenbluth

Spline

$$R_E^{R,1C} = 0.99 \pm 0.15\,\mathrm{fm}\,,$$

$$R_E^{S,1C} = 0.876 \pm 0.005\,\mathrm{fm}\,,$$

$$R_E^{R,2C} = 0.88 \pm 0.03\,\mathrm{fm}\,,$$

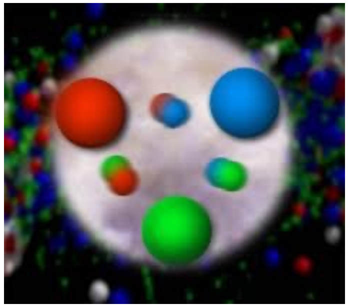
$$R_E^{S,2C} = 0.876 \pm 0.002\,\mathrm{fm}\,,$$

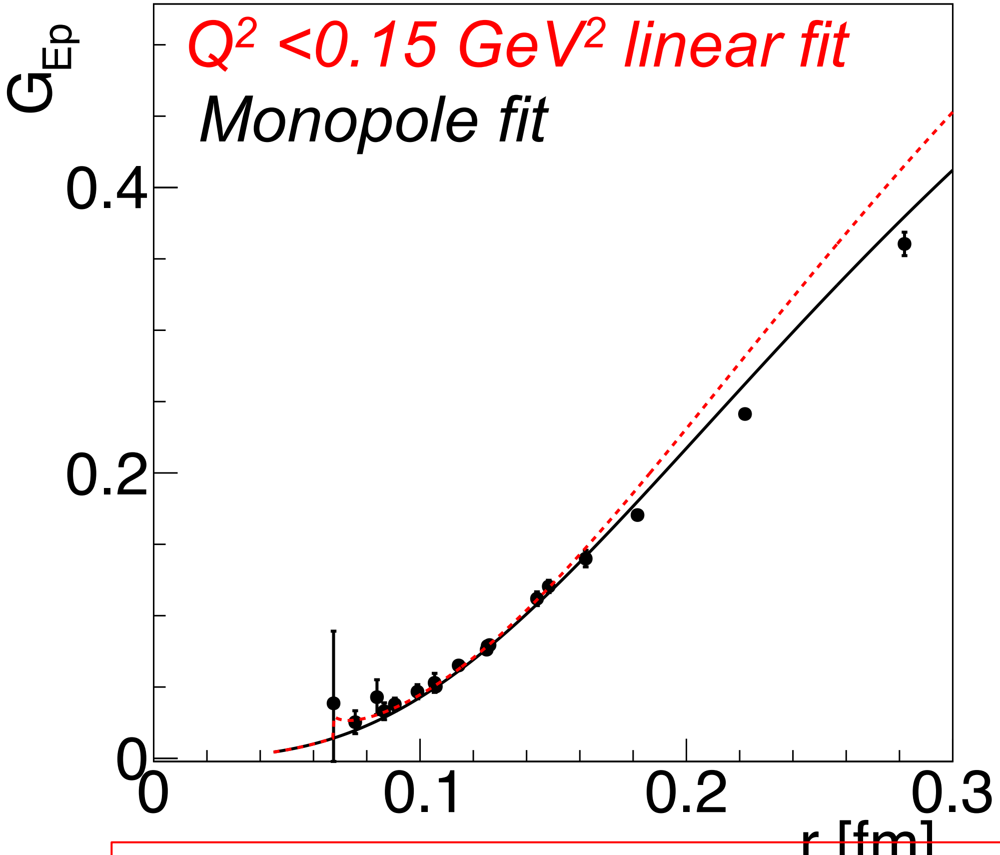
1C: derivative only

2C: 1 constraint -> derivative and radius

SL- the most precise ruler

$$r\ [\mathrm{fm}] = \lambda = \hbar c/\sqrt{Q^2} = 0.197\ [\mathrm{GeV\ fm}]/\sqrt{Q^2}[\mathrm{GeV}],$$





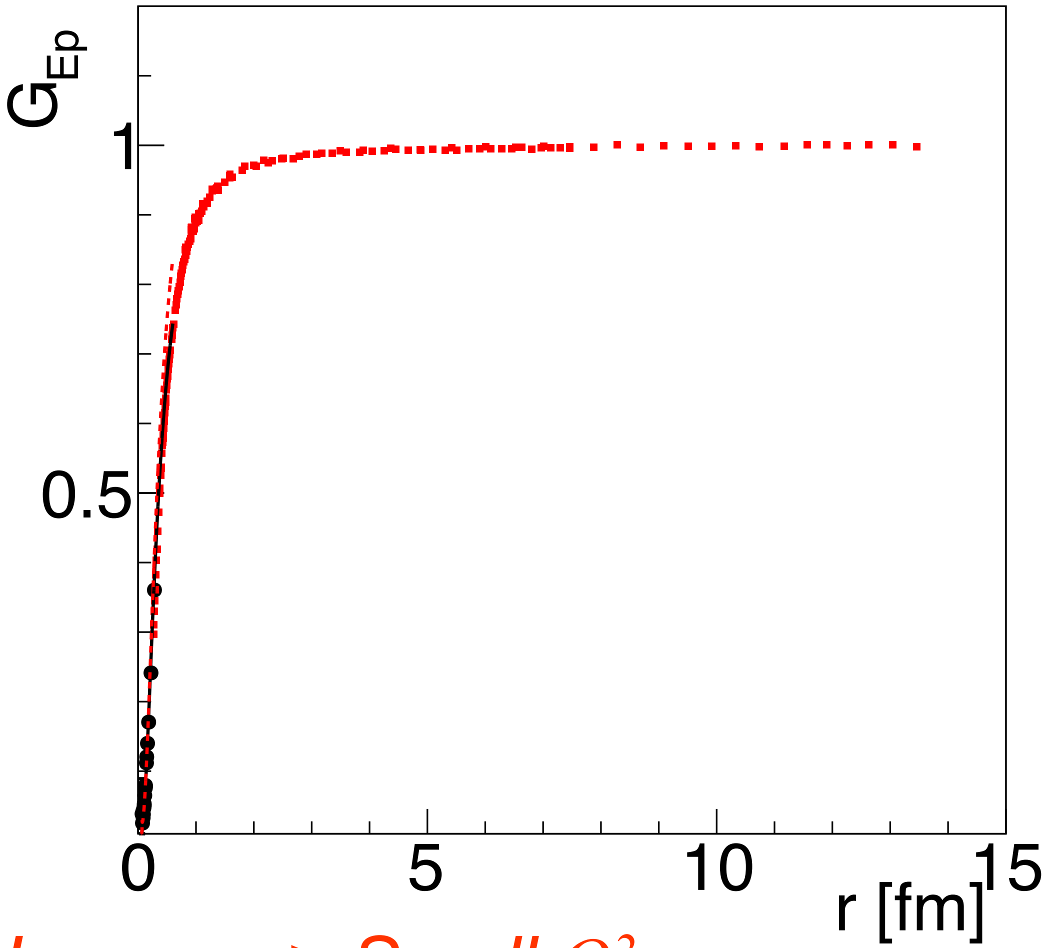
$$\mathcal{R} = \mu_p \frac{G_{Ep}}{G_{Mp}} = \left(1 + \frac{Q^2}{m_r^2}\right)^{-1}.$$

Zero crossing? *E.A. Kuraev, E.T-G., A. Dbeyssi, PLB 712(2012)240*

The photon 'sees' the neutral, screened, small region

Proton radius

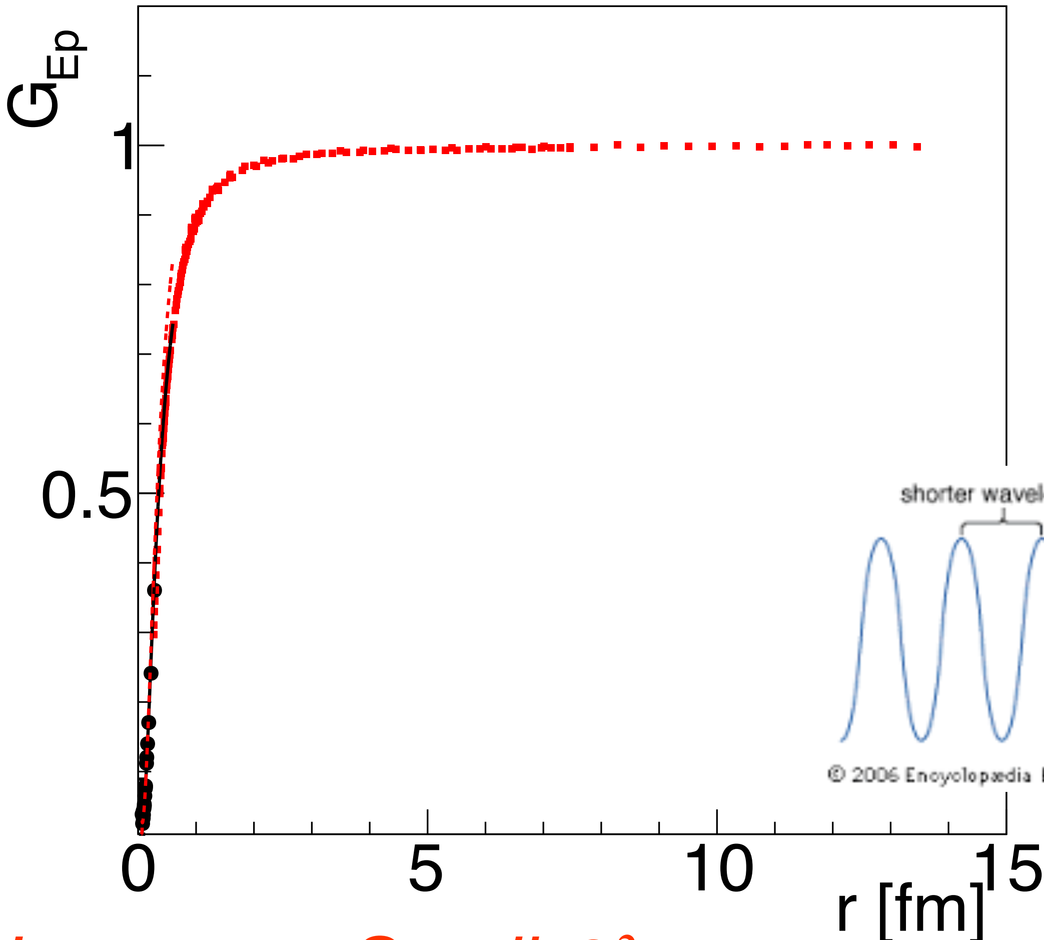
Data from Mainz,
PRC 90, 015206 (2014)



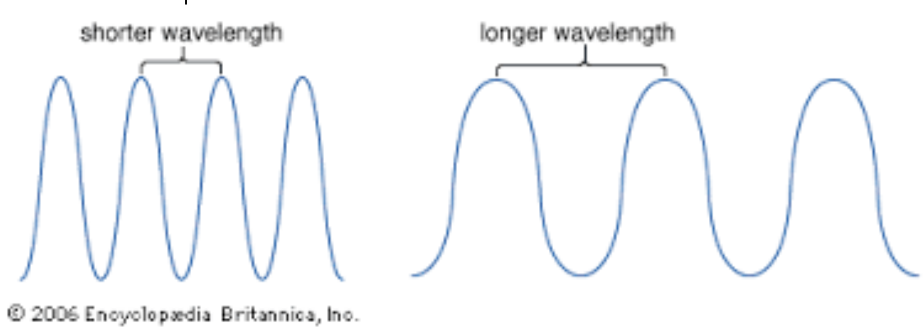
Large r -> Small Q^2

Proton radius

Data from Mainz, CLAS…



How can a photon with wavelength ~ 15 fm distinguish between a proton size of 0.84 or 0.87 fm?



Large r -> Small Q^2

Conclusions

- at $Q^2 << 10^{-3}$ GeV^2, the wavelength of the photon is much too large to 'see' dimensions of fractions of fm

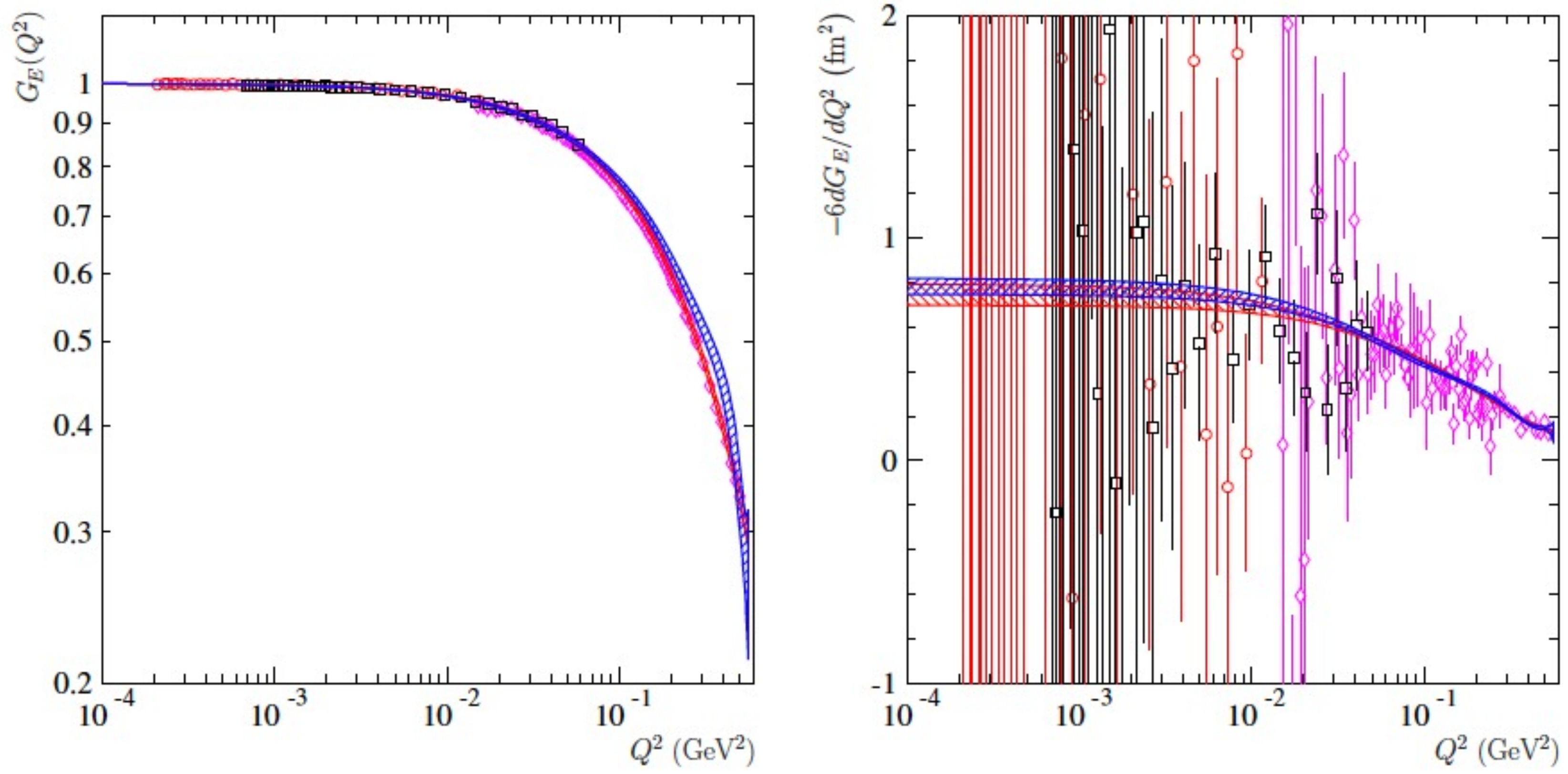
- *the scattering formalism is derived for a two-body process: the extrapolation to 'compound nucleus' is dangerous*

- limit of a (logarithmic) derivative: the error blows up (Δq^2 at the denominator)

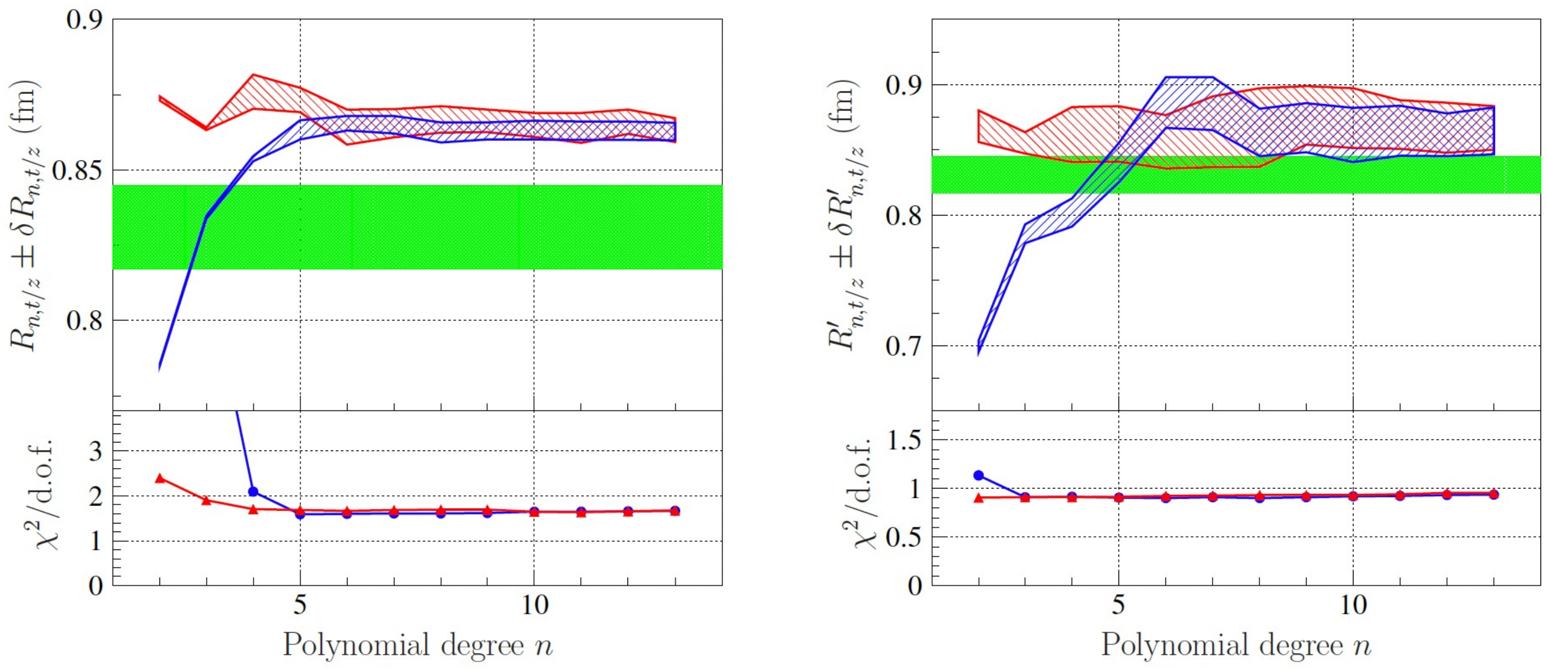
Fact: *the extrapolation* for different series of data, for different functions, for different Q^2 ranges *gives different results*

The error from ep elastic scattering may be much larger !

Mainz & CLAS



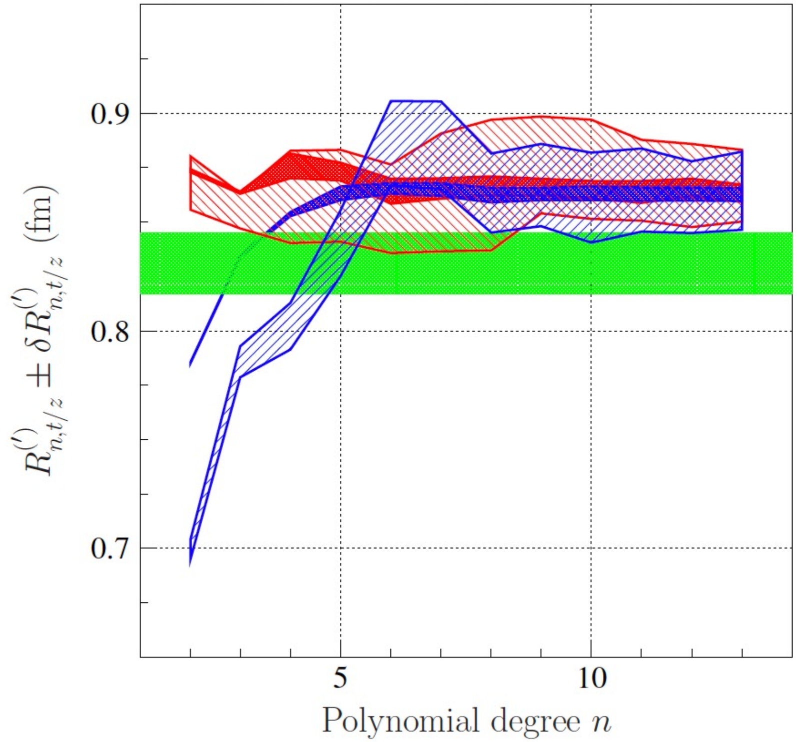
Mainz & CLAS



R(MAINZ) = (0.879+-0.005stat +-0.004syst+-0.002model+-0.004group) fm.

R(CLAS) = (0.831+-0.007stat +- 0.012syst)

Mainz & CLAS (summary)



R(MAINZ) = (0.879+-0.005stat +-0.004syst+-0.002model+-0.004group) fm.

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